Planetary Dynamos and the Effects of Density and Electrical Conductivity Stratifications

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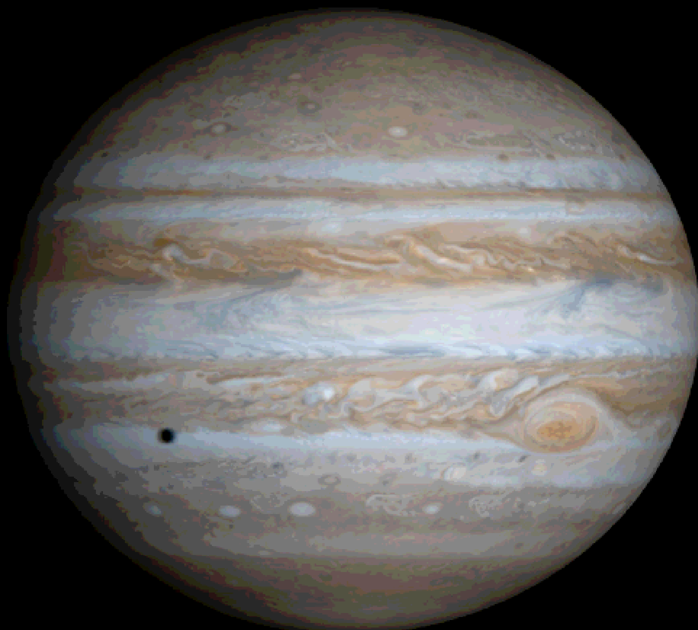
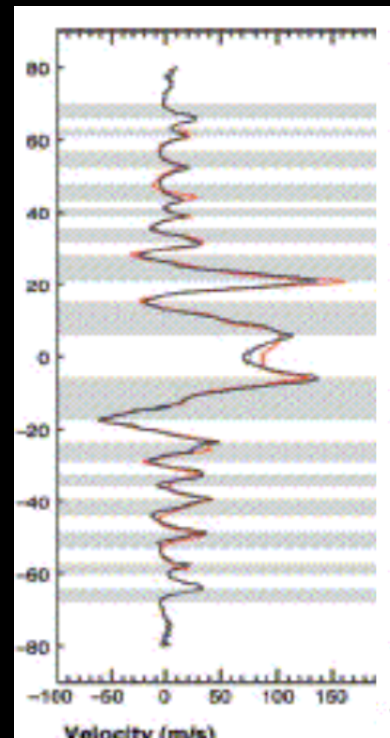
Surface observations of Jupiter





Luminosity = 3×10^{24} ergs/s with an internal heat flux minimum at equator.

Dipolar magnetic field with a surface intensity of ~10g.

Numerical method

-3D MHD dynamo simulations using the anelastic approximation

- poloidal / toroidal decomposition of mass flux and magnetic field

- spherical harmonics and Chebyshev polynomials

- spectral transform method, Chebyshev collocation and a semi-implicit time integration

- parallel (MPI)

Anelastic approximation

Subsonic: $v^2 << c^2$

Small thermodynamic perturbations: $T = \overline{T}(r) + T'(r,\theta,\phi,t) \qquad |T'| << \overline{T}$

Reference state: only a function of r,
hydrostatic equilibrium,
adiabatic (usually)

$$\nabla \bar{p} = -\bar{\rho} \nabla \overline{\Phi}$$

$$\nabla \overline{S} = 0$$

mass conservation

$$\nabla \cdot \bar{\rho} \mathbf{v} = 0$$

momentum conservation
(subtract out hydrostatic eq)
(assuming constant dynamic viscosity)

$$\bar{\rho} \frac{d\mathbf{v}}{dt} = -\nabla p' - \bar{\rho} \nabla \Phi' - \rho' \nabla \overline{\Phi} + 2\bar{\rho} \mathbf{v} \times \boldsymbol{\Omega} + + \bar{\rho} \bar{\nu} (\nabla^2 \mathbf{v} + 1/3 \nabla (\nabla \cdot \mathbf{v})) + \mathbf{J} \times \mathbf{B}$$

$$\nabla^2 \overline{\Phi} = 4\pi G \bar{\rho} - 2\Omega^2$$

$$\nabla^2 \Phi' = 4\pi G \rho'$$

heat equation

$$\bar{\rho} \overline{T} \frac{dS'}{dt} = \nabla \cdot (c_P \bar{\rho} \bar{\kappa}_{rad} \nabla (\overline{T} + T')) + \nabla \cdot (\overline{T} \bar{\rho} \bar{\kappa}_{turb} \nabla (\overline{S} + S')) - \bar{\rho} \overline{T} (\mathbf{v} \cdot \nabla) \overline{S} + \text{(heating)}$$

