Astrophysical & Geophysical
Fluid Dynamics
DEPARTMENT OF APPLIED MATHEMATICS

UNIVERSITY OF LEEDS

# Magnetic buoyancy: a mechanism for the formation of coherent structures

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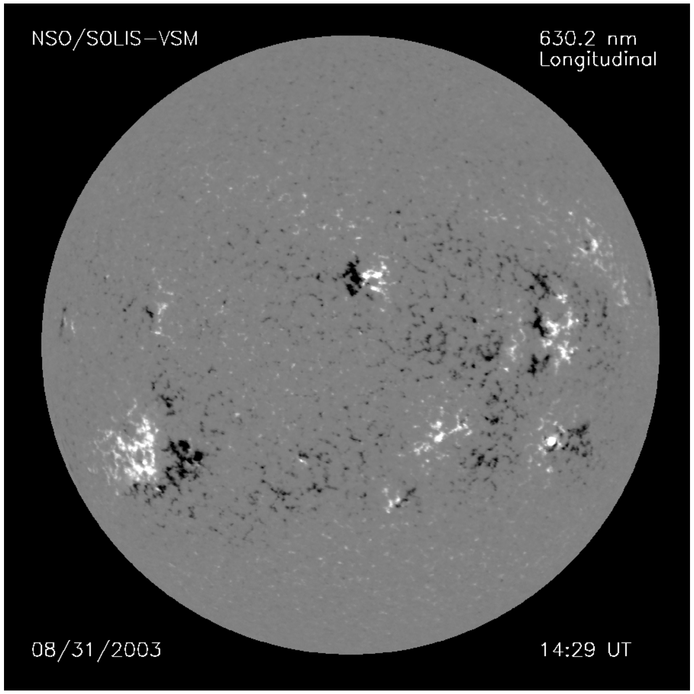
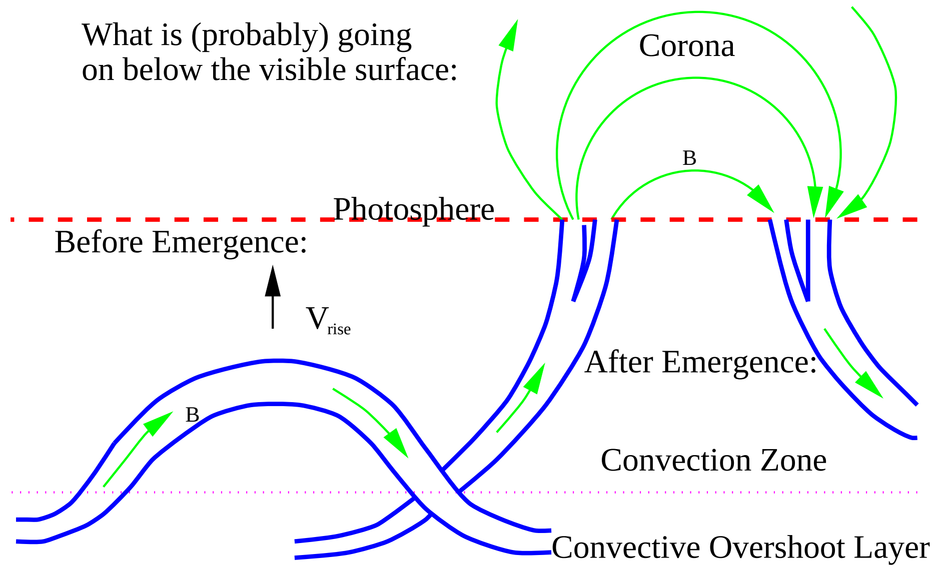
ApJ **663**, L113–L116

# The Sun's magnetic field




G.H. Fisher

Coronal magnetic field resulting from the emergence of a strong toroidal field stored deep within the Sun

Magnetic flux escapes from below the convection zone via magnetic buoyancy instability

# Magnetic buoyancy instability

A stratified horizontal magnetic field that increases with depth supports more gas than would be possible in its absence

Potential energy can be released, and hence instability arises, when the gradient of magnetic field exceeds a certain threshold (Newcomb 1961, Parker 1966)

