Quantum quenches in integrable models

A Pedagogical talk ?

Fabian Essler (Oxford)

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Outline

1. Quantum quenches & thermalisation
2. Quantum integrability in D=1
3. Free theories vs interacting integrable models
4. Generalized thermalisation in transl. inv. systems
5. Generalized hydrodynamics in inhomogeneous systems

I. Global Quantum Quenches

Simplest protocol for non-equilibrium dynamics

A. Many-particle system; Hamiltonian H.

B. Initial (lowly entangled) state $|\psi(0)\rangle$ that has non-zero overlap with exponentially many (in system size) eigenstates of H

C. Time evolution $|\psi(t)\rangle = \exp(-iHt)\, |\psi(0)\rangle$

D. Study expectation values of **local operators** $\langle\psi(t)|\mathcal{O}_A|\psi(t)\rangle$ in the thermodynamic limit.

Def.: a local operator acts as the identity outside a finite spatial region A in the infinite volume limit. Lattice spin models:

$\mathcal{O}_A = \sigma_{j_1}^{\alpha_1} \ldots \sigma_{j_\ell}^{\alpha_\ell}$ where $j_k \in A$

Global quantum quenches deposit an **extensive** amount of energy in the system:

$$\lim_{L\to\infty} \frac{\langle \Psi(0) | H | \Psi(0) \rangle}{L} > \lim_{L\to\infty} \frac{\langle \mathrm{GS} | H | \mathrm{GS} \rangle}{L}$$

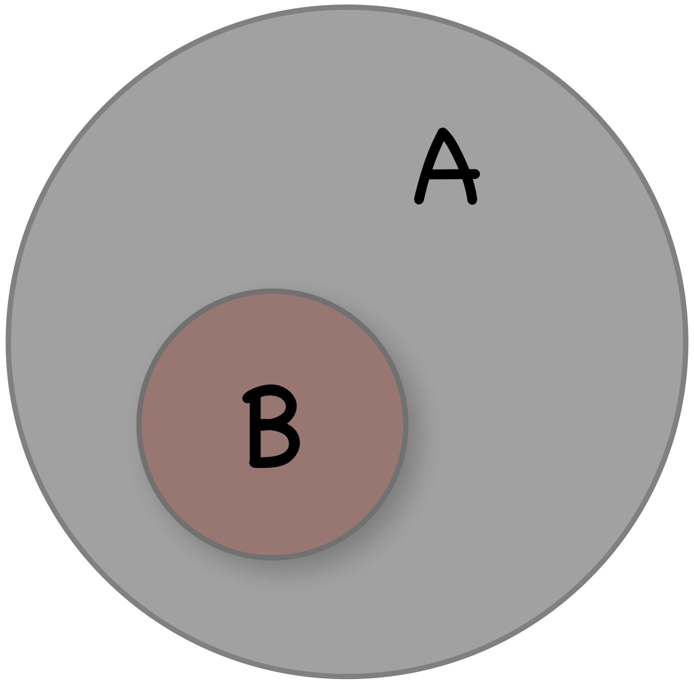
Probe physics far away from the GS.

For the time being focus on

A. Lattice models with finite local Hilbert spaces (e.g. lattice spins).

B. Hamiltonians + initial states invariant under translations.

II. Local relaxation after quantum quenches



Only consider **local properties** in thermodyn. limit: A **infinite**, B **finite**

$$\lim_{t\to\infty}\lim_{L\to\infty}\langle\Psi(t)|\mathcal{O}_B|\Psi(t)\rangle \text{ exists} \,\forall\, \mathcal{O}_B$$

$$= \lim_{L\to\infty}\mathrm{Tr}\left[\rho_{SS}\,\mathcal{O}_B\right]$$

Stationary values described by density matrix ρ_{ss} (not unique)

Physical Picture: A acts like a bath for B.

The system can never relax as a whole:

Expand in basis of energy eigenstates $|n\rangle$:

$$|\Psi(t)\rangle = \sum_n \langle n|\Psi(0)\rangle e^{-iE_n t}|n\rangle$$

"Observable" $O=O^\dagger= |1\rangle\langle 2|+|2\rangle\langle 1|$ does not relax:

$$\langle\psi(t)|O|\psi(t)\rangle = A\cos([E_1-E_2]t+\varphi)$$

But this is a horribly non-local operator...

What determines ρ_{ss} ?

Only consider local operators $\Longrightarrow$

Principle: in thermodyn. limit ρ_{ss} retains minimal possible amount of **local** information $\langle\Psi(0)|\mathcal{O}_B|\Psi(0)\rangle$ on initial state

"Thermalization"

Deutsch '91, Srednicki '94,....

Isolated system → energy conserved

$$e_0 = \lim_{L\to\infty} \frac{\langle\Psi(0)|H|\Psi(0)\rangle}{L} = \lim_{L\to\infty} \langle\Psi(0)|H_j|\Psi(0)\rangle = \lim_{L\to\infty} \langle\Psi(t)|H_j|\Psi(t)\rangle$$

This is the minimal local info on the initial state that must be retained; no other conserved quantities ⟹ system **thermalizes**

Stationary state described by e.g. micro-canonical ensemble

$$\lim_{t\to\infty}\lim_{L\to\infty} \langle\Psi(t)|\mathcal{O}_B|\Psi(t)\rangle = \lim_{L\to\infty} \langle E|\mathcal{O}_B|E\rangle$$

$|E\rangle$ any typical energy eigenstate at energy density e_0

Nonequilibrium Steady States and Conservation Laws

Local conservation laws (=those with local densities) are clearly important because

$$[H, I^{(n)}] = 0 \Rightarrow \langle \Psi(t) | I^{(n)} | \Psi(t) \rangle \text{ time independent}$$

Translational invariance $\Longrightarrow$ $\lim_{L\to\infty} \langle \Psi(t) | I_m^{(n)} | \Psi(t) \rangle$ time independent

$I_m^{(n)}$ local $\longrightarrow$ $\lim_{L\to\infty} \langle \Psi(0) | I_m^{(n)} | \Psi(0) \rangle = \mathrm{Tr}\left[\rho_{SS} I_m^{(n)}\right]$

ρ_{ss} retains **local** information about initial state !

will not thermalize unless fine-tuned!