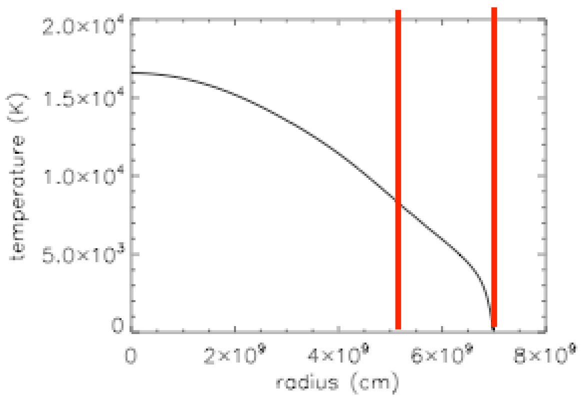
equation of state

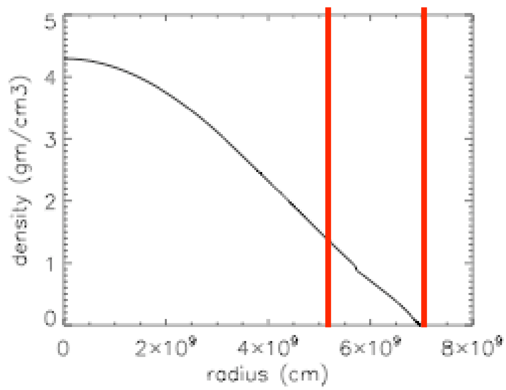
$$\rho' = \overline{\left(\frac{\partial \rho}{\partial S}\right)_p} S' + \overline{\left(\frac{\partial \rho}{\partial p}\right)_S} p'$$

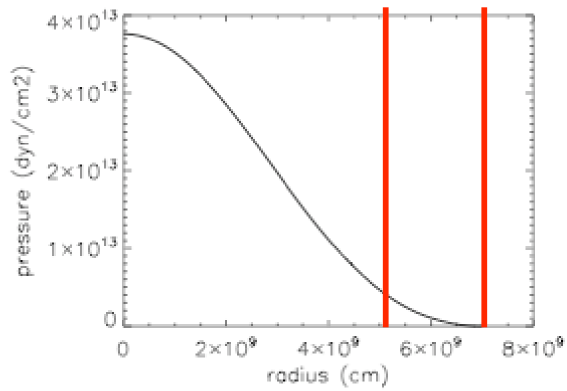
same magnetic equations

Internal (1D) evolutionary models of Jupiter

(T. Guillot)







temperature **density** **pressure**

for $r > 0.8\ R_J$: ~ perfect gas, $p \sim \rho T$

for $r < 0.8\ R_J$: ~ electron degeneracy, $p \sim \rho^{5/3}$

3D anelastic model

top boundary: 7.0×10^9cm, 0.04 gm/cm^3

bottom boundary: 5.2×10^9cm, 1.41 gm/cm^3

Experimental measurements of conductivity

(W. Nellis)

$$\sigma = \sigma_o \exp(-E(\rho)/(2\,k_B T))$$

Metallization of hydrogen
occurs at $1.4x10^{12}$ dynes/cm² ($0.84\ R_J$)
with $\eta = 1/(\mu\sigma) = 4x10^4$ cm²/s

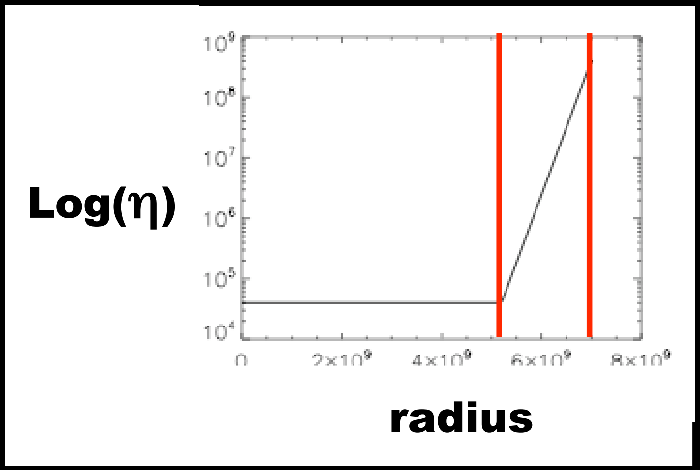
Liu, Goldreich and Stevenson (2008) Icarus

3D model of semi-conducting region

$\eta \sim \exp(r - r_{bot})$

$\eta_{top} = 4x10^8$ cm²/s

$\eta_{bot} = 4x10^4$ cm²/s



Problem: **3D simulations are forced to use greatly enhanced, viscous and thermal "turbulent" diffusivities.**

So, in order to achieve reasonable convective flows, differential rotation and magnetic field, the model's luminosity needs to be greater than observed.

However, if realistic flows and fields are the goals, these can be compared at the surface to observations and constrained below the surface by maintaining a total rate of entropy production by ohmic heating

$$\int (J^2/\sigma T)\, dV$$

less than the observed luminosity * $(1/T_{out} - 1/T_{in})$.

If most of the ohmic heating occurs near T_{in}, it needs to be less than the luminosity * $(T_{in}/T_{out} - 1)$.

Rate entropy flows out	=	Rate entropy flows in	+	Rate of production of entropy by dissipation

$$\frac{F_{out}}{T_{out}} = \frac{F_{in}}{T_{in}} + \int \frac{J^2}{\sigma T} dV + \ldots$$

