Properties of transient superfluids

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Quantum quench

$$\left|\Psi(t)\right\rangle = Te^{-i\int dt' H_f(t')}\left|\Phi_{H_i}\right\rangle$$

Parameters of the Hamiltonian changed in time at some arbitrary rate

Does the system thermalize to some effective temperature?

What happens when Hi is a band insulator but Hf is topological, what if Hi supports one phase (metallic/paramagnetic/normal….) and Hf supports another phase (insulating/magnetic/superconducting……).

THIS TALK: Initial system normal, final Hamiltonian can support superconductivity

Recent reviews:

Quantum quenches in 1+1 dimensional conformal field theories
Pasquale Calabrese, John Cardy **arXiv:1603.02889**

Quantum quench dynamics, Aditi Mitra, Annual Reviews of Condensed Matter Physics, 2018.

An optically stimulated superconducting-like phase in K_3C_{60} far above equilibrium T_c

Nature **530**, 461 (2016).

M. Mitrano[1], A. Cantaluppi[1], D. Nicoletti[1], S. Kaiser[1], A. Perucchi[2], S. Lupi[3], P. Di Pietro[2], D. Pontiroli[4], M. Riccò[4], A. Subedi[1], S. R. Clark[5,6], D. Jaksch[5,6], A. Cavalleri[1,5]

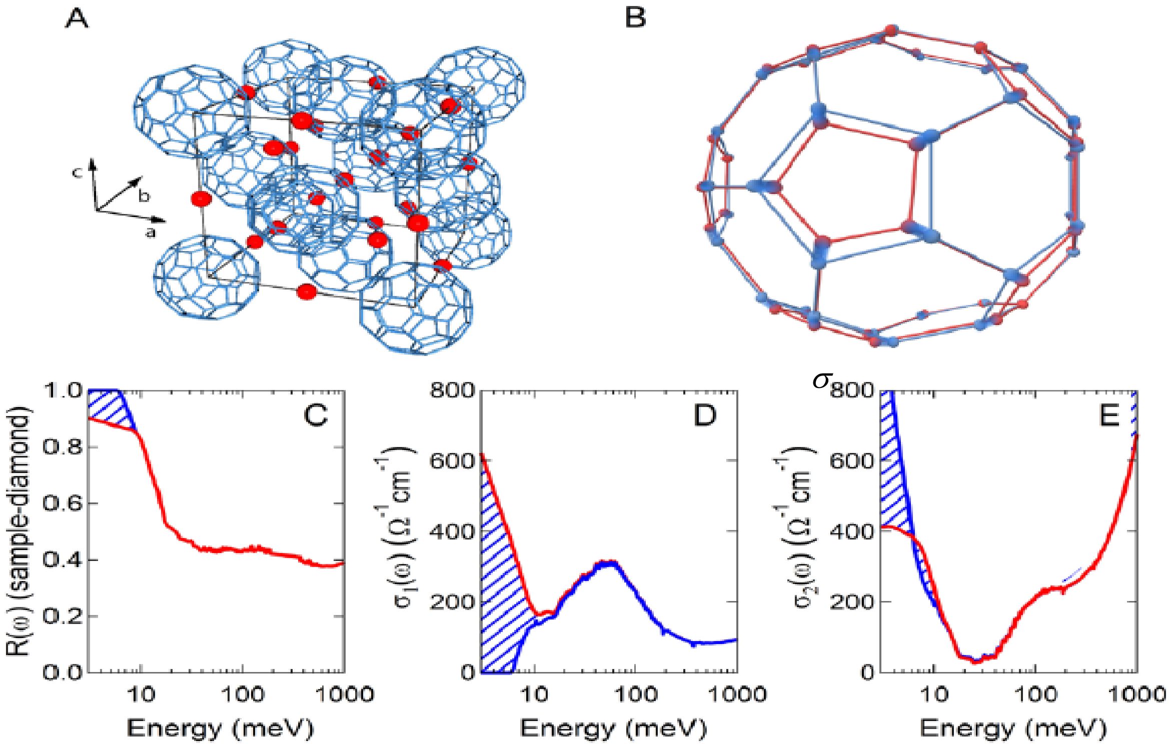


Fig. 1. Structure and equilibrium optical properties of K_3C_{60}. (A) Face centered cubic (fcc) unit cell of K_3C_{60}[xxxviii]. Blue bonds link the C atoms on each C_{60} molecule. K atoms are represented as red spheres. **(B)** C_{60} molecular distortion (red) along the $T_{1u}(4)$ vibrational mode coordinates. Equilibrium structure is displayed in blue. The displacement shown here corresponds to ~12% of the C-C bond length. **(C-E)** Equilibrium reflectivity and complex optical conductivity of K_3C_{60} measured at T = 25 K (red) and T = 10 K (blue).

Drude picture:

$$\sigma(\omega)=\frac{\sigma_0}{1-i\omega\tau}$$

$$\sigma_0=\frac{ne^2\tau}{m}$$

$$\mathrm{Re}\,\sigma=\frac{\sigma_0}{1+\omega^2\tau^2}$$

$$\mathrm{Im}\,\sigma=\frac{\sigma_0\omega\tau}{1+\omega^2\tau^2}$$

Nature **530**, 461 (2016).

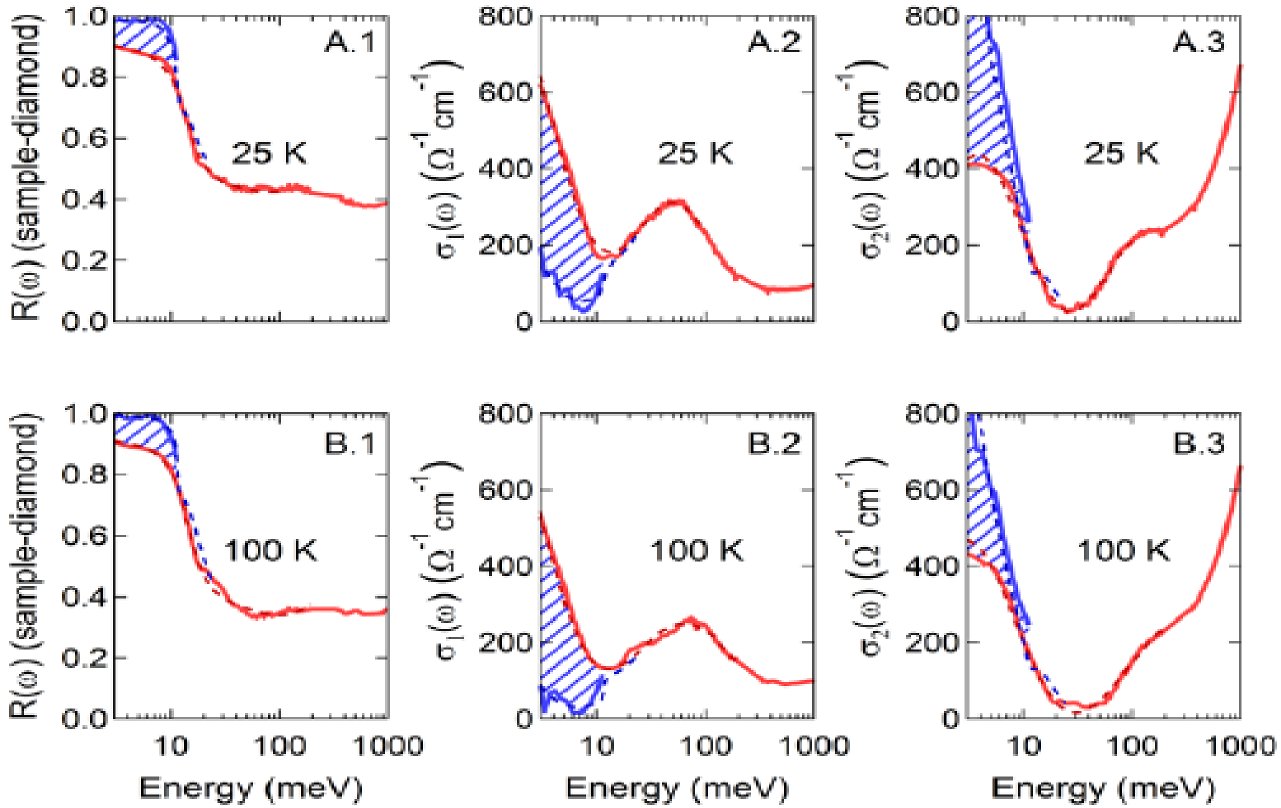


Fig. 2. Transient optical response of photo-excited K_3C_{60} at T = 25 K and T = 100 K. Reflectivity and complex optical conductivity of K_3C_{60} at equilibrium (red) and 1 ps after photo-excitation (blue) with a pump fluence of 1.1 mJ/cm², measured at base temperatures T = 25 K **(A.1-3)** and T = 100 K **(B.1-3)**. Fits to the data are displayed as dashed lines. Those at equilibrium were performed with a Drude-Lorentz model, while those for the excited state using a model describing the optical response of a superconductor with a gap of 11 meV. The band at 55 meV was assumed to stay unaffected.

State lives for 2-10 pico seconds after the initial pump

Theoretical proposals:

Pumped phonons affect the electrons by modifying the effective electron band structure/electron-phonon couplings/attractive Hubbard-U.

- Coulthard, J., Clark, S. R., Al-Assam, S., Cavalleri, A. & Jaksch, D. Enhancement of super-exchange pairing in the periodically-driven Hubbard model. *arXiv:1608.03964* (2016).
- M. Knap, M. Babadi, G. Refael, I. Martin, and E. Demler, Phys. Rev. B **94**, 214504 (2016).
- D. M. Kennes, E. Y. Wilner, D. R. Reichman, and A. J. Millis, Nature Physics **13**, 479 (2017).
- M. A. Sentef, A. Tokuno, A. Georges, and C. Kollath, Phys. Rev. Lett. **118**, 087002 (2017).
- M. A. Sentef, Phys. Rev. B **95**, 205111 (2017).

Non-superconducting scenarios:

Chiriaco, Millis, Aleiner, arxiv:1806.06645

Goal 1: How can one identify the onset of superconductivity in short lived (few ps) states?

Goal 2: Make predictions with as few material dependent fitting parameters as possible.

Outline: Transient properties of the correlated electron system

The pump acts like a "quench" where electrons are subjected to a time-dependent attractive interaction.

NO TRUE LONG RANGE ORDER, YET SUPERCONDUCTING FLUCTUATIONS ARE IMPORTANT

1. **Signatures in time-resolved angle resolved photoemission tr-ARPES.**

   Yonah Lemonik and Aditi Mitra, *Time-resolved spectral density of interacting fermions following a quench to a superconducting critical point*, Phys. Rev. B 96, 104506 (2017).

2. **Signatures in transport such as optical conductivity for a clean system.**

   Yonah Lemonik and Aditi Mitra, *Model Predictions for Time-Resolved Transport Measurements Made near the Superfluid Critical Points of Cold Atoms and K_3C_{60} Films*, arxiv:1711.10023, PRL in print.

