Viscous, Resistive MRI Modes (and their stability)

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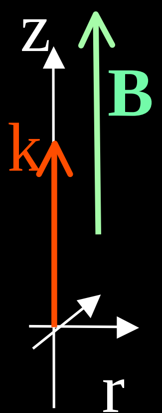
MRI and MHD Turbulence

Differential Rotation

+

Weak Magnetic Field

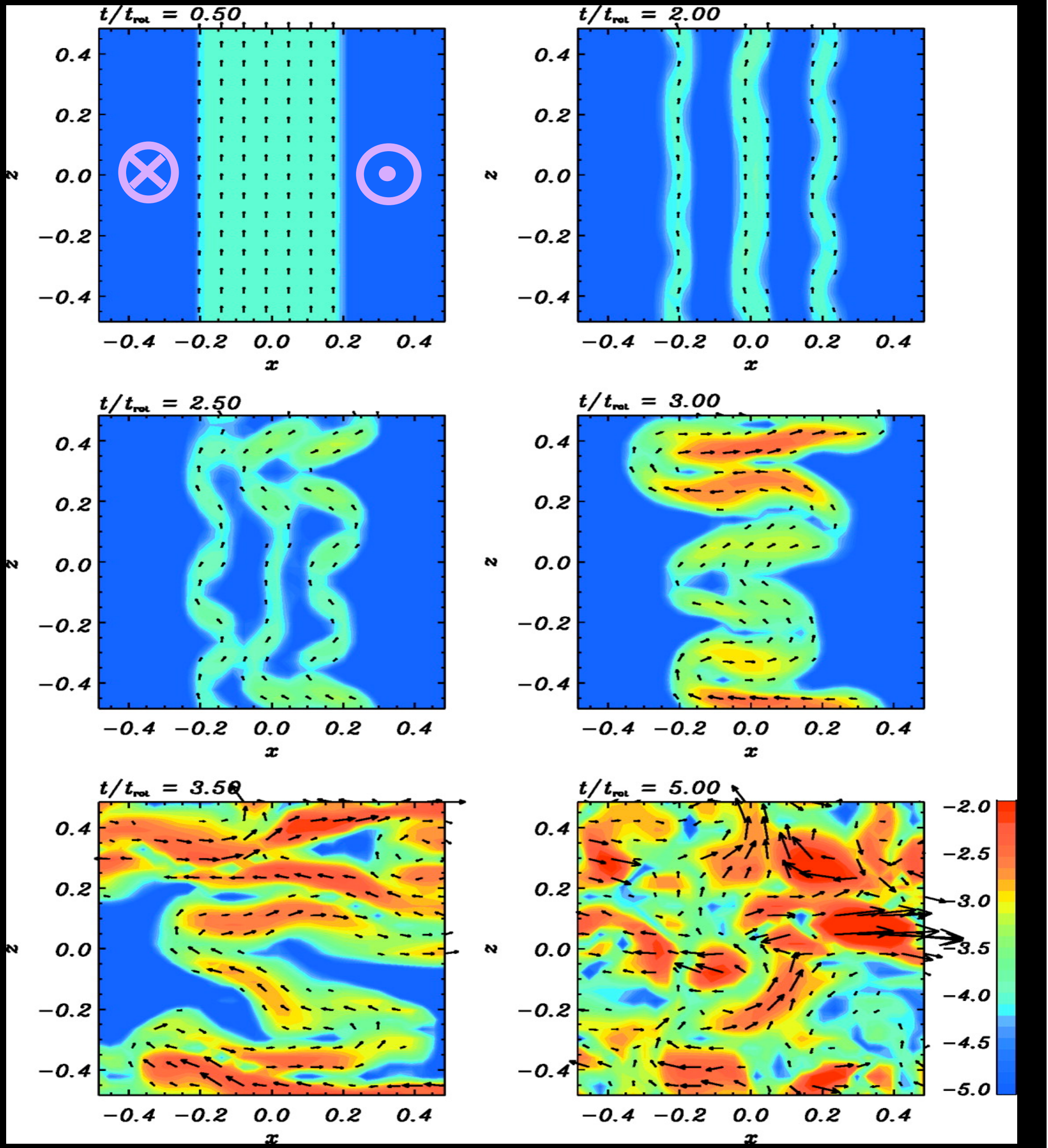
Magnetorotational Instability



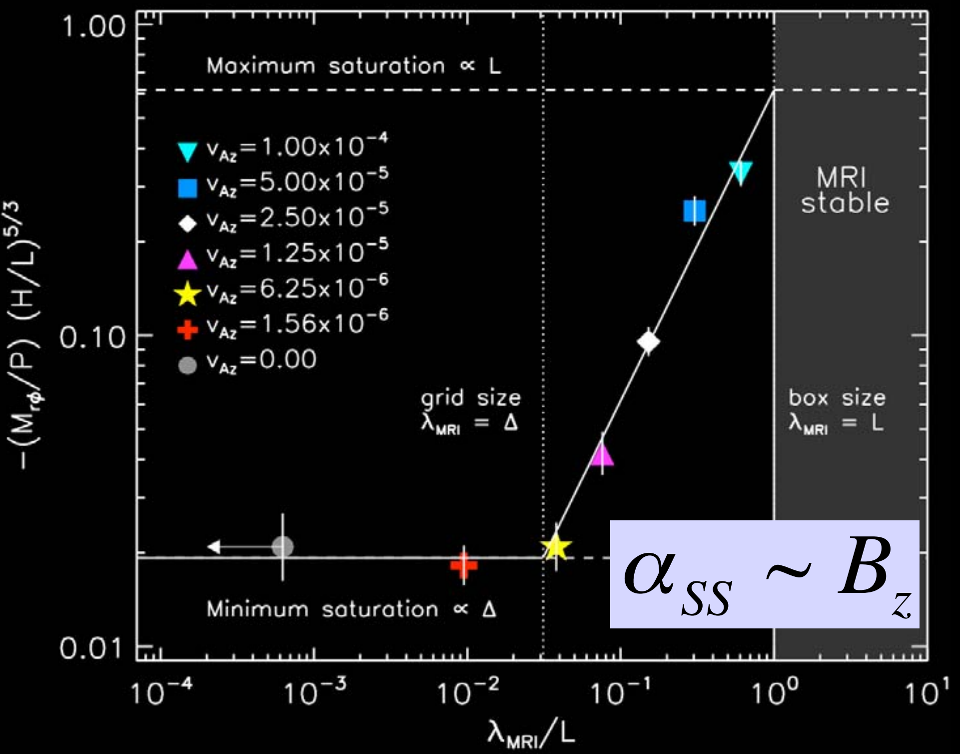
$$k^2 v_{Az}^2 < -2\Omega^2 \frac{d\ln\Omega}{d\ln r}$$