Instability if: (see Gilman 1970; Acheson 1979)

$$g\frac{c_a^2}{c_s^2}\frac{\mathrm{d}}{\mathrm{d}z}\ln(B/\rho) > \frac{\eta}{\kappa}N^2 \text{ (interchange)} \qquad g\frac{c_a^2}{c_s^2}\frac{\mathrm{d}}{\mathrm{d}z}\ln B > \frac{\eta}{\kappa}N^2 \text{ (undular)}$$

Important questions: origin of the 3-D nature of magnetic structures, strength of the field, mechanism responsible for non-linear saturation and properties of turbulent transport

# Compressible MHD

**Non-dimensional equations** : $T_0, \rho_0, l_0$ (box height) and $t_0 = l_0/\sqrt{(c_p - c_v)T_0}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \qquad P = \rho T$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} - \mathcal{F}\mathbf{B}\mathbf{B}) = -\nabla \left( \rho T + \mathcal{F}\frac{B^2}{2} \right) + \theta(m+1)\rho \mathbf{e}_z + C_\kappa \sigma \left[ \nabla^2 \mathbf{u} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{u}) \right]$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (T\mathbf{u}) = -(\gamma - 2)T\nabla \cdot \mathbf{u} + \gamma C_\kappa \frac{\nabla^2 T}{\rho} + (\gamma - 1)C_\kappa \sigma \frac{\Phi}{\rho} + (\gamma - 1)C_\kappa \mathcal{F}\zeta \frac{(\nabla \times \mathbf{B})^2}{\rho}$$

where $\Phi = \frac{\partial u_i}{\partial x_j}\left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \right)$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + C_\kappa \zeta \nabla^2 \mathbf{B}, \qquad \nabla \cdot \mathbf{B} = 0$$

**with non-dimensional parameters** :

$\theta = \Delta d/T_0$ with $\Delta = \mathrm{d}T/\mathrm{d}z$, $m = -1 + g/\Delta(c_p - c_v)$ is the polytropic index

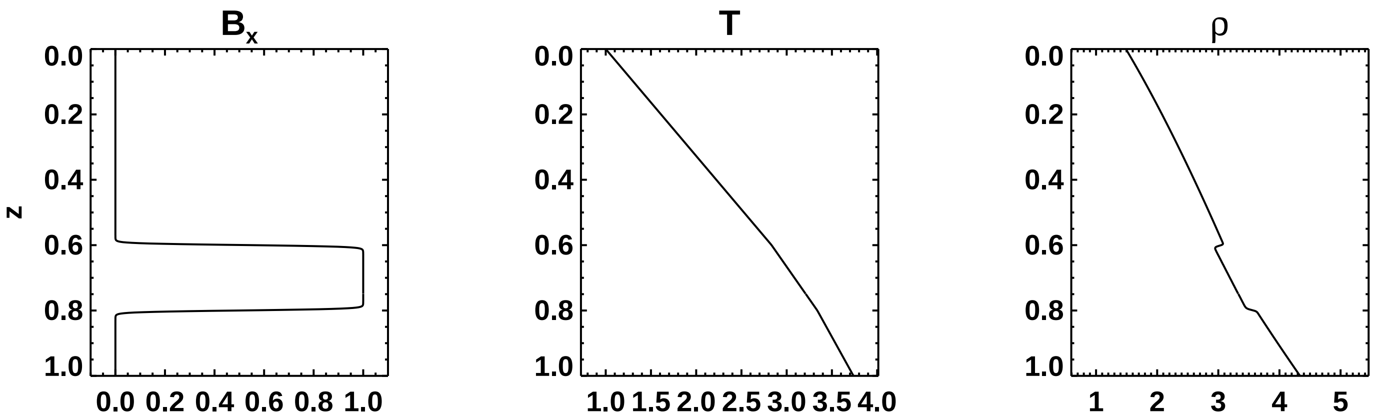
$C_\kappa = \frac{\kappa/d\rho_0 c_p}{\sqrt{(c_p - c_v)T_0}}$, $\sigma = \frac{\mu c_p}{\kappa}$, $\zeta = \frac{\eta \rho_0 c_p}{\kappa}$ and $\mathcal{F} \equiv \frac{2}{\beta} = \frac{B_0^2}{\mu_0 p_0}$

