Information propagation and entanglement generation with long-range interactions

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Apply for JQI postdoc fellowship (theory, expt) & QuICS postdoc fellowship (theory)

KITP Program "The Dynamics of Quantum Information"
KITP
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Motivation

Typical condensed matter systems:

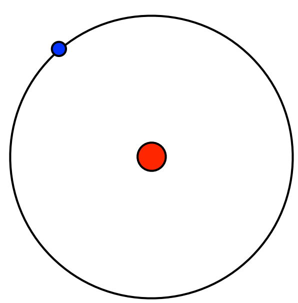
- short-range interactions = finite range or exponentially decaying

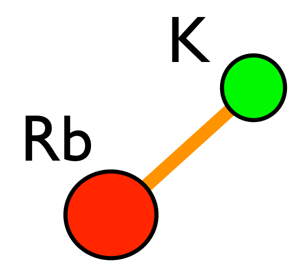
AMO and other synthetic quantum systems:

- long-range = not short-range (e.g. decaying as $1/r^\alpha$)

Examples:

- $1/r^3$: Rydberg or magnetic atoms, excitons, NV centers, polar molecules
- $1/r^6$: Rydberg atoms





- $\sim 1/r^\alpha$ & other forms: ion crystals, atoms in multimode cavities or along waveguides

Motivation

• among the strongest & most tunable interactions available in AMO

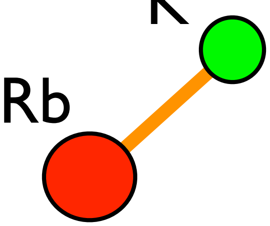
$\Rightarrow$ ideal for studying strongly interacting quantum many-body physics

Examples:

e.g. Lev

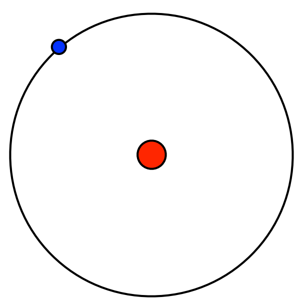
e.g. Abanin

• $1/r^3$: Rydberg or magnetic atoms, excitons, NV centers, polar molecules



e.g. Lukin

• $1/r^6$: Rydberg atoms



• $\sim 1/r^\alpha$ & other forms: ion crystals, atoms in multimode cavities or along waveguides

e.g. Rey

Features of long-range interactions

- faster quantum state transfer, faster quantum computing, faster preparation of entangled states
- mask dimensionality [e.g. Peter, Müller, Wessel, Büchler, PRL 2012; Maghrebi, Gong, AVG, PRL 2017]
- unusual ground-state entanglement properties [e.g.: Koffel, Lewenstein, Tagliacozzo, PRL 2012 Vodola, Lepori, Ercolessi, AVG, Pupillo, PRL 2014]
- ...

Long-range interactions: active research areas

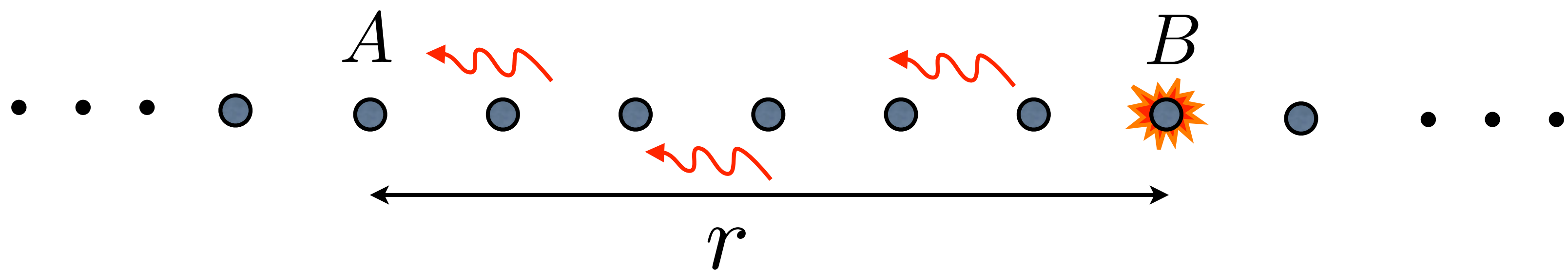
- short-time dynamics after quench: speed limit?
- long-time dynamics after quench: thermalization? localization?
- topological phases in the presence of long-range interactions?
- new phases and phase transitions?
- …

Today

- short-time dynamics after quench: speed limit?
- long-time dynamics after quench: thermalization? localization?
- topological phases in the presence of long-range interactions?
- new phases and phase transitions?
- …

Lieb-Robinson bounds

- lattice in arbitrary dimension (draw 1D for simplicity)



observable A $\qquad B^\dagger B = 1$

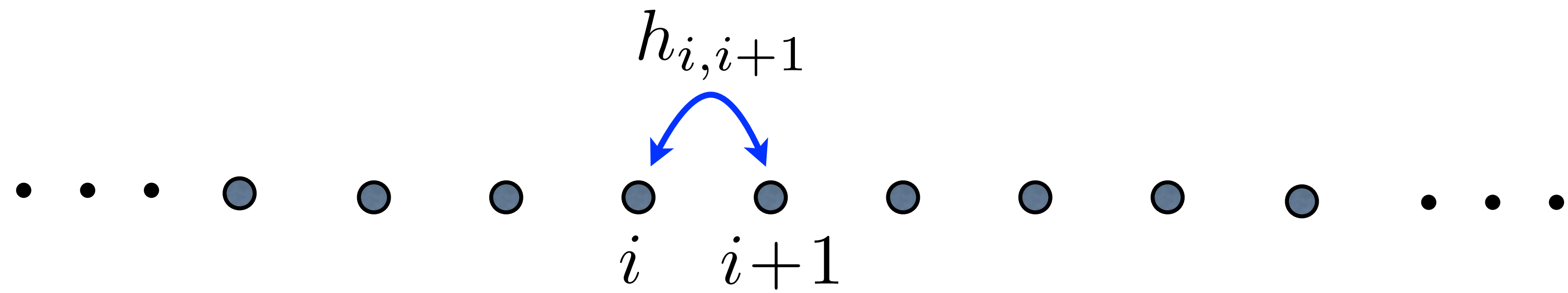
- arbitrary initial state $|\psi\rangle$ $\qquad A(t) = e^{iHt} A e^{-iHt}$

- effect on A due to disturbance B :

$$|\langle\psi|B^\dagger A(t) B|\psi\rangle - \langle\psi|A(t)|\psi\rangle| = |\langle\psi|B^\dagger A(t) B - B^\dagger B A(t)|\psi\rangle|$$

$$= |\langle\psi|B^\dagger [A(t), B]|\psi\rangle| \le ||B^\dagger [A(t), B]|| = ||[A(t), B]|| \equiv Q(r, t)$$

