Quantum Information, Tensor Networks & MBL

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MPQ
Max Planck Institut
of Quantum Optics
(Garching)

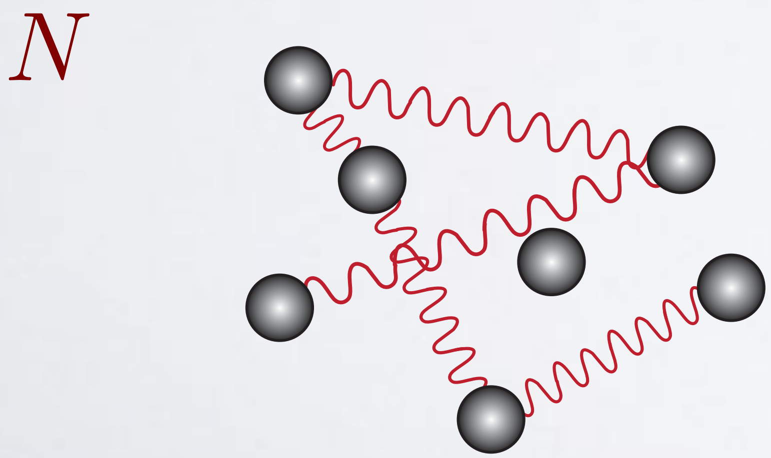
KITP September 2018

What are TNS?

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$



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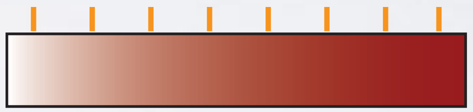
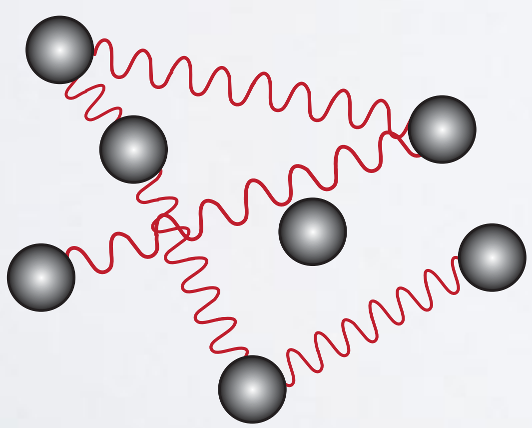


N-legged tensor

d^N

N



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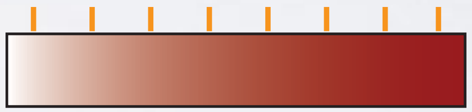

N-legged tensor

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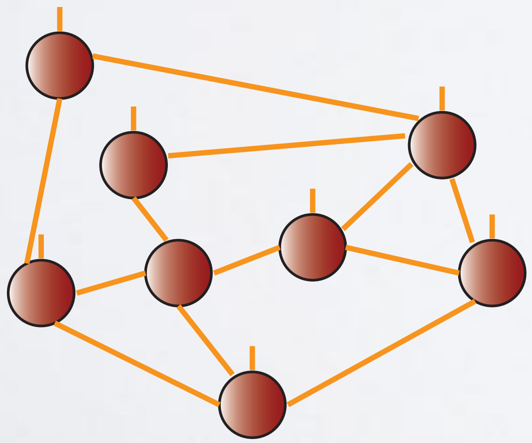
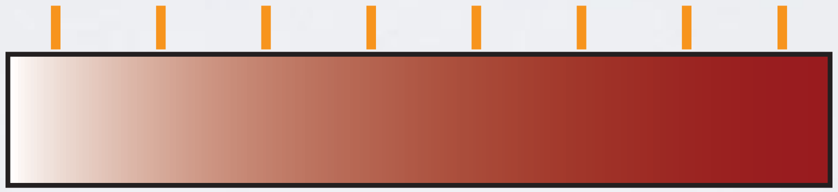
A TNS has only a polynomial number of parameters



$\mathrm{poly}(N)$

Paradigmatic: MPS

- MPS = Matrix Product States



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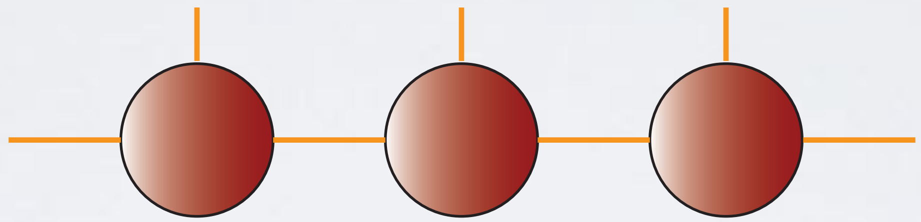

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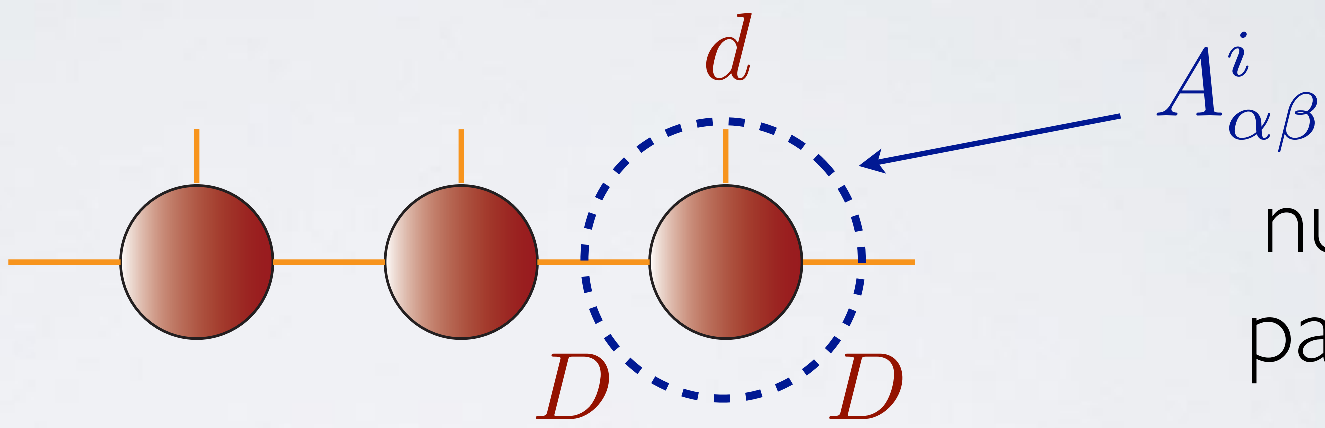

number of parameters

NdD^2

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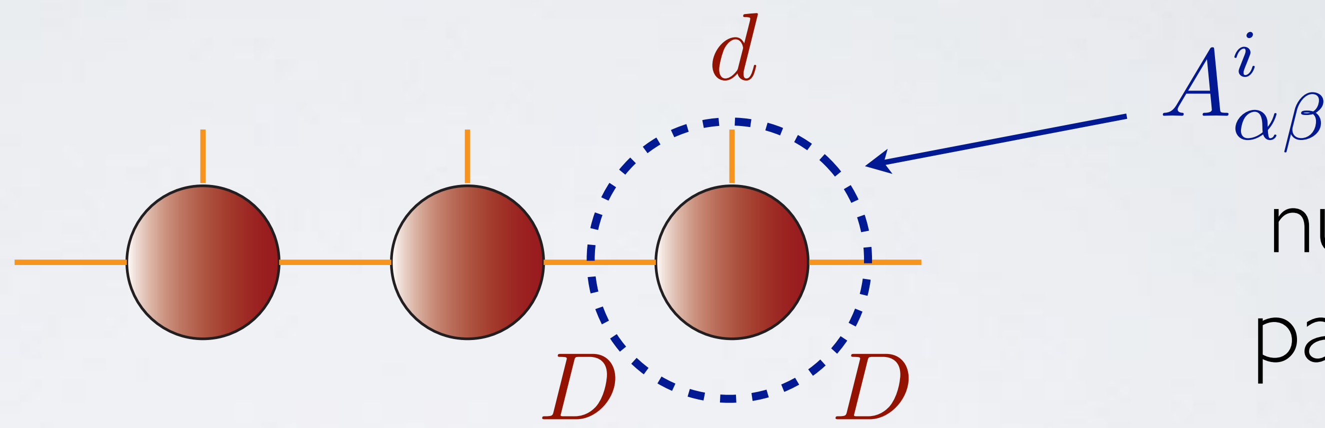

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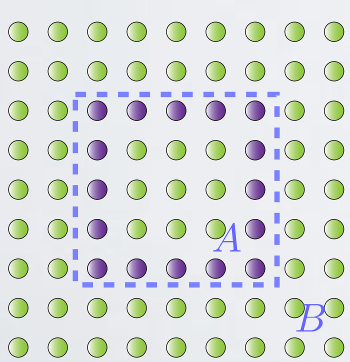

Area law by construction

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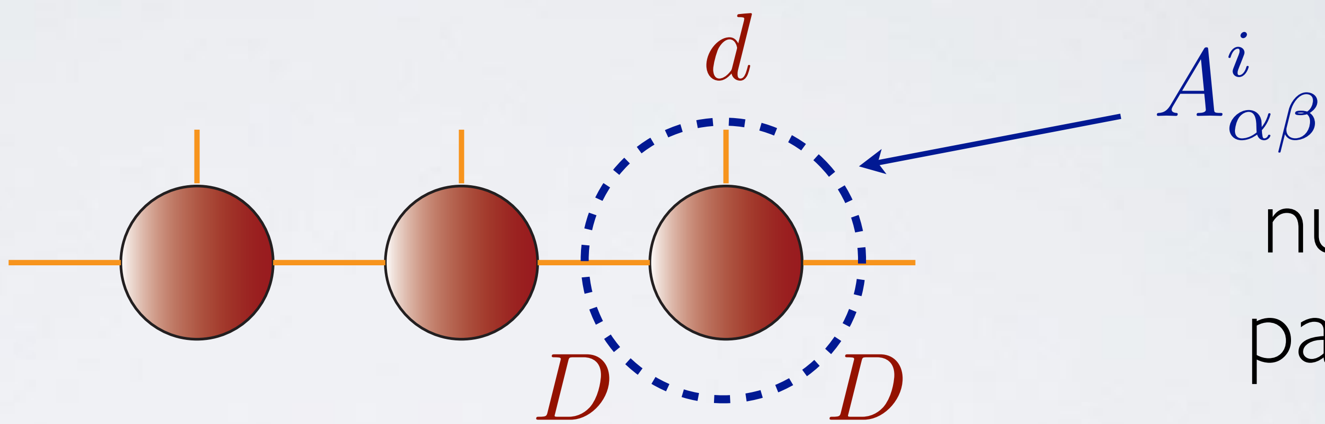

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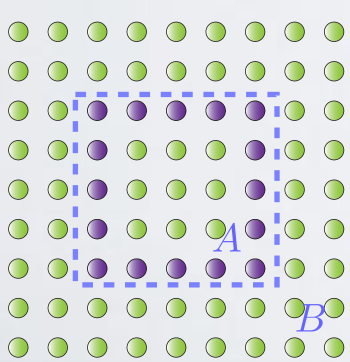

Area law by construction

Bounded entanglement

$S(L/2) \leq \log D$

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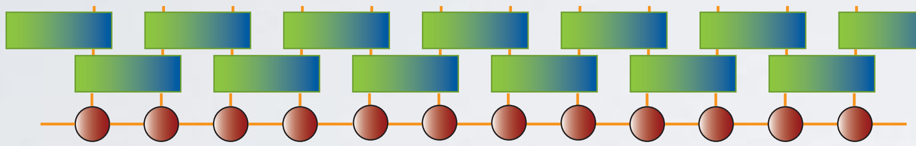
time evolution with MPS

evolving the (pure state) ansatz



entanglement can grow fast!

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White, Feiguin, PRL 2004
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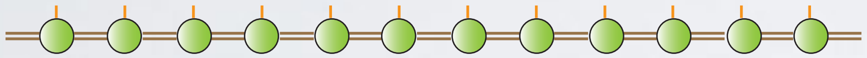

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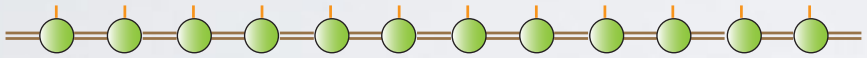

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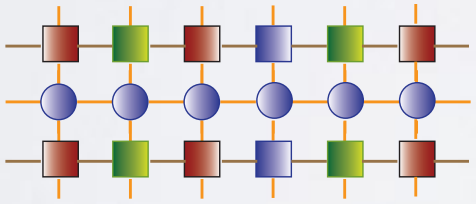

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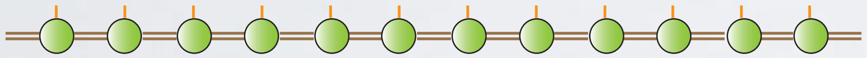

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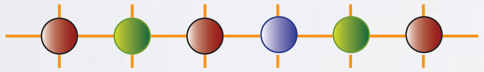

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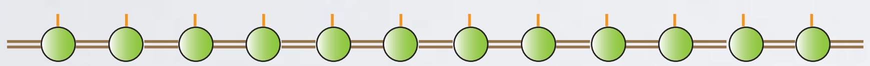

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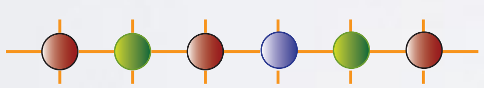

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operator space entanglement

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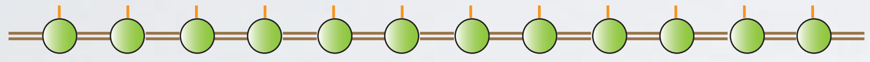

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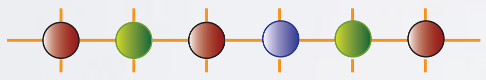

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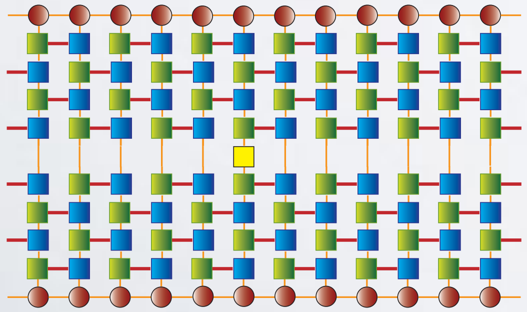

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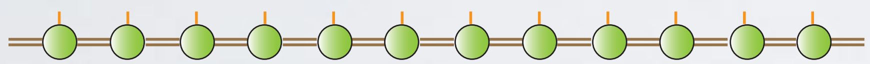

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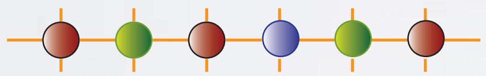

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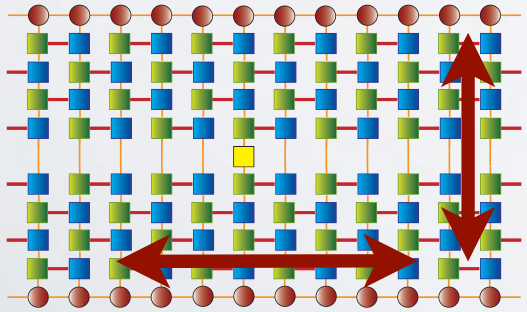

different *entanglement* quantities

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TNS: mixed states & evolution

$$\rho = \sum_{i_1, j_1 \ldots i_N, j_N} \boxed{C^{i_1 j_1 \ldots i_N j_N}} |i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

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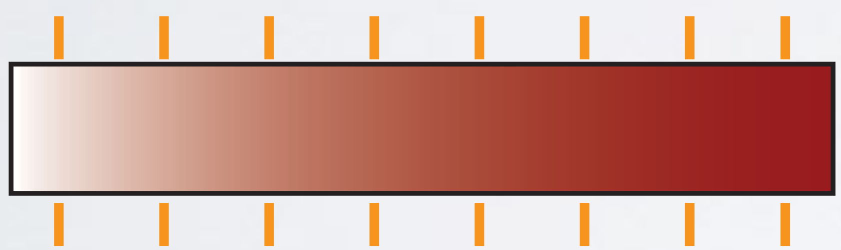

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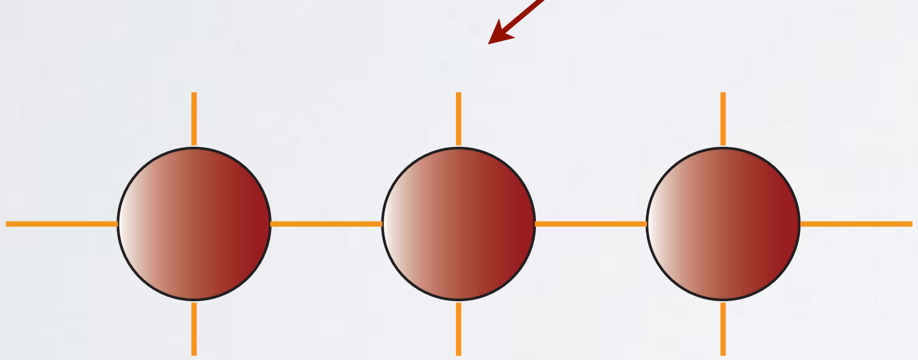

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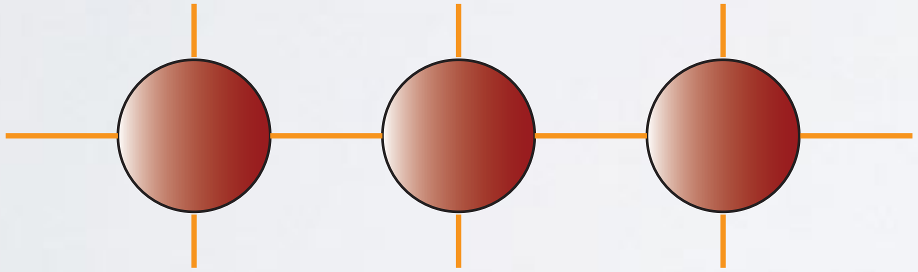

can describe mixed states and operators

e.g. thermal equilibrium

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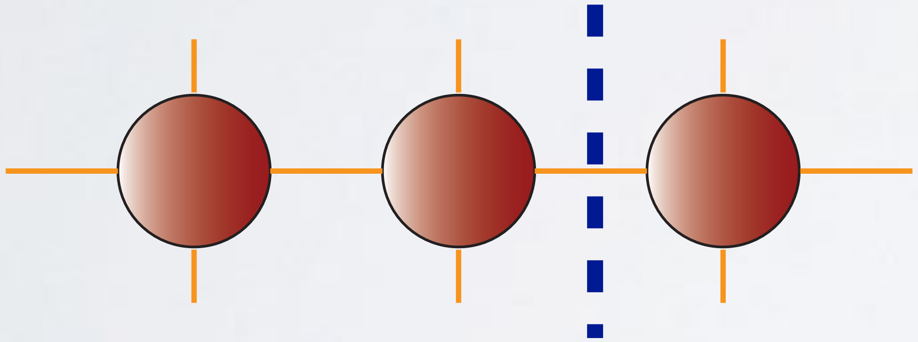

bounded operator space
entanglement entropy

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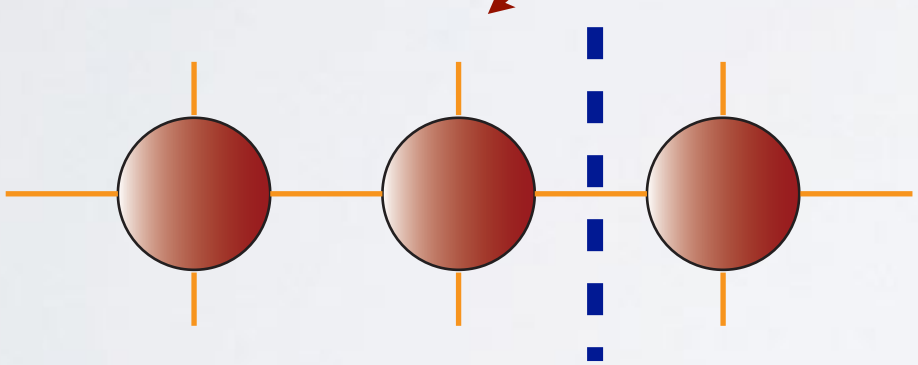

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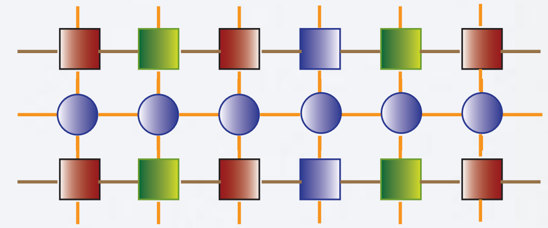
time evolution: unitary and non-unitary



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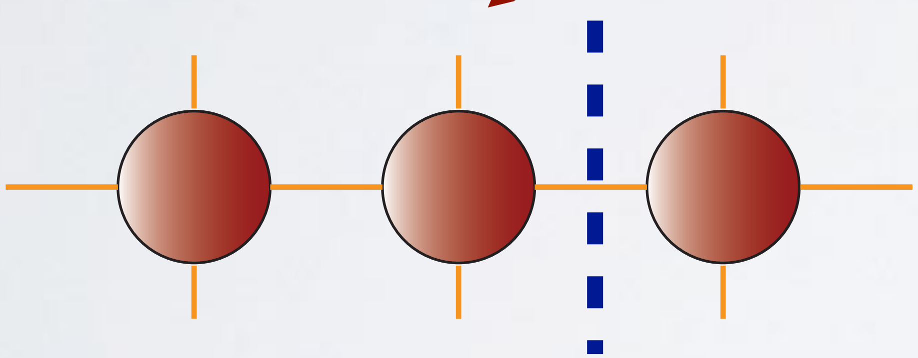

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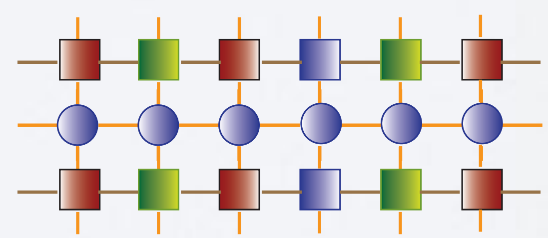

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long-time properties: slowest operators

Kim et al, PRE 2015

FOCUS: MBL dynamical scenario

interactions + strong disorder ⇒ localizing regime

absence of thermalization

Basko, Aleiner, Altshuler, Ann. Phys. 2006
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Bauer, Nayak, JStatMech 2013;
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here: some quantum information + TN perspectives

Some questions we are asking

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propagation of correlations

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quantum memory features

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dynamics of mixed states

- propagation of correlations
- quantum memory features
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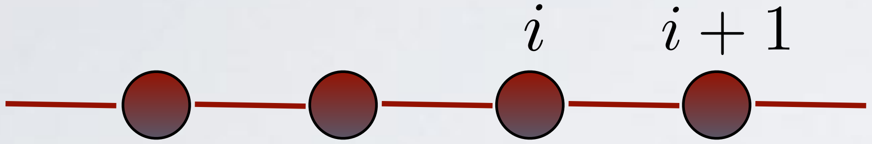
Hamiltonian properties

- local conserved quantities

the model

$$H = \sum \left(S_x^{[i]} S_x^{[i+1]} + S_y^{[i]} S_y^{[i+1]} + J S_z^{[i]} S_z^{[i+1]} + h_i S_i^z \right)$$



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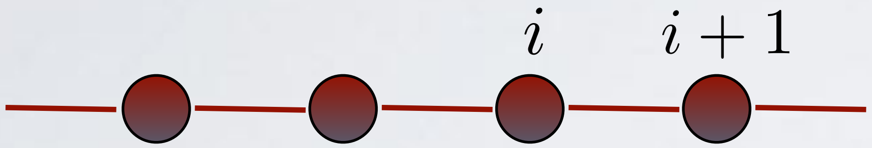

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J=0 ⇒ non-interacting XY

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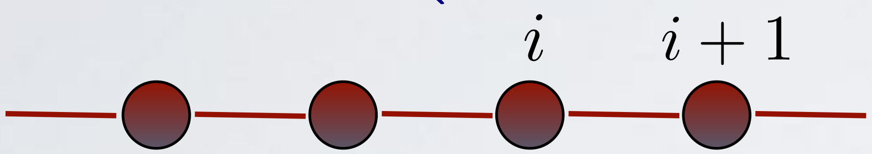

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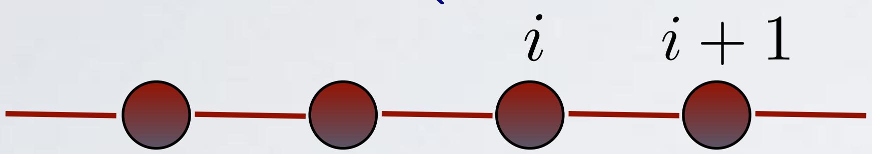

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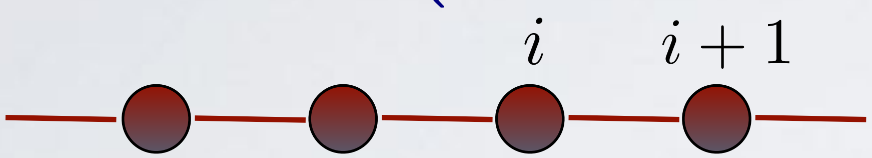

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initial states

mixed states at high T

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

propagation of correlations

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usual scenario: global quenches
for pure states

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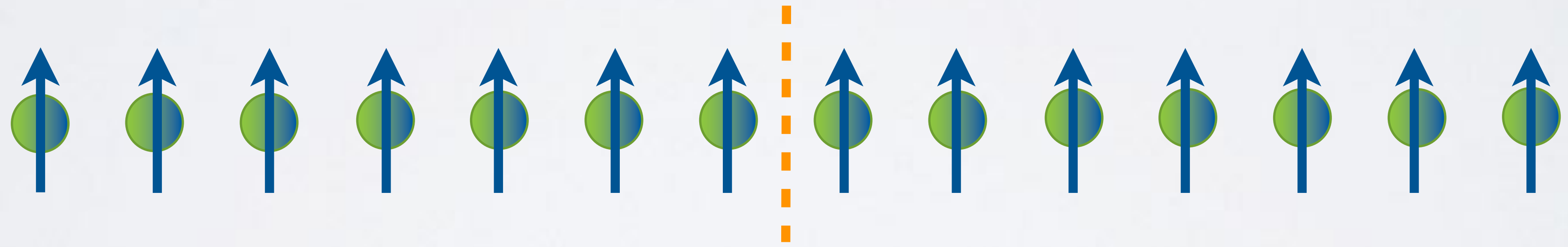
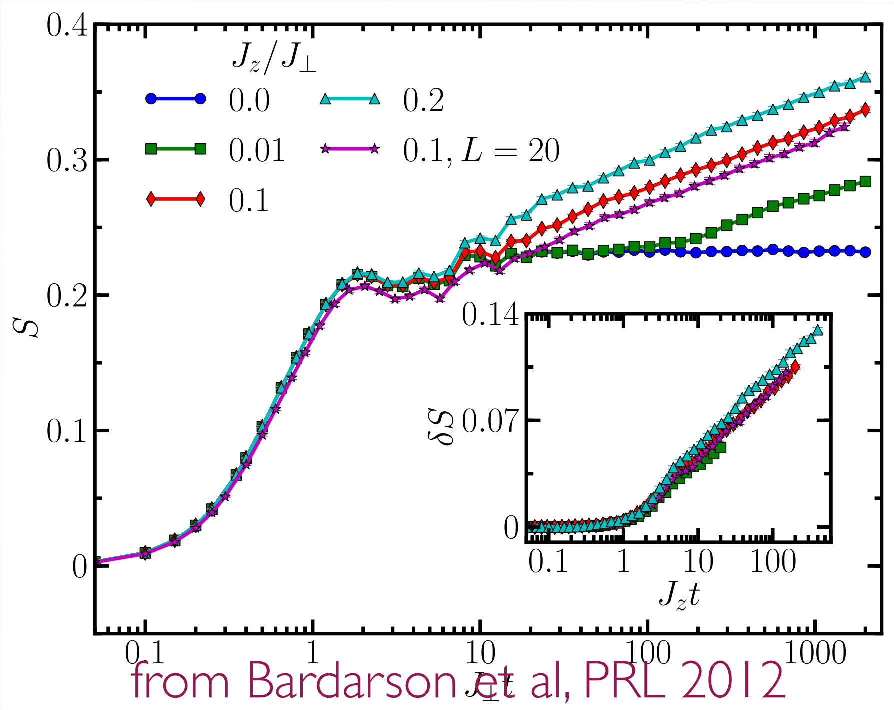
usual scenario: global quenches
for pure states





from Bardarson et al, PRL 2012

propagation of correlations

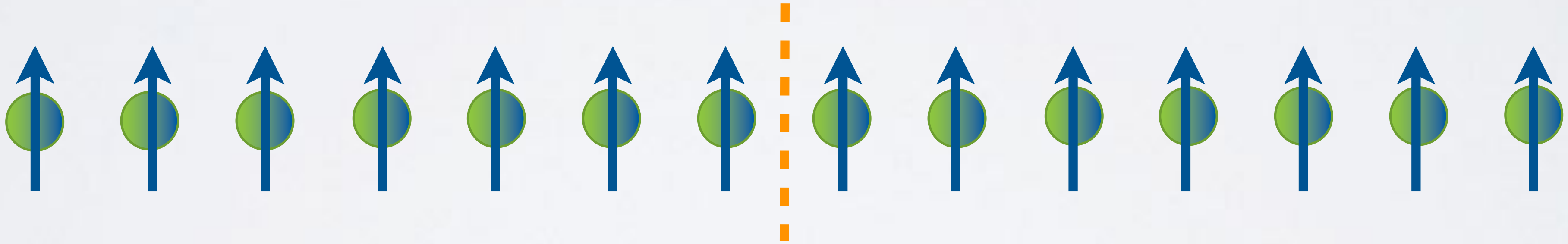
usual scenario: global quenches for pure states

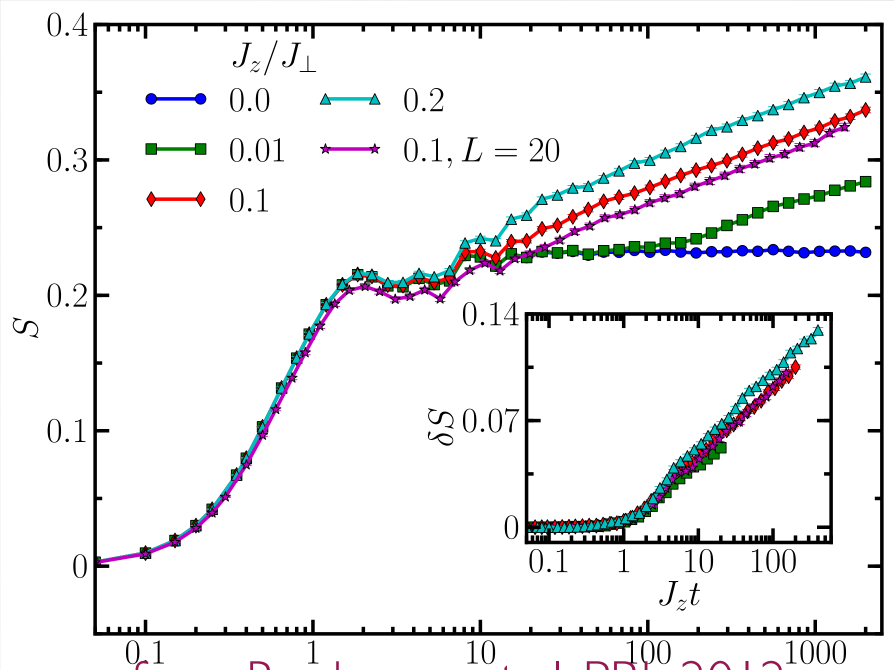
single particle localization → saturation of S

MBL → logarithmic growth of S

explained by l-bit model

Bardarson et al, PRL 2012
Serbyn et al, PRL 2013
Huse et al, PRB 2014





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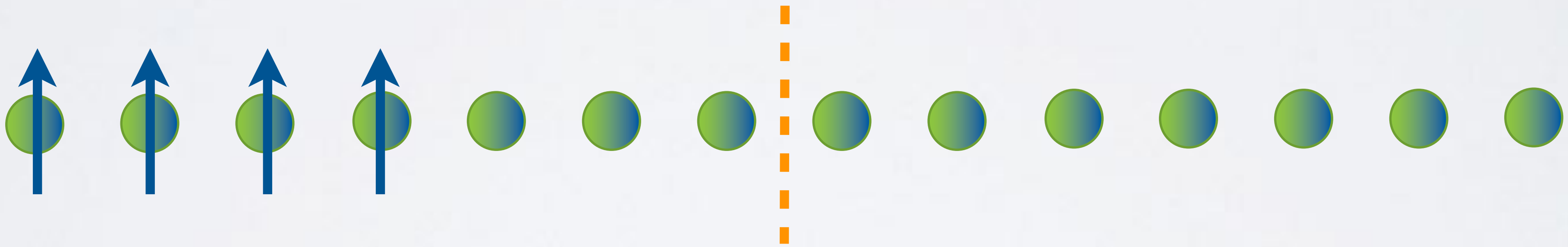
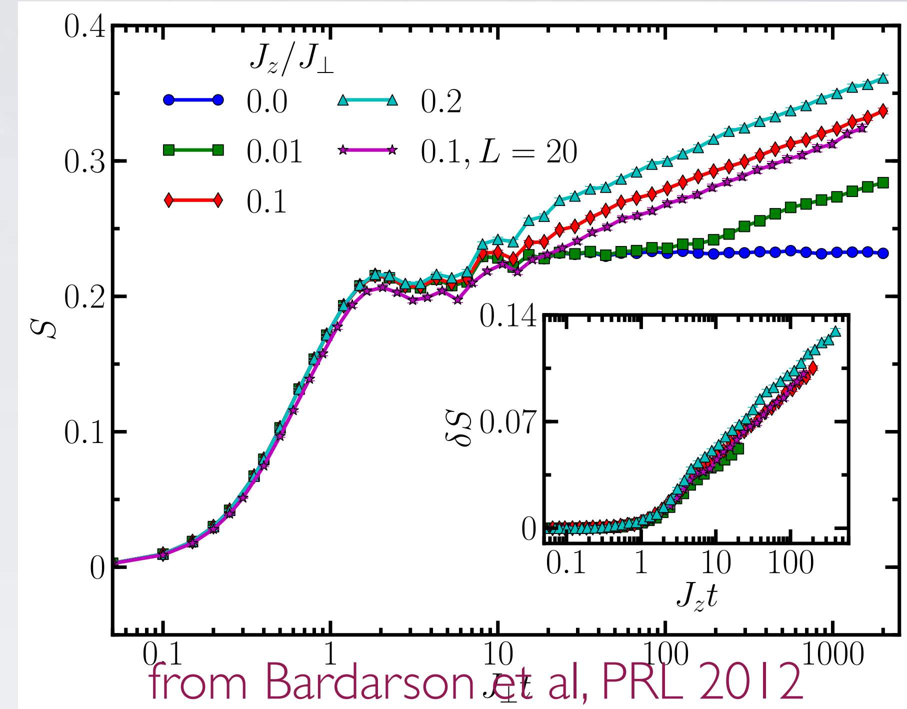


propagation of correlations

usual scenario: global quenches for pure states



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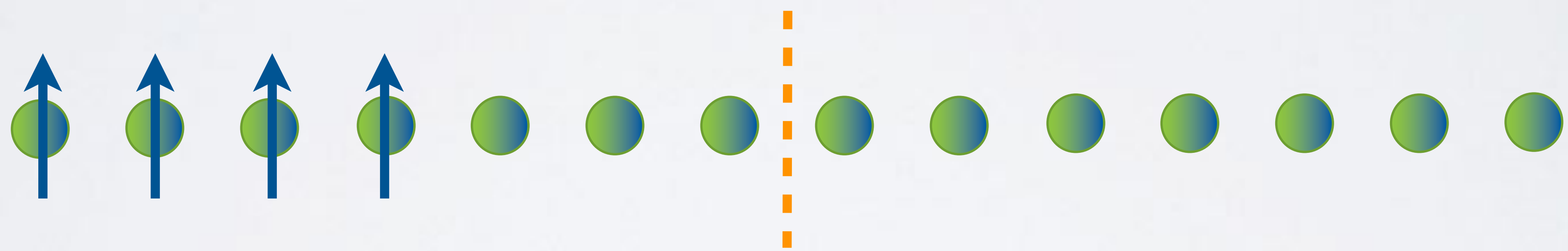


