20 August 2010

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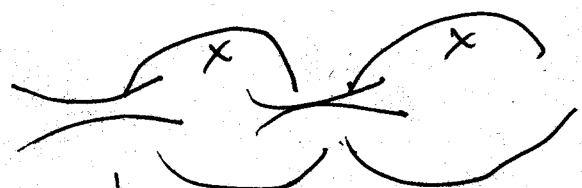
Liouville correlation functions from four-dimensional gauge theories, II

2 M5 branes



$U(2)$

$$\longrightarrow \bigsqcup_{k=0}^{\infty} \mathcal{M}_{2,k}^{\epsilon_1 \epsilon_2 \quad SO(4)\times U(2) \ \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}} \Rightarrow \mathcal{H}_a : |Y^{(1)} Y^{(2)}\rangle$$

Young diagrams

$\frac{Q}{2}, -\frac{Q}{2}$

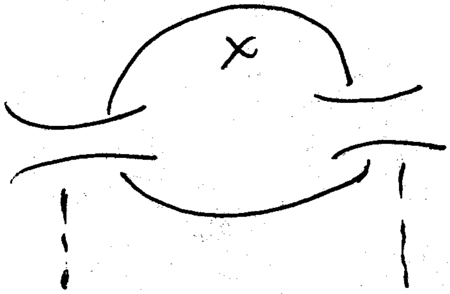
$$\mathcal{H}_a \simeq \text{Fock} \otimes V_{\Delta(a)}$$

$$\mathcal{H}_a = \bigoplus_k H^{*}_{U(1)^2 \times U(1)}(\mathcal{M}_{2,k})$$

$V_{\Delta(a)}$: Verma module of Virasoro

$$c = 1 + 6 \frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1 \epsilon_2}$$

$$\Delta(a) = \frac{1}{4} \frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1 \epsilon_2} - a^2$$



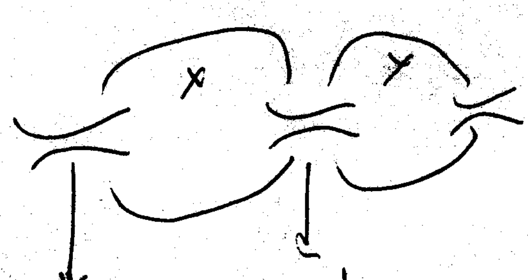
$$\tilde{\rho} = \mathbf{2}_1 \otimes \bar{\mathbf{2}}_2 \otimes \mathbb{1} \oplus \bar{\mathbf{2}}_1 \otimes \mathbf{2}_2 \otimes \bar{\mathbb{1}}$$

$U(2)_1 \times U(2)_2 \times U(1)$
↑
extra puncture

$$\begin{array}{ccc} U(2)_1 & & U(2)_2 \\ \begin{pmatrix} a_1 & \\ & -a_1 \end{pmatrix} & & \begin{pmatrix} a_2 & \\ & -a_2 \end{pmatrix} \\ \Downarrow & & \Downarrow \\ \mathcal{H}_{a_1} & \xleftarrow{\Phi_{a_1, m a_2}} & \mathcal{H}_{a_2} \end{array}$$

$$\Phi_{a_1 m a_2} \simeq e^{m\phi(z)} \otimes V_{\Delta(m)}$$

N M5 branes



$U(N)$ $\quad$ $U(N)$

$SO(4) \times U(N)$
$\epsilon_1, \epsilon_2, \vec{a} = (a_1 \ldots a_N)$. $\quad \Sigma a_i = 0$

$$\bigtimes_{k=0}^{\infty} \tilde{\mathcal{M}}_{N,k}$$
↑
desingularization of instanton moduli space

$$\mathcal{H}_{\vec{a}} \ni |Y^{(1)}, Y^{(2)}, \ldots, Y^{(N)}\rangle = |\vec{Y}\rangle$$