"Generalized Thermalization"

Rigol et al '07

If we have additional conservation laws with local densities $I_m^{(n)}$ the minimal local information that must be retained is

$$\lim_{L\to\infty} \langle \Psi(0) | I_m^{(n)} | \Psi(0) \rangle = \mathrm{Tr}\left[\rho_{SS} I_m^{(n)}\right] \equiv i^{(n)}$$

What should the ensemble describing the steady state be?

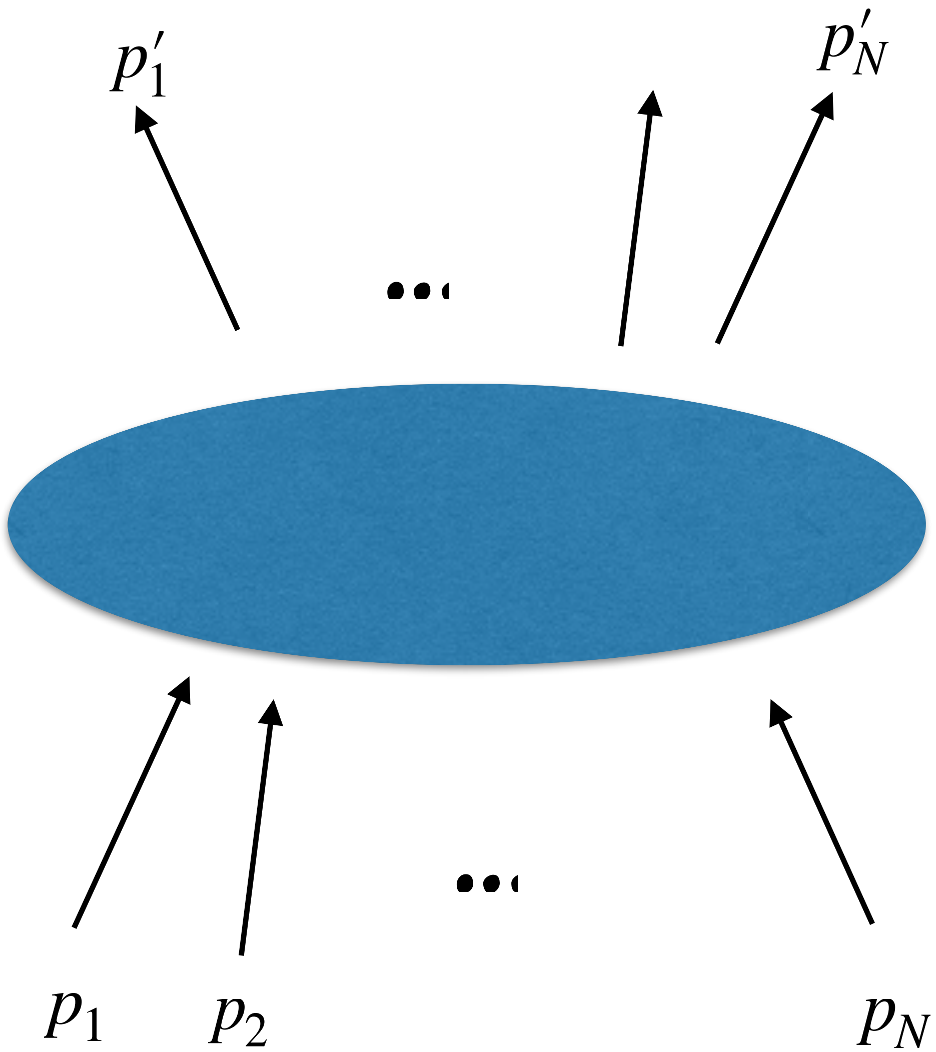
$$\lim_{t\to\infty} \lim_{L\to\infty} \langle \Psi(t) | \mathcal{O}_B | \Psi(t) \rangle = \lim_{L\to\infty} \langle \rho | \mathcal{O}_B | \rho \rangle$$

$|\rho\rangle$ any typical simultaneous eigenstate of H and $I^{(n)}$ with eigenvalues Le_0 & $L\, i^{(n)}$ (Generalized Micro-canonical Ensemble)

Cassidy et al '11
Caux&Essler '13

II. 1D Quantum Integrable Models

Definition 1: models with elementary excitations (generally not simply related to microscopic DOF) that scatter purely elastically



$$\{p_1, \ldots, p_N\} \equiv \{p_1', \ldots, p_N'\}$$

$$\psi(x_1, \ldots, x_N) = e^{i\sum_{j=1}^{N} p_j x_j} \longrightarrow \psi(x_1, \ldots, x_N) = \sum_{Q \in S_N} A(Q) e^{i\sum_{j=1}^{N} p_{Q_j} x_j}$$

Definition 2: models with extensive numbers of (quasi) local integrals of motion.

$$[H, I^{(n)}] = 0 = [I^{(n)}, I^{(m)}] \ , \qquad I^{(n)} = \sum_j I_j^{(n)}$$

↑ (quasi) local operators

Example: For a spin-1/2 chain $I^{(n)} = \sum_j I_j^{(n)}$

$$I_j^{(n)} = f_{\alpha_1}^{(1)} \sigma_j^{\alpha_1} + f_{\alpha_1 \alpha_2}^{(2)} \sigma_j^{\alpha_1} \sigma_{j+1}^{\alpha_2} + f_{\alpha_1 \alpha_2 \alpha_3}^{(3)} \sigma_j^{\alpha_1} \sigma_{j+1}^{\alpha_2} \sigma_{j+2}^{\alpha_3} + \ldots \qquad \alpha_j = 0, x, y, z$$

is quasi-local, if coefficients $f^{(k)}_{\alpha_1 \alpha_2 \ldots \alpha_k}$ decay fast enough (exponentially) in k

Simplest integrable models are **free theories**, but they are **special:** ∃ basis s.t.

$$H = \sum_k \epsilon(k)\gamma^\dagger(k)\gamma(k) \qquad [\gamma(k), \gamma^\dagger(q)\} = \delta_{k,q}$$

Problem separates into uncoupled harmonic oscillator modes

Quantization cond. (PBC) $\quad e^{ikL} = 1 \ , \Rightarrow k_n = \dfrac{2\pi n}{L}$

Mode occ. # are conserved $\quad [\gamma^\dagger(k)\gamma(k), H] = 0$

and in 1-1 correspondence with local conservation laws.

