

Quantum Glassiness

Claudio Chamon



U.S. DEPARTMENT OF
ENERGY

Office of
Science



Preliminaries

Fractons

see Nandkishore & Hermele review

Vijay, Haah, Fu

see Pretko — symmetric tensor gauge theories

Fractons and quantum (and classical) glassiness

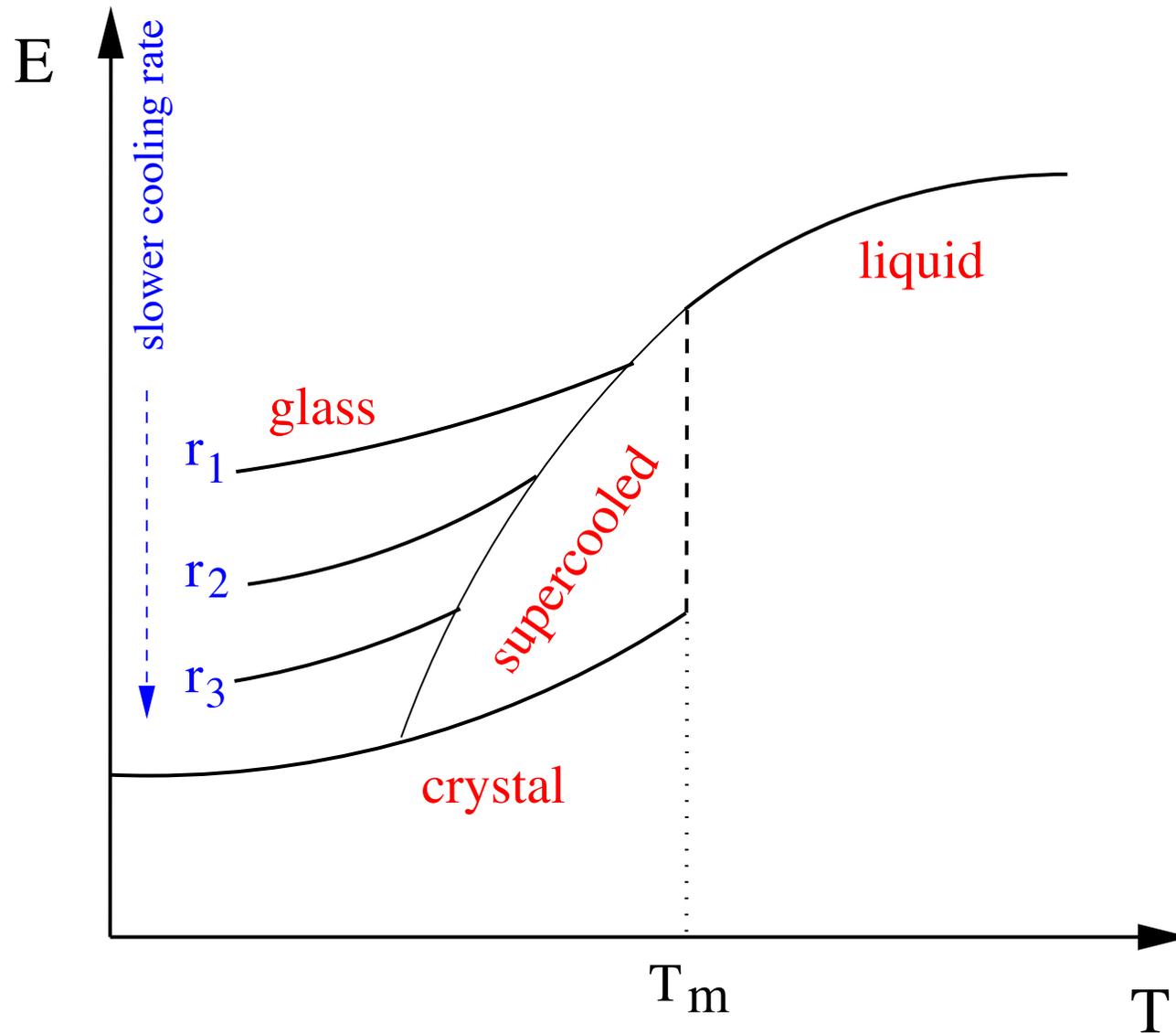
w/ Claudio Castelnovo

w/ Claudio + David Sherrington (X-cube model from gonihedric model)

Restricted mobility excitations and ultra-slow systems w/o finite T thermodynamic transitions

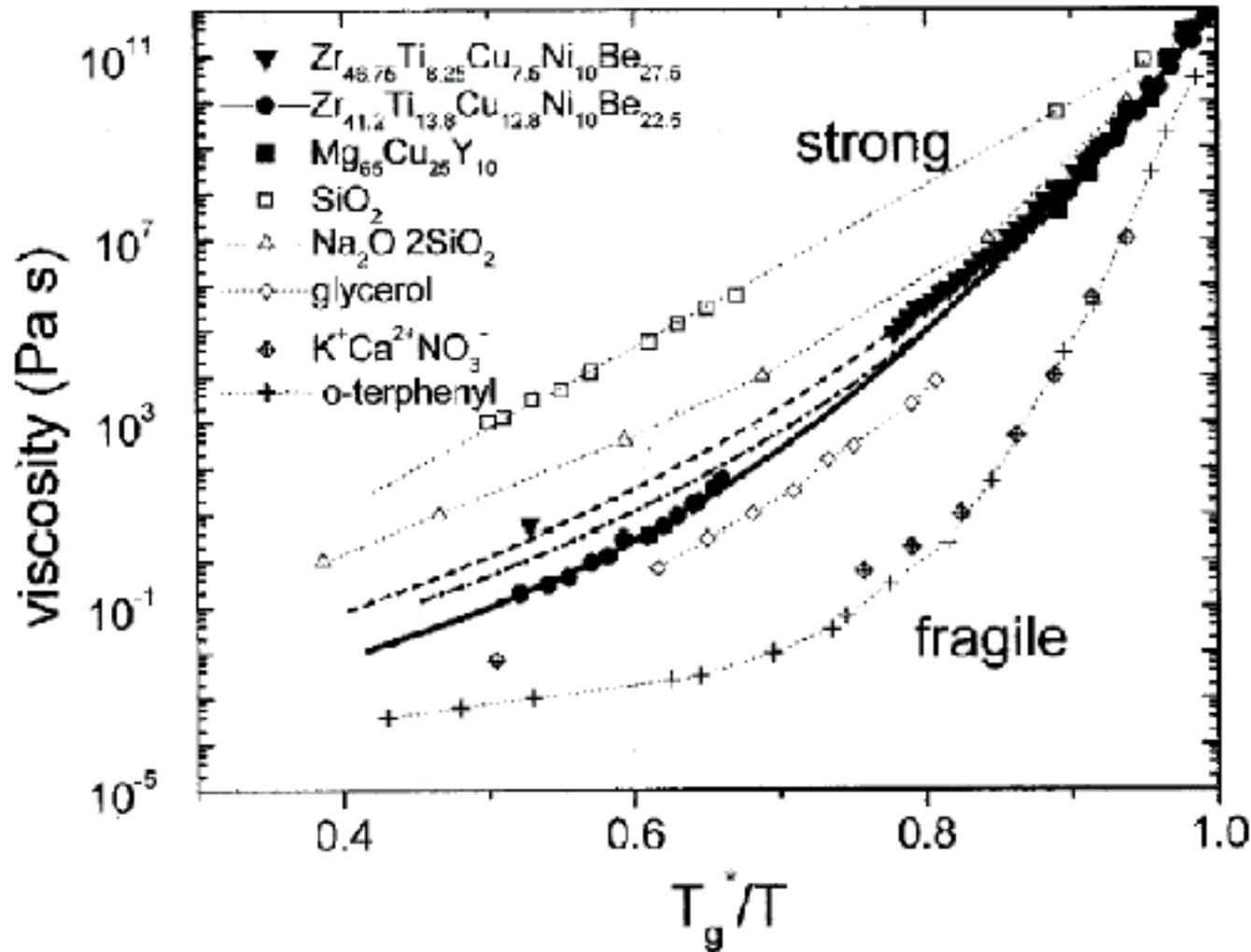
w/ Lei Zhang, Stefanos Kourtis, Eduardo Mucciolo, and Andrei Ruckenstein

Classical glassiness

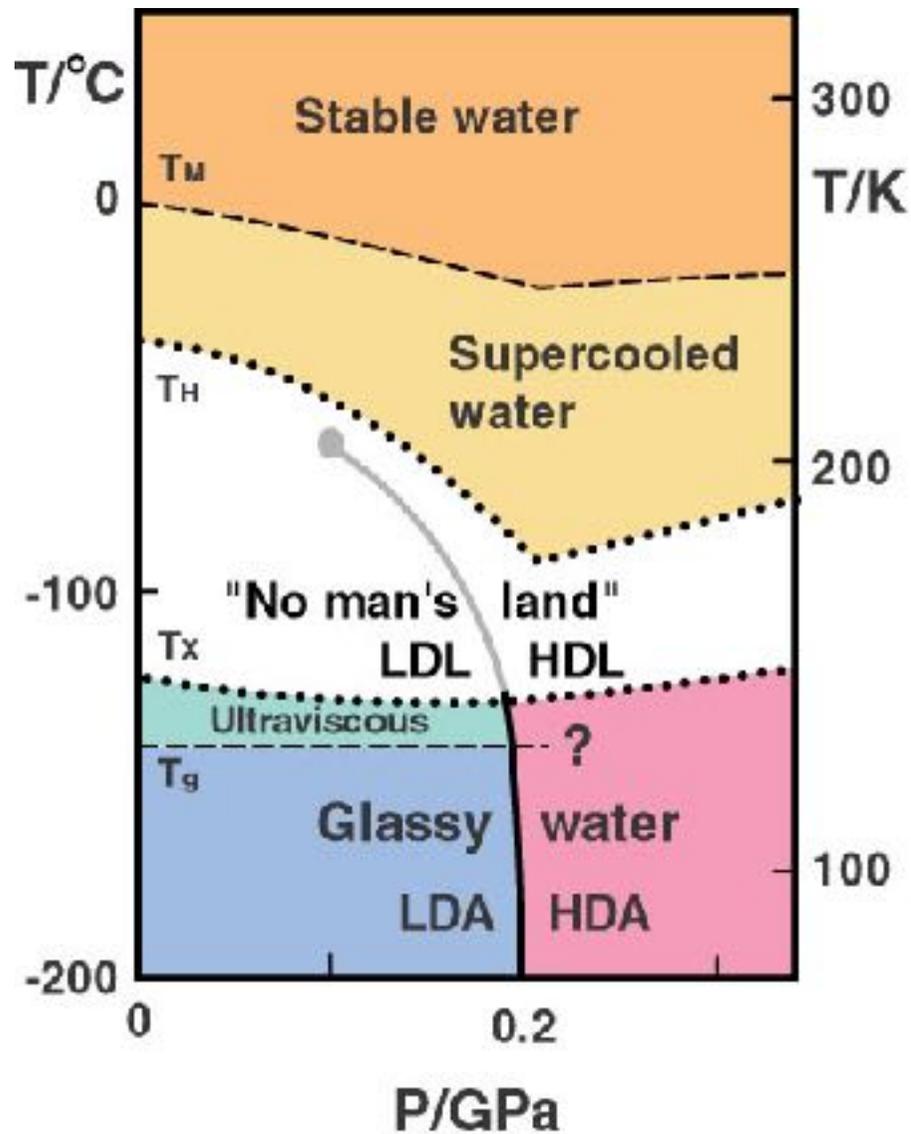


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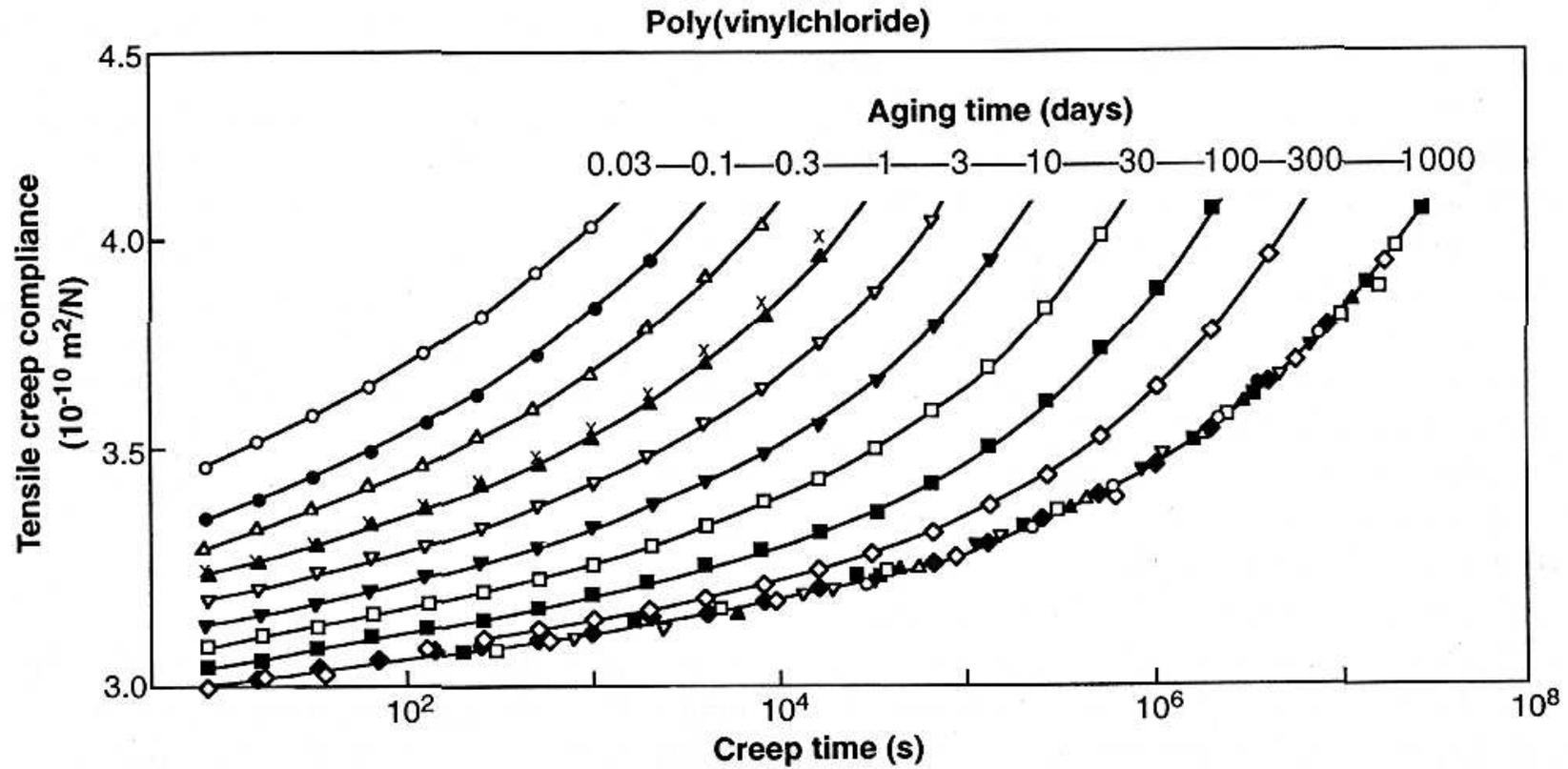
viscosity