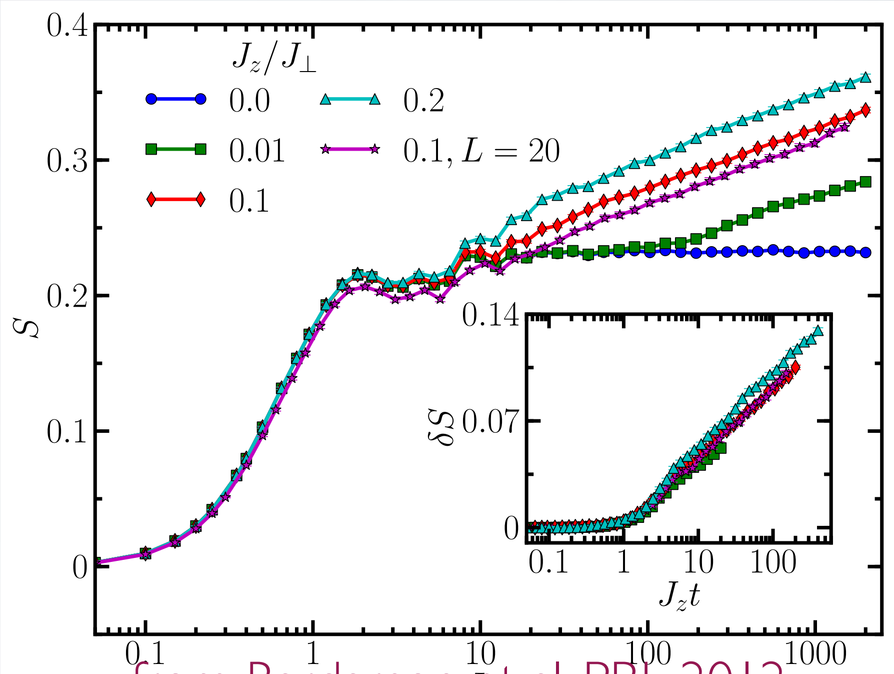
different for mixed states

$$I(\text{left } L_c \text{ sites : rest})$$

measures correlations between subsystems

propagation of correlations

usual scenario: global quenches for pure states



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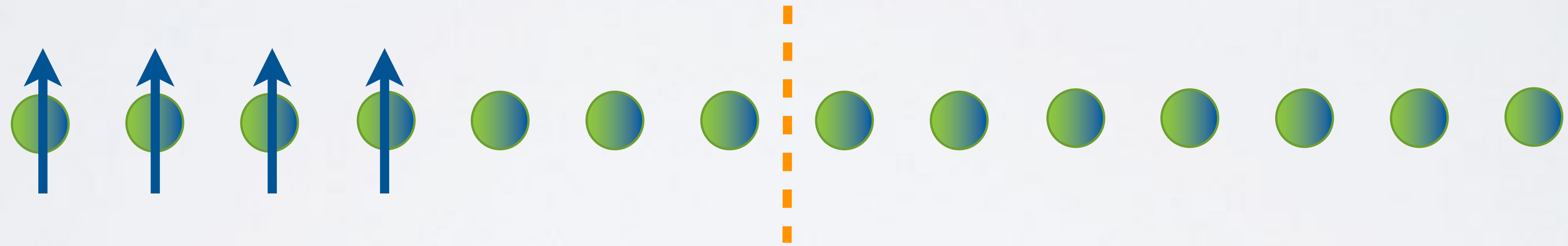


different for mixed states

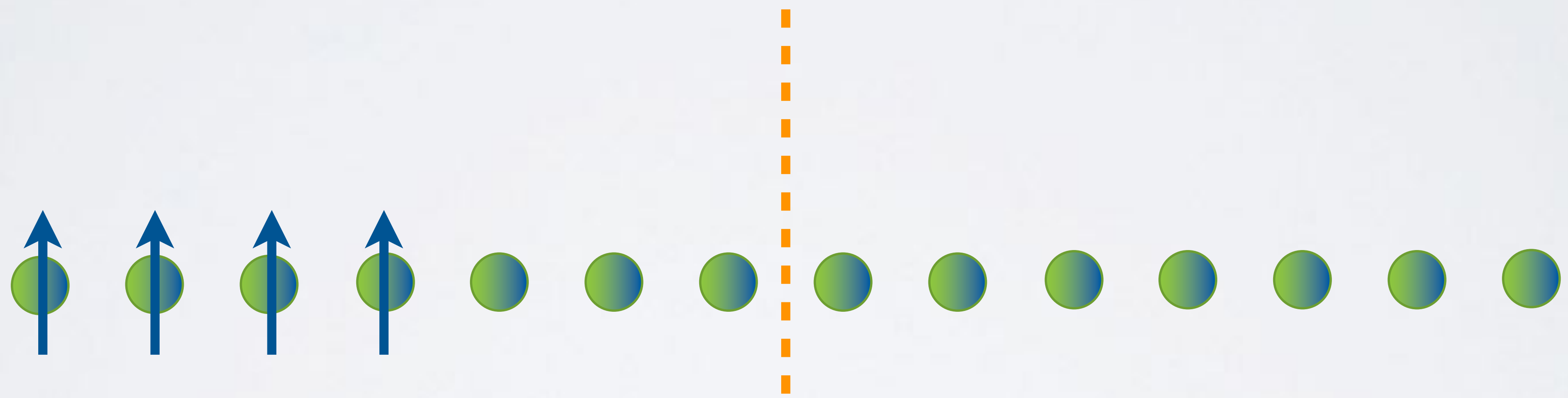
$$I(A:B) = S(A) + S(B) - S(AB)$$

measures correlations between subsystems

propagation of correlations

upper bounded

$$I(\ell : N-\ell) = S(\ell) + S(N-\ell) - S(N)$$

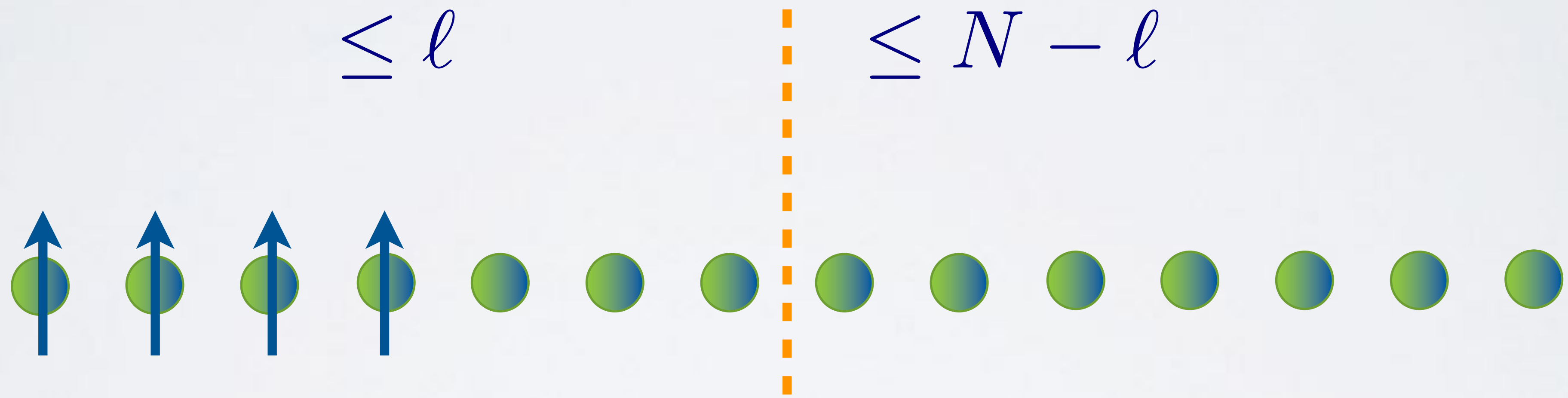


propagation of correlations

upper bounded

$$I(\ell : N - \ell) = S(\ell) + S(N - \ell) - S(N)$$

$$\leq \ell \qquad \leq N - \ell$$



propagation of correlations

upper bounded

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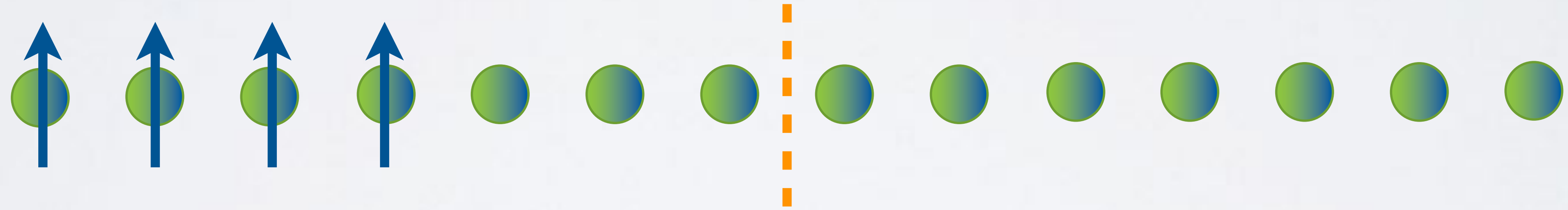
$$\leq \ell \qquad\qquad \leq N-\ell$$



initially

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$$L_0 = 1$$

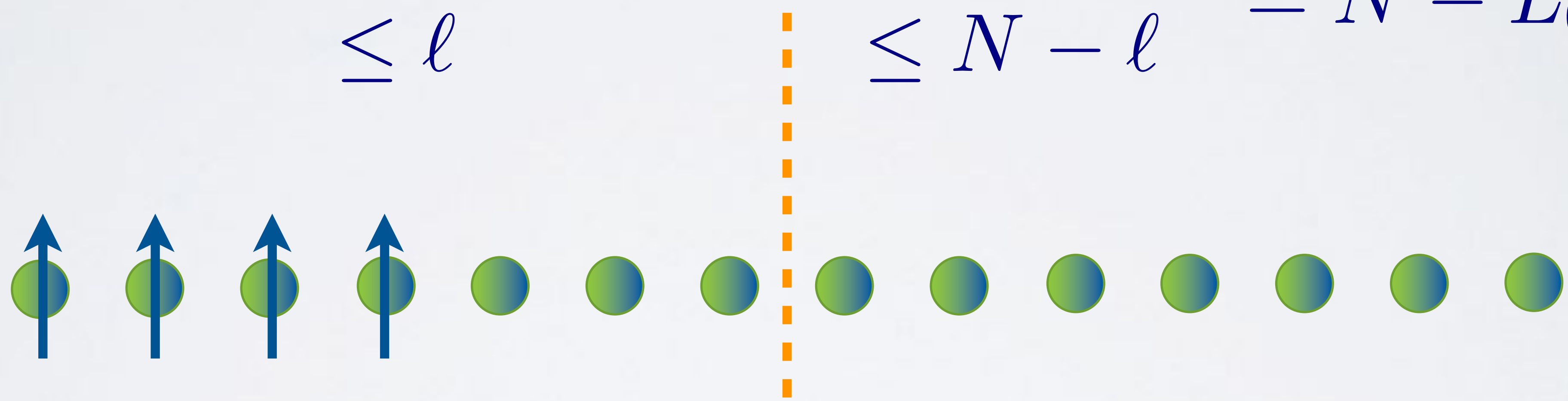
$$|\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$$

propagation of correlations

upper bounded

$$I(\ell : N-\ell) = S(\ell) + S(N-\ell) - S(N)$$

$\leq \ell$ $\quad$ $\leq N-\ell$ $\quad$ $= N - L_0$



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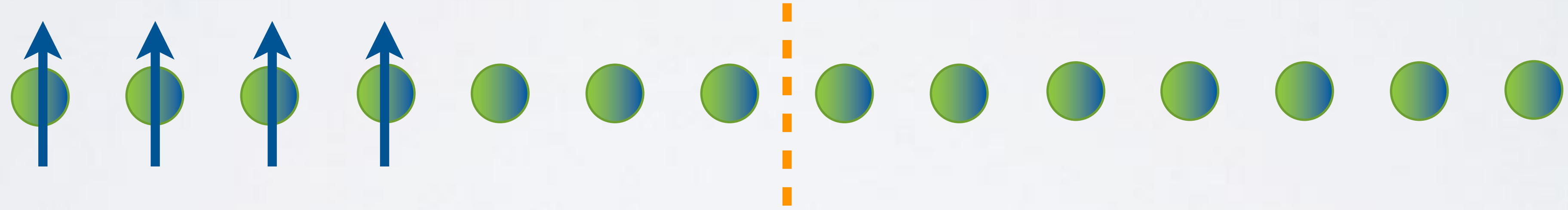
propagation of correlations

upper bounded

$$I(\ell : N-\ell) = S(\ell) + S(N-\ell) - S(N) \leq L_0 = 1$$

$\leq \ell$ $\qquad$ $\leq N-\ell$ $\qquad$ $= N - L_0$



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propagation of correlations

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$$I(\ell : N-\ell) = S(\ell) + S(N-\ell) - S(N) \leq L_0 = 1$$

$S(\ell) \leq \ell$, $\quad S(N-\ell) \leq N-\ell$, $\quad S(N) = N - L_0$

yet shows difference between many-body and single particle localized

initially

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propagation of correlations

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$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$$L_0 = 1$$

$$|\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$$

exact calculation possible in the non-interacting and l-bit models

propagation of correlations

non-interacting case: quadratic fermionic model

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}} \qquad |\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$$

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propagation of correlations

non-interacting case: quadratic fermionic model

exact evolution: single parameter $\mathcal{V}_\ell = \sum_{r=0}^{\ell-1} |\langle r|U(t)|0\rangle|^2$

probability to the left

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}} \qquad |\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$$

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propagation of correlations

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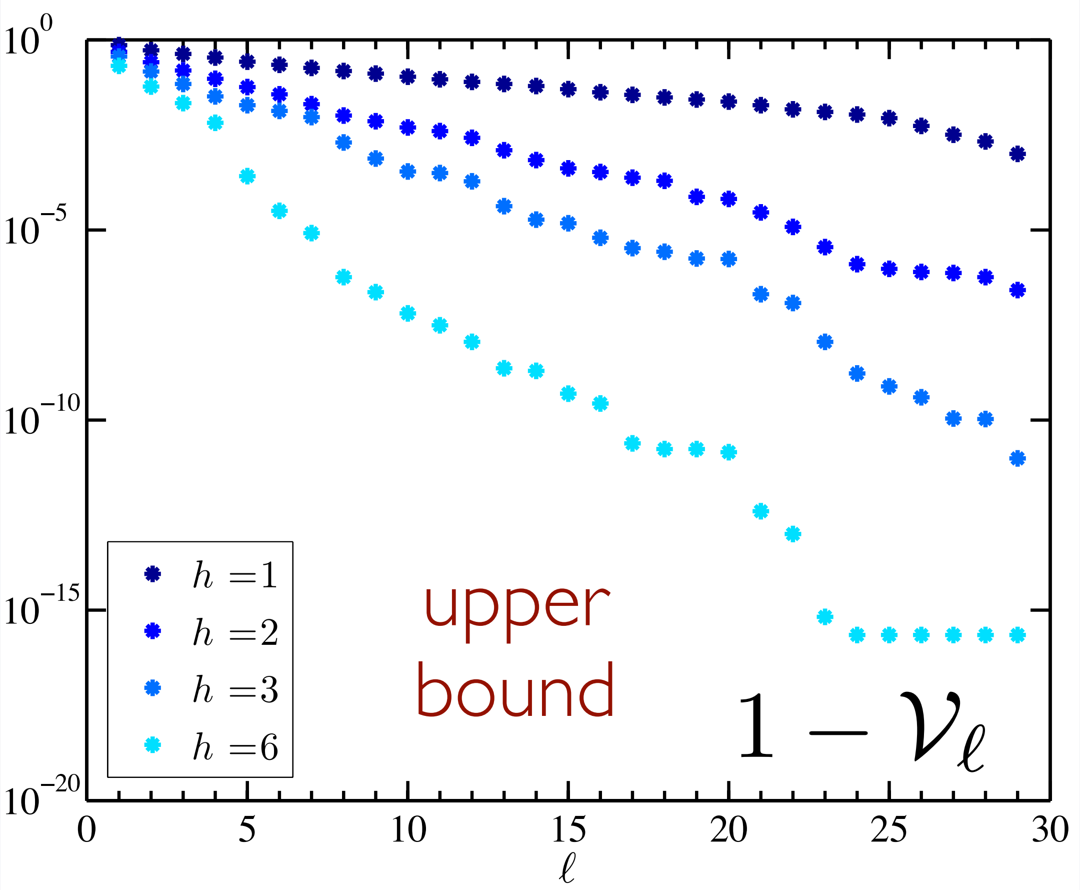
probability to the left

probability to the right



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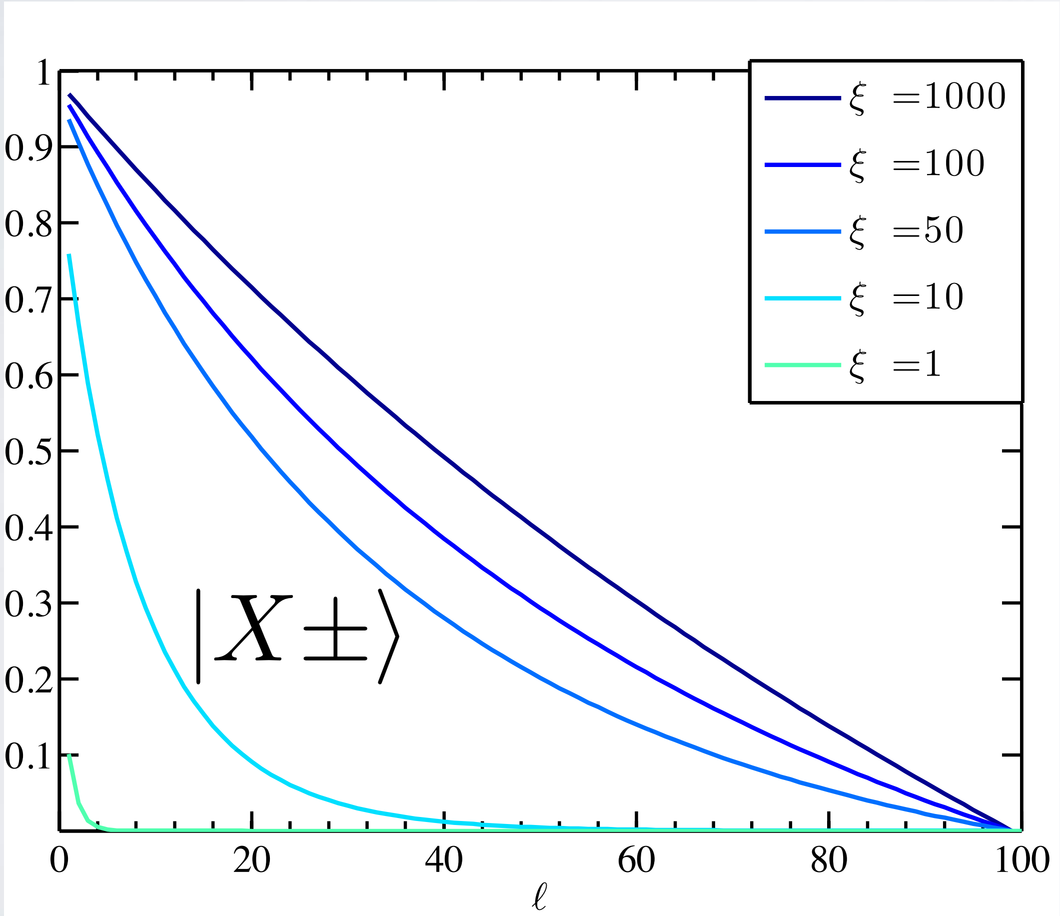
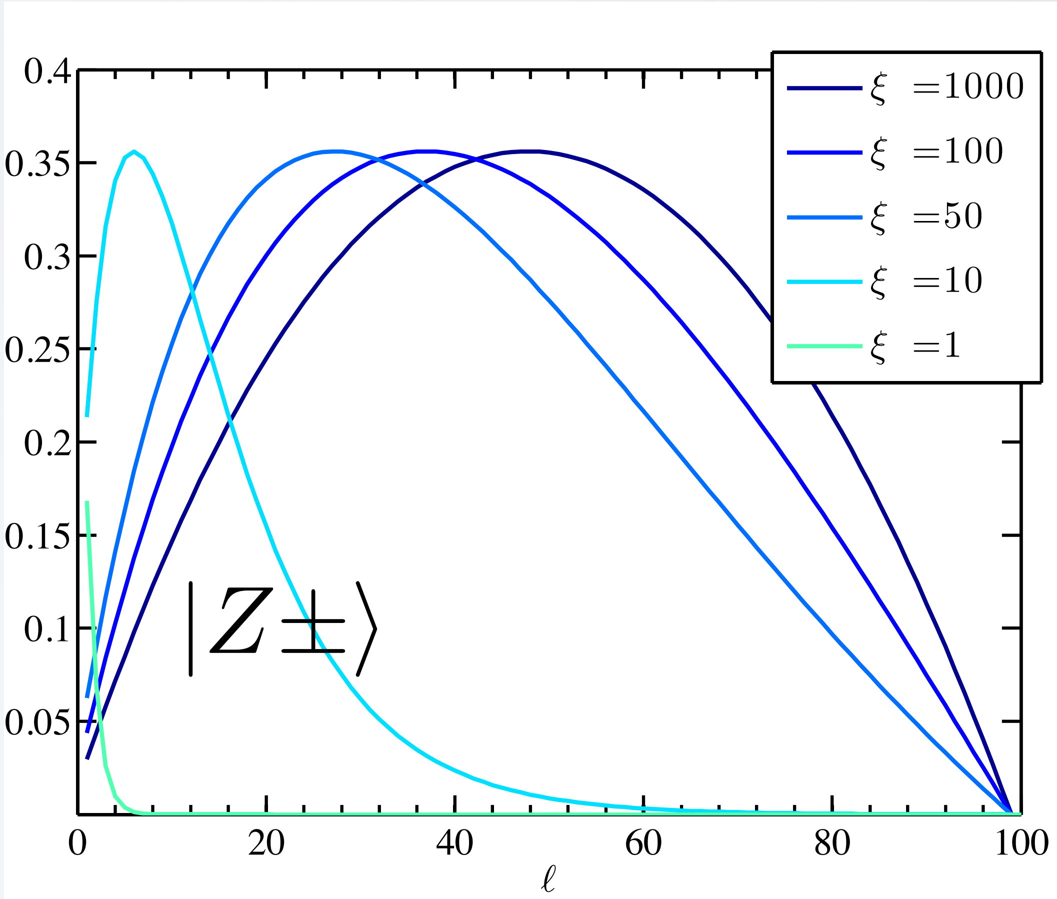
propagation of correlations

non-interacting case: localization $h>0$

asymptotic value of mutual information

assuming exponential decay with ξ





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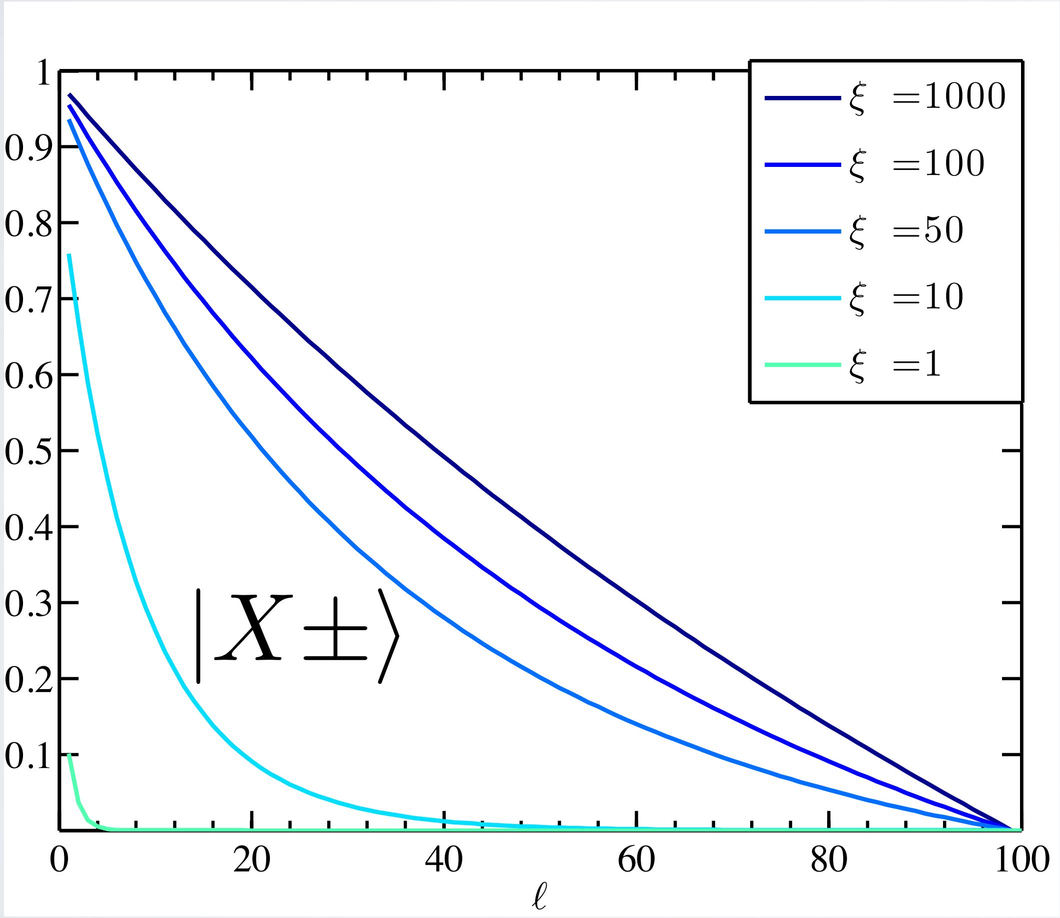
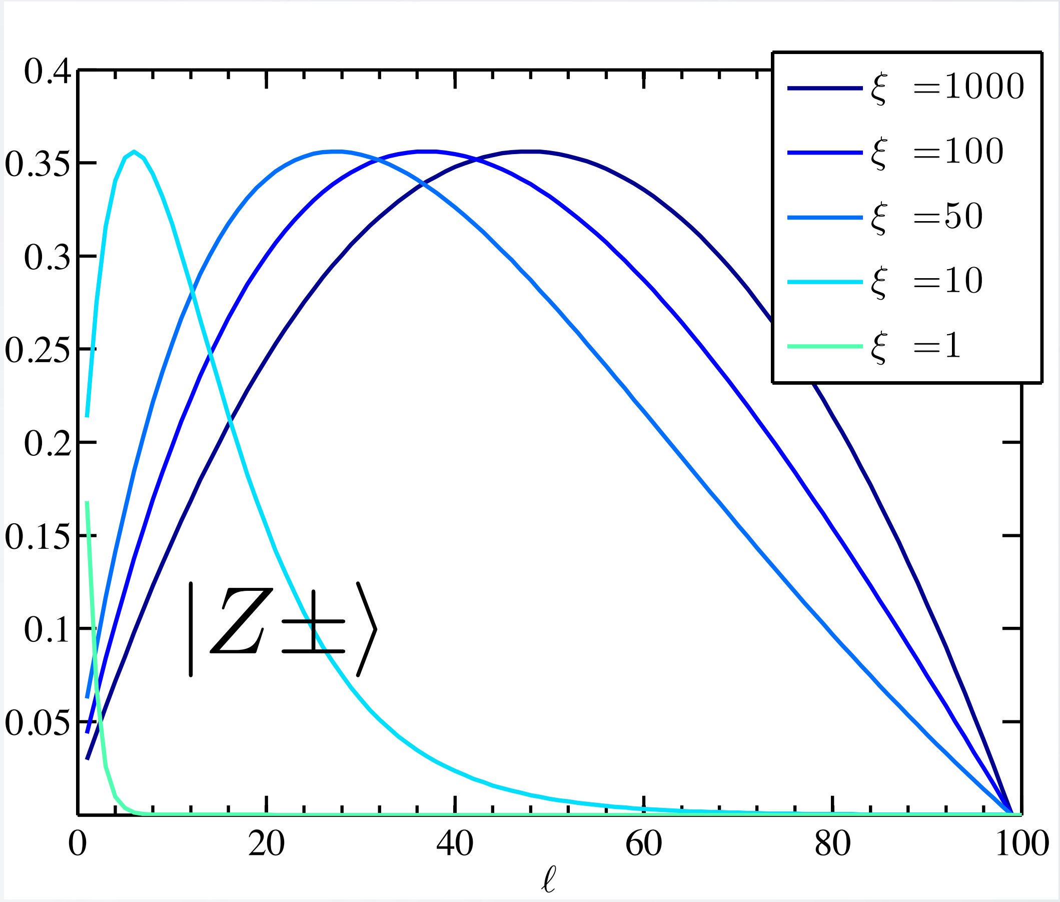
propagation of correlations

non-interacting case: localization $h>0$

$L = 50, h = 1$





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propagation of correlations

non-interacting case: localization $h>0$

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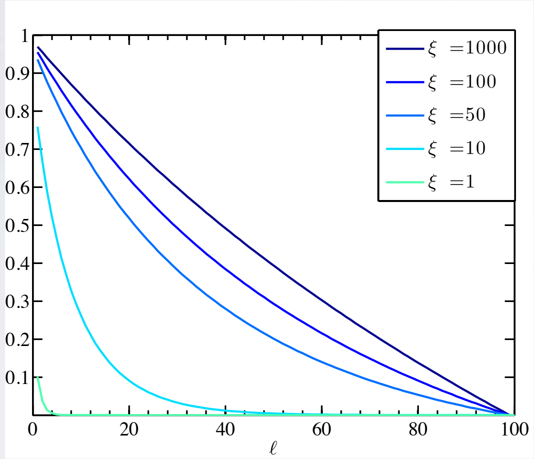
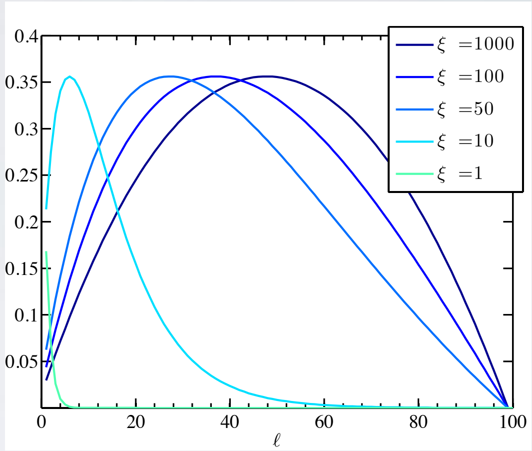
$|X\pm\rangle$



$|Z\pm\rangle$



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propagation of correlations

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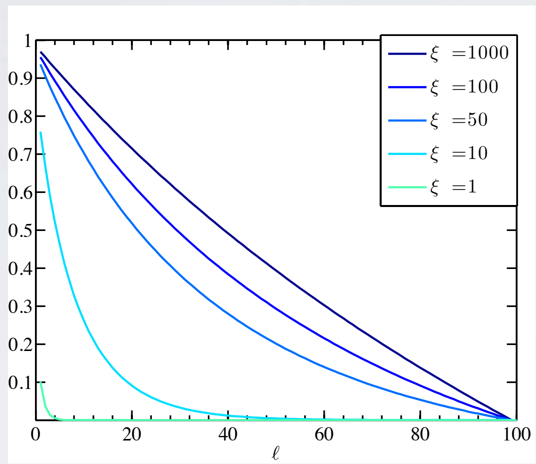
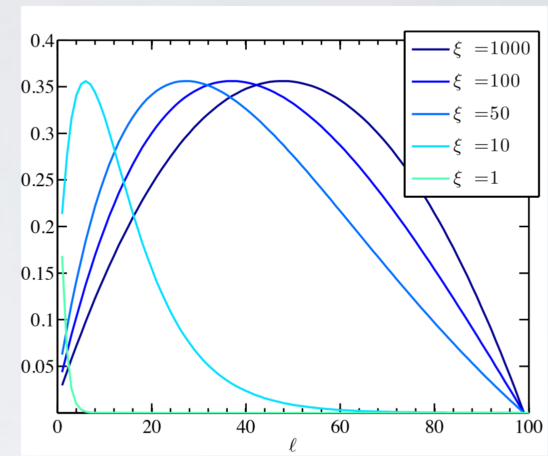
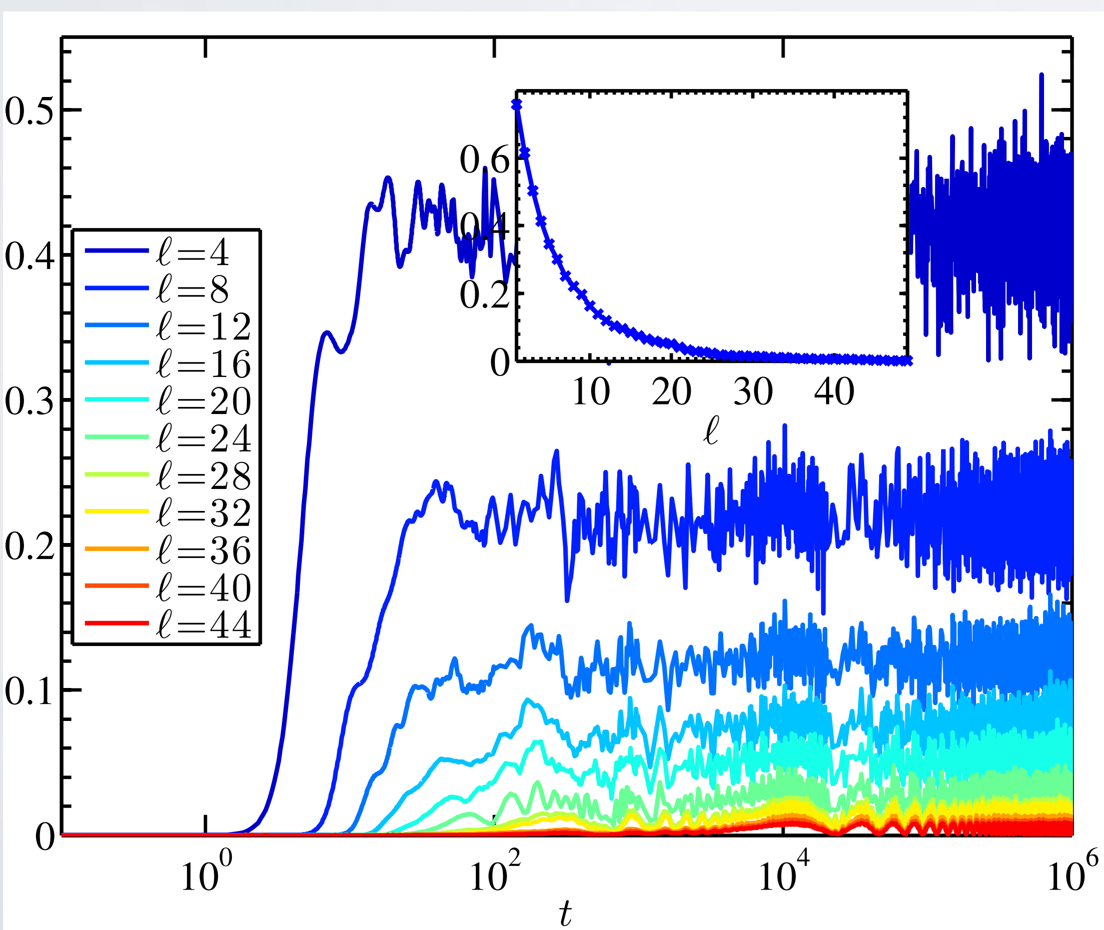


$|Z\pm\rangle$





$\frac{L-L_0}{2}$

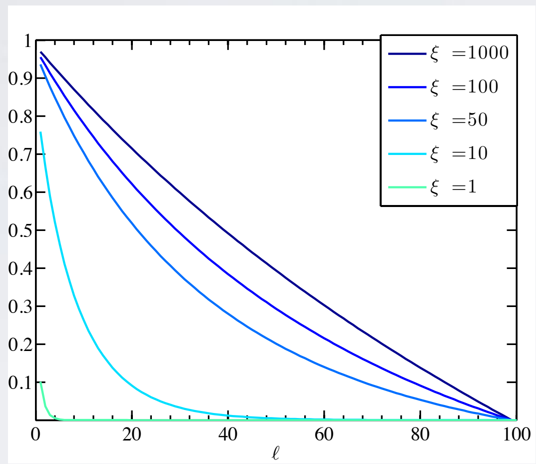
$|\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$

propagation of correlations

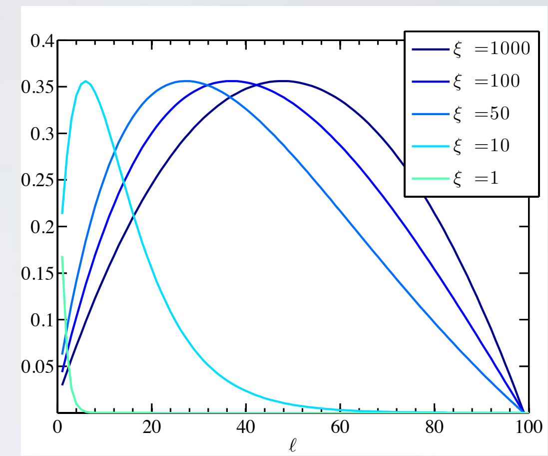
non-interacting case: localization $h>0$

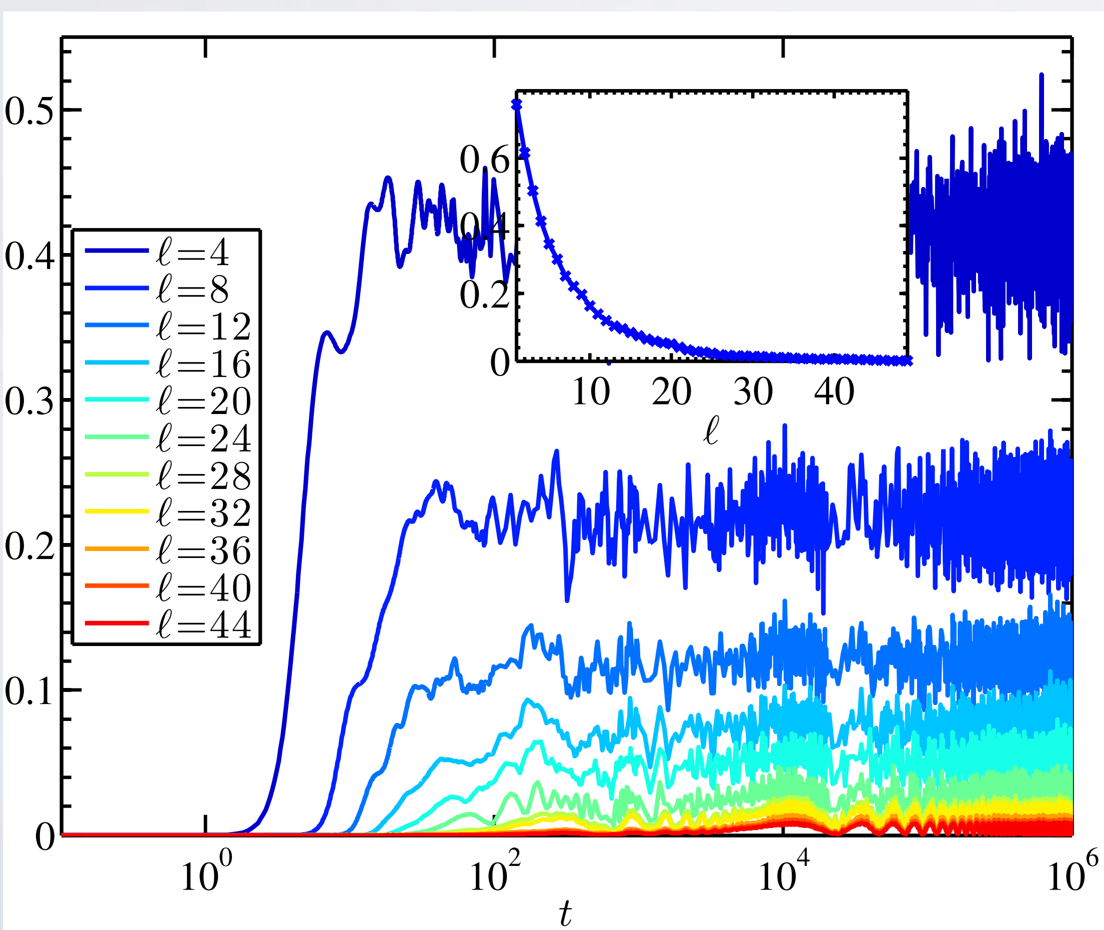
$L = 50, h = 1$

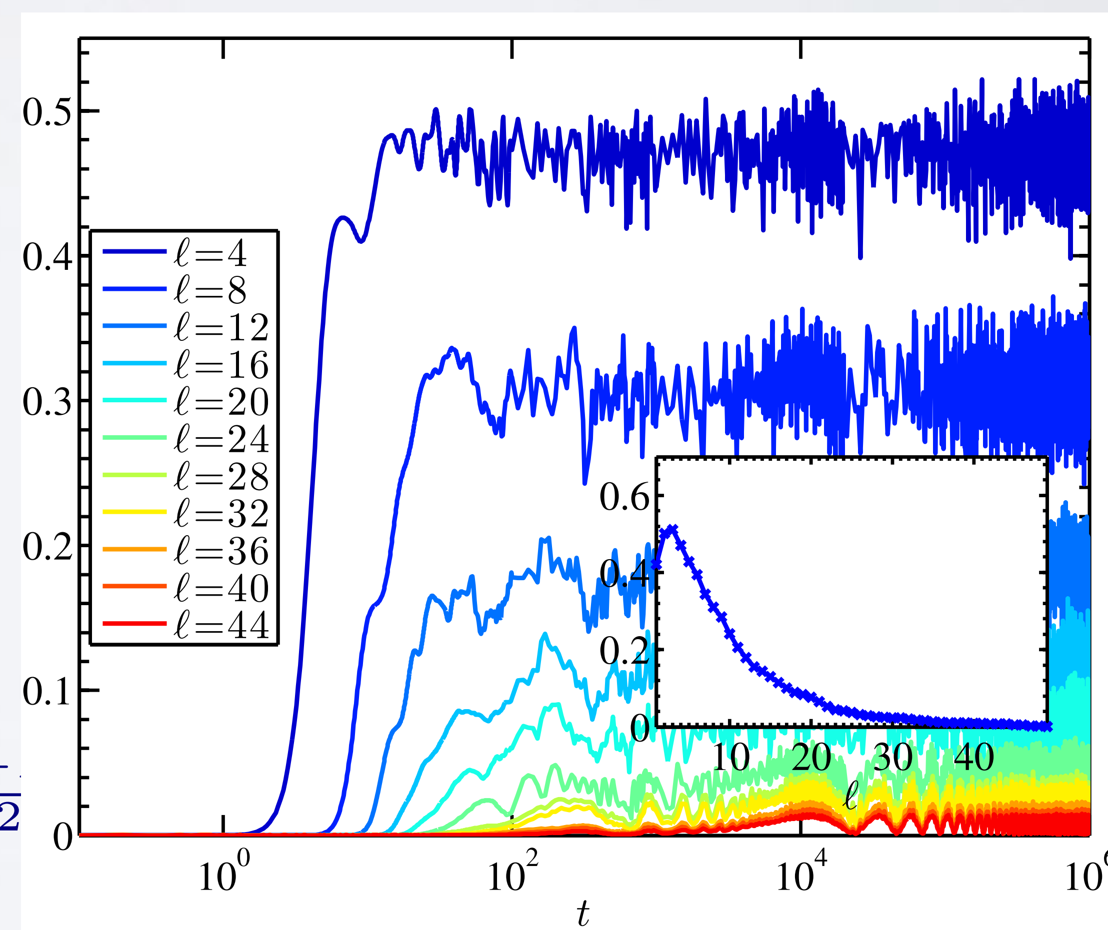
$|X\pm\rangle$



$|Z\pm\rangle$





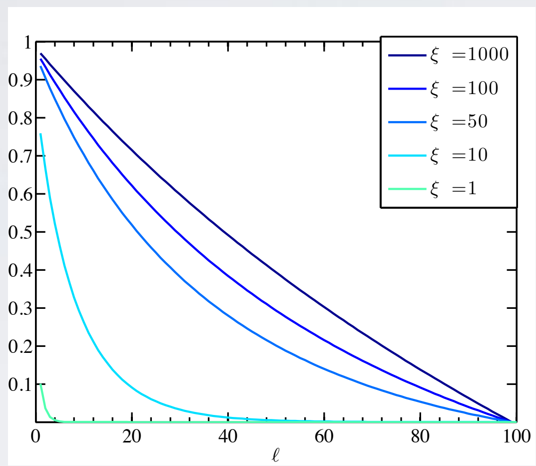


propagation of correlations

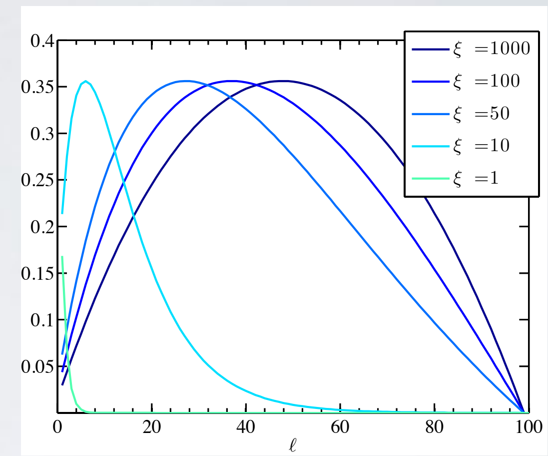
non-interacting case: localization *h*>0