Stability criterion involving shear and characteristic scale

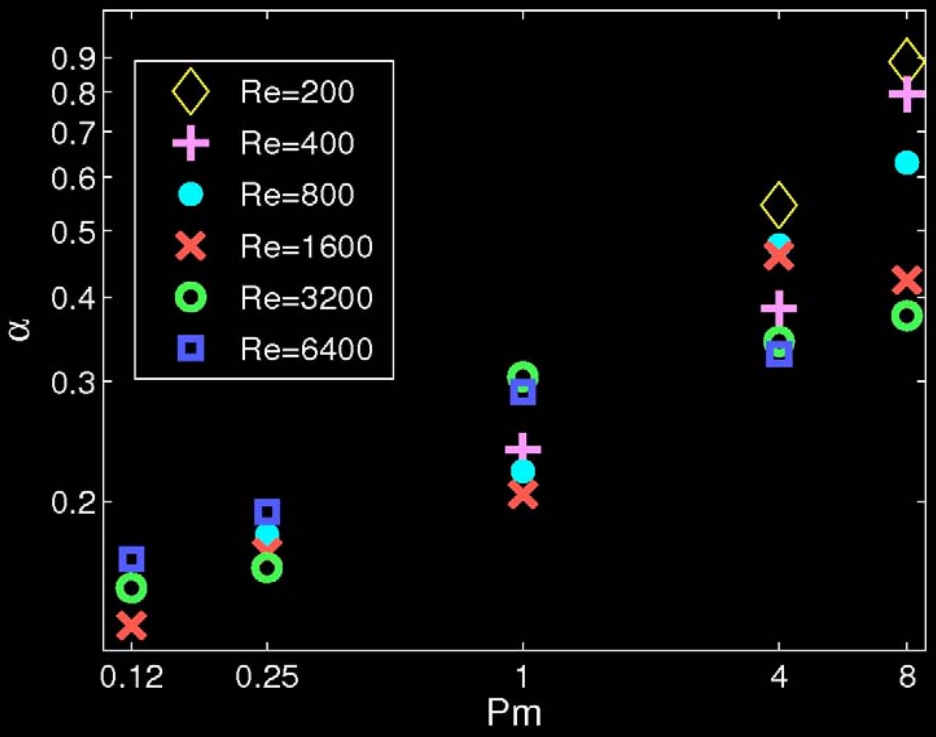
Sano et al. 2004



Outstanding Question: MRI Saturation



$$\alpha_{SS} \sim B_z$$



Pessah, Chan, & Psaltis, 2007; Sano et al. 2004

Lesur & Longaretti, 2007

What sets the saturation level of the MRI?

(Goodman & Xu ‘94, Knobloch & Julien, Umurhan, Regev, & Menou, …
Parasitic instabilities; Asymptotic analysis; Weakly non-linear analysis, …)

Local -shearing box- MHD Equations

$$\begin{aligned}\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} &= -2\boldsymbol{\Omega}_0\times\mathbf{v} + q\Omega_0^2\nabla(r-r_0)^2 \\ &- \frac{1}{\rho}\nabla\left(P+\frac{\mathbf{B^2}}{8\pi}\right) + \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{v}\end{aligned}$$

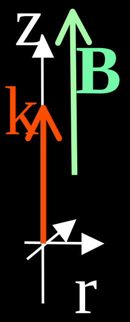
$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{B} = (\mathbf{B}\cdot\nabla)\,\mathbf{v} + \eta\nabla^{\mathbf{2}}\mathbf{B}$$

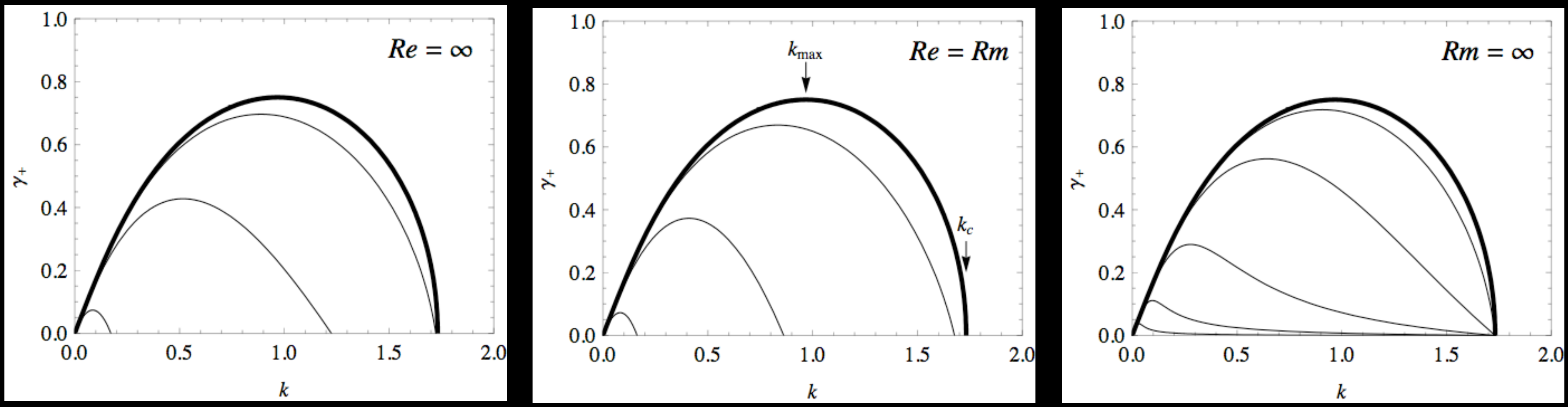
$$\mathrm{Re} \equiv \frac{v_{\mathrm{A}z}^2}{\nu\Omega_0} \equiv \frac{1}{\nu} \qquad \mathrm{Rm} \equiv \frac{v_{\mathrm{A}z}^2}{\eta\Omega_0} \equiv \frac{1}{\eta} \qquad \mathrm{Pm} \equiv \frac{\nu}{\eta}$$