# Stability of a slab of magnetic field

Basic state: hydrostatic equilibrium

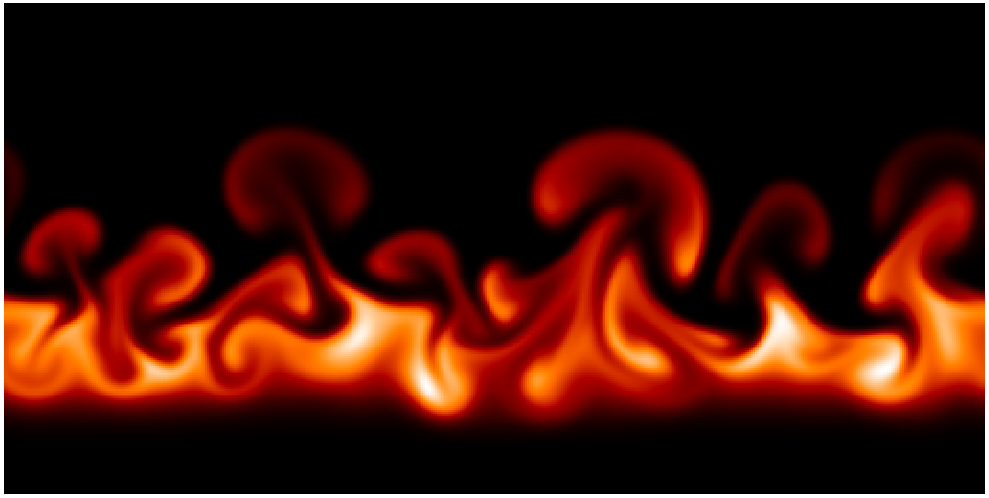
$$B_x = \tanh(z - z_1)/2h - \tanh(z - z_2)/2h,$$

$$\gamma\partial^2_{zz}T = -2(\gamma - 1)\beta^{-1}\zeta(\partial_z B)^2 \quad \text{and} \quad \partial_z\left(\rho T + B^2/\beta\right) = \theta(m+1)\rho$$

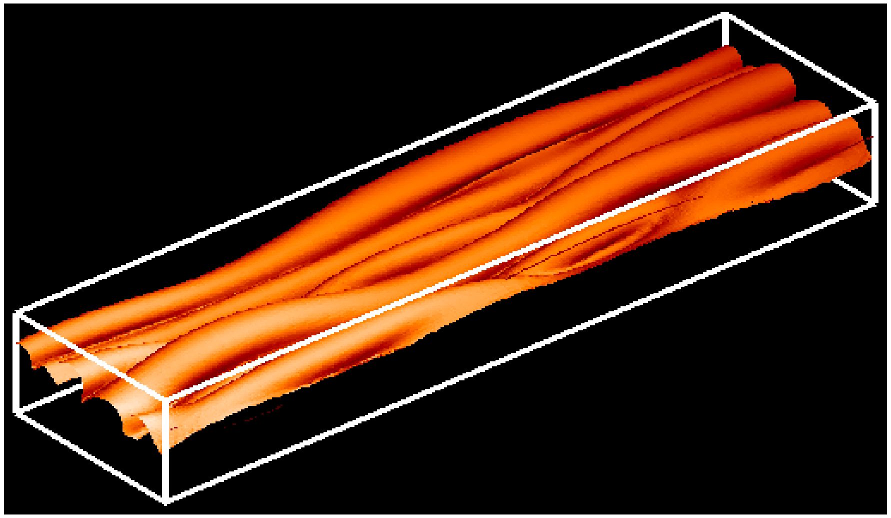



For sufficiently narrow jumps at top and bottom interfaces 2-D modes are preferred

# Cattaneo & Hughes 88; Matthews et al. 95; Wissink et al. 00



Dynamics is dominated, first by the rise of buoyant fluid, then by strong vortex-vortex interactions and finally by dissipation