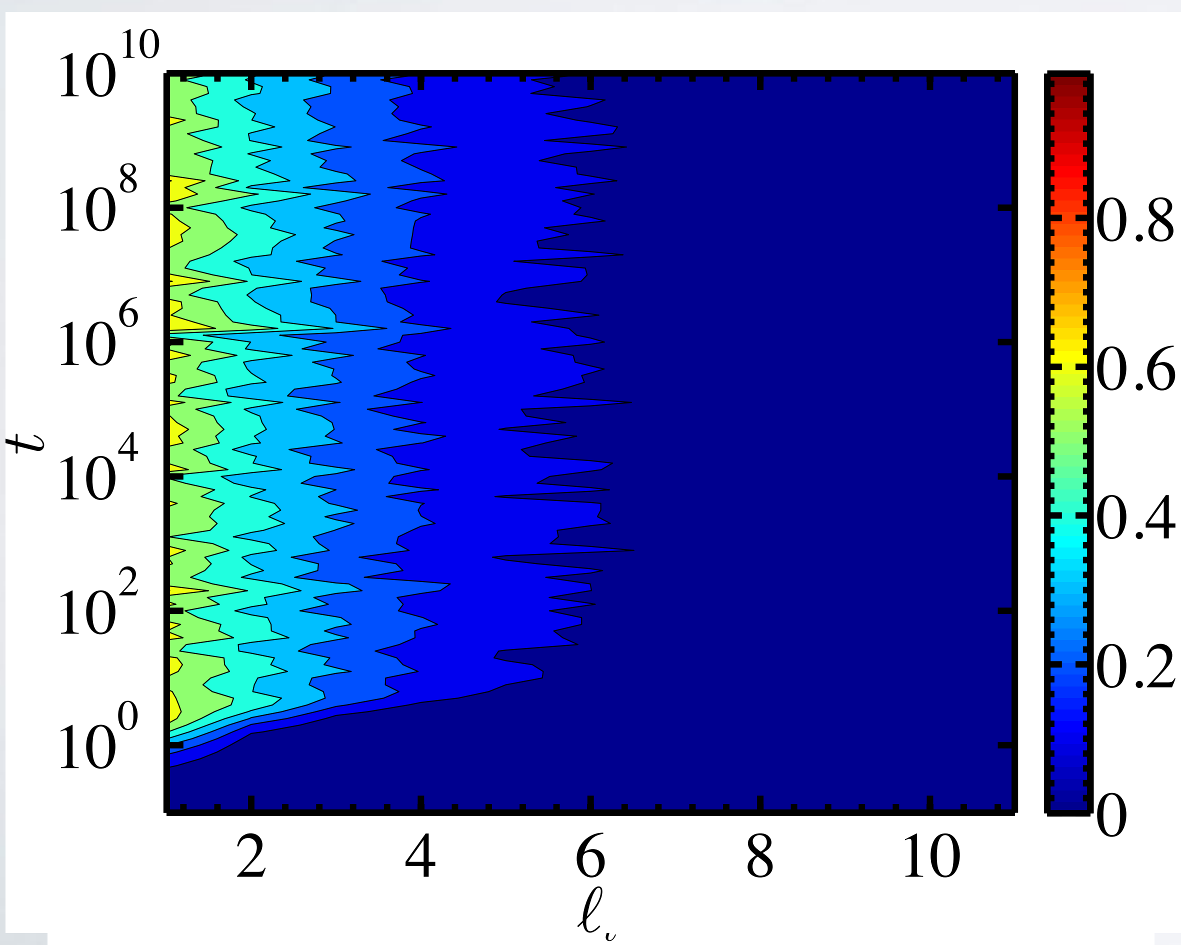
$L = 50, h = 1$

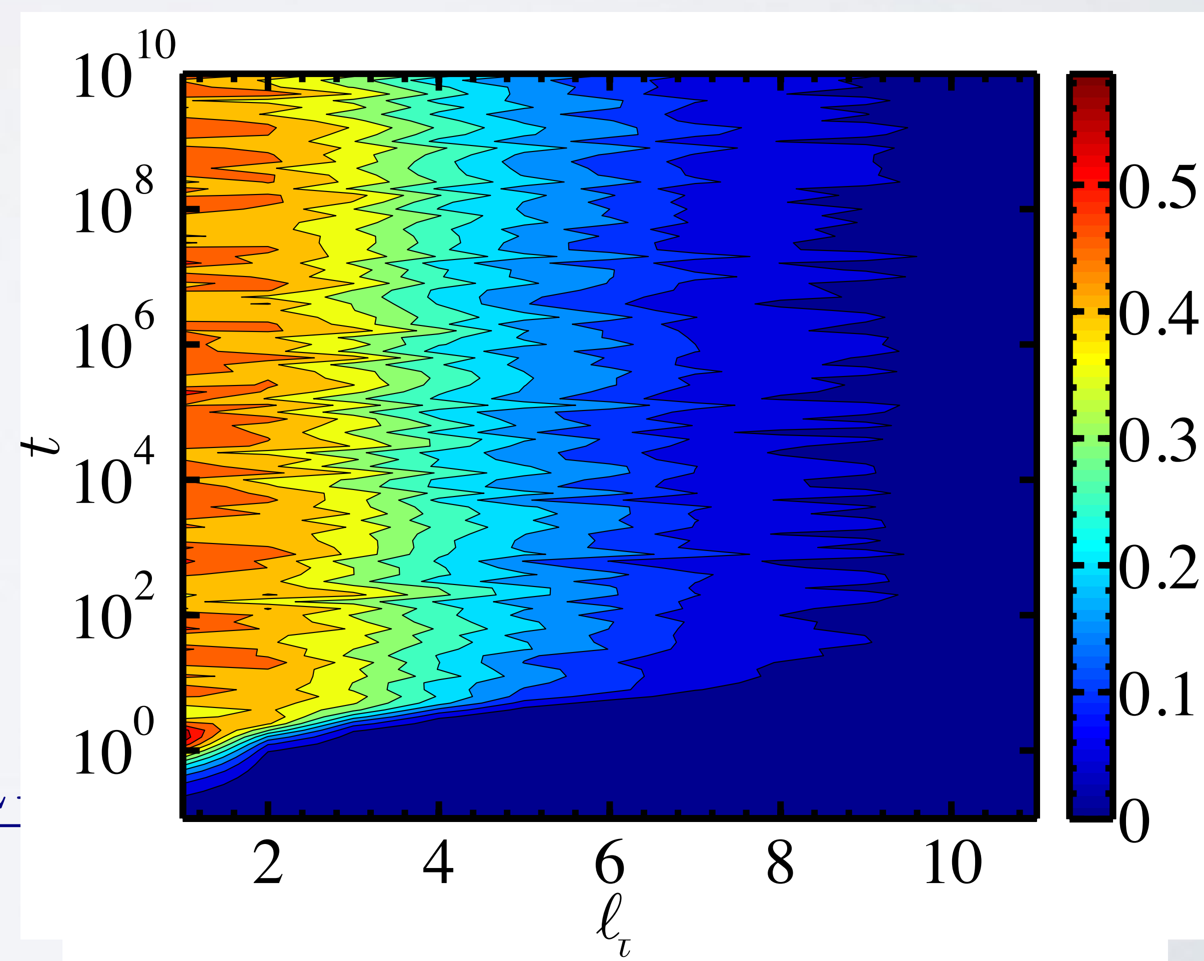
$|X\pm\rangle$



$|Z\pm\rangle$







propagation of correlations

interacting case: l-bit model

propagation of correlations

interacting case: l-bit model

$$H_{\text{eff}} = \sum_{i=0}^{N-1} \epsilon_i \tau_z^{[i]} + \sum_{i,j=0}^{N-1} K_{ij}^{(2)} \tau_z^{[i]} \tau_z^{[j]} + \sum_{i,j,k=0}^{N-1} K_{ijk}^{(3)} \tau_z^{[i]} \tau_z^{[j]} \tau_z^{[k]} + \ldots,$$

propagation of correlations

interacting case: l-bit model

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initial states

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}} \qquad |\Phi\rangle = |Z\pm\rangle, |X\pm\rangle$$

propagation of correlations

interacting case: l-bit model

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initial states: simple model

$$\rho_{X+} \approx \frac{1 + \tau_x^{[0]}}{2} \otimes Id^{\otimes \frac{L-L_0}{2}}$$

propagation of correlations

interacting case: l-bit model

$$H_{\text{eff}} = \sum_{i=0}^{N-1} \epsilon_i \tau_z^{[i]} + \sum_{i,j=0}^{N-1} K_{ij}^{(2)} \tau_z^{[i]} \tau_z^{[j]} + \sum_{i,j,k=0}^{N-1} K_{ijk}^{(3)} \tau_z^{[i]} \tau_z^{[j]} \tau_z^{[k]} + \dots,$$

exponentially decreasing

initial states: simple model

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propagation of correlations

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only terms involving $\tau_z^{[0]}$

exponentially decreasing

initial states: simple model

$$\rho_{X+} \approx \frac{1+\tau_x^{[0]}}{2} \otimes Id^{\otimes \frac{L-L_0}{2}}$$

propagation of correlations

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single parameter

$$x(\ell, t) = \prod_{k=\ell}^{N-1} \cos(2tK_{0k}^{(2)})$$

propagation of correlations

interacting case: l-bit model

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exponentially decreasing

initial states: simple model

$$\rho_{X+} \approx \frac{1 + \tau_x^{[0]}}{2} \otimes Id^{\otimes \frac{L - L_0}{2}}$$

single parameter $\quad x(\ell, t) = \prod_{k=\ell}^{N-1} \cos(2tK_{0k}^{(2)})$

$$K_{0k}^{(2)} \approx e^{-k/\xi} \Rightarrow x(\ell, t) \approx 1 - 2t^2(N - \ell)e^{-2\ell/\xi}$$

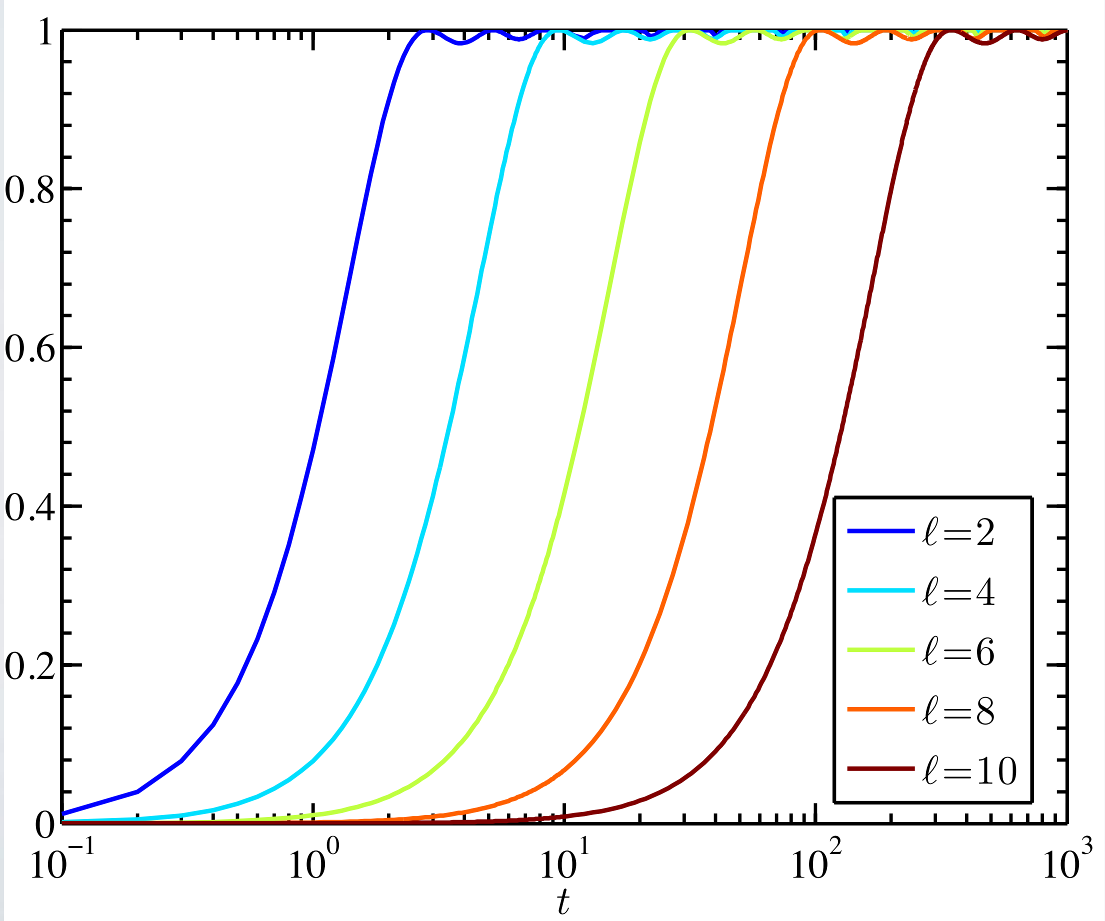
can be close to 0
takes exponential time

propagation of correlations

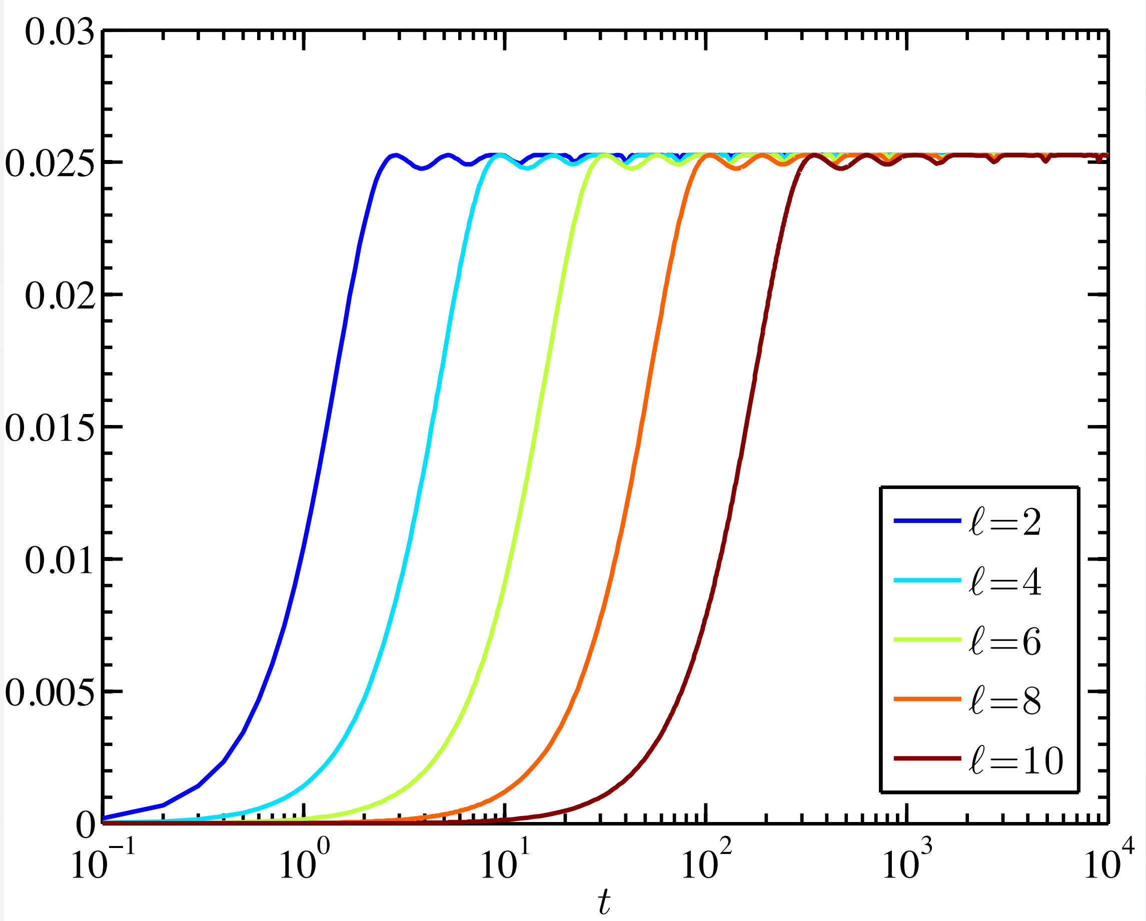
interacting case: l-bit model

$\xi = 10$

$|X\pm\rangle$



$|Z\pm\rangle$



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propagation of correlations

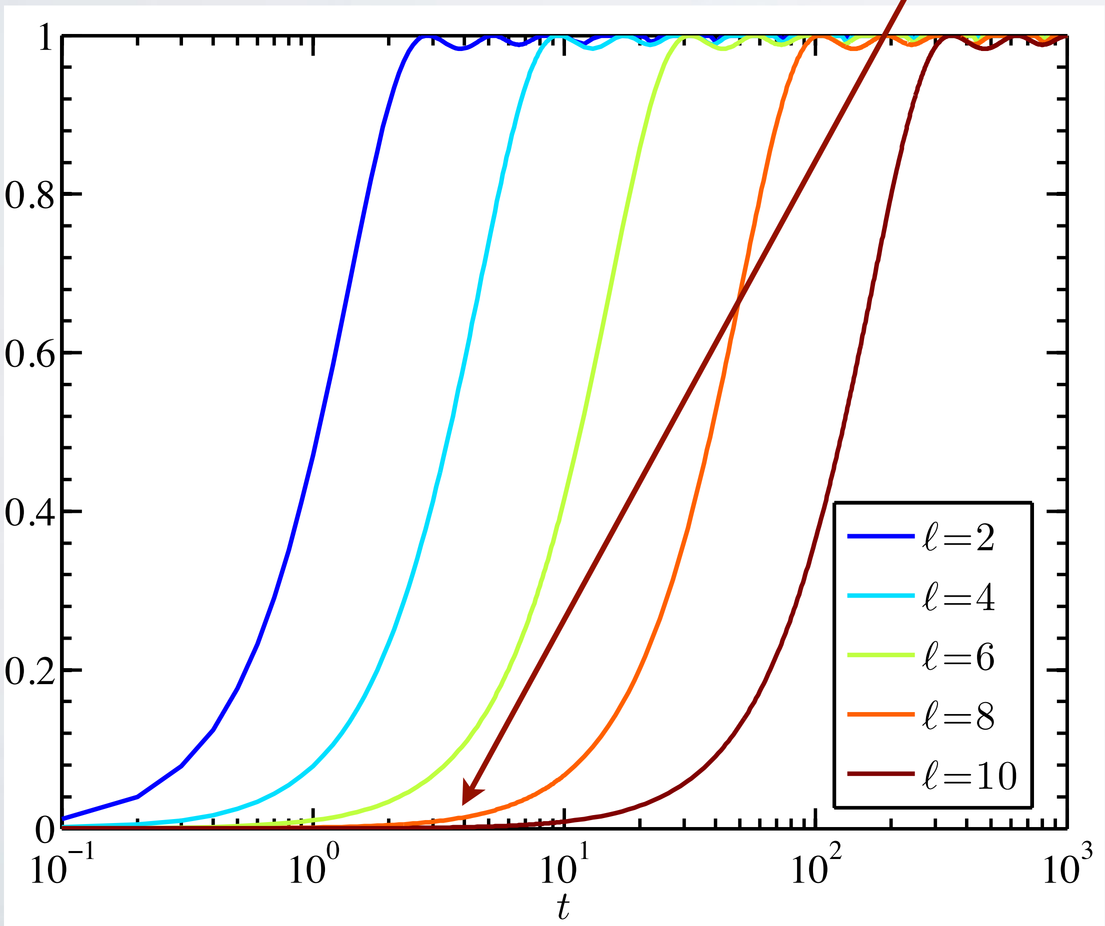
interacting case: l-bit model

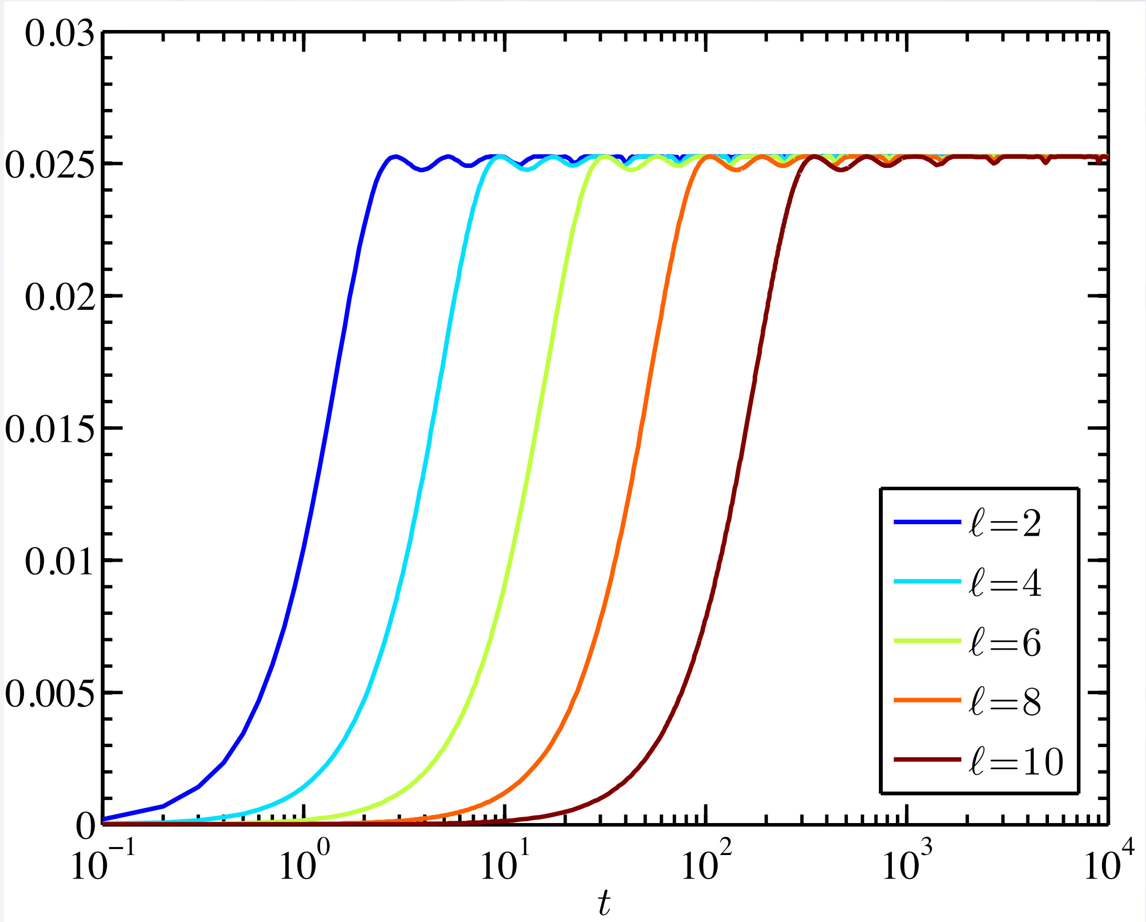
$\xi = 10$

exponential time
to reach a cut

$|X\pm\rangle$

$|Z\pm\rangle$





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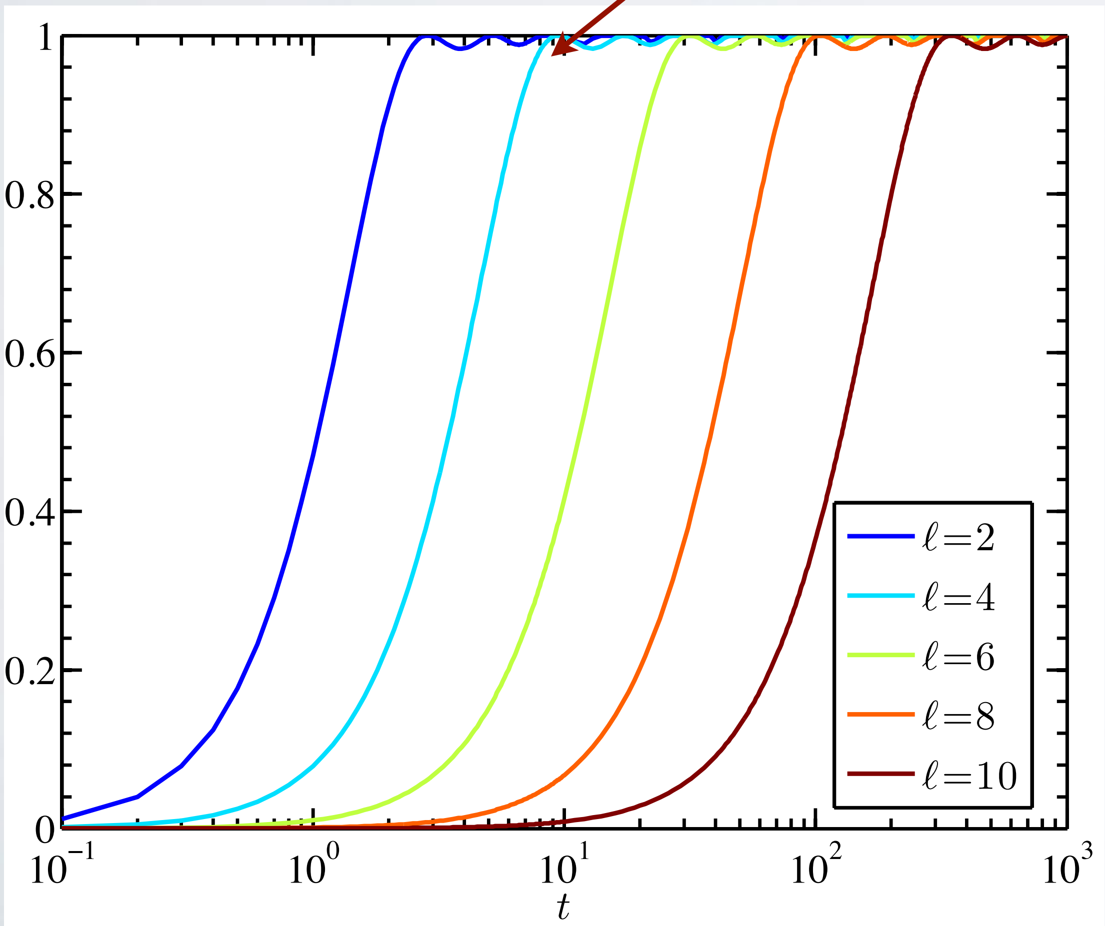
propagation of correlations

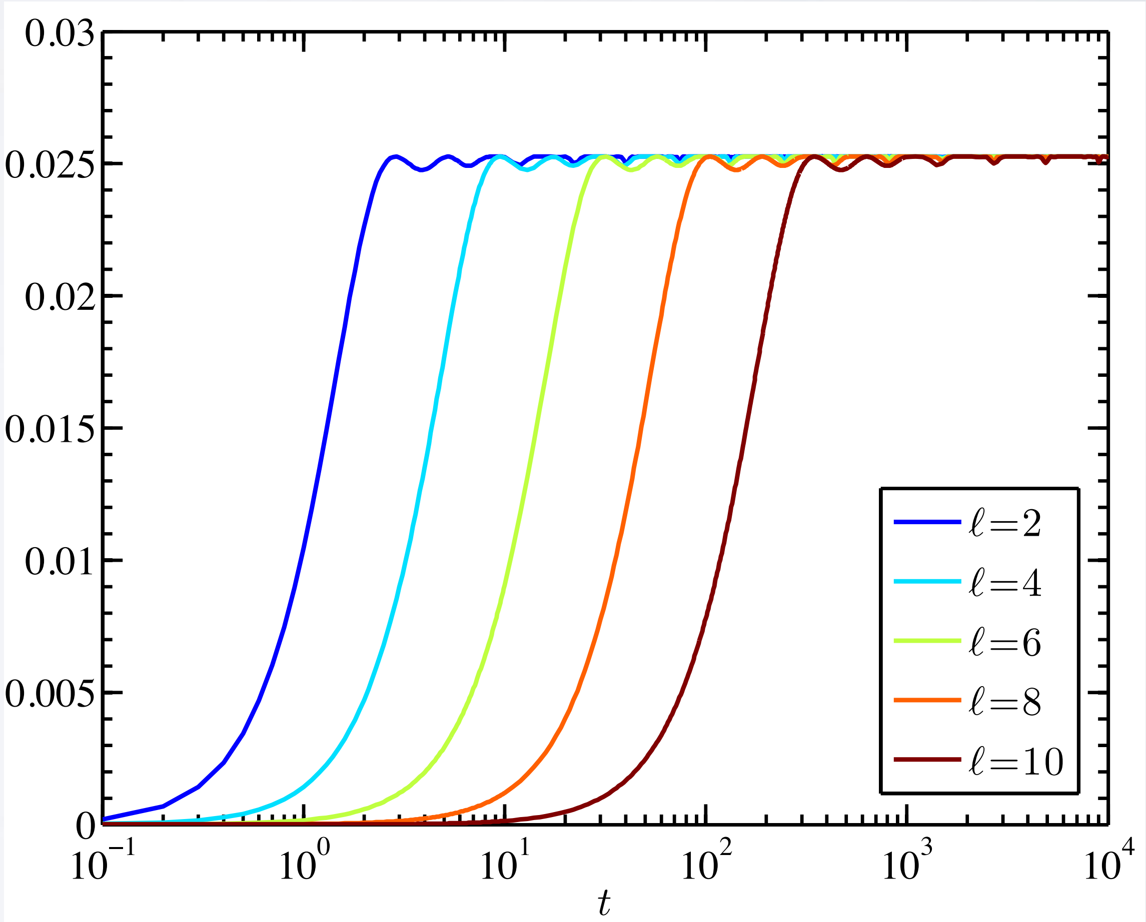
interacting case: l-bit model $\xi = 10$