Short-range interactions



$$H = \sum_i h_{i,i+1}$$

$$||h_{i,i+1}|| \leq 1$$

- arbitrary time dependence allowed
- arbitrary time-dependent on-site terms allowed

E. Lieb & D. Robinson, 1972

Short-range interactions

$$Q(r,t) \leq \mathcal{J}_1(r)t + \mathcal{J}_2(r)\frac{t^2}{2!} + \mathcal{J}_3(r)\frac{t^3}{3!} + \dots$$

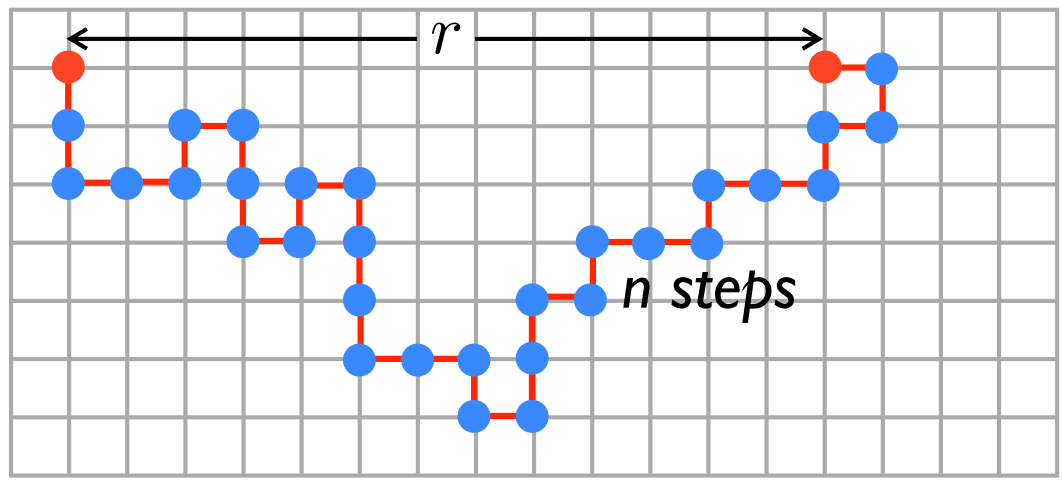
E. Lieb & D. Robinson, 1972

Short-range interactions

$$Q(r,t) \le \mathcal{J}_1(r)t + \mathcal{J}_2(r)\frac{t^2}{2!} + \mathcal{J}_3(r)\frac{t^3}{3!} + \ldots$$

(kind of like a path integral, but all contributions positive)

$$\mathcal{J}_n(r) = \sum$$



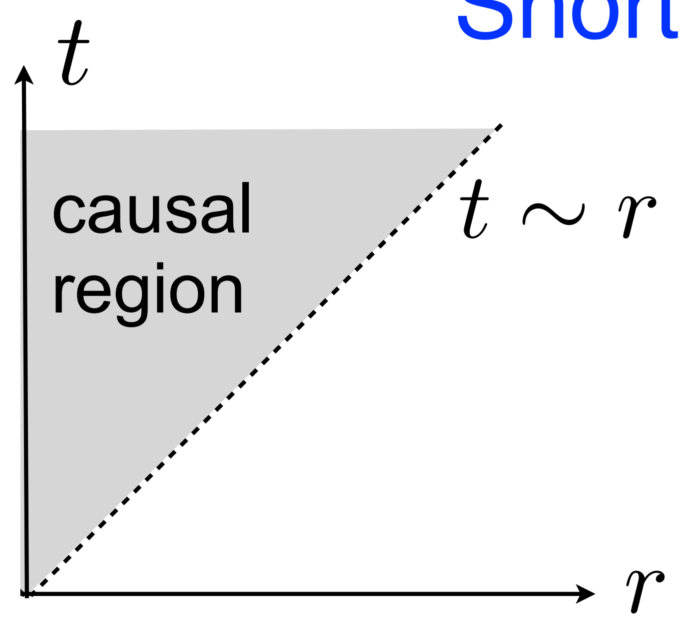
short-range Lieb-Robinson bound

- signal after time t distance r away: $Q(r,t) \lesssim e^{vt-r}$

$v \sim 1$

E. Lieb & D. Robinson, 1972

Short-range interactions



- shortest time t to send quantum info over distance r is $t \gtrsim r$

• observed in cold atoms:
Cheneau et al (Bloch, Kuhr), Nature (2012)

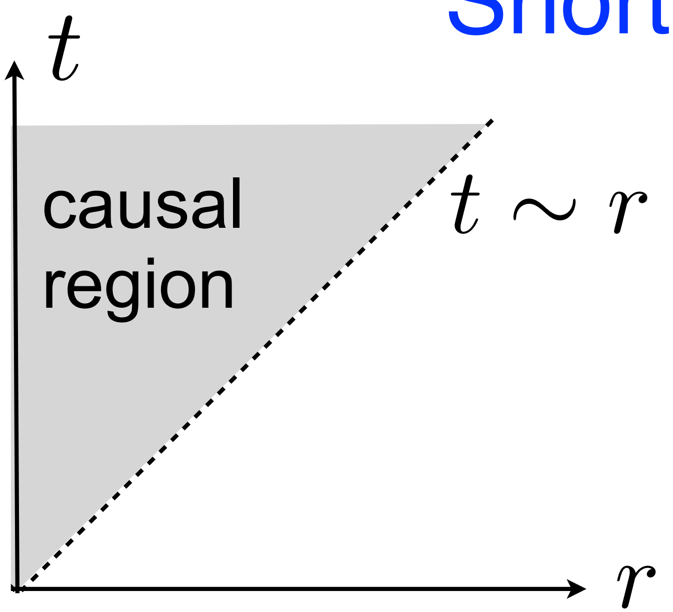
short-range Lieb-Robinson bound

- signal after time t distance r away: $Q(r,t) \lesssim e^{vt-r} = \epsilon$

$v \sim 1$

E. Lieb & D. Robinson, 1972

Short-range interactions



- shortest time t to send quantum info over distance r is $t \gtrsim r$

Applications:

- quantum communication through spin chains
- entanglement growth after quenches or under other (possibly time-dependent) unitary dynamics
- speed of quantum computers
- thermalization rates
- entanglement and correlations in gapped ground states