Interacting integrable models are different:

- No simple notion of eigenmodes (in finite volume)
- Generically there are (hierarchies of) bound states ("strings")

Example: spin-1/2 Heisenberg ferromagnet

$$H = -J\sum_{j=1}^{L} \mathbf{S}_j \cdot \mathbf{S}_{j+1}$$

2-part. eigenstates:

$$|k_1, k_2\rangle = \sum_{x_1<x_2} \psi(x_1, x_2) S^-_{x_1} S^-_{x_2} |\uparrow\uparrow \ldots \uparrow\rangle$$

Wave function:

$$\psi(x_1, x_2) = e^{ik_1x_1+ik_2x_2} + S(k_2, k_1)e^{ik_1x_2+ik_2x_1}$$

$$S(k_2, k_1) = -\frac{e^{ik_1+ik_2} + 1 - 2e^{ik_2}}{e^{ik_1+ik_2} + 1 - 2e^{ik_1}}$$

Energy

$$E = J\sum_{j=1}^{2}[1 - \cos(k_j)] - \frac{JL}{4}$$

Periodic bc's:

$$e^{ik_jL} = \prod_{l\neq j} S(k_j, k_l)$$

Non-trivial quant. cond. — allowed values of k_1 depend on k_2 (interactions!)

This generalises to n particles: "Bethe ansatz equations"

A priori k_j are complex

Bound states: go to pole of scattering phase $e^{ik_1+ik_2}+1-2e^{ik_1}=0$

$$\psi(x_1,x_2)=e^{iP\frac{x_1+x_2}{2}}e^{-2\gamma(x_2-x_1)}$$

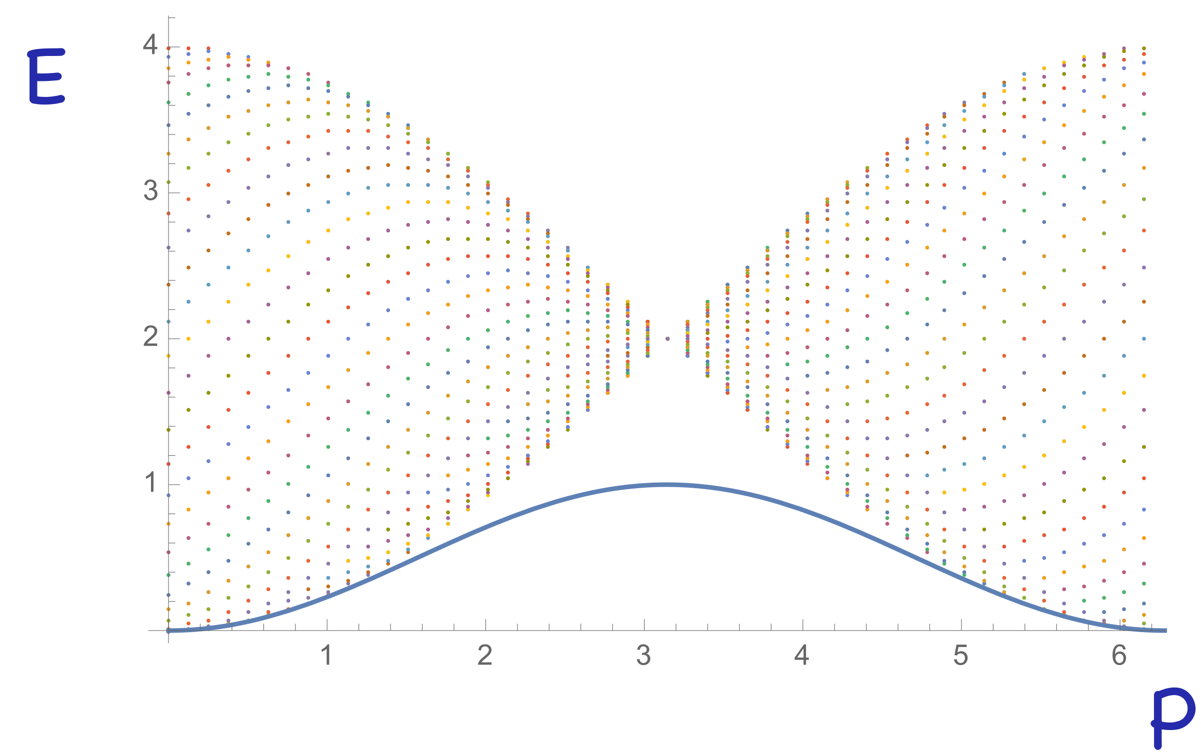
$$e^{-\gamma}=\cos(P/2)$$

$x_2>x_1$

Wave-fn decays exponentially wrt to distance $|x_2-x_1|$

Energy

$$E=\frac{J}{2}[1-\cos(P)]-\frac{JL}{4}$$



Non-equilibrium steady states in free theories

Want thermodynamic limit $N, L \to \infty$, $\frac{N}{L} = n$ fixed

$\Longrightarrow$ work with macro states

Hamiltonian: $$H = \sum_k \epsilon(k)\gamma^\dagger(k)\gamma(k) \qquad [\gamma(k), \gamma^\dagger(q)\} = \delta_{k,q}$$

Energy eigenstates (finite L) $$\prod_{j=1}^{N} \gamma^\dagger(p_j)\,|\,0\rangle$$

Define mode occ. density $\rho_p(k)$ by coarse graining: $$\rho_p(k)\frac{L\Delta k}{2\pi} = \#\ \text{of}\ p_j\ \text{in}\ [k, k+\Delta k]$$

In the thermodyn. limit each function 0≤n(k)≤1 with

$$\int_0^{2\pi} \frac{dk}{2\pi}\,\rho_p(k) = n \qquad \text{defines a macro state}$$