Growth Rates with Dissipation

$$(k^2+\sigma_\nu\sigma_\eta)^2+\kappa^2(k^2+\sigma_\eta^2)-4k^2=0$$

$$\sigma_\nu=\sigma+\nu k^2 \quad \sigma_\eta=\sigma+\eta k^2$$

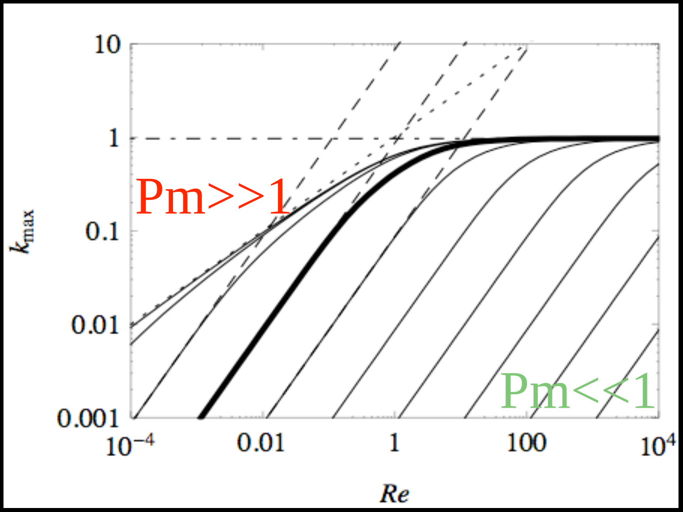


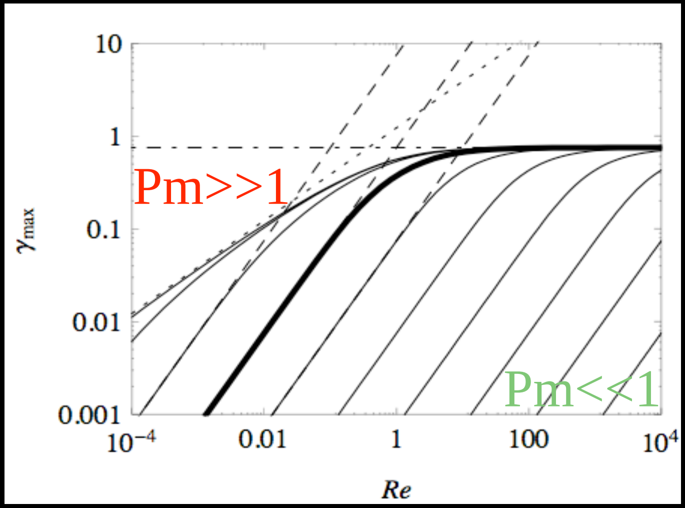


$$\gamma_{\max}(\nu,\eta);\ k_{\max}(\nu,\eta);\ k_{\mathrm{c}}(\nu,\eta)$$

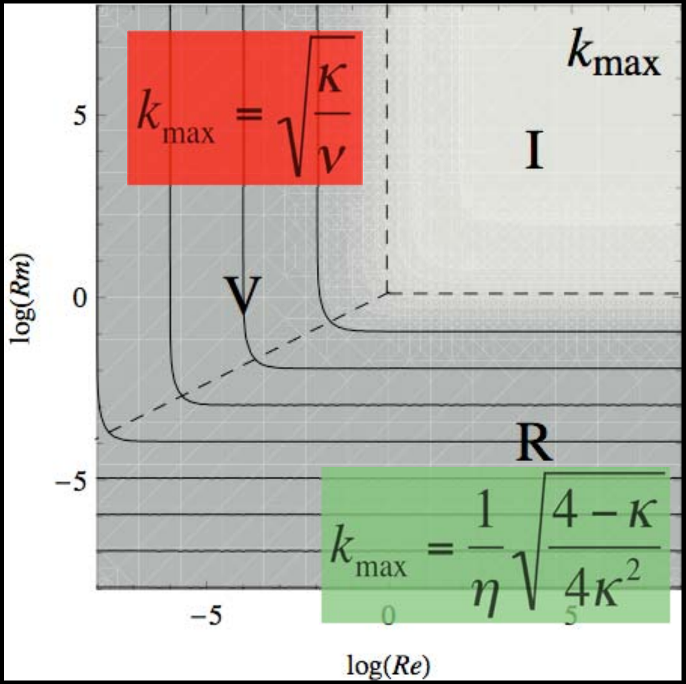
(Various limits studied by Sano et al., Lesaffre & Balbus, Lesur & Longaretti, and many others)