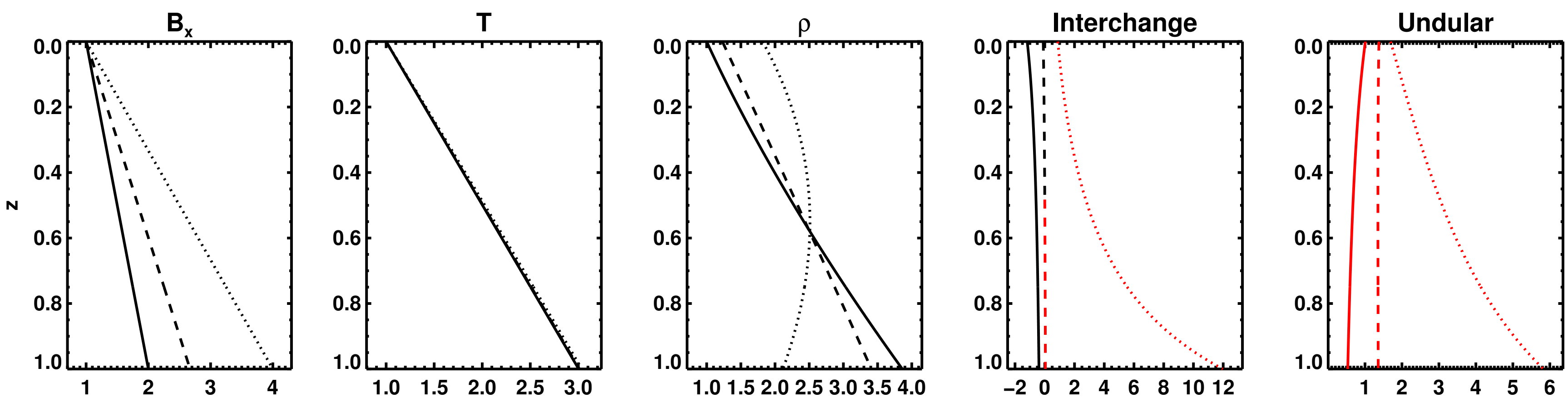


3-D structures due to secondary instability (nonlinear interactions of counter-rotating lines of vortices)

# Continuously magnetised atmosphere

Basic state: magnetohydrostatic equilibrium solution of

$$
\begin{aligned}
\partial^2_{zz} B_x &= 0 \\
\gamma \partial^2_{zz} T &= -2(\gamma - 1)\beta^{-1}\zeta(\partial_z B_x)^2 \\
\partial_z \left(\rho T + B_x^2/\beta\right) &= \theta(m+1)\rho
\end{aligned}
$$



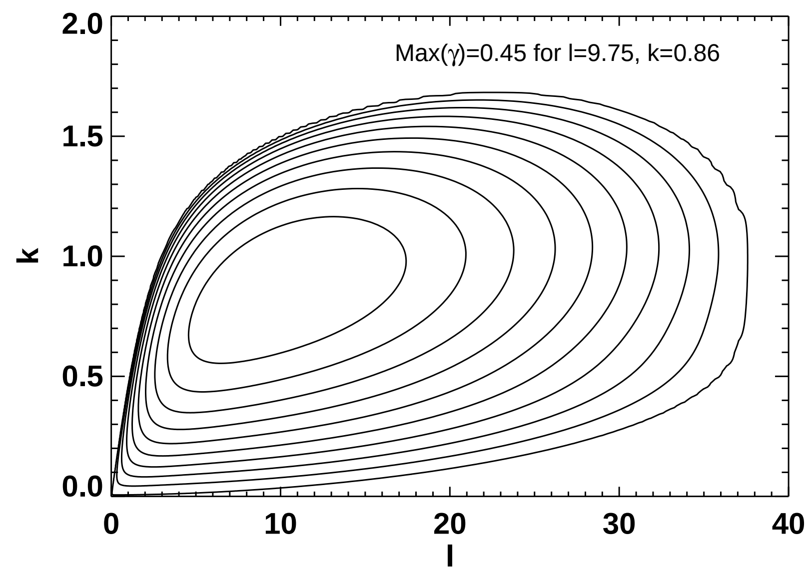

Instability continually driven from the boundaries:

$\partial_z B_x =$ const. or $B_x =$ const.

