The importance of density stratification in simulations of giant planets

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Anelastic approximation

Subsonic: $v^2 << c^2$

Small thermodynamic perturbations: $T = \overline{T}(r) + T'(r,\theta,\phi,t) \qquad |T'| << \overline{T}$

Reference state: only a function of r,
hydrostatic equilibrium,
adiabatic (usually)

$$\nabla \bar{p} = -\bar{\rho}\nabla\overline{\Phi}$$

$$\nabla \overline{S} = 0$$

mass conservation

$$\nabla\cdot\bar{\rho}\mathbf{v} = 0$$

momentum conservation
(subtract out hydrostatic eq)
(assuming constant dynamic viscosity)

$$\bar{\rho}\frac{d\mathbf{v}}{dt} = -\nabla p' - \bar{\rho}\nabla\Phi' - \rho'\nabla\overline{\Phi} + 2\bar{\rho}\mathbf{v}\times\mathbf{\Omega} + + \bar{\rho}\bar{\nu}(\nabla^2\mathbf{v} + 1/3\nabla(\nabla\cdot\mathbf{v})) + \mathbf{J}\times\mathbf{B}$$

$$\nabla^2\overline{\Phi} = 4\pi G\bar{\rho} - 2\Omega^2$$

$$\nabla^2\Phi' = 4\pi G\rho'$$

heat equation

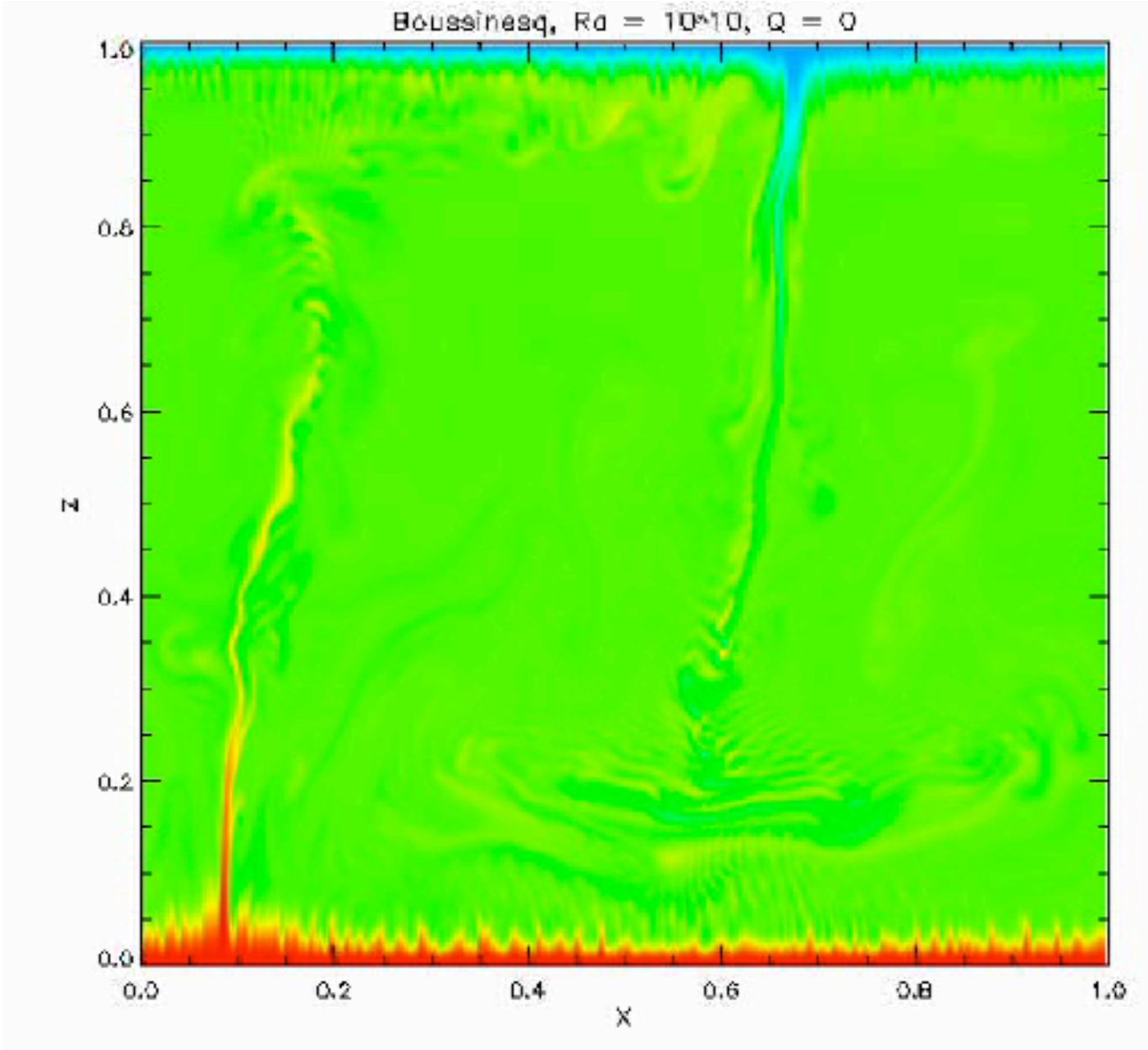
$$\bar{\rho}\overline{T}\frac{dS'}{dt} = \nabla\cdot(c_P\bar{\rho}\bar{\kappa}_{rad}\nabla(\overline{T}+T')) + \nabla\cdot(\overline{T}\bar{\rho}\bar{\kappa}_{turb}\nabla(\overline{S}+S')) - \bar{\rho}\overline{T}(\mathbf{v}\cdot\nabla)\overline{S}$$