Mode structure: each mtm state either occupied ("particle") or empty (hole)

$$\rho_p(k) + \rho_h(k) = 1 = \rho_t(k)$$

Entropy (# micro states)

$$S \approx L \int_0^{2\pi} \frac{dk}{2\pi} \left[\rho_t(k)\ln[\rho_t(k)] - \rho_p(k)\ln[\rho_p(k)] - \rho_h(k)\ln[\rho_h(k)]\right]$$

Typical (max ent) state at given energy density:

$$\rho_p(k) = \frac{1}{e^{\beta\epsilon(k)} \pm 1}$$

Bose-Einstein/ Fermi-Dirac

$$e(\beta) = \int \frac{dk}{2\pi}\, \rho_p(k)\epsilon(k)$$

Stationary State after Quantum Quenches

(1) Fix a macro-state $\rho_p(k)$ by requiring (mode occ.$\Leftrightarrow I^{(n)}$)

$$\lim_{L\to\infty} \langle \Psi(0) | \gamma^\dagger(k)\gamma(k) | \Psi(0) \rangle = \rho_p(k)$$

(2) Take a micro-state $|\Phi\rangle$ corresponding to $\rho_p(k)$

Then

$$\lim_{t\to\infty} \lim_{L\to\infty} \langle \Psi(t) | \mathcal{O}_B | \Psi(t) \rangle = \lim_{L\to\infty} \langle \Phi | \mathcal{O}_B | \Phi \rangle$$

Rigorous result.

Gluza et al '16

Relaxation to Stationary State

Driven by "excitations" over SS

Caux&Essler '13

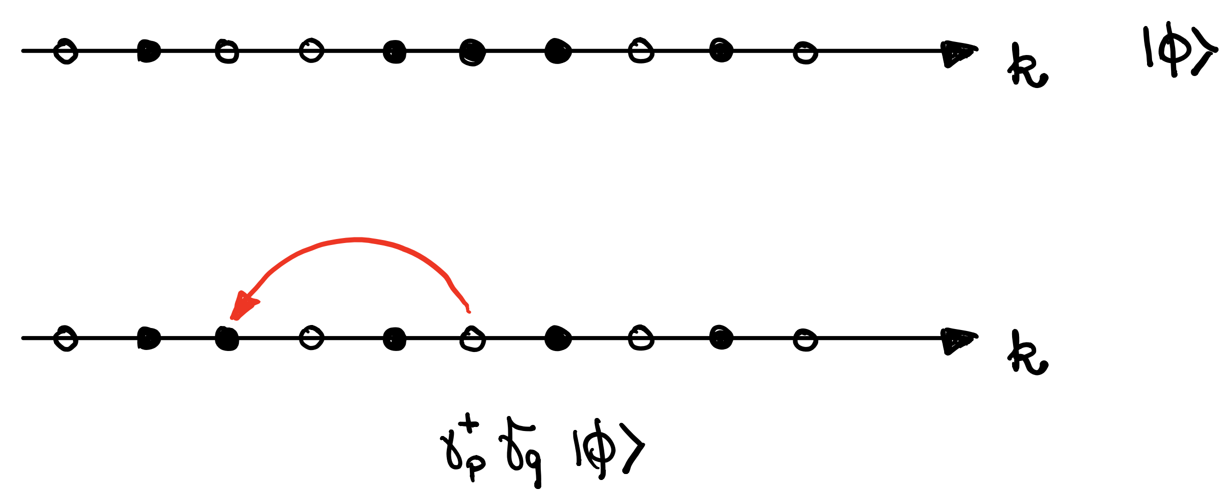
$$\langle \Psi(t) | \mathcal{O}_A | \Psi(t) \rangle = \lim_{L\to\infty} \sum_{\chi} \left[e^{\mathscr{E}^*_\Phi - \mathscr{E}^*_\chi + i(E_\chi - E_\Phi)t} \frac{\langle \chi | \mathcal{O} | \Phi \rangle}{2} + e^{\mathscr{E}_\Phi - \mathscr{E}_\chi - i(E_\chi - E_\Phi)t} \frac{\langle \Phi | \mathcal{O} | \chi \rangle}{2} \right]$$

$$e^{-\mathscr{E}_\chi} \equiv \langle \Phi | \Psi(0) \rangle$$

e.g. particle-hole ex

$$|\chi\rangle = \gamma^\dagger(p)\gamma(q)|\Phi\rangle$$

$$E_\chi - E_\Phi = \epsilon(p) - \epsilon(q)$$



"Quantum information" spreads with velocities

$$\mathrm{v}(p) = \epsilon'(p)$$

Macro states in interacting integrable theories

Starting point: quantisation conditions in large, finite L, e.g.

$$e^{ip(\lambda_j)L} = \prod_{l\neq j=1}^{N} S(\lambda_j - \lambda_l) \ , \quad j = 1,\ldots,N$$

λ_j rapidity variables

$$e^{ik_j} = \frac{\lambda_j + i}{\lambda_j - i}$$ for XXX chain

Step 1: Need to deal with complex solutions (bound states)

- For large L,N these do not correspond (precisely) to poles of $S(\lambda)$
- **Assume** that deviations are negligible ("string hypothesis") $\Longrightarrow$ each bound state of α particles parametrised by a single "centre-of-mass" rapidity $\lambda_j^\alpha \in \mathbb{R}$
- Bound states become like different **species** of particles

Quantisation conditions become

$$e^{ip_\alpha(\lambda_j^\alpha)L} = \prod_{(\beta,k)\neq(\alpha,j)} S_{\alpha\beta}(\lambda_j^\alpha - \lambda_l^\beta)\ ,\ j = 1\ldots N_\alpha\ ,\ \alpha = 1,\ldots$$

phase from taking bound state λ_j^α around ring

phases acquired by scattering off all other particles

Energy and momentum

$$E = \sum_{(n,\alpha)} \epsilon_\alpha(\lambda_n^\alpha)\ ,\quad P = \sum_{(n,\alpha)} p_\alpha(\lambda_n^\alpha)$$