# Linear theory

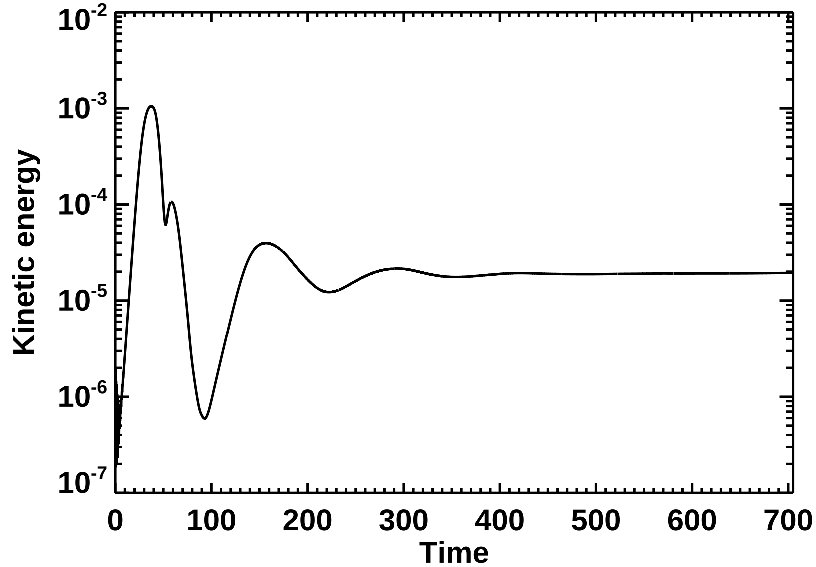
Dimensionless parameters: stably stratified atmosphere ($\gamma = 5/3, \theta = 2, m = 1.6$) with $C_k = 2.5 \times 10^{-2}, \sigma = \zeta = 2 \times 10^{-2}$ and $\beta = 2, 8$

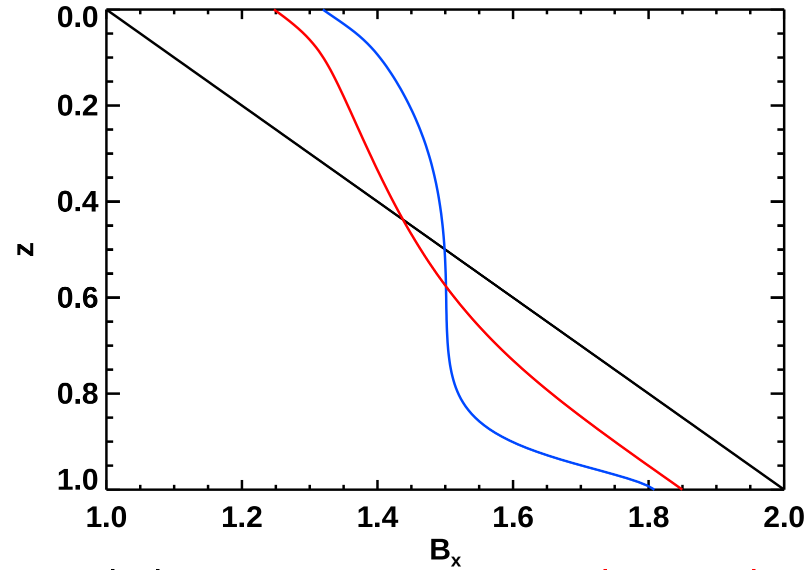
We choose the magnetic field gradient such that only undular modes are unstable ($B_x = 1 + z/H_b$ with $H_b = 1$)