$\vec{Q} \propto \frac{N}{2}, \frac{N}{2}-1, \ldots, -\frac{N}{2}$

$\mathcal{H}_{\vec{a}} \simeq \text{Fock} \otimes V_{\vec{a}}$

Verma module of W-algebra of type A_{N-1}
(also called W_N-algebra)

$W_2(z) = T(z) \rightsquigarrow L_n$

$W_3(z) \rightsquigarrow W_{3,n}$

$W_4(z) \rightsquigarrow$

$\vdots \qquad \vdots$

$W_N(z) \rightsquigarrow W_{N,n}.$

$V_{\vec{a}} \quad W_{k,n}|\vec{a}\rangle = 0 \quad , n > 0$

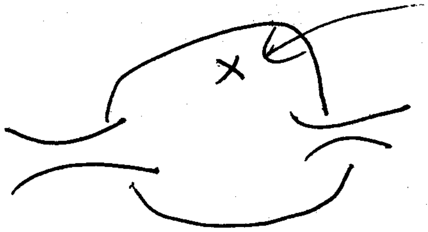
$W_{k,0}|\vec{a}\rangle = S_k(\vec{a})|\vec{a}\rangle$

basically elementary deg-k sym. poly. constructed from $\vec{a}$.

$\vec{a} \quad w\vec{a}$

Weyl reflection of A_{N-1}.

$V_{\vec{a}} \simeq V_{w\vec{a}}$



Simple puncture

$U(1)$ flavor symmetry

$$\tilde{\rho} = N_1 \otimes \overline{N_2} \otimes \mathbb{1}$$
$$\oplus\ \overline{N_1} \otimes N_2 \otimes \overline{\mathbb{1}}$$

$$\underbrace{\overline{N_1}}_{\vec{a}_1}\quad \underbrace{N_2}_{\vec{a}_2}\quad \underbrace{\overline{\mathbb{1}}}_{m}$$

$$\mathcal{H}_{\vec{a}_1} \xleftarrow[\Phi_{\vec{a}_1 m \vec{a}_2}]{} \mathcal{H}_{\vec{a}_2}$$

$$\Phi_{\vec{a}_1 m \vec{a}_2} \simeq e^{m\phi(z)} \otimes V_{m\vec{\chi}}$$

$\vec{\chi}$: weight vector of ~~AF~~ vect ~~fnd. rep.~~

① $\langle \vec{a}_1 | w w w \cdots V_{m\vec{\chi}}\ w \cdots w | \vec{a}_2 \rangle$

$\downarrow$ can reduce

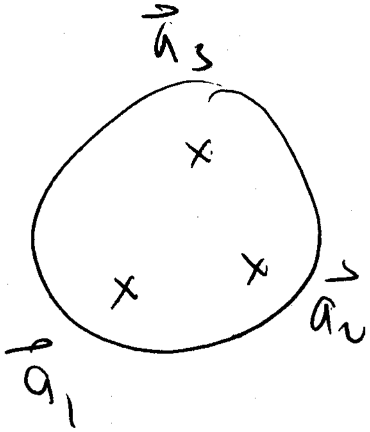
$$\langle \vec{a}_1 | V_{m\vec{\chi}} | \vec{a}_2 \rangle$$

$$W_{k,1} | V_{m\vec{\chi}} \rangle \propto L_1 | V_{m\chi} \rangle$$

② Liouville $\rightsquigarrow$ $\mathrm{Toda}_{A_{N-1}}$

$\quad$ " Toda(A_1)

$C(\vec{a}_1, m\vec{\chi}, \vec{a}_2)$: known products of Γ_2



$N=3$

1-parameter family of conformal blocks

M-N theory E_6 has one parameter u.