$$F_{in} = F_{out}$$

$$\int \frac{J^2}{\sigma T} dV \approx \frac{Q_{dis}}{T_{in}}$$

$$\frac{Q_{dis}}{F_{out}} < \left(\frac{T_{in}}{T_{out}} - 1 \right)$$

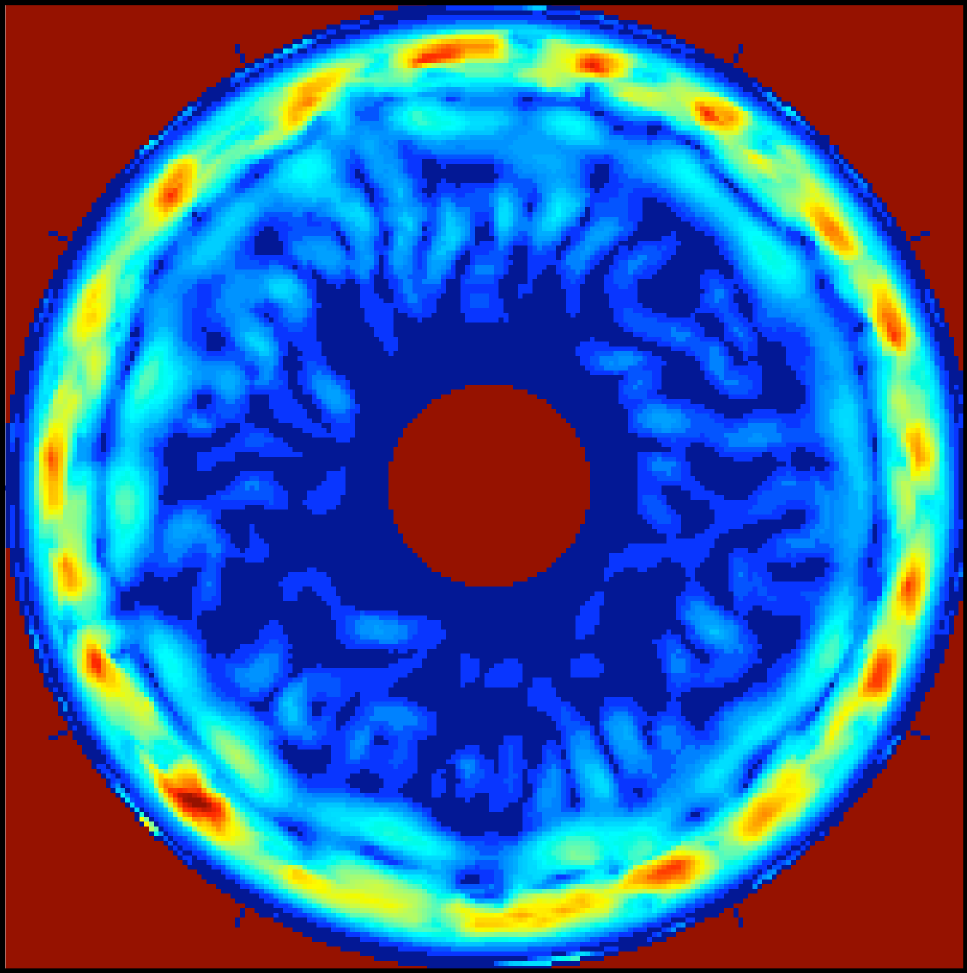
Backus, G.E. (1975) *Proc. Nat. Acad. Sci. Wash*. 72, 1555-1558.

Hewitt, J.M., McKenzie, D.P. and Weiss, N.O. (1975) *JFM* 68, 721-738.

Most of the magnetic field is likely generated in a thin layer deep below the surface where the local magnetic Reynolds number (v d σ μ) is maximum, since velocity, v, decreases with depth and conductivity, σ, increases with depth.