Viscous, Resistive MRI



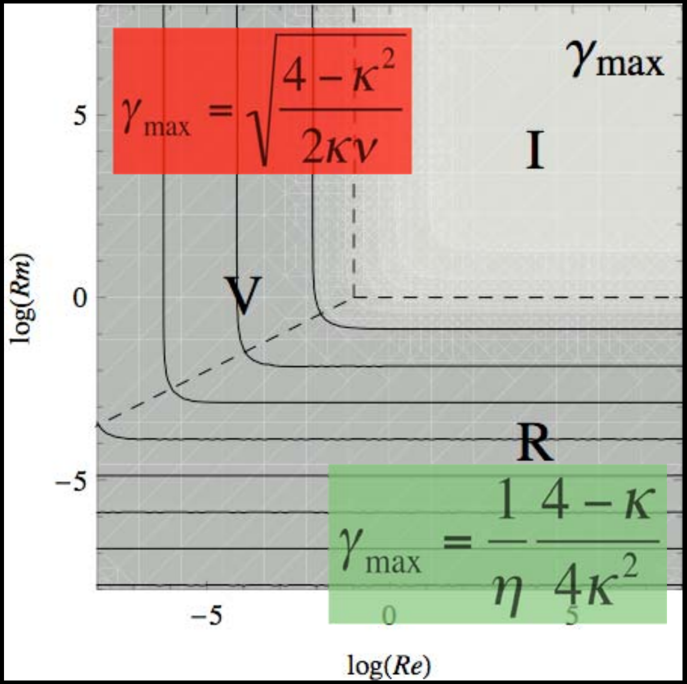


Pessah & Chan, 2008



$$k_{max} = \sqrt{\frac{\kappa}{\nu}}$$

$$k_{max} = \frac{1}{\eta}\sqrt{\frac{4-\kappa}{4\kappa^2}}$$

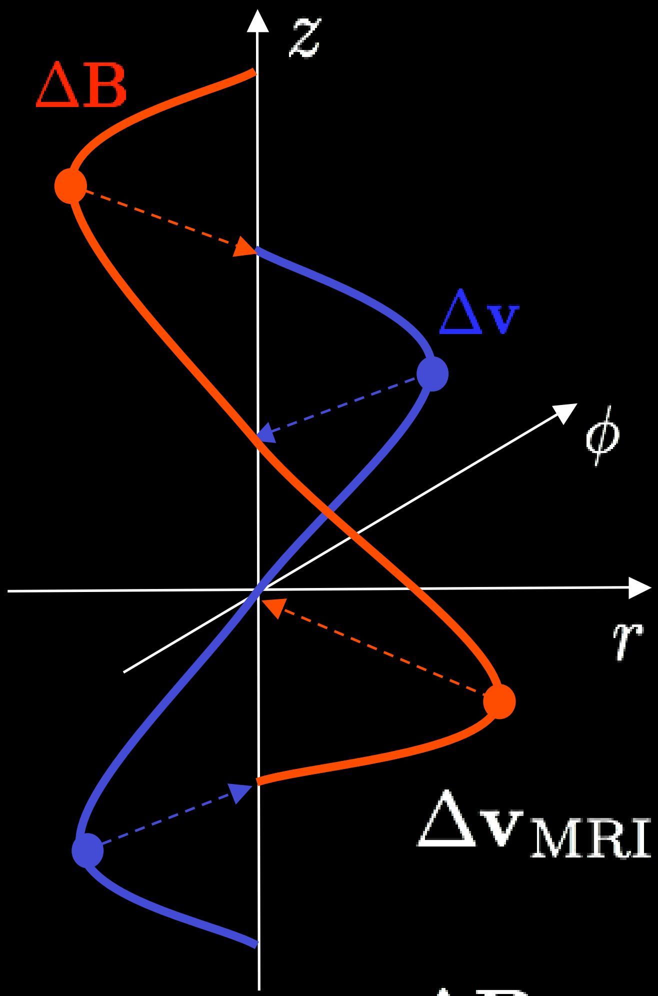


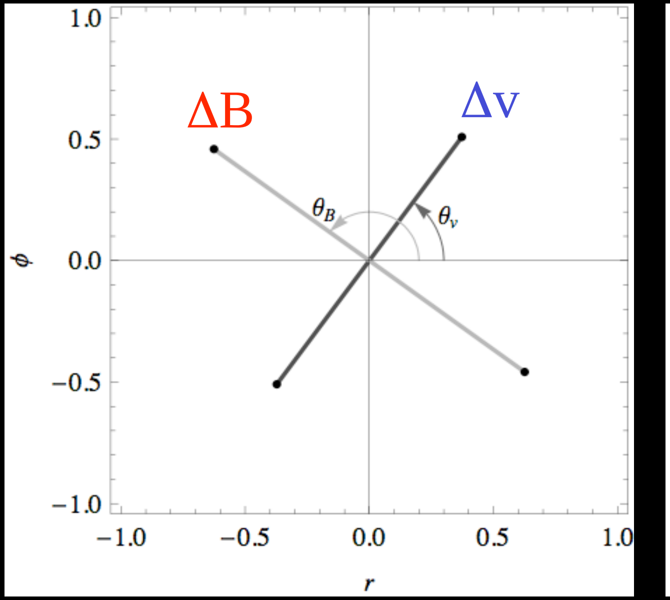
$$\gamma_{max} = \sqrt{\frac{4-\kappa^2}{2\kappa\nu}}$$

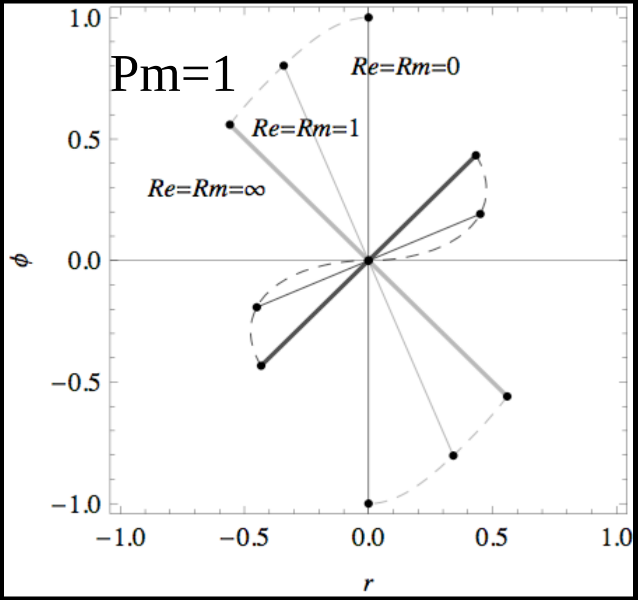
$$\gamma_{max} = \frac{1}{\eta}\frac{4-\kappa}{4\kappa^2}$$

Viscous, Resistive MRI Modes

How do the MRI modes look like in physical space?