Glassy H₂O



Physical aging



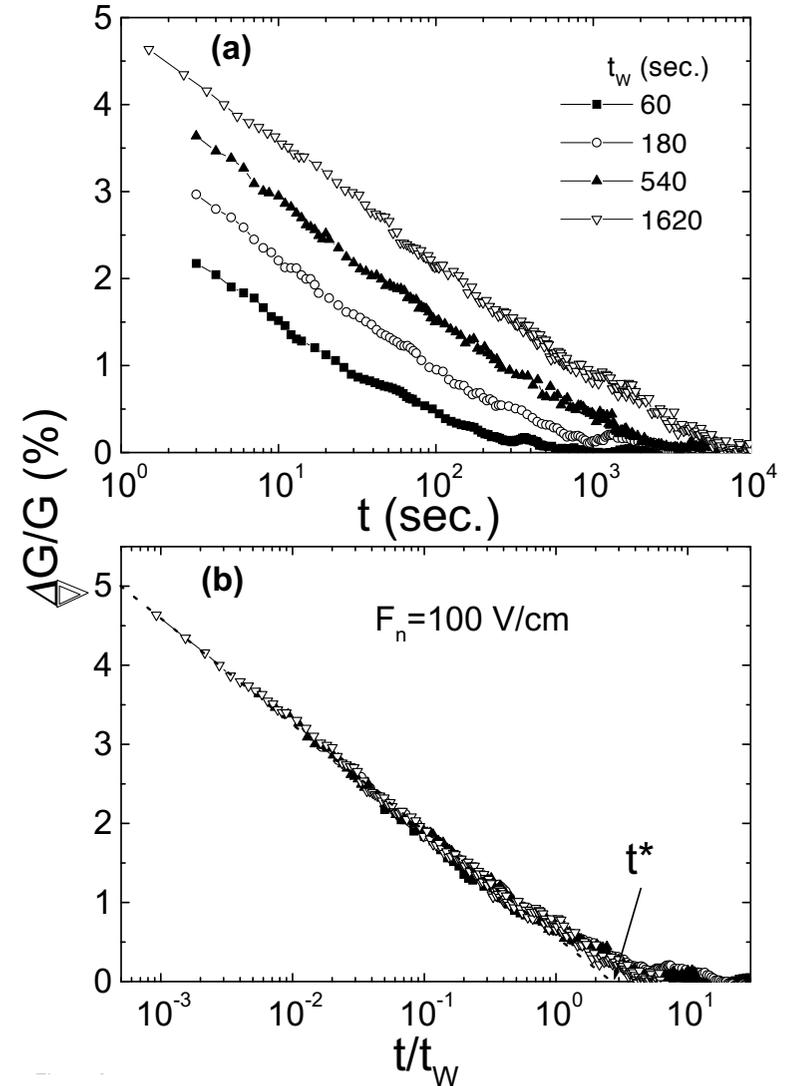
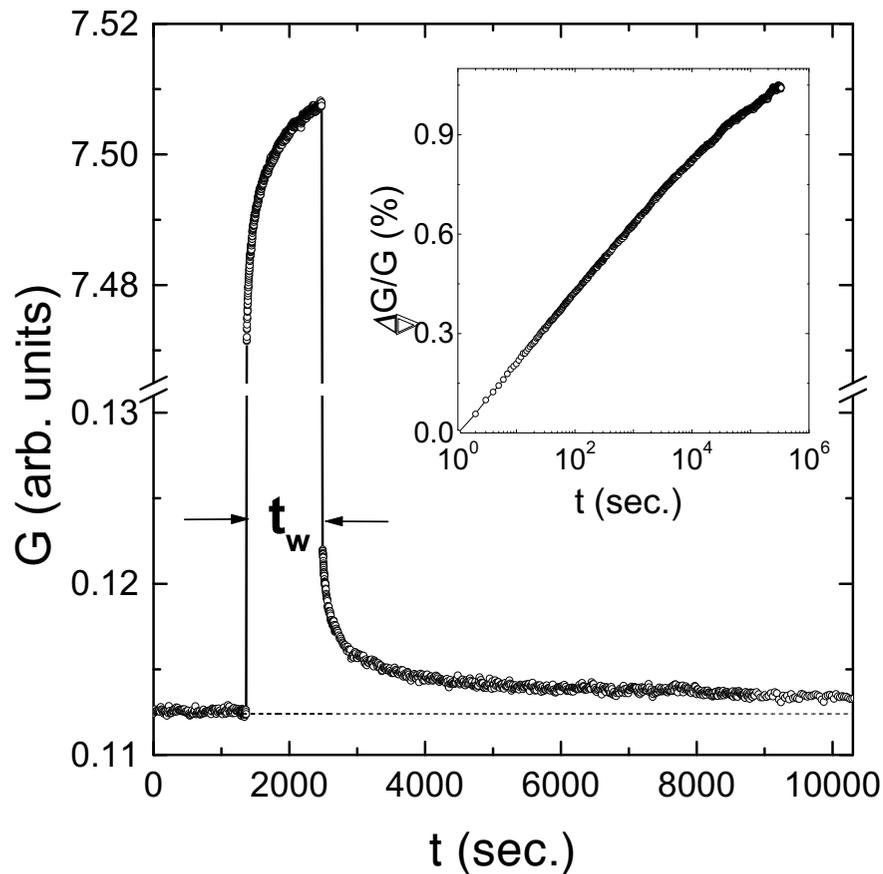
Source: L.C.E. Struik, *Physical aging in amorphous polymers and other materials*, Elsevier, Amsterdam (1978)

Physical aging II

2D electron glass

Orlyanchik & Ovadyahu, PRL (2004)

2D thin films of crystalline $\text{In}_2\text{O}_{3-x}$



Source: Orlyanchik & Ovadyahu, PRL (2004)

Quantum glassy systems

disordered systems

eg. quantum spin glasses

Bray & Moore, J. Phys. C (1980)

Sachdev & Ye, PRL (1993)

Read, Sachdev, and Ye, PRB (1995)

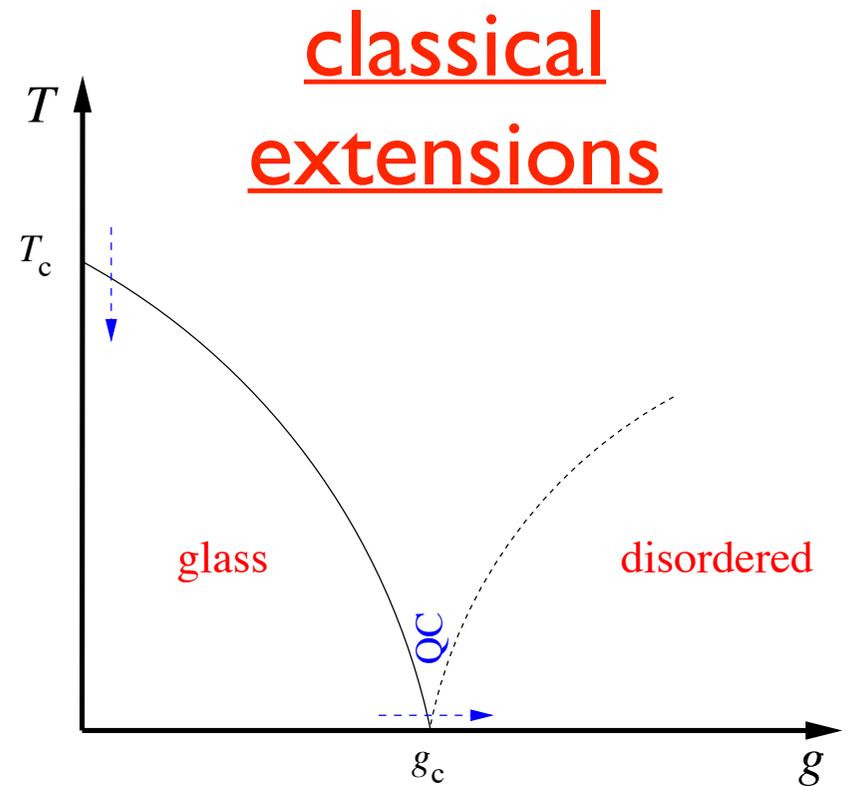
frustrated systems

eg. I frustrated Josephson junctions
with long-range interactions

Kagan, Feigel'man, and Ioffe, ZETF/JETP (1999)

eg. II self-generated mean-field glasses

Westfahl, Schmalian, and Wolynes, PRB (2003)



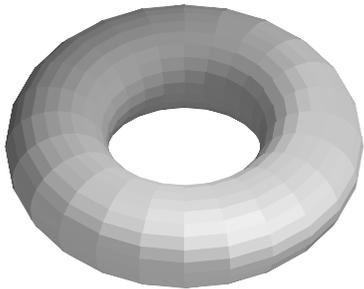
Topological quantum glasses

strongly correlated systems with
topological order

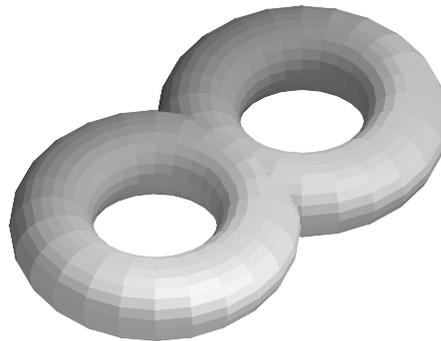
Ground state degeneracy
eg. fractional quantum Hall effect

Wen, Int. J. Mod. Phys. B (1991), Adv. Phys. (1995)

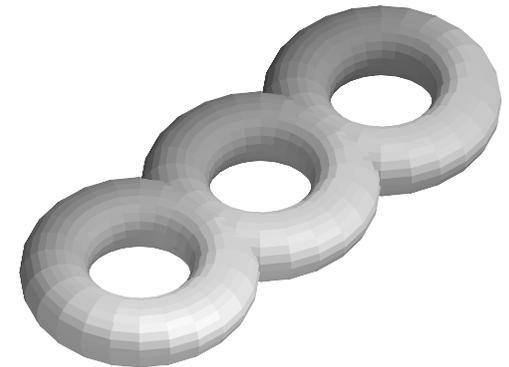
$$\nu = 1/3 \Rightarrow N_{\text{GS}} = 3^g$$



$$N_{\text{GS}} = 3^1$$



$$N_{\text{GS}} = 3^2$$



$$N_{\text{GS}} = 3^3$$

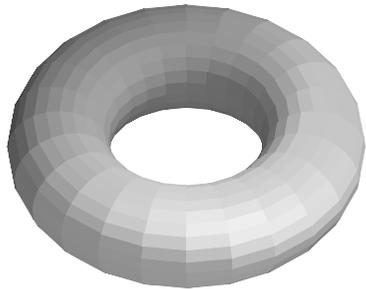
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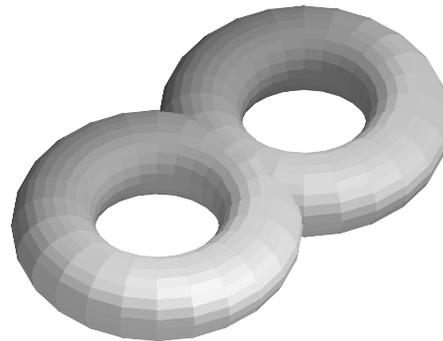
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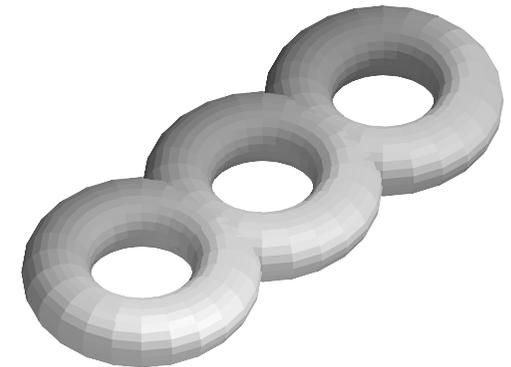
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Interestingly enough, strong correlations that can lead to these exotic quantum spectral properties can in some instances also impose kinetic constraints, similar to those studied in the context of classical glass formers.

Why solvable examples are important?

The dynamics of classical glasses can be efficiently simulated in a computer; but real time simulation of a quantum system is hard!

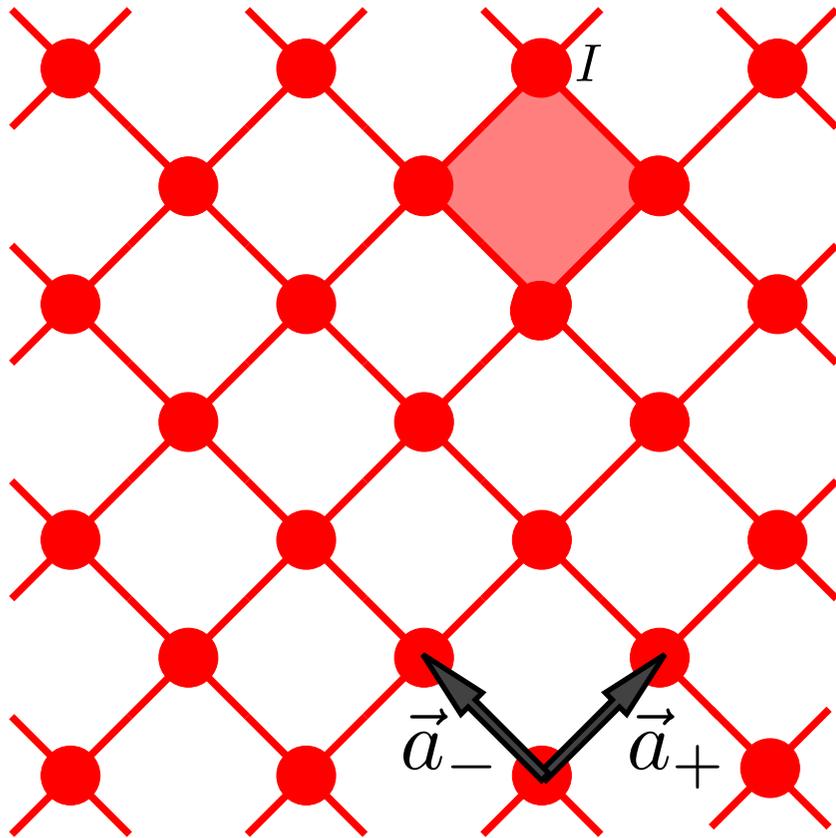
Even a quantum computer does not help; quantum computers are good for unitary evolution. One needs a “quantum supercomputer”, with many qubits dedicated to simulate the bath.

Solvable toy model can show unambiguously and without arbitrary or questionable approximations that there are quantum many-body systems without disorder and with only local interactions that are incapable (in accessible times) of reaching their quantum ground states.

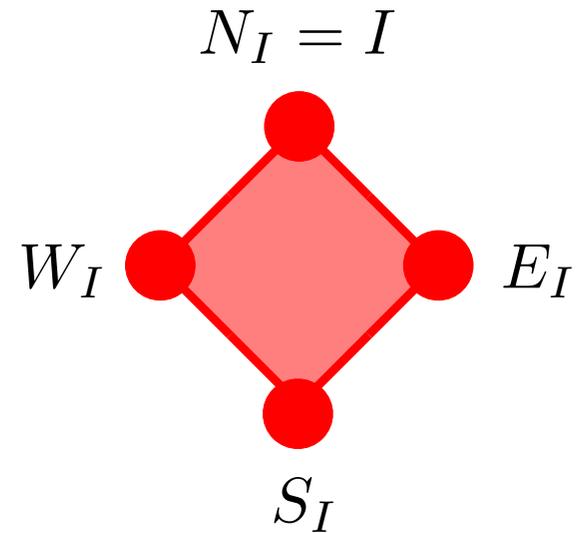
2D example (not glassy yet)

Toric code (in Wen's plaquette formulation)

Kitaev, Ann. Phys. (2003) - quant-physics/97
Wen, PRL (2003)



$$\vec{R} = i\vec{a}_+ + j\vec{a}_- \quad I \equiv (i, j)$$



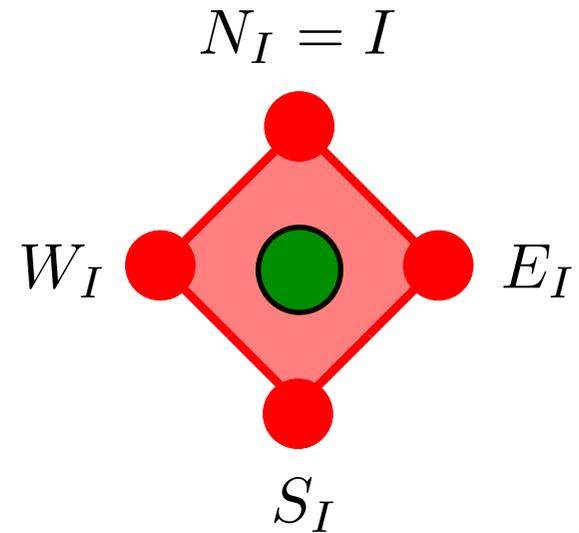
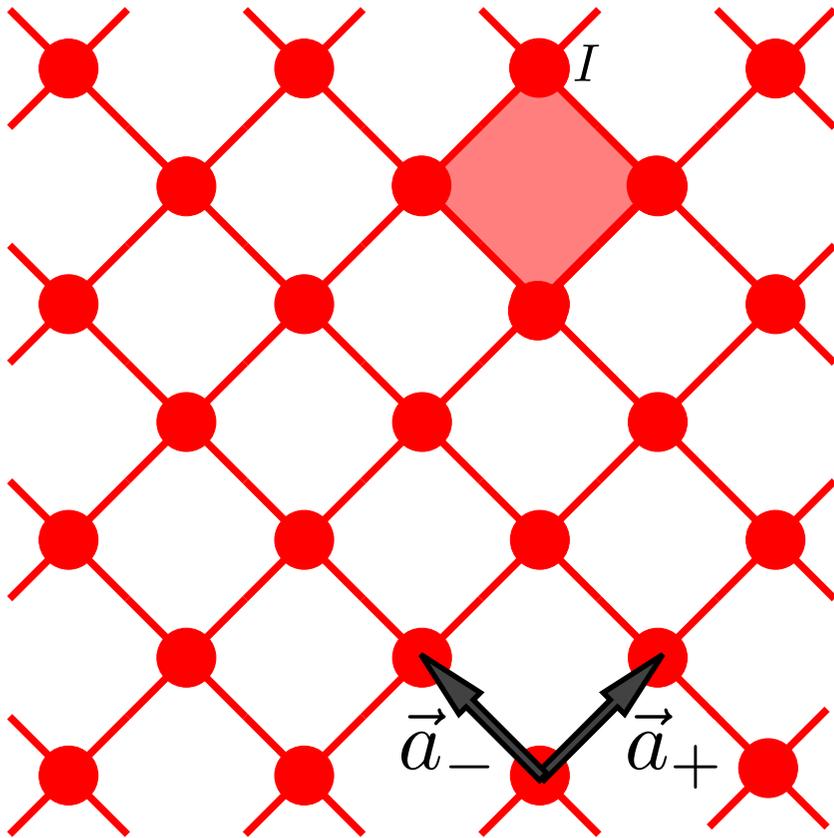
$$H = -\frac{\hbar}{2} \sum_I \sigma_{N_I}^y \sigma_{W_I}^x \sigma_{S_I}^y \sigma_{E_I}^x$$