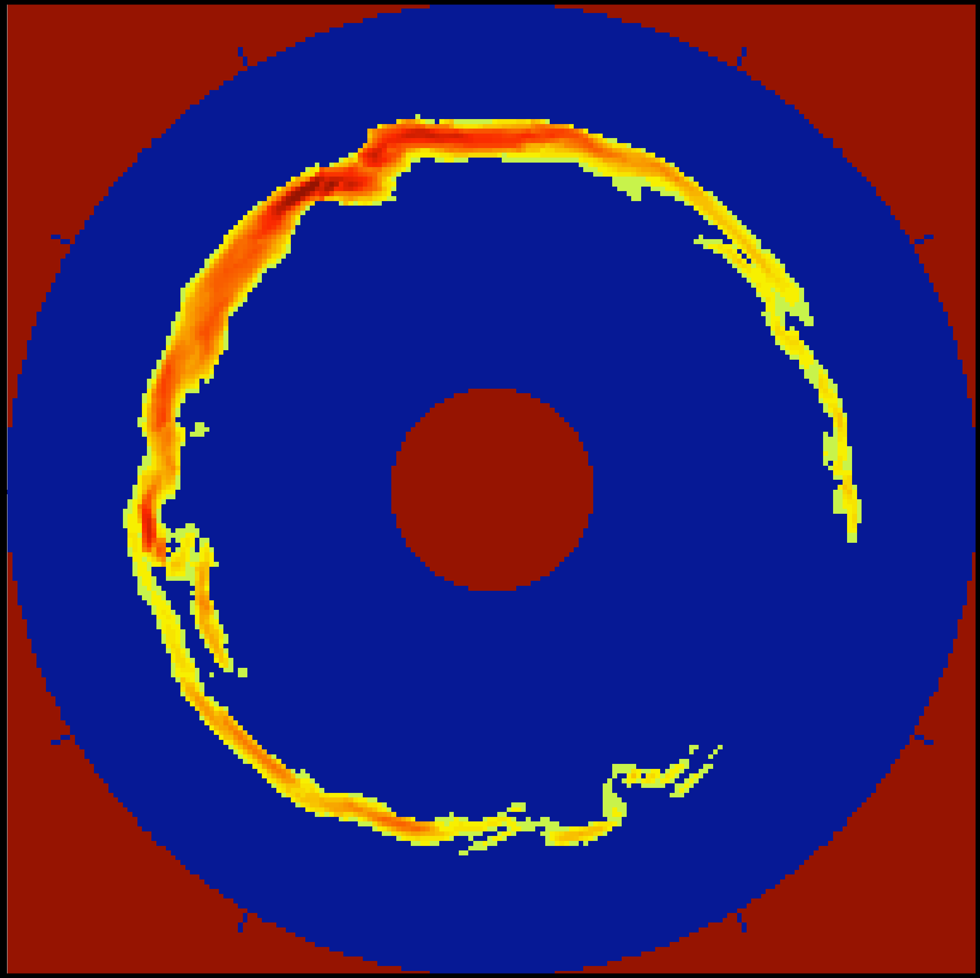
Kinetic energy

giant
planet
simulation



3D - viewed in equatorial plane

Magnetic energy



3D - viewed in equatorial plane

3D MHD model

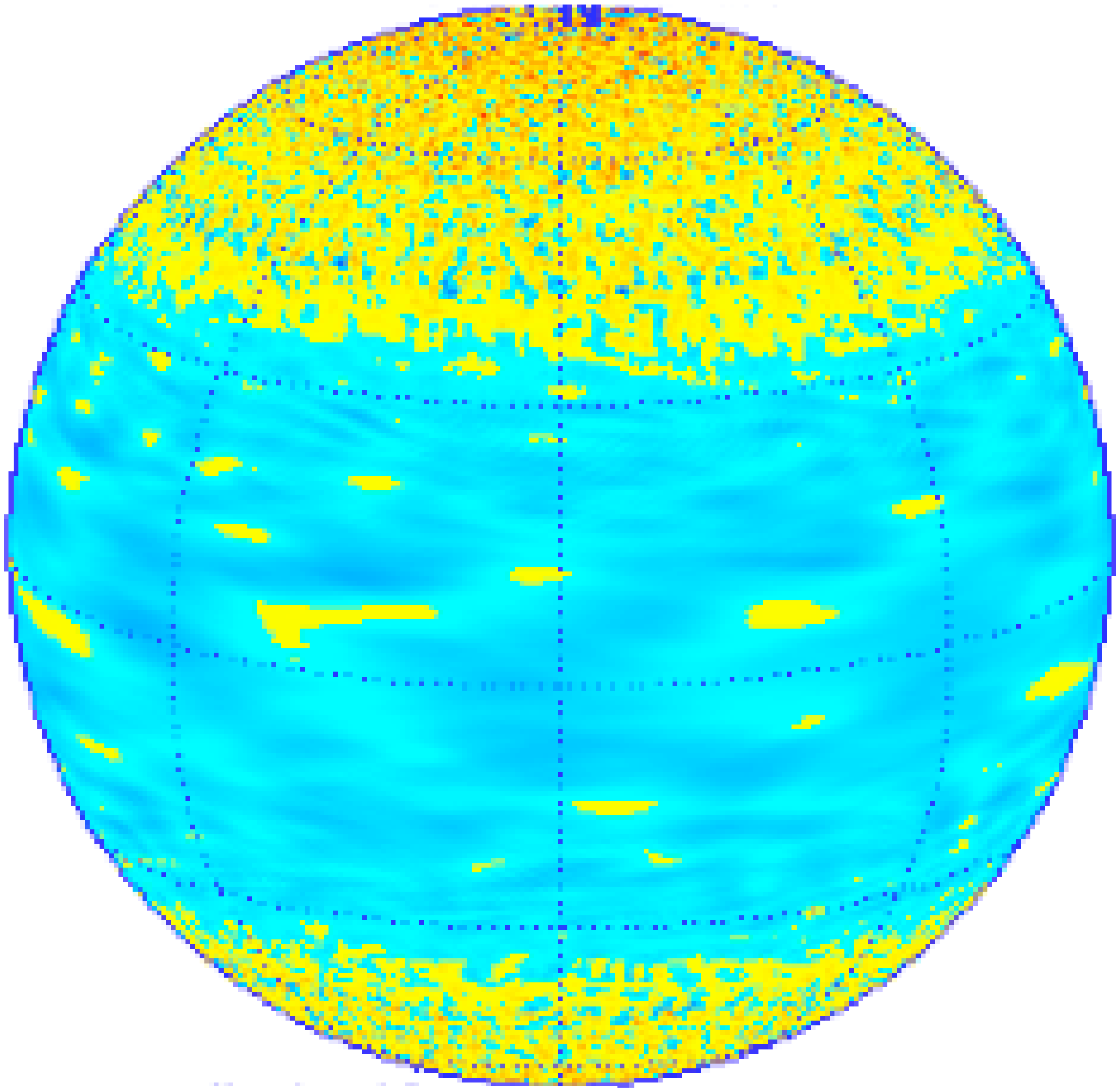
Anelastic, perfect gas EOS, $\rho_{bot}/\rho_{top} = 32$

$\nu = 10^9$ cm²/s, $\kappa = 3 \times 10^9$ cm²/s, $\eta = 4 \times 10^4 - 4 \times 10^8$ cm²/s