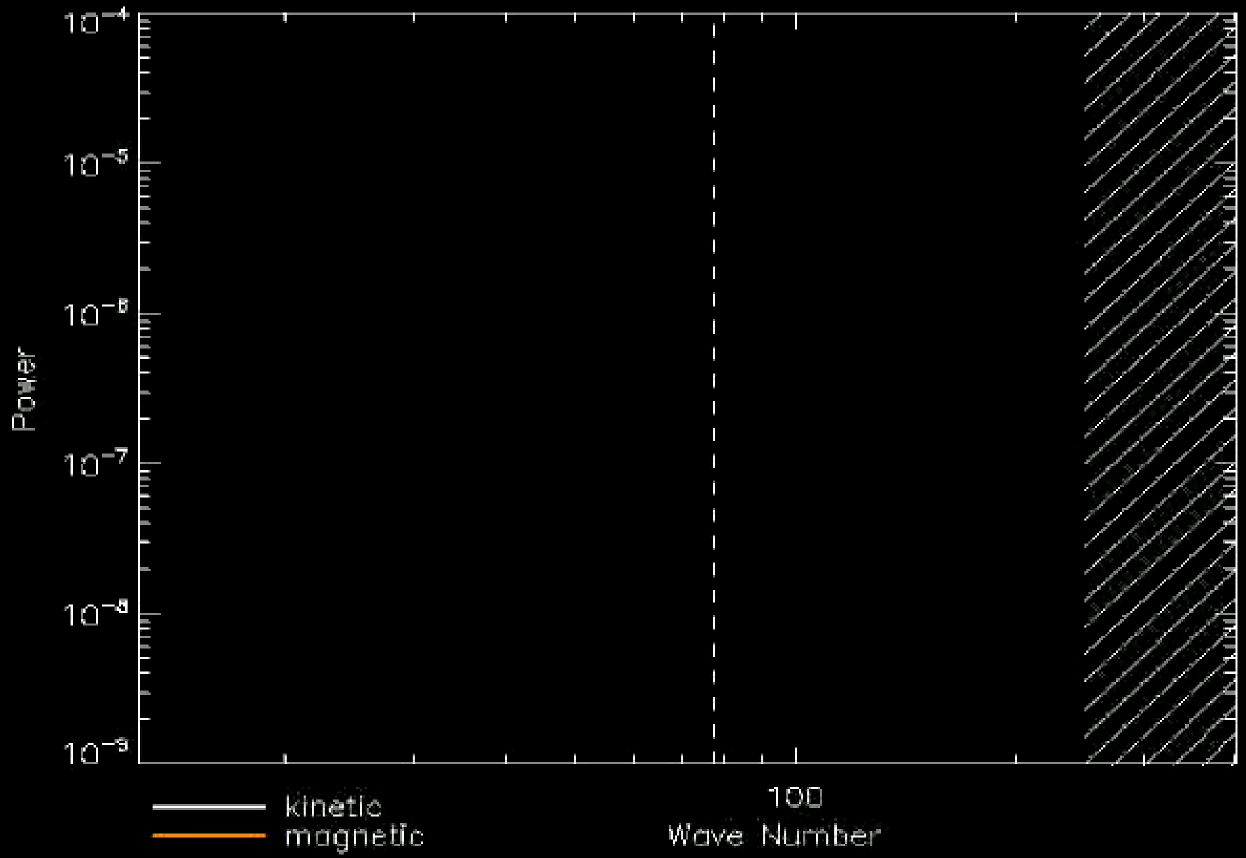


$$\Delta \mathbf{v}_{\mathrm{MRI}}(z,t) = e^{\gamma t} v_0 \sin(kz) \begin{bmatrix} \cos\theta_{\mathrm{v}}(k,\nu,\eta) \\ \sin\theta_{\mathrm{v}}(k,\nu,\eta) \end{bmatrix}$$

$$\Delta \mathbf{B}_{\mathrm{MRI}}(z,t) = e^{\gamma t} b_0 \cos(kz) \begin{bmatrix} \cos\theta_{\mathrm{b}}(k,\nu,\eta) \\ \sin\theta_{\mathrm{b}}(k,\nu,\eta) \end{bmatrix}$$

Long-Term Evolution of MRI

large
scales



small
scales

MRI modes are exact solutions of shearing-box equations

What halts the exponential growth?

Parasitic Instabilities

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla\left(\mathbf{P} + \frac{\mathbf{B}^2}{8\pi}\right) + \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{B} = (\mathbf{B}\cdot\nabla)\,\mathbf{v} + \eta\nabla^2\mathbf{B}$$