3. **Signatures in the optical conductivity for a disordered system.**

   Yonah Lemonik and Aditi Mitra, *Quench dynamics of superconducting fluctuations and optical conductivity in a disordered system*, arxiv:1804.09280

4. **An example where the symmetry of the superconducting order-parameter can be controlled by "quench" amplitude.**

   Hossein Dehghani and Aditi Mitra, *Dynamical generation of superconducting order of different symmetries in hexagonal lattices*, Phys. Rev. B 96, 195110 (2017).

Model (clean system)

$$H_i = \sum_{k,\sigma=\uparrow,\downarrow,\tau=1\ldots N} \epsilon\left(k + A(t)\right) c^\dagger_{k\sigma\tau} c_{k\sigma\tau}.$$

Weak probe A(t)

Effect of pumping the phonons:

$$H_f = H_i + \frac{U(t)}{N} \sum_q \Delta^\dagger_q \Delta_q,$$

$$\Delta_q = \sum_{k\tau} c_{k,\uparrow,\tau} c_{-k+q,\downarrow,\tau}; \Delta^\dagger_q = \sum_{k,\tau} c^\dagger_{-k+q,\downarrow,\tau} c^\dagger_{k\uparrow\tau}$$

In a superconducting phase $\langle \Delta(\vec{q} = 0) \rangle$ iis non-zero

For us on the other hand $\langle \Delta(\vec{q} = 0) \rangle$ will be zero. However fluctuations in this quantity will be large.

Fluctuations measured by: $F(q) = \left\langle \left|\Delta(q,t)\right|^2 \right\rangle$

Superconducting Fluctuations in Equilibrium

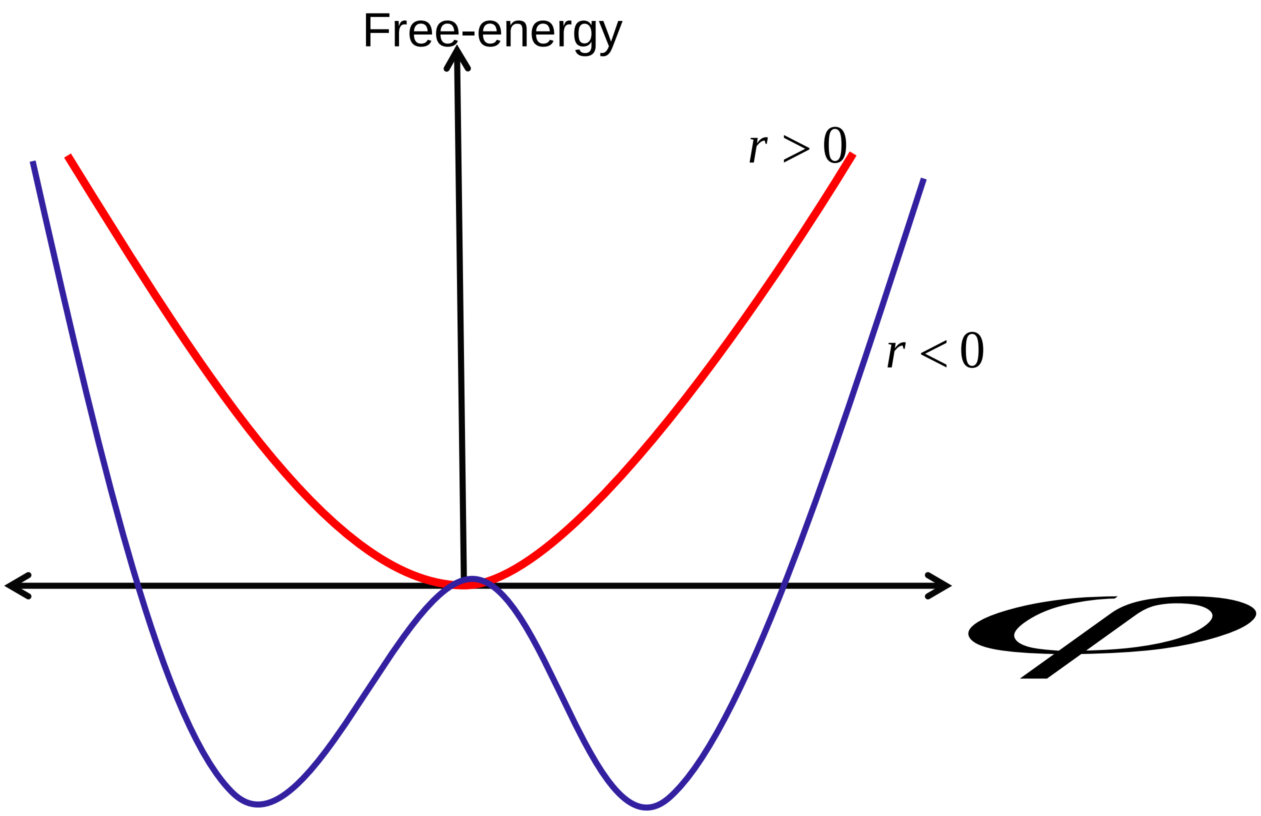
$$\Delta \equiv \varphi$$

$$r \propto U - U_c(T)$$

r: detuning from critical point

$$H_f = \int d^d x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2} r\varphi^2 + \frac{u}{4!N}\varphi^4 \right]$$

$$[\Pi, \varphi] = i$$



$$F_{\mathrm{eq}}(q) = \frac{T}{v^2|q|^2/T + r}; \quad \begin{array}{c} r \propto U - U_c(T) \\ vq \ll T, \quad r \ll T, \end{array}$$

Universal power-laws at critical point (r=0)

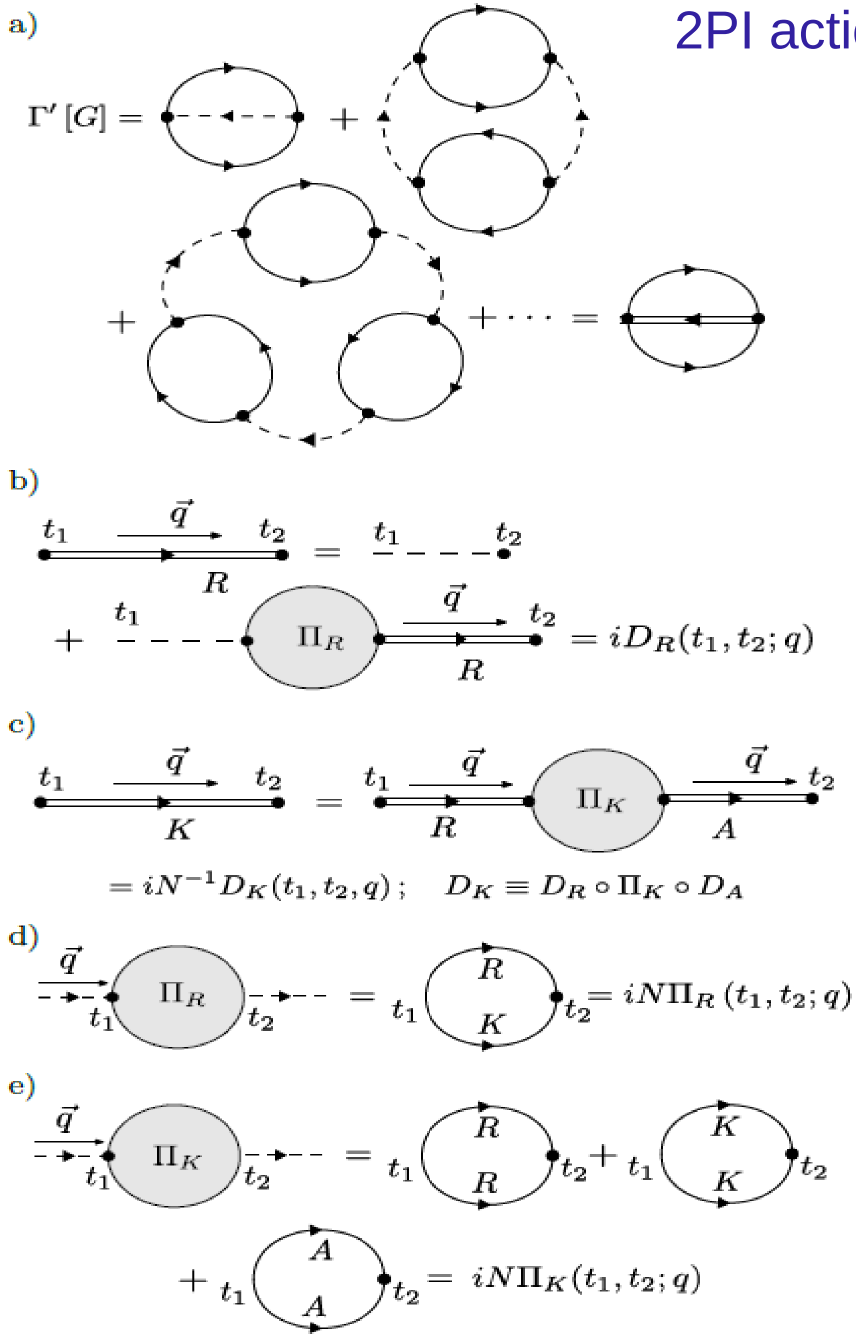
Fluctuations of the order-parameter at two different positions and time are strongly correlated in that they show power-law correlations:

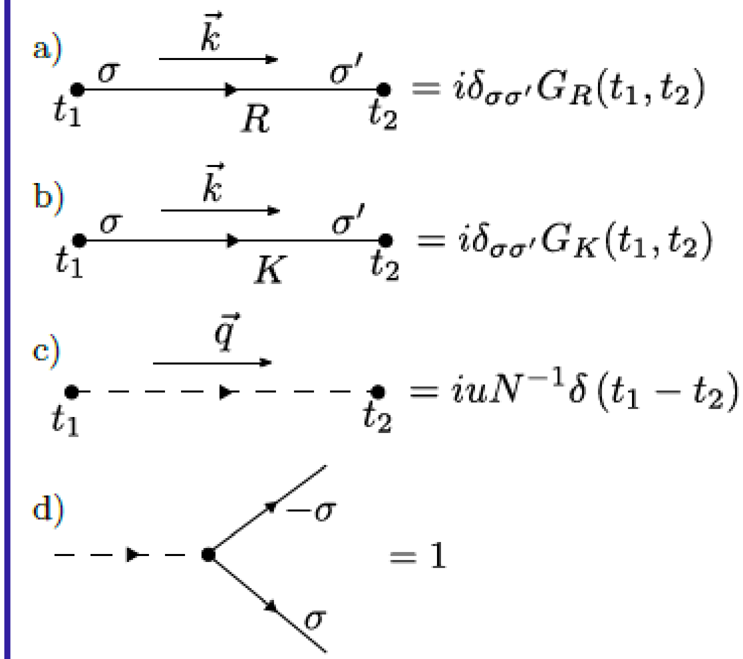
$$\langle \varphi(xt)\varphi(x't) \rangle \approx \frac{1}{|x - x'|^{-2+d+\eta}}$$

$$\langle \varphi(xt)\varphi(xt') \rangle \approx \frac{1}{|t - t'|^{(-2+d+\eta)/z}}$$