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Defect dynamics

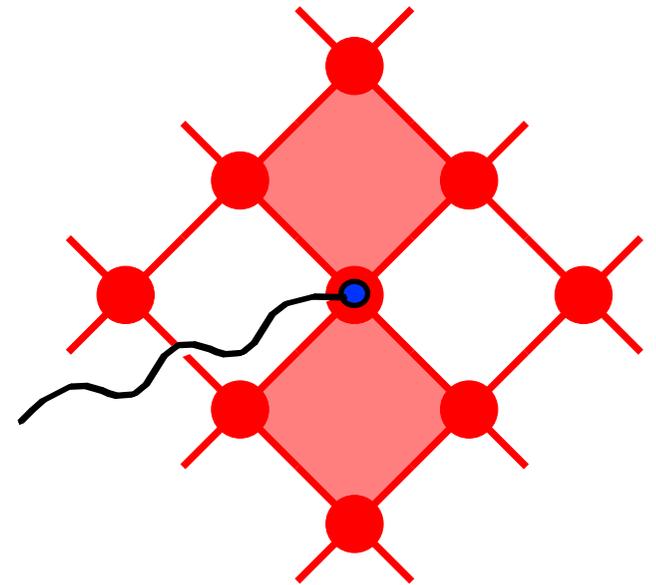
bath of quantum oscillators;
acts on physical degrees of freedom

Caldeira & Leggett, Ann. Phys. (1983)

Feynman & Vernon, Ann. Phys. (1963)

$$\hat{\mathcal{H}} = \hat{H} + \hat{H}_{\text{bath}} + \hat{H}_{\text{spin/bath}}$$

$$\hat{H}_{\text{spin/bath}} = \sum_{I,\alpha} g_{\alpha} \sigma_I^{\alpha} \sum_{\lambda} \left(a_{\lambda,I}^{\alpha} + a_{\lambda,I}^{\alpha \dagger} \right)$$



Defect dynamics

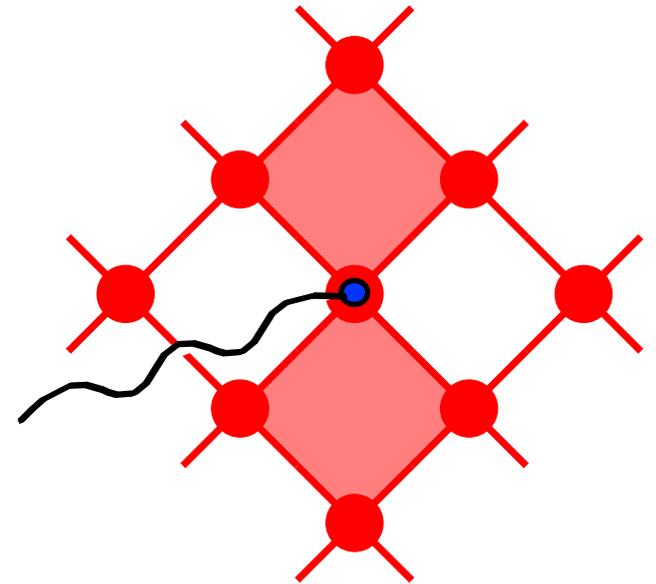
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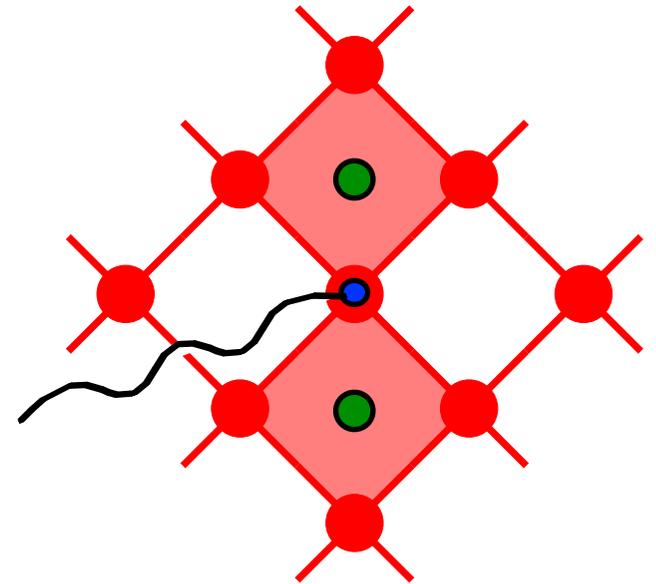
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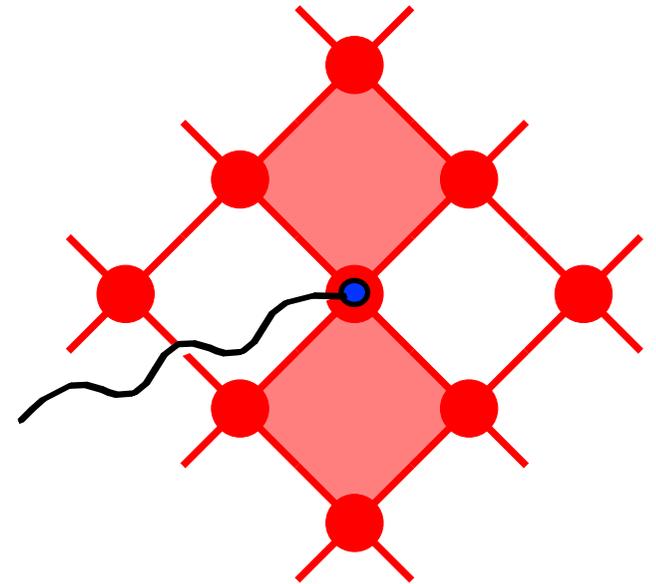
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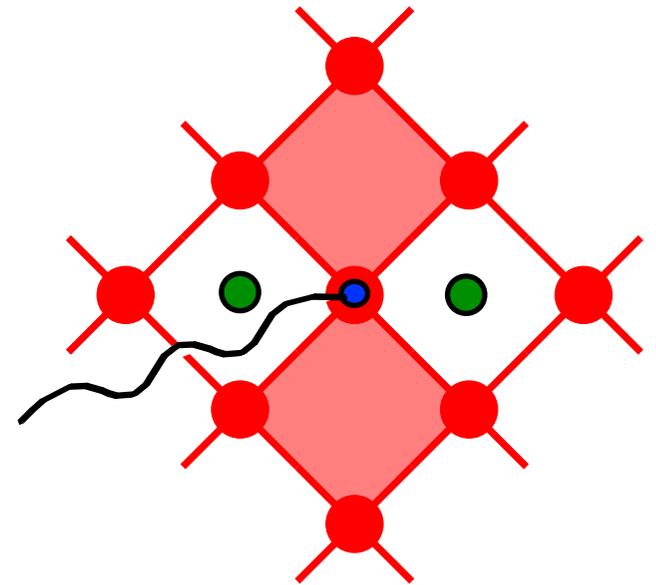
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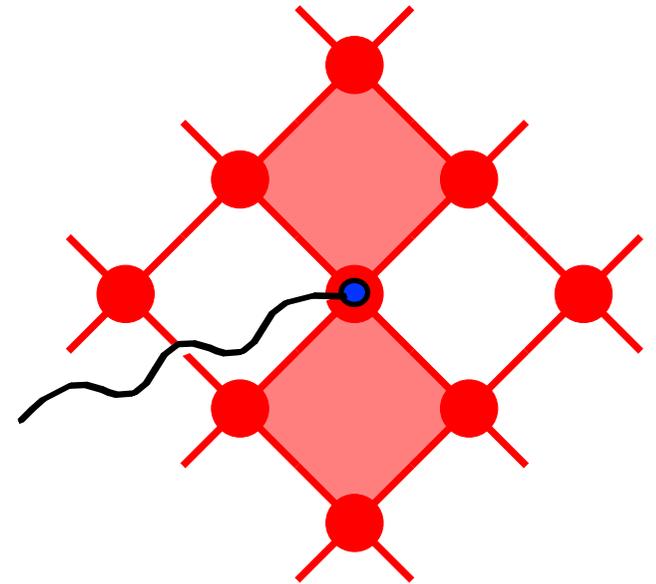
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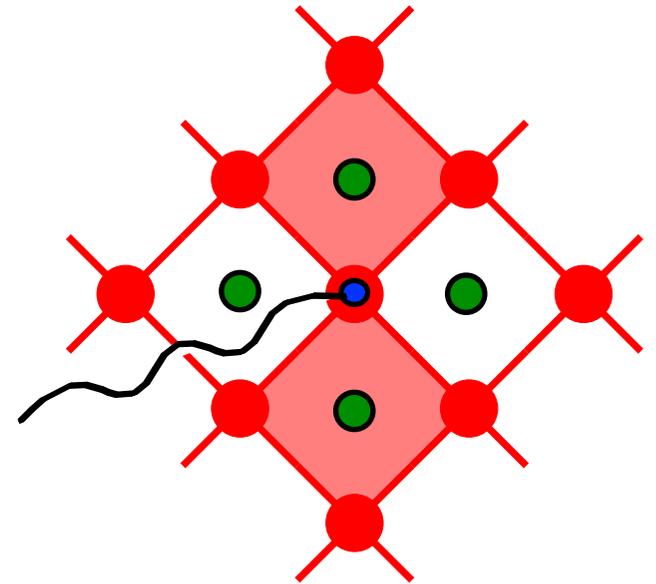
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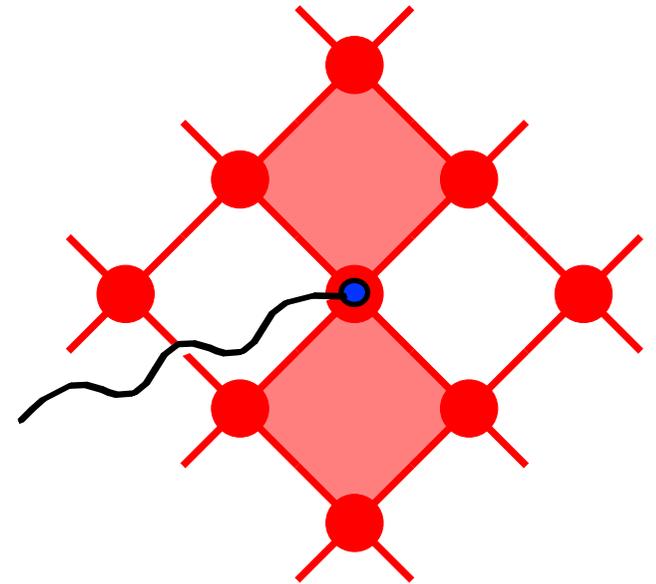
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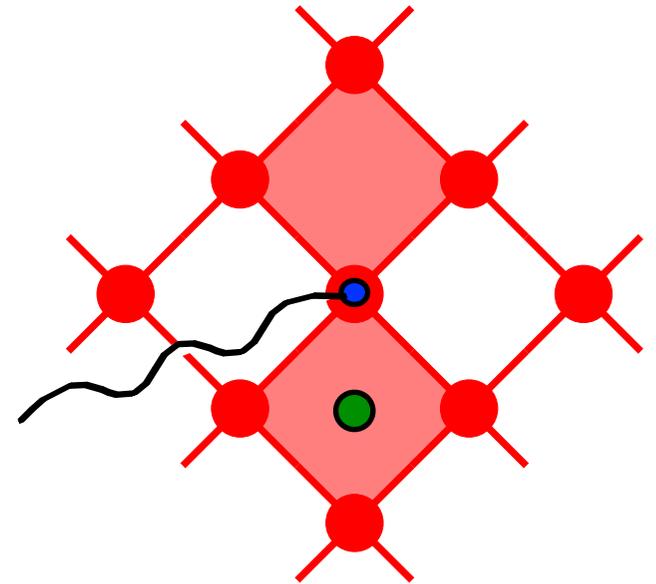
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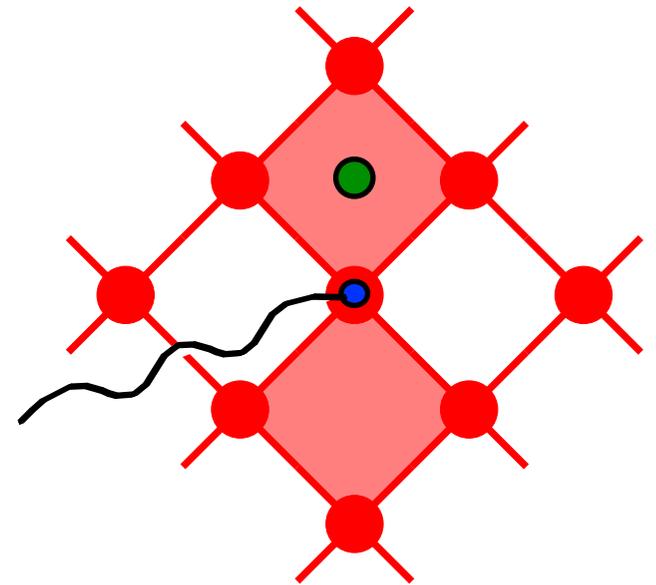
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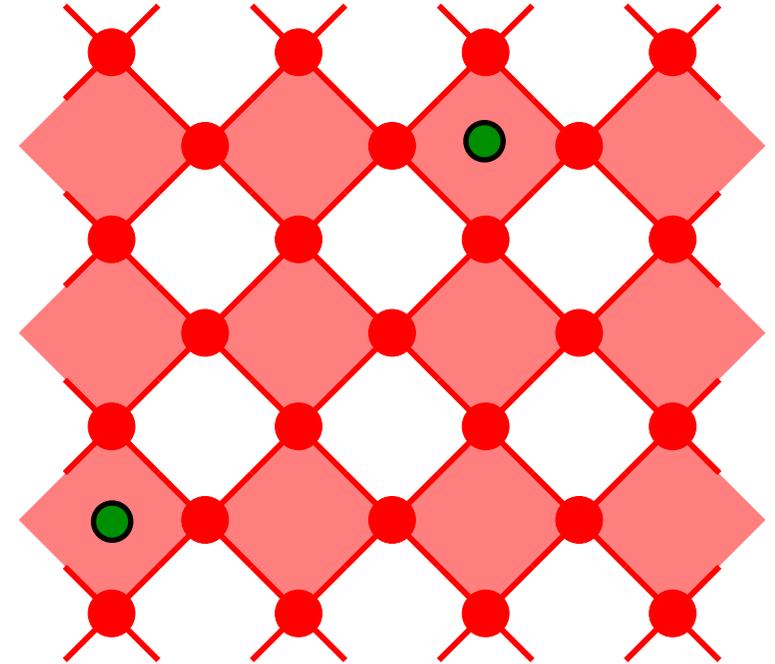


Defect dynamics

defects must go away to reach a GS

equilibrium concentration: $c \approx e^{-h/T}$

defects cannot be annihilated;
must be recombined



$\sigma_I^{x,y} \Rightarrow$ simple defect diffusion (escapes glassiness)

$\sigma_I^z \Rightarrow$ activated diffusion $t_{\text{seq.}} \sim \tau_0 \exp(2h/T)$ (Arrhenius law)

equivalent to classical glass model by

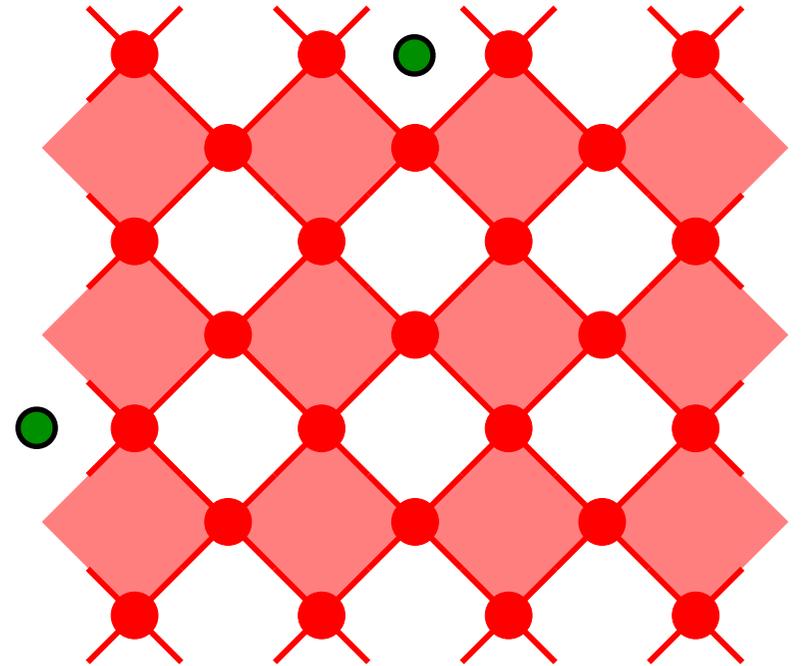
Garrahan & Chandler, PNAS (2003)
Buhot & Garrahan, PRL (2002)