Take logs

$$p_\alpha(\lambda_j^\alpha)L = 2\pi I_j^\alpha + \sum_{(\beta,k)\neq(\alpha,j)} \theta_{\alpha\beta}(\lambda_j^\alpha - \lambda_l^\beta)$$

integer "quantum numbers"

$$\{I_\alpha^n\} \Leftrightarrow \{\lambda_\alpha^n\} \Leftrightarrow \{\lambda_j\} \Leftrightarrow \psi_{k_1,\ldots,k_N}(x_1,\ldots,x_N)\ .$$

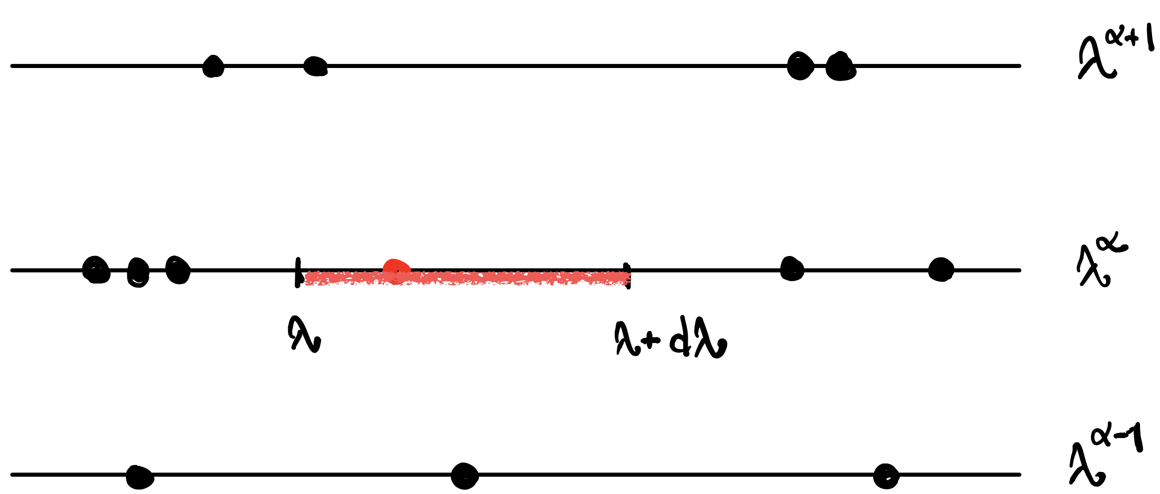
Step 2: Macro states in thermodyn limit

In thermodyn limit $\lambda_{j+1}^{\alpha} - \lambda_j^{\alpha} = \mathcal{O}(L^{-1})$

$\Longrightarrow$ can describe macro states by **densities** of particles/holes

$\rho_{\alpha,p}(\lambda)d\lambda = \#$ of λ_j^{α} in $[\lambda, \lambda + \Delta\lambda]$

Complication: $\rho_{\alpha,p}(\lambda) + \rho_{\alpha,h}(\lambda) = \rho_{\alpha,t}(\lambda) \neq 1$



\# of vacancies in $[\lambda, \lambda + \Delta\lambda]$

$\rho_{\alpha,t}(\lambda)d\lambda$

depends on all other particles because of interactions

Quantisation conditions $p_\alpha(\lambda_j^\alpha)L = 2\pi I_j^\alpha + \sum_{(\beta,k)\neq(\alpha,j)} \theta_{\alpha\beta}(\lambda_j^\alpha - \lambda_l^\beta)$

Use $\lambda_{j+1}^\alpha - \lambda_j^\alpha = \mathcal{O}(L^{-1})$ to turn sums into integrals; massage

$$\rho_{\alpha,p}(\lambda) + \rho_{\alpha,h}(\lambda) = p'_\alpha(\lambda) - \sum_{\beta=1}^{\infty} \int_{-\infty}^{\infty} d\lambda'\, T_{\alpha\beta}(\lambda-\lambda')\, \rho_{\beta,p}(\lambda'),$$

$$T_{\alpha\beta}(\lambda) = -i\frac{d}{d\lambda}\ln S_{\alpha,\beta}(\lambda)$$

"Thermodynamic limit of Bethe ansatz equations"

System of linear integral eqns relating particle and hole densities.

Each set of positive functions $\{\rho_{\alpha,p}(\lambda), \rho_{\alpha,h}(\lambda)\,|\,\alpha = 1,\ldots\}$ satisfying the TLBAE defines a macro state.

Notations: $|\vec{\rho}\rangle$

Typical states at a given energy density

Energy/entropy densities of macro state $|\vec{\rho}\rangle$

$$e[\{\rho_{\alpha,p},\rho_{\alpha,h}\}] = \sum_{\alpha=1}^{\infty}\int_{-\infty}^{\infty} d\lambda\ \rho_{\alpha,p}(\lambda)\epsilon_\alpha(\lambda)$$

$$s[\{\rho_{\alpha,p},\rho_{\alpha,h}\}] = \sum_{\alpha=1}\int d\lambda\left[\rho_{\alpha,t}(\lambda)\ln[\rho_{\alpha,t}(\lambda)] - \rho_{\alpha,p}(\lambda)\ln[\rho_{\alpha,p}(\lambda)] - \rho_{\alpha,h}(\lambda)\ln[\rho_{\alpha,h}(\lambda)]\right]$$

Typical state at e(T): maximise e-Ts wrt to $\rho_{\alpha,p}(\lambda)$

Thermodynamic Bethe Ansatz (TBA) equations

$$\ln\left(1+\frac{\rho_{\alpha,h}(\lambda)}{\rho_{\alpha,p}(\lambda)}\right) = \frac{\epsilon_\alpha(\lambda)}{T} + \sum_{m=1}^{\infty}\int_{-\infty}^{\infty} d\mu\left[\delta_{\alpha,\beta}\delta(\lambda-\mu) + T_{\alpha\beta}(\lambda-\mu)\right]\ln\left(1+\frac{\rho_{\beta,p}(\lambda)}{\rho_{\beta,h}(\lambda)}\right)$$