$C = (N-1) + (N-1)N(N+1)$ central charge of W_N

$$C = (\mathrm{rank}\, G) + (\dim G)\, h^\vee(G) \left(\frac{(\varepsilon_1+\varepsilon_2)^2}{\varepsilon_1\varepsilon_2}\right)$$

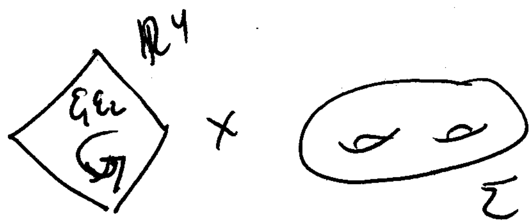
<u>anomaly polynomial</u> of $\mathcal{N}=(0,2)$ 6d theory :

$\uparrow$ A, D, E

~ 1998 Minasian Harvey Moore.

$$I_8 = \mathrm{rank}(G) \times \sim + (\dim G) \times h^\vee(G) \times \frac{p_2(NW)}{24}$$

W = worldvolume
N = normal bundle



: Kaluza-Klein equivariant integral $\int_{\mathbb{R}^4}$

$$\frac{p_2(NW)}{24} \to \frac{(\varepsilon_1+\varepsilon_2)^2}{\varepsilon_1\varepsilon_2}$$

$$\sim \to 1$$

Pure gauge theory

$G = U(N)$, $\tilde{\rho}$ = empty

$$Z_{Nek} = \sum_k q^k \int_{\mathcal{M}_{N,k}} 1 = \sum_k q^k \sum_{\vec{Y}} \frac{1}{Z_V(\vec{Y},\vec{a})}$$

$$= \langle \text{pure} | q^k | \text{pure} \rangle$$

$$|\text{pure}\rangle = \sum_{\vec{Y}} |\vec{Y}\rangle \in \mathcal{H}_{\vec{a}}$$

$$\mathcal{H}_{\vec{a}} \simeq \text{Fock} \otimes V_{\vec{a}}$$

$$|\text{pure}\rangle \simeq |\text{vac}\rangle \otimes |W\rangle$$

Whittaker vector

$$W_{k,n} |W\rangle = 0 \text{ unless } (k,n) = (N,1) \quad n>0,$$

$$W_{N,1} |W\rangle = \frac{1}{(\varepsilon_1 \varepsilon_2)^{N/2}} |W\rangle$$

$$\langle W | q^{L_0} | W \rangle = \sum_k q^k \int_{M_{N,k}} 1$$

$SO(4) \times U(N)$
$H_1, H_2 \quad G_1, \dots, G_N$

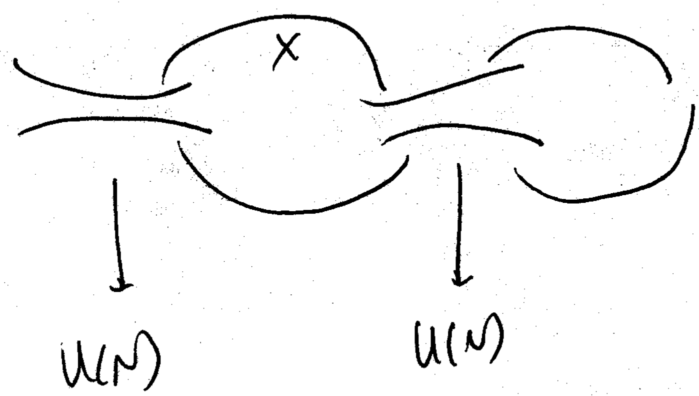
$$= \sum_k q^k \int_{M_{N,k}} d\text{vol}\; e^{-(\varepsilon_1 H_1 + \varepsilon_2 H_2 + \vec{a} \cdot \vec{G})}$$

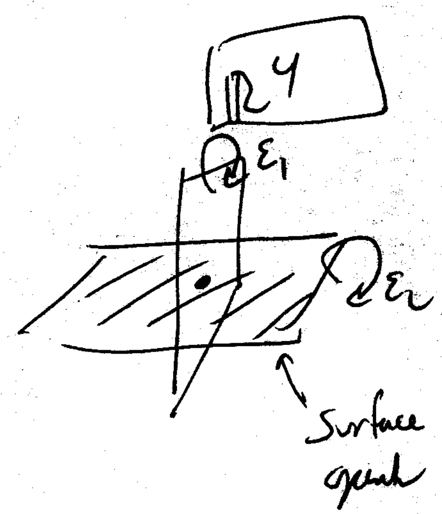
A, D, E

For general G, use Whittaker vector $w \in W({}^L(\hat{\mathfrak{g}}))$.