Halperin & Hohenberg, Theory of dynamic critical phenomena, Rev. Mod. Phys. 1977

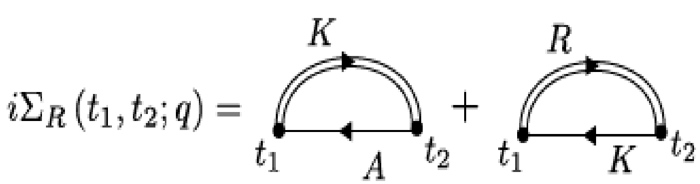
2PI action





$$\Sigma_R[G] \equiv \delta\Gamma'/\delta G_A,$$

$$G_R{}^{-1} = g_R{}^{-1} - \Sigma_R[G],$$



Thermalizing system

Significant simplification possible by exploiting a separation of time scales: Fermi Energy >> Temperature >> r (distance from critical point)

The dynamics for the fluctuations are,

$$\left(\partial_t + 2r(t) + 2\frac{v^2|q|^2}{T}\right) F(q,t) = T,$$

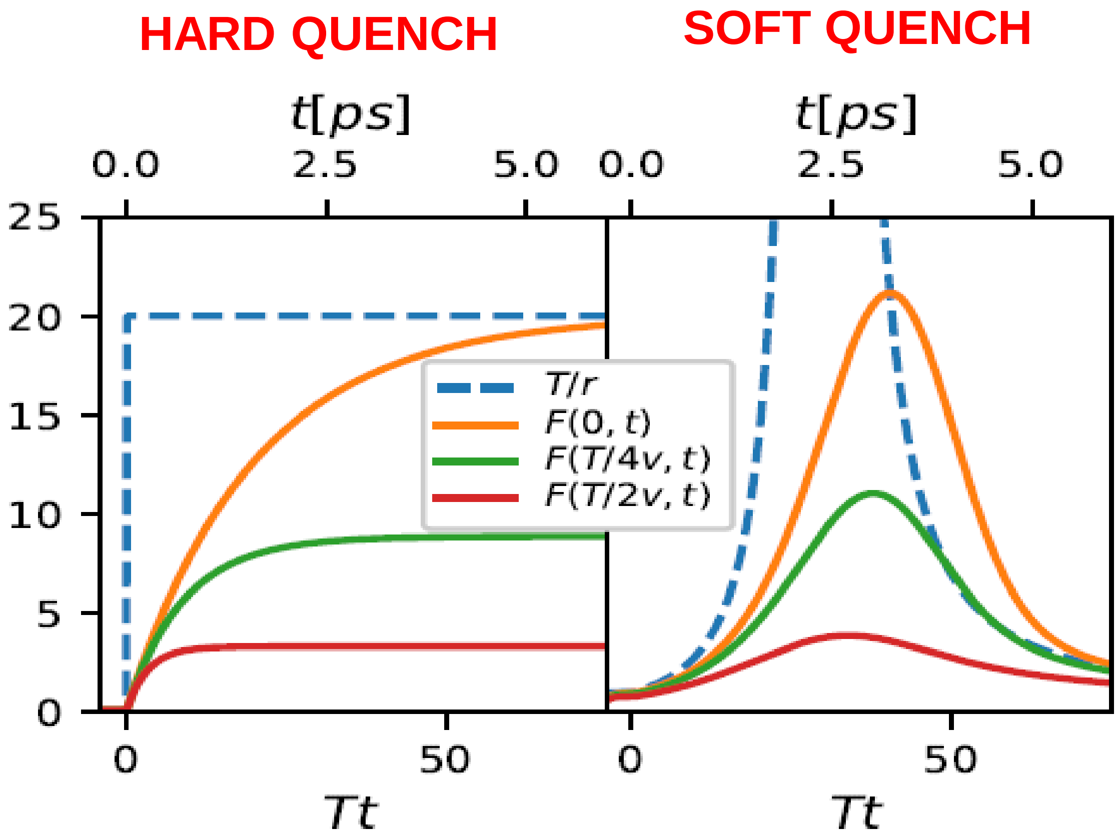
Absence of higher order terms in F is due to a non-equilibrium version of the Ginzburg-Levanyuk criterion:

$$F(q=0,t) \ll E_F/T$$

In equilibrium, this criterion is:

$$r \gg T^2/E_F$$

Thus even if r(t>0)=0, the post quench transient dynamics could obey the Ginzburg-Levanyuk criterion.



100K=1/80fs
=8.6meV

FIG. 1. Growth of superfluid fluctuations $F(q,t)$ following an interaction quench. Fluctuations at several different momenta q are shown. The times are measured in units of T^{-1} (lower axis) and in terms of ps (upper axis) for $T \sim 100$K. The quantity T/r is the inverse of the detuning from the critical point, which at equilibrium is equal to $F(q=0)$. Left: hard quench from the normal state to $T/r = 20$. Right: Soft quench, with $r(t>0) = T[1-(t/t_*)e\exp(-t/t_*)]$, $t_*T = 30$.

HARD QUENCH in spatial dimension d>1

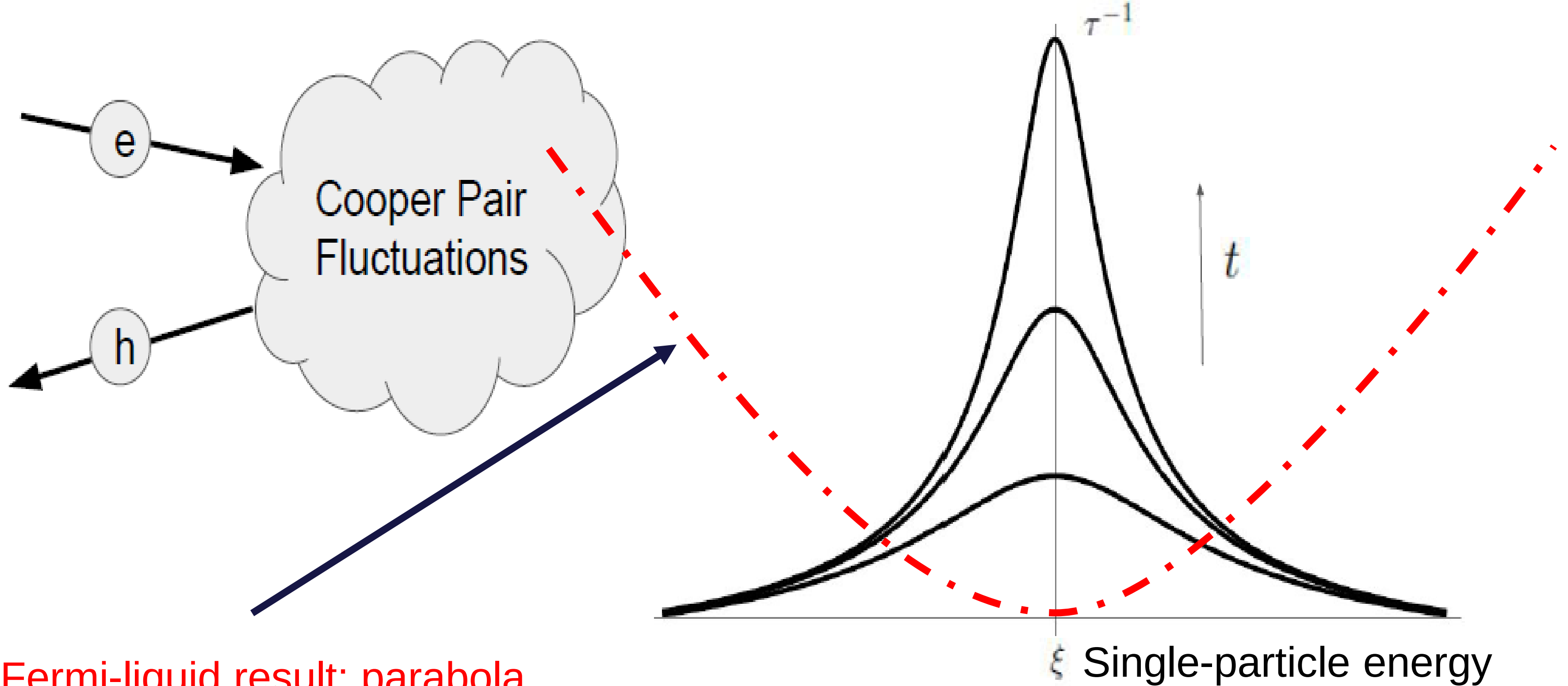
Electron Andreev reflects into a hole. This process is resonant for electrons at the Fermi energy.

Lifetime

Universal

Can we see this in observables?

$$Im\Sigma \equiv \tau^{-1} = t^{\frac{3-d}{2}} g_d(\xi^2 T t)$$



Fermi-liquid result: parabola

Non-Fermi liquid as life-time is shorter at the Fermi-energy.

Conductivity of transient state

$$J(t) = \int dt' \sigma(t, t') E(t'),$$

Optical conductivity $\sigma(\omega, t) = \int d\tau \, \sigma(t + \tau, t) \exp(i\omega\tau)$

Galilean invariant system: Momentum is proportional to velocity.
Since momentum is conserved, current never decays

$$\sigma(t, t') \propto \theta(t - t')$$

Broken Galilean invariance: Momentum and velocity no longer proportional. This implies a component of the current will decay.
(We neglect Umklapp processes)

Obtaining current dynamics from the 2PI formalism

Minimizing the 2PI action yields the quantum kinetic equation:

$$g_R^{-1} \cdot G_K - G_K \cdot g_A^{-1} = \Sigma_K \cdot G_A - \Sigma_R \cdot G_K - G_R \cdot \Sigma_K + G_K \cdot \Sigma_A$$

Put the electric field from the probe pulse in the 2PI action, and minimize it. The new saddle point yields the correct “conserving dynamics” of the system.

In the above equation, shift in the saddle point is encoded in shifts in the Green’s functions:

$$g \rightarrow g + \delta g$$

$$G \rightarrow G + \delta G$$

$$\delta(g, G) = O(E)$$

$$\Sigma \rightarrow \sum + \delta\Sigma$$

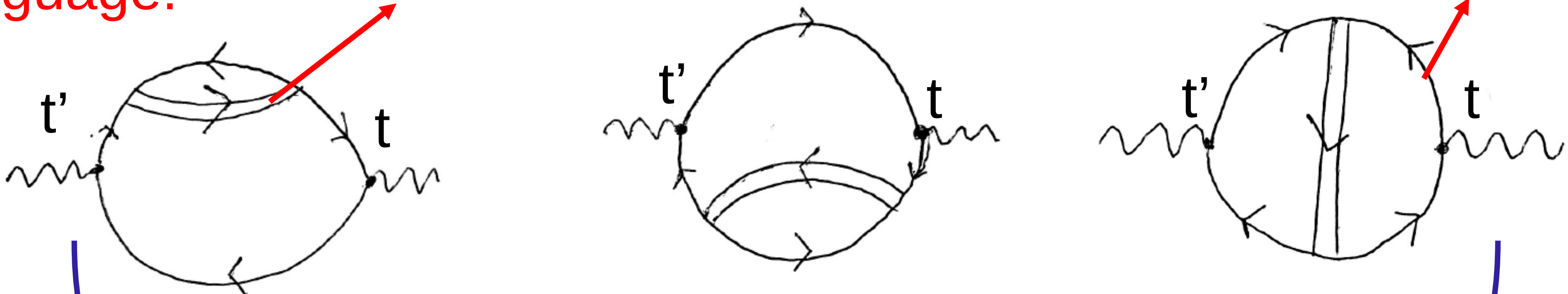
$$D \rightarrow D + \delta D$$

$$\Pi \rightarrow \Pi + \delta\Pi$$

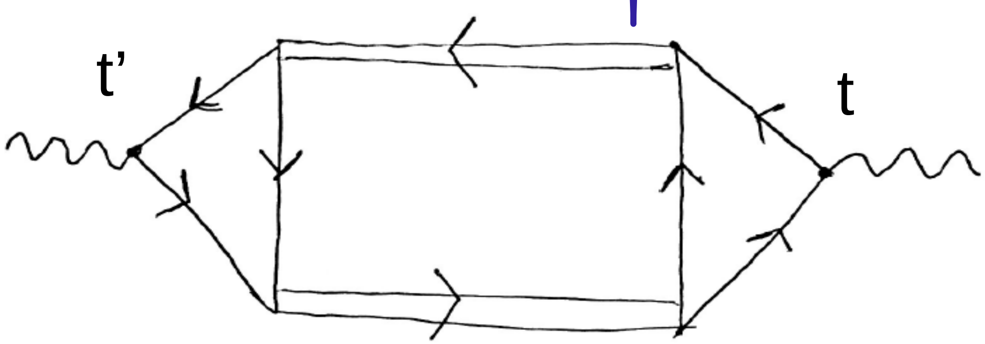
In diagrammatic language:

Fluctuation propagator

Electron propagator

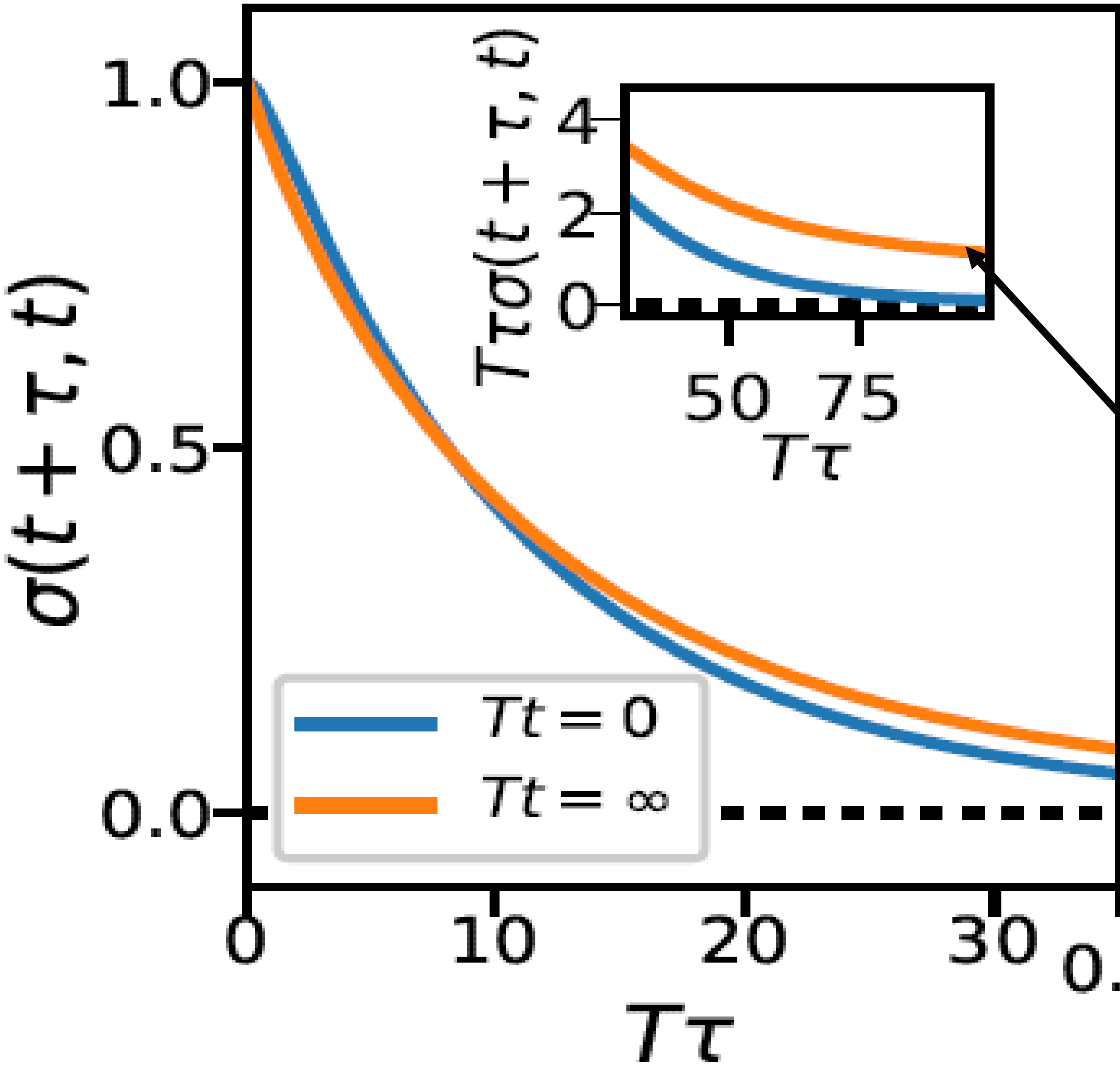


$$\partial_t J(t) - \frac{\rho}{\bar{m}} E(t) = -\frac{1}{\tau_r} \Bigg(A(t) J(t) - \alpha \int_0^t dt' \left[B(t,t') J(t') + \frac{\rho}{2\bar{m}} C(t,t') E(t') \right] \Bigg)$$



Azlamazov-Larkin diagram

Conductivity for the hard quench. $r = 0$



$$J(\tau) = \sigma(t+\tau, t),$$
$$E(t) = \delta(t)$$

t=Delay time between quench and probe

Fluctuations provide a low-resistance current carrying channel

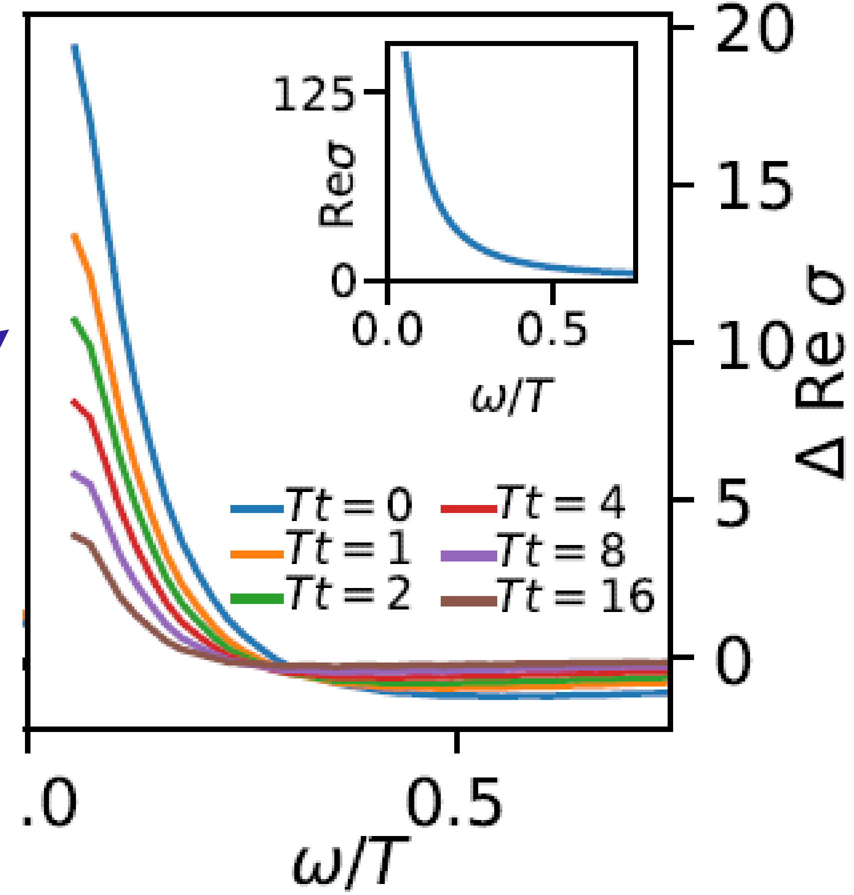
$T\tau_r = 5,\ \alpha = 0.5$

In the inset are the tails of $\sigma(t+\tau,t)$, $T\tau > 30$, plotted as $T\tau\sigma(t+\tau,t)$ to improve visibility.

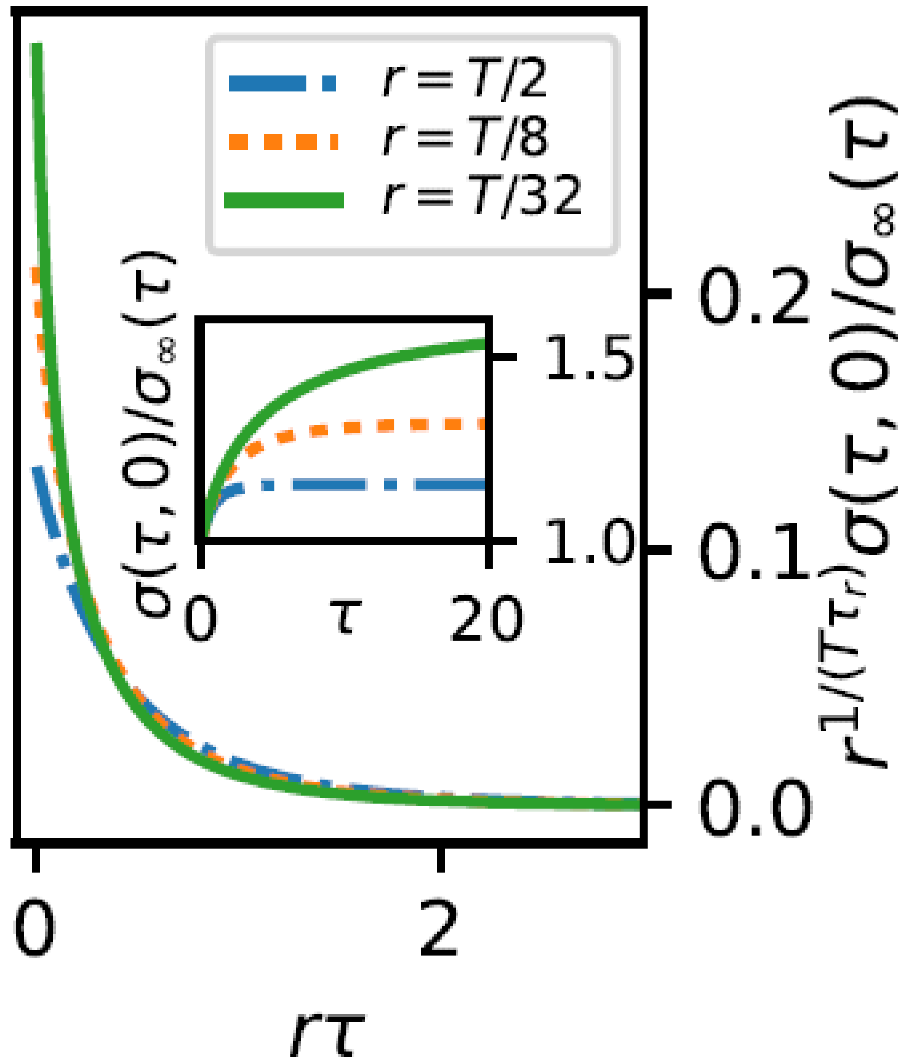
Time-evolution of the optical conductivity, for a critical quench

$$\sigma(\omega,t) = \int d\tau\, \sigma(t+\tau,t) \exp(i\omega\tau)$$

Low-frequency conductivity changes as log(t). Saturation at Log[min(frequency,1/t)]