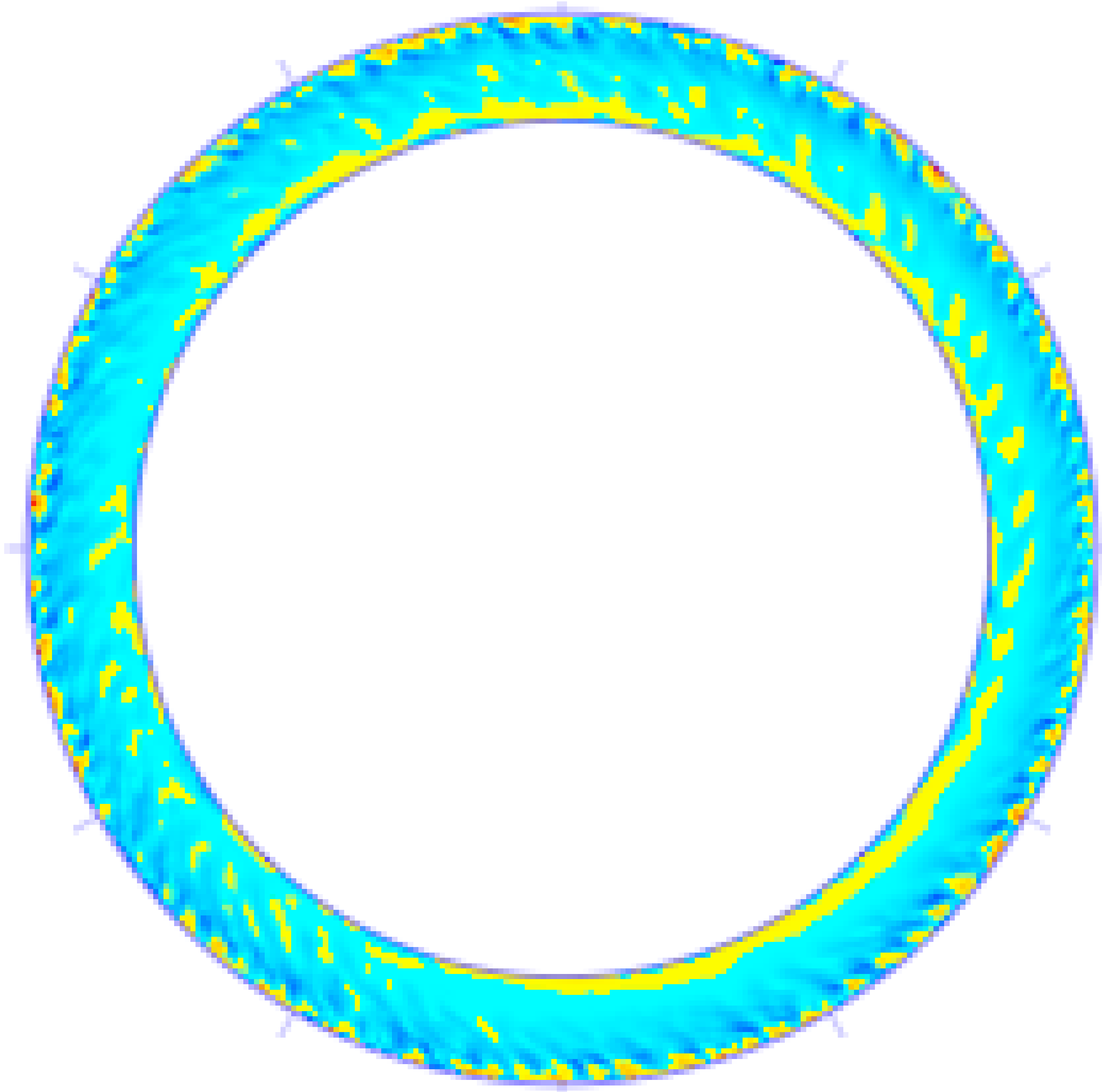
$Ra = 10^9$, $Ek = 10^{-6}$, $Pr = 1/3$, $Rc = (Ra/Pr)^{1/2}\,Ek = 0.05$

$Re = 10^4$, $Ro = 0.01$

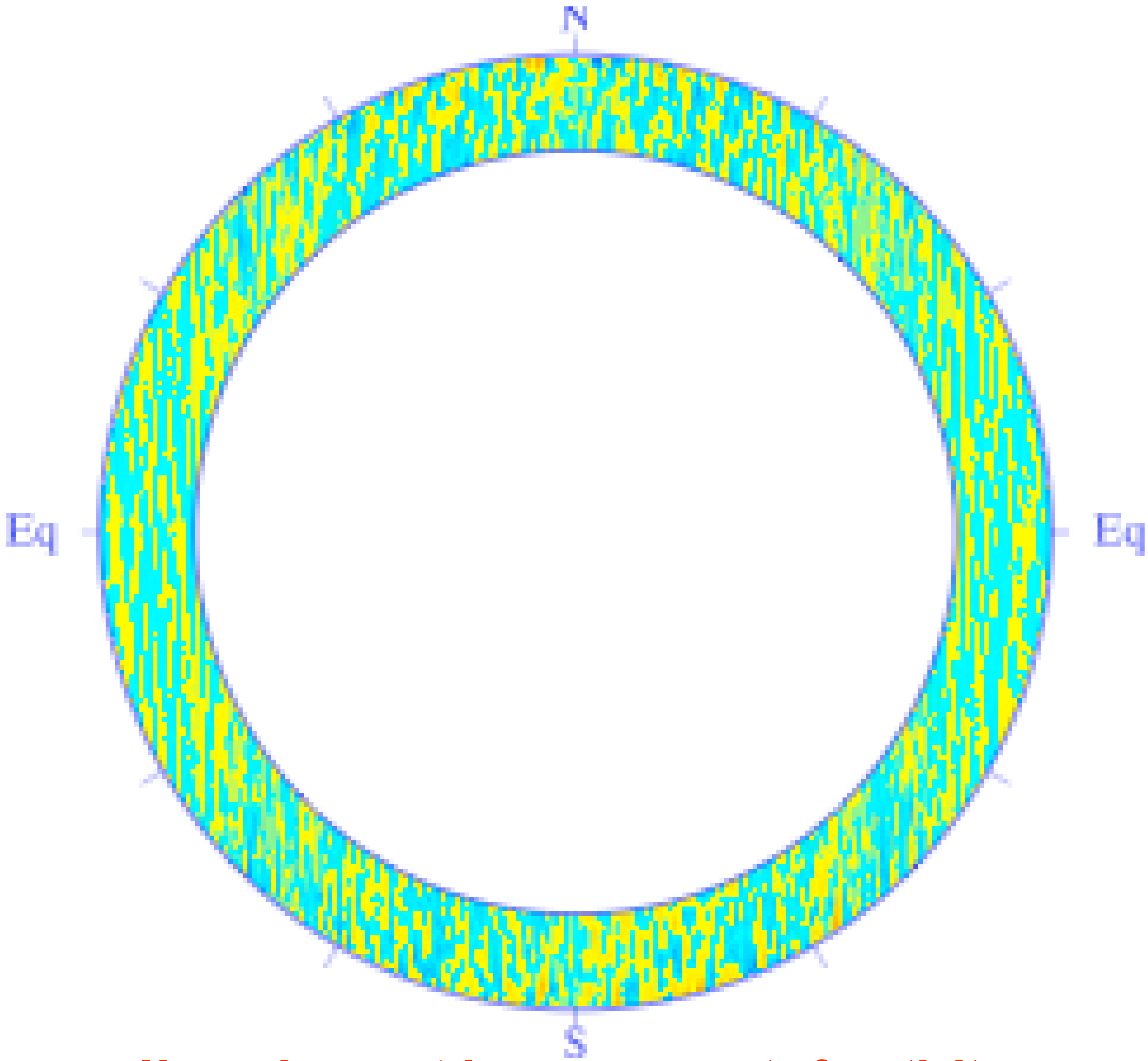
$241 \times 768 \times 768$, $l_{max} = 511$, $m_{max} = 255$