can reach the largest value

$|X\pm\rangle$



$|Z\pm\rangle$



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propagation of correlations

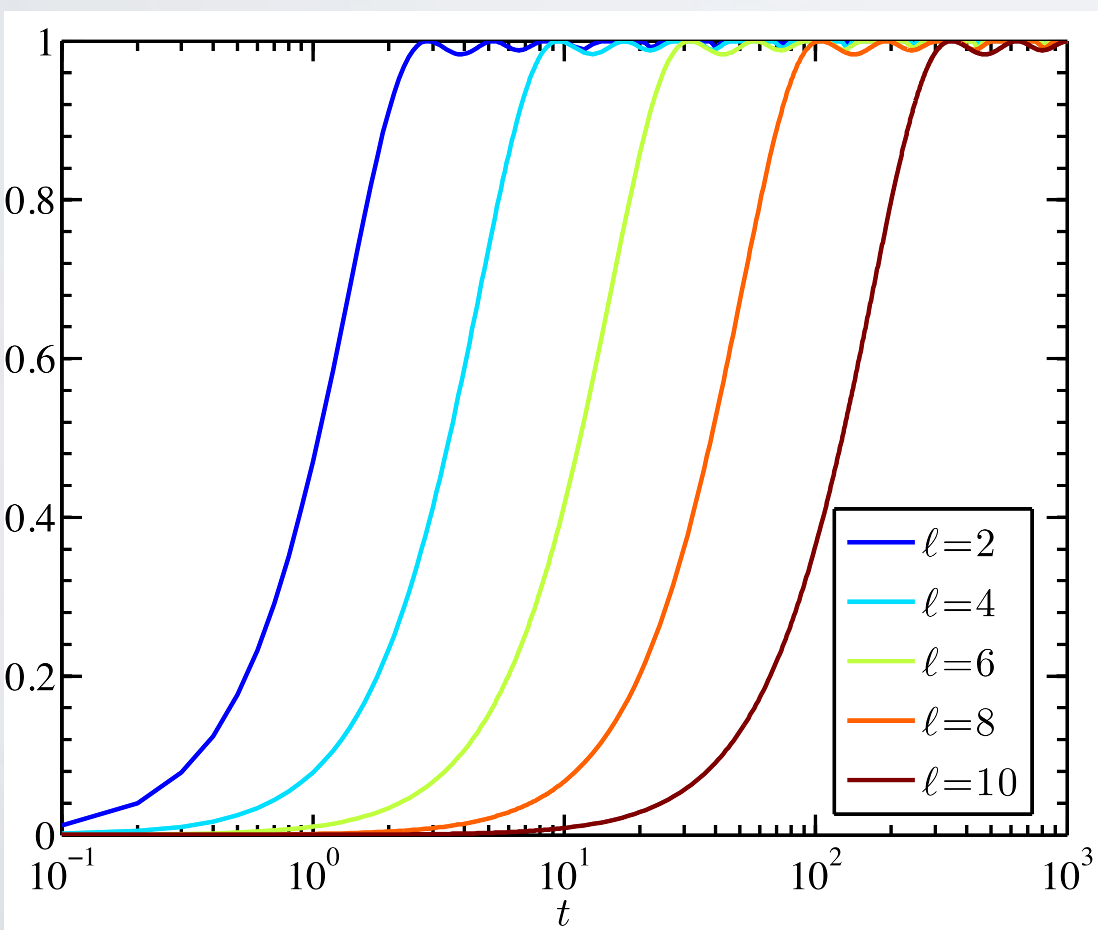
interacting case: l-bit model

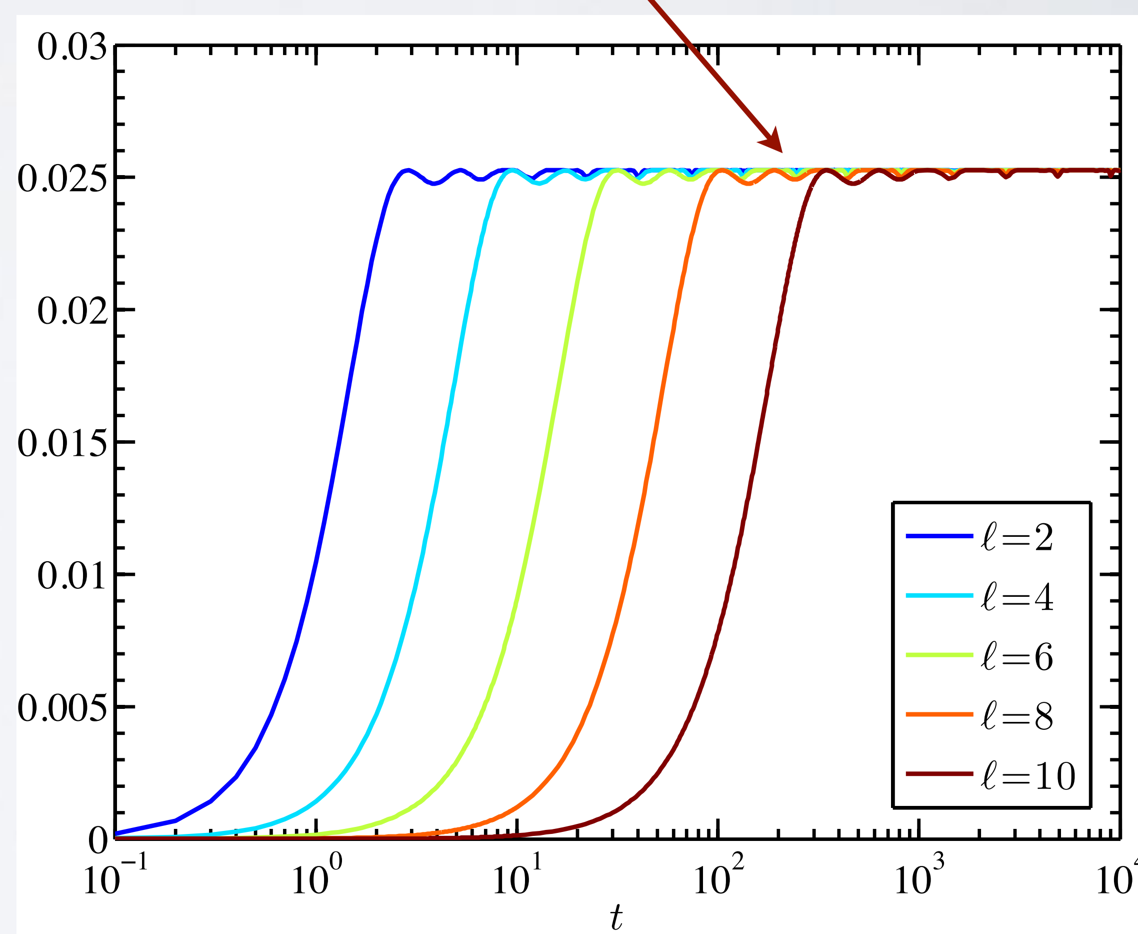
$\xi = 10$

Z case from $\tau_x^{[0]}$
upper bounded

$|X\pm\rangle$

$|Z\pm\rangle$





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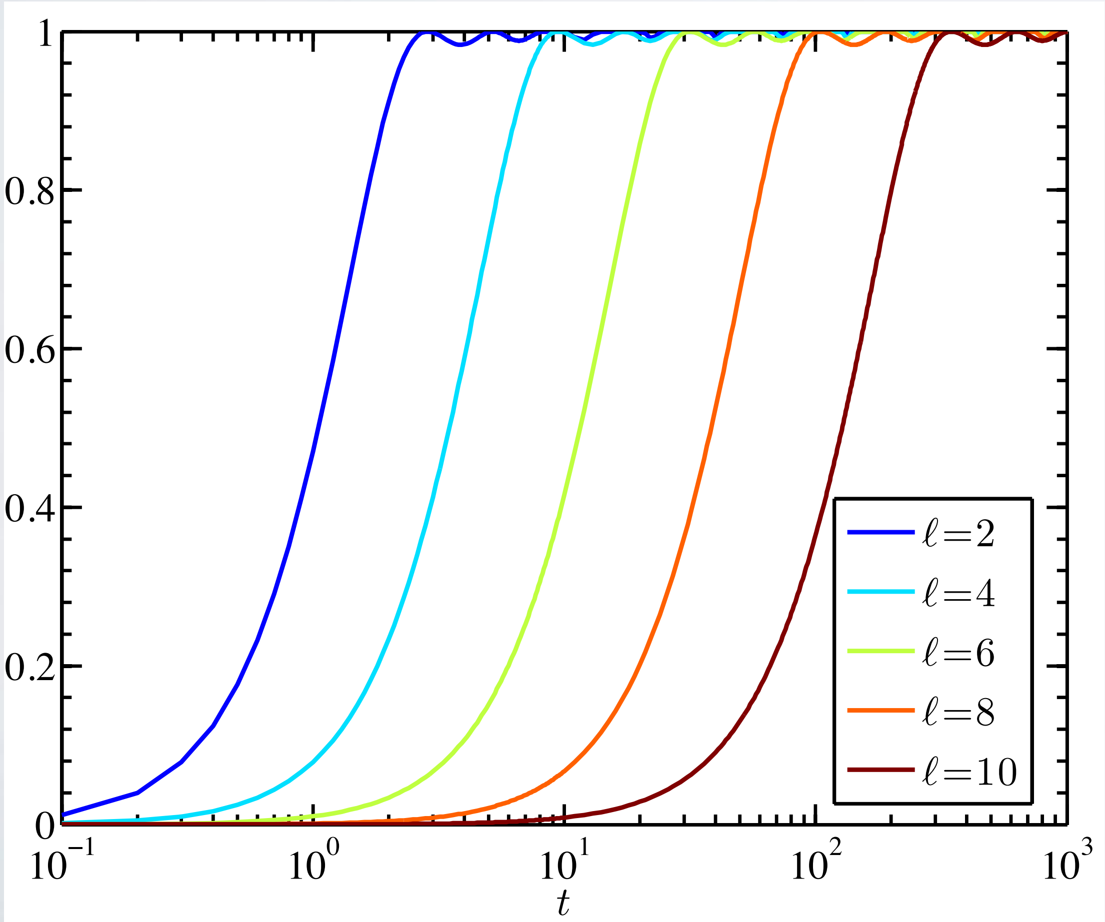
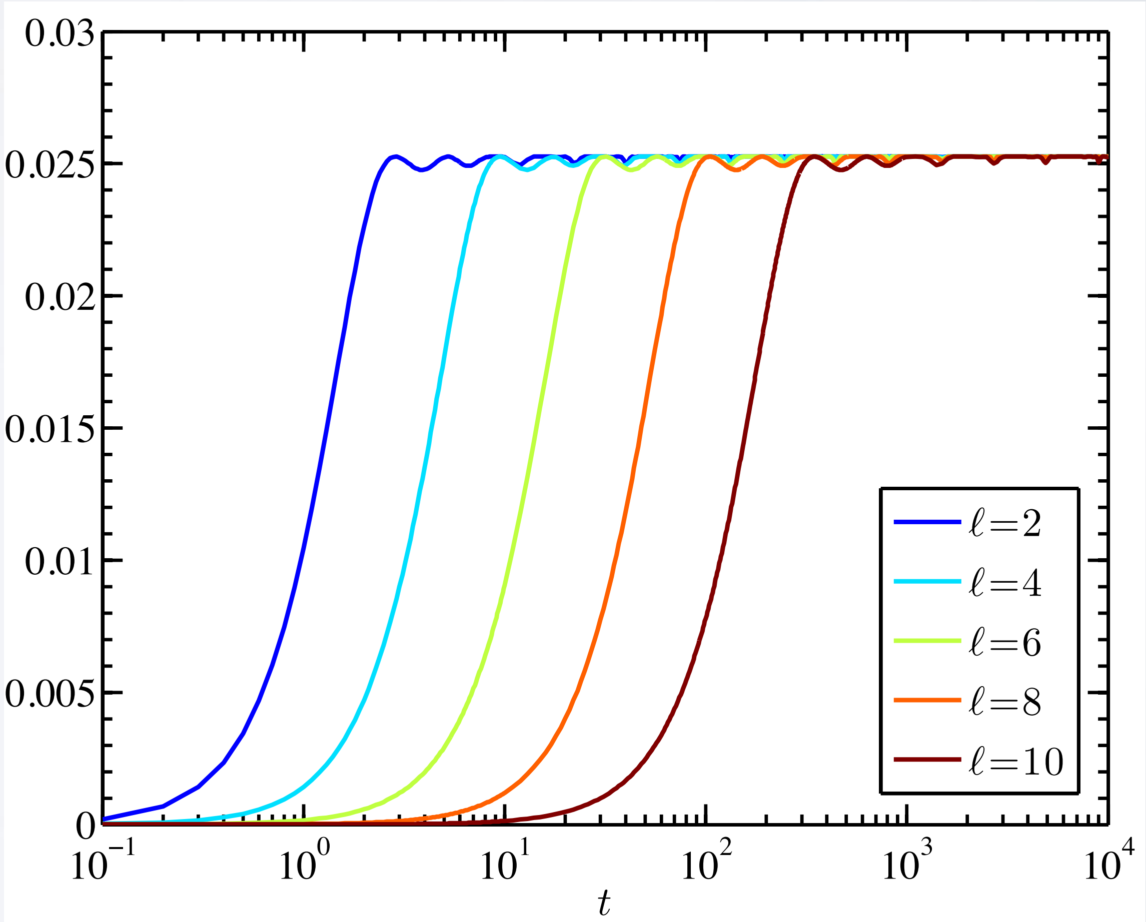
propagation of correlations

interacting case: l-bit model

$|X\pm\rangle$



$|Z\pm\rangle$



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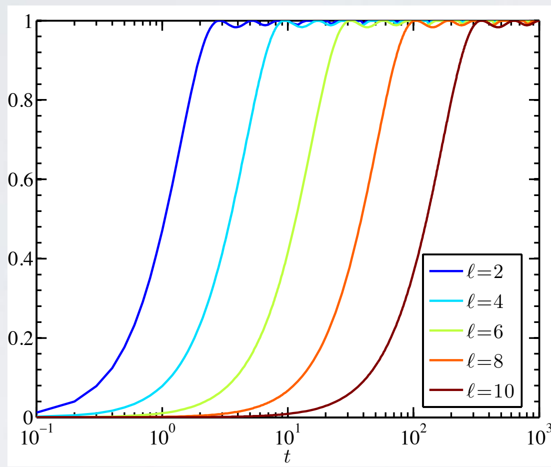
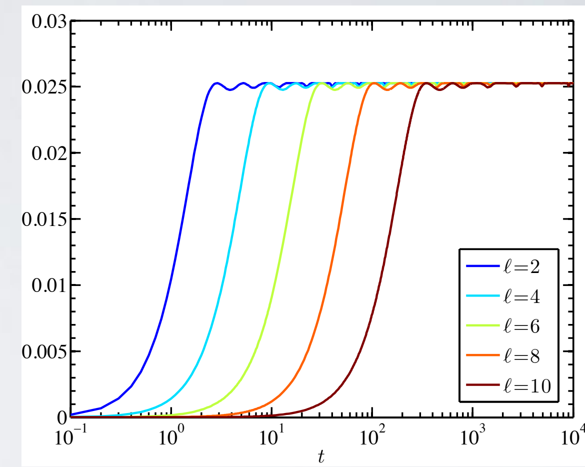
propagation of correlations

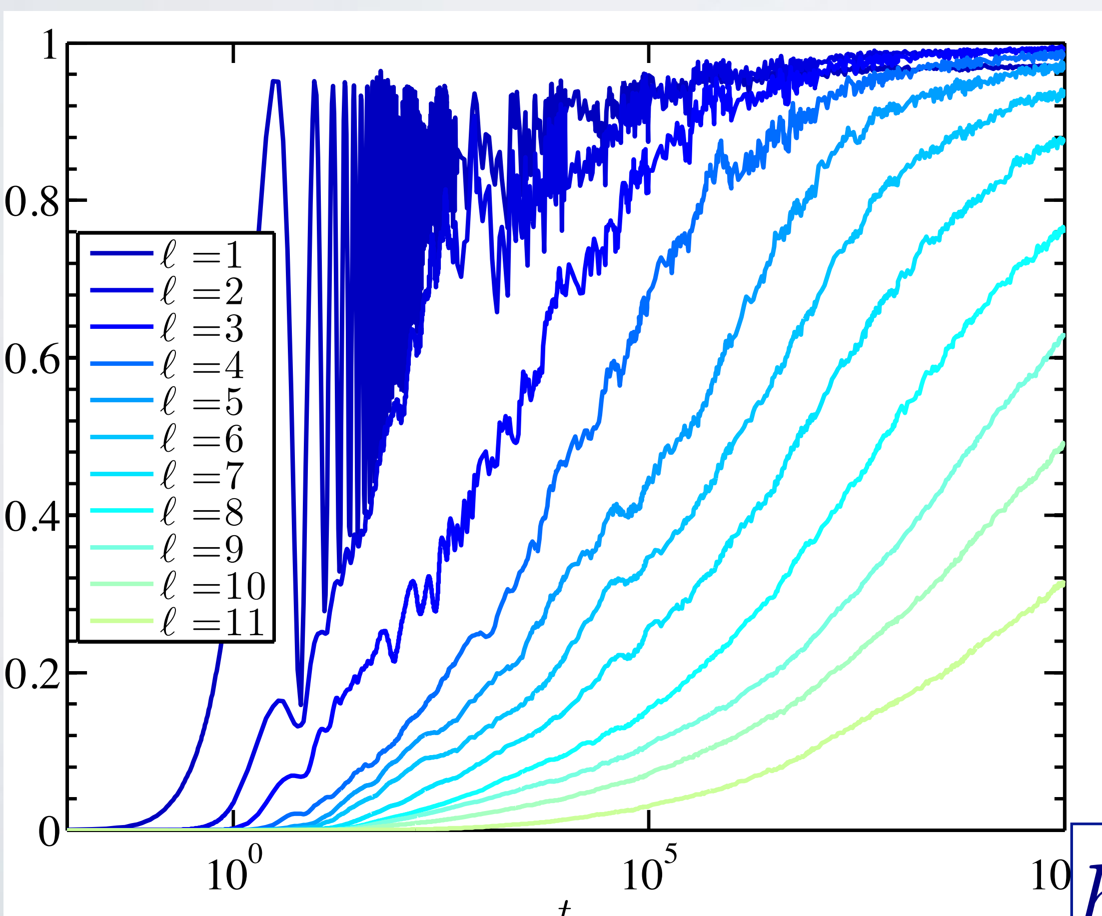
interacting case: l-bit model

$|X\pm\rangle$

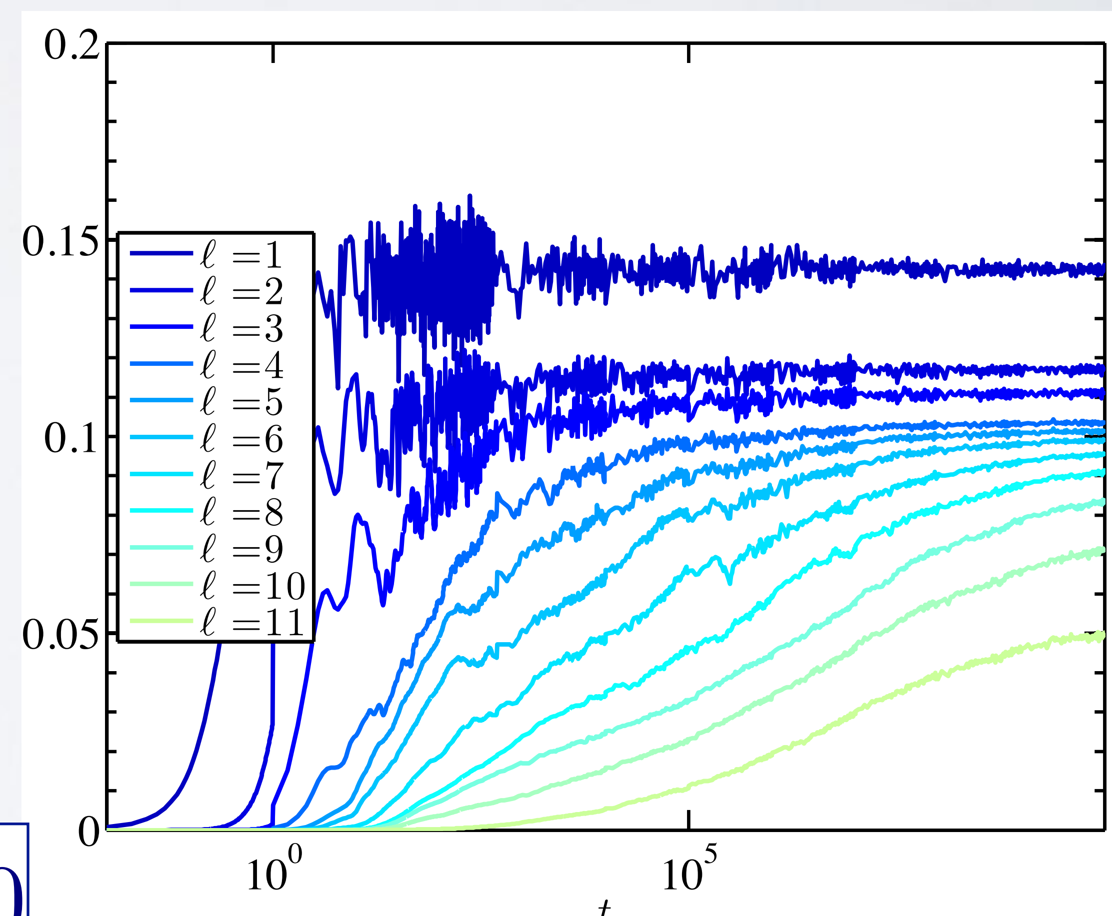


$|Z\pm\rangle$





$h = 10$



PRB 96,174201 (2017)

propagation of correlations

interacting case: l-bit model

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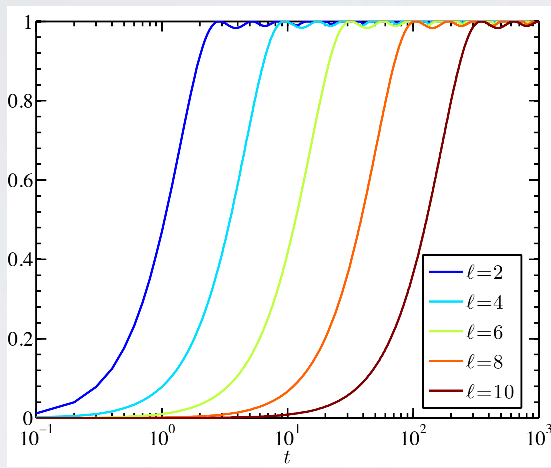
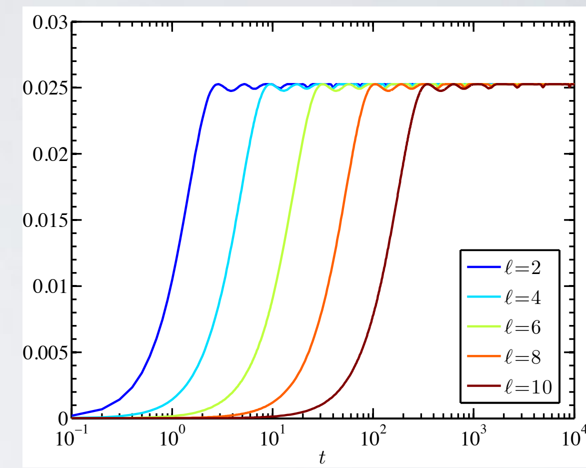
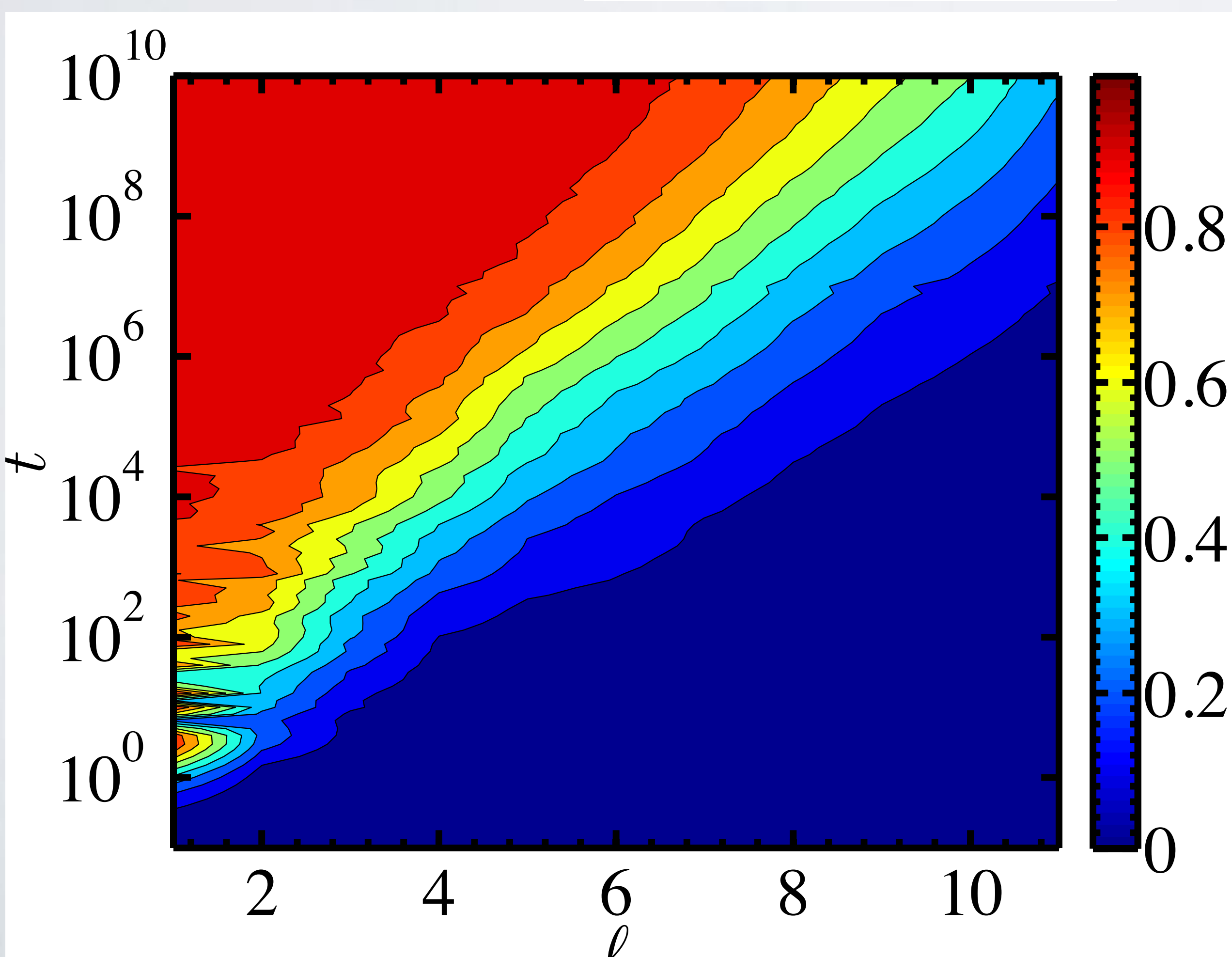
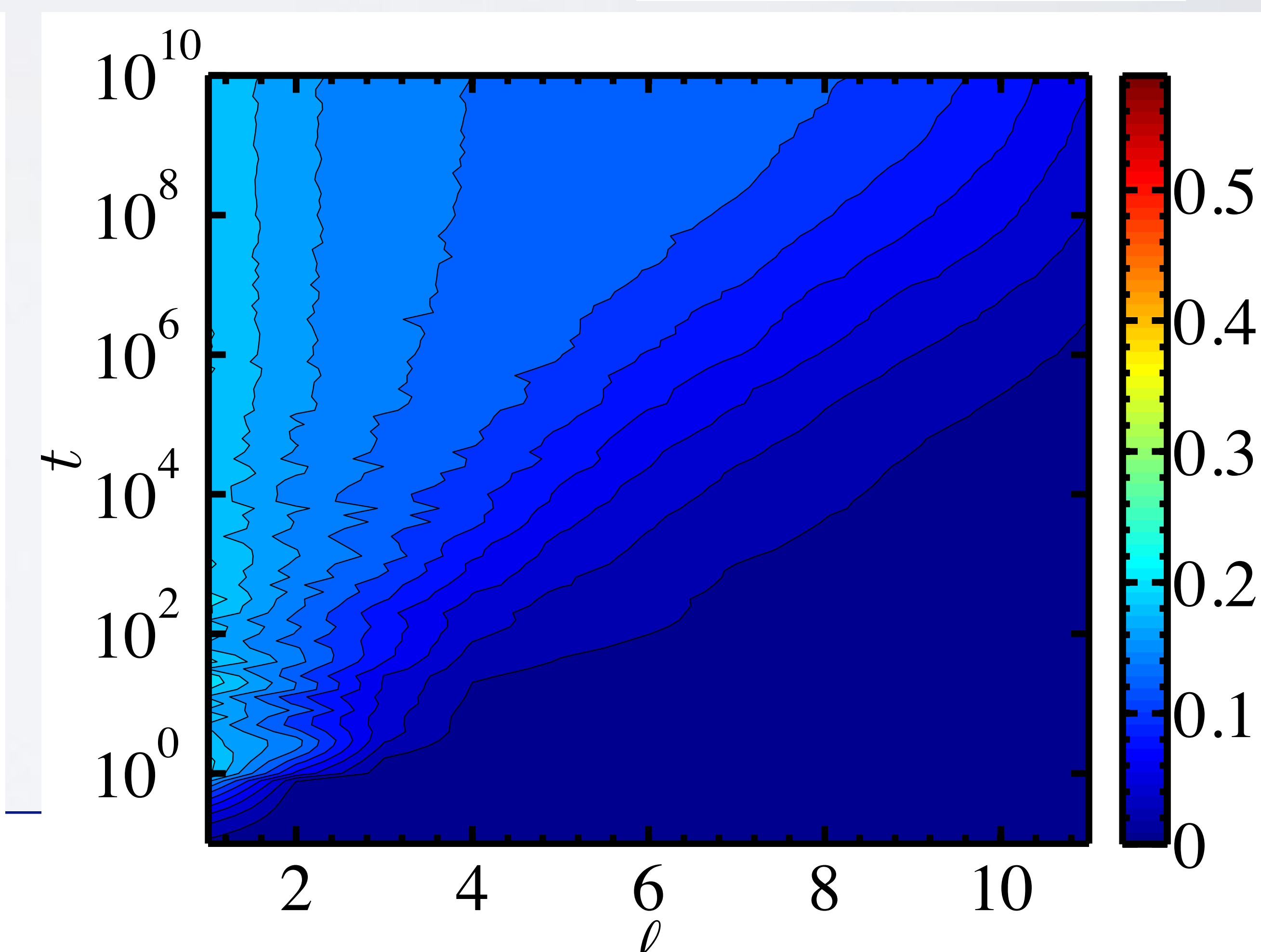


$|Z\pm\rangle$

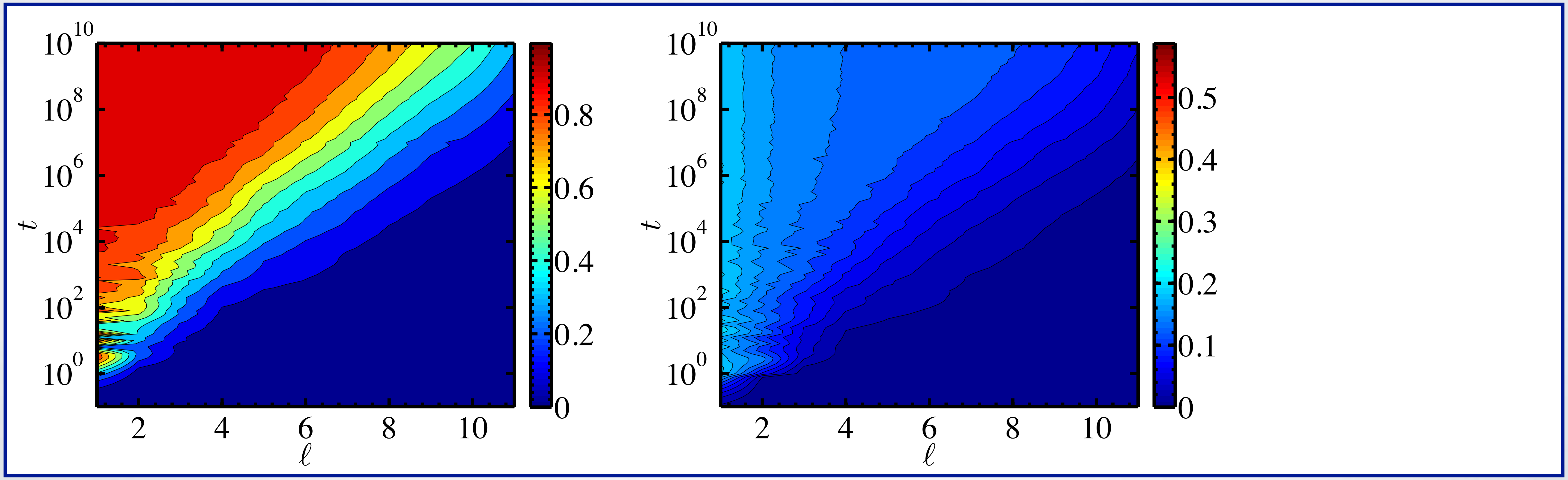






propagation of correlations

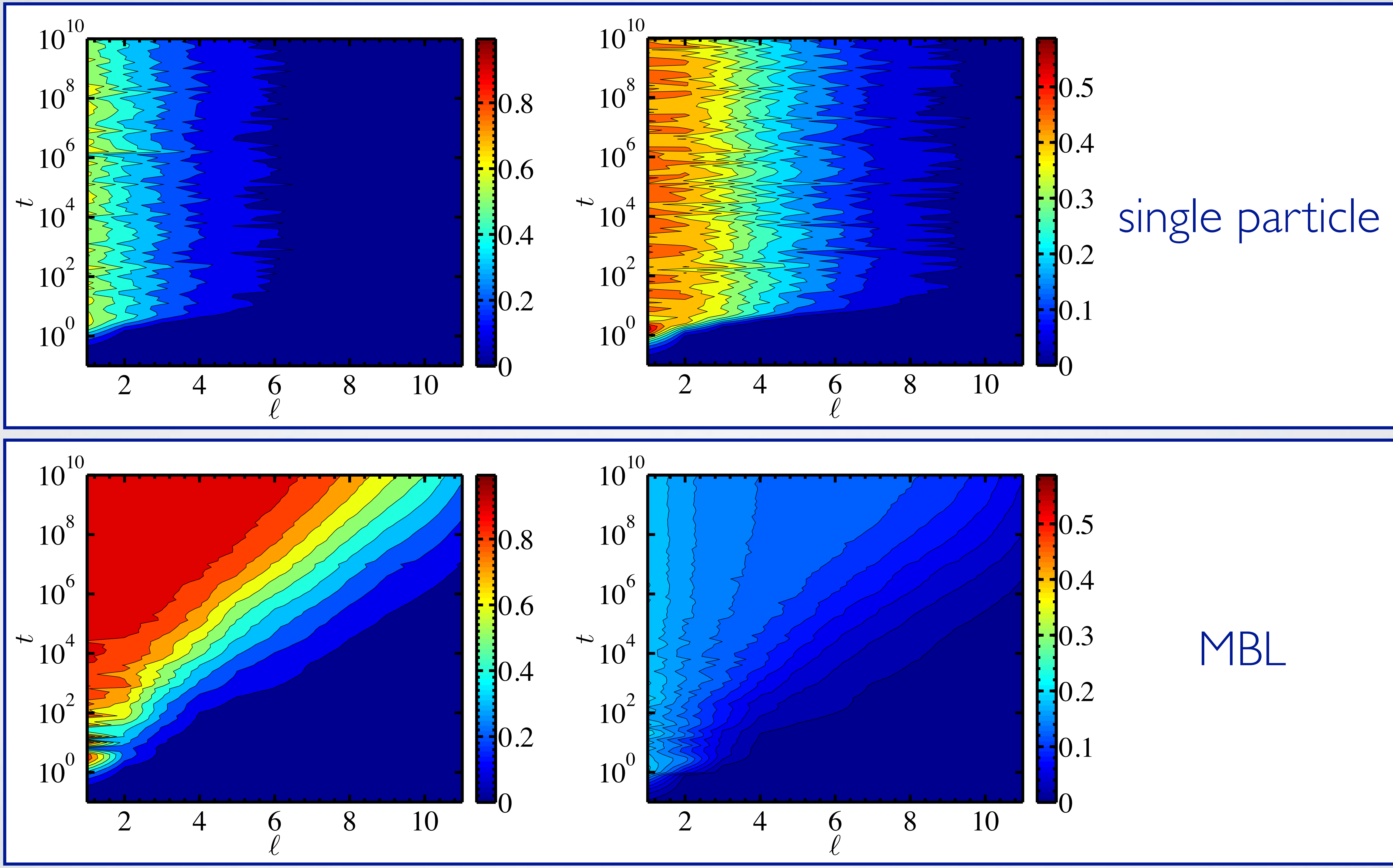
$|X\pm\rangle$ $|Z\pm\rangle$



PRB 96, 174201 (2017)

propagation of correlations

$|X\pm\rangle$ $|Z\pm\rangle$



PRB 96,174201 (2017)

Some questions we are asking

dynamics of mixed states

- propagation of correlations ✓
- quantum memory features ←
- simulability with MPO

Hamiltonian properties

- local conserved quantities

quantum memory

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

quantum memory

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$|0\rangle$

could be used to encode a qubit

quantum memory

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$|1\rangle$ 

could be used to encode a qubit

quantum memory

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$|1\rangle$ 

quantify potential as quantum memory

recovery fidelity

quantum memory

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quantify potential as quantum memory

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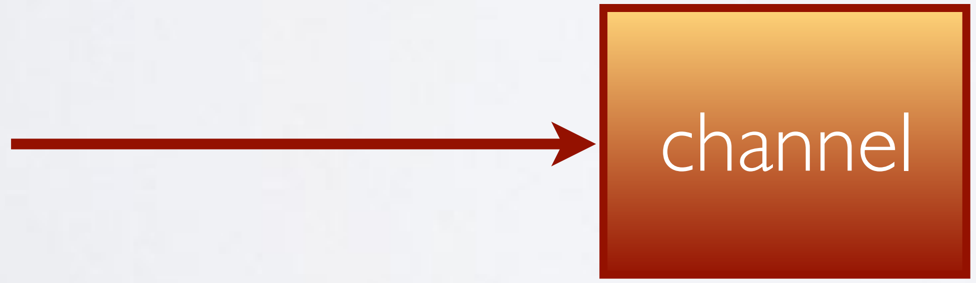
$|1\rangle$

quantify potential as quantum memory

recovery fidelity

$|\Phi\rangle$



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recovery fidelity

preparing
the state

quantum memory

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quantify potential as quantum memory

recovery fidelity

preparing the state

evolution and tracing out *N-1*

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$|1\rangle$

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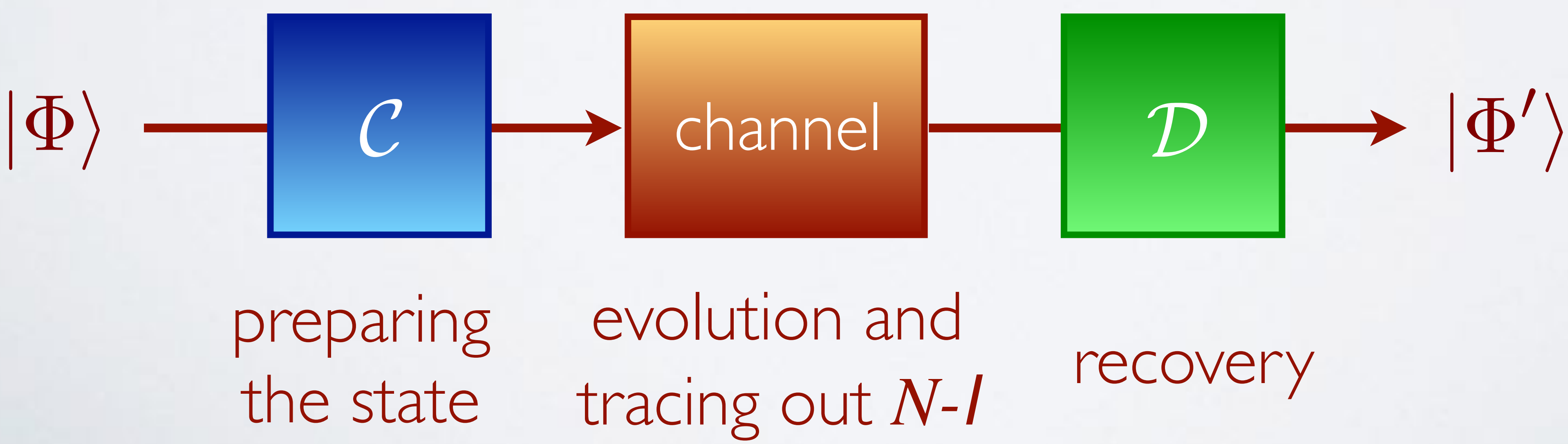
recovery fidelity



quantum memory

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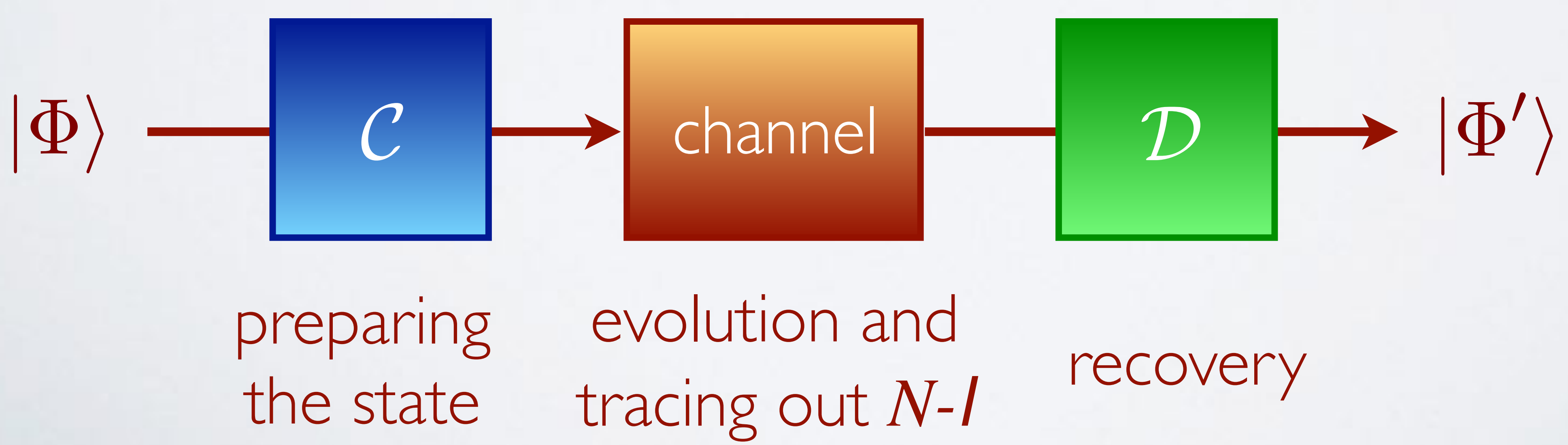
$|1\rangle$ 

quantify potential as quantum memory

recovery fidelity how well we can recover a qubit state



quantum memory

$$\rho_\Phi \propto |\Phi\rangle\langle\Phi| \otimes Id^{\otimes \frac{L-L_0}{2}}$$