\+ (heating)

equation of state

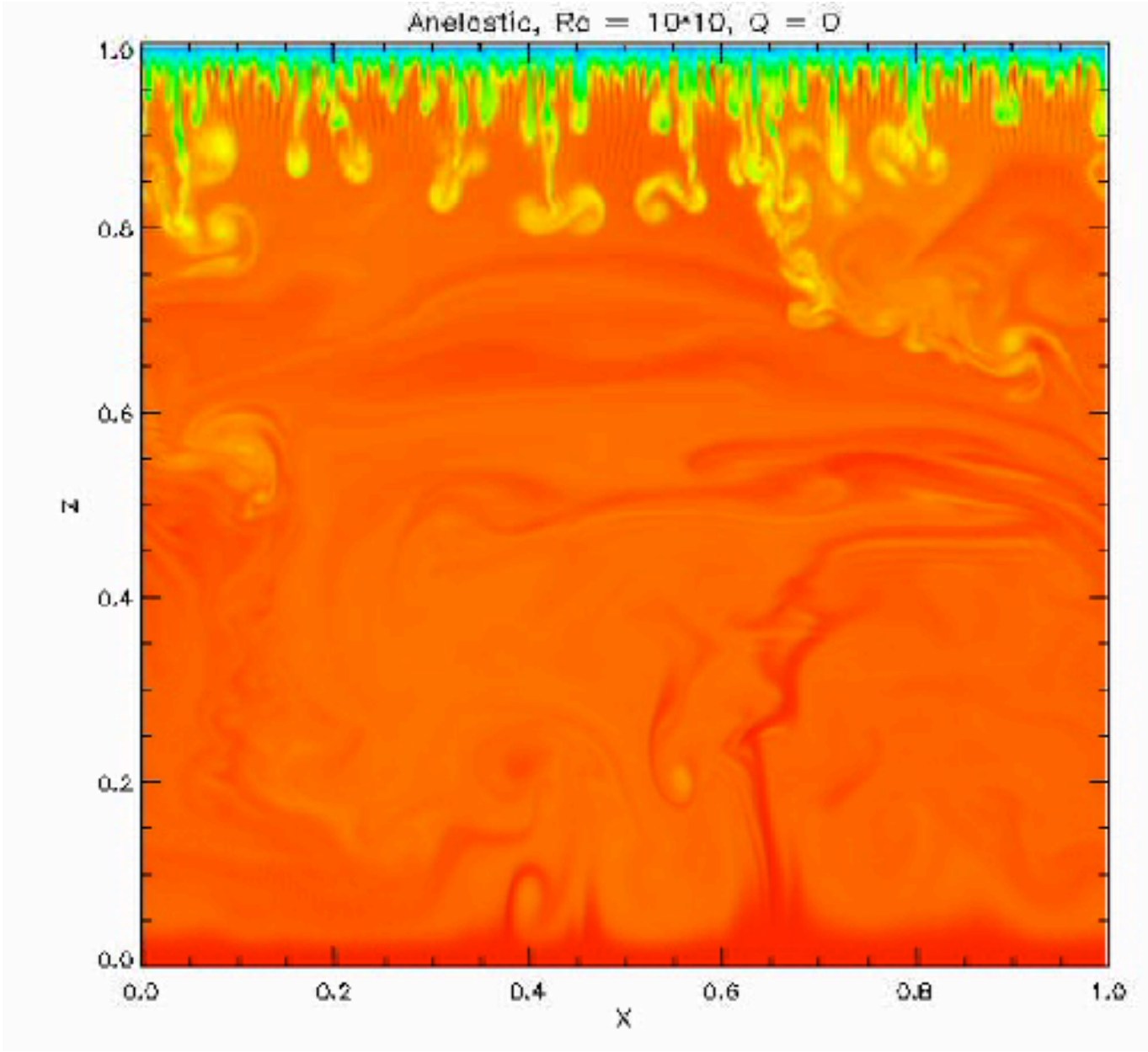
$$\rho' = \overline{\left(\frac{\partial\rho}{\partial S}\right)}_p S' + \overline{\left(\frac{\partial\rho}{\partial p}\right)}_S p'$$