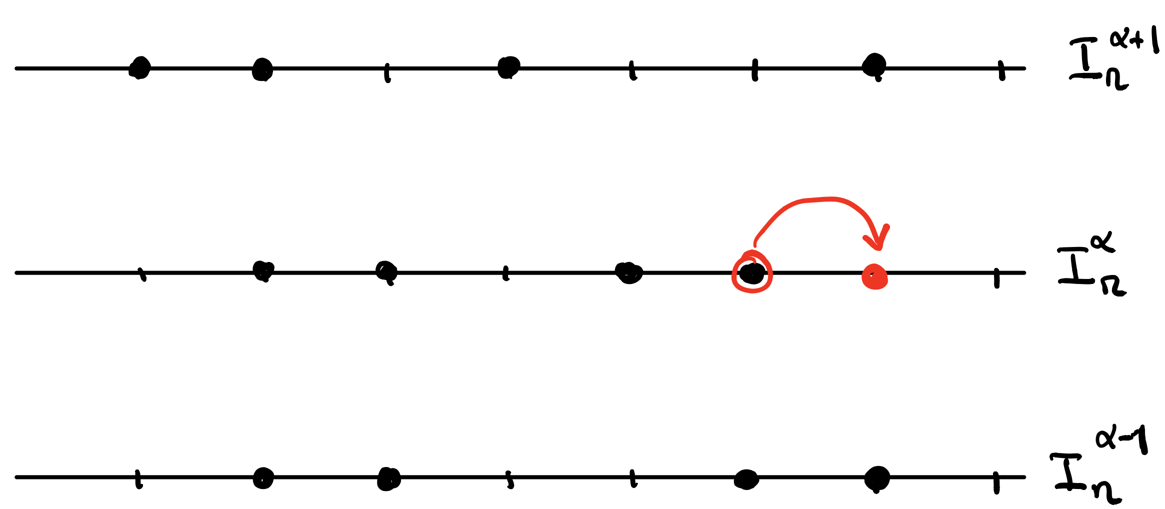
TBA equations and TLBAE together determine the state of thermal equilibrium.

"Excitations" over Macro States

Let $\{\lambda_j^\alpha\}$ be a micro state corresponding to $|\vec{\rho}\rangle$

$$p_\alpha(\lambda_j^\alpha)L = 2\pi I_j^\alpha + \sum_{(\beta,k)\neq(\alpha,j)} \theta_{\alpha\beta}(\lambda_j^\alpha - \lambda_l^\beta)$$

Can make e.g. "particle-hole excitations"



$\Longrightarrow \quad \{\tilde{\lambda}_j^\alpha\}$

$$p_\alpha(\tilde{\lambda}_j^\alpha)L = 2\pi \tilde{I}_j^\alpha + \sum_{(\beta,k)\neq(\alpha,j)} \theta_{\alpha\beta}(\tilde{\lambda}_j^\alpha - \tilde{\lambda}_l^\beta)$$

Excitation energy and momentum are additive!

$$E_{\text{ex}} = \mathscr{E}_\alpha(\lambda^p) - \mathscr{E}_\alpha(\lambda^h)\ , \quad P = \mathscr{P}_\alpha(\lambda^p) - \mathscr{P}_\alpha(\lambda^h)$$

- $\lambda^{p,h}$ rapidities of the particle/hole
- $\mathscr{E}_\alpha(\lambda)$, $\mathscr{P}_\alpha(\lambda)$ depend only on $|\vec{\rho}\rangle$

Macro state dependent quasi-particle picture!

Correlations/entanglement are spread by these quasiparticles!

Associated group velocities:

$$\mathrm{v}_{\alpha,|\vec{\rho}\rangle}(\lambda) = \frac{\frac{\partial \mathscr{E}_\alpha(\lambda)}{\partial \lambda}}{\frac{\partial \mathscr{P}_\alpha(\lambda)}{\partial \lambda}}$$

Bonnes, Essler, Läuchli '14

Alba& Calabrese '17

Summary of this part

- Integrable models have **atypical** finite-entropy macro states
- Described by sets of particle/hole densities for "fundamental" particles and bound states
- ∃ stable quasiparticle excitations over each macro state; their numbers and properties depend on the macro state

How to access atypical states? (they are very rare!)

- Energy eigenstates are also eigenstates of the (quasi) local conservation laws $I^{(n)}$
- Recall that by generalised thermalisation

$$\lim_{t\to\infty}\lim_{L\to\infty}\langle\Psi(t)|\mathcal{O}_B|\Psi(t)\rangle = \lim_{L\to\infty}\langle\rho|\mathcal{O}_B|\rho\rangle$$

$|\rho\rangle$ any typical simultaneous eigenstate of H and $I^{(n)}$ with eigenvalues Le_0 & $L\, i^{(n)}$

Stationary states after quantum quenches are automatically atypical, unless we fine-tune the initial conditions!

They are also interesting (different from thermal states). e.g. QDL.

How to construct the GMC after a QQ?

Let $|\rho\rangle$ be a micro state corresponding to $|\vec{\rho}\rangle$

Then
$$\lim_{L\to\infty}\frac{1}{L}\langle\rho|I^{(n)}|\rho\rangle = \sum_{\alpha}\int d\lambda\ \rho_{\alpha,p}(\lambda)\ \epsilon_{\alpha}^{(n)}(\lambda) \equiv i_{\text{stat}}^{(n)} \qquad (1)$$

known functions

Initial conditions $\lim_{L\to\infty}\frac{1}{L}\langle\Psi(0)|I^{(n)}|\Psi(0)\rangle = i^{(n)}$ Must have $i_{\text{stat}}^{(n)} = i^{(n)}$

- Calculate $i^{(n)}$ ("initial data") — possible for simple matrix-product initial states Fagotti&Essler '13
- Determine $\{\rho_{\alpha,p},\rho_{\alpha,h}|\alpha=1,\ldots\}$ from (1) Ilievski et al '16
- $\rho_{SS} = |\Phi\rangle\langle\Phi|$ where $|\Phi\rangle$ is any micro state corresponding to $|\vec{\rho}\rangle$

alternative way for special initial states: “Quench Action Approach”

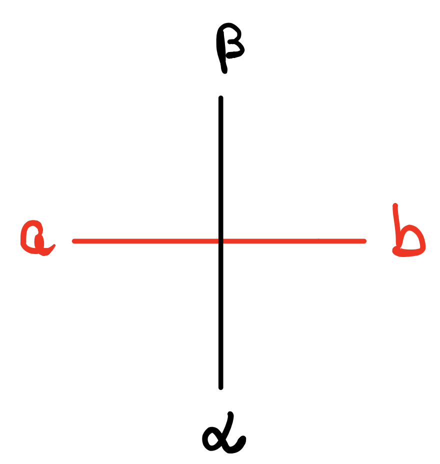
Caux&Essler ‘13

Brockmann et al ’14
Poszgay et al ’14
Bertini et al ’14
de Nardis et al ’14
...