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3D strong glass model

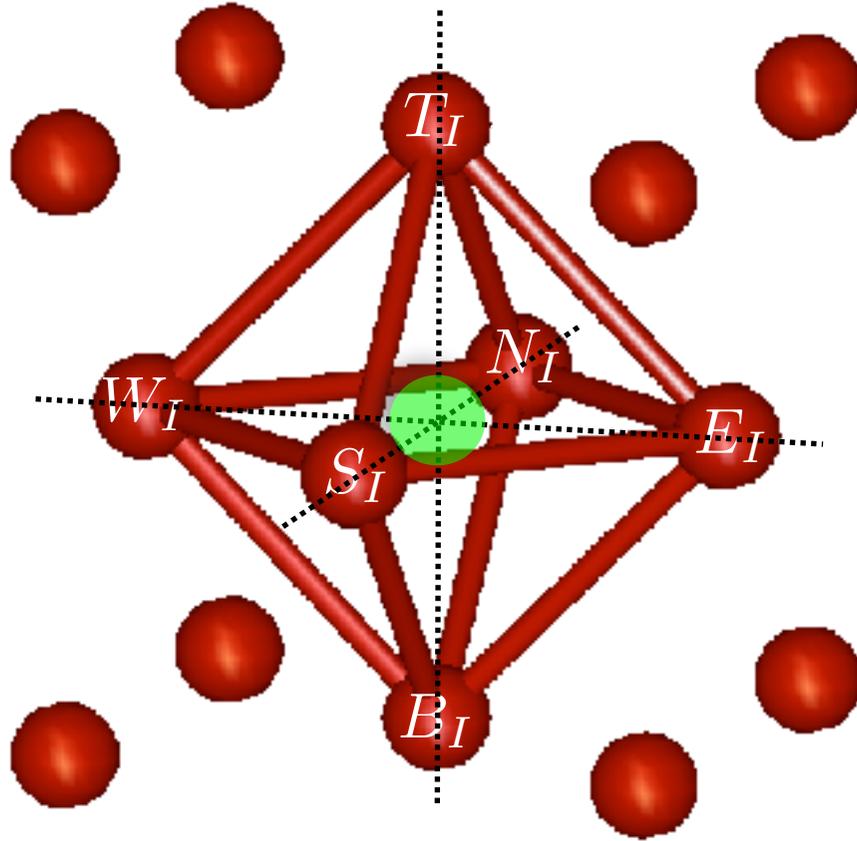
Chamon, PRL (2005)

Bravyi, Leemhuis, and Terhal, Ann. Phys. (2011)

ground state
degeneracy

$$L_x \times L_y \times L_z$$

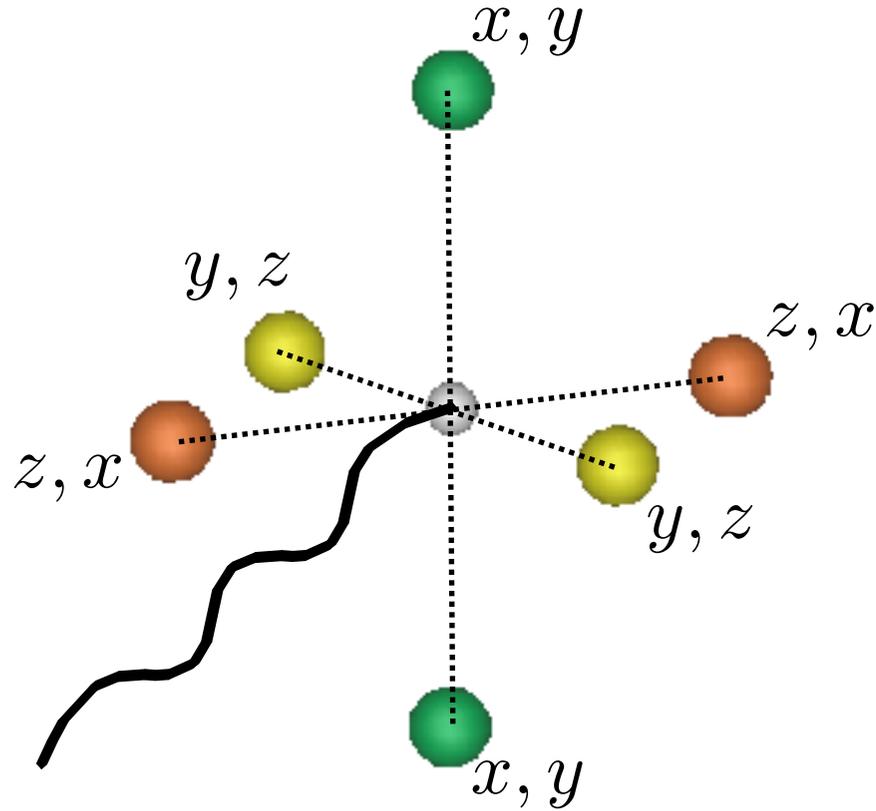
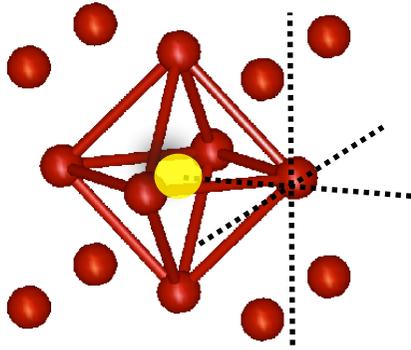
$$g = 2^4 \gcd(L_x, L_y, L_z)$$



$$\hat{O}_I = \sigma_{T_I}^z \sigma_{N_I}^y \sigma_{W_I}^x \sigma_{B_I}^z \sigma_{S_I}^y \sigma_{E_I}^x$$

$$H = -\frac{\hbar}{2} \sum_I \hat{O}_I$$

3D strong glass model



$$\hat{H}_{\text{spin/bath}} = \sum_{I,\alpha} g_{\alpha} \sigma_I^{\alpha} \sum_{\lambda} \left(a_{\lambda,I}^{\alpha} + a_{\lambda,I}^{\alpha \dagger} \right)$$

always flip 4 octahedra: never simple defect diffusion

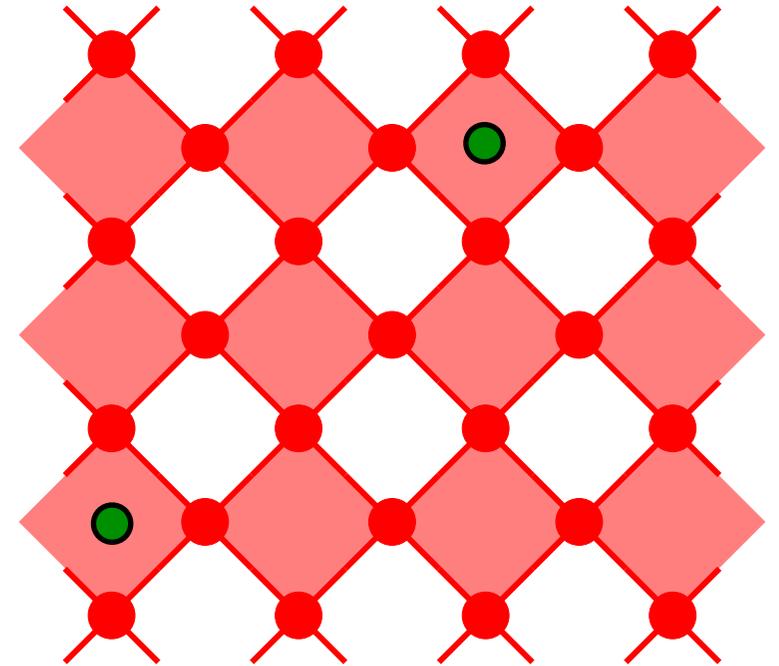
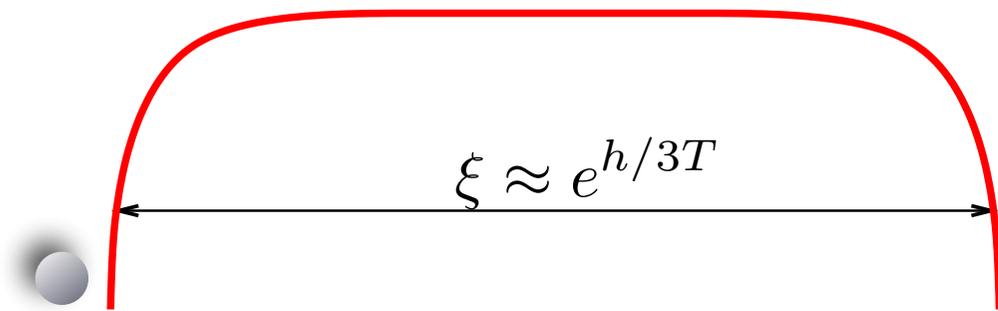
$$t_{\text{seq.}} \sim \tau_0 \exp(2h/T) \quad (\text{Arrhenius law})$$

What about quantum tunneling?

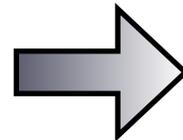
defect separation: $\xi \approx c^{-1/3} \approx e^{h/3T}$

virtual process: $\mathcal{O}[(g/h)^\xi]$

$$t_{\text{tun.}} \sim \tau_0 \exp \left[\ln(h/g) e^{h/3T} \right]$$



topological quantum protection



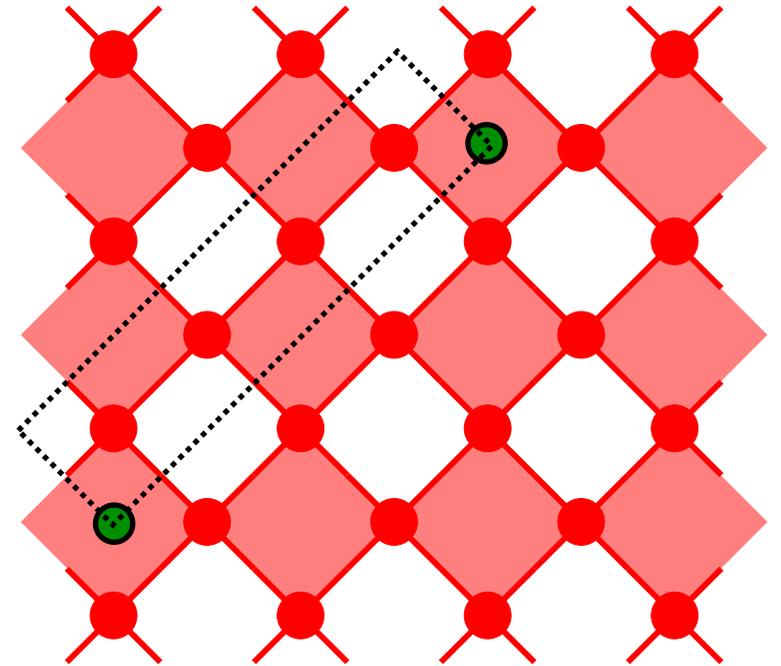
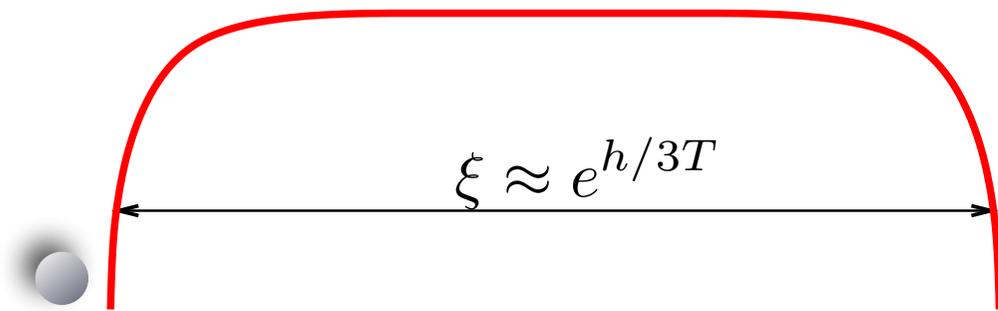
quantum OVER protection

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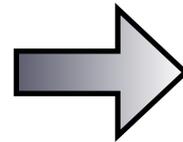
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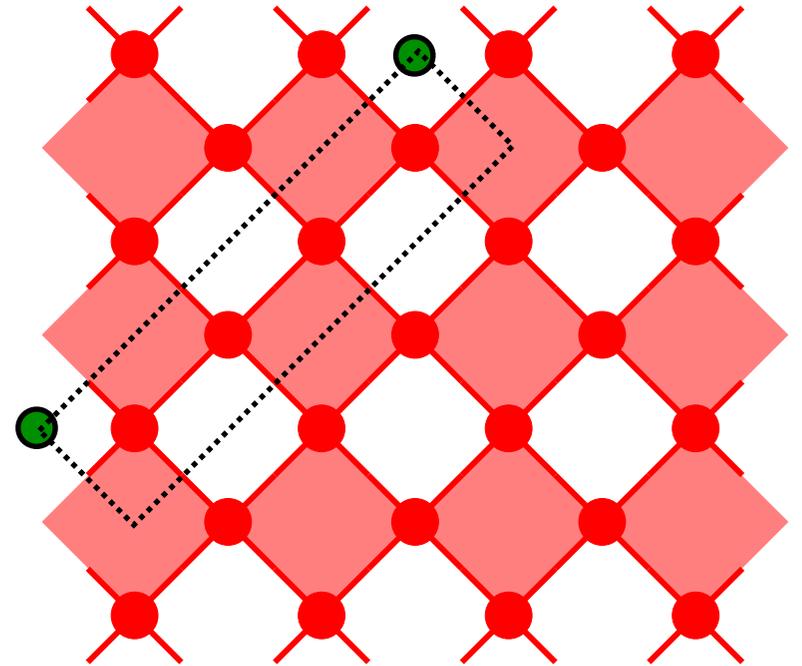
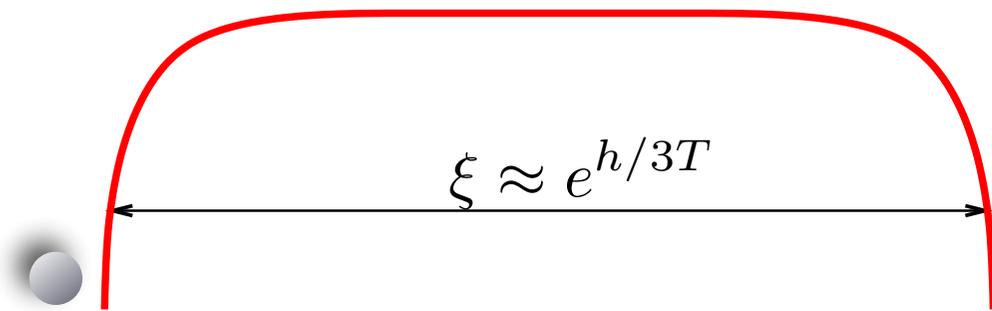
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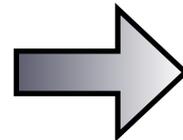
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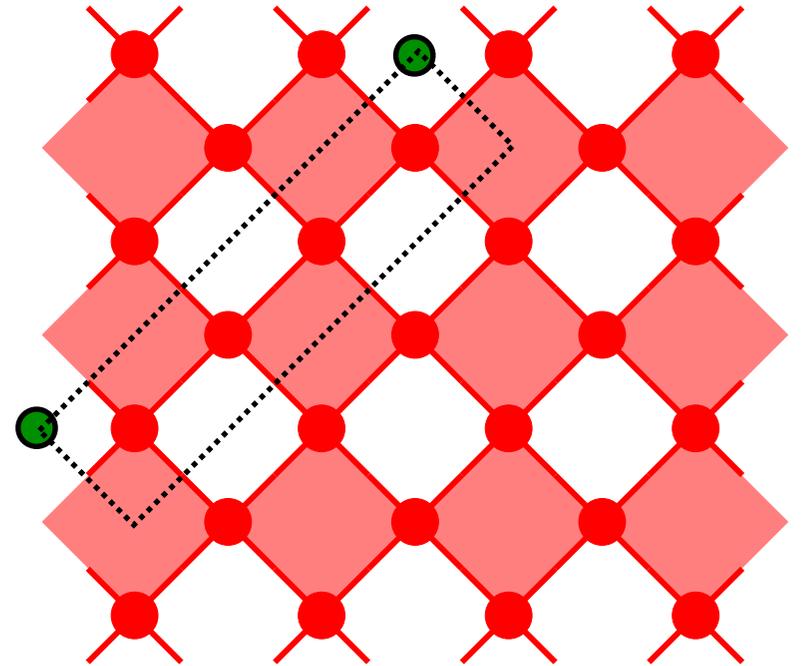
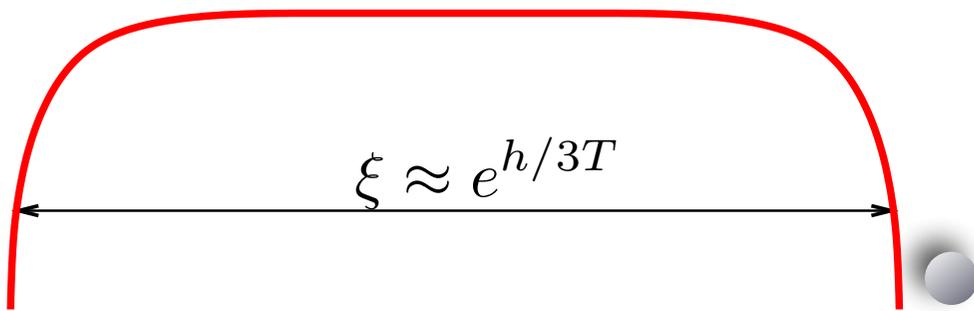
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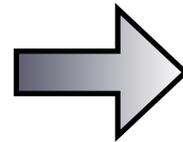
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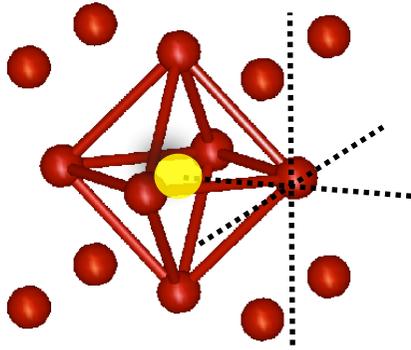