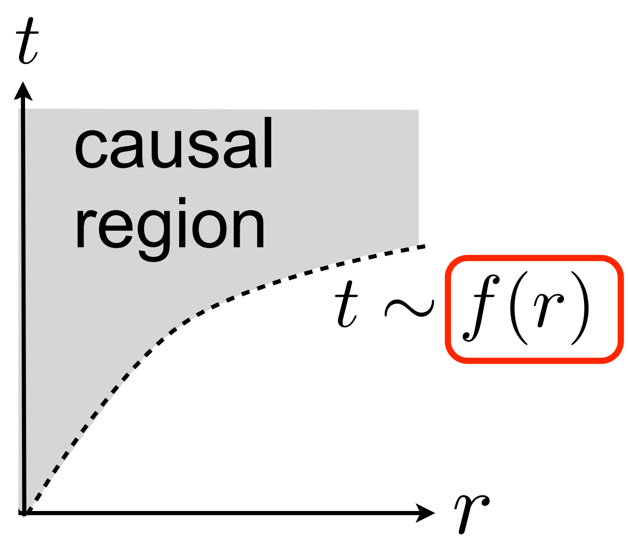
E. Lieb & D. Robinson, 1972

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

Interested in:

- shape of causal region (or "light cone")

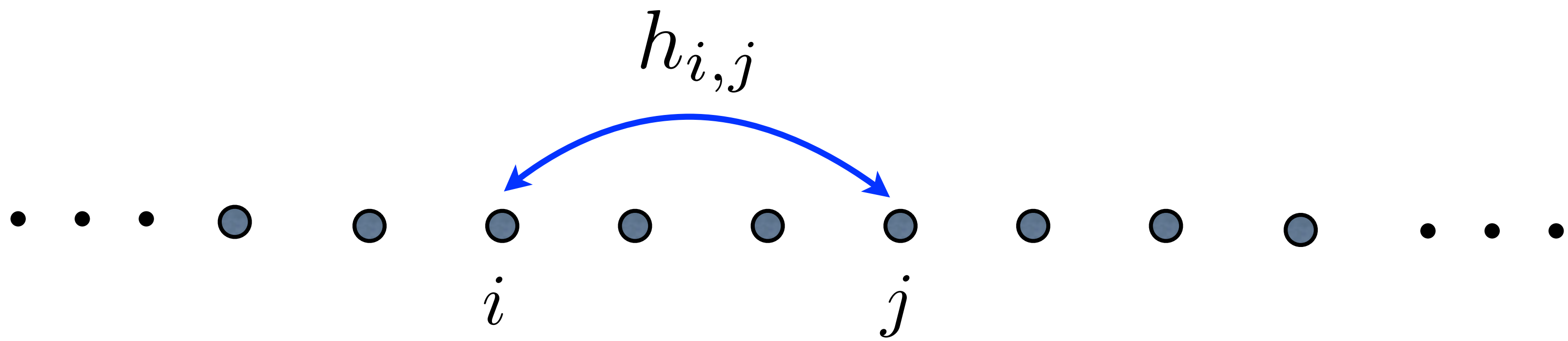
  $\rightarrow$ shortest time t to send quantum info over distance r is $t \gtrsim f(r)$



First theoretical work: Hazzard et al, 2013; Hauke & Tagliacozzo, 2013; Schachenmayer et al, 2013; Knap et al, 2013; Juenemann et al, 2013; Eisert et al, 2013; Hazzard et al, 2014; Storch et al, 2015; Rajabpour et al, 2014, 2015, …

First experiments: Richerme et al, Nature 2014; Jurcevic et al, Nature 2014

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions



$$H = \sum_{i<j} h_{i,j}$$

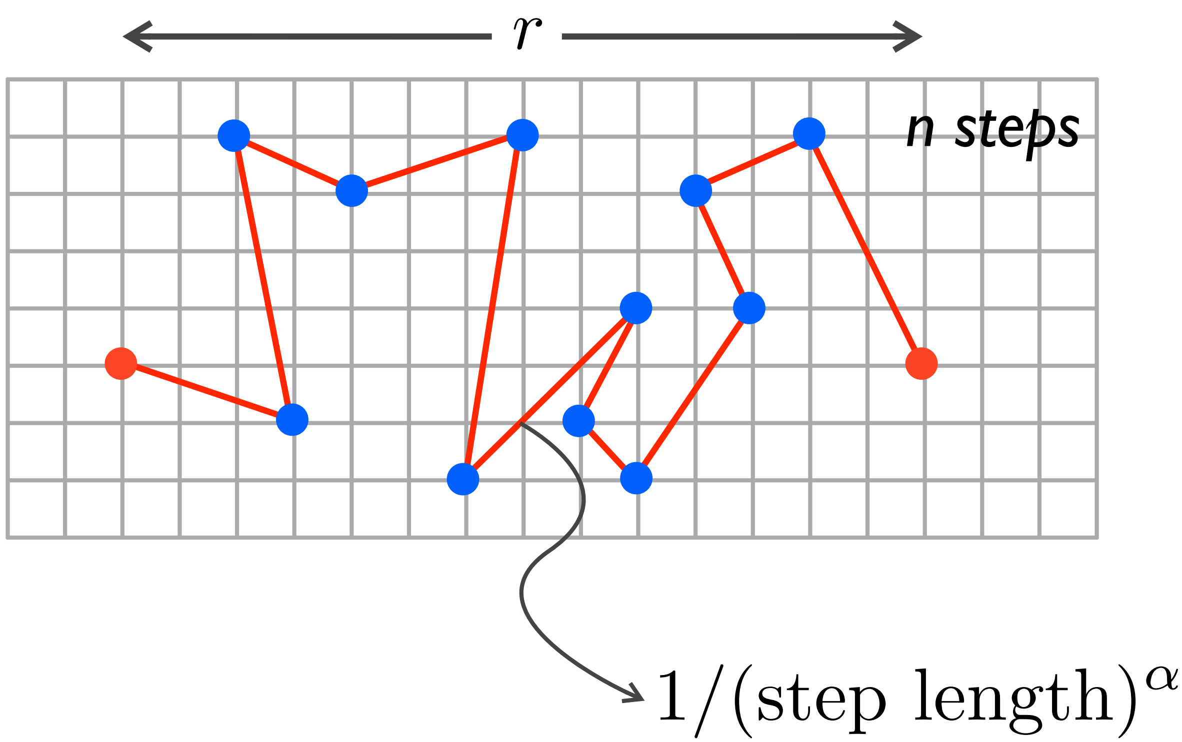
$$||h_{i,j}|| \leq \frac{1}{|i-j|^{\alpha}}$$

- arbitrary time dependence allowed
- arbitrary time-dependent on-site terms allowed
- consider all $\alpha \geq 0$
  (can include Kac normalization at the end if desired)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

$$Q(r,t) \leq \mathcal{J}_1(r)t + \mathcal{J}_2(r)\frac{t^2}{2!} + \mathcal{J}_3(r)\frac{t^3}{3!} + \ldots$$