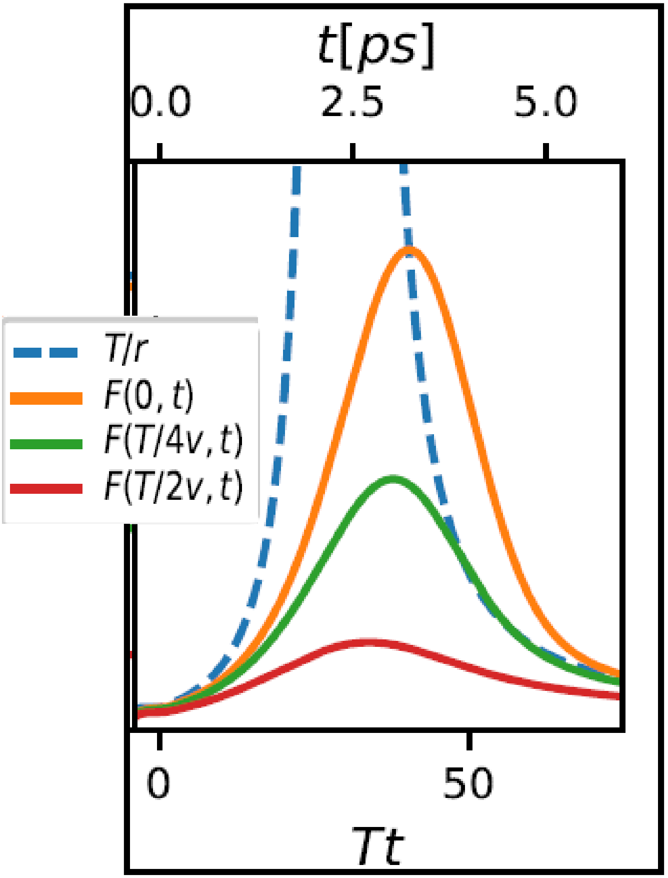


$\mathrm{Re}\left[\sigma(\omega,t) - \sigma(\omega,t\to\infty)\right]$ for different times since the quench. Note the times increase in a geometric fashion. Inset: $\mathrm{Re}\sigma(\omega,t\to\infty)$. This diverges as $\log\omega$ as $\omega\to 0$ so all curves are clipped at $\omega = .01T$



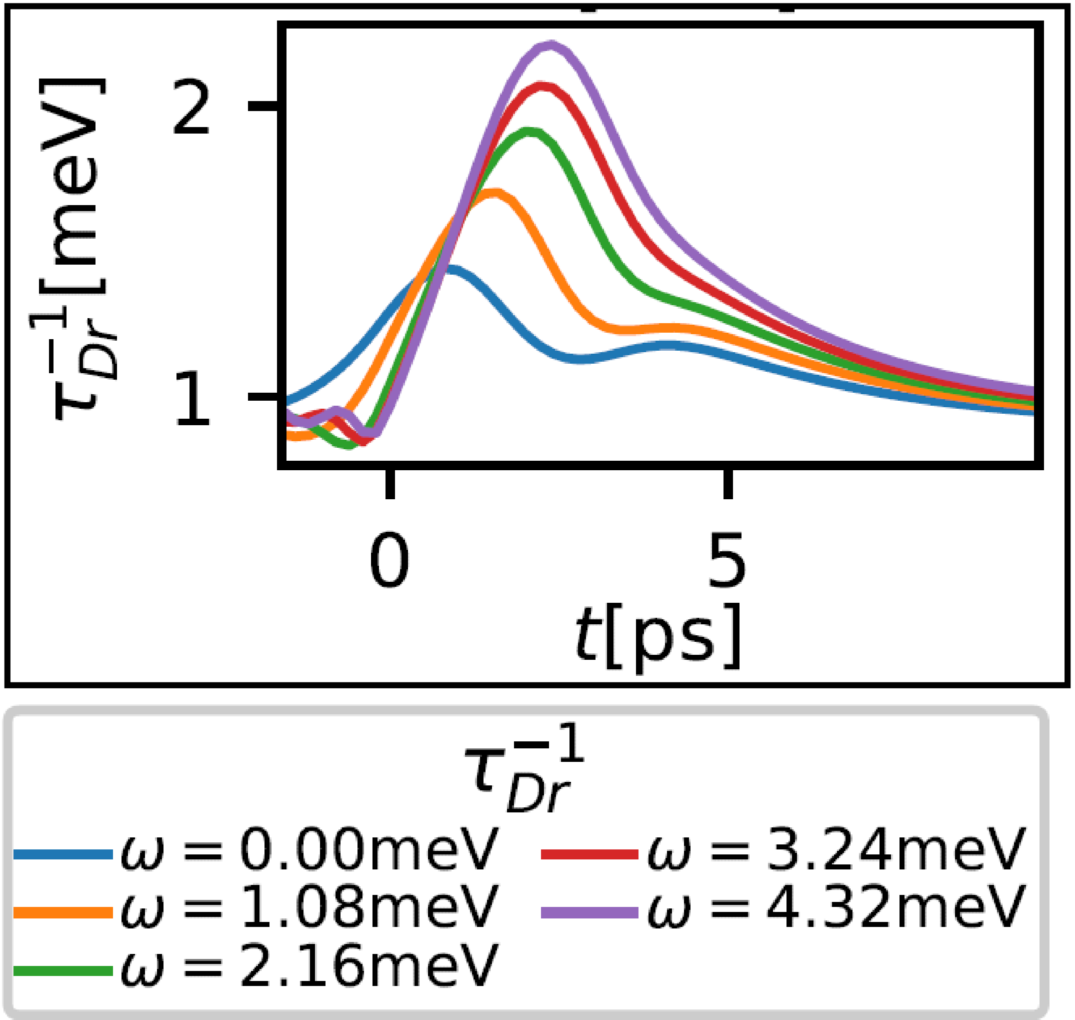
Scaling plot of the $\sigma(\tau, 0)$ at $\alpha = 0$ for different detunings r. Inset: unscaled $\sigma(\tau, 0)$. Plots shown for $T\tau_r = 5$.

Optical conductivity for the soft quench



Drude scattering time defined as:

$$\tau_{\rm Dr}(\omega) = {\rm Im}\left[\sigma\right] / \left[\omega {\rm Re}\sigma\right]$$



1. Real systems have disorder.

2. Fluctuation conductivity for a strongly disordered metal is well studied. Kubo formalism (linear response) for the optical conductivity is used.

A. I. Larkin and A. A. Varlamov, in *Handbook on Superconductivity: Conventional and Unconventional Superconductors*, edited by K.-H.Bennemann and J. Ketterson (Springer, 2002).

We generalize these studies to a quantum quench.

Disorder

$$H = \sum_{kk's}\Big[(\varepsilon_k \delta_{kk'} + V_{k-k'})\, c^{\dagger}_{ks} c_{k's}$$

$$+ U(t) \sum_{qs'} c^{\dagger}_{ks} c_{k-qs} c^{\dagger}_{k'-qs'} c_{k's'}\Big].$$

$$\langle V_q V_{-q'} \rangle \; = \; \delta_{qq'}/2\pi\nu\tau.$$

As before:
Electrons assumed to thermalize rapidly at temperature T,
Fluctuation propagators are slow due to proximity to critical point.

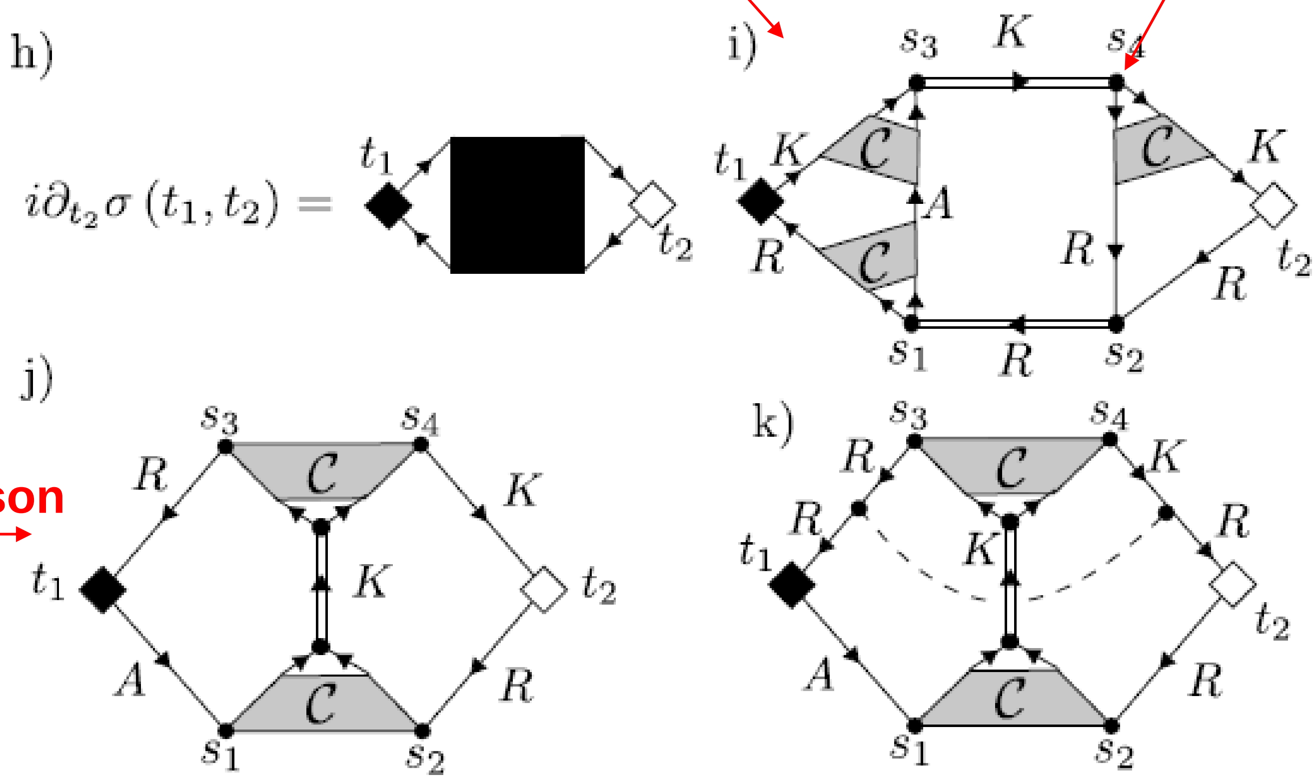
We also assume strong disorder: $T\tau/\hbar \ll 1$
(many scatterings within a de-Broglie wave-length)