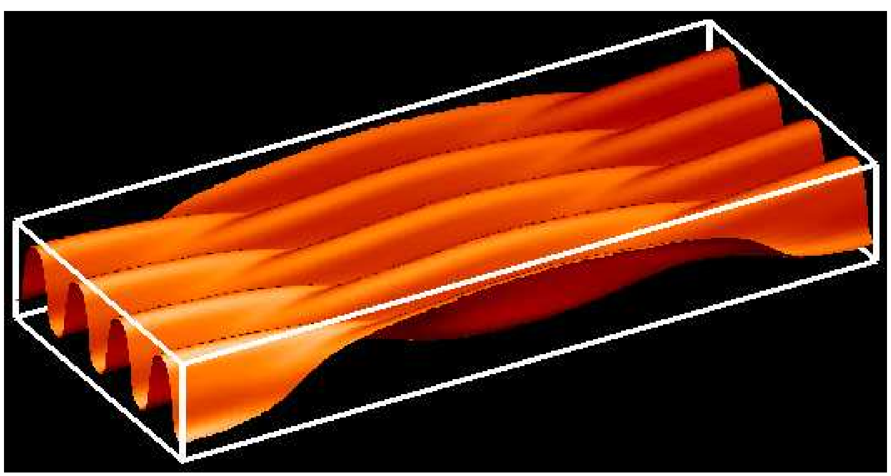

3-D unstable modes grow on large scales along the magnetic field and on smaller scales in the transverse direction

# Nonlinear saturation




Evolution of the 3-D magnetic buoyancy instability to a non-trivial steady state close to marginal ($\beta = 8$ and smooth initial perturbations)

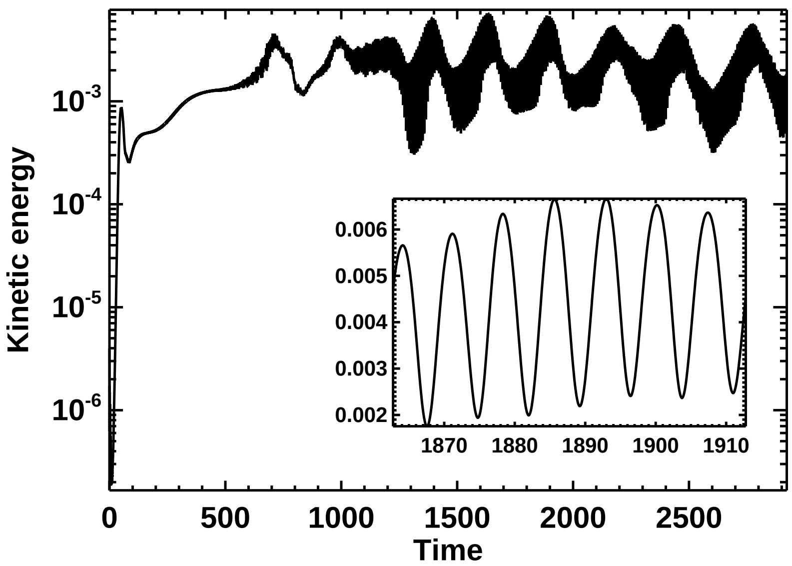


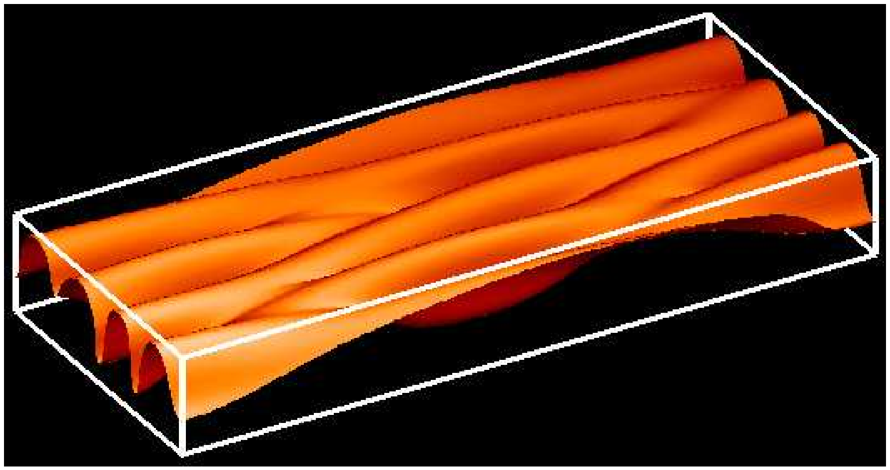
Broad upflows and narrow downflows carry magnetic fields, leading to transient arched structures