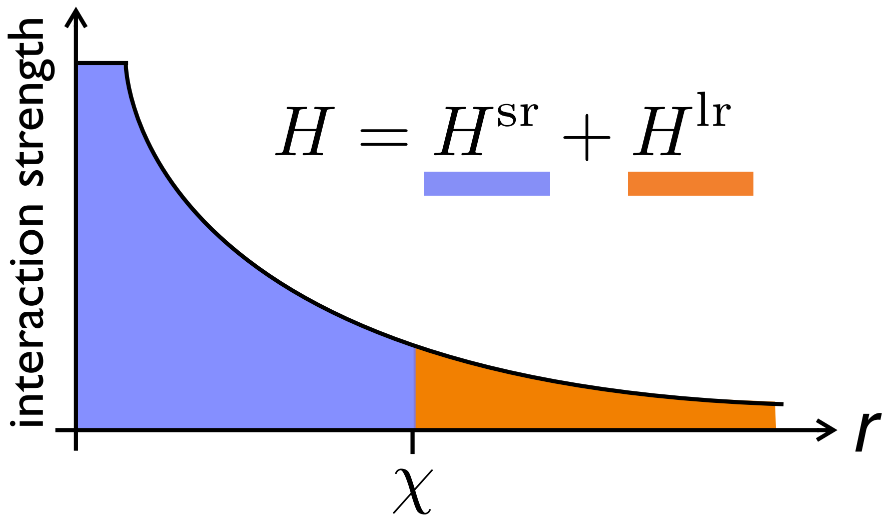
$$\mathcal{J}_n(r) = \sum$$



Hastings, Koma, Commun. Math. Phys. 265, 781 (2006)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

Additional trick #1:



- Hastings-Koma series bad at treating short-range physics
- work in interaction picture of $H^{\rm sr}$
- choose optimal χ at the end

$\|[A(t), B]\|$

$$A(t) = U_I^\dagger(t) A_I(t) U_I(t)$$

$$A_I(t) = e^{iH^{\rm sr}t} A e^{-iH^{\rm sr}t}$$

$$U_I(t) = \mathcal{T}\left(e^{-i\int_0^t d\tau H_I^{\rm lr}(\tau)}\right)$$

Foss-Feig, Gong, Clark, AVG, PRL 114, 157201 (2015)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

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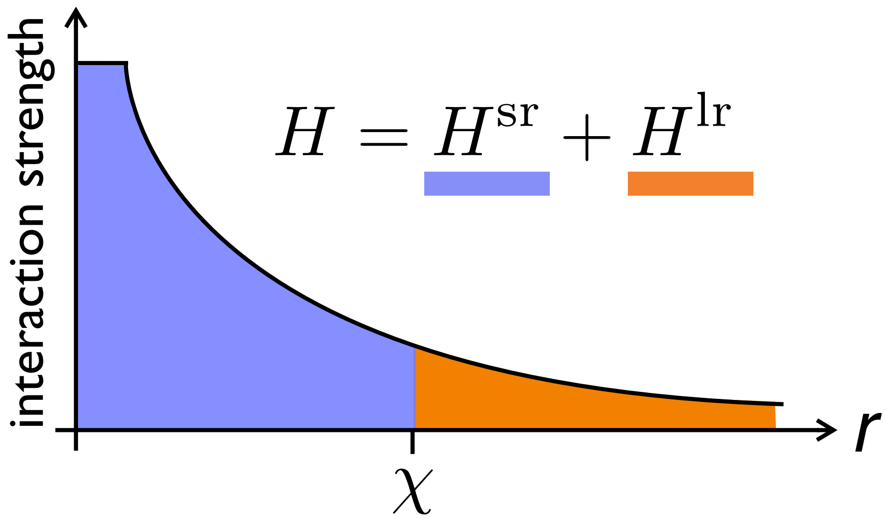
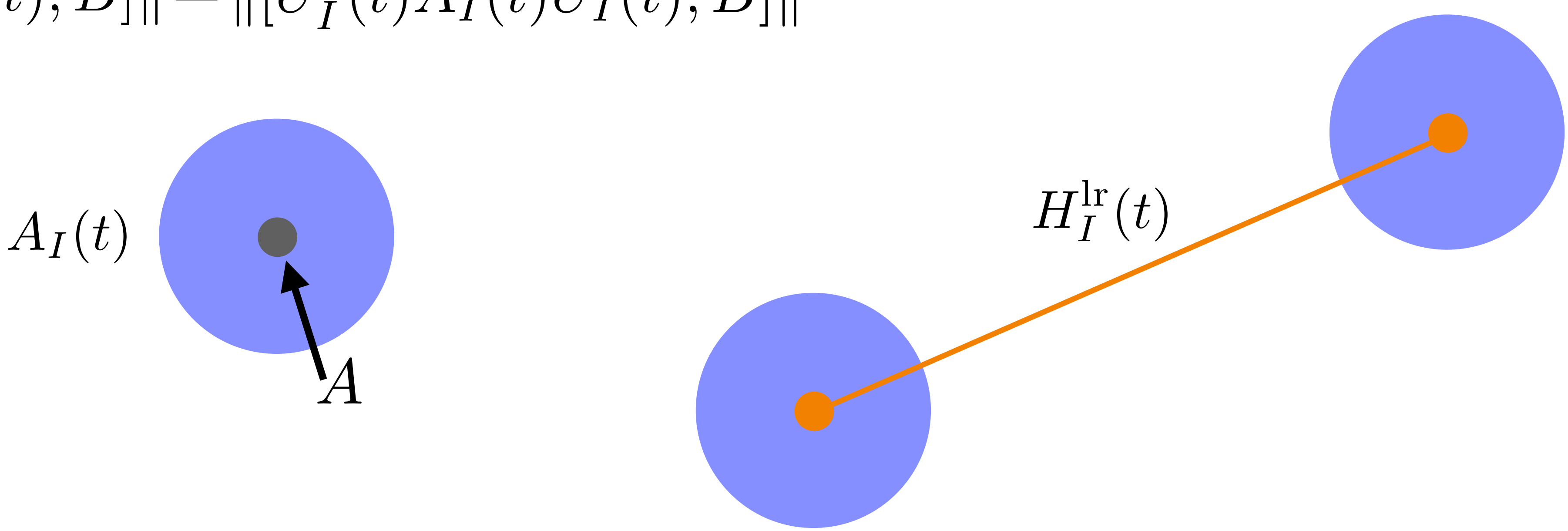


- Hastings-Koma series bad at treating short-range physics
- work in interaction picture of H^{sr}
- choose optimal χ at the end

$$\|[A(t), B]\| = \|[U_I^\dagger(t) A_I(t) U_I(t), B]\|$$



Foss-Feig, Gong, Clark, AVG, PRL 114, 157201 (2015)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