$|1\rangle$ 

quantify potential as quantum memory

recovery fidelity — how well we can recover a qubit state

determined by distinguishability of orthogonal pairs $|X\pm\rangle$ $|Z\pm\rangle$

preparing the state

evolution and tracing out *N-1*

recovery

non-interacting case

$$\text{disting}_X = 2\sqrt{\mathcal{V}_l}$$

$$\text{disting}_Z = 2\mathcal{V}_l$$

$$\mathcal{V}_\ell = \sum_{r=0}^{\ell-1} |\langle r|U(t)|0\rangle|^2$$

non-interacting case $h > 0 \Rightarrow \xi$

$$\text{disting}_X = 2\sqrt{\mathcal{V}_l} \quad \geq 2\sqrt{1 - 2N(N-\ell)e^{-\ell/\xi}}$$

$$\text{disting}_Z = 2\mathcal{V}_l \quad \geq 2 - 2CN(N-\ell)e^{-\ell/\xi}$$

quantum memory

non-interacting case $h > 0 \Rightarrow \xi$

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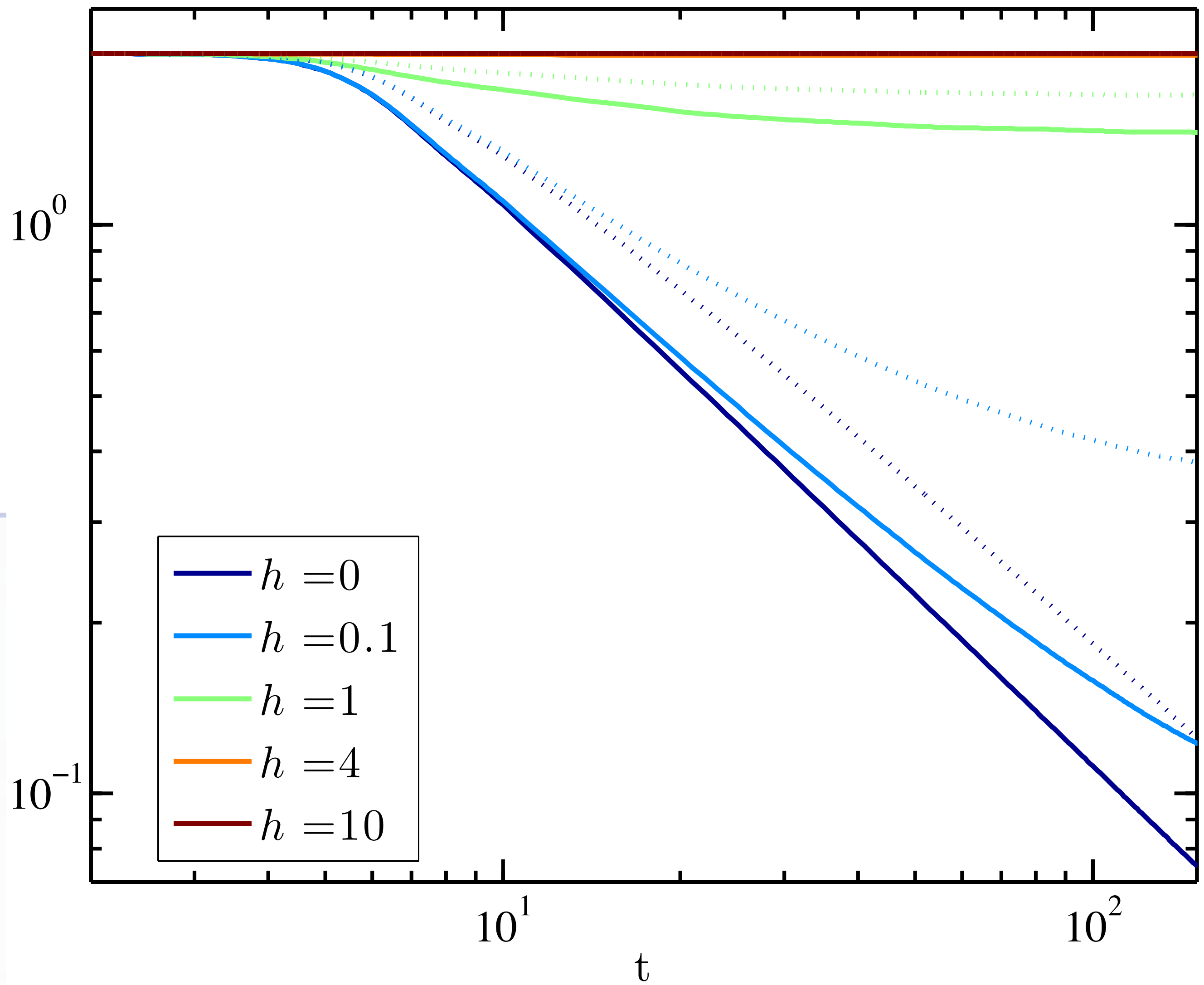
can store quantum information

quantum memory

non-interacting case $h > 0 \to \xi$

disting$_X$

disting$_Z$



$e^{-\ell/\xi}$

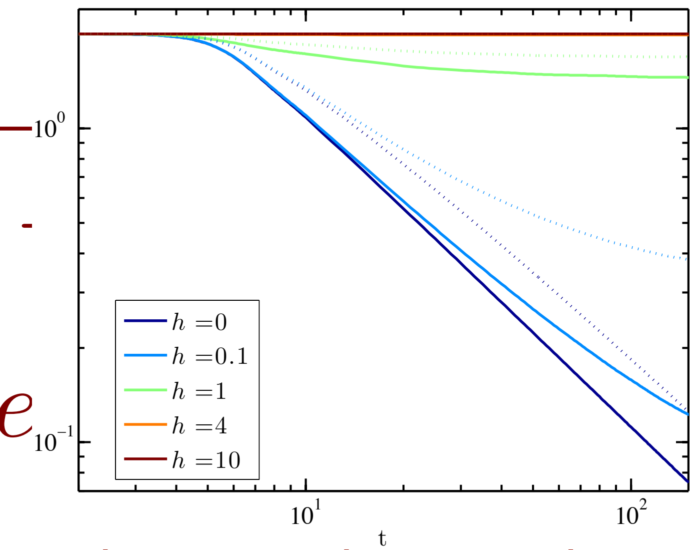
n information

quantum memory

non-interacting case $h > 0 \Rightarrow \xi$

$$\text{disting}_X = 2\sqrt{\mathcal{V}_l} \geq 2\sqrt{1 - 2N(N}$$

$$\text{disting}_Z = 2\mathcal{V}_l \geq 2 - 2CN(N - \ell)e$$



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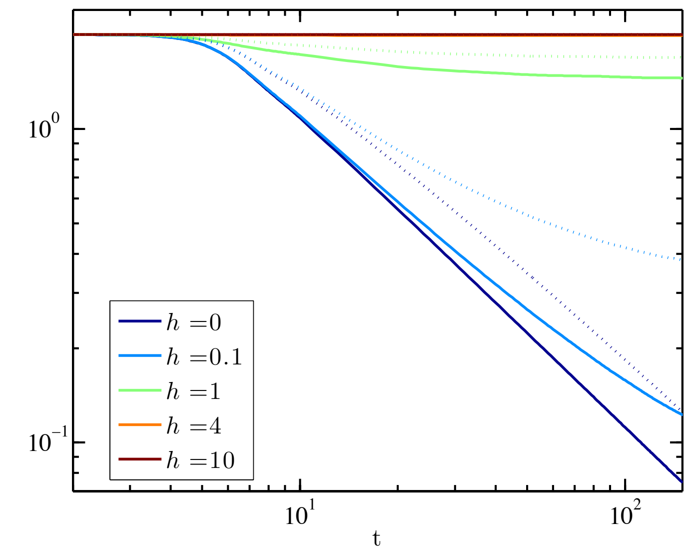

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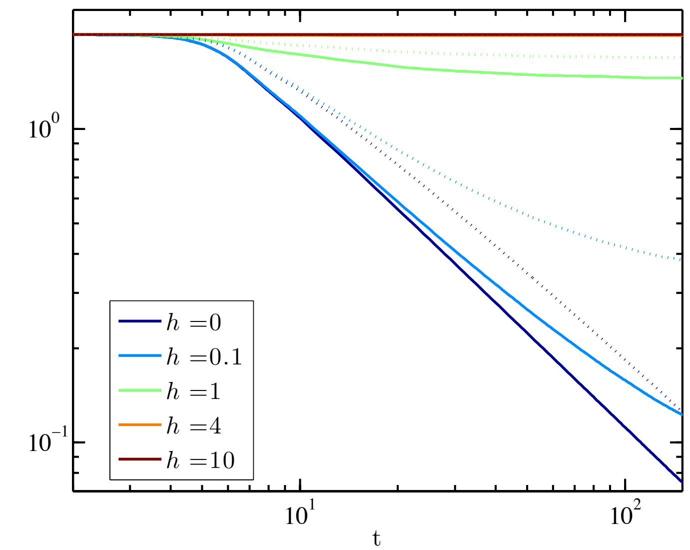

can store quantum information

interacting case: l-bits

$$\mathrm{disting}_X \approx 2|x(\ell,t)|$$

$$\mathrm{disting}_Z \approx 2\sqrt{1 - \alpha^2(1 - x(\ell,t)^2)}$$

$$x(\ell,t) = \prod_{k=\ell}^{N-1} \cos(2tK_{0k}^{(2)})$$

quantum memory

non-interacting case $h > 0 \Rightarrow \xi$

$$\text{disting}_X \geq 2\sqrt{1 - 2N(N-\ell)e^{-\ell/\xi}}$$

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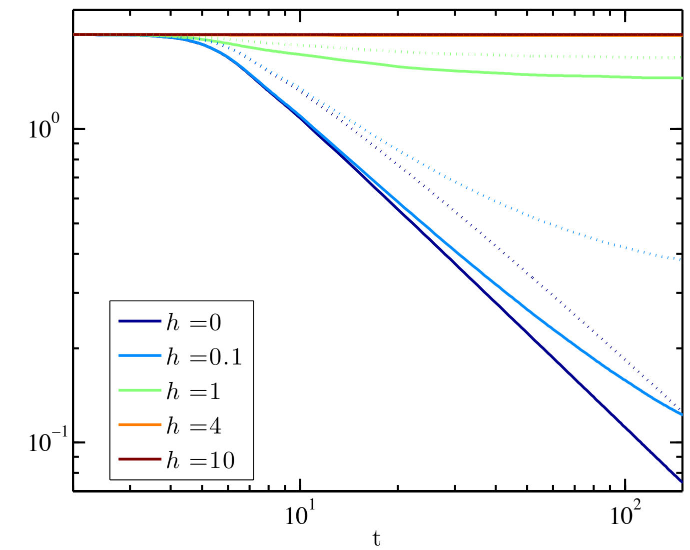

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interacting case: l-bits

$$\text{disting}_X \approx 2|x(\ell,t)| \approx 2|1 - 2t^2(N-\ell)e^{-2\ell/\xi}|$$

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quantum memory

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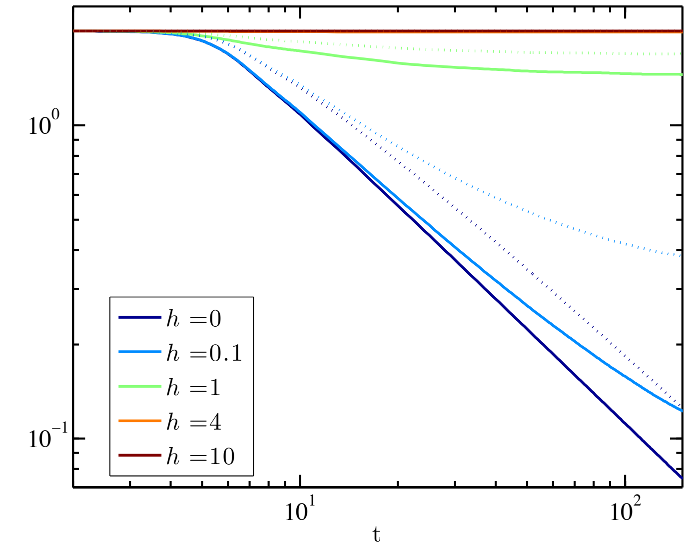

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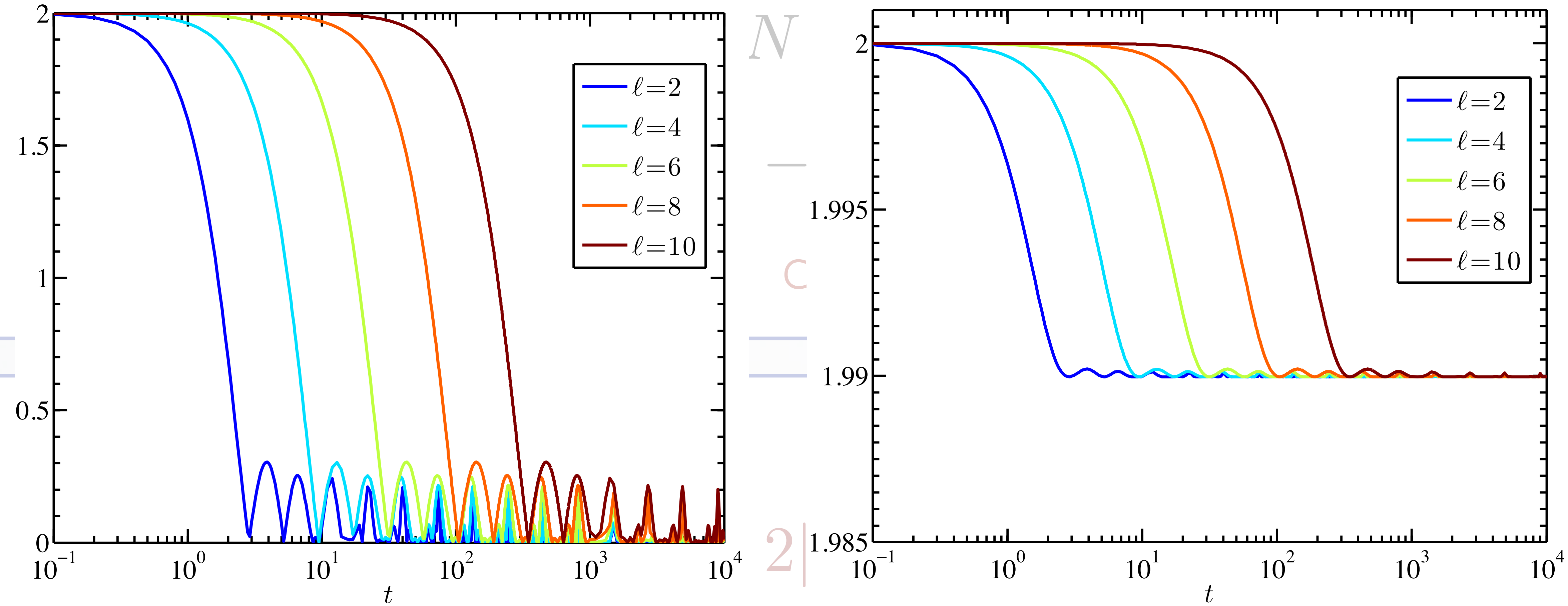
can store only classical information

quantum memory

non-interacting case $h > 0 \Rightarrow \xi$



$$\text{disting}_Z \approx 2\sqrt{1-\alpha^2(1-x(\ell,t)^2)} \geq 2\sqrt{1-\alpha^2}$$

can store only classical information

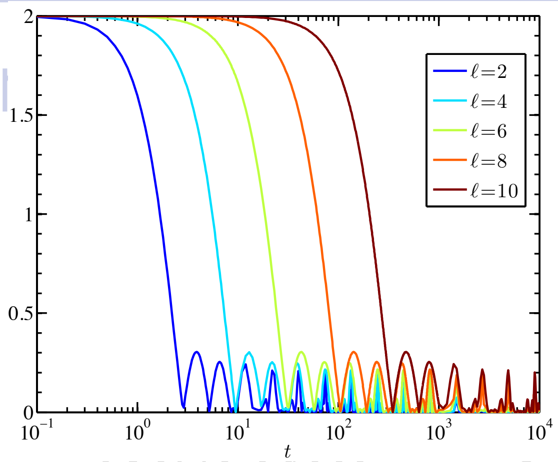
quantum memory

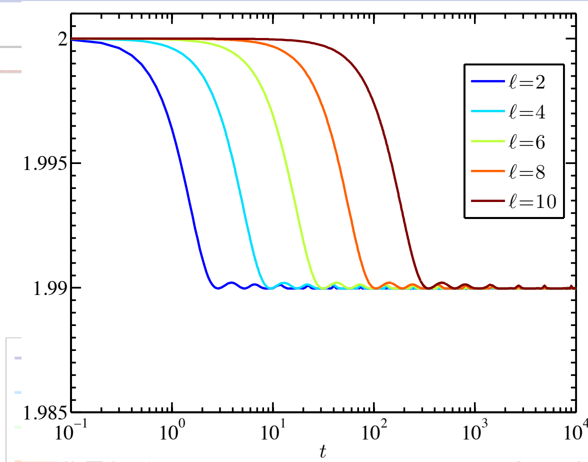
case $h > 0 \Rightarrow \xi$

$2\sqrt{1 - 2N(N-\ell)e^{-\ell/\xi}}$

$\mathrm{disting}_Z \leq 2 - 2CN(N-\ell)e^{-\ell/\xi}$

m information



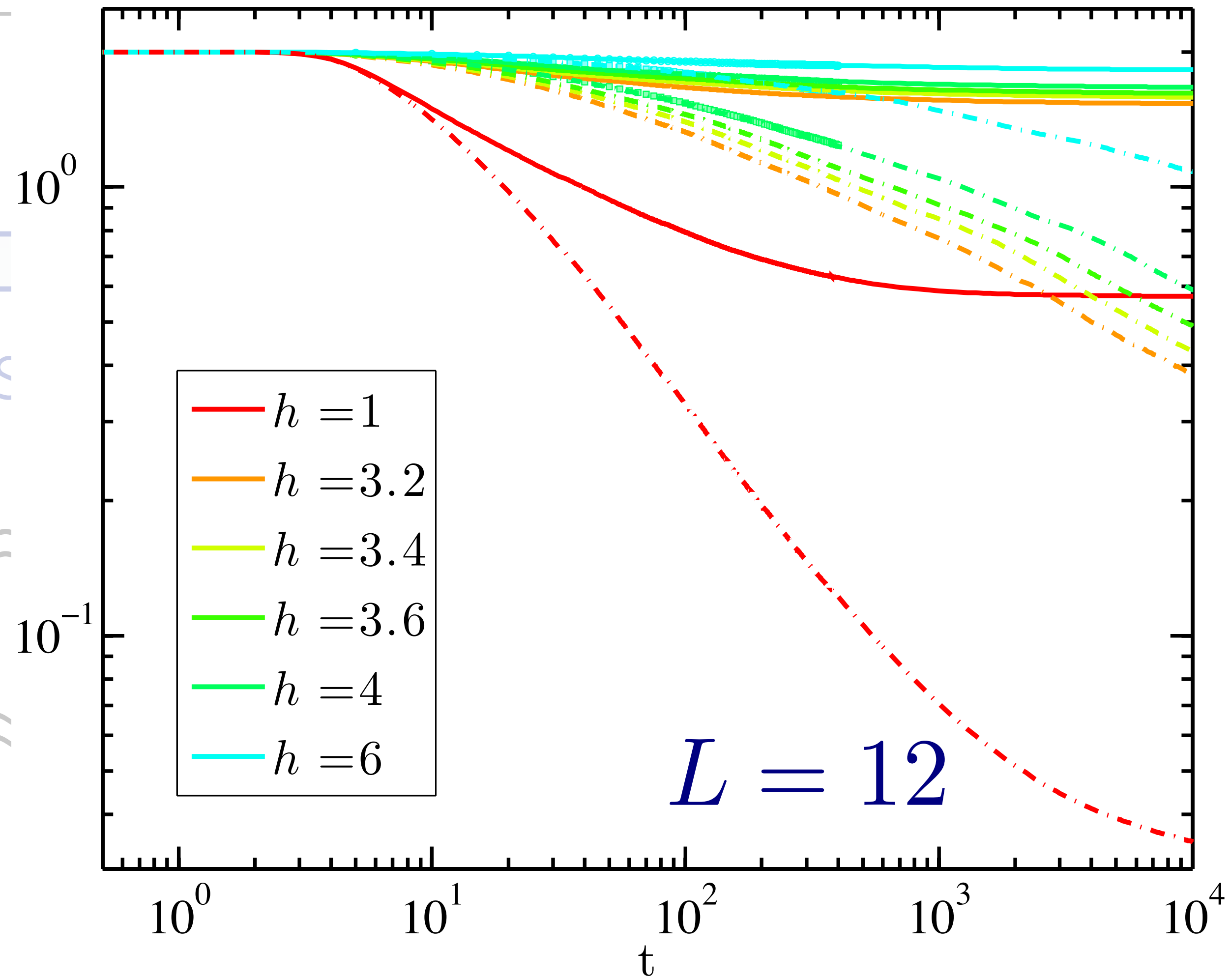


interacting cas

$\mathrm{disting}_X \approx$ … $)e^{-2\ell/\xi}|$

$\mathrm{disting}_Z \approx$ … $\sqrt{1-\alpha^2}$

cal information



quantum memory

non-interacting case $\quad h > 0 \Rightarrow \xi$

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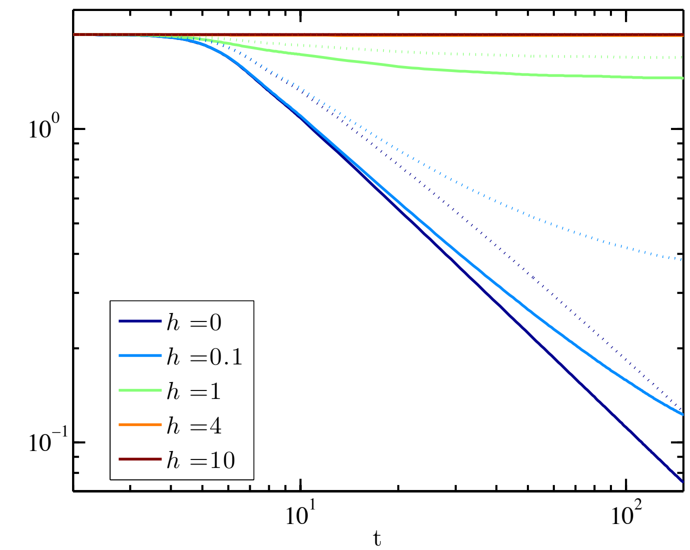

can store quantum information

interacting case: l-bits

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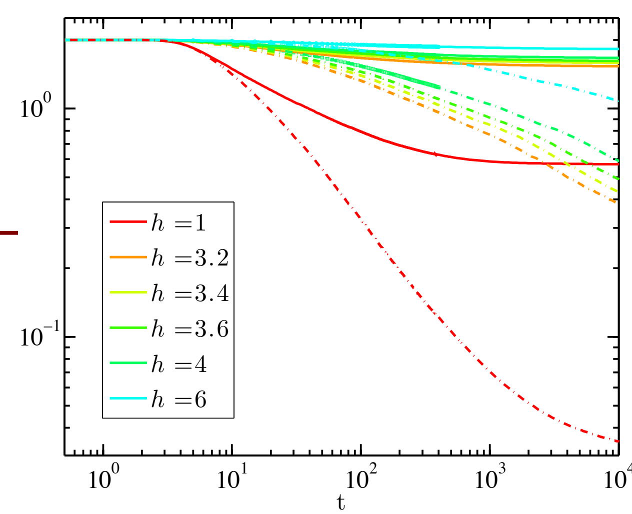

can store only classical information

Some questions we are asking

dynamics of mixed states

- propagation of correlations ✓
- quantum memory features ✓
- simulability with MPO ←

Hamiltonian properties

- local conserved quantities

simulability with MPO

simulability with MPO

Q: how do errors behave?

simulability with MPO

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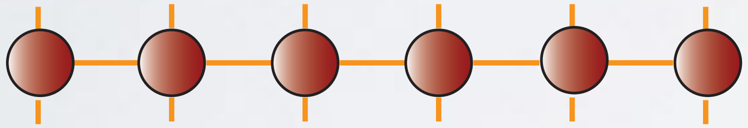
estimating error

best approximation



simulability with MPO

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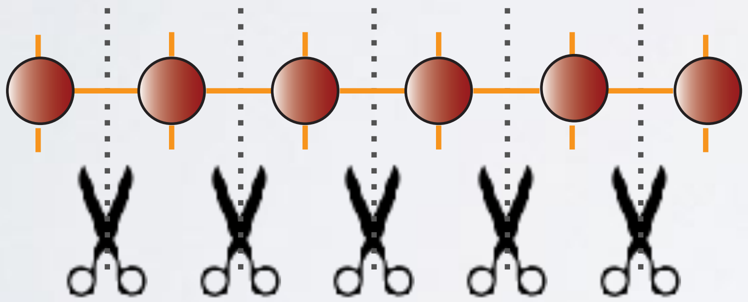

simulability with MPO

Q: how do errors behave?

estimating error

best approximation

MPO approximation with smaller D

for long chains only Frobenius norm

simulability with MPO

localization ⇔ errors in TN simulation

$$L = 20$$
$$D_{\max} = 120$$

simulability with MPO

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D for constant error

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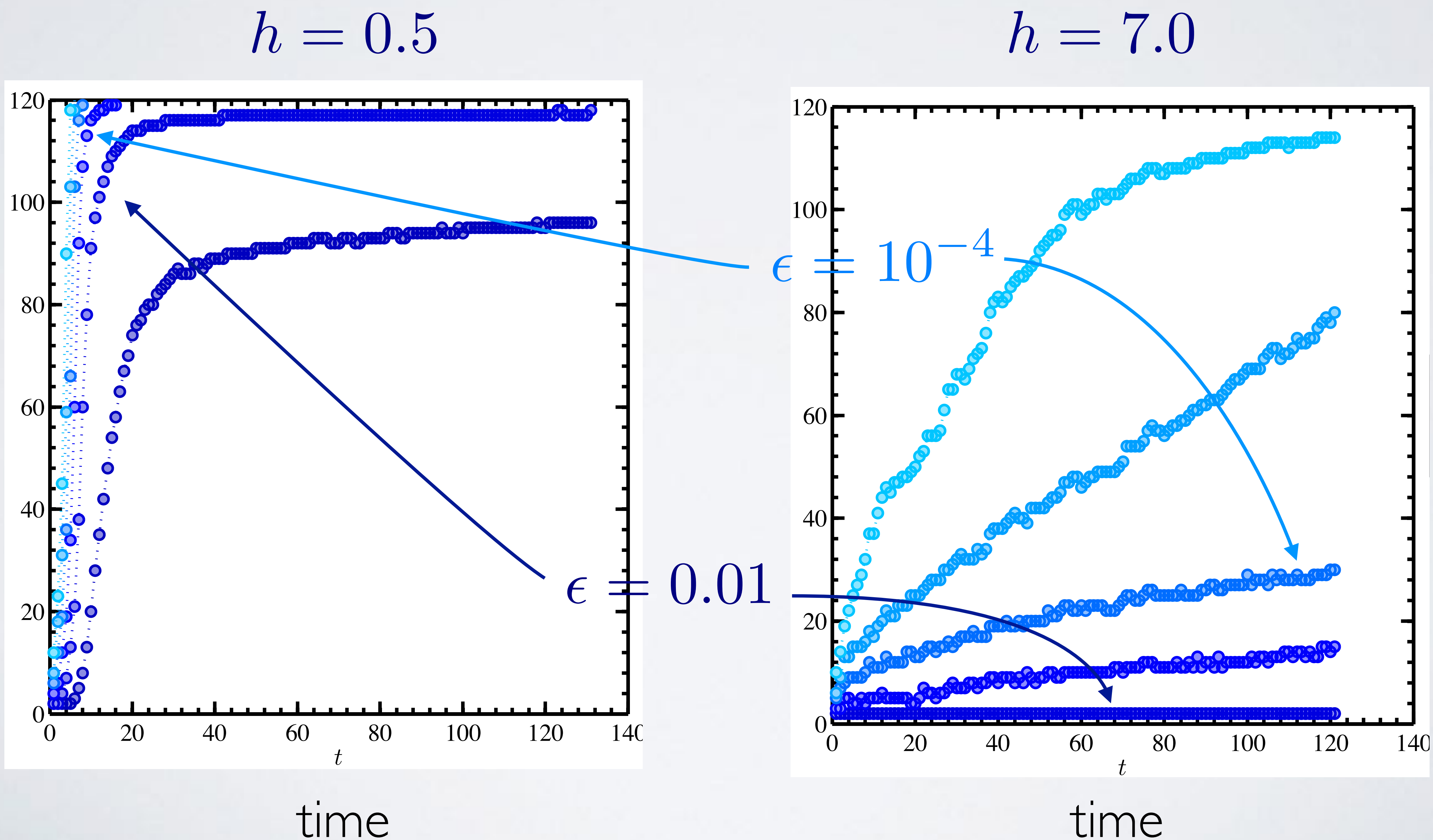


simulability with MPO

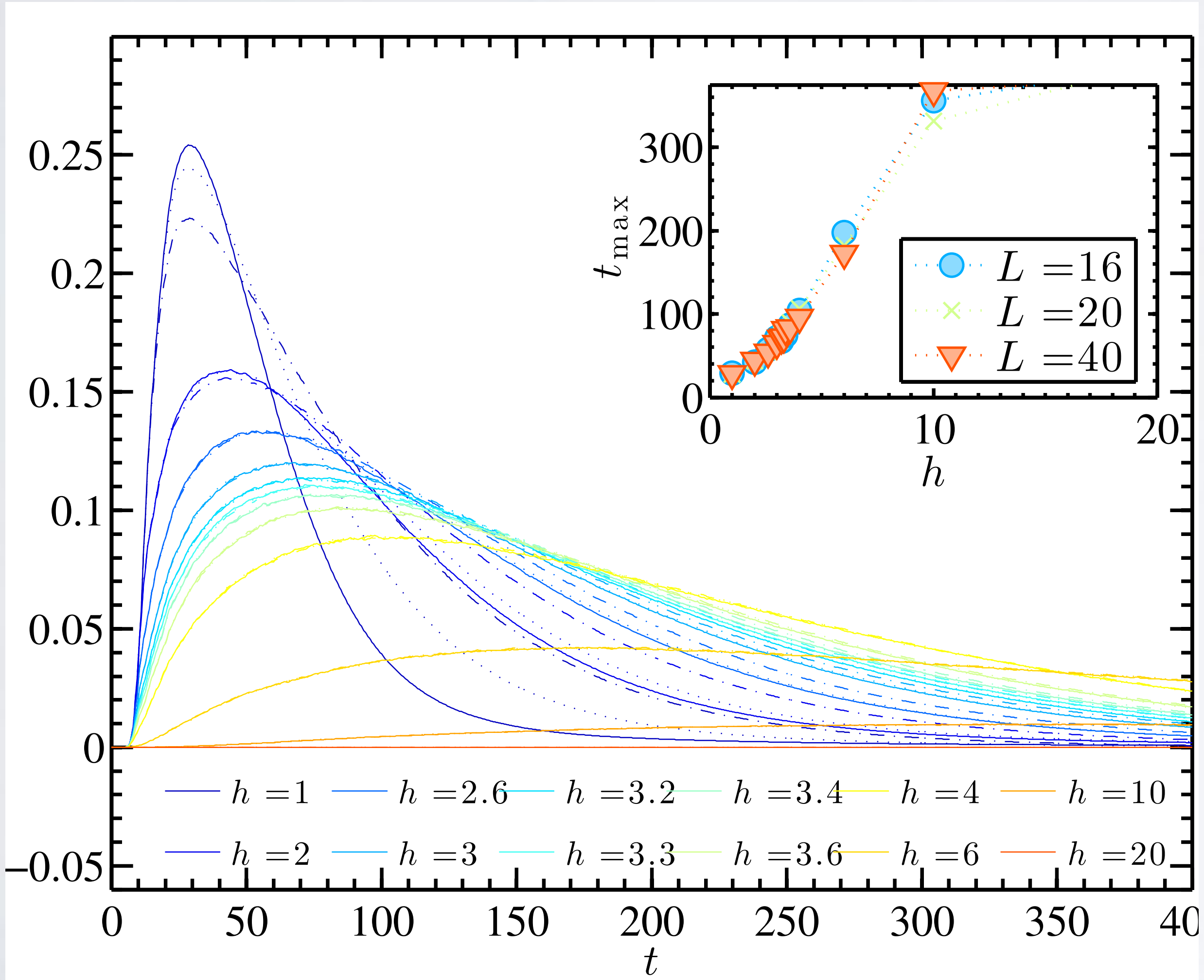
localization ⇔ errors in TN simulation

simulability with MPO

localization ⇔ errors in TN simulation

larger systems (Hilbert-Schmidt distance)

single instance



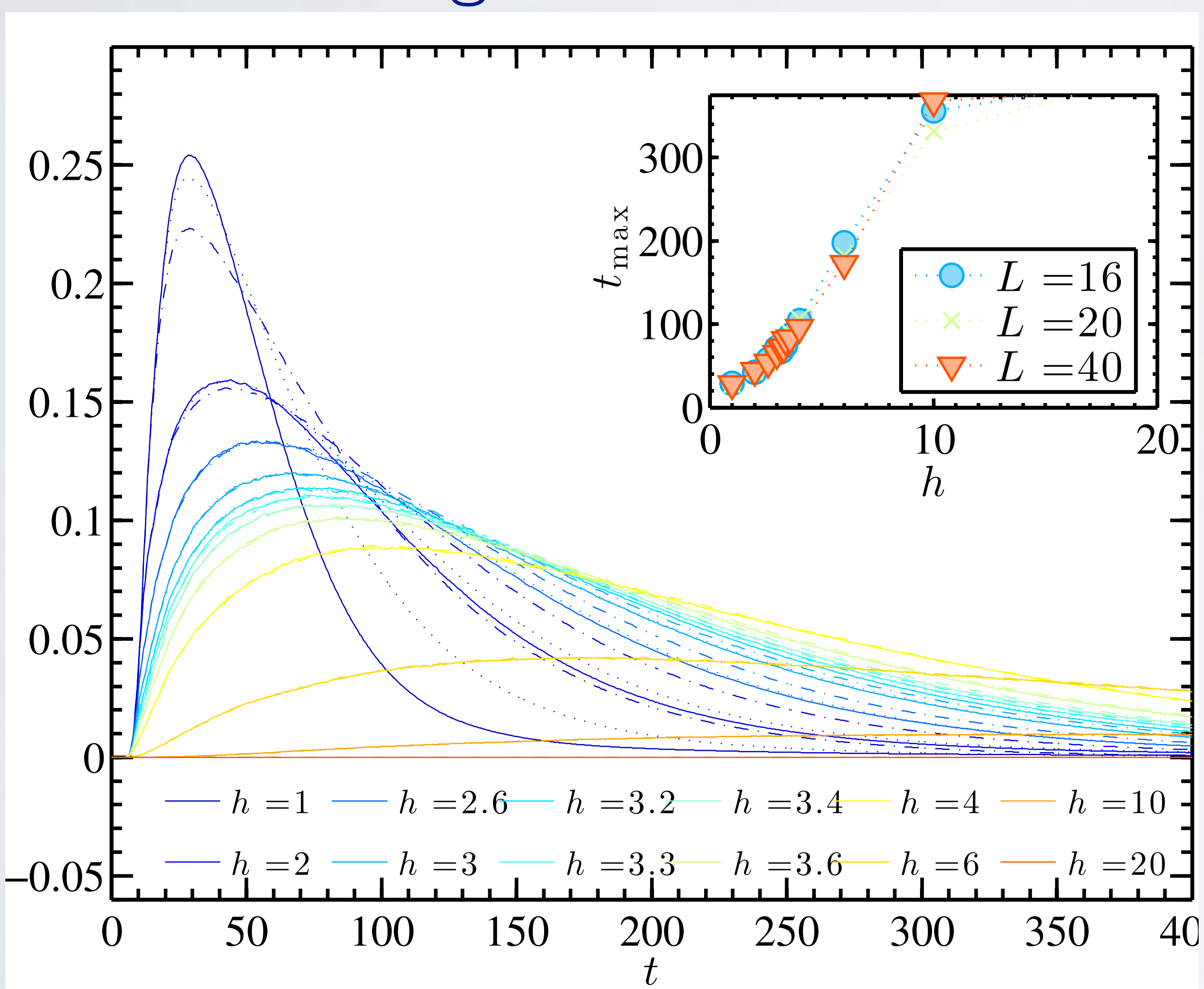
time

simulability with MPO

localization ⇔ errors in TN simulation

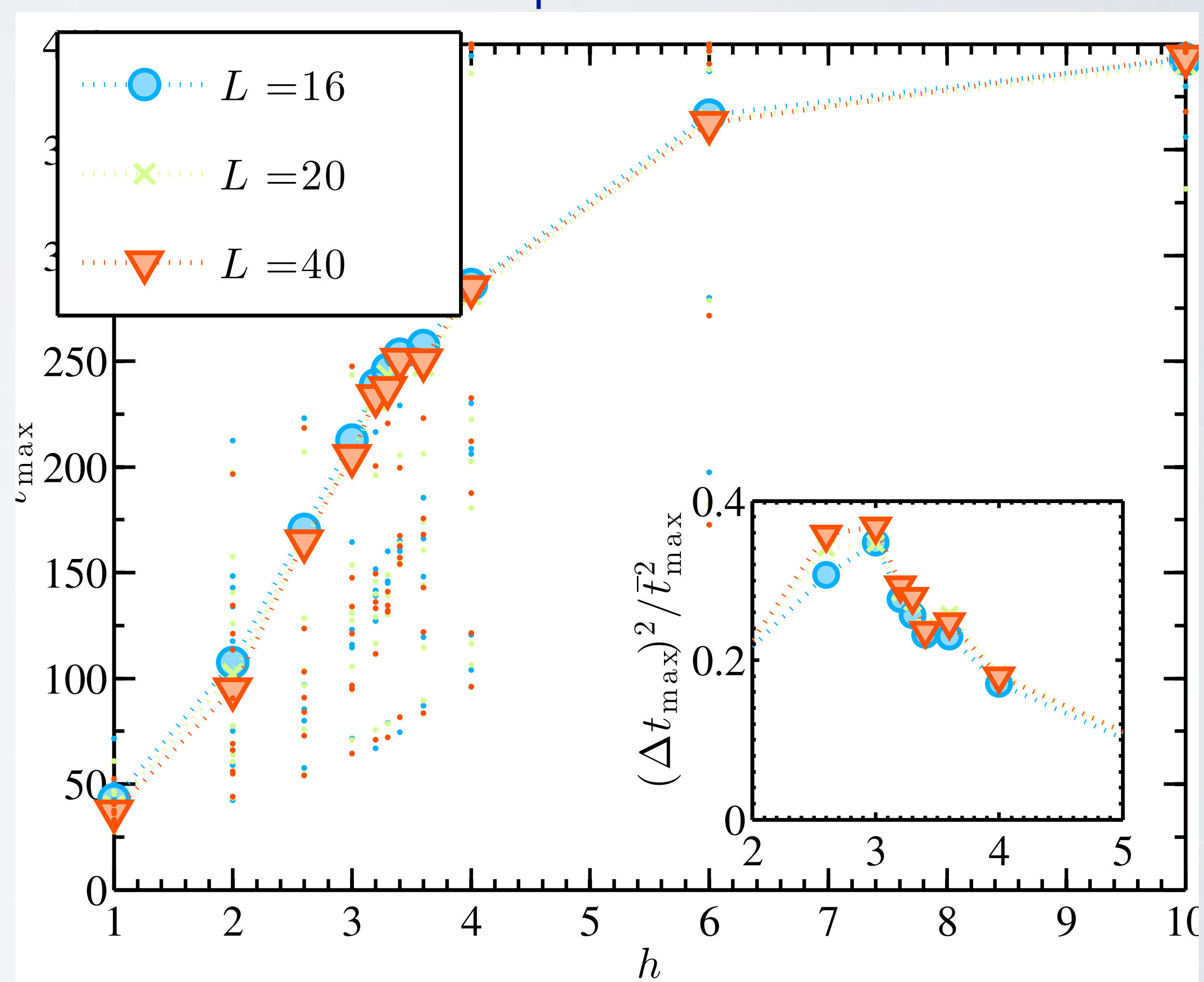
larger systems (Hilbert-Schmidt distance)

single instance



time

multiple realizations



disorder

Some questions we are asking

dynamics of mixed states

- propagation of correlations ✓
- quantum memory features ✓
- simulability with MPO ✓

Hamiltonian properties

- local conserved quantities ←

(ALMOST) LOCAL CONSERVED OPERATORS

What are the slowest evolving (local) operators?