Conductivity = Drude formula + fluctuation correction

$$\sigma = \frac{ne^2\tau}{m} + \ldots\ldots\ldots$$

Azlamazov-Larkin

Maki-Thompson



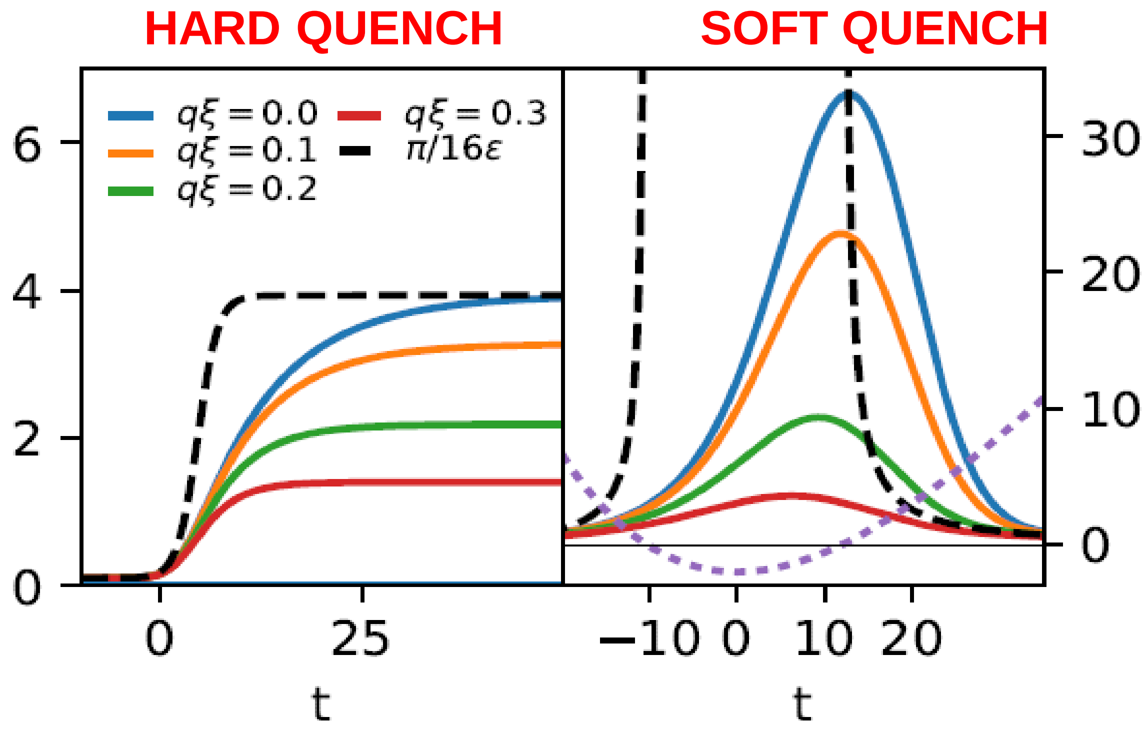


FIG. 1. The growth of the superconducting fluctuation at different lengths (q^{-1}) under a changing detuning $\epsilon(t)$ from the superconducting critical point. The time is given in units of $\pi\hbar/8T$. The dashed line shows $\pi/16\epsilon(t)$ which equals $B(q=0)$ in equilibrium. The dotted purple line in the right panel gives $\epsilon(t)$ at arbitrary scale. In the left panel the detuning saturates at the value $\epsilon = 0.05$. In the right panel, the detuning is $\epsilon(t) = \epsilon_0 + (\epsilon_{\min} - \epsilon_0)(t/t_* e)\exp(-t/t_*)\theta(t)$ with the parameters $t_* = 30$, $\epsilon_0 = 1$, $\epsilon_{\min} = -0.05$.

coherence length $\xi = \sqrt{\pi D/8T}$

$D = v_F^2 \tau / d$ is the diffusion constant

AGING FOR A CRITICAL QUENCH:

Decay of the current depends on when it was probed, with a power-law dependence on the two time-scales

Azlamazov-Larkin

Probe at t2, current at t1

$$\sigma(t_1,t_2) = \frac{2T}{\pi}\left[-\frac{t_2}{t_1} - \log\left(1-\frac{t_2}{t_1}\right) + e^{-2(t_1-t_2)/\tau_\phi}\log\left(1+\frac{1}{2}\frac{t_1+t_2}{t_1-t_2}\right)\right],$$

Maki-Thompson

Experimentally determine the phase-breaking time?

HARD QUENCH

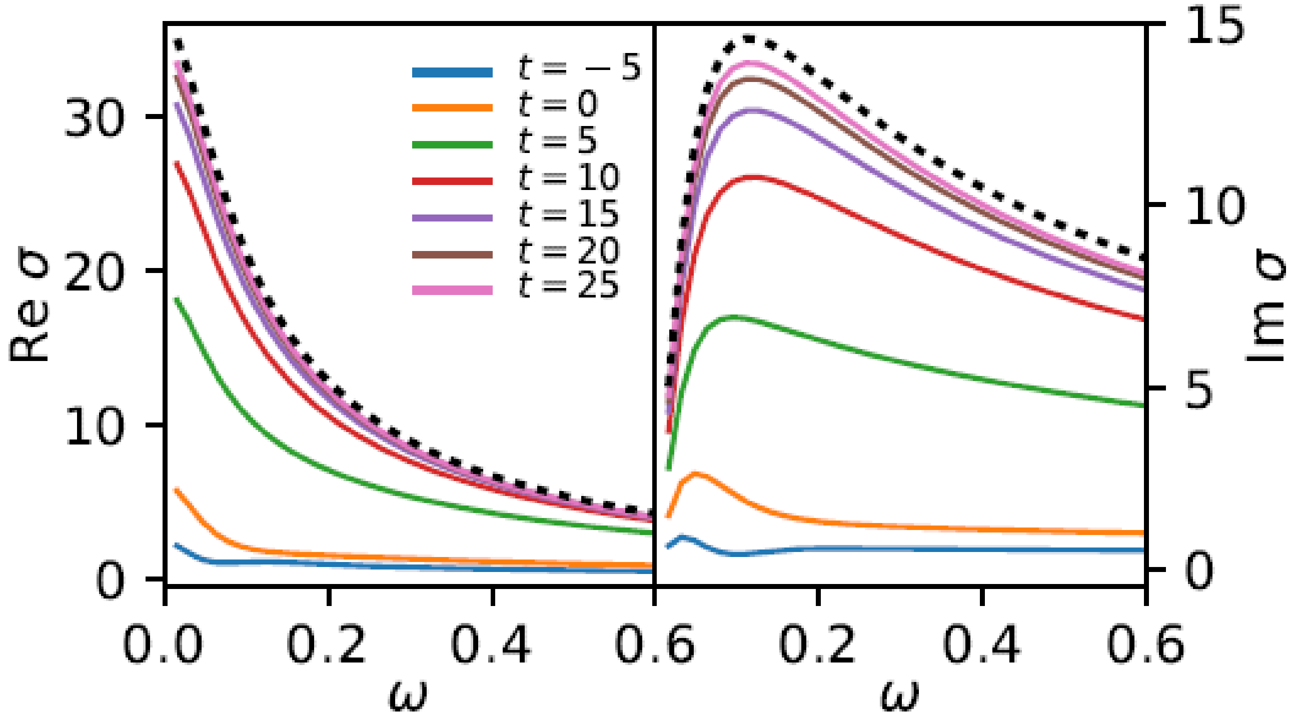


FIG. 3. Conductivity $[e^2/\hbar]$ as a function of frequency for several times. The left panel shows real part, the right panel shows the imaginary part. All times are in units of $\pi\hbar/8T$. The detuning ϵ varies according to Fig. 1, left panel and $\tau_\phi = 20 \times (\hbar\pi/8T)$. The dashed line gives the equilibrium result for the final value of the detuning $\epsilon = .05$.

SOFT QUENCH

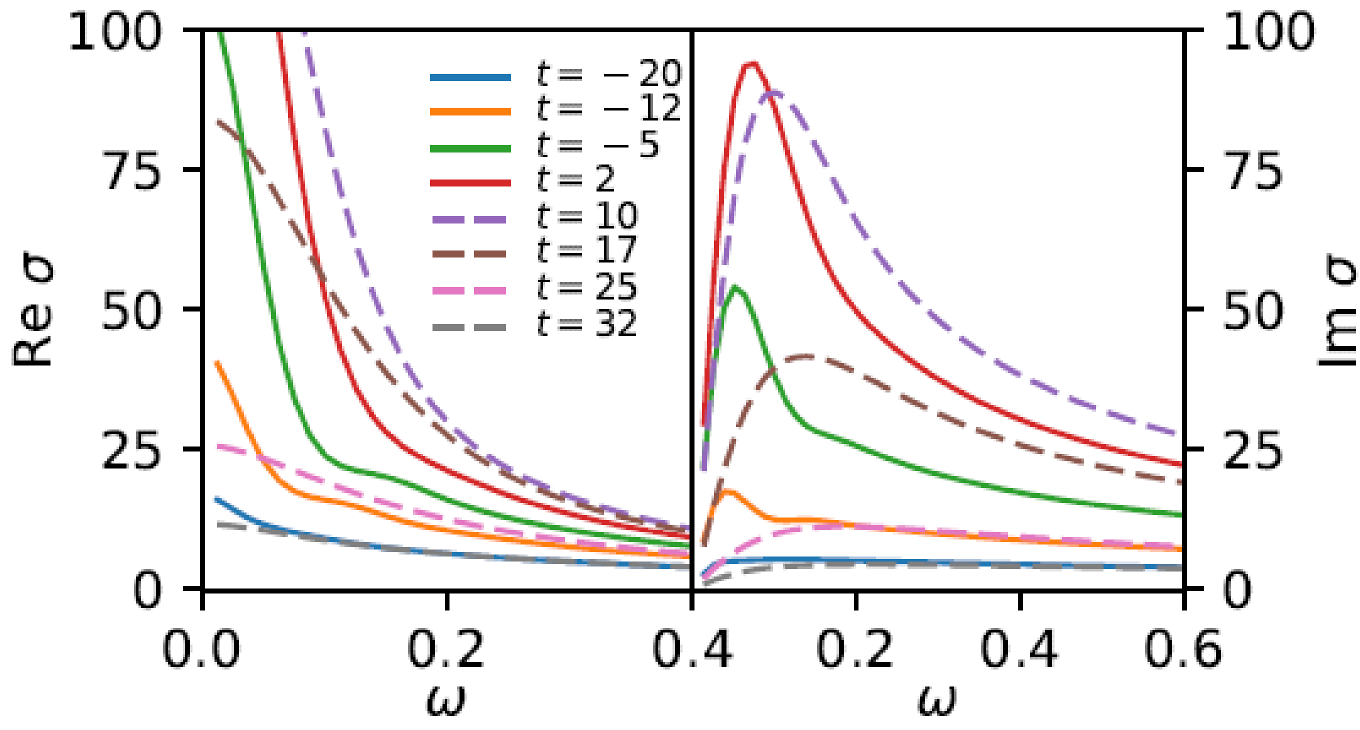
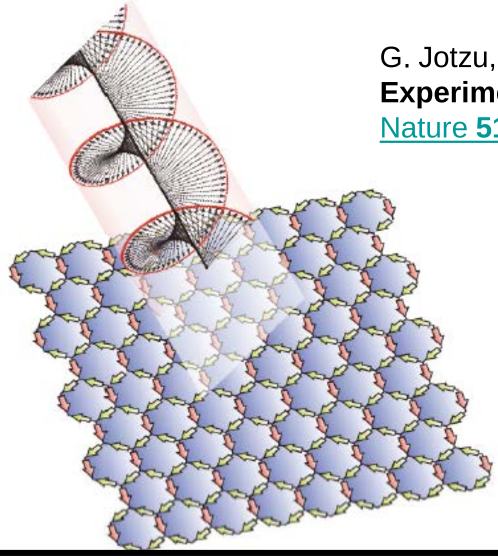


FIG. 4. Conductivity $[e^2/\hbar]$ as a function of frequency for several times, all in units of $\pi\hbar/8T$. The detuning is given in Fig. 1, right panel and $\tau_\phi = 20 \times (\hbar\pi/8T)$. In order to improve clarity, lines for $t \leq 2$ are shown with full lines and those for $t > 2$ are shown with dashed lines. The conductivity σ as $\omega \to 0$ reaches a maximum of ~ 175 at $t \approx 7$.