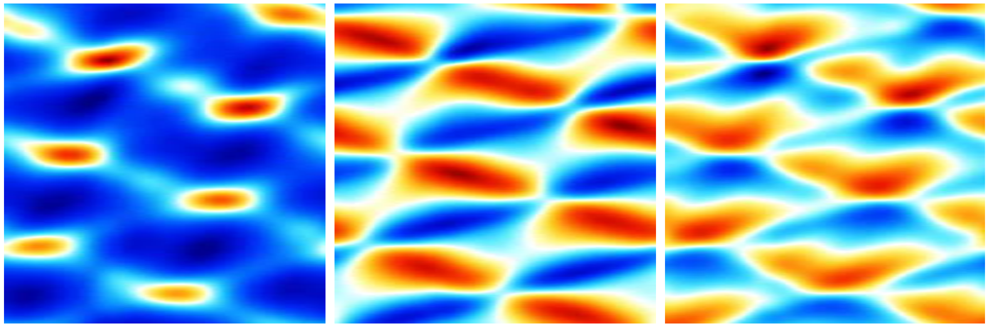
Significant reduction of $\partial_z \langle B_x \rangle_h$ (i.e. suppression of the driving mechanism)

# Coherent concentrations of magnetic energy

We perturb the basic state with small-amplitude random noise

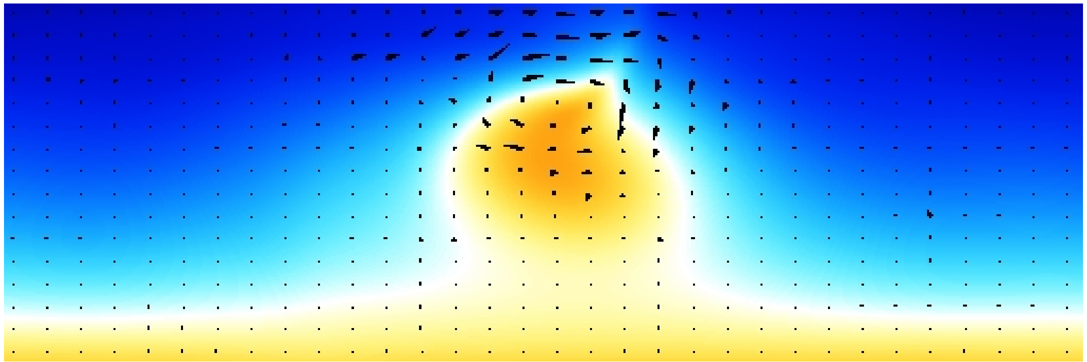




Development of a secondary oscillatory instability leading to the formation of concentrations of magnetic energy taking the form of a modulated travelling wave

# Mechanism for the continual formation of concentration of magnetic energy:



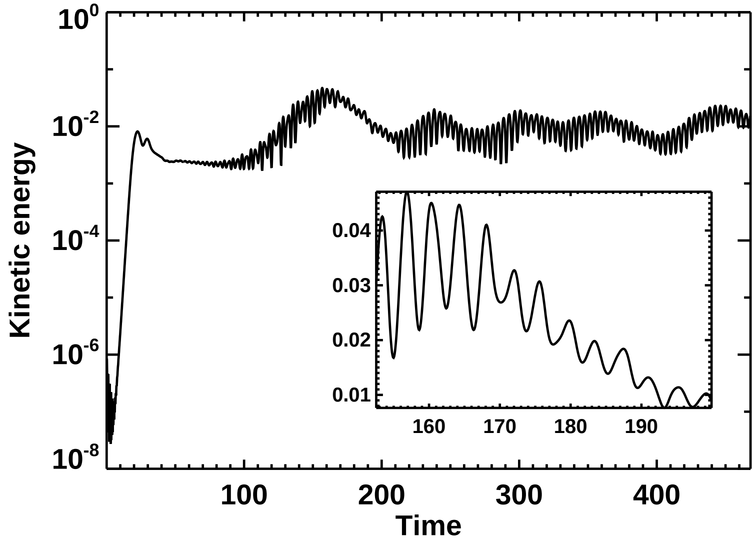
- Concentrations of magnetic energy result from convergent downflows associated with counter-rotating rolls