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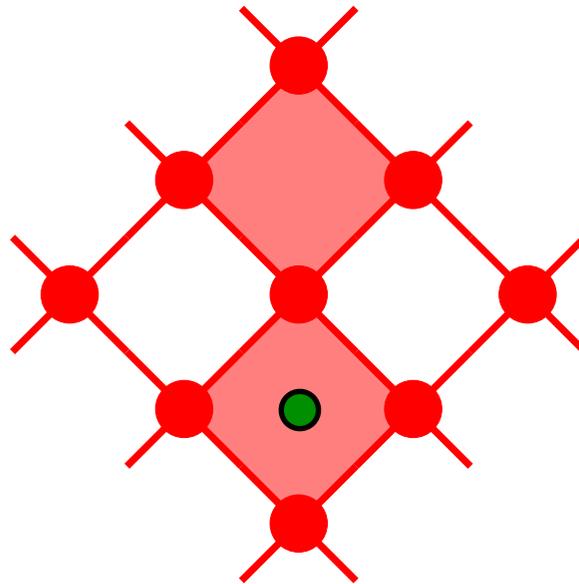
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3D strong glass model



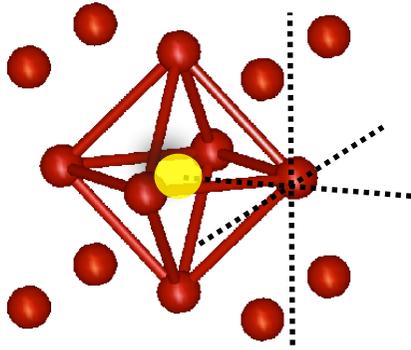
Strong glass

$$t_{\text{seq.}} \sim \tau_0 \exp(2h/T) \quad (\text{Arrhenius law})$$



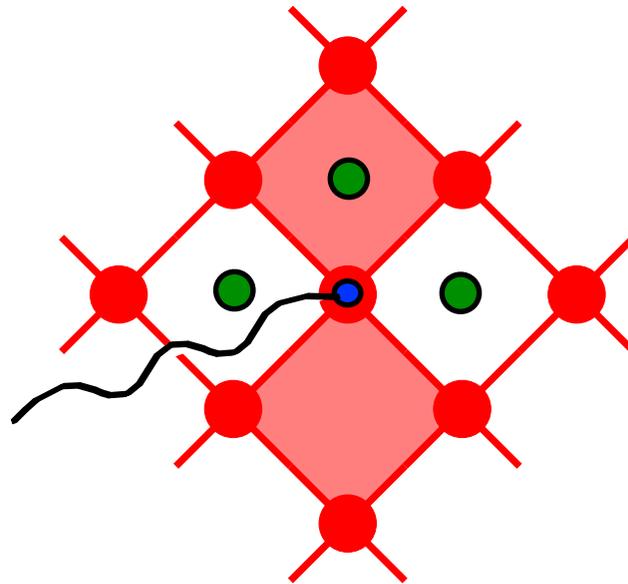
$$E_B = 2h$$

3D strong glass model



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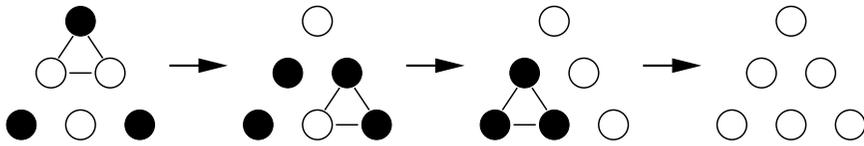


$$E_B = 2h$$

3D fragile glass model

Classical triangular plaquette model

Newman & Moore, PRE (1999)



$$t_{\text{seq.}} \sim \tau_0 \exp(\Delta^2/T^2)$$

(super Arrhenius law)

FIGURE 2 A triangle of side 2^k can be flipped by flipping three triangles of side 2^{k-1} . The solid circles represent the defects and the lines indicate the triangles to be flipped at each step.

$$E_B = \varepsilon k$$

$$t_{\text{seq.}} \sim \tau_0 \exp(E_B/T)$$

$$\xi \sim e^{\varepsilon/2T} \sim 2^k \Rightarrow k \sim \varepsilon/T \, 2 \ln 2$$

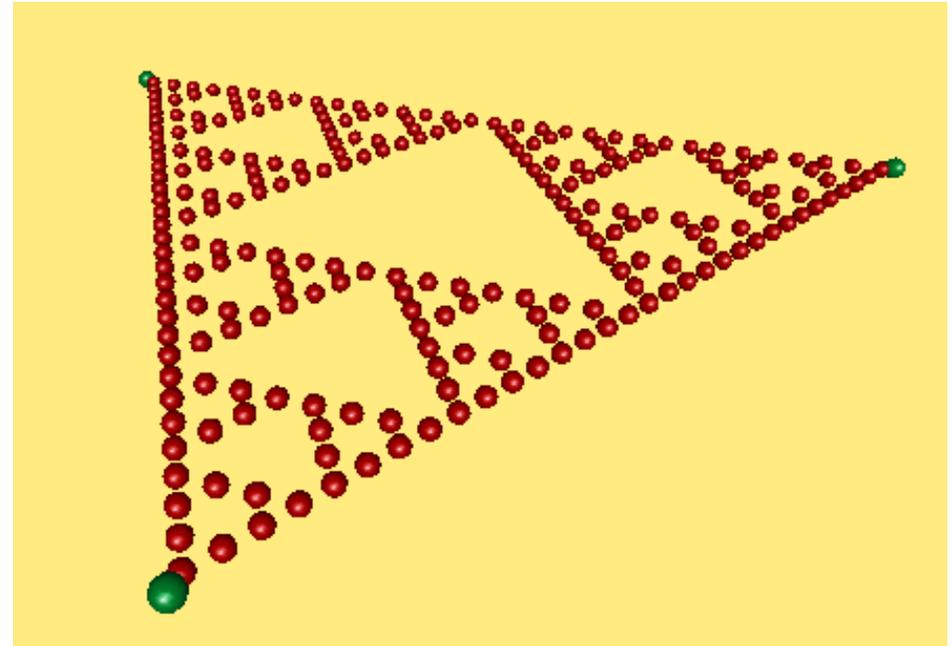
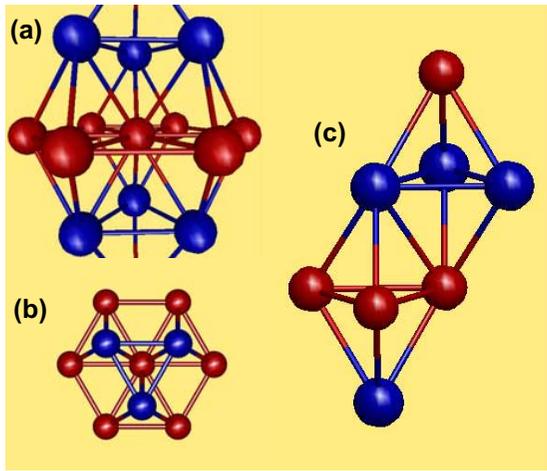
$$t_{\text{seq.}} = \tau_0 \exp(\varepsilon^2/T^2 \, 2 \ln 2)$$

3D fragile glass model

Quantum model

Lectures at ICTP 2009 Summer College on “Nonequilibrium Physics from Classical to Quantum Low Dimensional Systems”

Castelnovo & Chamon, Phil. Mag. (2011)



$$\mathcal{O}_I = \sigma_{J_1(I)}^z \sigma_{J_2(I)}^x \sigma_{J_3(I)}^x \sigma_{J_4(I)}^x \sigma_{J_5(I)}^z$$

\mathbb{Z}_2 charge

parity of defects on vertical lines

$$\tau_{i,j;q} = \prod_k \mathcal{O}_{(i,j,k;q)}$$

$$s_{i,j;q} = \prod_k \sigma_{(i,j,k;q)}^x$$

$$s_{i,j;q} = \prod_{mn} [\tau_{n,m;q}]^{\binom{j-n}{i-m}}$$

$$\tau_{i,j;q} = s_{i,j;q} s_{i+1,j;q} s_{i,j+1;q}$$

3D fragile glass model

Quantum model

Haah, PRA (2011)

fractal sponge

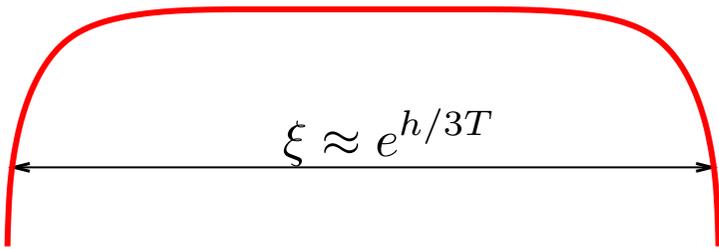


<https://quantumfrontiers.com/2018/02/16/fractons-for-real/>

Annealing times

Quantum annealing

$$t_{\text{tun.}} \sim \tau_0 \exp \left[\ln(h/g) e^{h/3T} \right]$$



$$t_{\text{tun.}} \sim \tau_0 \exp \left[\ln(h/g) \xi \right]$$

Thermal annealing

Ex. I: Arrhenius law

$$t_{\text{seq.}} \sim \tau_0 \exp(\Delta/T)$$

Ex. II super-Arrhenius law

$$t_{\text{seq.}} \sim \tau_0 \exp(\Delta^2/T^2)$$

$$t_{\text{seq.}} \sim \tau_0 \exp \left(\frac{\Delta}{h} \ln \xi \right)^\alpha$$

“Solving” a problem of size L

Annealing times

“Solving” a problem of size L

Quantum annealing

$$t_{\text{tun.}} \sim \tau_0 \exp [\ln(h/g) L]$$

exponential time in L

Thermal annealing

$$t_{\text{seq.}} \sim \tau_0 \exp \left(\frac{\Delta}{h} \ln L \right)^\alpha$$

$$\alpha = 1$$

Arrhenius

$$\alpha > 1$$

super-Arrhenius

$$t_{\text{seq.}} \sim \tau_0 L^{\frac{\Delta}{h}}$$

polynomial time in L

$$t_{\text{seq.}} \sim \tau_0 L \left(\frac{\Delta}{h} \right)^\alpha (\ln L)^{\alpha-1}$$

quasi-polynomial time in L

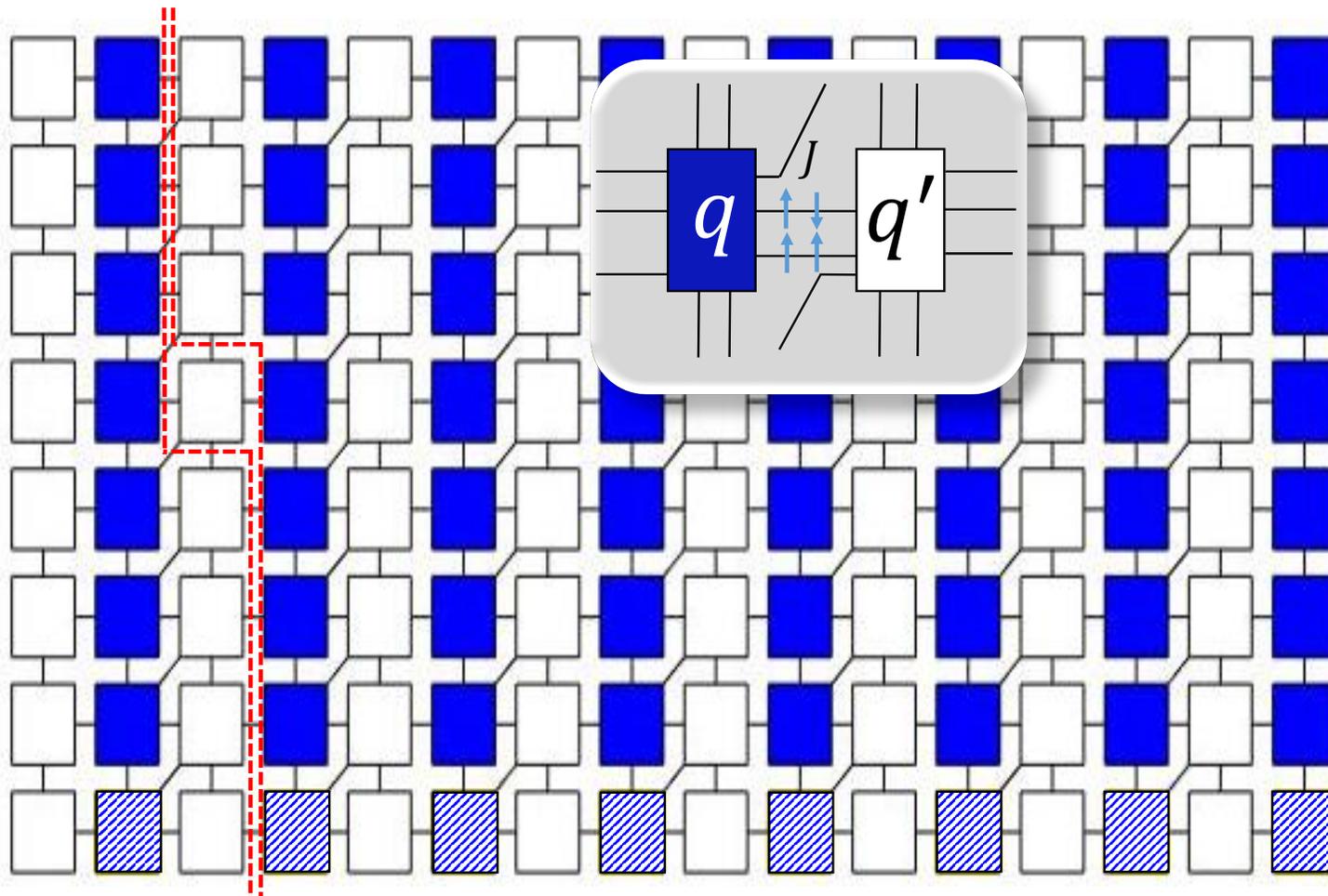
Example of exponential time for thermal annealing?