same magnetic equations



2D Boussinesq convection $\rho_{bot} / \rho_{top} = 1$

Martha Evonuk



2D Anelastic convection $\rho_{bot} / \rho_{top} = 148$

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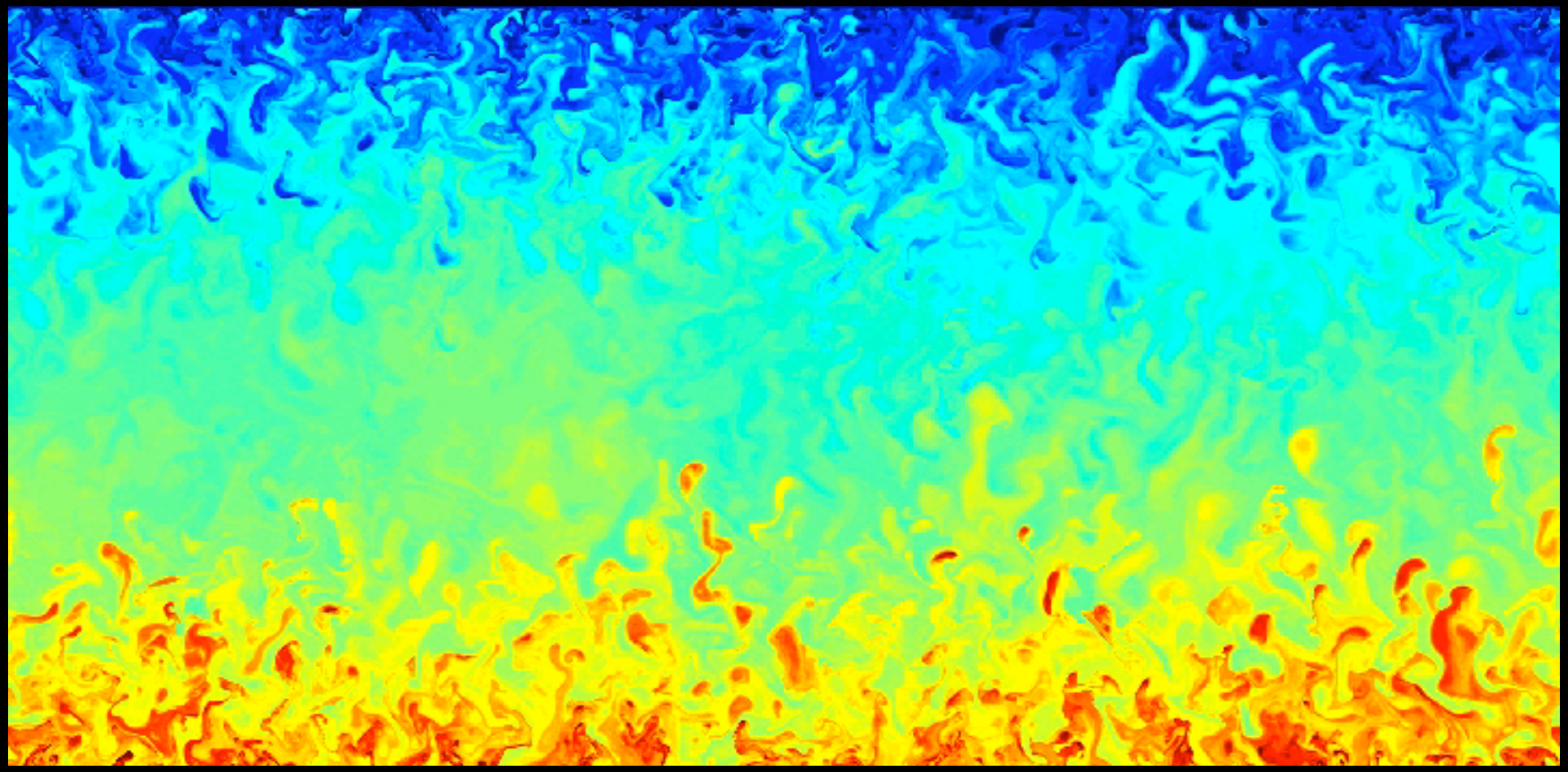
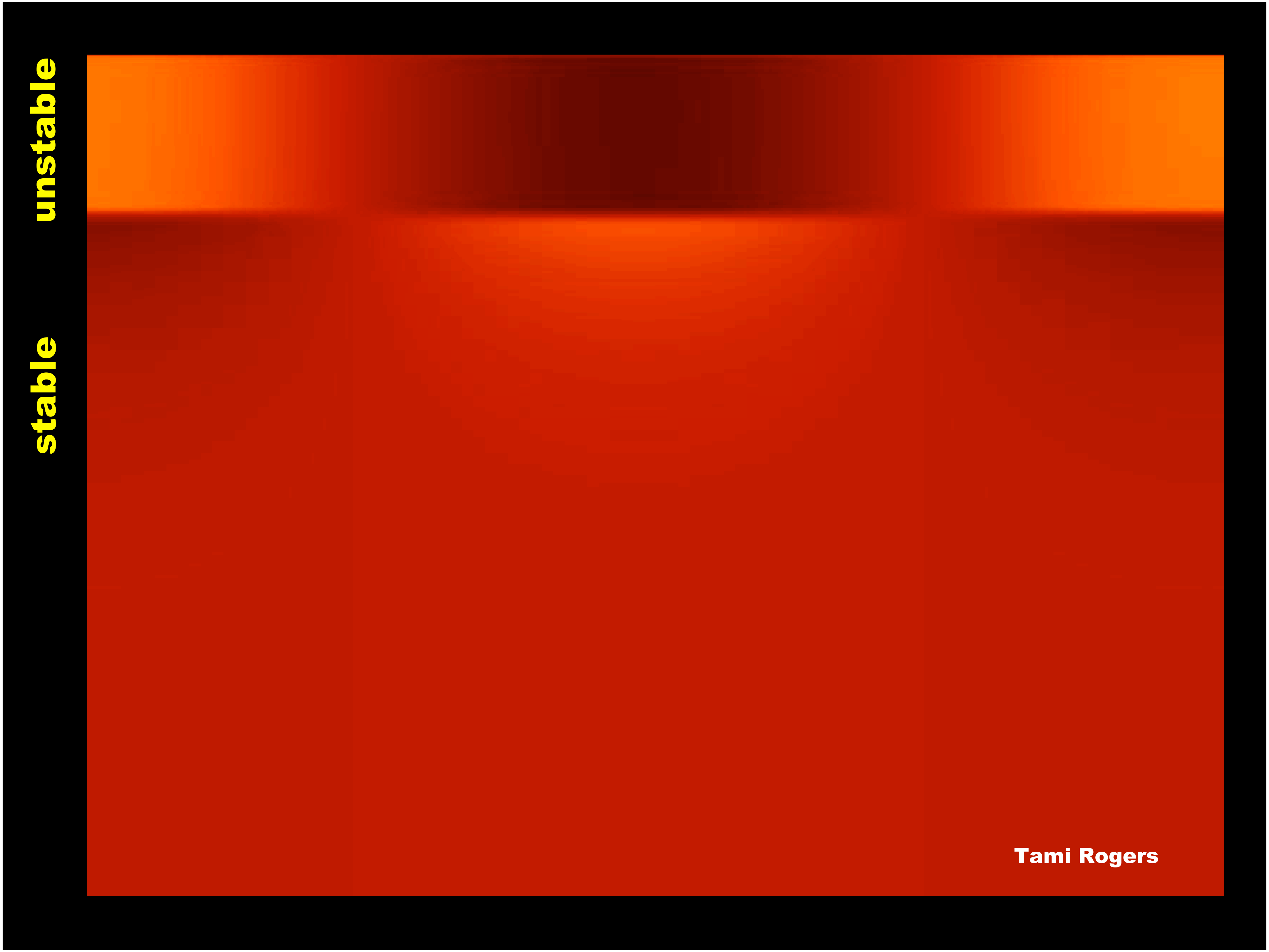
Turbulent convection