Kim, MCB, Cirac, Hastings, Huse, PRE92, 012128 (2015)

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$$\frac{dA(t)}{dt} = i[H, A(t)]$$

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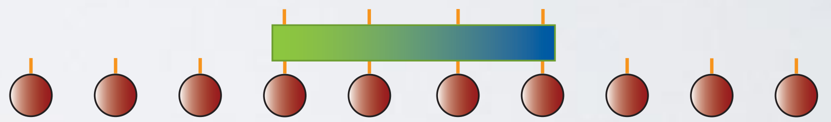
operator acting on M sites



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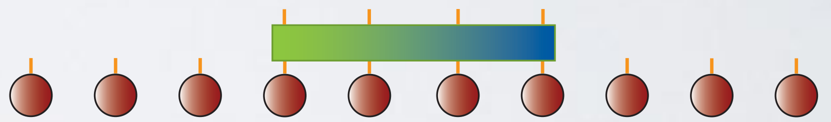

Goal: minimizing $\|[H, A_M]\|$

$$\lambda_M = \min_{A_M} \frac{\|[A_M, H]\|_2^2}{\|A_M\|_2^2}$$

Kim, MCB, Cirac, Hastings, Huse, PRE92, 012128 (2015)

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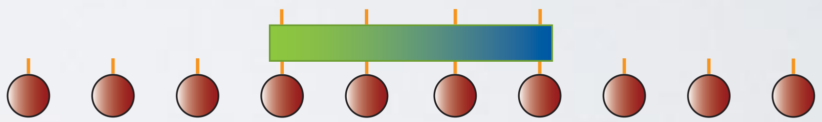

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numerically with ED and TNS

Kim, MCB, Cirac, Hastings, Huse, PRE92, 012128 (2015)

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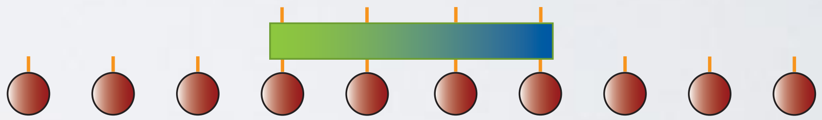

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N. Pancotti et al PRB 97, 094206 (2018)

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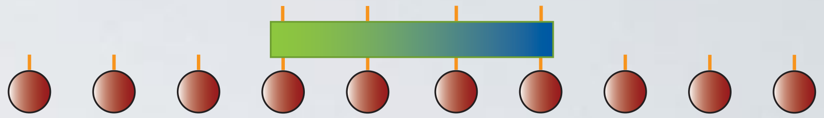

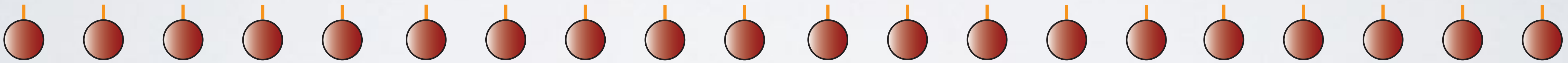

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N. Pancotti et al PRB 97, 094206 (2018)

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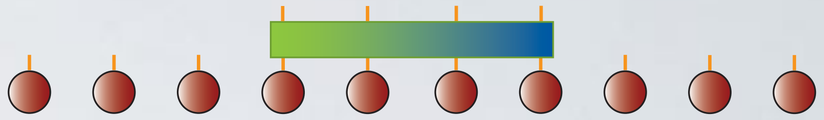

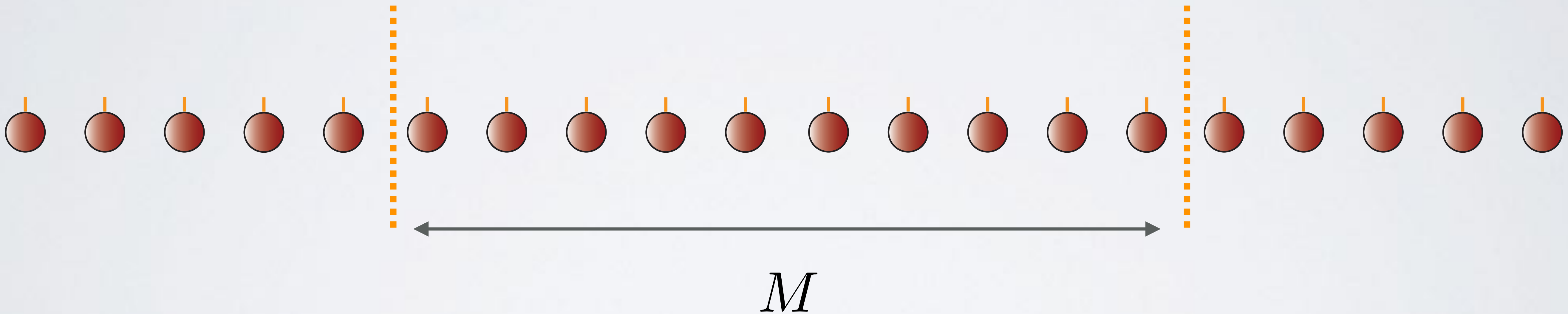

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N. Pancotti et al PRB 97, 094206 (2018)

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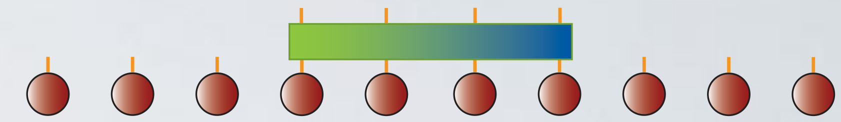

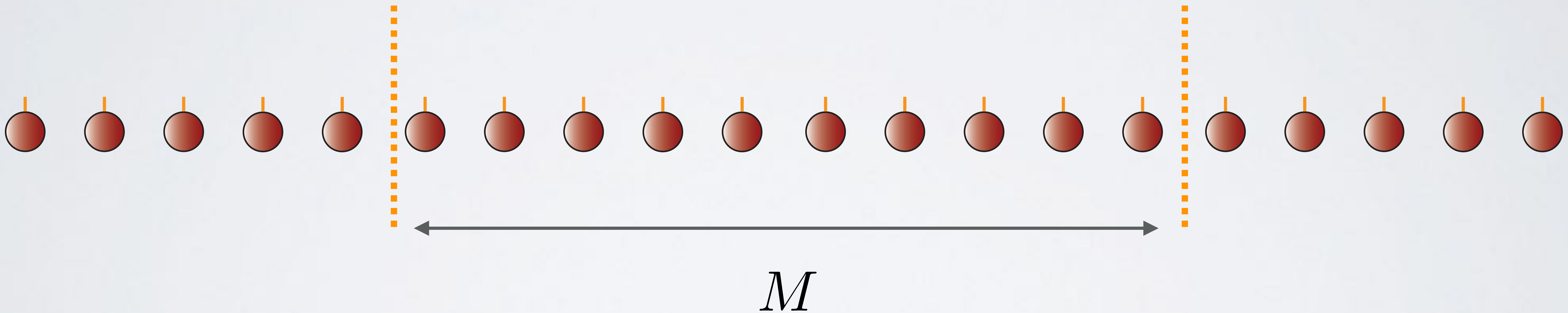

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N. Pancotti et al PRB 97, 094206 (2018)

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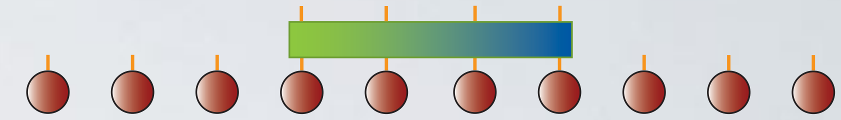

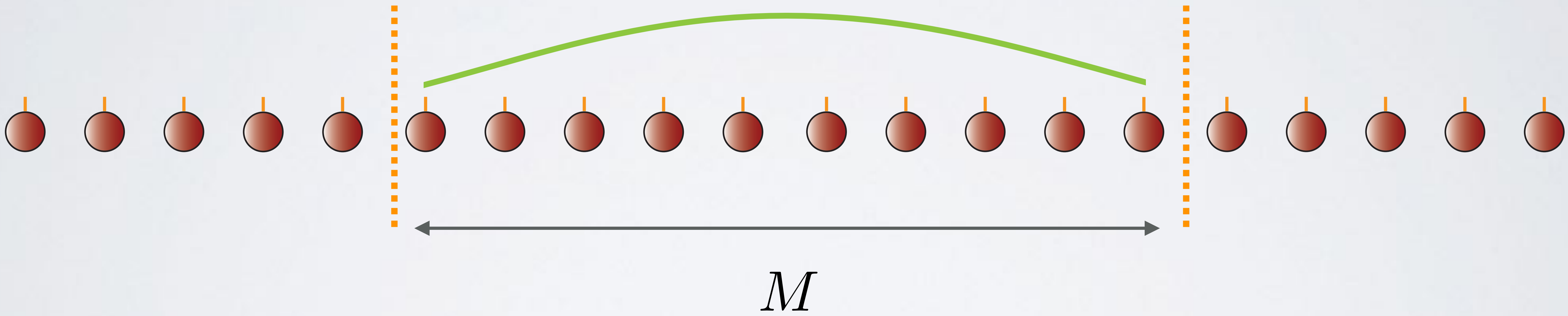

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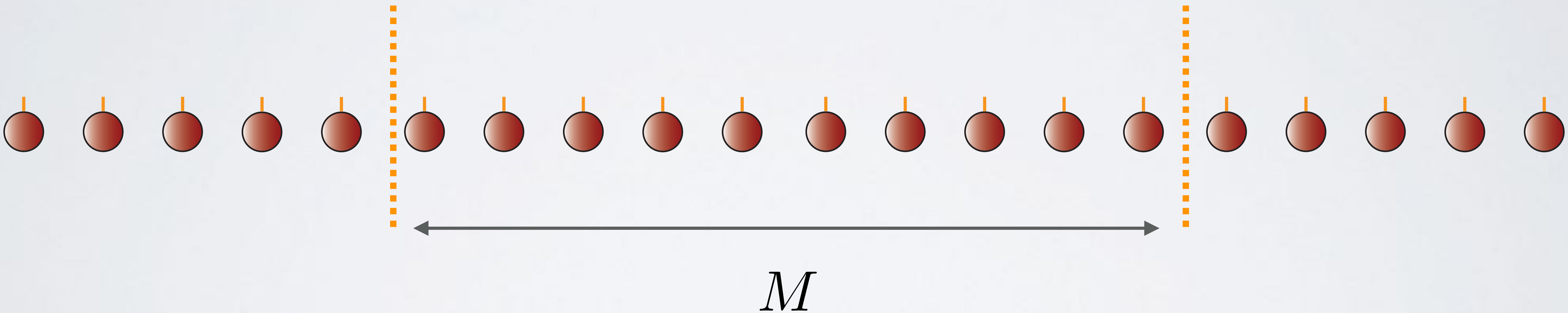
in the localized regime: l-bit model

$$H_{\text{eff}} = \sum_{i=0}^{N-1} \epsilon_i \tau_z^{[i]} + \sum_{i,j=0}^{N-1} K_{ij}^{(2)} \tau_z^{[i]} \tau_z^{[j]} + \sum_{i,j,k=0}^{N-1} K_{ijk}^{(3)} \tau_z^{[i]} \tau_z^{[j]} \tau_z^{[k]} + \ldots,$$



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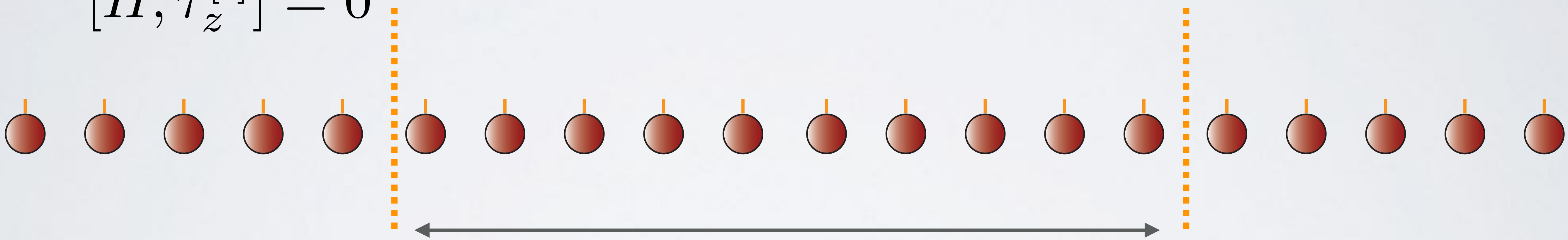
$\tau_z^{[i]}$ exponentially localized

$[H, \tau_z^{[i]}] = 0$



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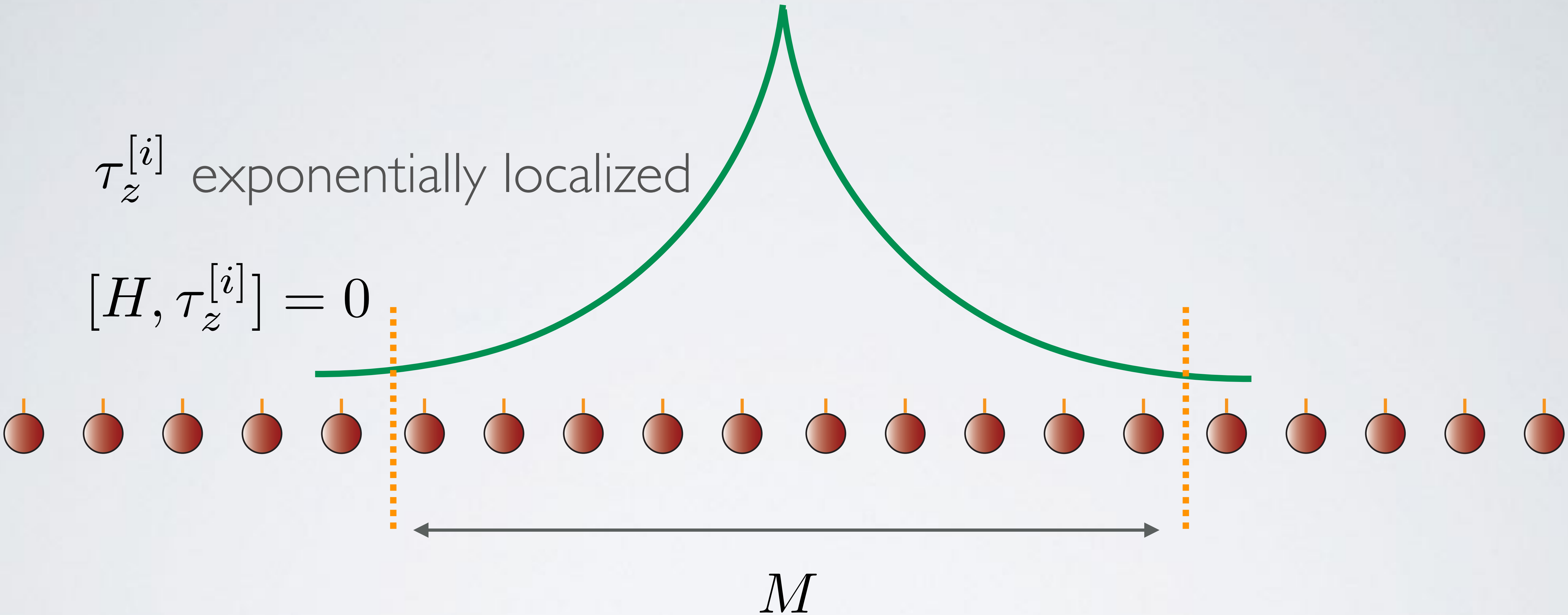

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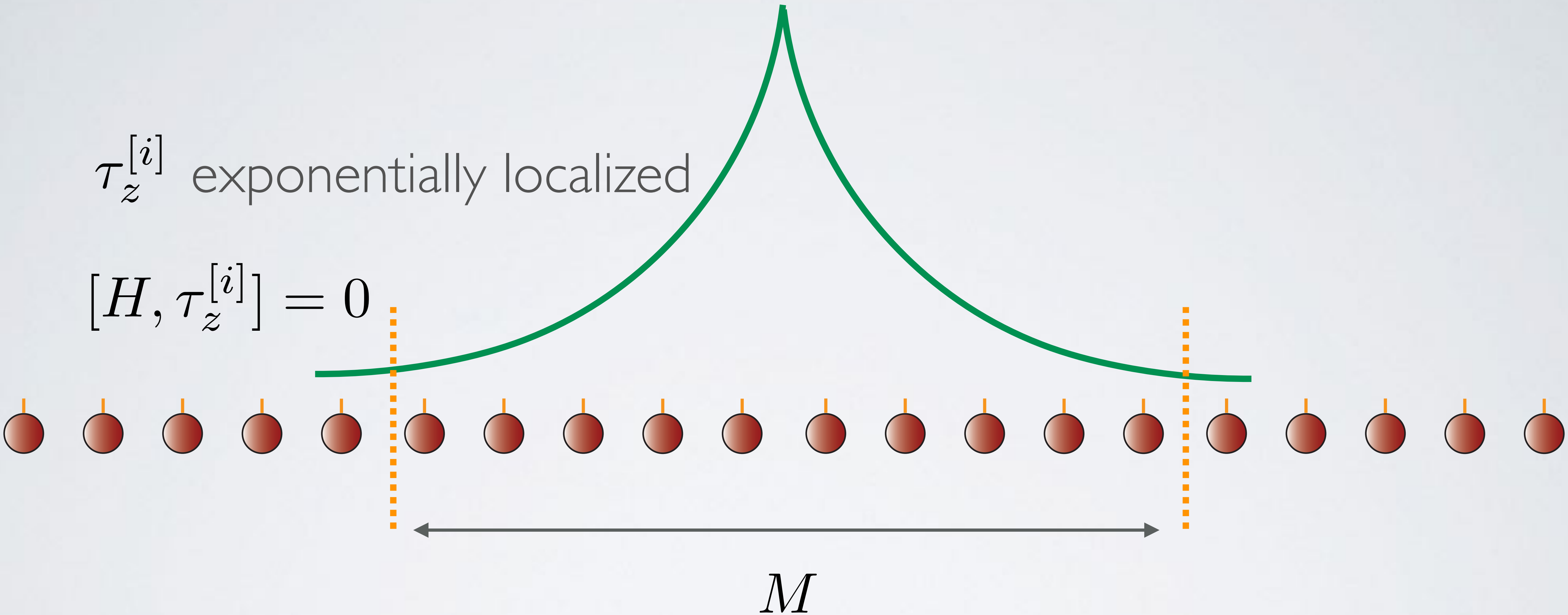

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truncated support → expect exponentially small

see also Chandran et at. PRB 2015

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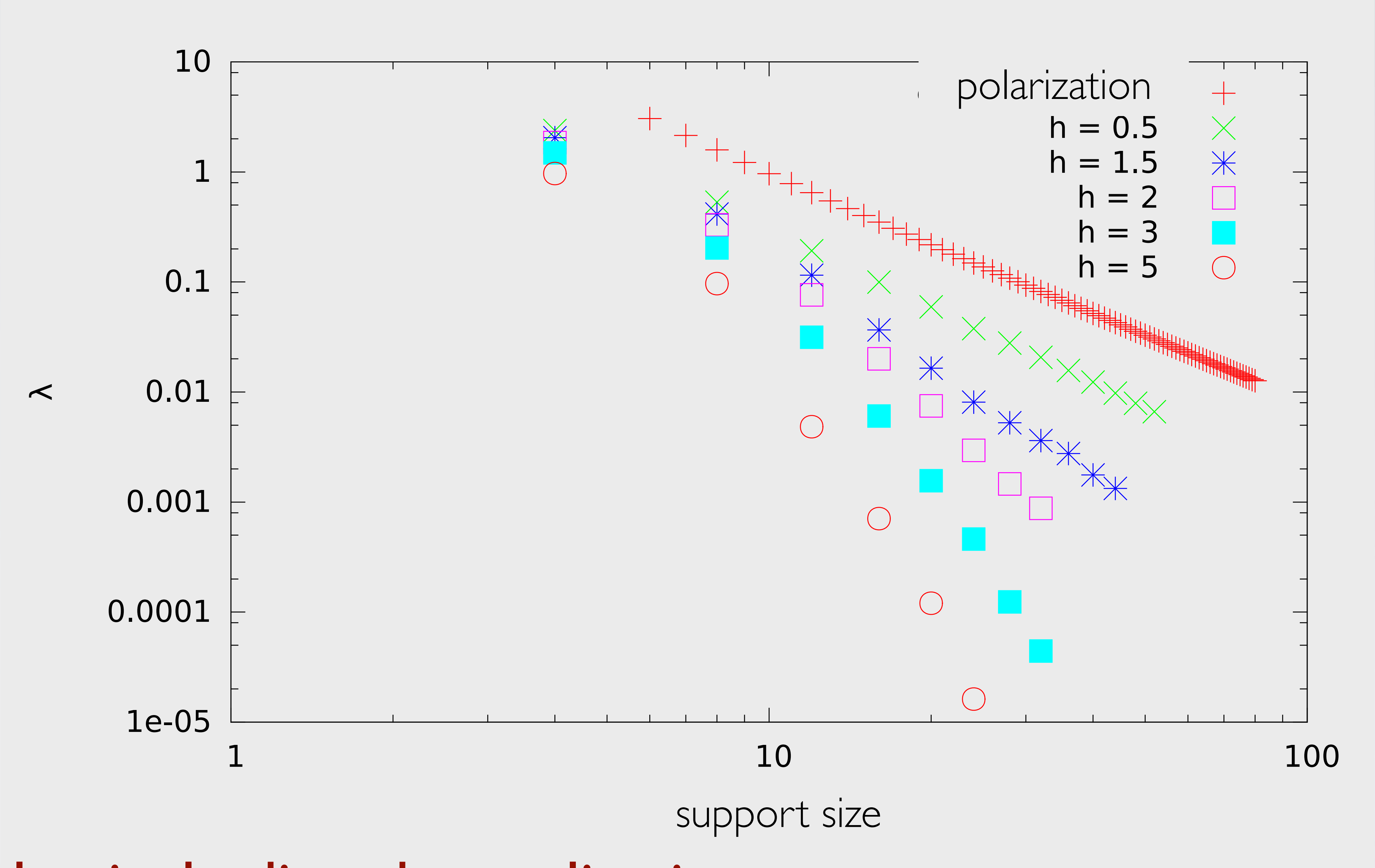
ALMOST CONSERVED QUANTITIES & MBL



example: single disorder realization

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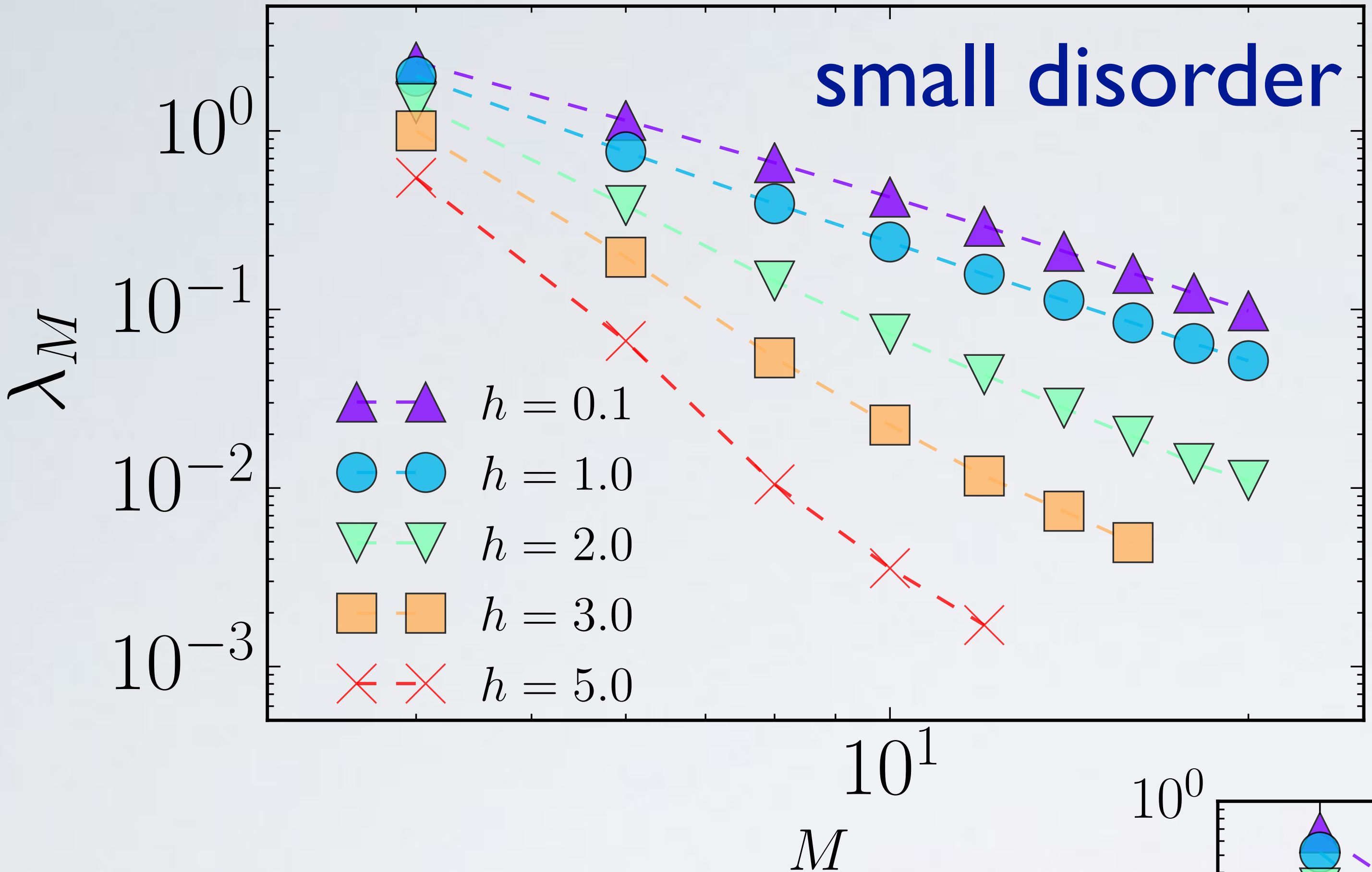
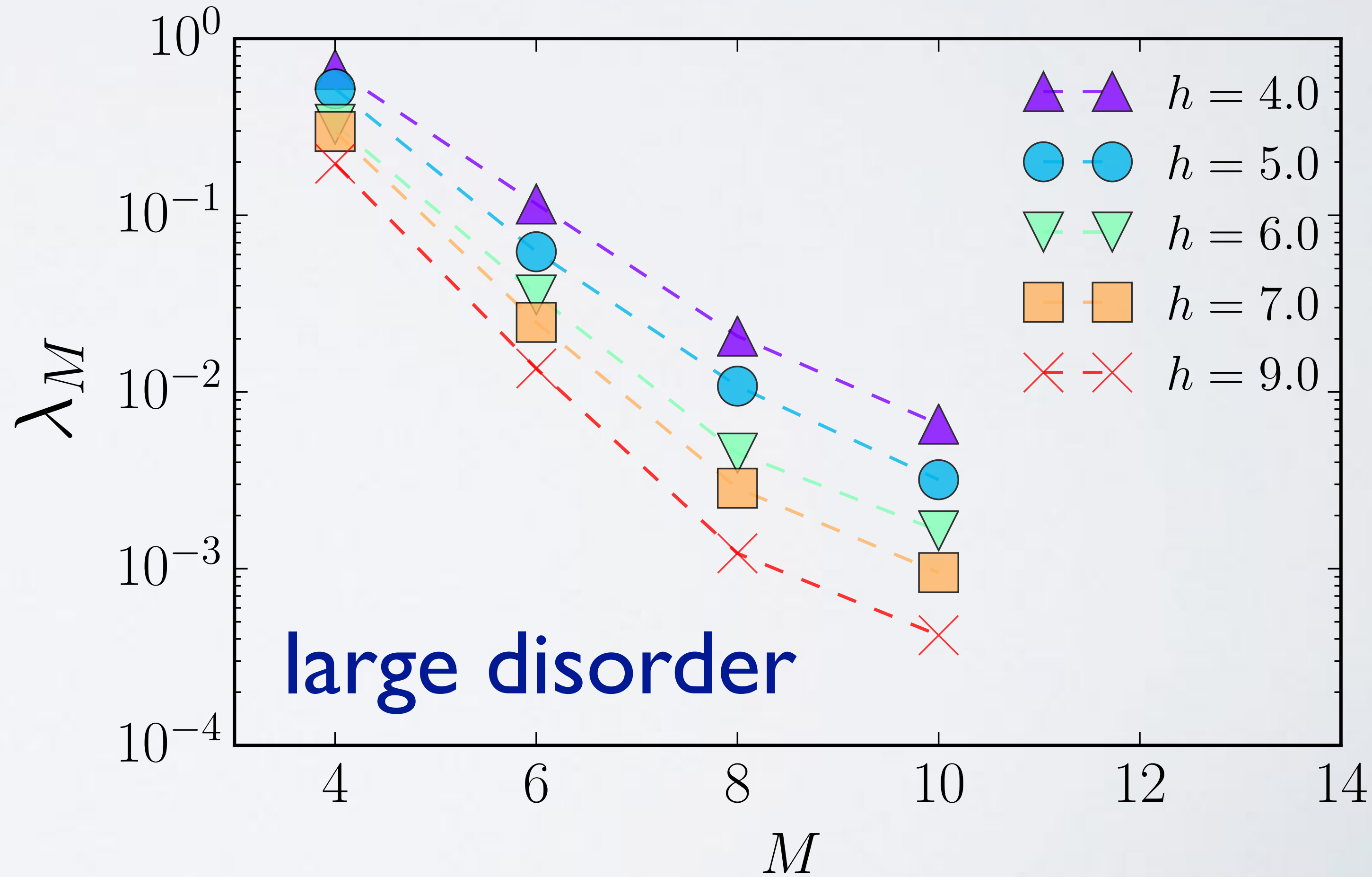
averaging over disorder





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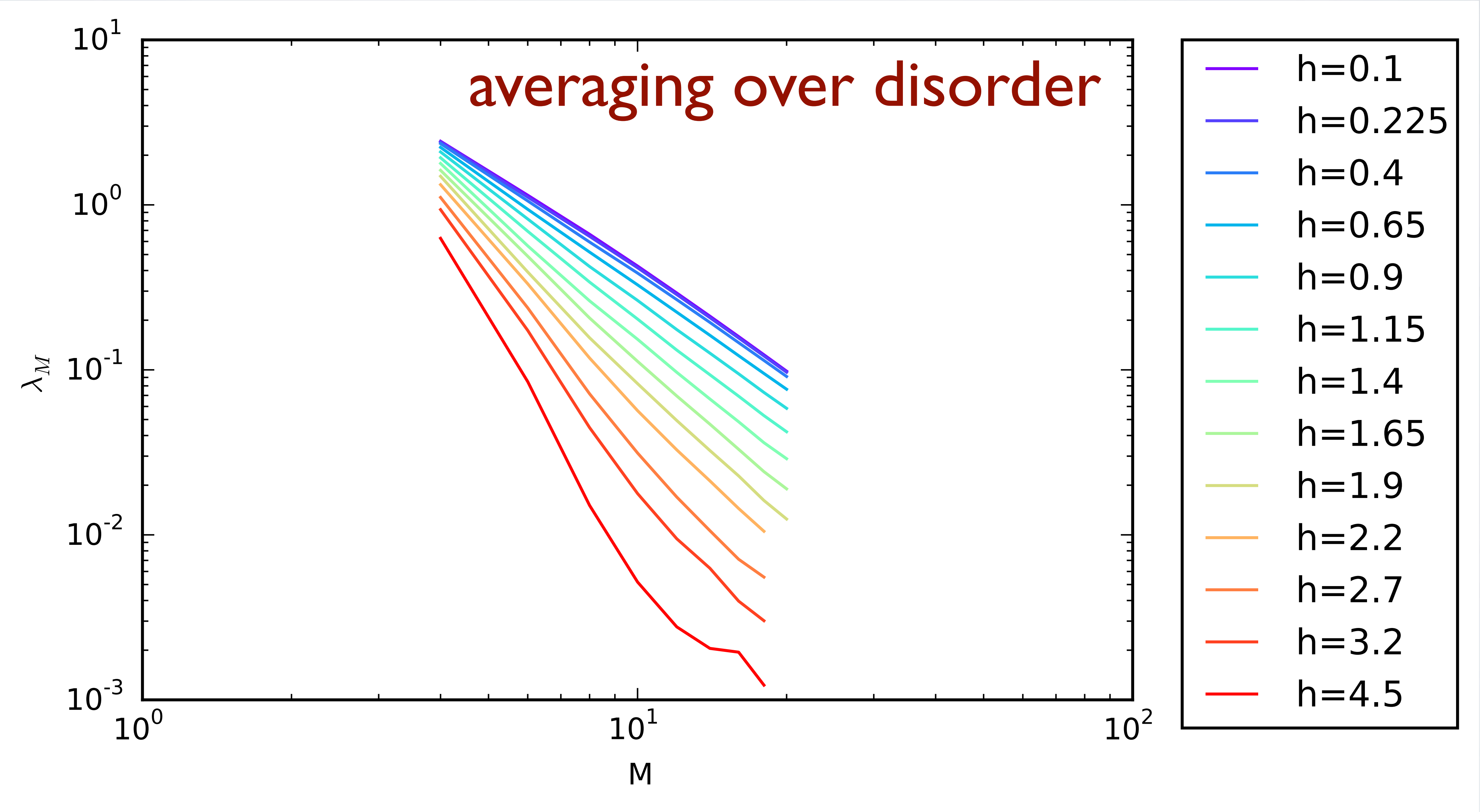

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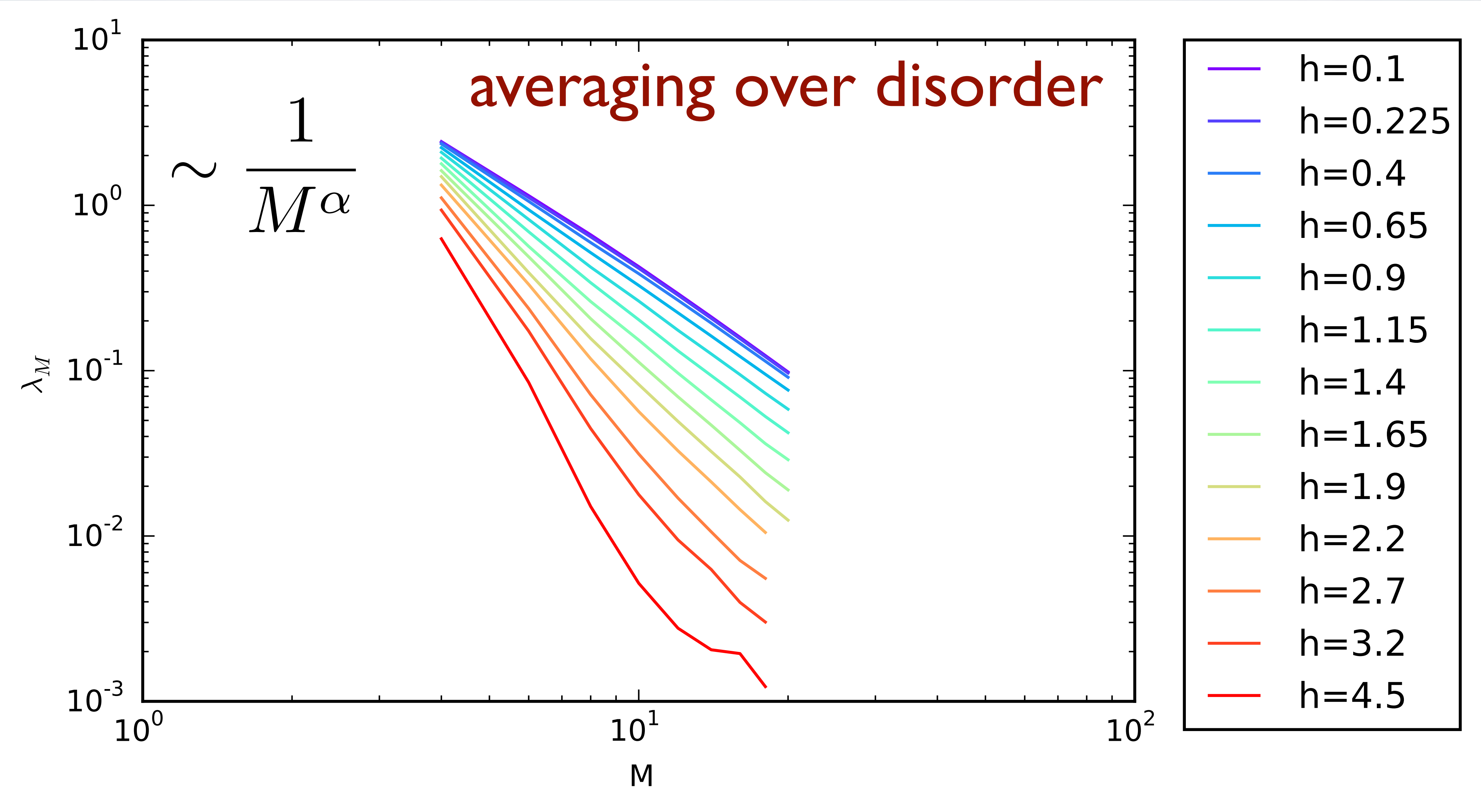

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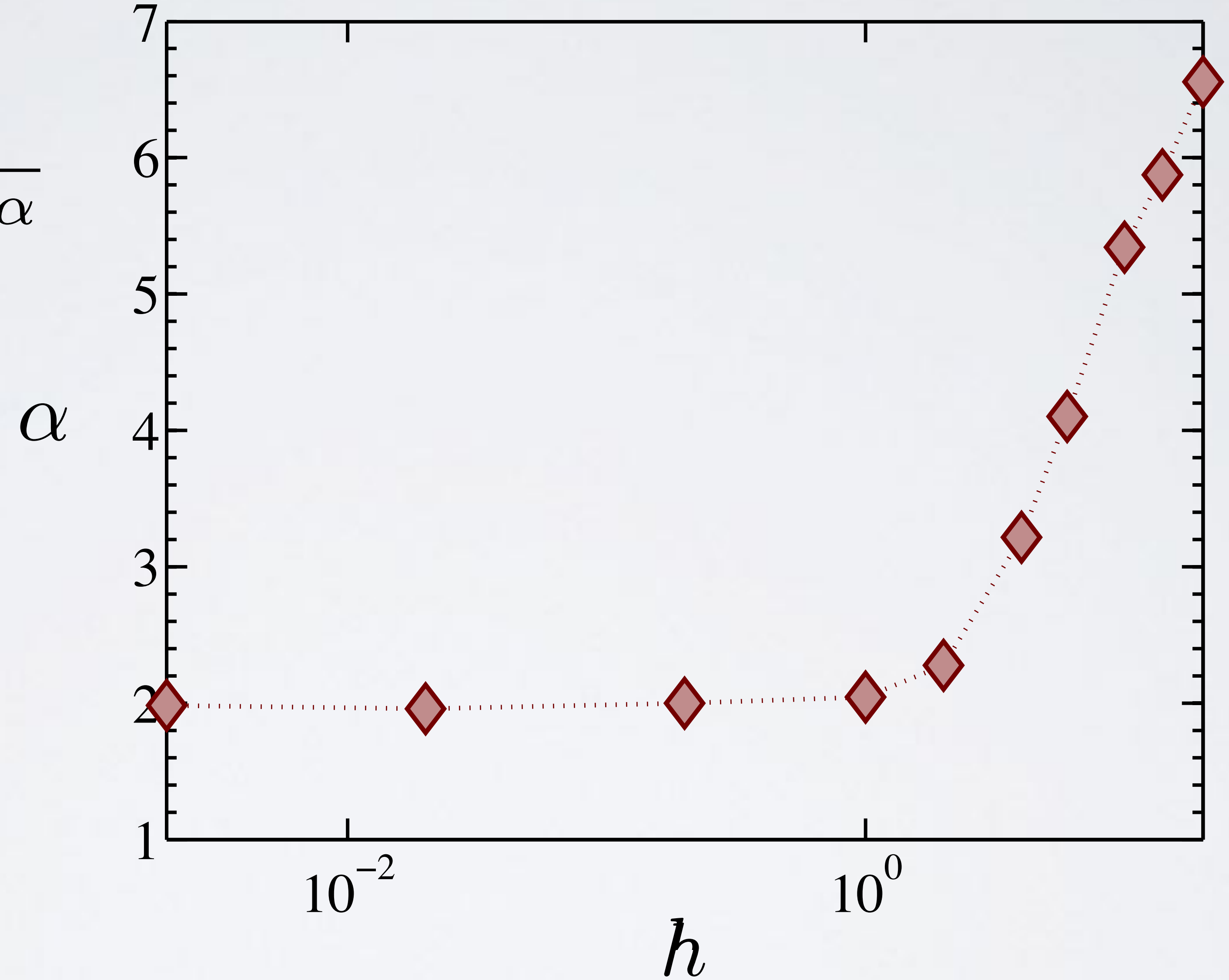
ALMOST CONSERVED QUANTITIES & MBL

$$\sim \frac{1}{M^{\alpha}}$$



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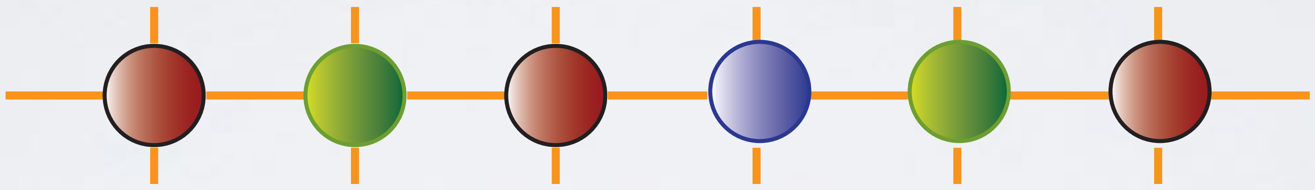
constructive method