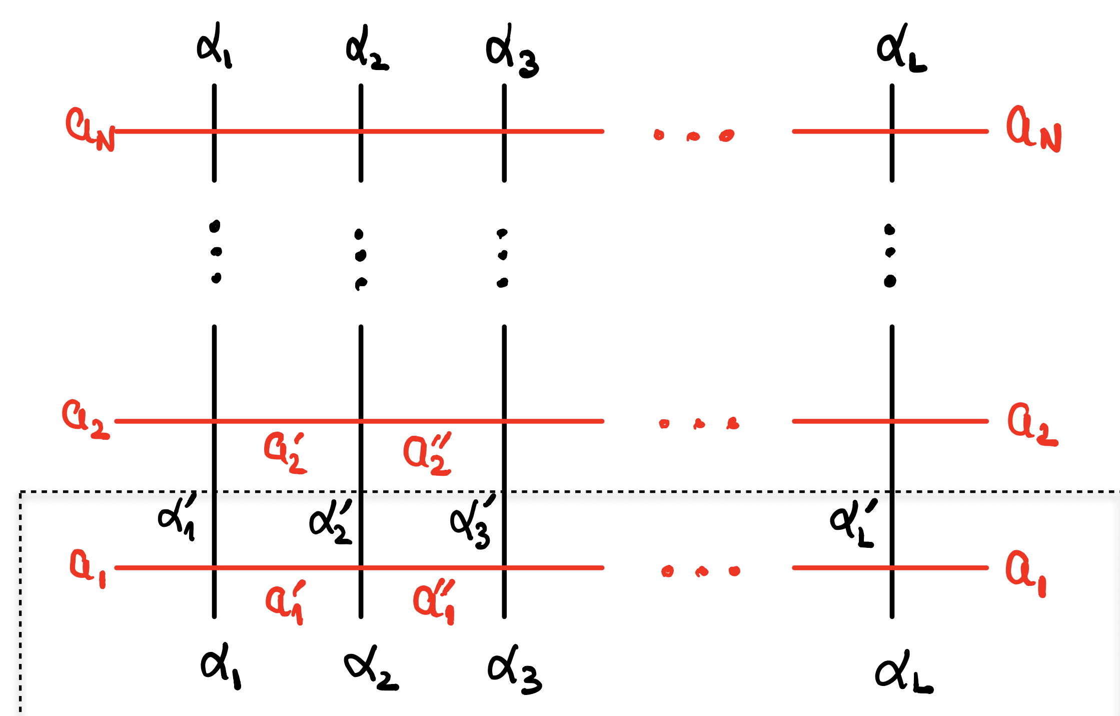
How to find the conservation laws?

Use connection between D-dim QM & D+1-dim classical Stat. Mech.

Vertex weights



$(L(\mu))^{ab}_{\alpha\beta}$



Partition fn

$$\left[\tau(\mu)\right]^{\alpha'_1\ldots\alpha'_L}_{\alpha_1\ldots\alpha_L}$$

$$Z = \sum_{\{\alpha_j\},\{\alpha'_j\}\ldots} \; \sum_{\{a_j\},\{a'_j\}\ldots} \left[L(\mu)\right]^{a_1,a'_1}_{\alpha_1\alpha'_1} \left[L(\mu)\right]^{a'_1,a''_1}_{\alpha_2\alpha'_2} \cdots \quad = \mathrm{Tr}\left[\tau(\mu)^N\right]$$

$$H = \frac{d}{d\mu}\bigg|_{\mu=\mu_0} \ln\left[\tau(\mu)\right]$$

Integrability:

$$\left[\tau(\mu), \tau(\lambda)\right] = 0$$

Heisenberg model $\Longleftrightarrow$ 6-vertex model   a,b=1,2 and α,β=1,2

Higher conservation laws:
$$I^{(\frac{1}{2},n)} = \frac{d^n}{d\mu^n}\bigg|_{\mu=\mu_0} \ln\left[\tau_{\frac{1}{2}}(\mu)\right]$$

These are "ultra-local"
$$I^{(\frac{1}{2},n)} = \sum_j I^{(\frac{1}{2},n)}_{j,j+1,\ldots,j+n}$$

But $\exists$ much larger family of commuting transfer matrices: take "auxiliary space" 2S+1 dim
$$[\tau_S(\mu), \tau_{S'}(\lambda)] = 0$$

Kulish & Reshetikhin '83

Quasi-local conservation laws
$$I^{(S,n)} = \frac{d^n}{d\mu^n}\bigg|_{\mu=\mu_S} \ln\left[\tau_S(\mu)\right]$$