with rotation and magnetic field

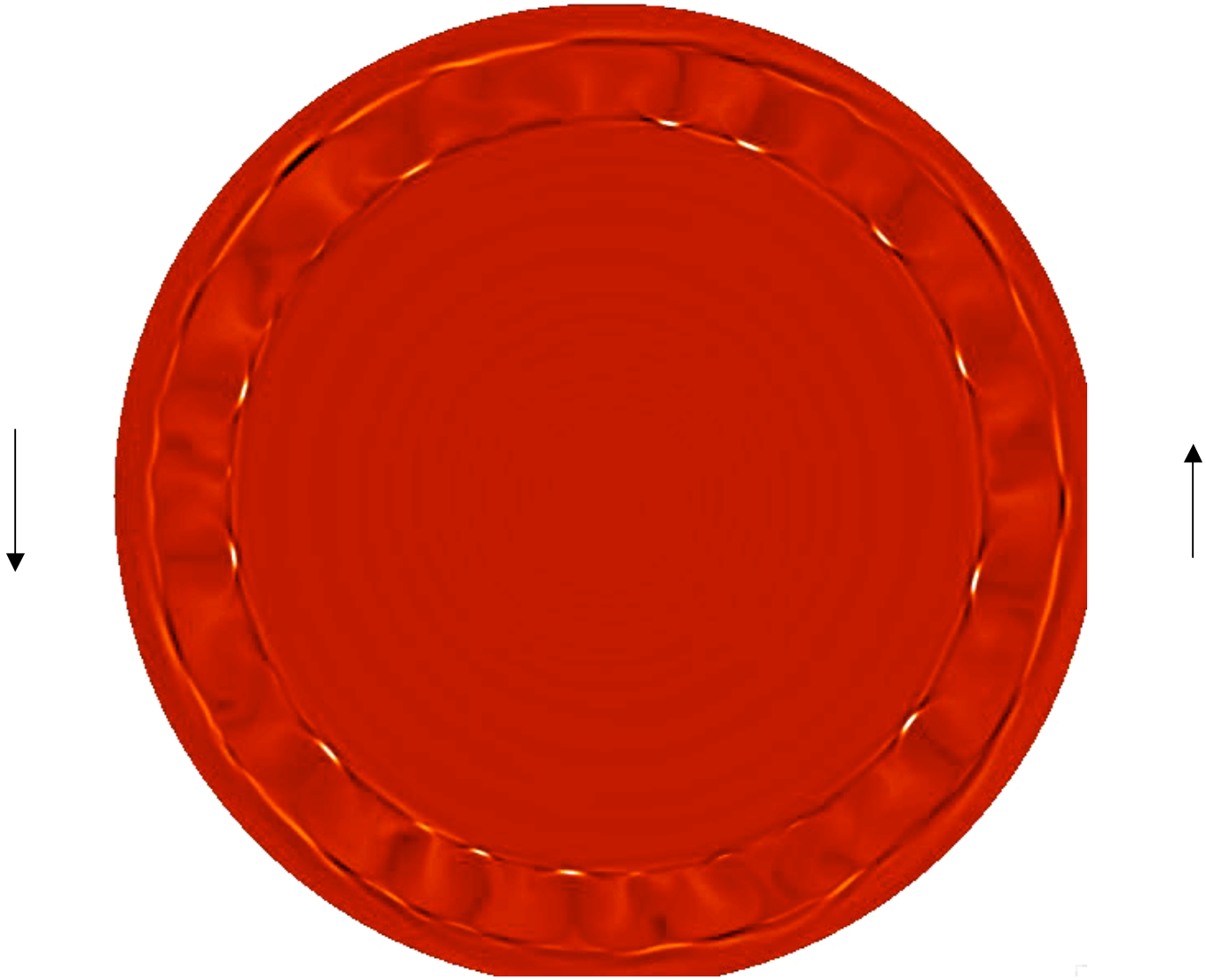


$\nu = \kappa = \eta$ $\quad \rho_{bot} / \rho_{top} = 12$ $\quad E_k = \nu / 2\Omega D^2 = 10^{-9}$

$Ra = g\alpha\Delta T D^3 / \nu\kappa = 10^{12}$ $\quad Q = B_o^2 D^2 / \mu\rho\nu\eta = 10^6$


unstable
stable
Tami Rogers

Stable radiative interior and unstable outer convection zone

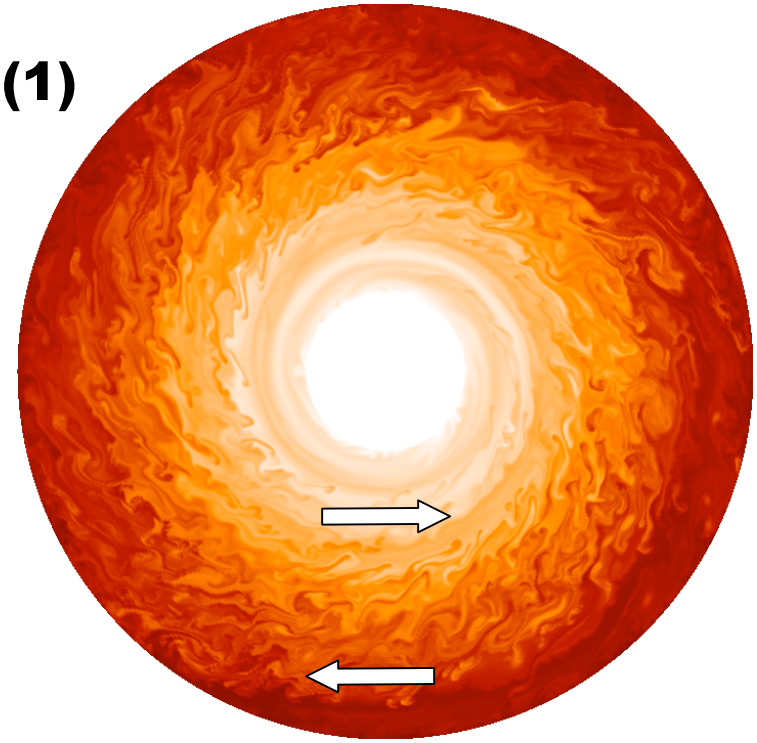


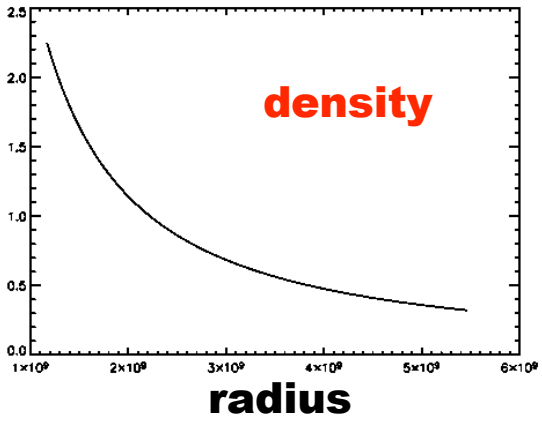
Entropy perturbations in the equatorial plane