$$\dim M_{G,k} = 4k\, h^\vee(G)$$

$$W(G) \quad \ldots \quad W_{h(G)}$$





$$M^S_{N,(k_1,k_2\ldots k_N)}$$

$$\downarrow$$

$$\mathcal{H}^S_{\vec a}$$

$$A_\mu dx^\mu = \underbrace{\text{diag}(\alpha_1,\alpha_2 \ldots \alpha_N)}_{\text{generic}}\, d\theta$$

$$\mathcal{H}^S_{\vec a} \simeq \text{Fock} \otimes V_{\vec a/\varepsilon_1} \leftarrow \text{spin}$$

Verma module of $\widehat{sl(N)}$

$$k = -N - \frac{\varepsilon_2}{\varepsilon_1}$$

quantum Drinfeld-Sokolov

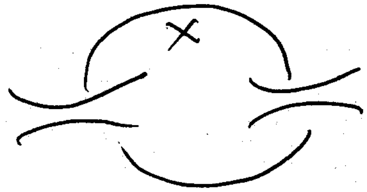
$$\downarrow$$

W-algebra rep. (int of $\widehat{sl(N)}$)

The level k above gives $c = (N-1) + (N-1)N(N+1)\frac{(\epsilon_1+\epsilon_2)^2}{\epsilon_1\epsilon_2}$

$\mathcal{H}^S_{\vec a}$ level come by considering moduli space of instantons with surface operator

W-algebra - - - - - without surface operators



$$\mathcal{H}^S_{\vec a} \xleftarrow{\Phi^S_{\vec a m \vec a}} \mathcal{H}^S_{\vec a}$$

$$\Phi^S_{\vec a_1 m \vec a_2} = e^{m\phi(z)} \otimes \tilde V_{m\vec x}$$

$$Z_{Nek} \simeq \mathrm{tr}\left[V_{m\vec x}(z_1) V_{m\vec x}(z_2) \cdots V_{m\vec x}(z_N) \right] \leftarrow \text{KZB.}$$

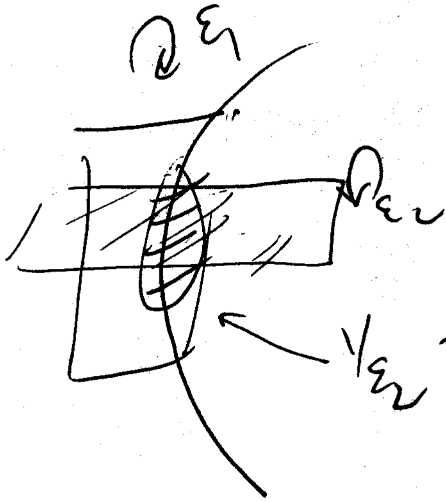
$$\xrightarrow{\epsilon_2 \to 0} e^{W(m_1,\dots,m_N, z_1 - z_N)/\epsilon_2}\, \Psi(m_1 - m_N, z_1 - z_N, \tau)$$

$Z_{Nek} \simeq tr [V \cdots V \quad]$

W-algebra

$\xrightarrow{\varepsilon_2 \to 0} e^{W(m_1 - w_n; z_1, \ldots, z_n)/\varepsilon_2}$

The finite piece Ψ = simultaneous eigenfunction of quantized Hitchin.



$1/\varepsilon_2$ = effective area

$Z_{Nek} \simeq e^{\quad /\varepsilon_2}$

$Z^S_{Nek} \simeq e^{\quad /\varepsilon_2 + \quad}$

bulk

surface operator insertion

(insertion at origin)

For pure gauge theory,

$Z^S_{Nek} = \langle W | q^k | W \rangle$

whittaker vector

$L(\hat{\mathfrak{g}})$

proved in 2004 (Braverman)