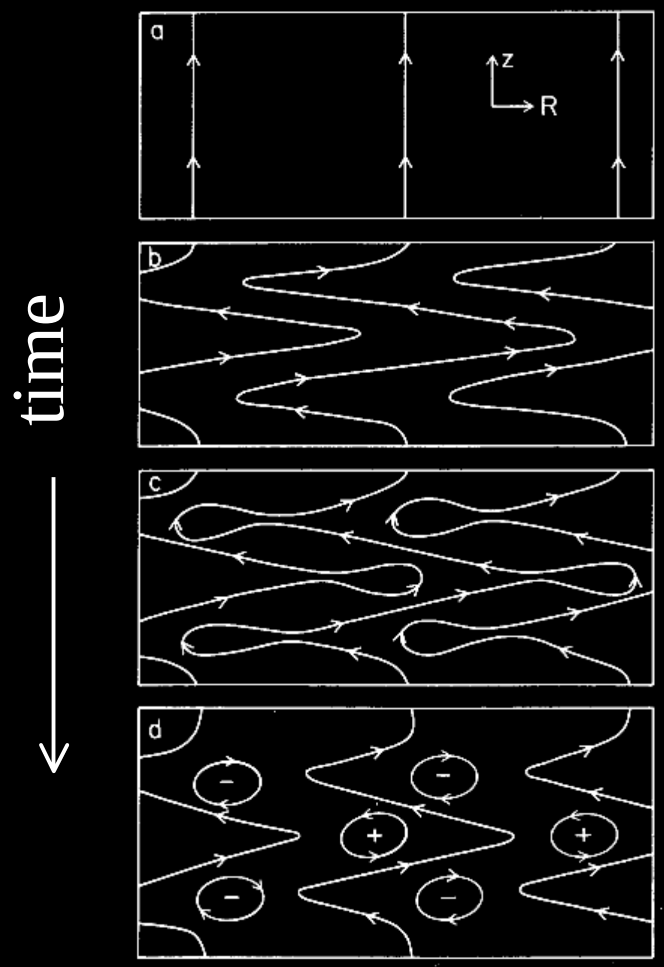
$$\begin{aligned} \mathbf{v}(\mathbf{x},t) &= \Delta\mathbf{v}_{\mathrm{MRI}}(z) + \delta\mathbf{v}(x,y,z,t) \\ \mathbf{B}(\mathbf{x},t) &= \Delta\mathbf{B}_{\mathrm{MRI}}(z) + \delta\mathbf{B}(x,y,z,t) \end{aligned}$$

Seek Bloch-type eigenfunctions (like for a particle in a periodic potential)

$$\delta\mathbf{v}(x,y,z,t) = \delta\mathbf{v}_0(z)\ \exp\left[st - i(k_x x + k_y y)\right]$$

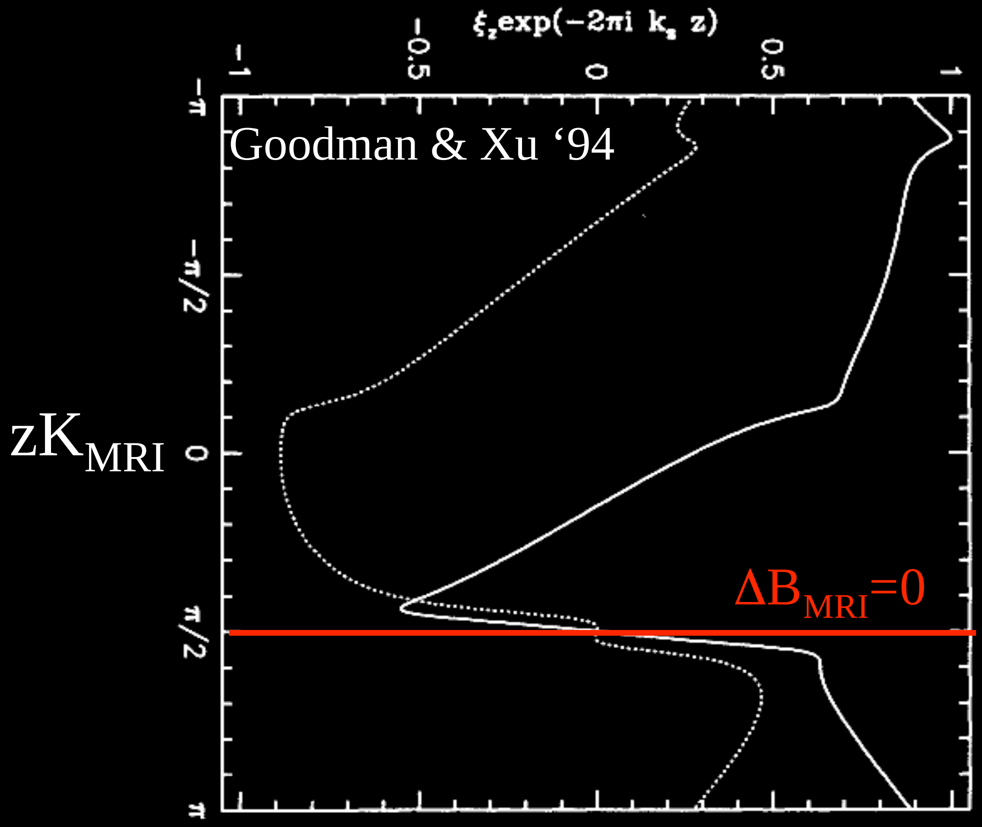
$$\delta\mathbf{v}_0(z + 2\pi/K_{\mathrm{MRI}}) = \delta\mathbf{v}_0(z)\exp(i2\pi k_z/K_{\mathrm{MRI}})$$