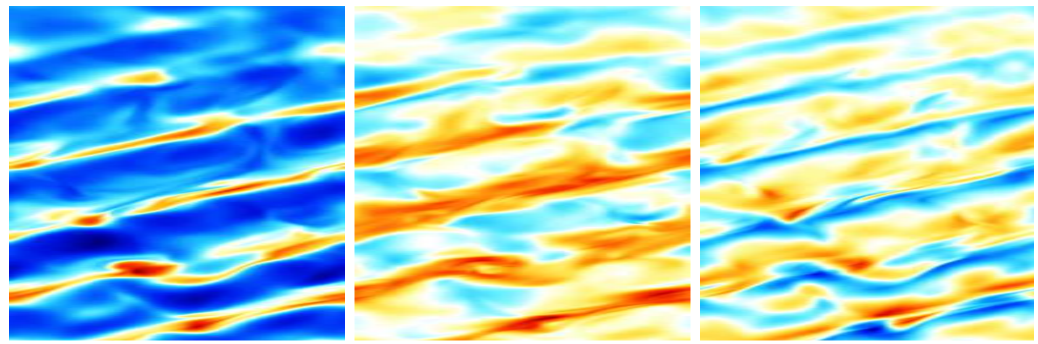
- Magnetic flux becomes buoyant and rises rapidly, driving a counter cell

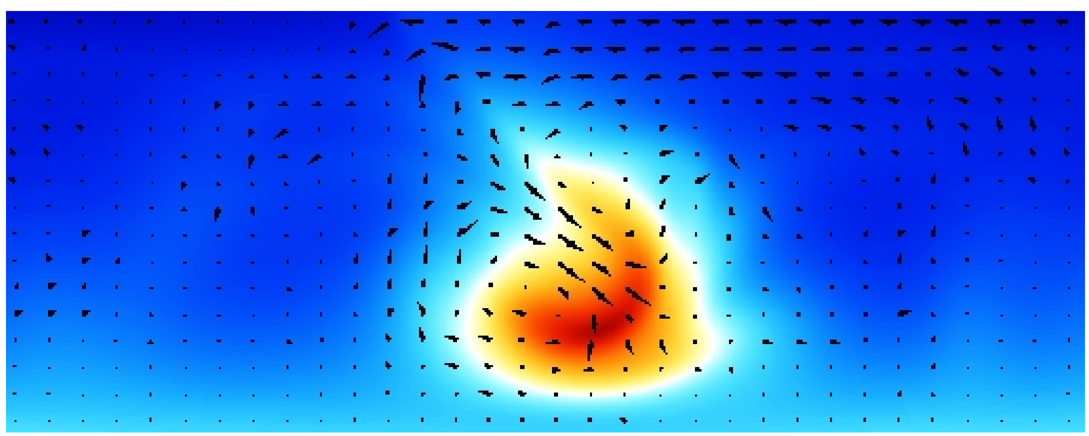
- Flow diverges at the flux concentration leading to its disruption

- Cellular flow re-establishes itself and the entire process is repeated

For increased initial magnetic field strength $(\beta = 2)$ the instability is more vigorous and presents shorter characteristic timescales





# Long-term projects

Basic principles of magnetic buoyancy instabilities well understood but theory still to be developed for the solar interior

Bulk of the toroidal magnetic field stored in the tachocline: transport properties and topology of the field determined by the interplay between:

- magnetic buoyancy instabilities
- shear flows
- shear instabilities

More realistic description in the context of solar dynamo if we let be $B$ evolving in response to some forcing:
$\Rightarrow$ Study the stability of equilibria involving shear flows or boundary motions (tachocline) in the presence of a vertical field (pumping)