Example of a system w/o disorder and double-exponential in T relaxation

w/ Lei Zhang, Stefanos Kourtis, Eduardo Mucciolo, and Andrei Ruckenstein

- Provable absence of a thermodynamic phase transition
- Sub-extensive ground state degeneracy (scaling with the boundary)
- Relaxation times to (a) ground state is double-exponential in T

Example of a system w/o disorder and double-exponential in T relaxation

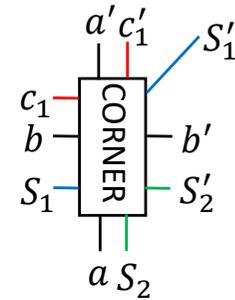
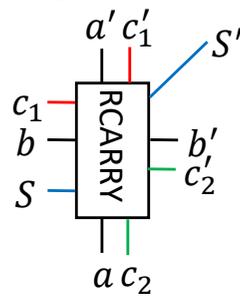
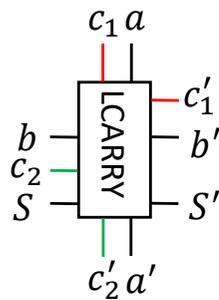
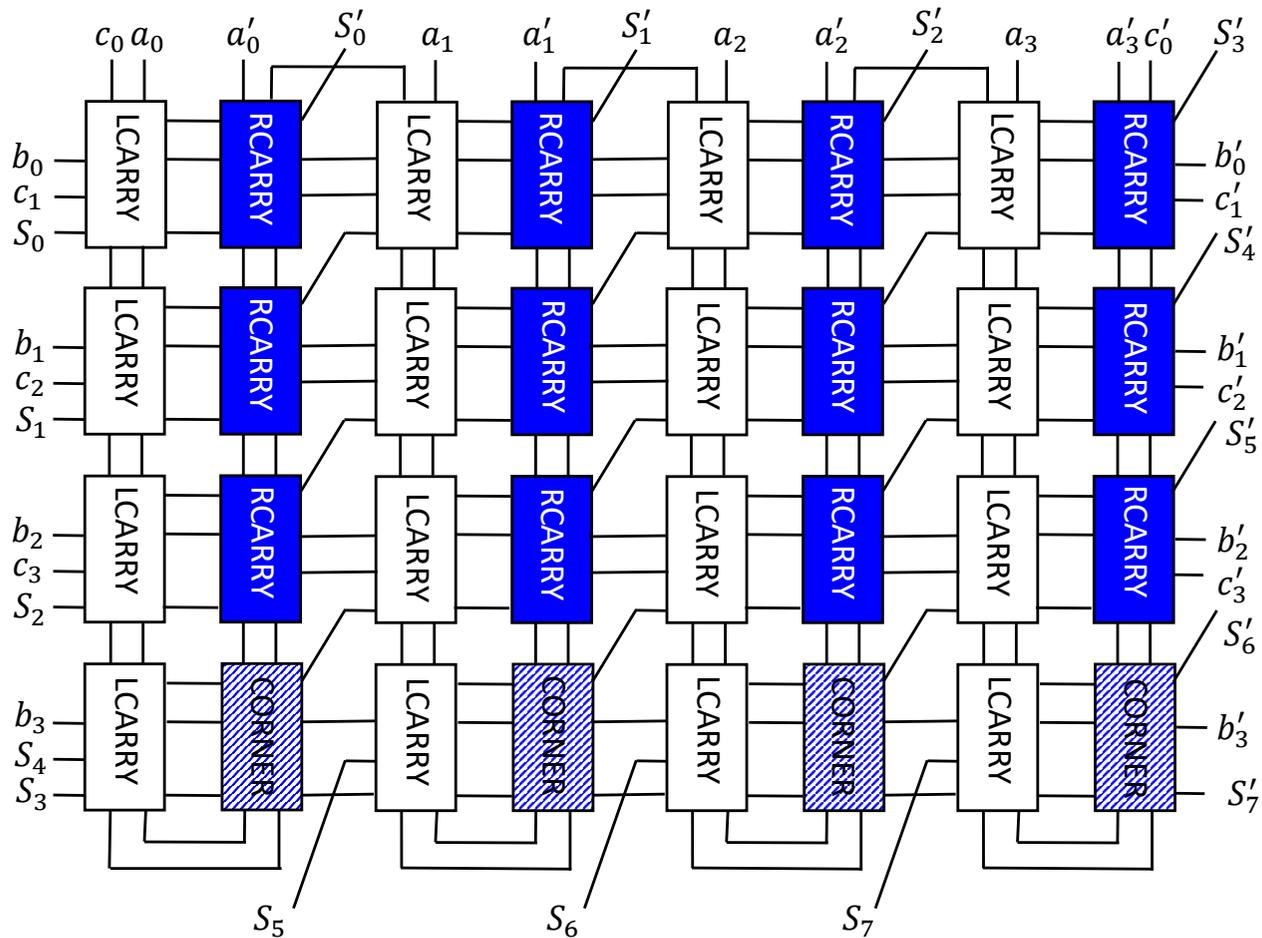


$$q = 0, 1, \dots, 2^n - 1$$

vertex model w/ twin spins on the bonds

array multiplier

$$S' = a \times b + S$$



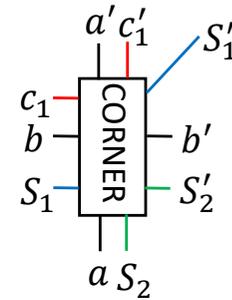
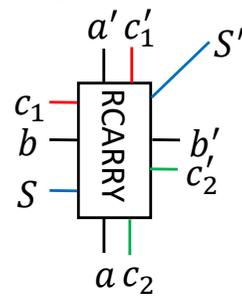
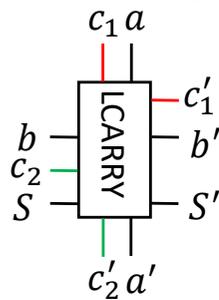
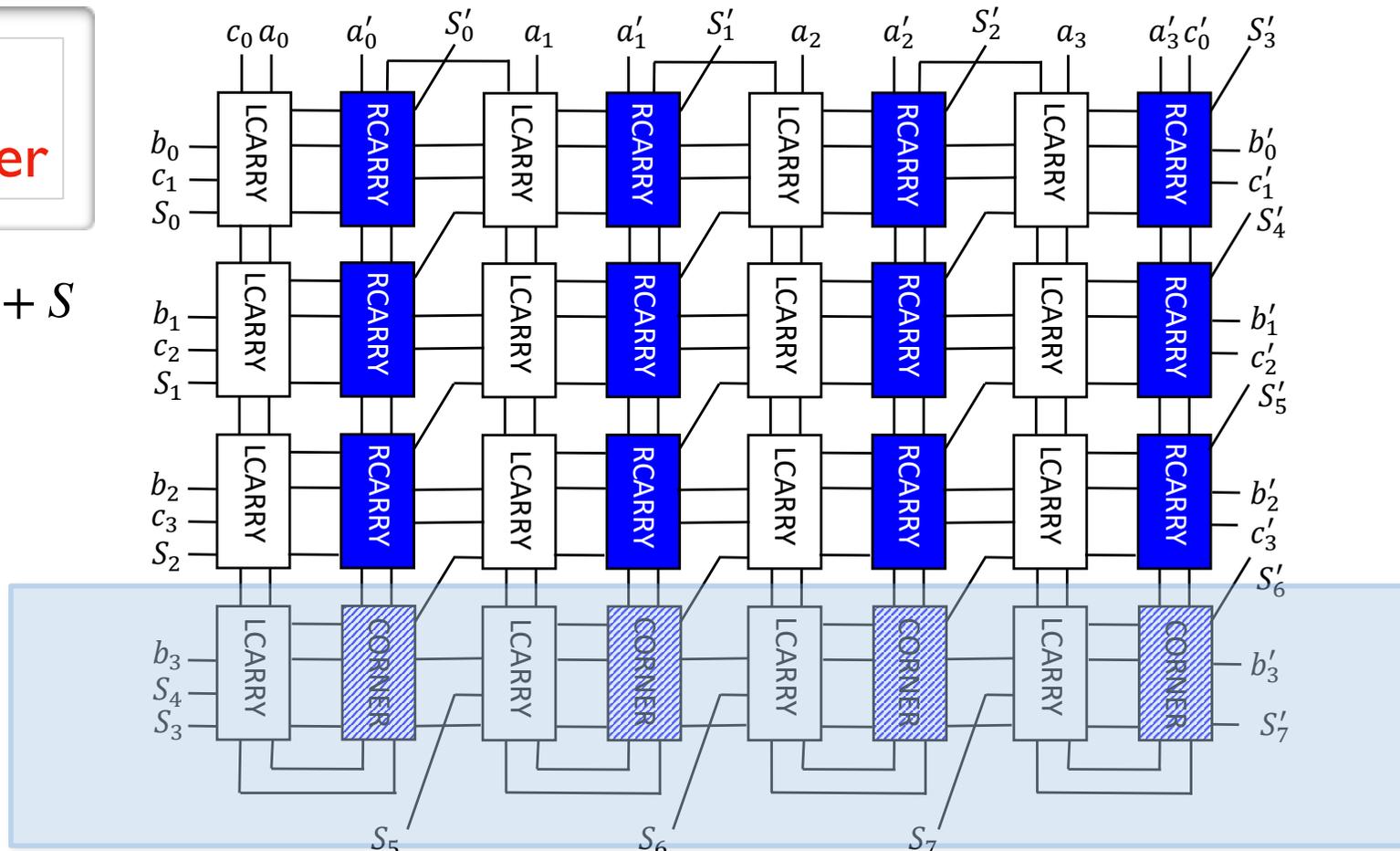
$$\text{LCARRY: } c'_2 = c_2 \oplus a b s \oplus a c_1 (s \oplus a b) \\ s' = s \oplus a b$$

$$\text{RCARRY: } c'_2 = c_2 \oplus a c_1 s \oplus a b (s \oplus a b) \\ s' = s \oplus a c_1$$

$$\text{CORNER: } s'_1 = s_1 \oplus a c_1$$

array multiplier

$$S' = a \times b + S$$

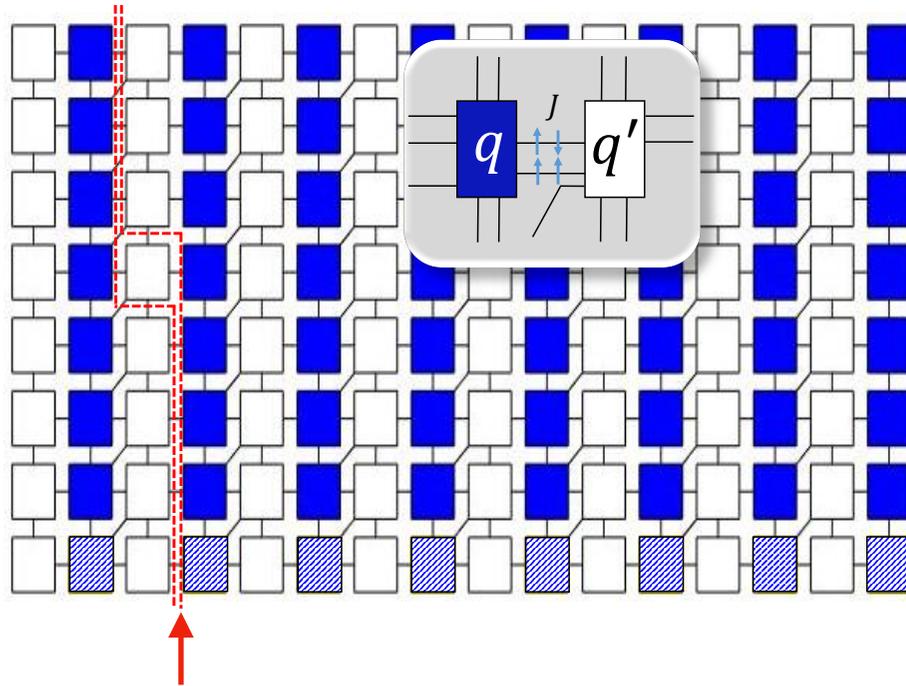


$$\text{LCARRY: } c'_2 = c_2 \oplus a b s \oplus a c_1 (s \oplus a b) \\ s' = s \oplus a b$$

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Thermodynamics: absence of a phase transition



$$\mathcal{T}_g = t_g \otimes \mathbb{1}_{\bar{\sigma}(g)}$$

$$\mathcal{T}_g |\Sigma\rangle = \lambda |\Sigma\rangle$$

transfer matrix

free b.c. $|\Sigma\rangle = \sum_{\{\sigma\}_{5L}} |\{\sigma\}_{5L}\rangle$

$$Z = \langle \Sigma | \left(\prod_{g=1}^{N_{\text{gates}}} \mathcal{T}_g \right) | \Sigma \rangle$$

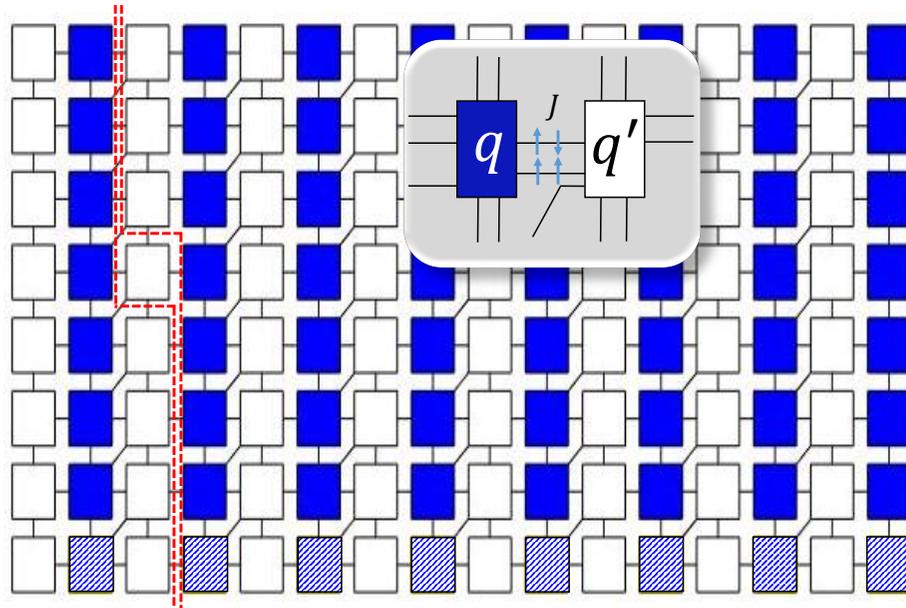
$$H_{\text{gates}} = \Delta \sum_{g=1}^{N_{\text{gates}}} \bar{T}_g[\sigma^{\text{in}}(g), \sigma^{\text{out}}(g)],$$

$$\lambda = \sum_{\sigma_{\ell_i}^{\text{in}}, \sigma_{\ell_i}^{\text{out}} = \pm 1} e^{-\beta \Delta \bar{T}_g[\sigma^{\text{in}}(g), \sigma^{\text{out}}(g)]} e^{\beta J \sum_{\ell \in w^{\text{in}}(g)} \sigma_{\ell}^{\text{in}} \sigma_{\ell}^{\text{out}}}$$

$$H_{\text{links}} = -J \sum_{\ell} \sigma_{\ell}^{\text{in}} \sigma_{\ell}^{\text{out}},$$

$$= (1 + 31e^{-\beta \Delta}) (2 \cosh \beta J)^5,$$

Thermodynamics: absence of a phase transition



transfer matrix

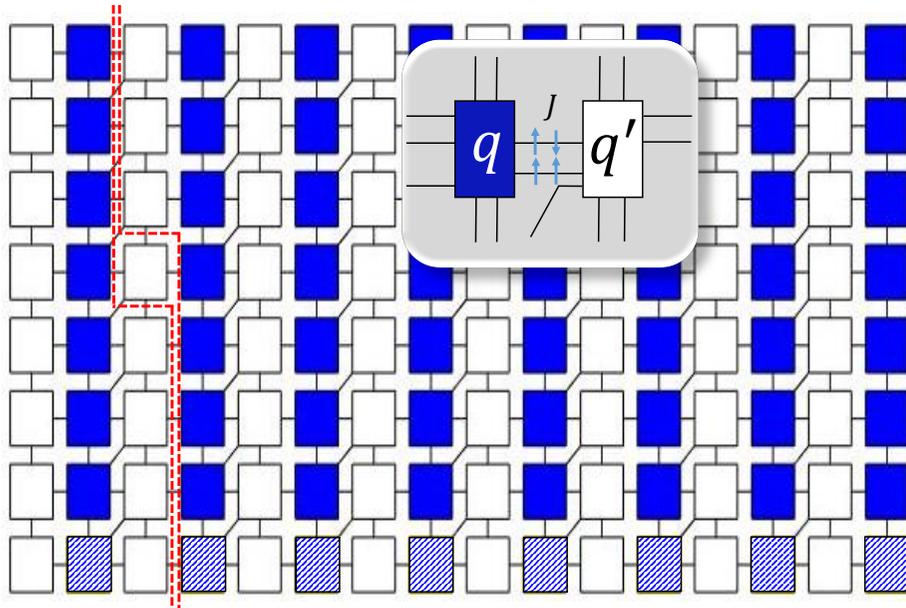
$$Z = (1 + 31e^{-\beta\Delta})^{10L^2} (2 \cosh \beta J)^{10L^2 - 5L} 2^{5L}$$

paramagnet-like:
no phase transition

$$H_{\text{gates}} = \Delta \sum_{g=1}^{N_{\text{gates}}} \bar{T}_g[\sigma^{\text{in}}(g), \sigma^{\text{out}}(g)],$$

$$H_{\text{links}} = -J \sum_{\ell} \sigma_{\ell}^{\text{in}} \sigma_{\ell}^{\text{out}},$$

Thermodynamics: absence of a phase transition



transfer matrix

$$Z = (1 + 31e^{-\beta\Delta})^{10L^2} (2 \cosh \beta J)^{10L^2 - 5L} 2^{5L}$$

paramagnet-like:
no phase transition

$$\langle \Sigma | \Sigma \rangle = 2^{5L}$$

$$H_{\text{gates}} = \Delta \sum_{g=1}^{N_{\text{gates}}} \bar{T}_g[\sigma^{\text{in}}(g), \sigma^{\text{out}}(g)],$$

$$H_{\text{links}} = -J \sum_{\ell} \sigma_{\ell}^{\text{in}} \sigma_{\ell}^{\text{out}},$$

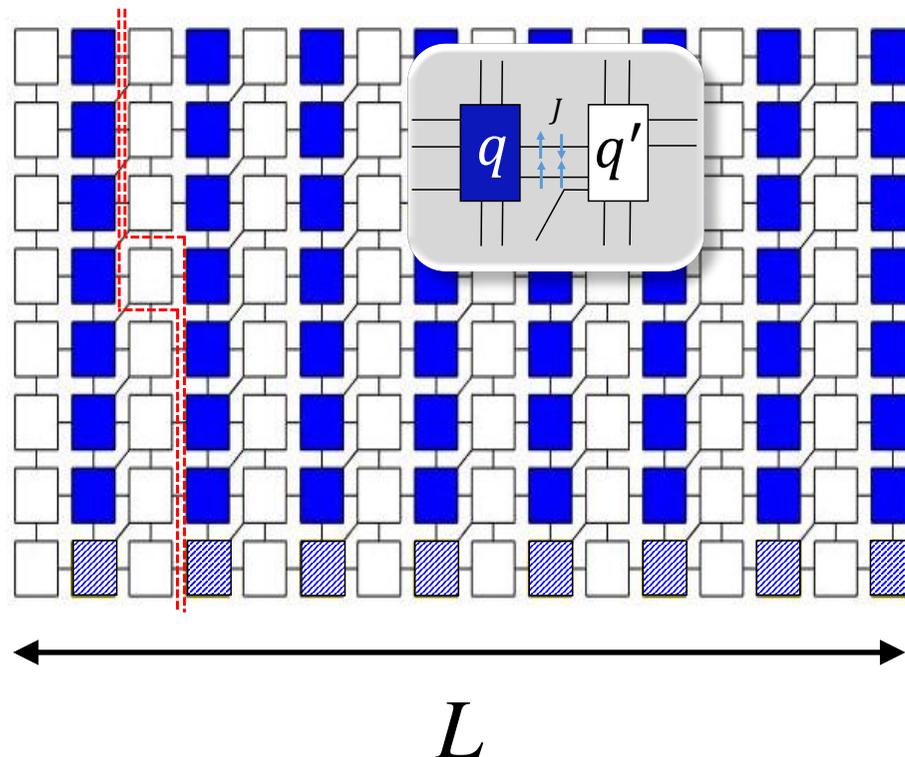
ground state degeneracy
“holographic”