Ideal MHD Parasitic Instabilities



Balbus & Hawley ‘92

Cartoon evolution of a primary MRI mode

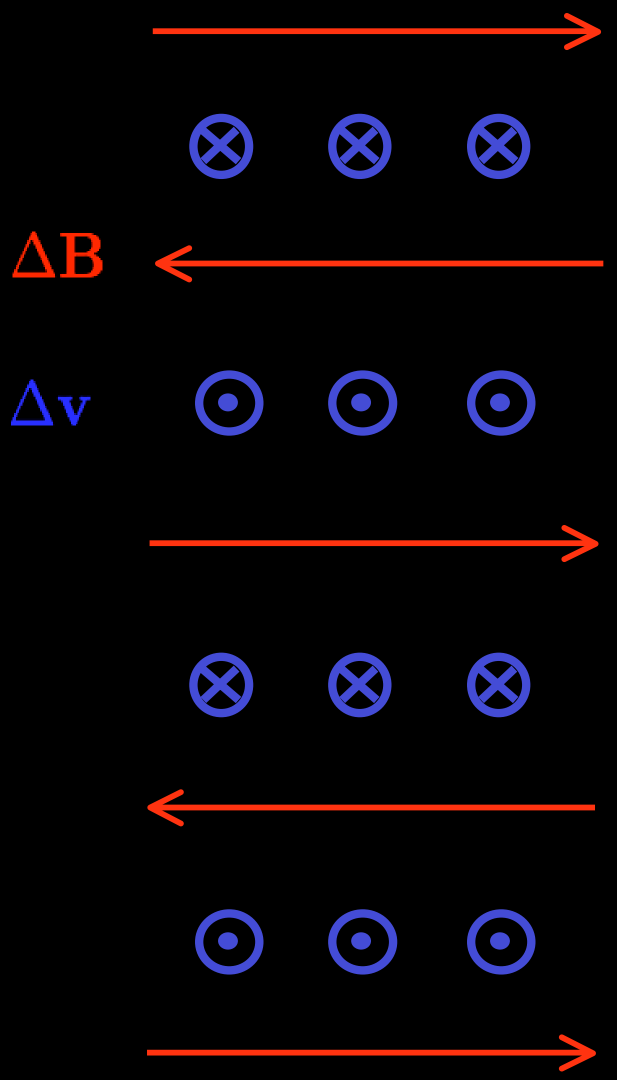


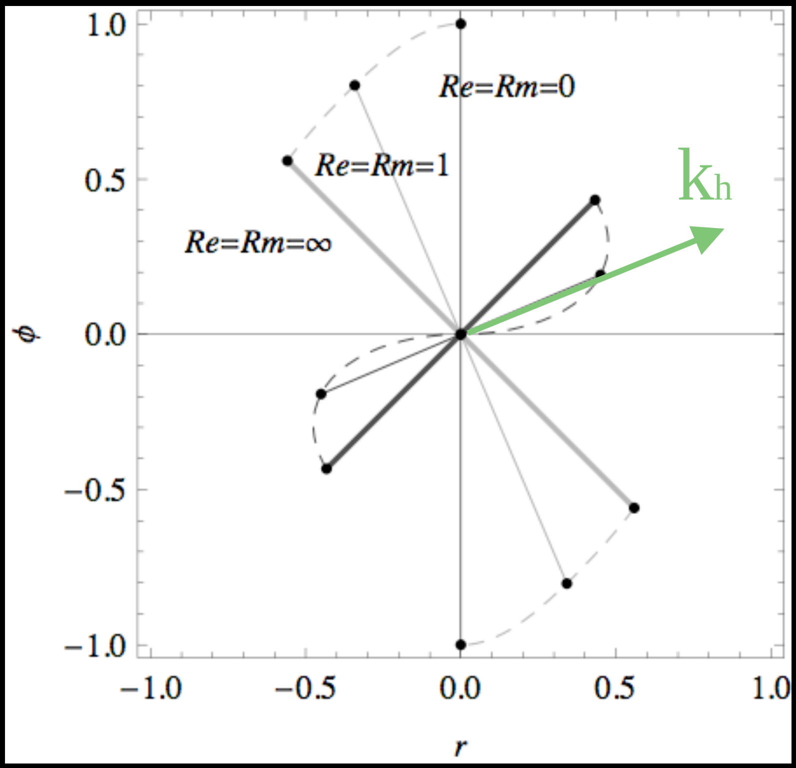
Vertical Lagrangian displacement

Some parasitic instabilities tend to promote reconnection of the MRI field

Viscous, Resistive MRI Modes

Ideal MHD

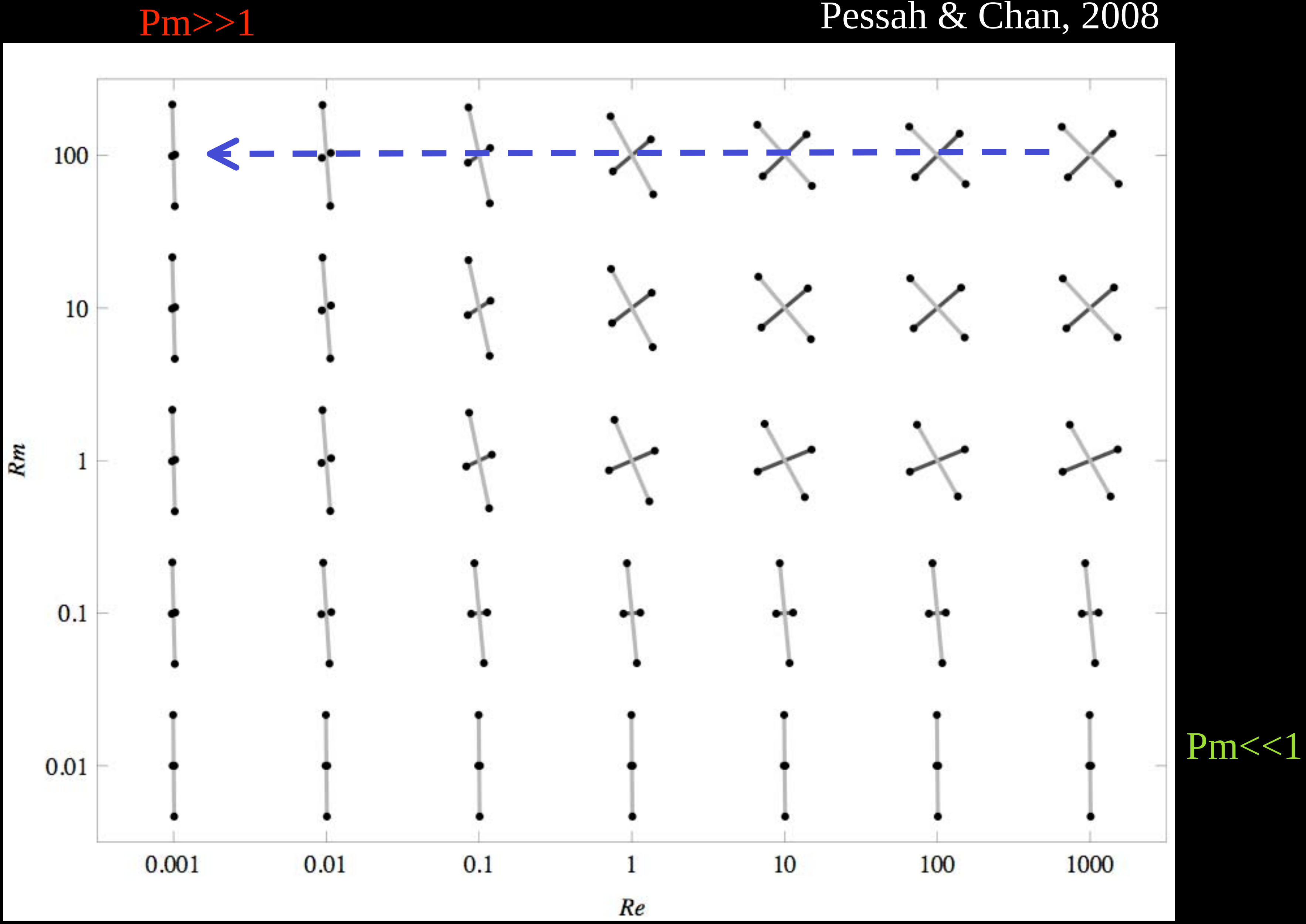




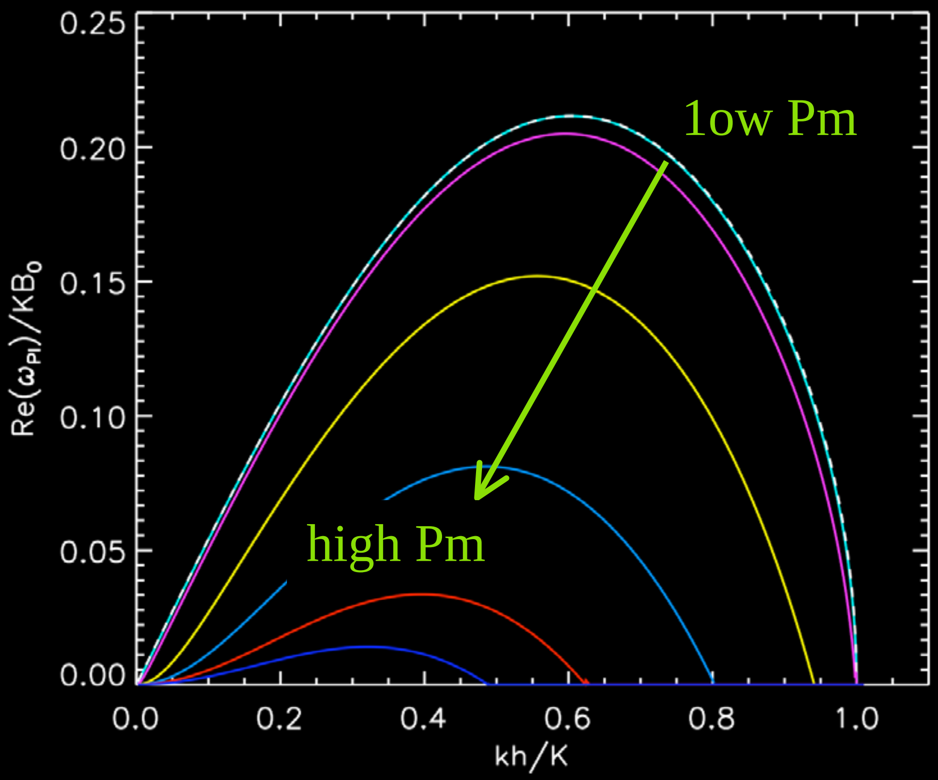