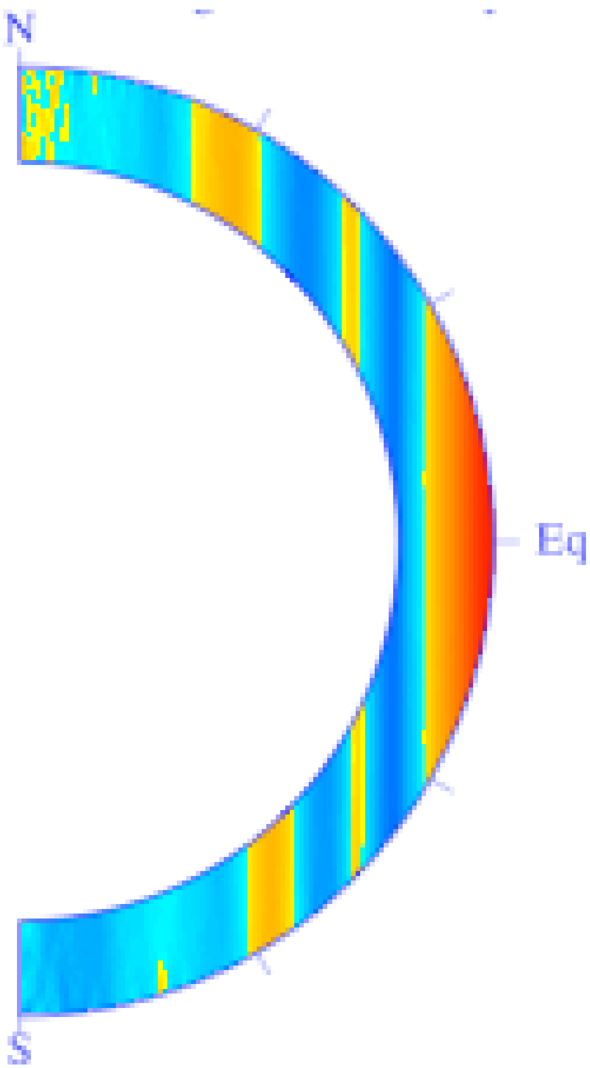
Entropy perturbations on outer boundary



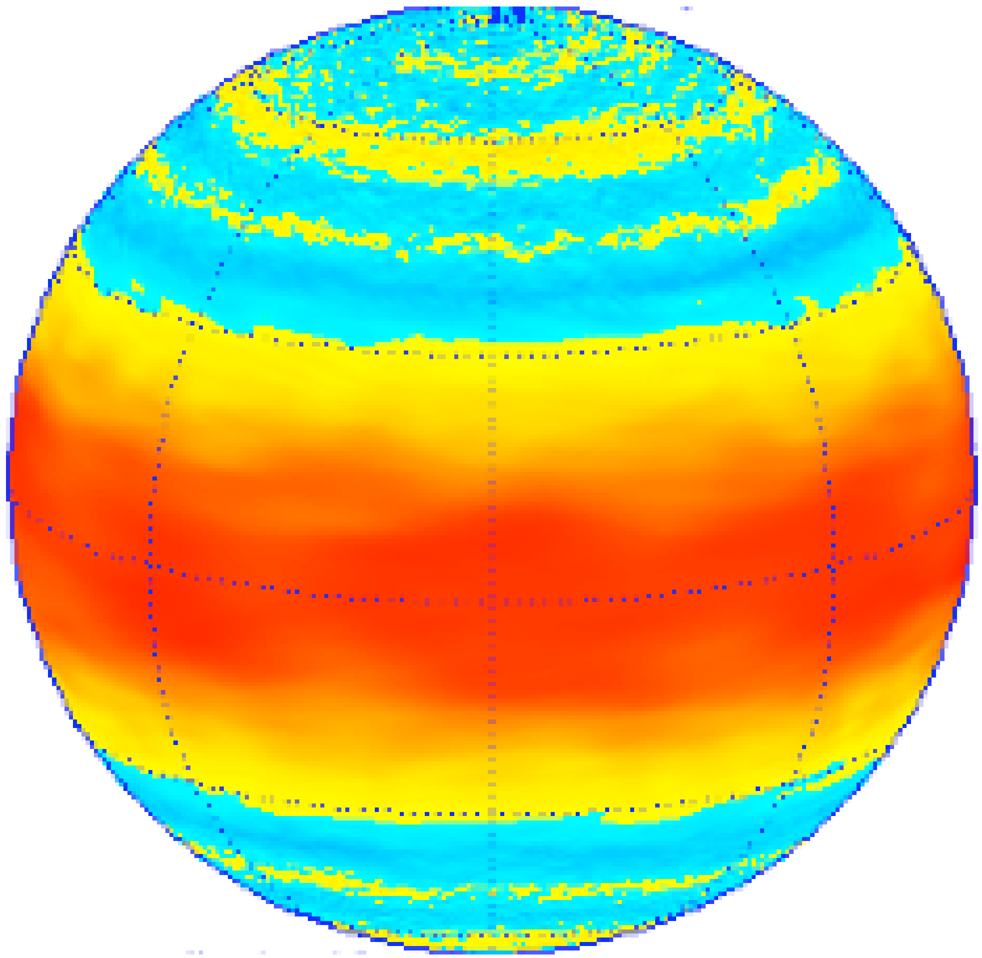
Entropy perturbations in the equatorial plane