Thermodynamics: absence of a phase transition

$$\xi_T \sim e^{K/2T}$$

Temperature needed is *not* very low! Only *logarithmic* in the size of the system.

$$T \sim K/\ln L$$

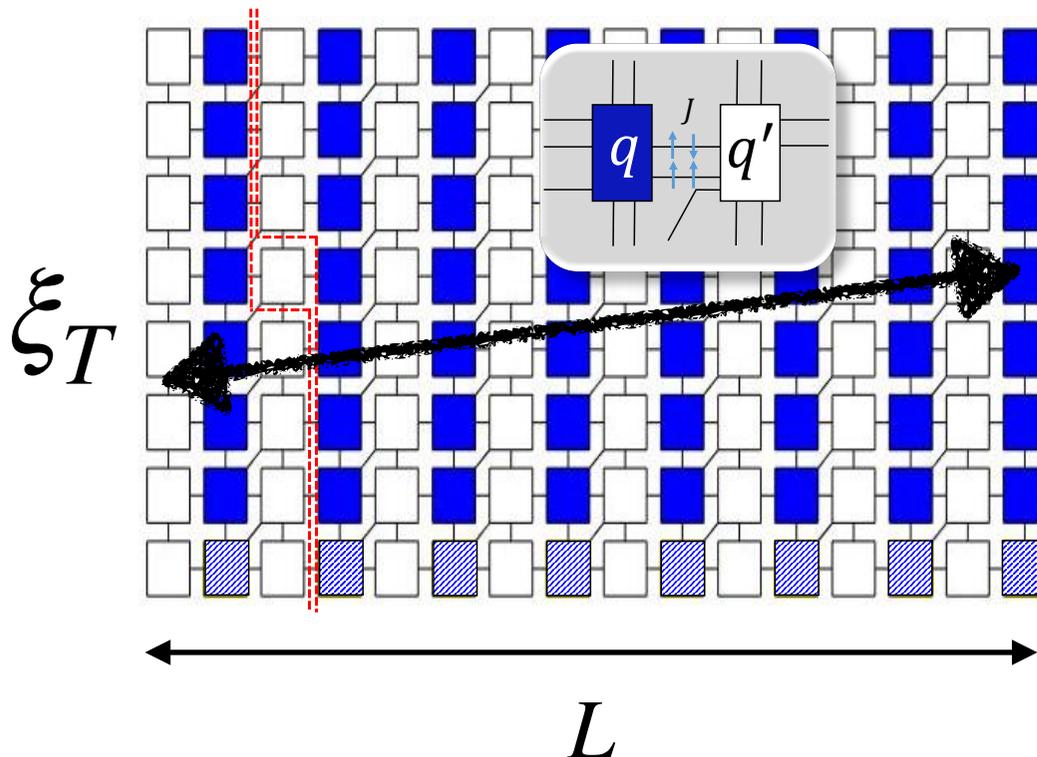


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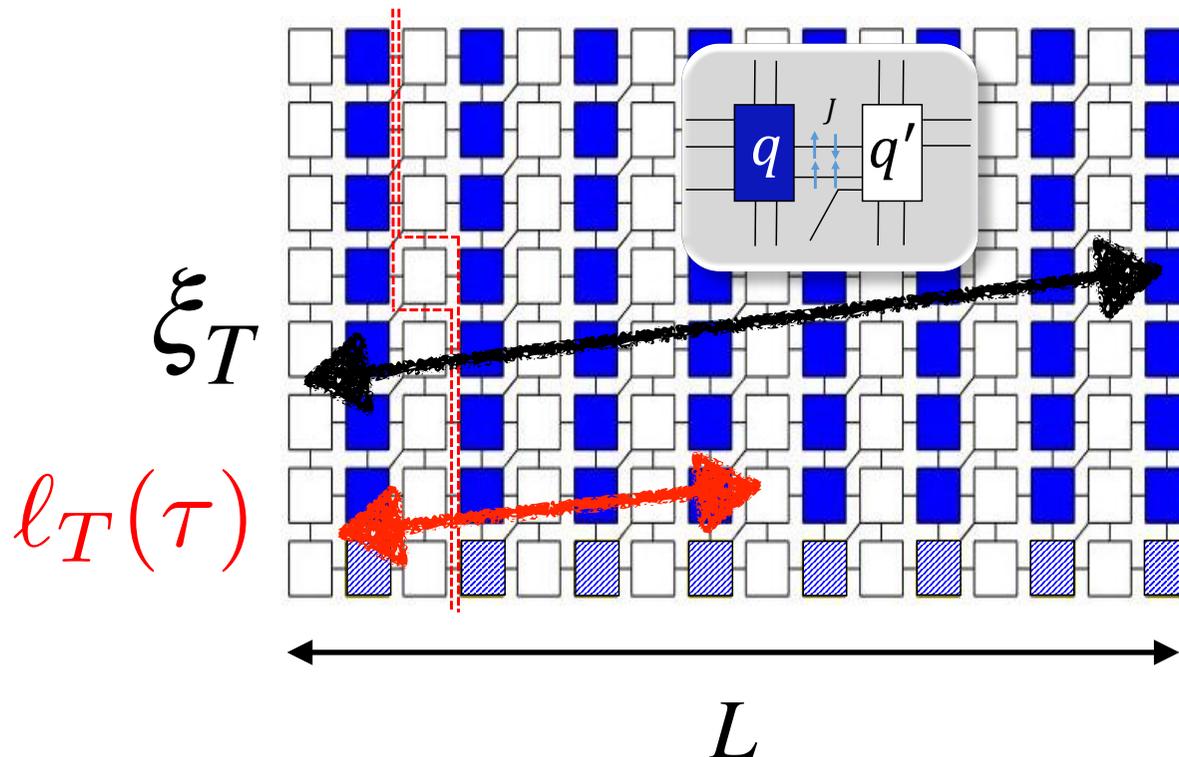


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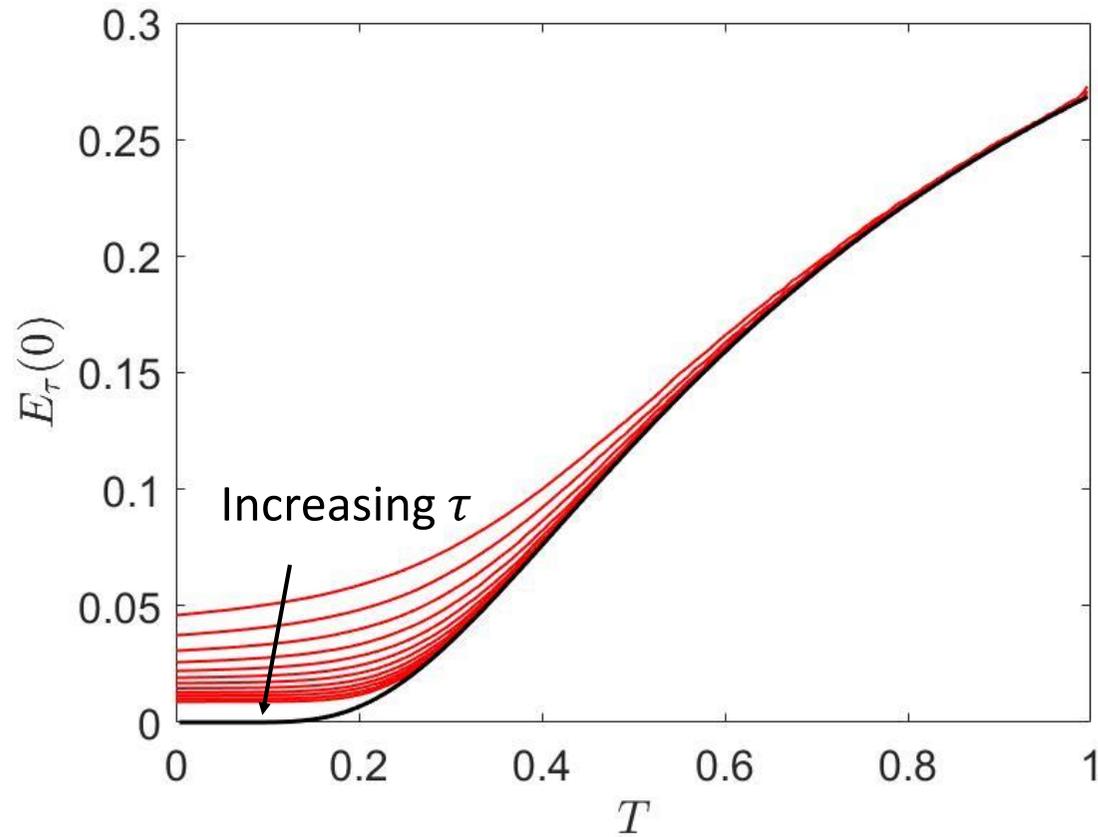
$$T \sim K/\ln L$$



Dynamics is what matters!

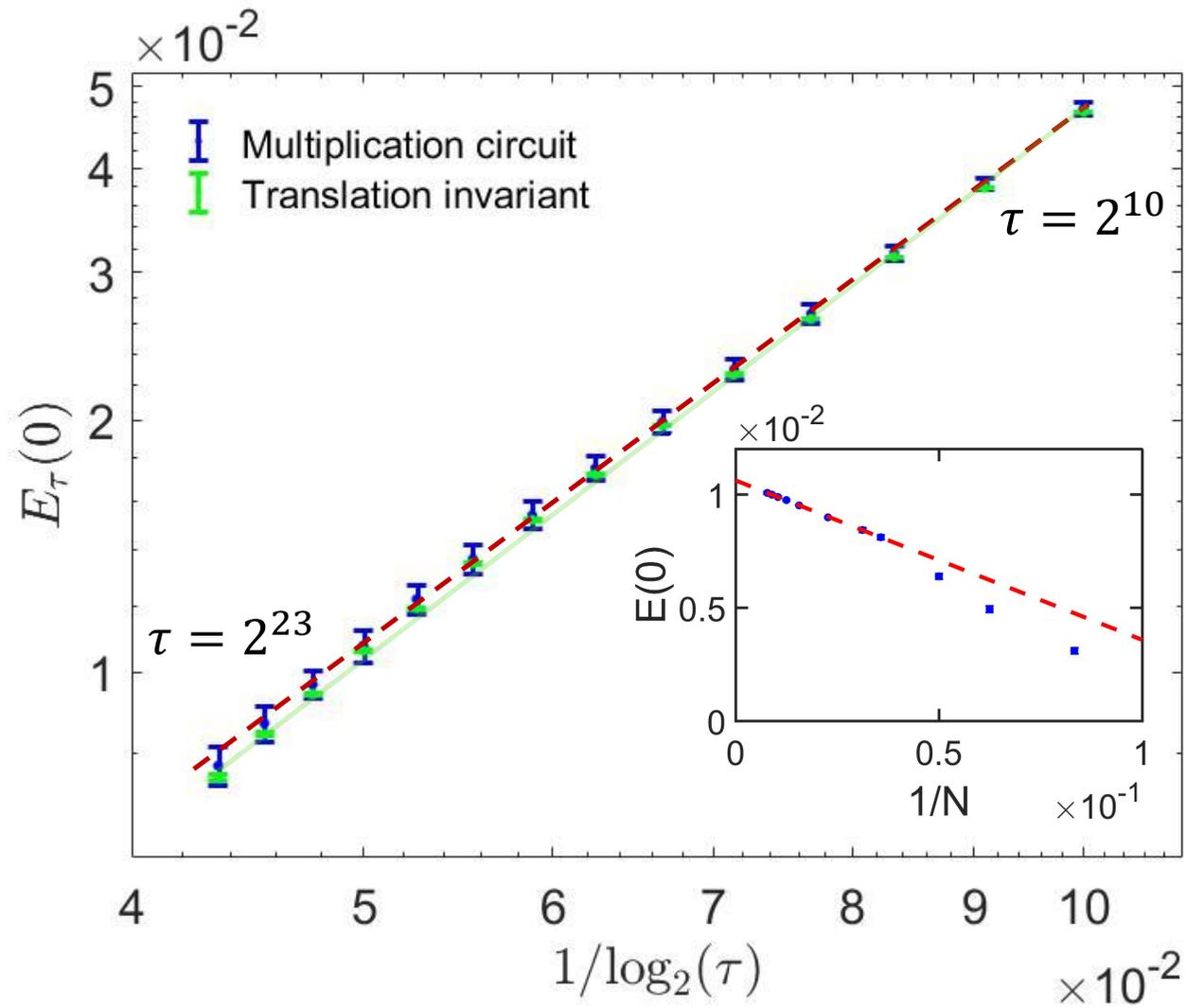
How long does it take to thermalize?