Ilievski, Medenjak & Prosen '16

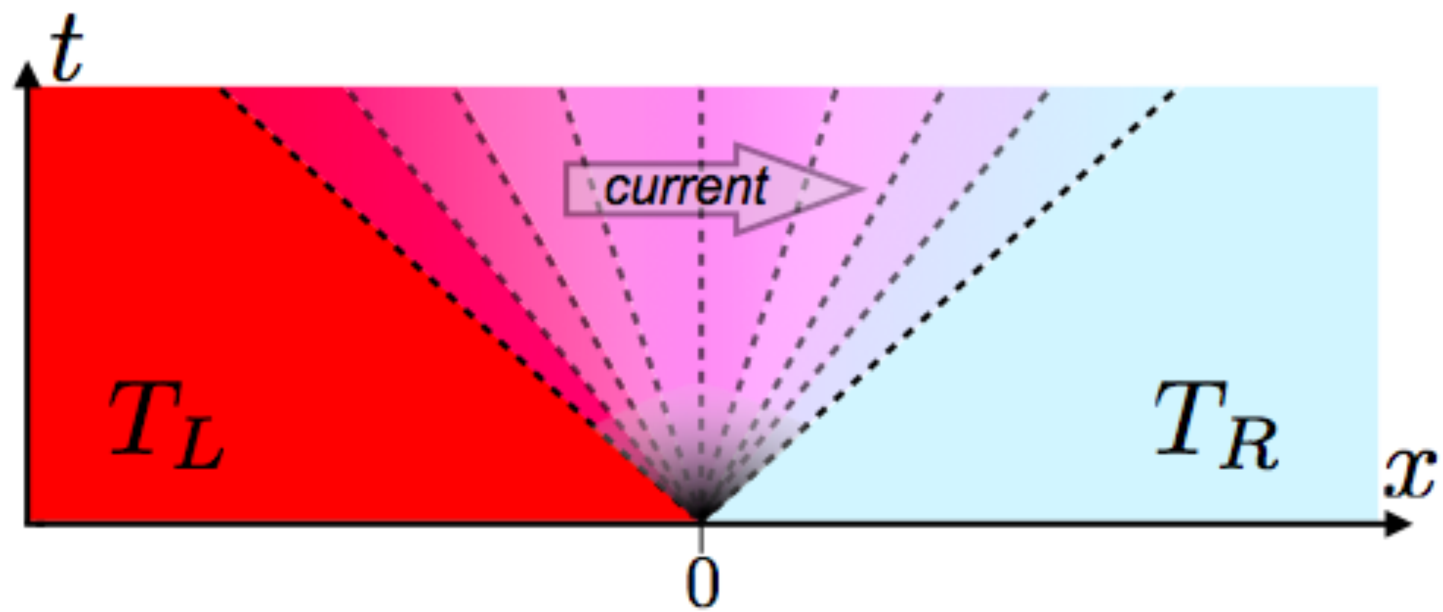
Generalized Hydrodynamics

Bertini et al '16

Castro-Alvarado et al '16

Translational invariant model with **inhomogeneous** initial state

Simplest setup



Basic idea: at late times a current-carrying NESS develops along each ray x/t=ξ inside the light cone.

$$\langle \Psi(t) | \mathcal{O}_x | \Psi(t) \rangle \longrightarrow \mathrm{Tr}\left[\rho_{SS}(\xi)\ \mathcal{O}_x\right]$$

$\mathcal{O}_x$ acts non-trivially only around x

Generalize ideas from homogenous case:

$\rho_{SS}(\xi)$ described in terms of macro state $\{\rho_{\alpha,p}(\xi,\lambda), \rho_{\alpha,h}(\xi,\lambda)\}$

How to determine these macro states?

Use continuity eqns for densities of cons. laws

$$\frac{d}{dt} I_j^{(n)} = i[H, I_j^{(n)}] = J_j^{(n)} - J_{j+1}^{(n)}$$

currents

Expectation values in the stationary states?

Homogeneous case: $$\lim_{L\to\infty} \langle \rho \,|\, I_j^{(n)} \,|\, \rho \rangle = \sum_\alpha \int d\lambda \; \rho_{\alpha,p}(\lambda) \; \epsilon_\alpha^{(n)}(\lambda)$$

Inhomogeneous case: $$\lim_{L\to\infty} \langle \rho_\xi \,|\, I_j^{(n)} \,|\, \rho_\xi \rangle = \sum_\alpha \int d\lambda \; \rho_{\alpha,p}(\xi,\lambda) \; \epsilon_\alpha^{(n)}(\lambda)$$

$$\lim_{L\to\infty} \langle \rho_\xi \,|\, J_j^{(n)} \,|\, \rho_\xi \rangle = \sum_\alpha \int d\lambda \; \mathrm{v}_{\alpha,|\vec{\rho}\rangle}(\lambda) \; \rho_{\alpha,p}(\xi,\lambda) \; \epsilon_\alpha^{(n)}(\lambda)$$

quasiparticle group velocities

This gives $$\sum_\alpha \int d\lambda \; \epsilon_\alpha^{(n)}(\lambda) \left[\partial_t \rho_{\alpha,p}(\xi,\lambda) + \partial_x \left(\mathrm{v}_{\alpha,|\vec{\rho}\rangle}(\lambda) \; \rho_{\alpha,p}(\xi,\lambda) \right) \right] = 0$$

"Completeness" of conservation laws $\Longrightarrow$

$$\partial_t \rho_{\alpha,p}(\xi,\lambda) + \partial_x \left(\mathrm{v}_{\alpha,|\vec{\rho}\rangle}(\lambda) \; \rho_{\alpha,p}(\xi,\lambda) \right) = 0$$

GHD equations

Given some initial conditions (special states) these can be integrated $\Longrightarrow$ description of the NESS.

$$\lim_{L\to\infty} \langle \rho_\xi | I_j^{(n)} | \rho_\xi \rangle = \sum_\alpha \int d\lambda \; \rho_{\alpha,p}(\xi,\lambda) \; \epsilon_\alpha^{(n)}(\lambda)$$

$$\lim_{L\to\infty} \langle \rho_\xi | J_j^{(n)} | \rho_\xi \rangle = \sum_\alpha \int d\lambda \; \mathrm{v}_{\alpha,|\overrightarrow{\rho}\rangle}(\lambda) \; \rho_{\alpha,p}(\xi,\lambda) \; \epsilon_\alpha^{(n)}(\lambda)$$

$\Longrightarrow$ profiles of current and charge densities.

Bertini et al
Doyon et al
Bulchandani et al
...

Summary

- Lot of progress in understanding non-equilibrium dynamics in integrable systems
- Interesting physics (e.g. non-thermal NESS)
- Important differences between interacting and free theories