Tami Rogers

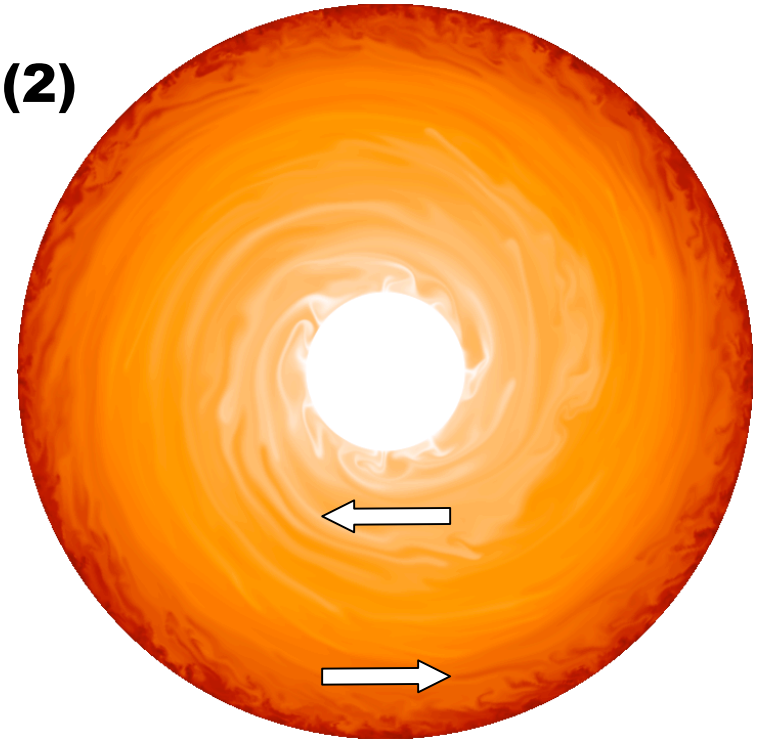
Opposite patterns of differential rotation in the equatorial plane maintained by convection in different profiles of density stratification

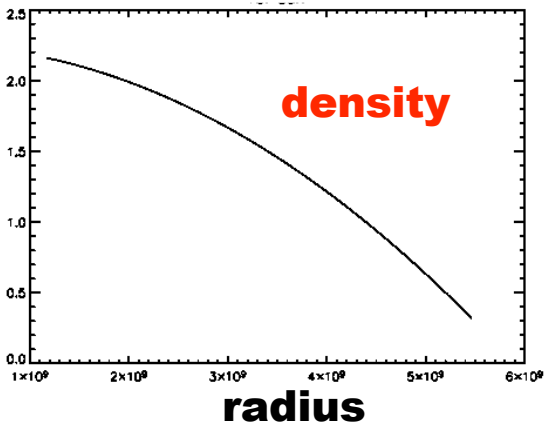
(1)