Non-axisymmetric, z-component of vorticity
in a meridian plane



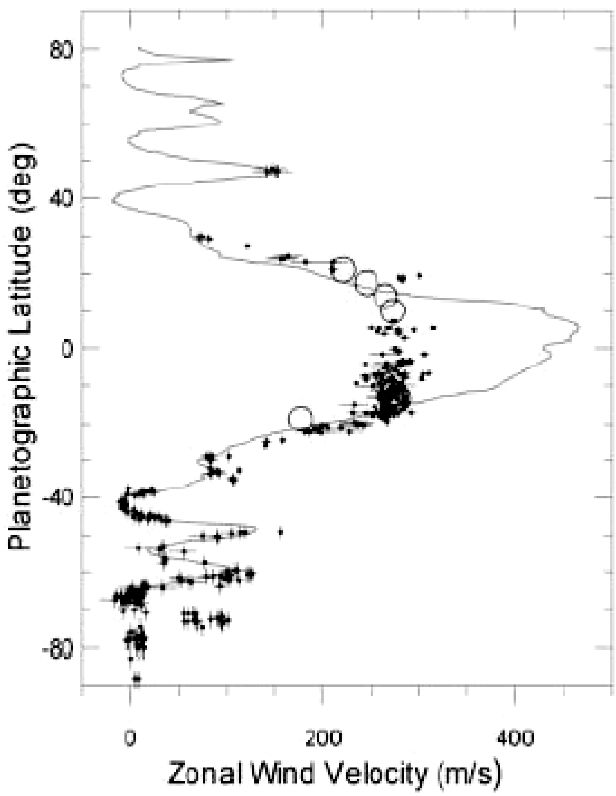
Zonal winds



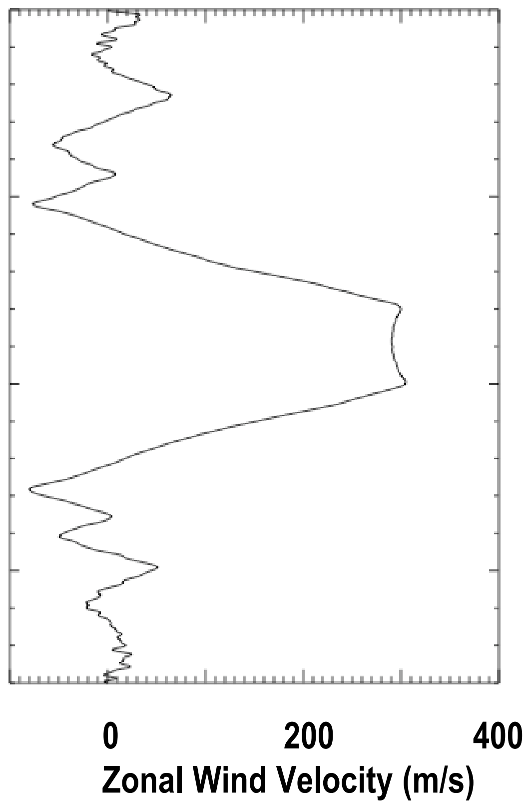
Longitudinal velocity at surface

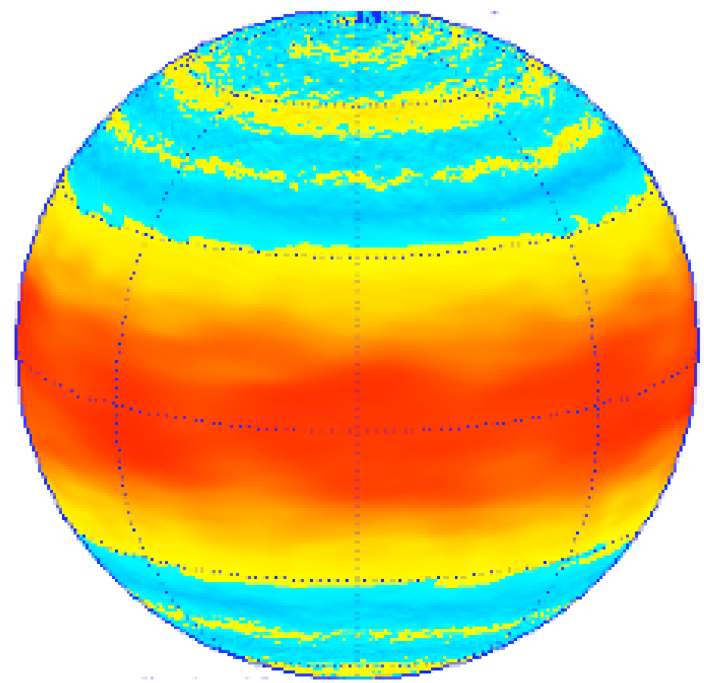
Surface differential rotation

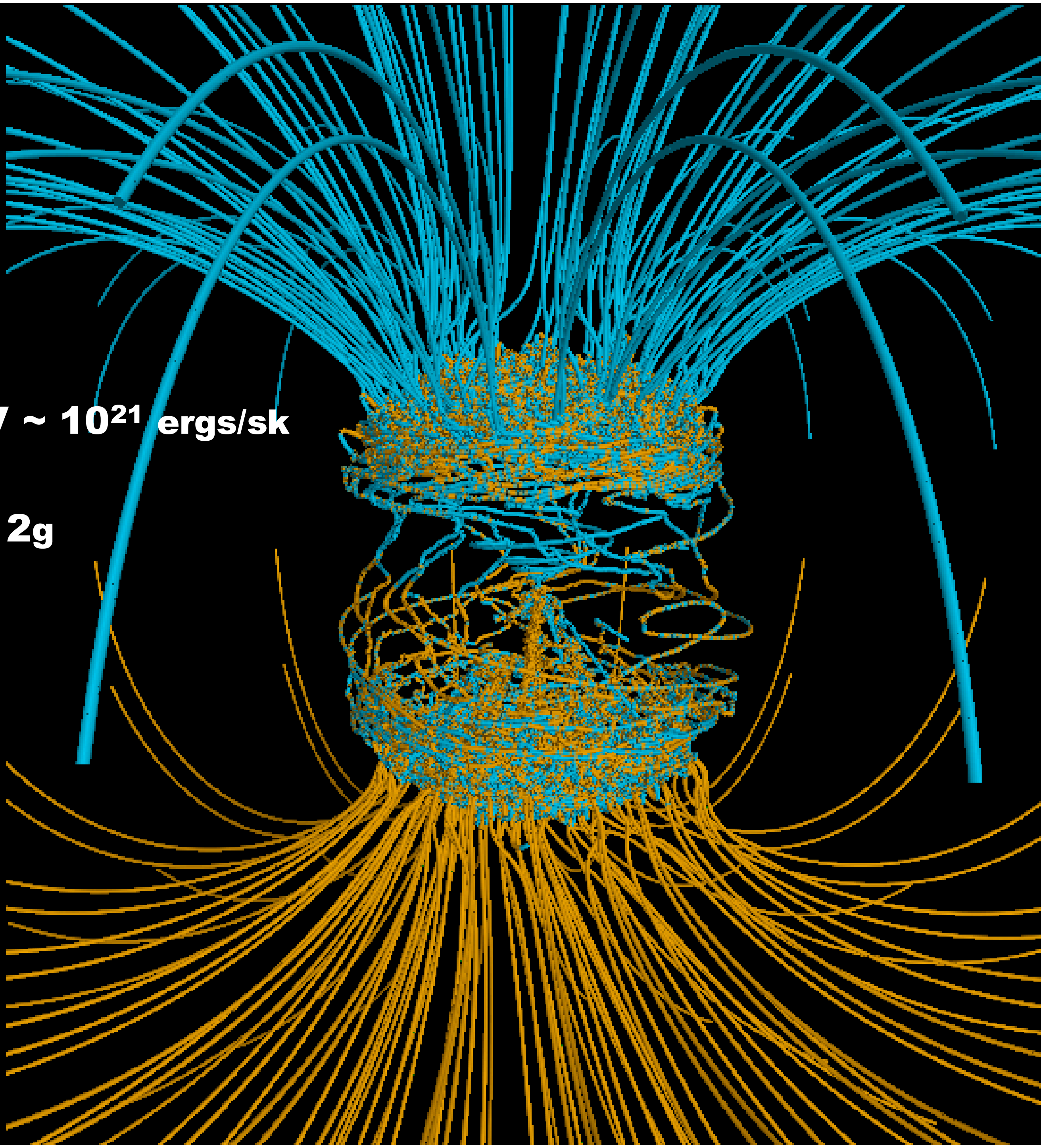
Saturn observed **3D simulation**



Sanchez-Lavega et al., 2003







$\int (J^2/\sigma T)\, dV \sim 10^{21}$ **ergs/sk**

Surf field ~ 2g

Main Conclusion

If realistic flows and fields are the goals, these can be compared at the surface to observations and constrained below the surface by maintaining a total rate of entropy production by ohmic heating

$$\int (J^2/\sigma T)\, dV$$

less than the observed luminosity * $(1/T_{out} - 1/T_{in})$.