Thermal annealing

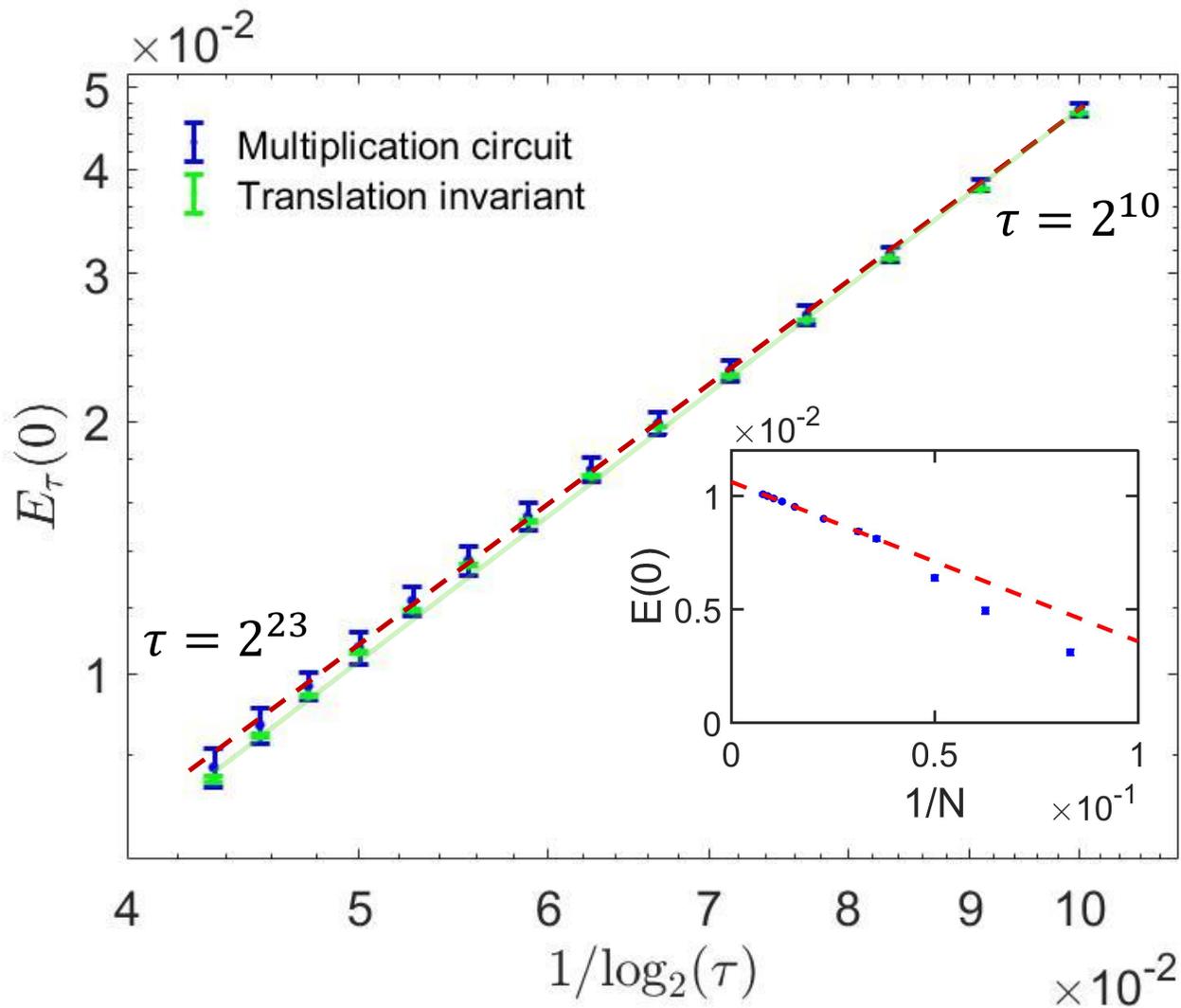


$$T(t) = J (1 - t/\tau)$$

Thermal annealing

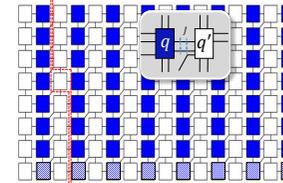


Thermal annealing

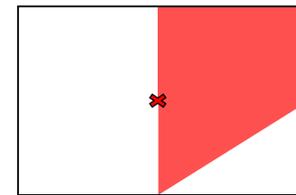
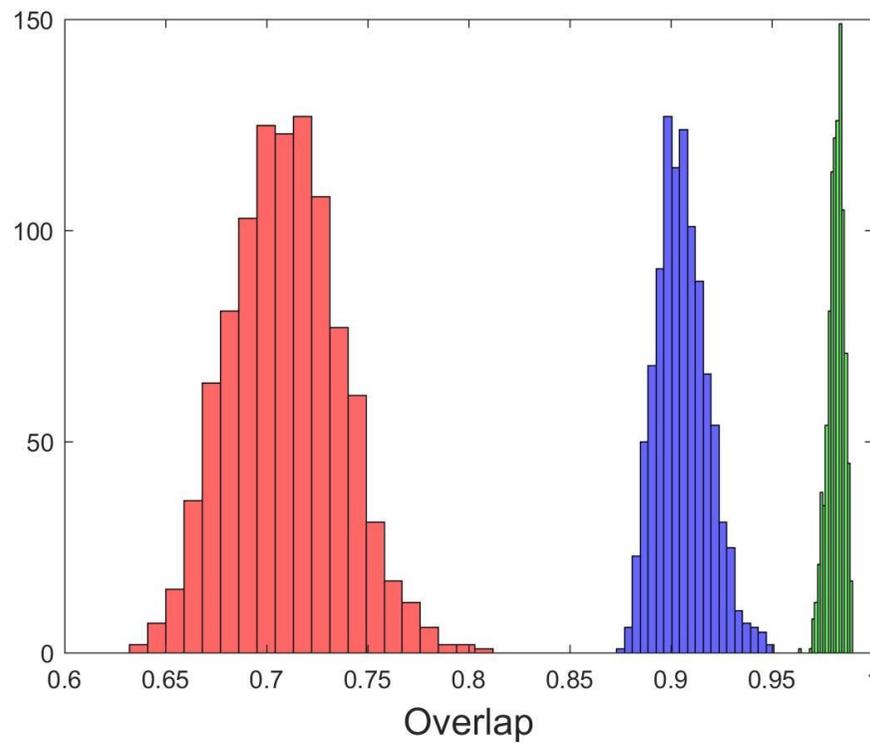


$$\tau \sim \tau_0 e^{e^{J/2T}/T}$$

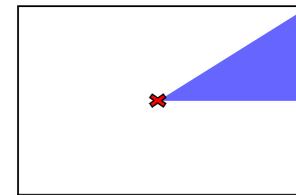
Thermal annealing



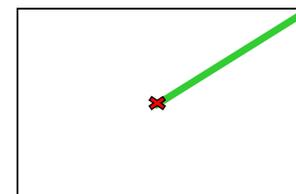
error in
bit type



a



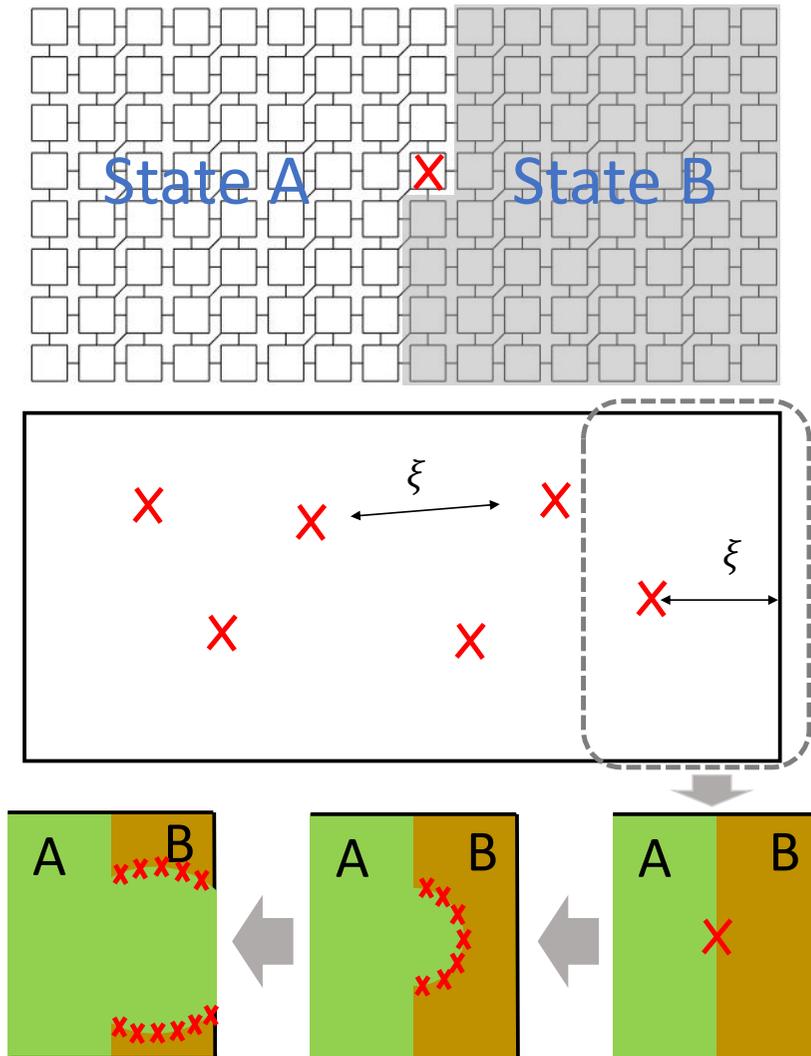
b or c



S

Red shows the distribution when flipping spin *a*, blue for spins *b* and *c*, and green for spin *S*

Thermal annealing



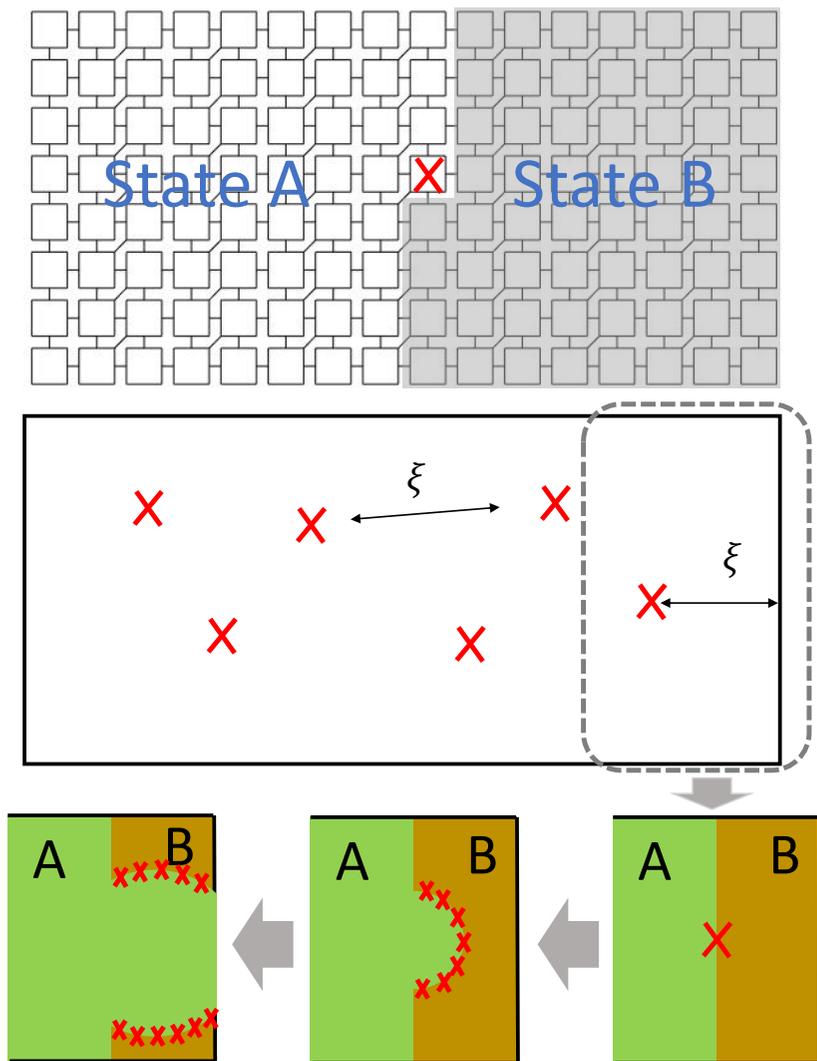
$$\tau \sim \tau_0 \exp(E_B/T)$$

$$E_B \propto \xi \sim \exp(J/2T)$$



$$\tau \sim \tau_0 e^{e^{J/2T}/T}$$

Thermal annealing



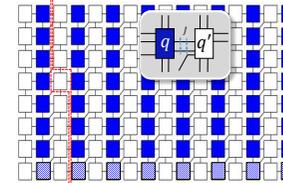
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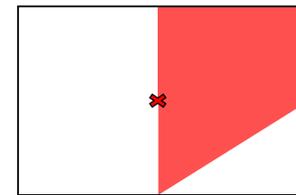
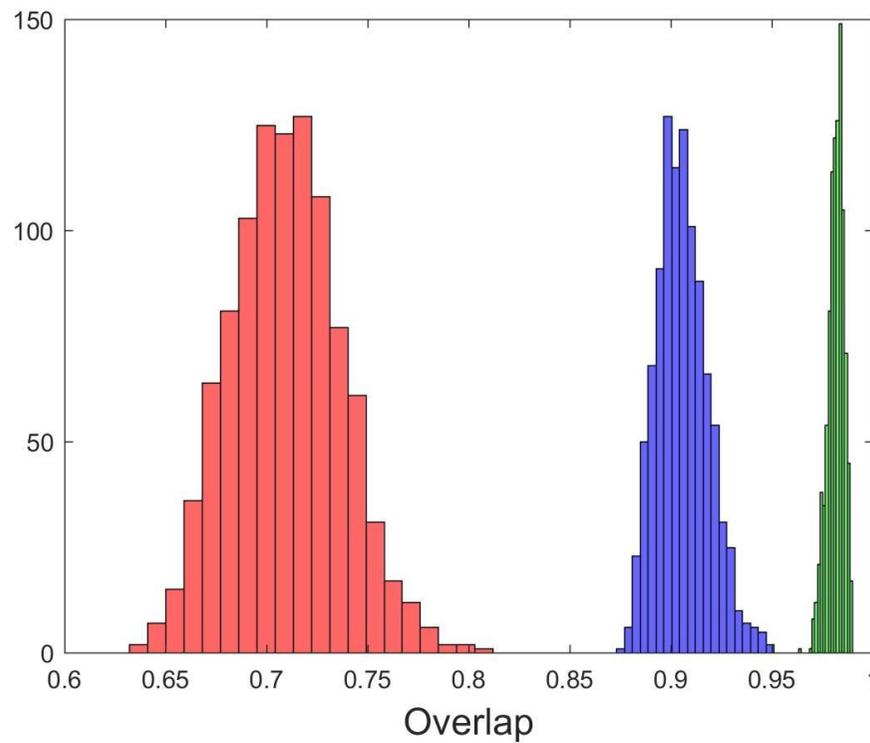


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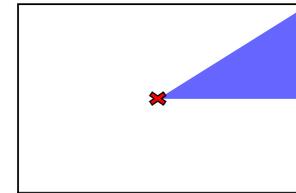
Thermal annealing



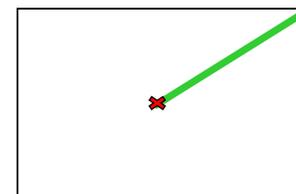
error in
bit type



a



b or *c*

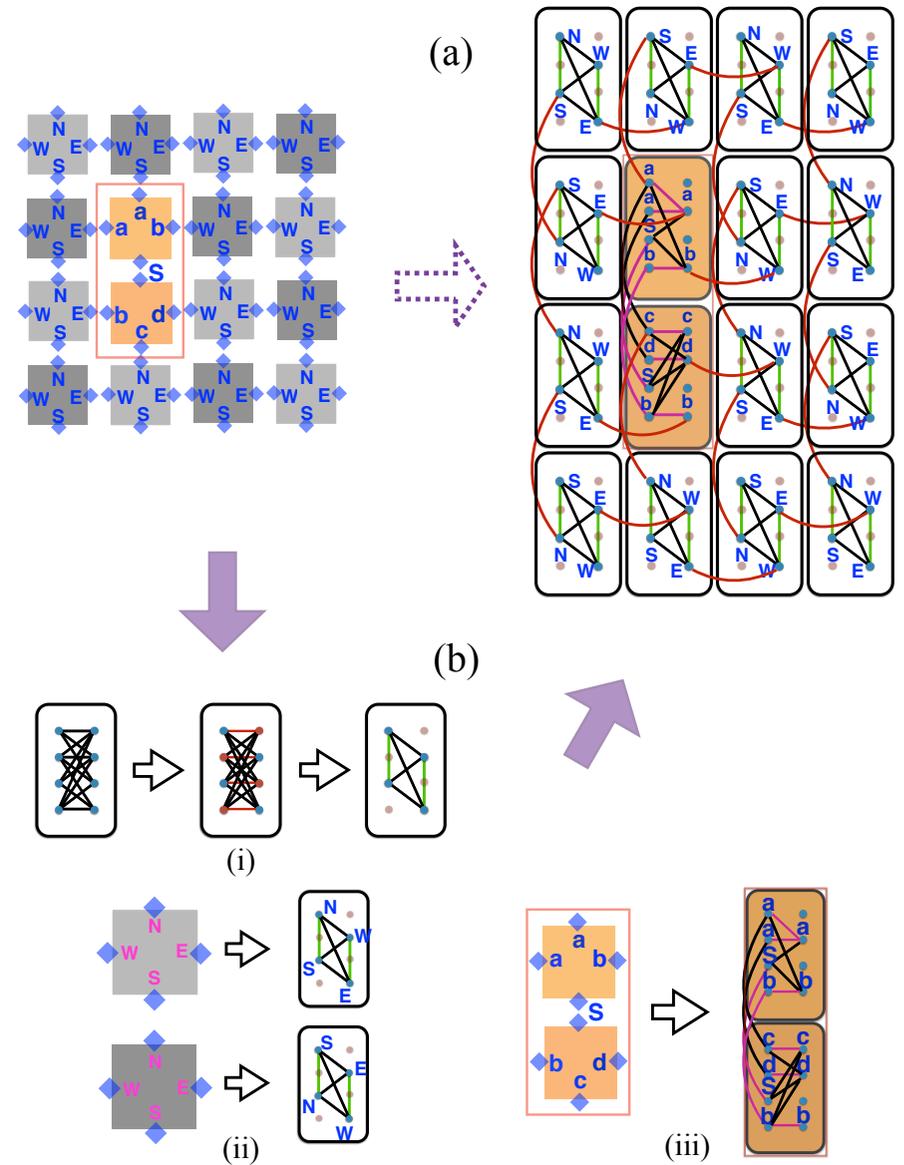
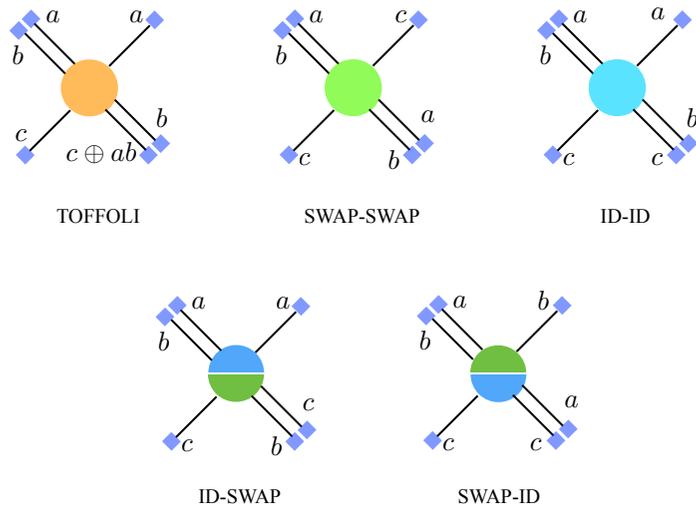


S

Red shows the distribution when flipping spin *a*, blue for spins *b* and *c*, and green for spin *S*

Realizing the vertex model in the chimera architecture

w/ Zhi-Cheng Yang, Stefanos Kourtis, Eduardo Mucciolo, and Andrei Ruckenstein
 Nat. Comm. (2017)



Conclusion

Presented solvable examples of quantum many-body Hamiltonians of systems with exotic spectral properties (topological order) that are unable to reach their ground states as the environment temperature is lowered to absolute zero.

Presented solvable example of a classical/quantum many-body Hamiltonians with ultra-slow thermal relaxation (double exponential in temperature). This system would take time exponential in system size to thermal or quantum anneal.

Out-of-equilibrium strongly correlated quantum systems is **still** an open frontier!

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