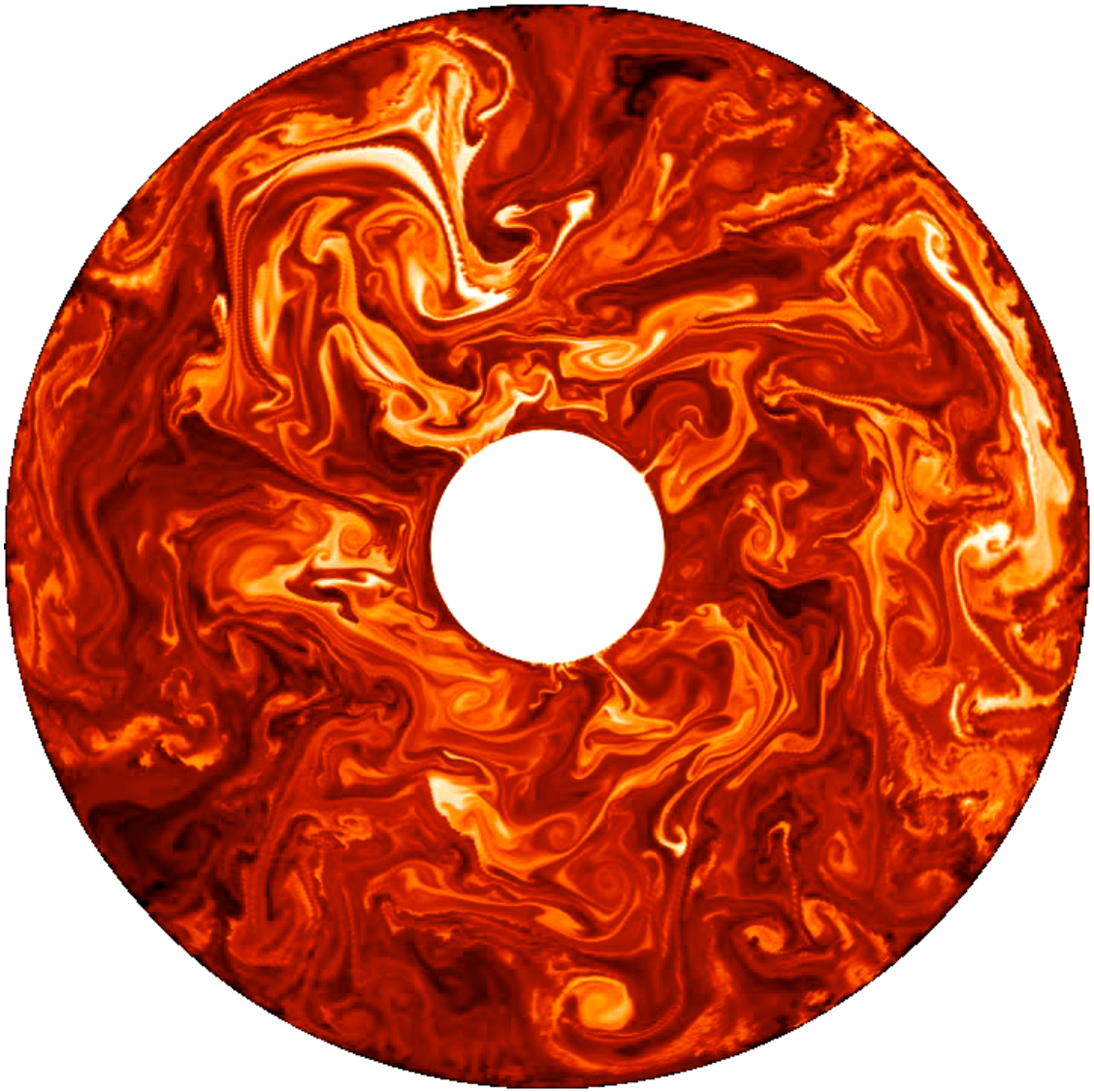
(2)





Differential rotation maintained by density-stratified turbulent convection

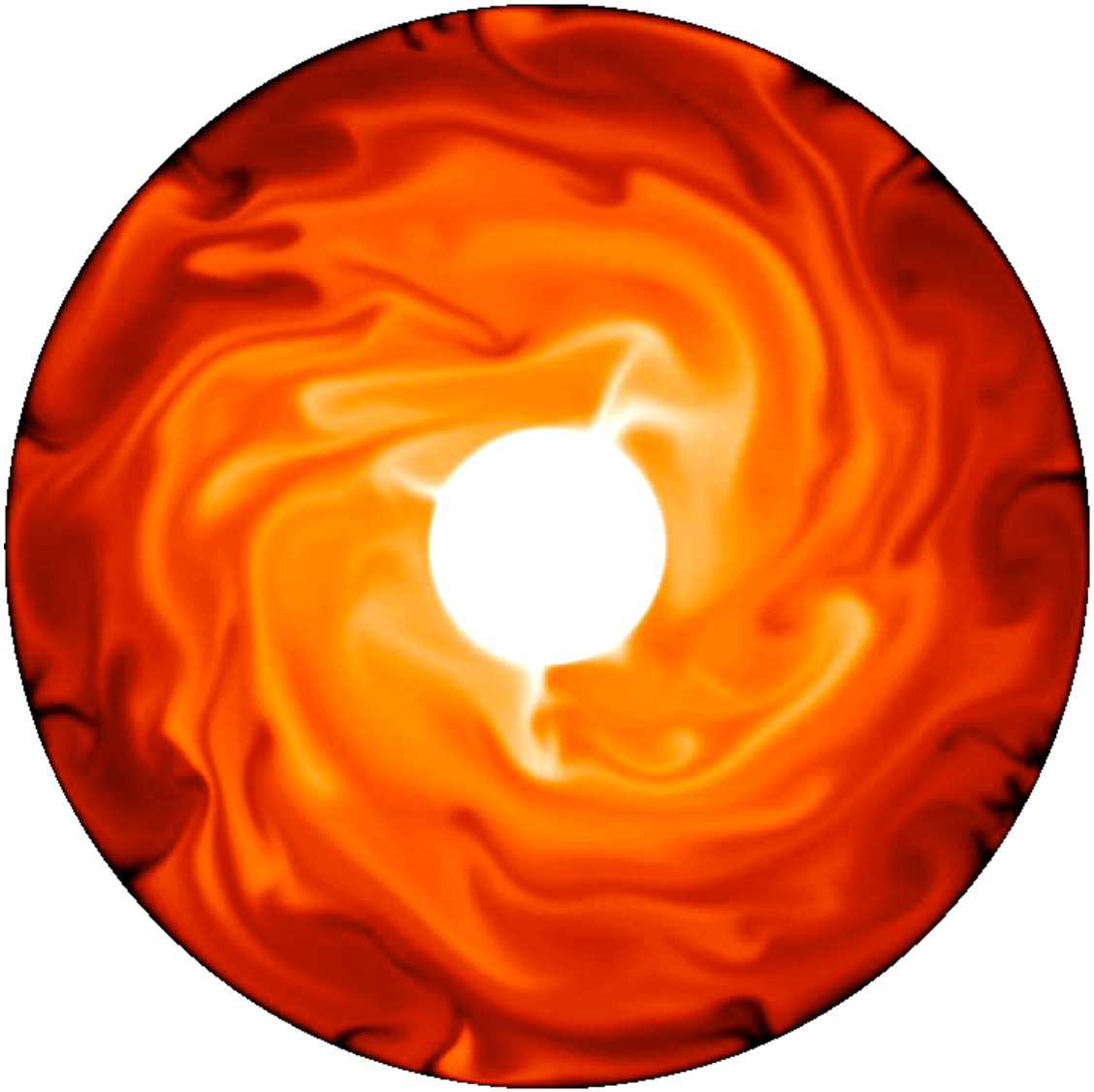
(1)