Parasitic Instabilities sensitive to viscosity and resistivity

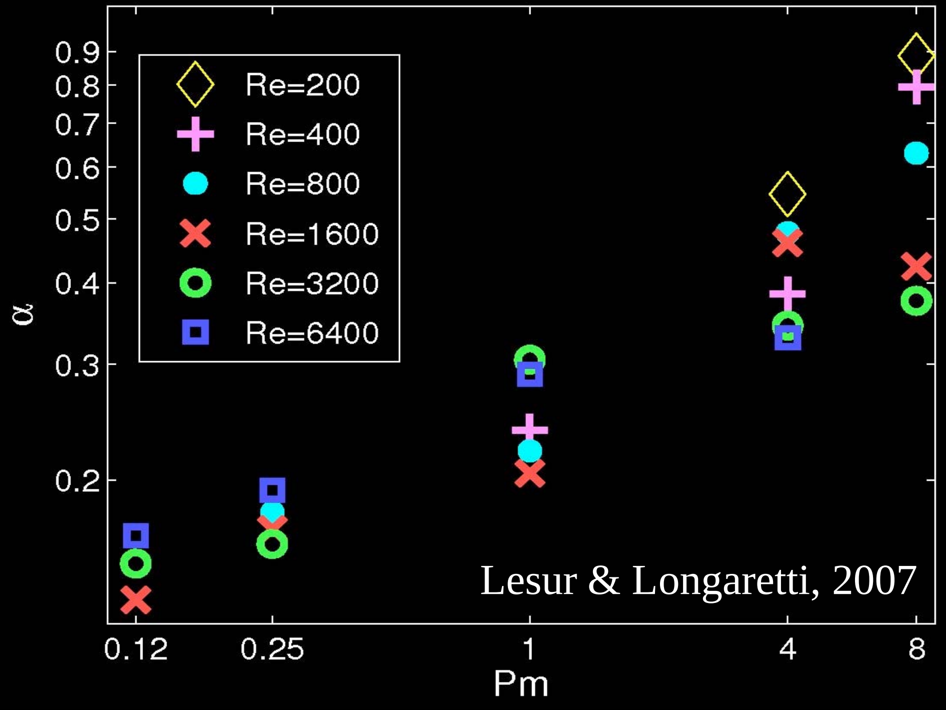
Viscous, Resistive MRI Modes

Pessah & Chan, 2008



Viscous, Resistive Parasitic Instabilities





Lesur & Longaretti, 2007

Parasitic Instabilities seem to be weaker at high Pm

Hint for difficulties to drive MRI at low Pm?

Can the destruction of primary MRI modes by parasitic instabilities explain (Re, Pm) dependencies?