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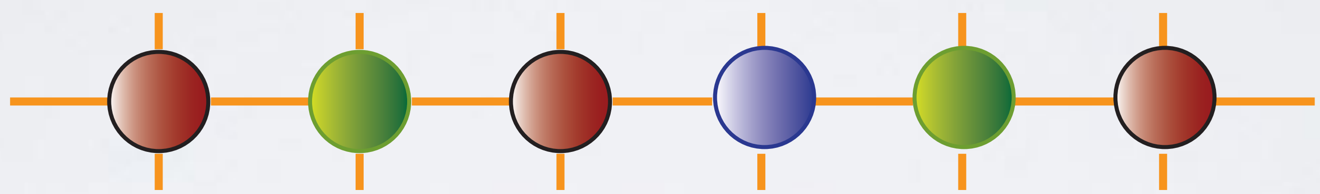

analyze weight of components with different support

$$\sigma_i^{[m]} \otimes \cdots \otimes \sigma_j^{[m+d]}$$

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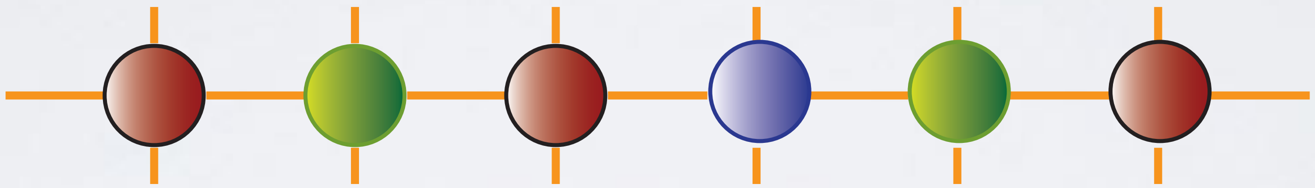
constructive method

efficient! → simple projector on MPO



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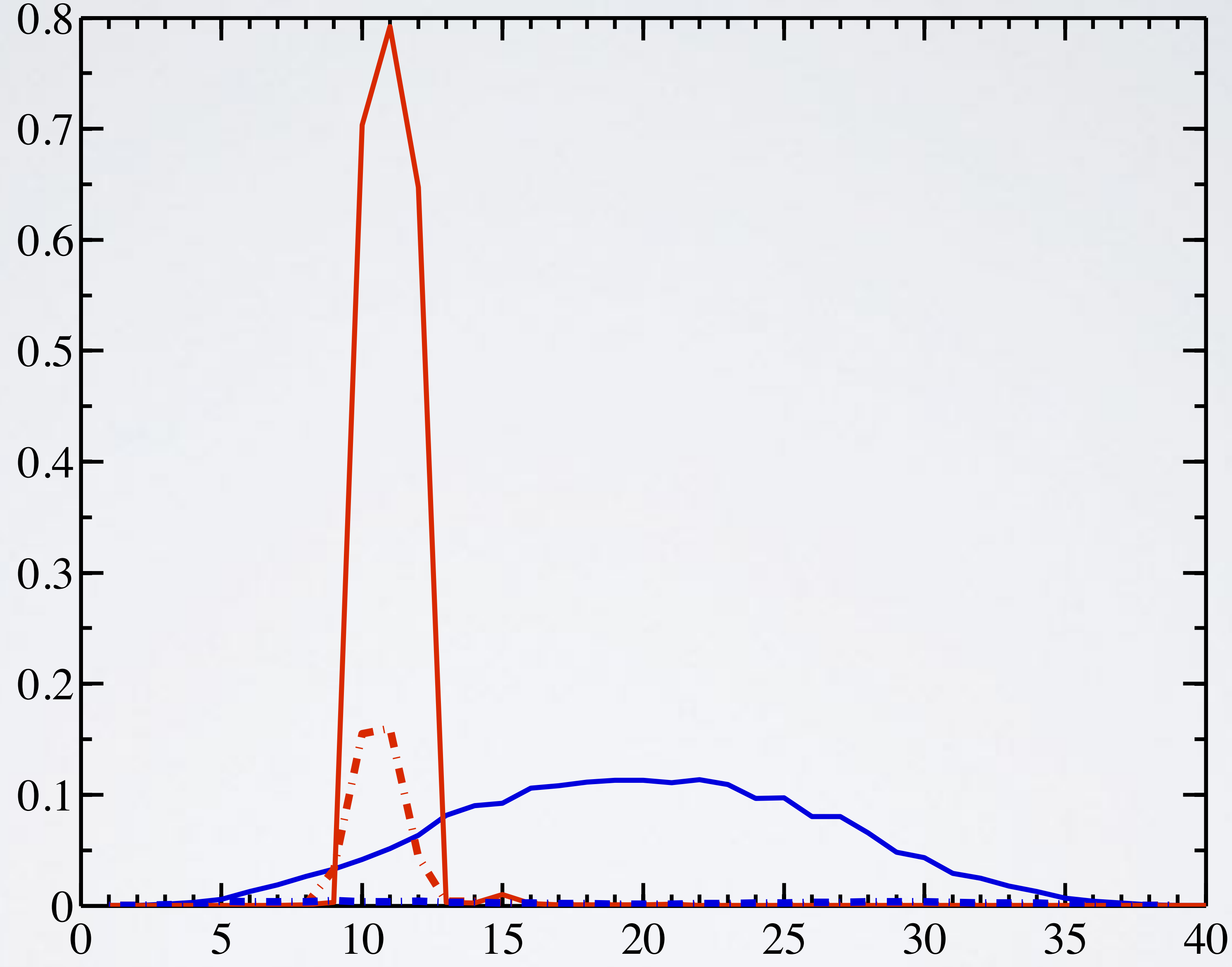
composition of slow operators: how local?

landscape of terms with fixed range



single realization $M = 40$

N. Pancotti et al PRB 97, 094206 (2018)

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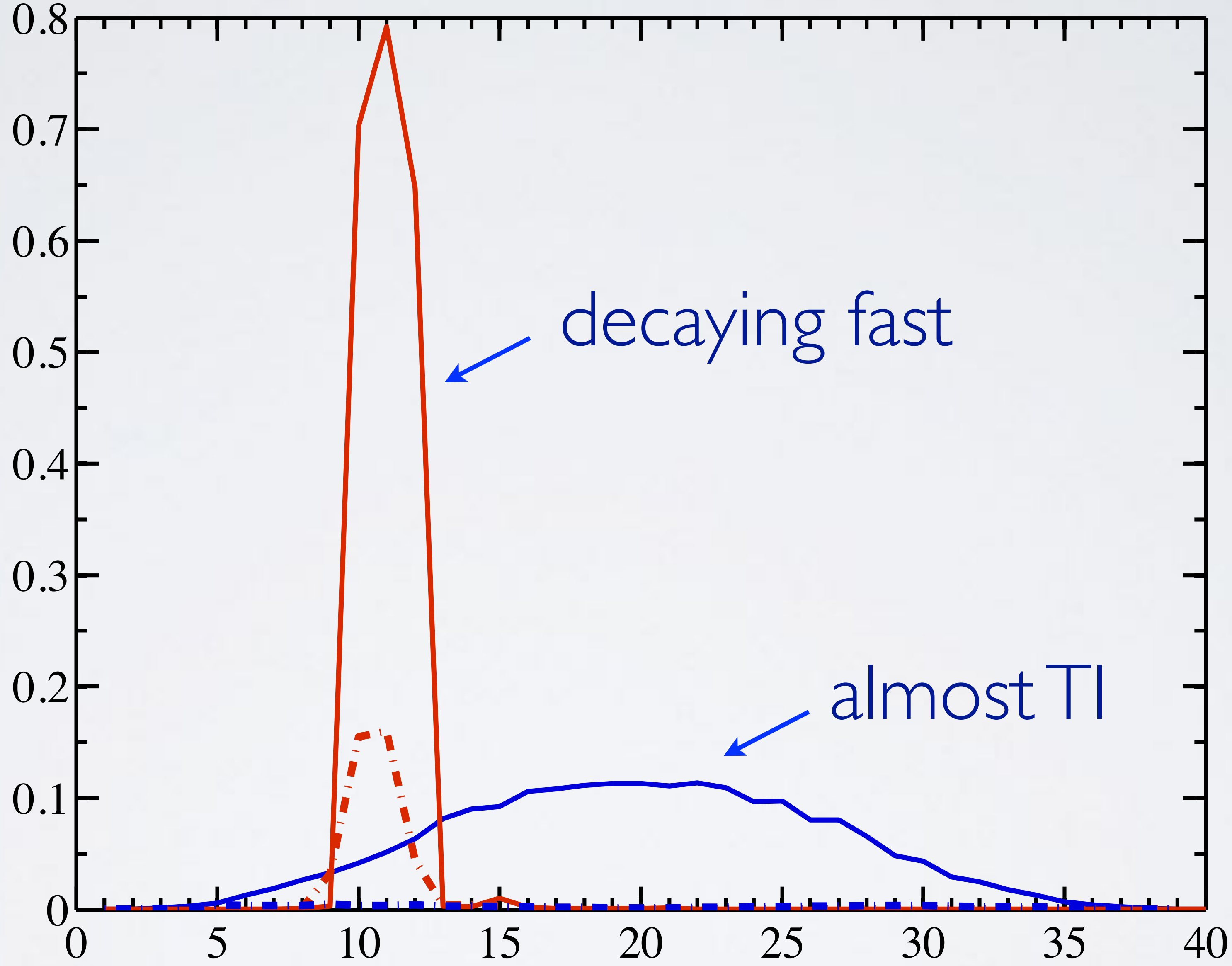

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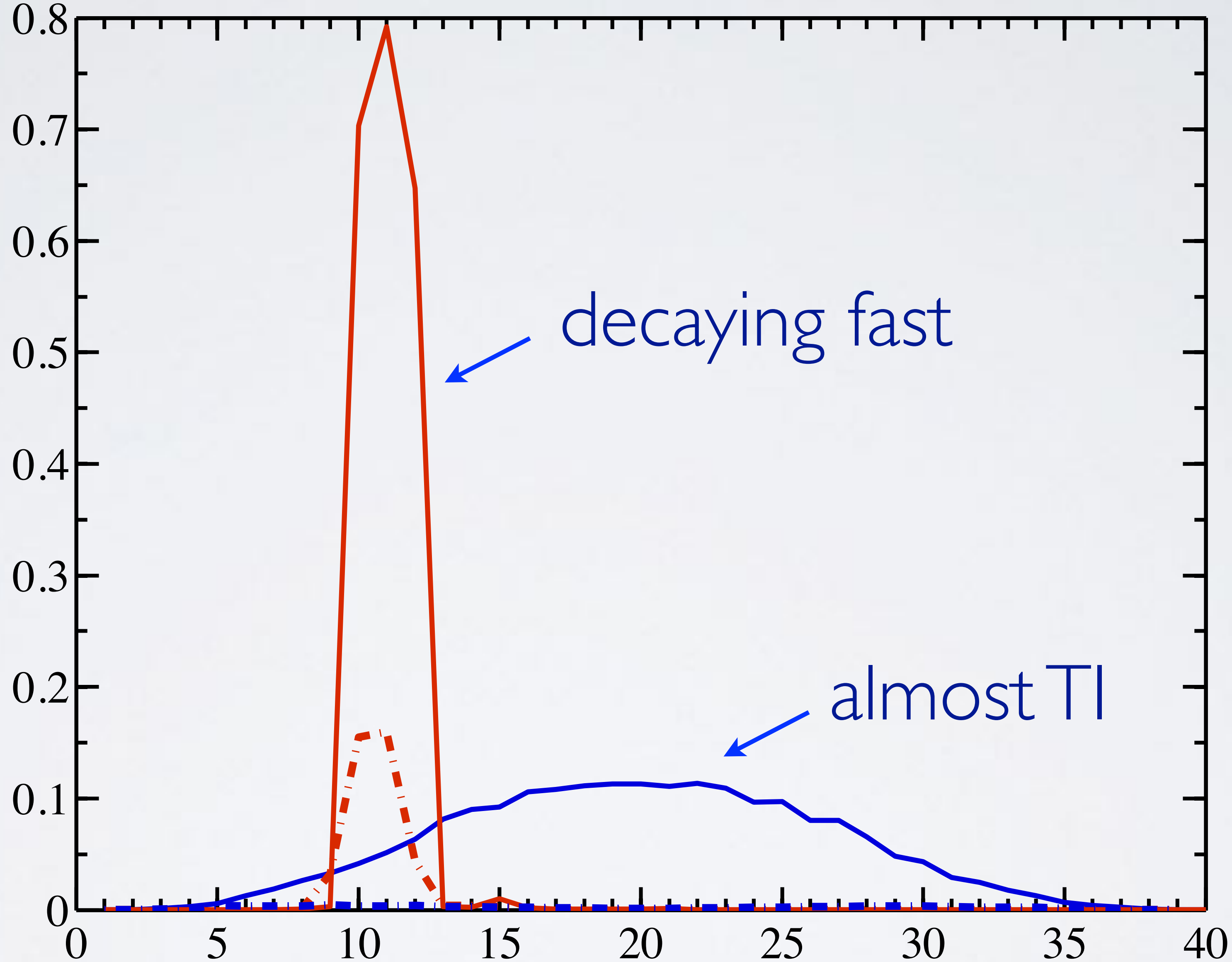

and much more information in the statistics!

single realization $M = 40$

N. Pancotti et al PRB 97, 094206 (2018)

Statistics of small commutators

Well described by Extreme Value Theory

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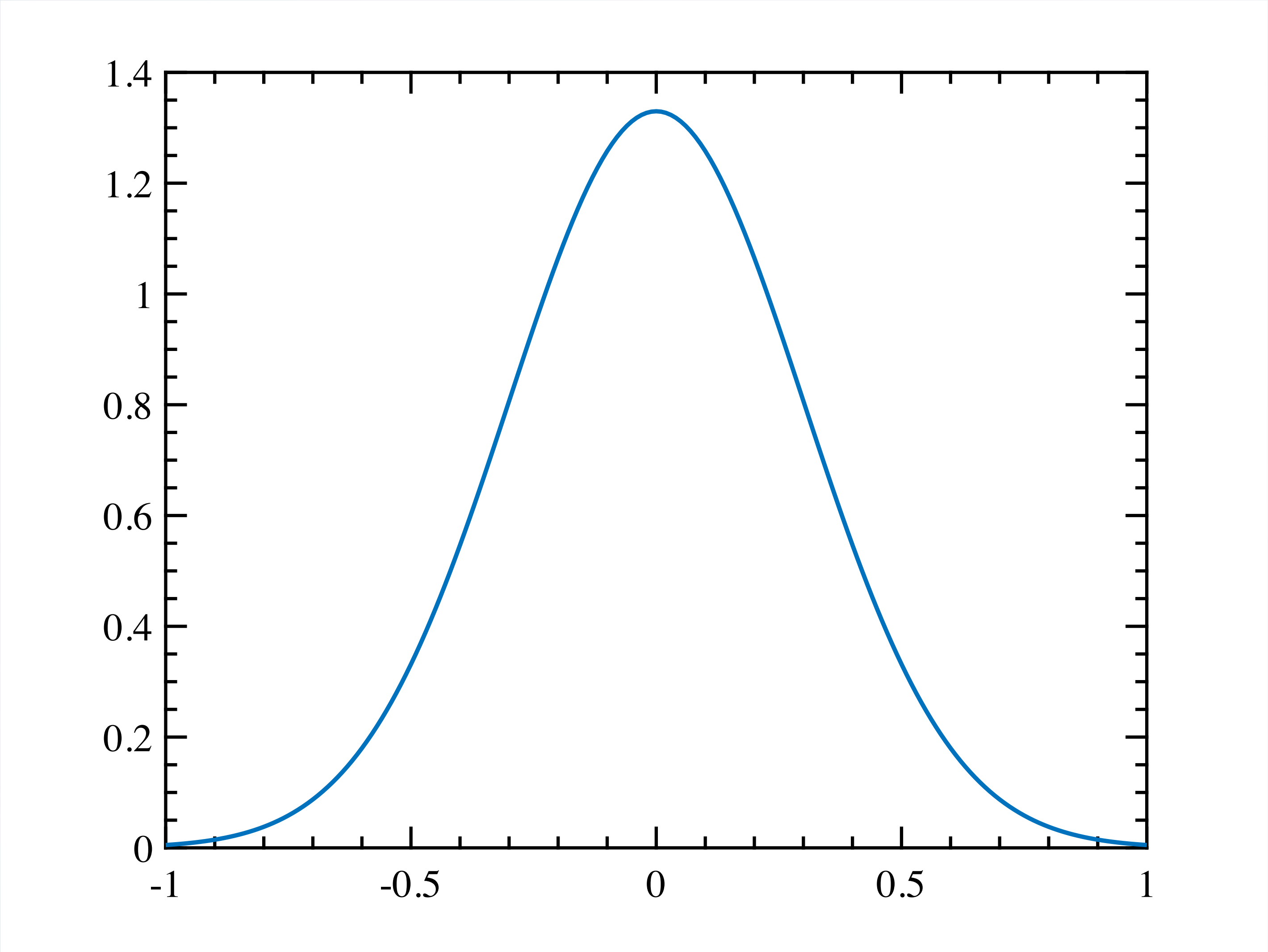
Q: extreme values when sampling from a pdf



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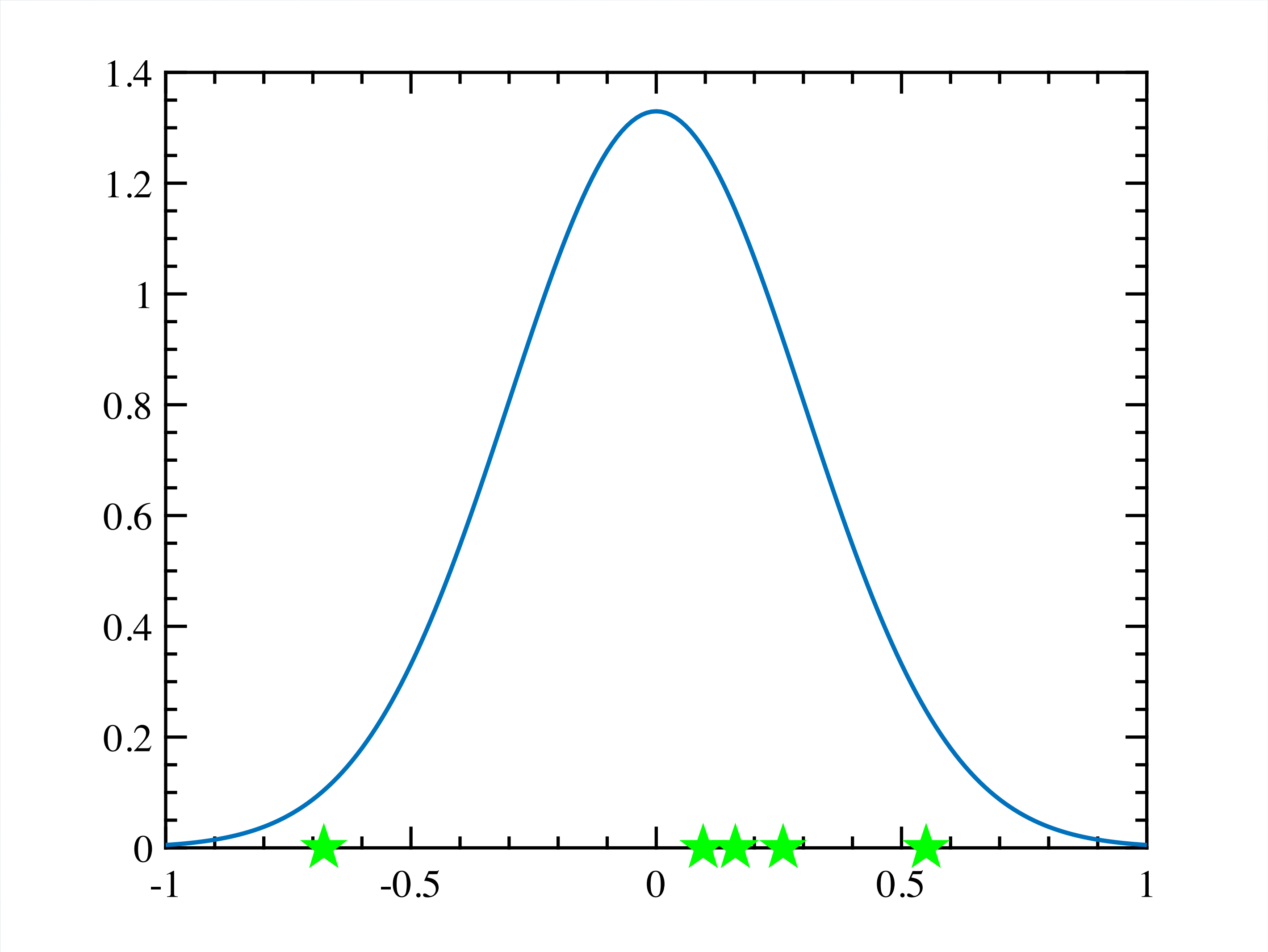

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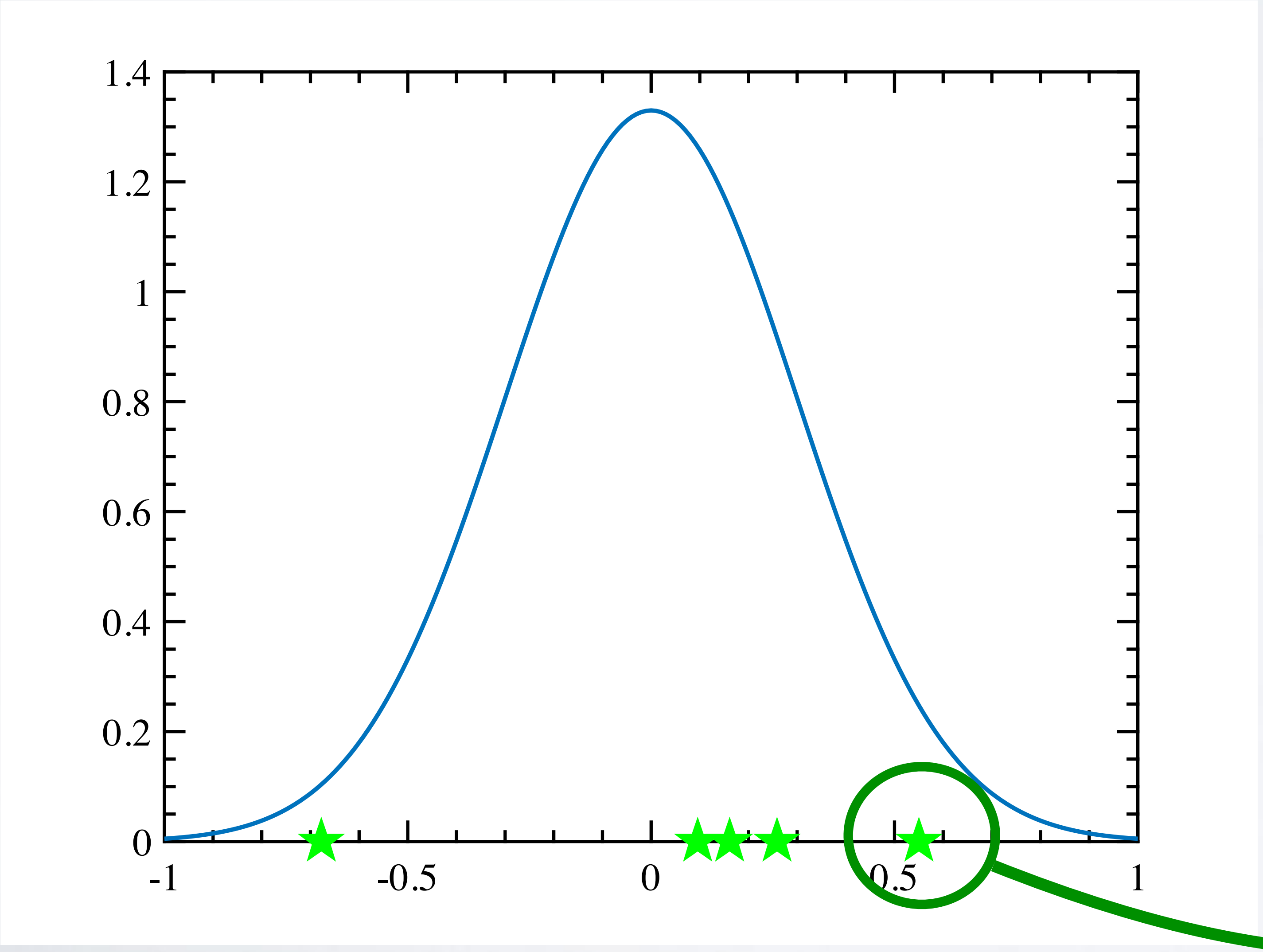
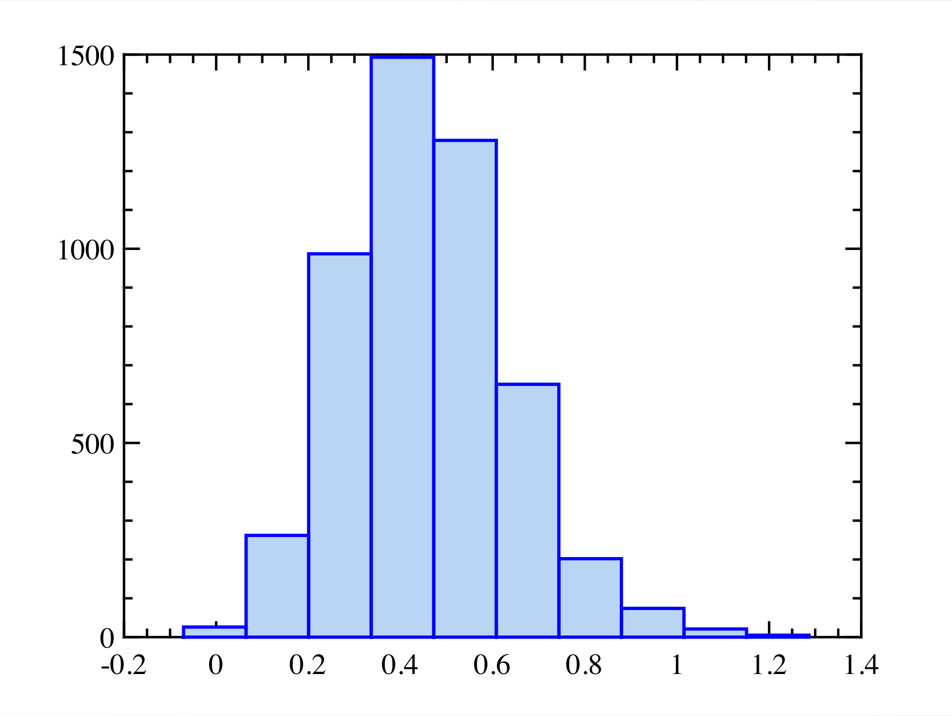


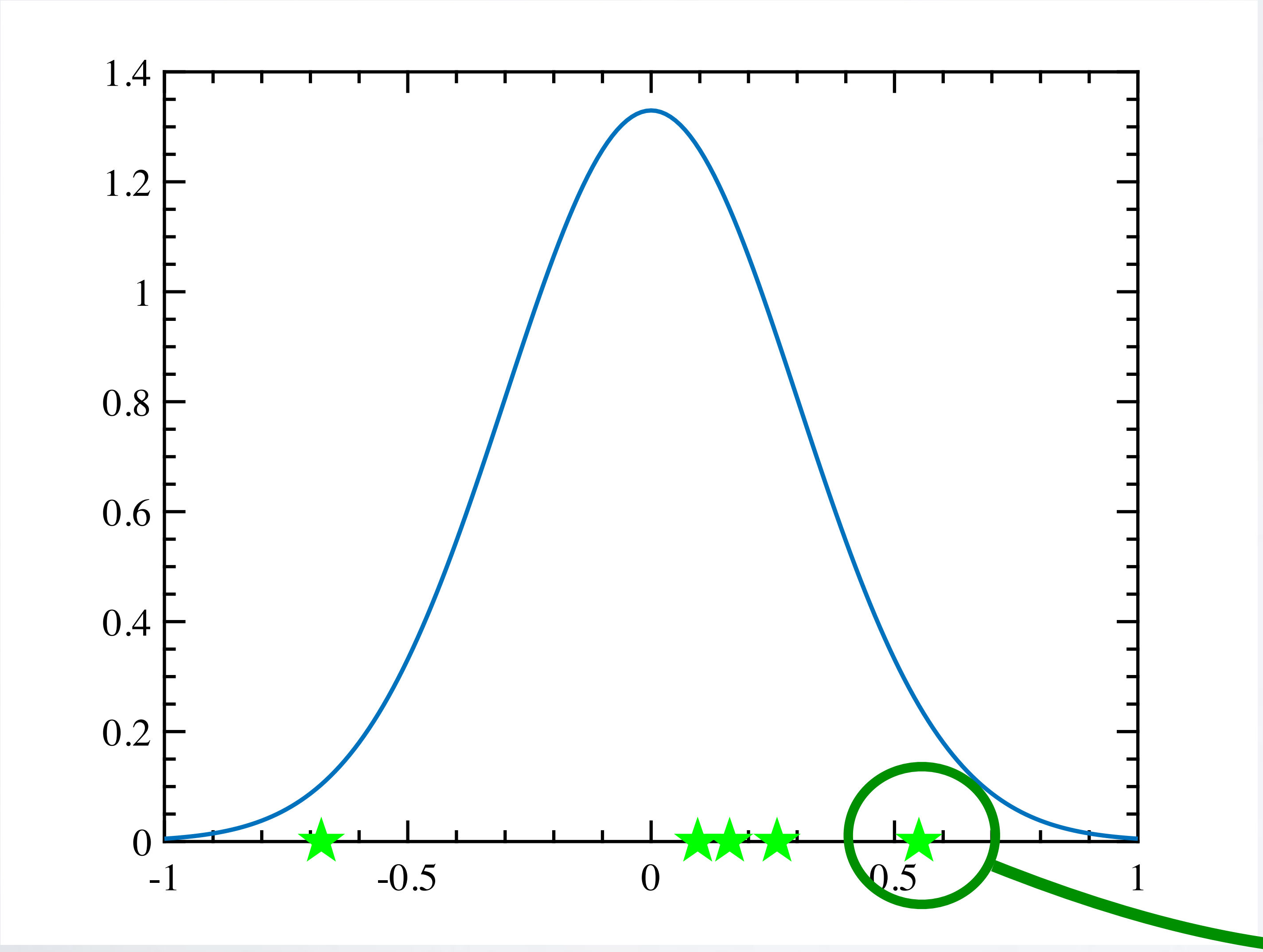


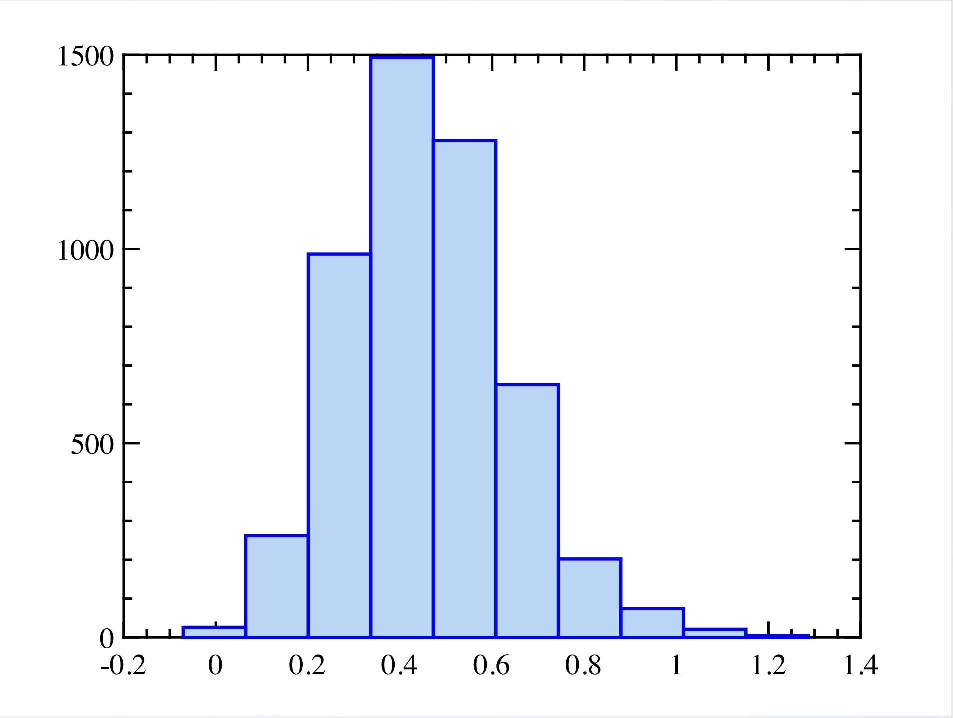
N. Pancotti et al PRB 97, 094206 (2018)

Statistics of small commutators

EVT

when is there a limit,
it is of the form





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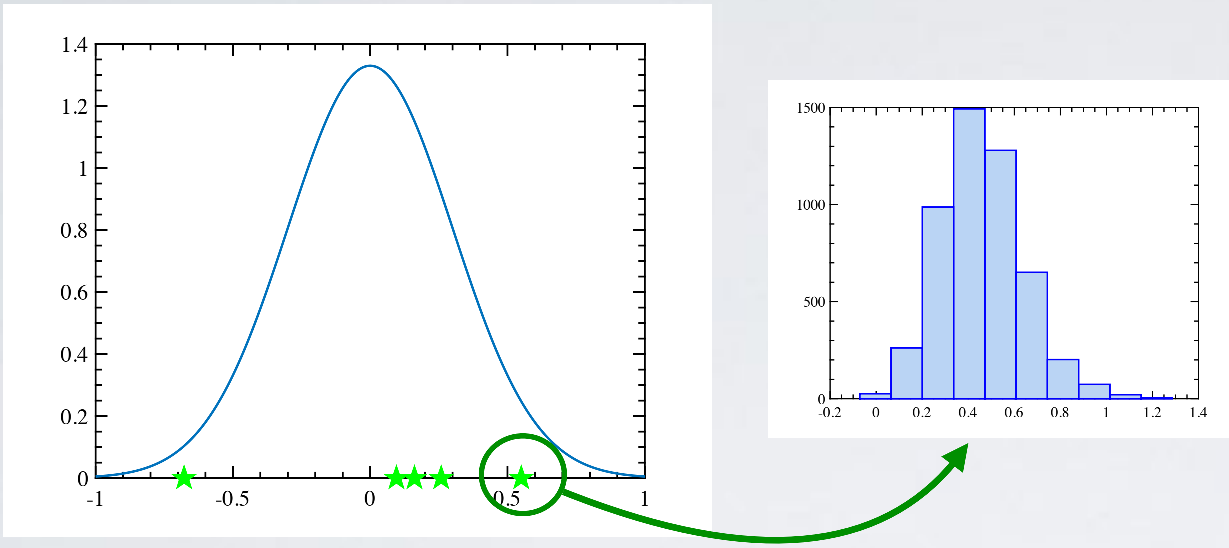

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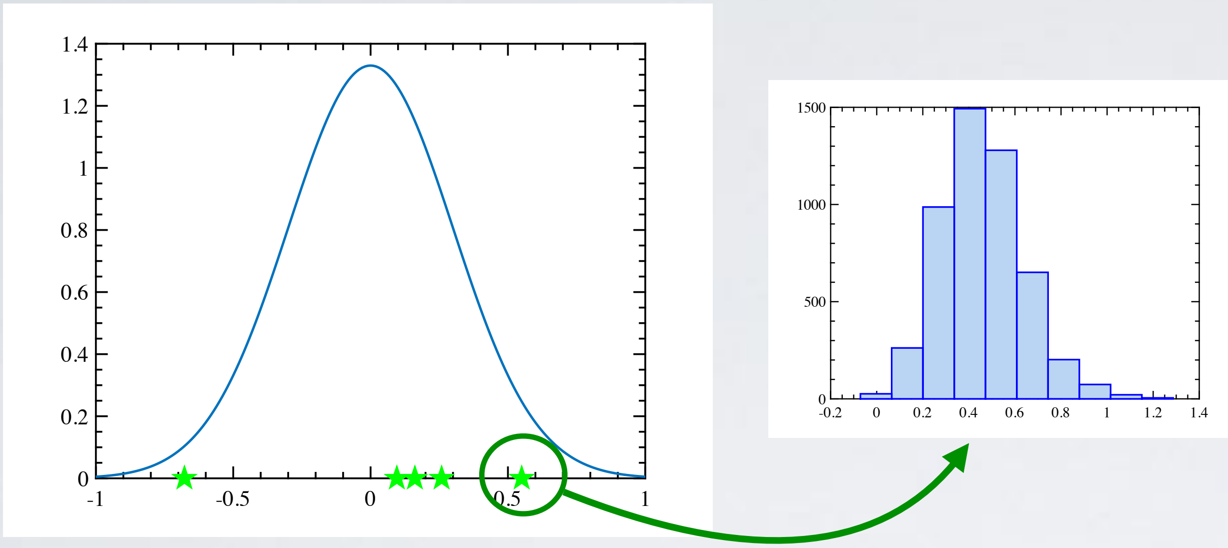

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GEV

$$G_\zeta(y) = \exp\left[-(1+\zeta y)^{-\frac{1}{\zeta}}\right]$$

rescaled and centered

CDF for extrema

N. Pancotti et al PRB 97, 094206 (2018)

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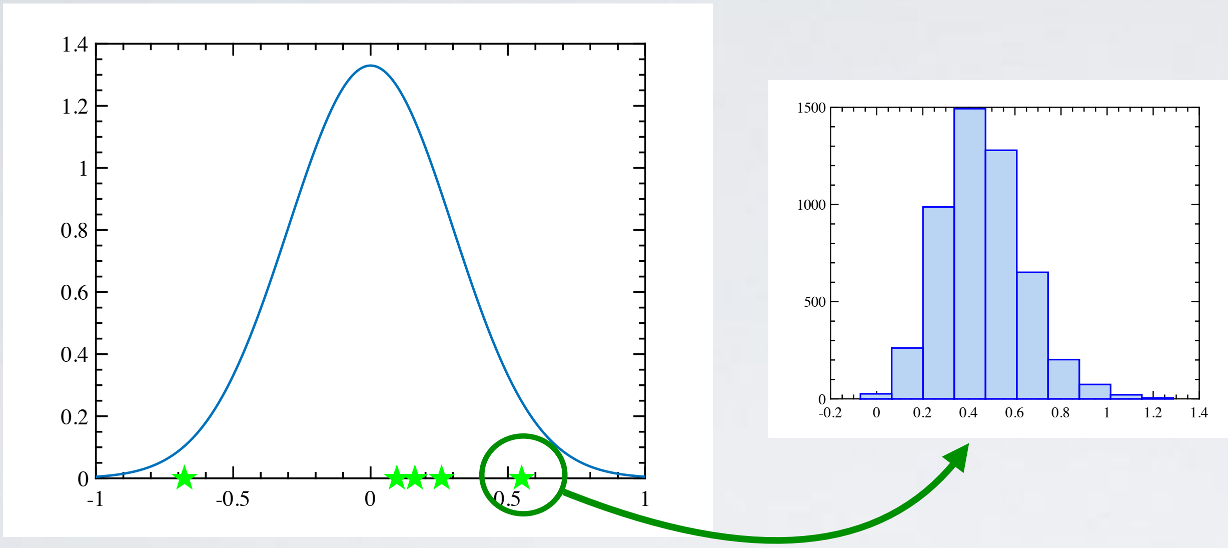