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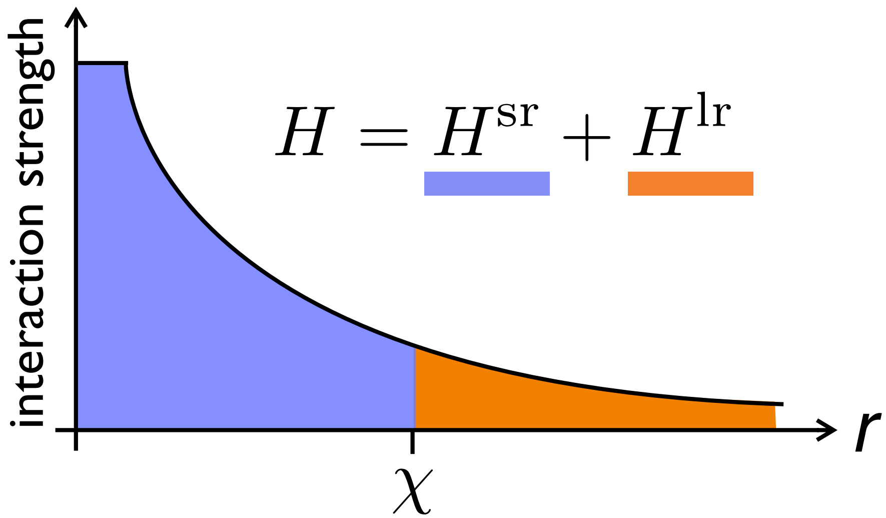
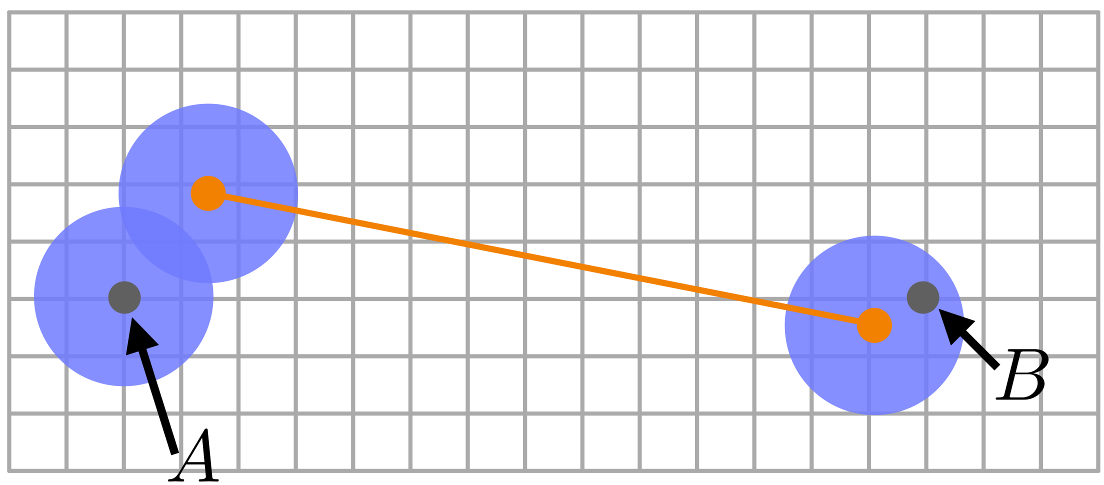


- Hastings-Koma series bad at treating short-range physics
- work in interaction picture of H^{sr}
- choose optimal χ at the end

$$|||[A(t), B]||| \equiv Q(r,t) \leq \mathcal{J}_1(r)t + \mathcal{J}_2(r)\frac{t^2}{2!} + \mathcal{J}_3(r)\frac{t^3}{3!} + \dots$$

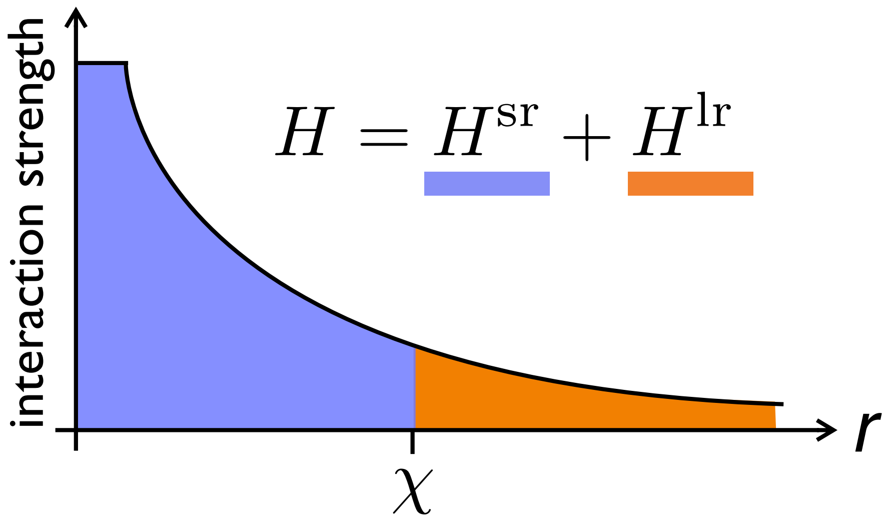
$\mathcal{J}_1(r) = \sum$



Foss-Feig, Gong, Clark, AVG, PRL 114, 157201 (2015)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

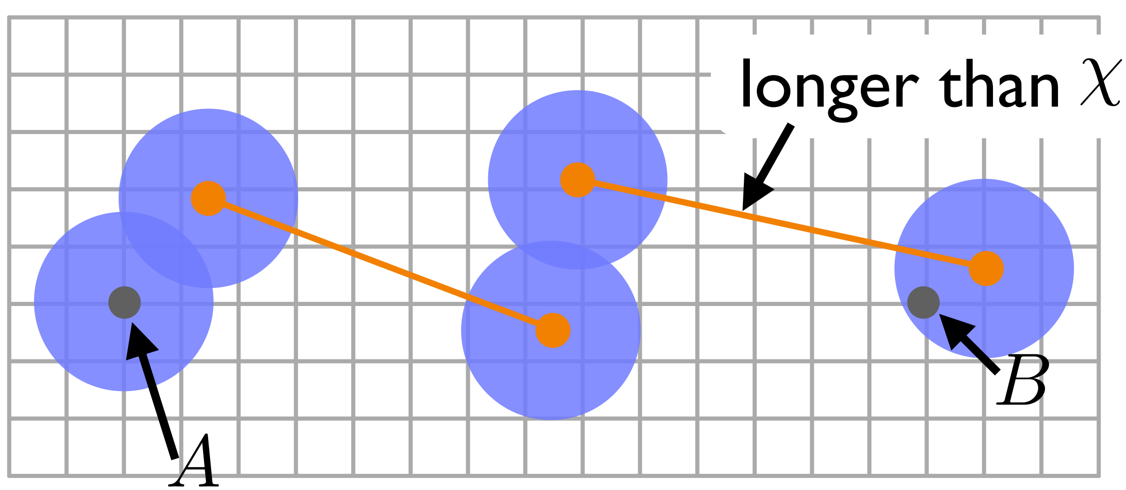
Additional trick #1:



- Hastings-Koma series bad at treating short-range interactions
- work in interaction picture of H^{sr}
- choose optimal χ at the end

$$|||[A(t), B]||| \equiv Q(r,t) \leq \mathcal{J}_1(r)t + \mathcal{J}_2(r)\frac{t^2}{2!} + \mathcal{J}_3(r)\frac{t^3}{3!} + \ldots$$

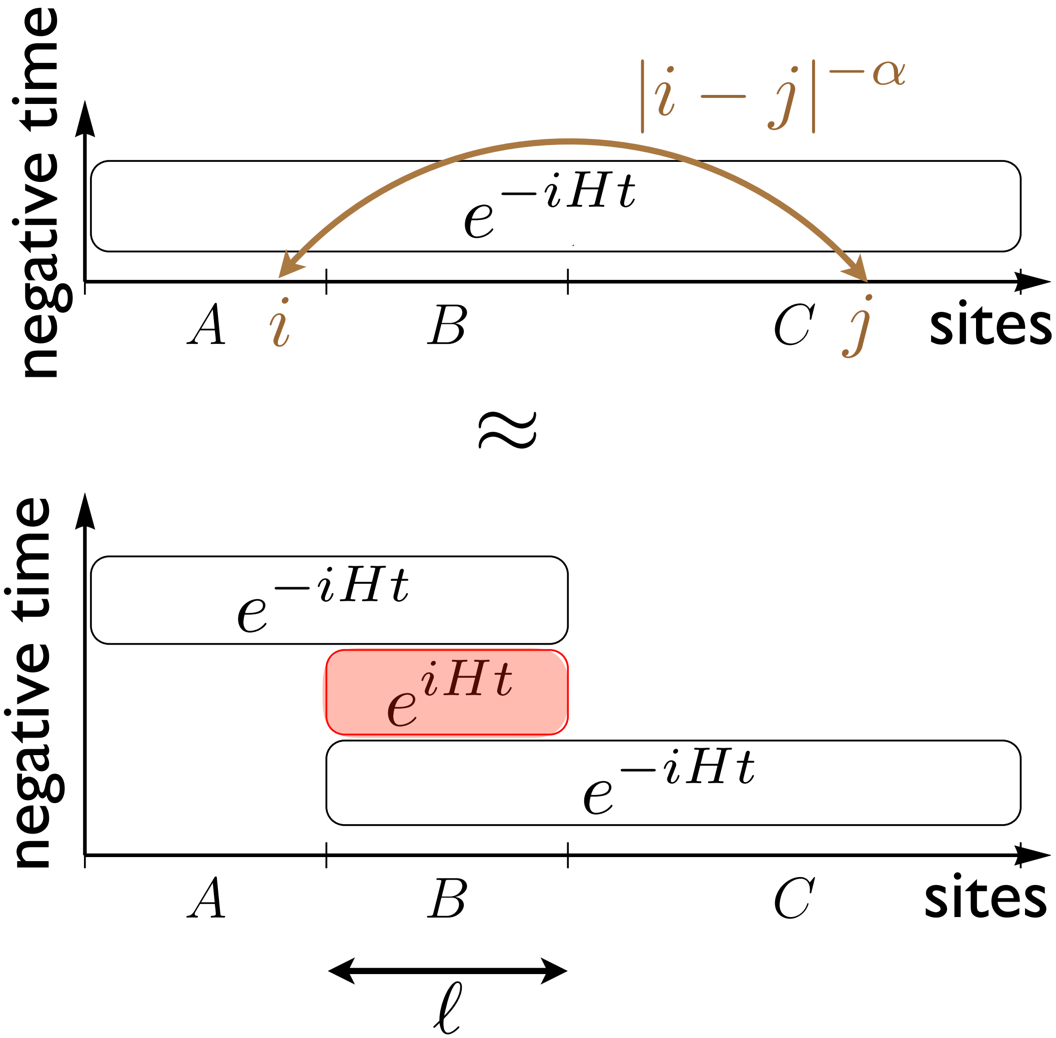
$\mathcal{J}_2(r) = \sum$



Foss-Feig, Gong, Clark, AVG, PRL 114, 157201 (2015)

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

Additional trick #2:



Two errors:

- ignore $A-C$ interactions
- Hamiltonians on AB, B, BC don't commute

Both vanish as $\ell \to \infty$

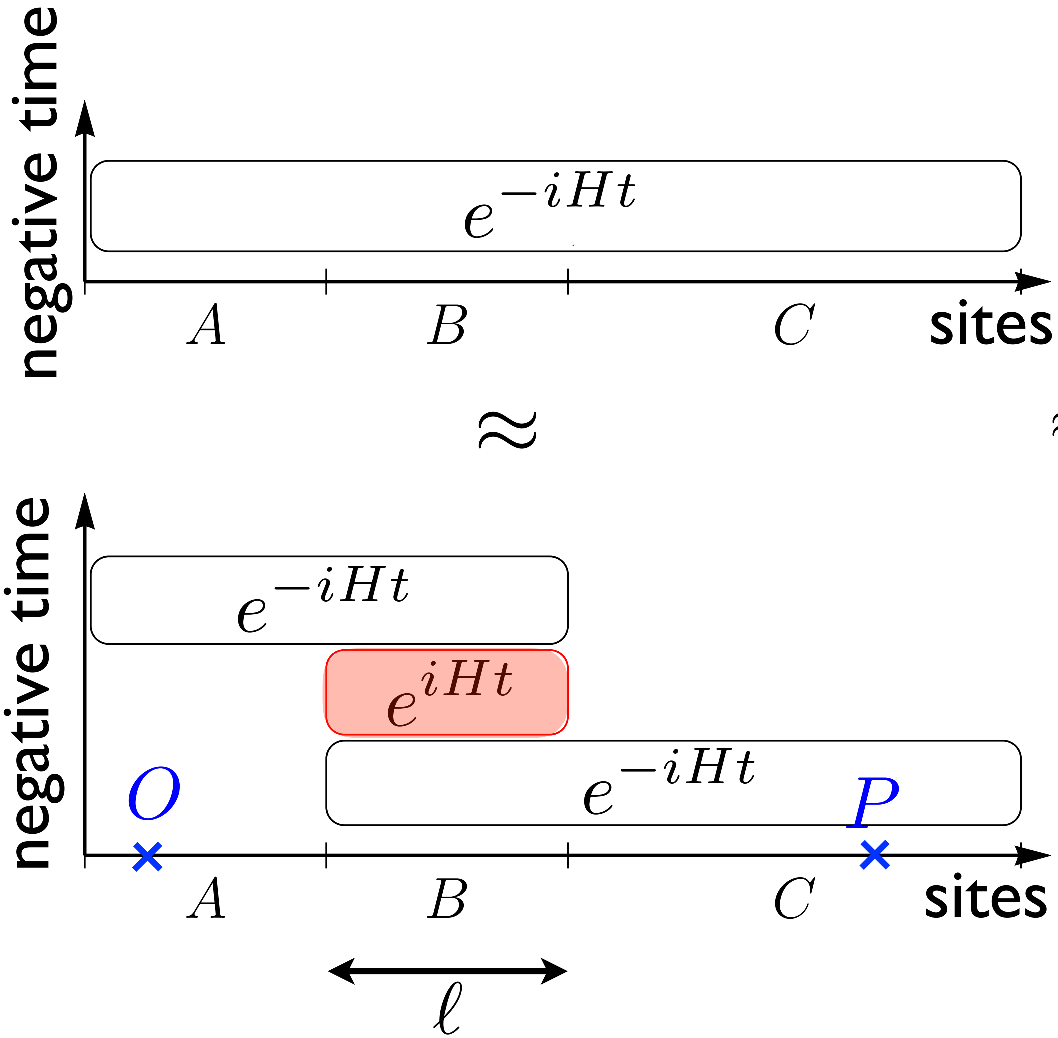
For large ℓ, same scaling with ℓ

error for $t \sim 1$ $\lesssim \sum_{i \in A} \sum_{j \in C} \frac{1}{|i-j|^{\alpha}} \sim \frac{1}{\ell^{\alpha-2}}$

Tran, Guo, Su, Garrison, Eldredge, Foss-Feig, AVG, Childs, arXiv:1808.05225
[Based on Haah, Hastings, Kothari, Low, arXiv:1801.03922]

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

Additional trick #2:



$$O(t) = (U^{ABC})^\dagger \, O \, U^{ABC}$$

$$\approx (U^{AB})^\dagger \, U^B \, (U^{BC})^\dagger \, O \, U^{BC} \, (U^B)^\dagger \, U^{AB}$$

$$= (U^{AB})^\dagger \, O \, U^{AB}$$

has no support on C!

$$||[O(t), P]|| \approx 0$$

error for $t \sim 1$ $\sim \frac{1}{\ell^{\alpha-2}}$

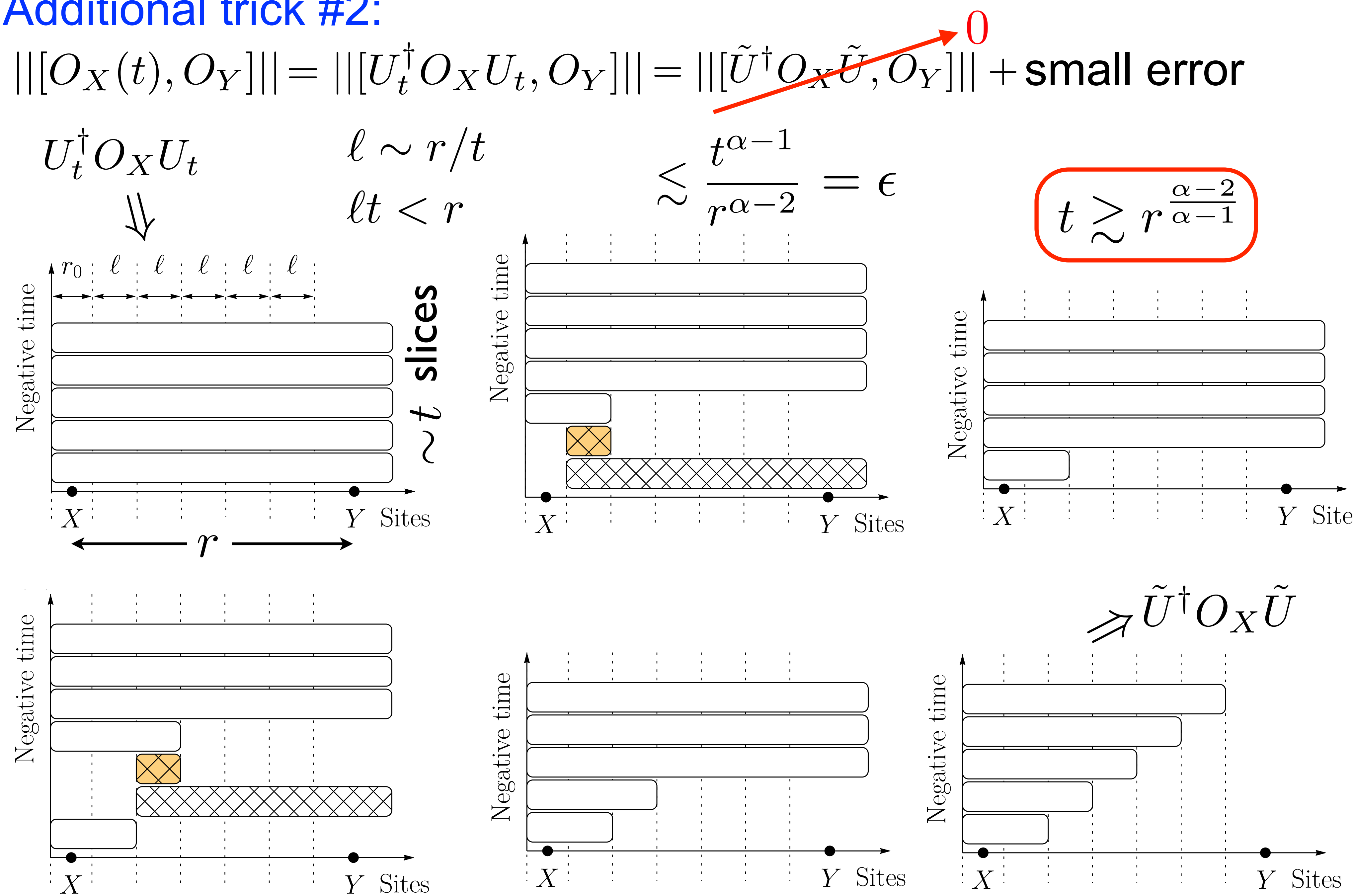
Tran, Guo, Su, Garrison, Eldredge, Foss-Feig, AVG, Childs, arXiv:1808.05225

[Based on Haah, Hastings, Kothari, Low, arXiv:1801.03922]

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

Additional trick #2:

$$\|[O_X(t), O_Y]\| = \|[U_t^\dagger O_X U_t, O_Y]\| = \|[\tilde{U}^\dagger O_X \tilde{U}, O_Y]\| + \text{small error}$$



Tran, Guo, Su, Garrison, Eldredge, Foss-Feig, AVG, Childs, arXiv:1808.05225

Lieb-Robinson-type bounds for $r^{-\alpha}$ interactions

~ "shortest time t to send quantum info over distance r"

D = dimension

N = total number of sites

(formulas shown for $N \sim r^D$)

$\alpha = 0$ — D — $2D$ — $\alpha = \infty$

$\alpha = 0$: $t \gtrsim \frac{\log N}{N}$

Lashakari et al, JHEP (2013)

$t \gtrsim \frac{\log N}{N^{1-\alpha/D}}$

Guo et al, in prep [improved over Storch et al (2015)]