Initially free system of electrons. Quench involves turning on attractive BCS interactions.

Many competing superconducting order-parameters in realistic systems. By tuning the quench amplitude, order parameters of different symmetries can be realized.

Graphene irradiated with circularly polarized laser



G. Jotzu, T. Esslinger et al.:
Experimental realization of the topological Haldane model
Nature **515**, 237-240 (2014).

Oka and Aoki PRB 2009 in graphene
Other models:
Kitagawa et al PRB 2011,
Lindner et al Nature 2011
Yao, MacDonald, Niu et al PRL 2007

$$U(T+t,t)=e^{-iH_{eff}T}$$

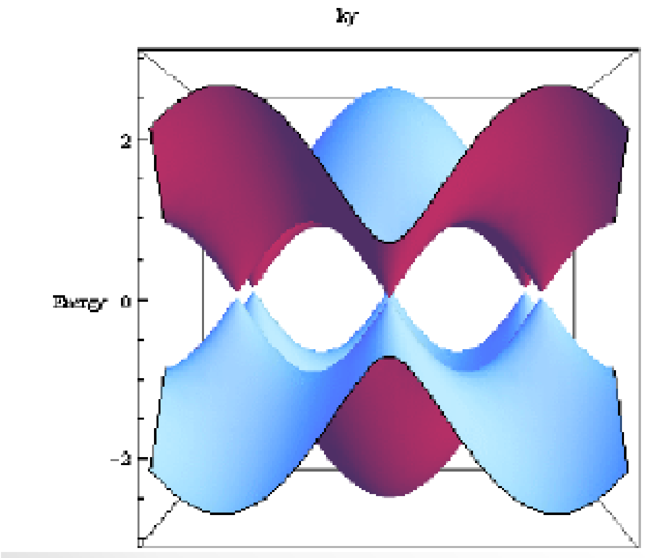
$$H_{eff}\approx k_x\sigma_x\tau_z+k_y\sigma_y+\frac{A_0^2}{\Omega}\sigma_z\tau_z$$

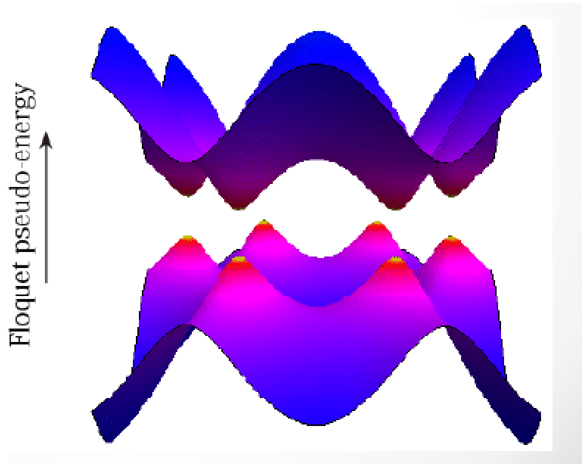
σ: Sublattice index

τ: K,K' points

Breaks time-reversal

H_{eff} **Maps onto the Haldane model**





Initial Hamiltonian Heff: Graphene and/or Haldane model

Final Hamiltonian H: Attractive BCS interactions

$$V = J \sum_{<ij>} \left[\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j \right]$$

$$\Delta_\alpha = J \left\langle b_{i+\alpha\downarrow} a_{i\uparrow} - b_{i+\alpha\uparrow} a_{i\downarrow} \right\rangle$$

$\alpha = 1, 2, 3$ denote the three nearest neighbor bonds

$$H = H_{\text{eff}} - \sum_{k,\alpha} \left[\Delta_\alpha e^{i\vec{k}\cdot\vec{a}_\alpha} \left(a^\dagger_{k\uparrow} b^\dagger_{-k\downarrow} - a^\dagger_{k\downarrow} b^\dagger_{-k\uparrow} \right) + h.c. \right].$$

We will monitor how an initial superconducting fluctuation evolves in time.

$$\Delta_\alpha(t) = J \sum_\beta \int_0^t dt' \Pi^R_{\alpha\beta}(q=0, t, t') \Delta_\beta(t').$$

SC in doped graphene in equilibrium

R. Nandkishore, L. Levitov, and A. Chubukov, Nature Physics **8**, 158 (2012).

A. Black-Schaffer and C. Honerkamp, Journal of Physics Condensed Matter **26** (2014).

$$\Delta_s = \frac{1}{\sqrt{3}}\,[1,1,1]$$

$$\Delta_{d+id} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ e^{2\pi i/3} \\ e^{4\pi i/3} \end{pmatrix}$$

$$\Delta_{d-id} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ e^{4\pi i/3} \\ e^{2\pi i/3} \end{pmatrix}$$

Graphene: C3 symmetry and time-reversal symmetry

$$\Pi^R(t) = \begin{pmatrix} A(t) & B(t) & B(t) \\ B(t) & A(t) & B(t) \\ B(t) & B(t) & A(t) \end{pmatrix}.$$

Eigenvalues (EV): s-wave. And two degenerate EV: d+id and d-id

Haldane model: C3 symmetry but broken time-reversal symmetry

$$\Pi^R(t) = \begin{pmatrix} A(t) & B(t) & C(t) \\ C(t) & A(t) & B(t) \\ B(t) & C(t) & A(t) \end{pmatrix}.$$

3 Non-degenerate eigenvalues: s-wave, d+id and d-id.

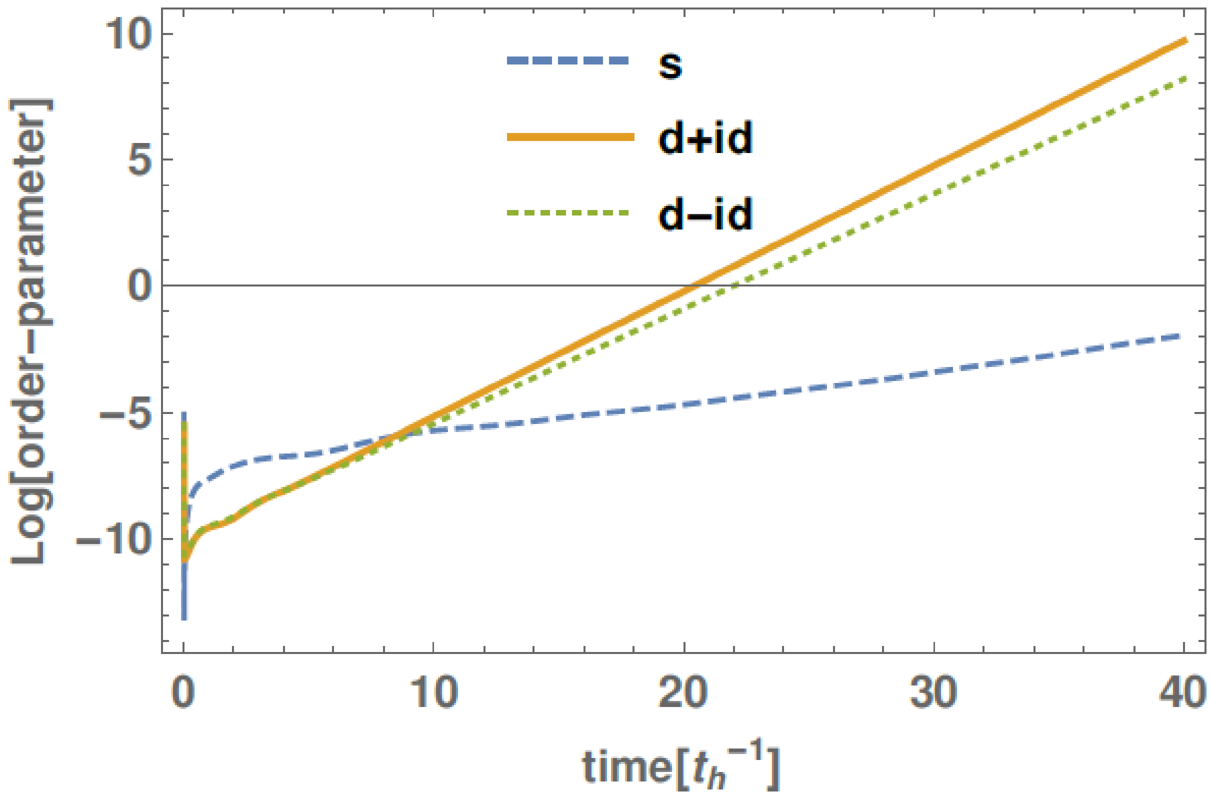


FIG. 1: Haldane model

$A_0 a = 0.5, \Omega = 10t_h, J = 1.82t_h, T = 0.01t_h$ and doping $\delta = 0.1$. Time-evolution of the logarithm of an initial random vector. The time-evolution is projected along the three orthogonal directions with s, $d + id$, $d - id$ symmetry. The slopes indicate that for the chosen parameters $d + id$ is the fastest growing instability, followed by $d - id$ and then s. Time is in units of t_h^{-1}.