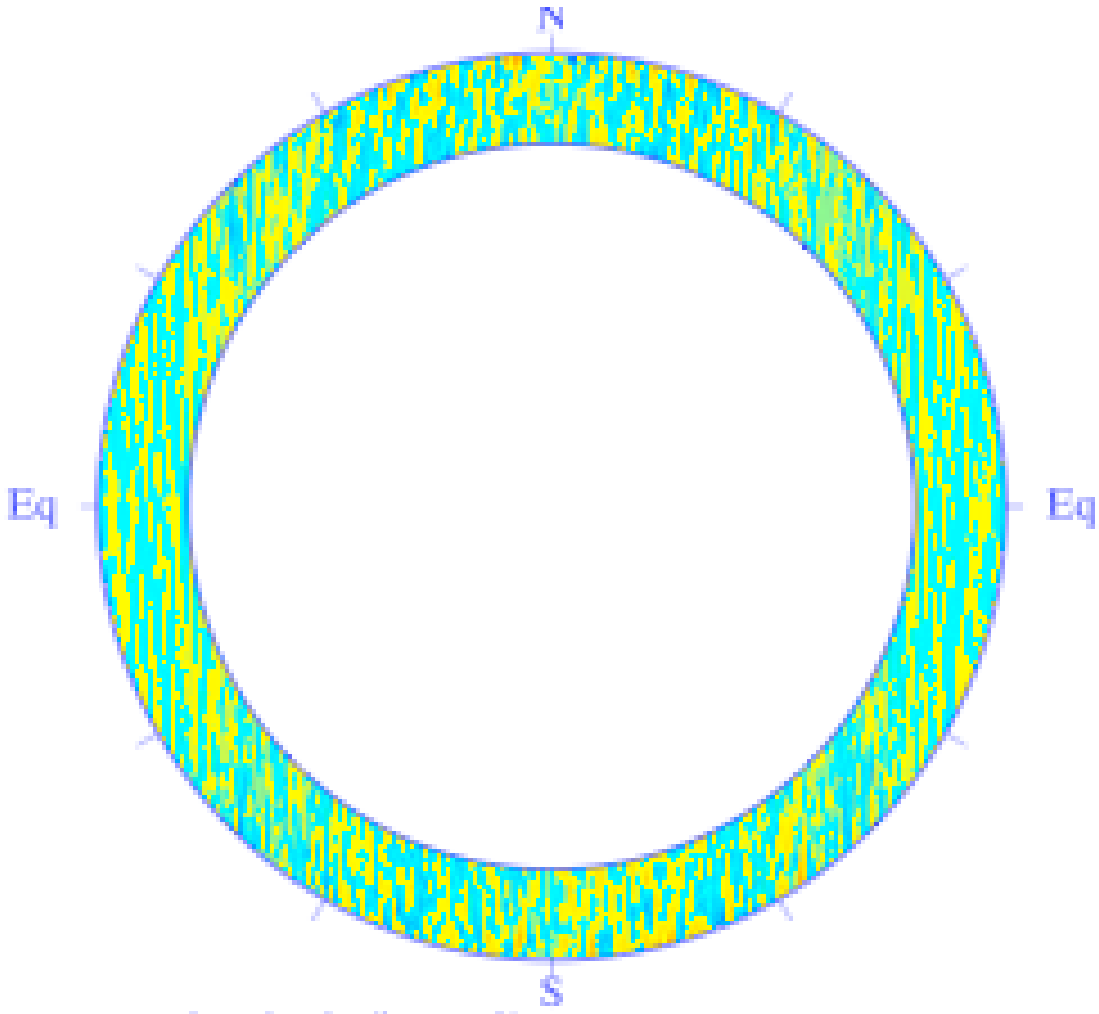
Entropy perturbations

Differential rotation maintained
by density-stratified turbulent convection

(2)



Entropy perturbations



Non-axisymmetric, z-component of vorticity in a meridian plane