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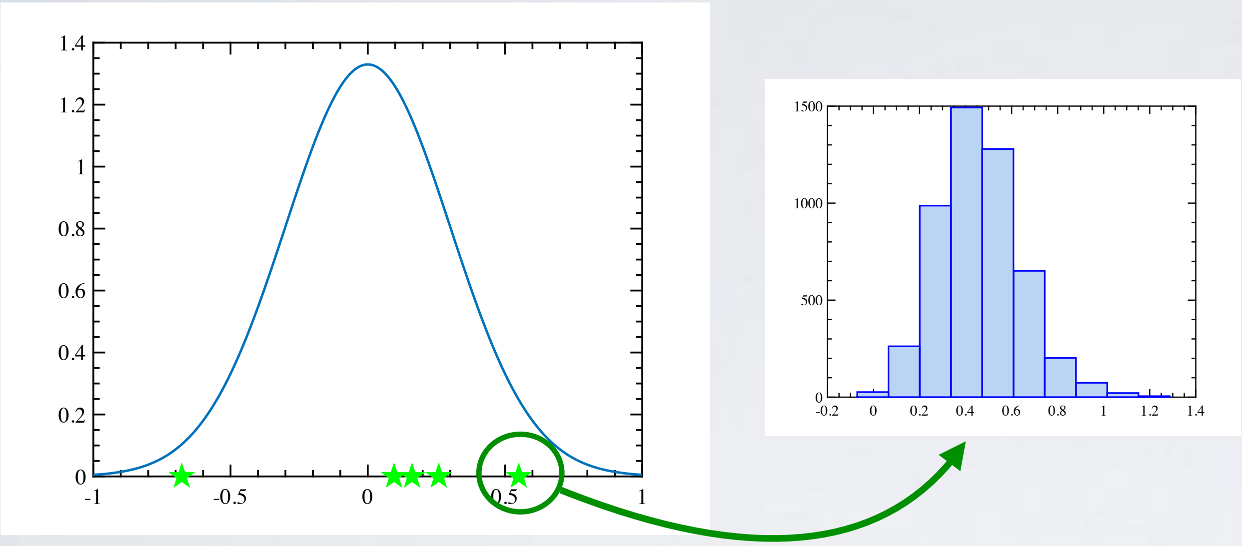

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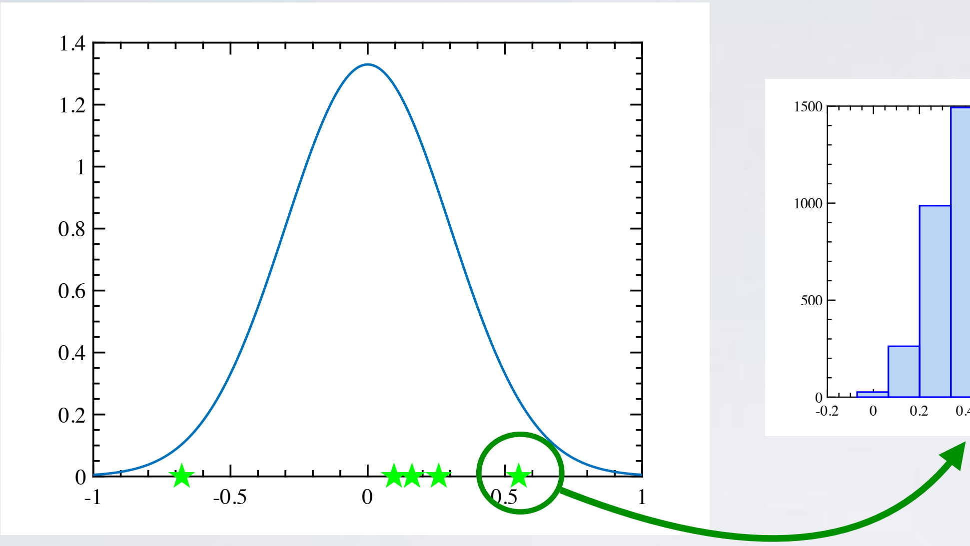

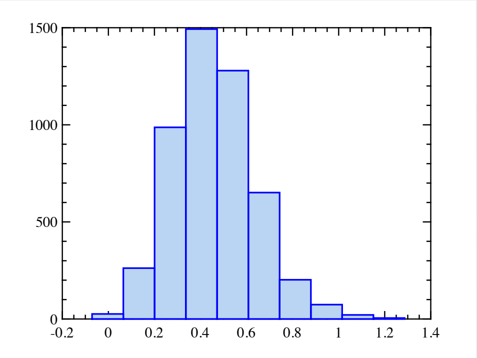

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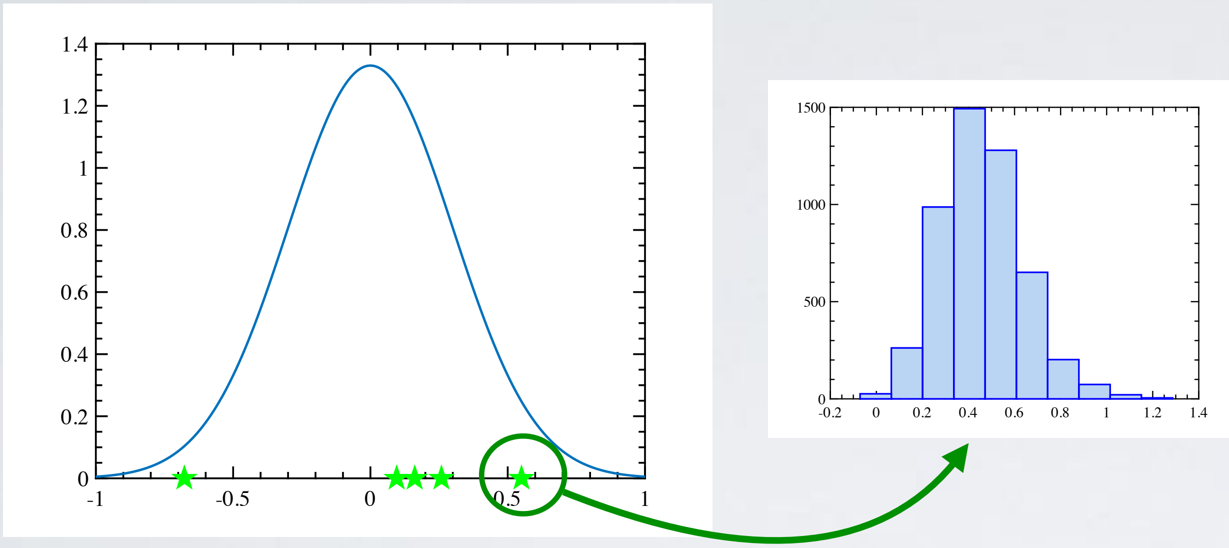

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$\zeta < 0$ Weibull: bounded light tails

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Statistics of small commutators

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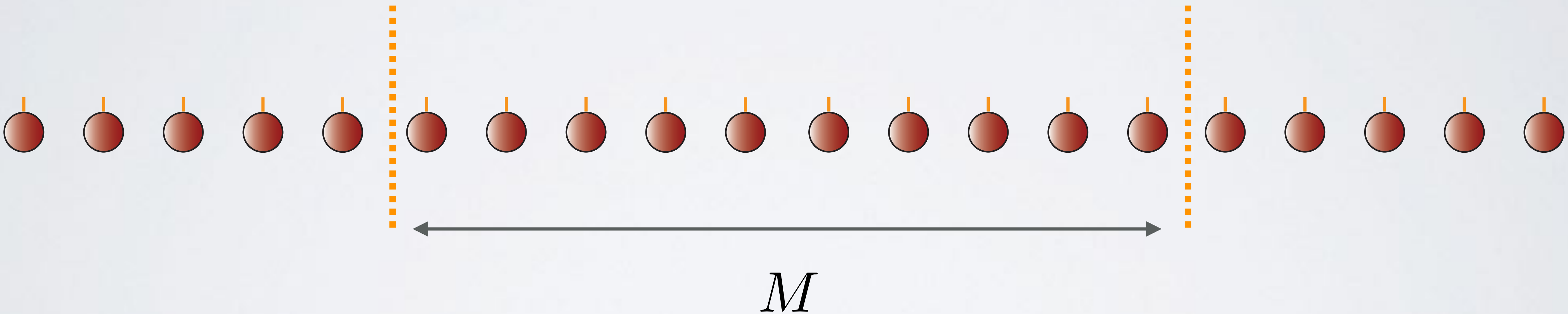

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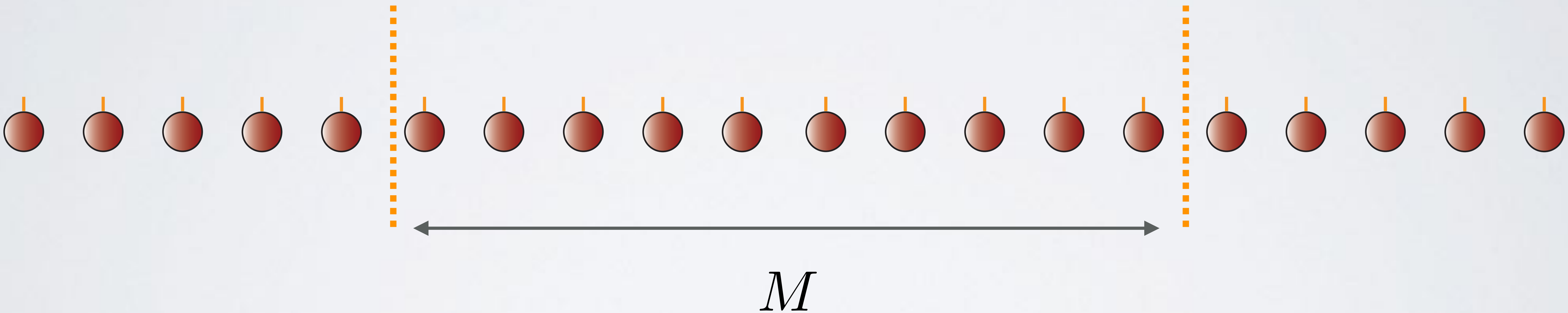

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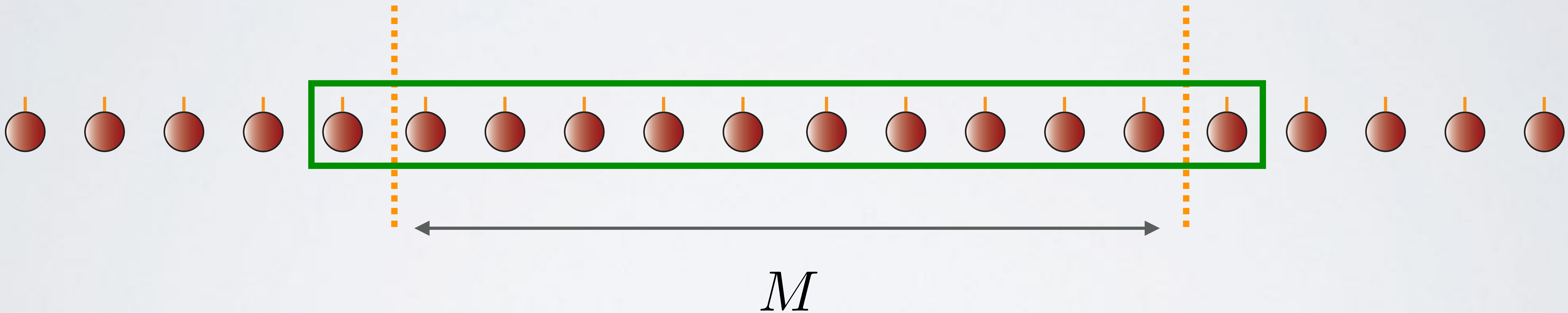
Statistics of small commutators

minimum eigenvalue of an effective
Hamiltonian on vectorized operators

$$H_{\text{eff}} \approx \left(H \otimes \mathbb{I} - \mathbb{I} \otimes H^T\right)^2_{M+2}$$



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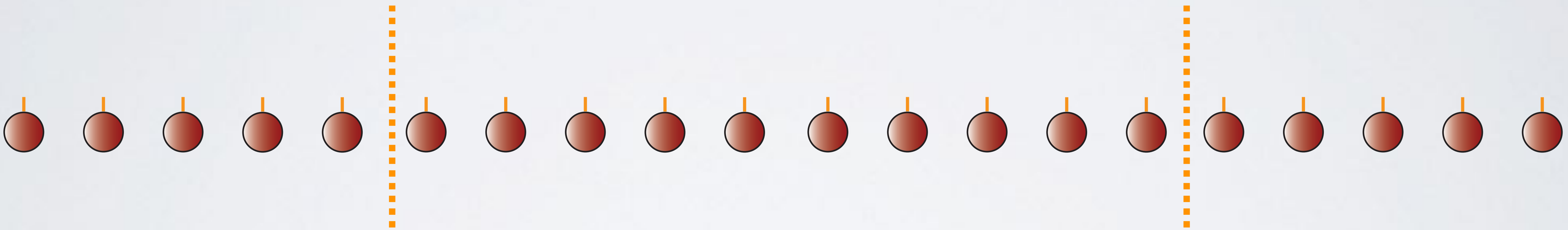
Statistics of small commutators

EVT   rare regions affect the distribution of commutators



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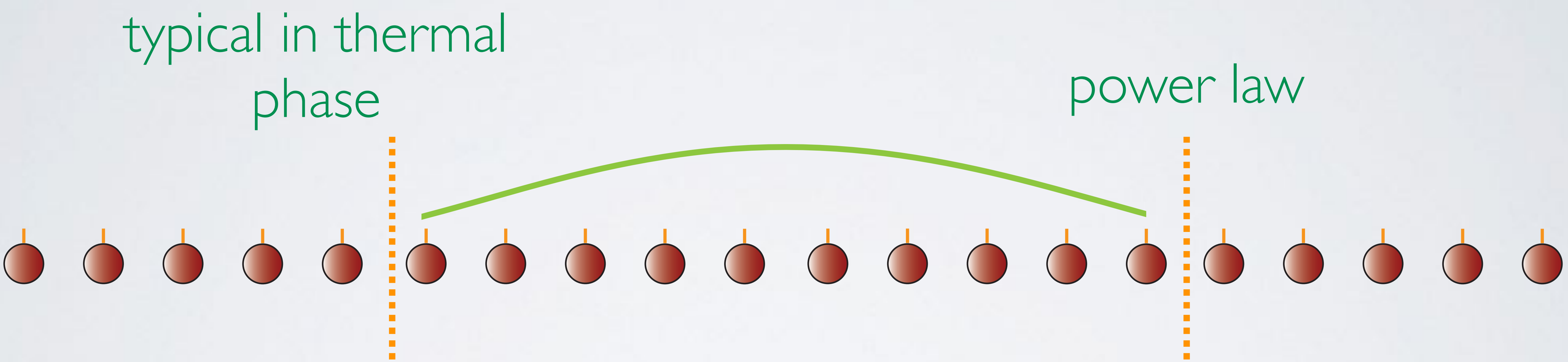

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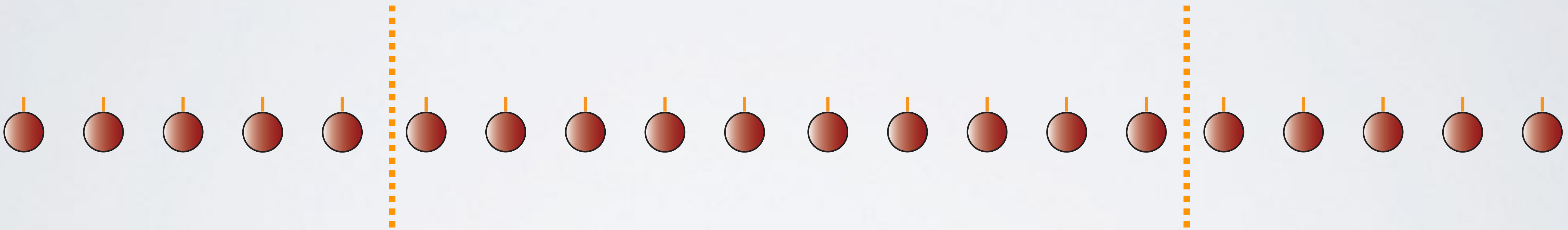

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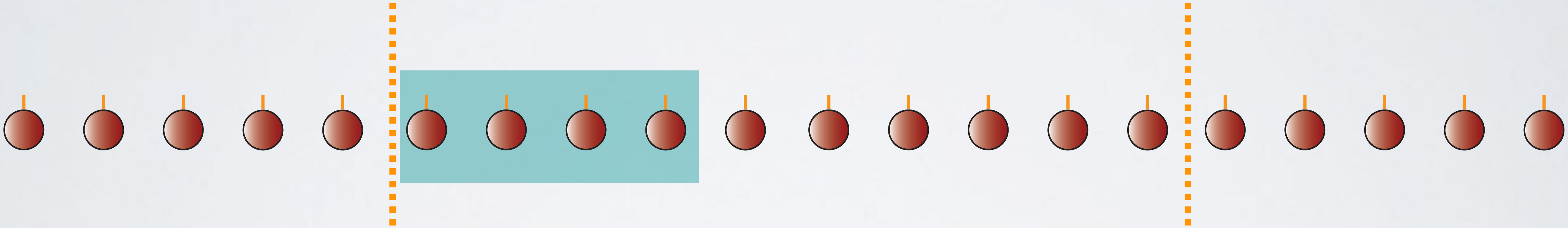

higher effective disordered

$$p(\ell < M) \sim c^{\ell}$$

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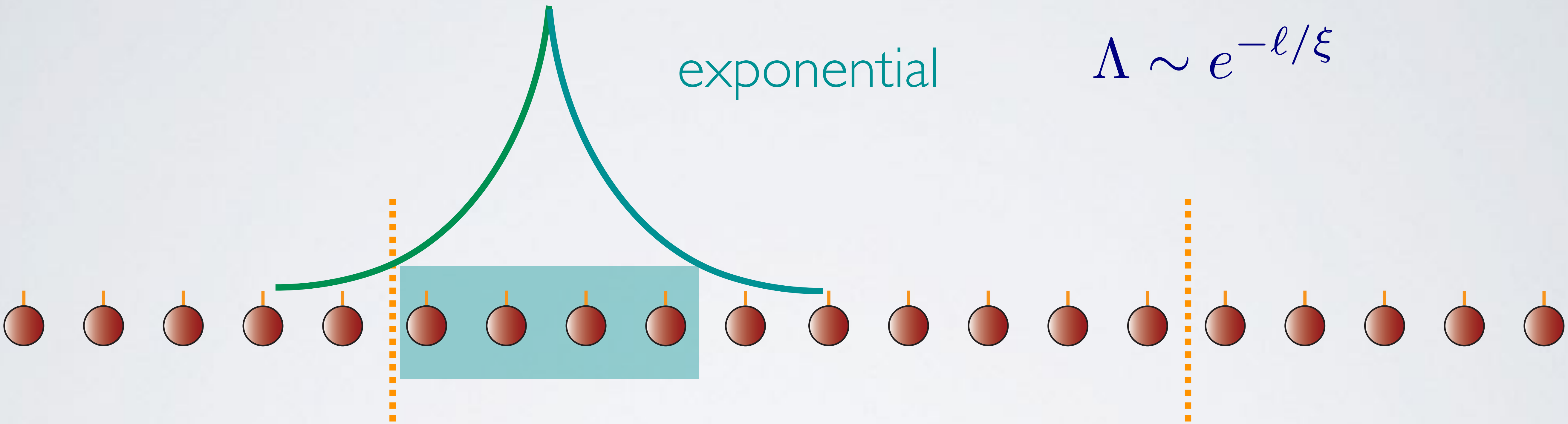

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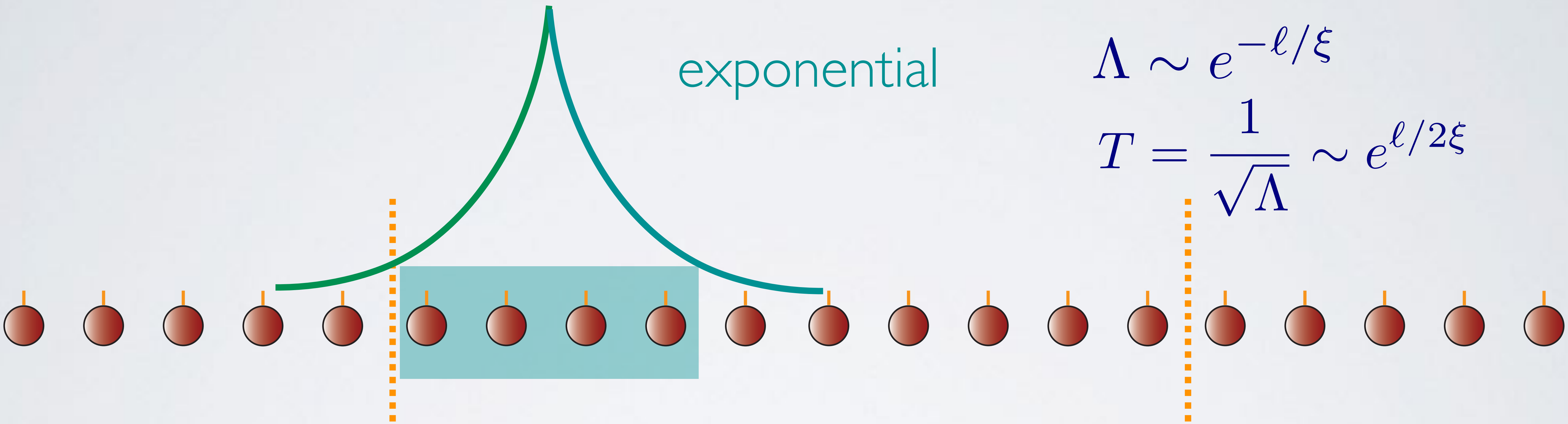

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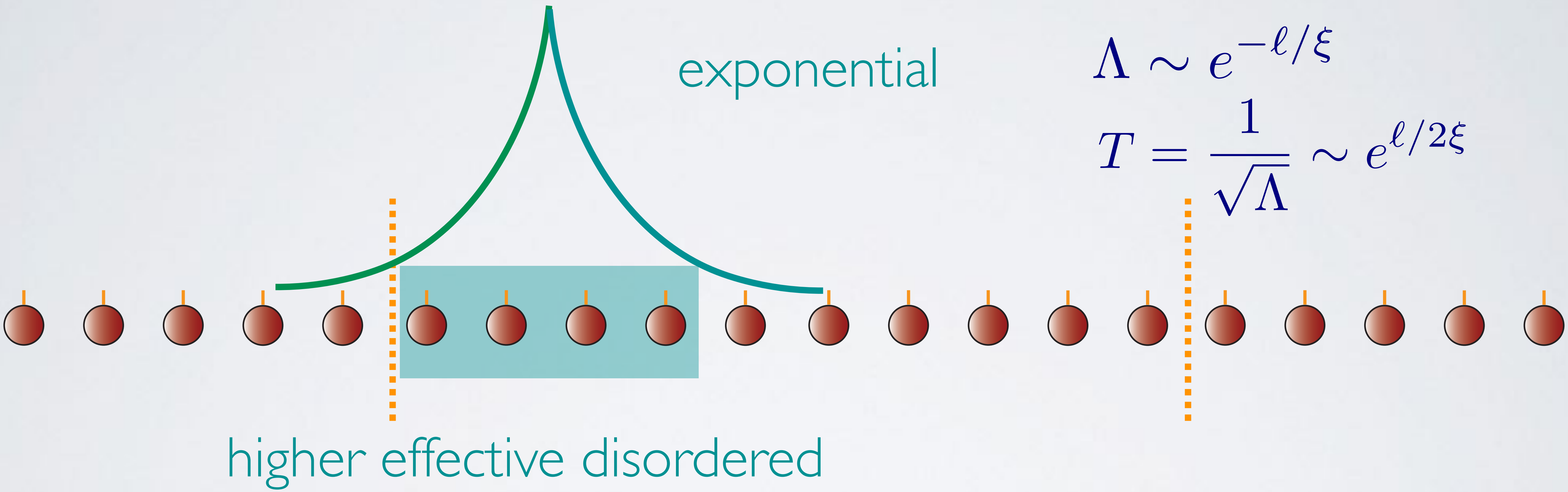

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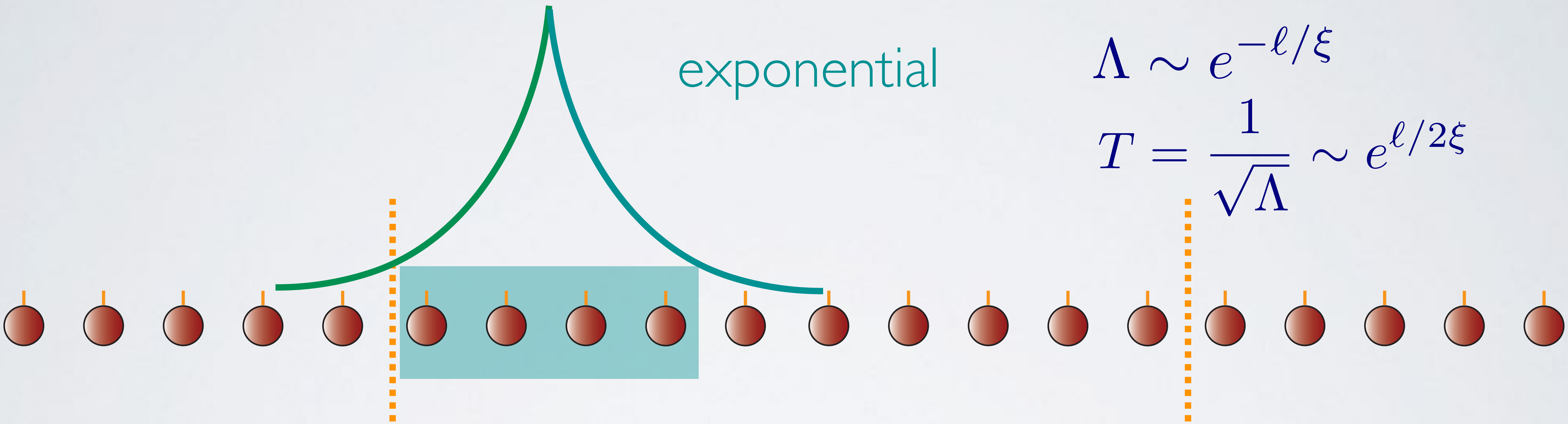

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Fréchet distribution

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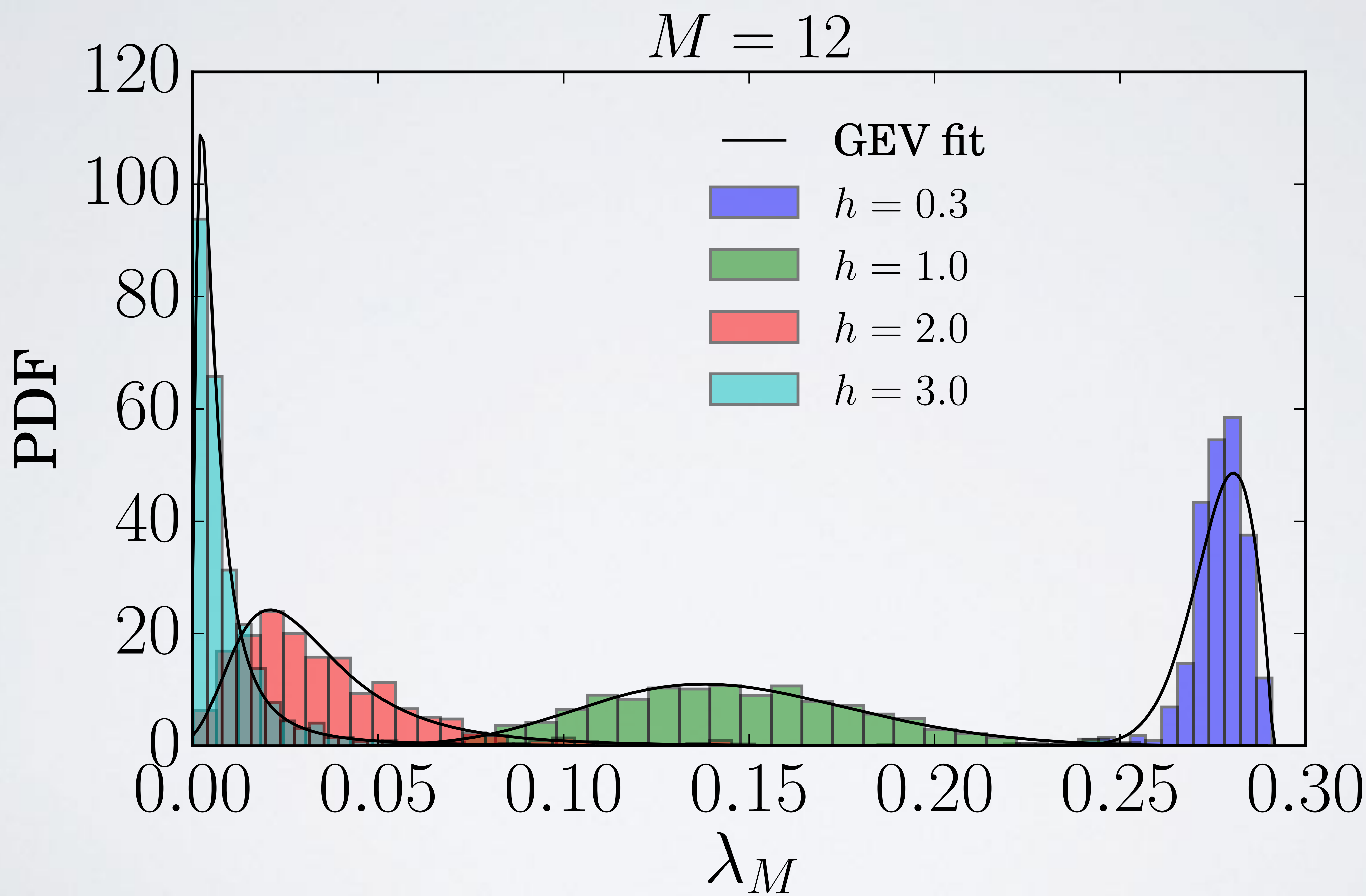
Statistics of small commutators

good fit to generalised extreme value distribution



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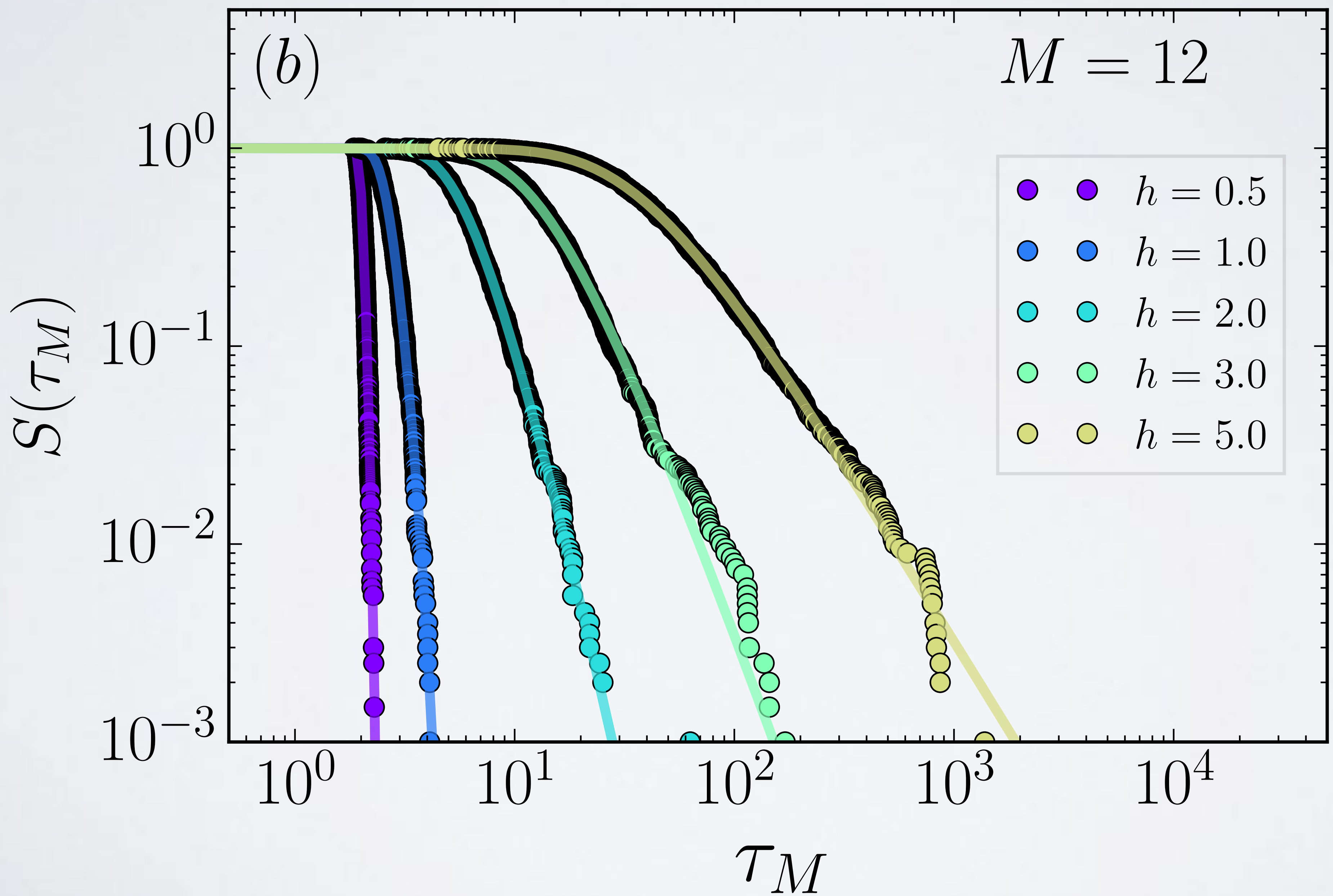
Statistics of small commutators

survival rate (probability of extremely large values)



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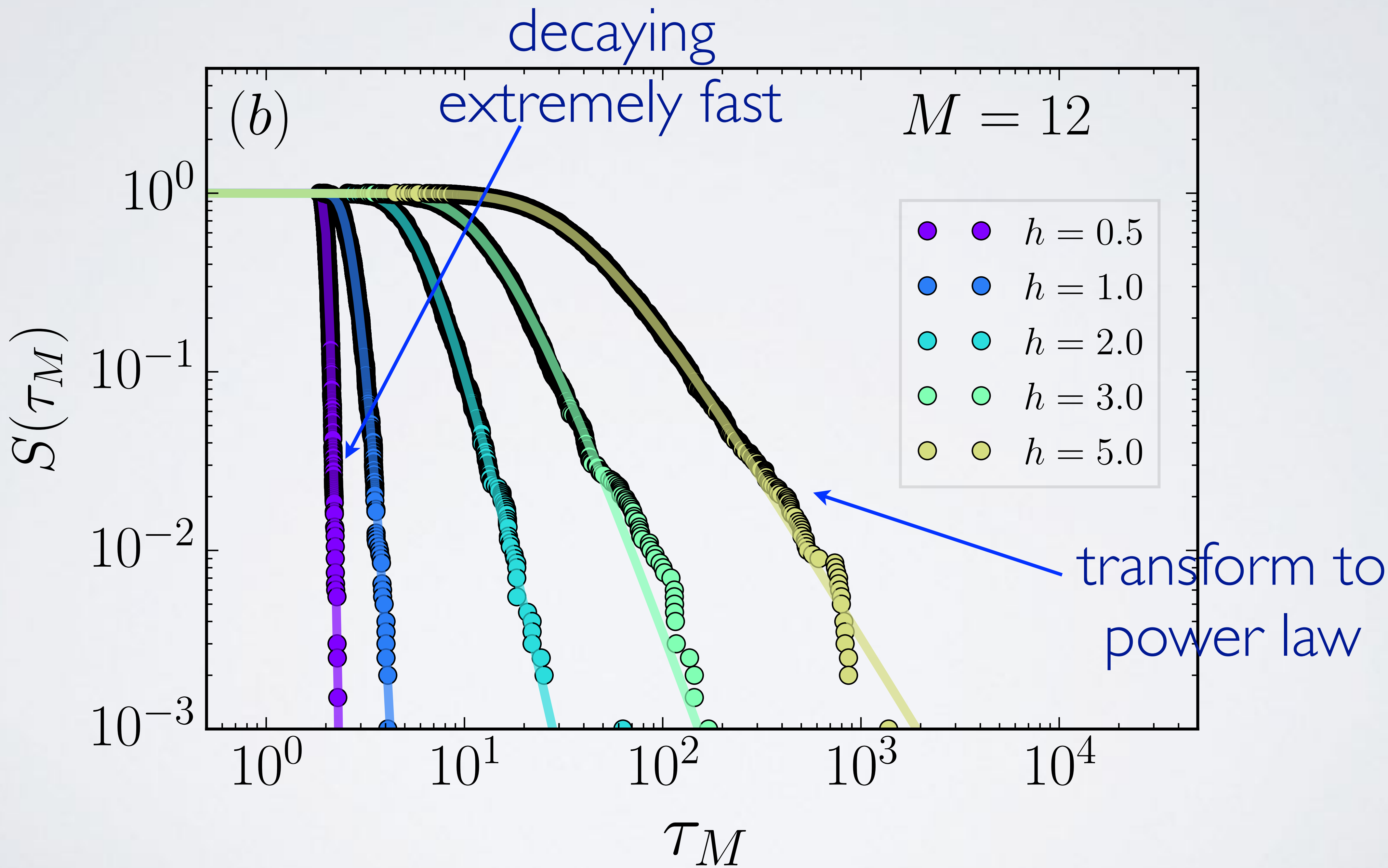

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TO CONCLUDE

MBL effects on the dynamics of very mixed states: quantum information perspective

MCB, N. Yao, S. Choi. M. Lukin, J.I. Cirac PRB 96,174201 (2017)

N. Pancotti et al PRB 97, 094206 (2018)

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almost conserved operators → signatures of localization, and rare regions in the statistics

N. Pancotti et al PRB 97, 094206 (2018)

THANKS

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propagation of correlations → mutual information bounded yet sensitive to $\log t$

quantum memory features → classical information localized quantum information eventually lost

MCB, N.Yao, S. Choi. M. Lukin, J.I. Cirac PRB 96,174201 (2017)

almost conserved operators → signatures of localization, and rare regions in the statistics

N. Pancotti et al PRB 97, 094206 (2018)