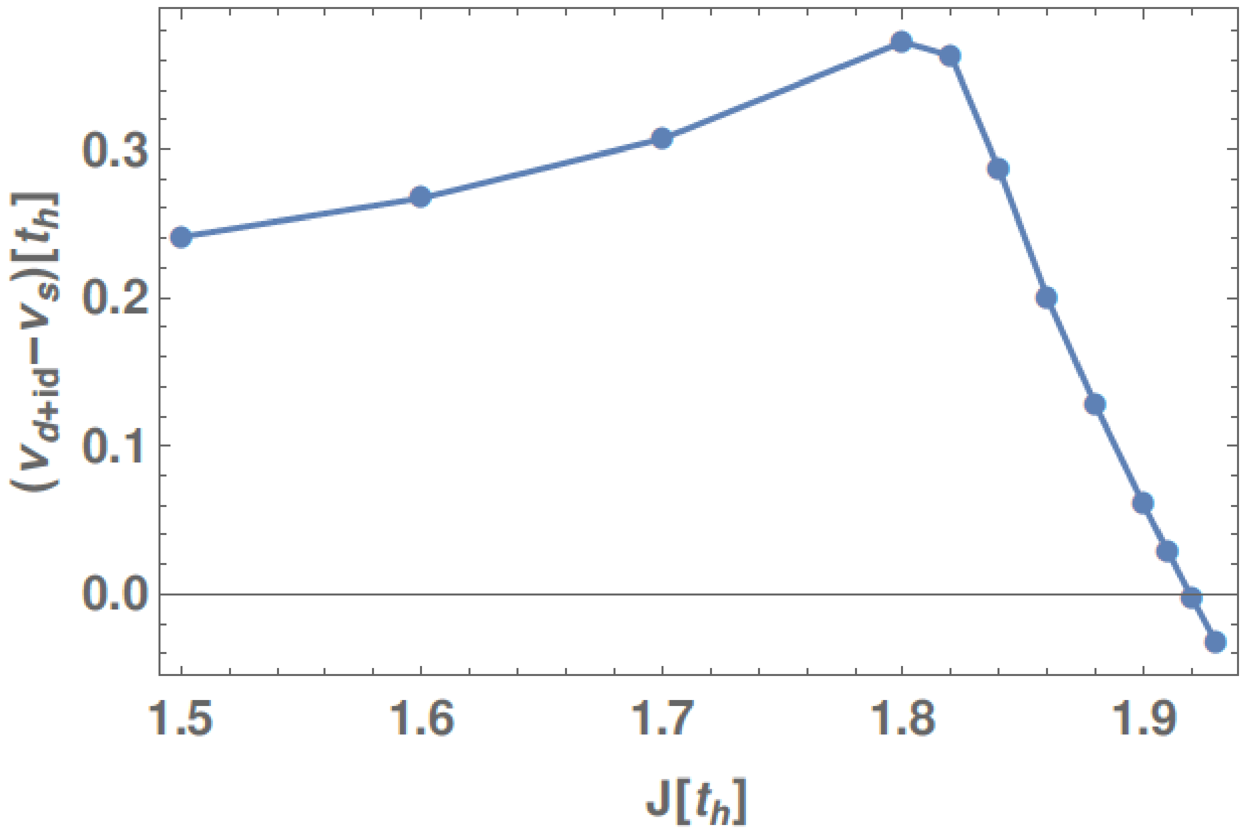


FIG. 2: Haldane model $A_0 a = 0.5, \Omega = 10t_h, T = 0.01t_h$ and doping $\delta = 0.1$ (same as Fig. 1). As the quench amplitude J is increased, the difference between the growth rate of chiral d-wave (ν_{d+id}) and s-wave (ν_s) varies as shown above. The difference first increases, and then decreases rapidly. For quench amplitudes larger than $J_c \sim 1.9$ the s-wave is preferred.

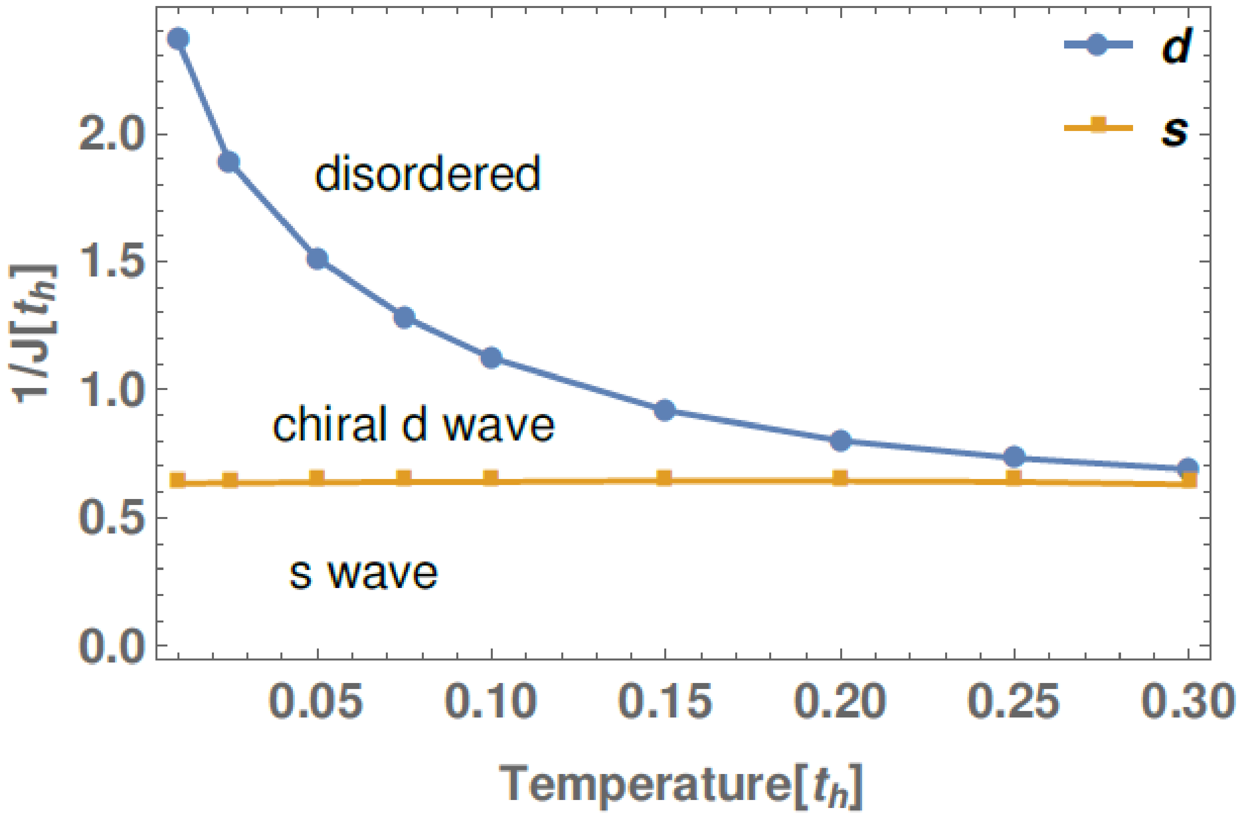
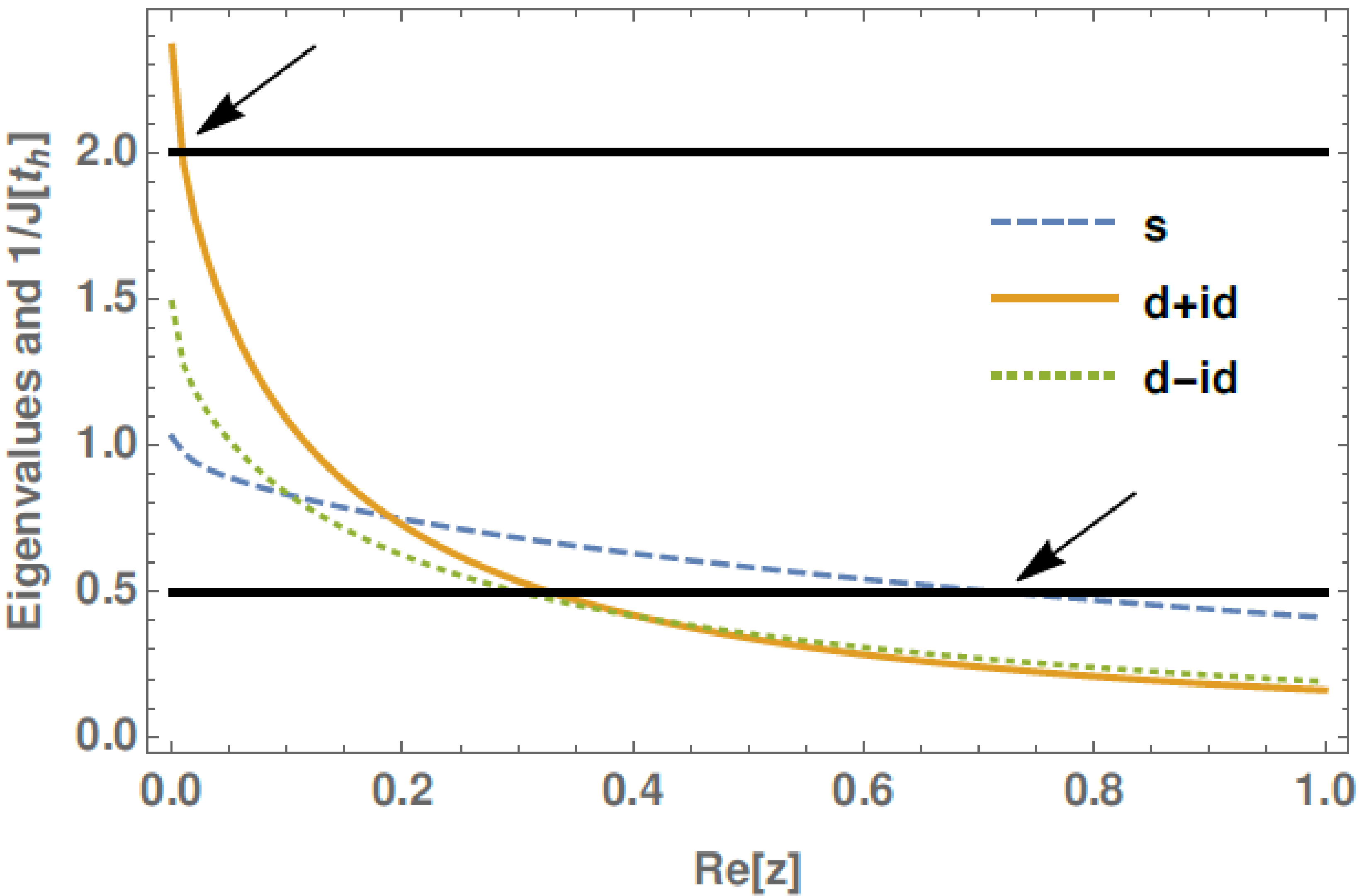


FIG. 5: Haldane model $A_0 a = 1.0, \Omega = 10 t_h$ and doping $\delta = 0.1$. The phase diagram determined by the fastest growing order parameter. The line corresponding to the transition from the disordered (normal) phase to the chiral d-wave phase coincides with the equilibrium phase diagram. As the quench amplitude is increased, the s-wave phase is preferred.

Dephasing effects due to excited quasi-particles detrimental for d-wave pairing



Eigenvalues and 1/J[t_h]
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Re[z]
s
d+id
d−id

Conclusions:

1. Pump-probe spectroscopy of correlated materials, and near unitary dynamics of cold-atomic gases, has opened up exciting new regimes of non-equilibrium physics of interacting systems.
2. Results were presented for an interacting electron gas that traverses arbitrarily close to a superconducting critical point. Even though the system is not ordered in the conventional sense, time-dependent behavior is strongly influenced by superconducting fluctuations.
3. Time resolved optical conductivity and time resolved ARPES show clear features of approaching a superconducting critical point.
4. Strongly disordered case reveals power-law scaling of the conductivity for critical quenches whose functional form depends on the relative importance of Azlamazov-Larkin and Maki-Thompson terms.
5. An example was also presented of how the symmetries of a superconducting order parameter may be influenced by the interaction quench amplitude.