D: $t \gtrsim 1$

Guo et al, in prep

$t \gtrsim \log r$

Hastings, Koma (2006)

$2D$: $t \gtrsim r^{\frac{\alpha-2D}{\alpha-D}}$

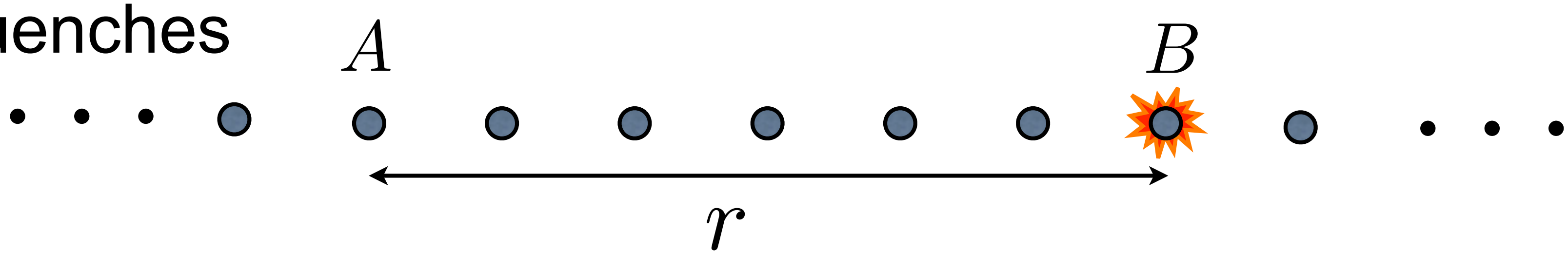
Tran et al, arXiv:1808.05225 & Foss-Feig, Gong, Clark, AVG, PRL (2015)

$\alpha = \infty$: $t \gtrsim r$

Lieb, Robinson (1972)

Applications

- local quenches



$$|\langle\psi|B^\dagger A(t) B|\psi\rangle - \langle\psi|A(t)|\psi\rangle|$$

Gong, Foss-Feig, Michalakis, AVG, PRL 113, 030602 (2014)

- growth of connected correlations after a global quench

$$\langle A(t)B(t)\rangle - \langle A(t)\rangle\langle B(t)\rangle$$

Gong, Foss-Feig, Michalakis, AVG, arXiv:1401.6174v1

- correlations in gapped ground states fall off no slower than $\dfrac{1}{r^\alpha}$

Foss-Feig, Gong, Clark, AVG, PRL 114, 157201 (2015)

- entanglement area laws for dynamics & gapped ground states

Gong, Foss-Feig, Brandão, AVG, PRL 119, 050501 (2017)

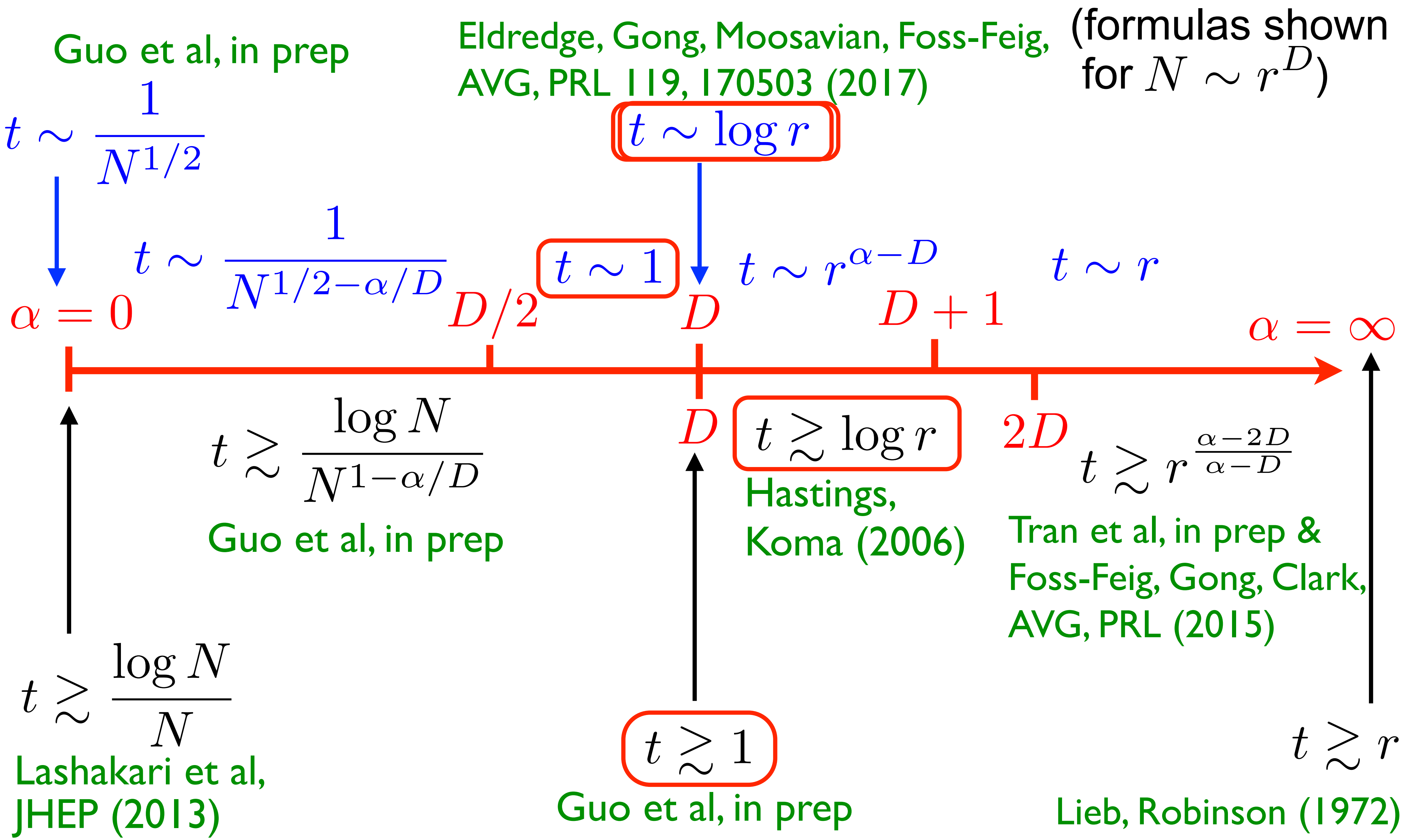
- more gate-efficient quantum simulation protocols

Tran, Guo, Su, Garrison, Eldredge, Foss-Feig, AVG, Childs, arXiv:1808.05225

Fastest known protocols

Shortest time t to send quantum info over distance r

$1/r^\alpha$ interactions in D dimensions N = total number of sites



State transfer over distance L in time $T \sim \log L$ using $1/r^D$ in D dimensions

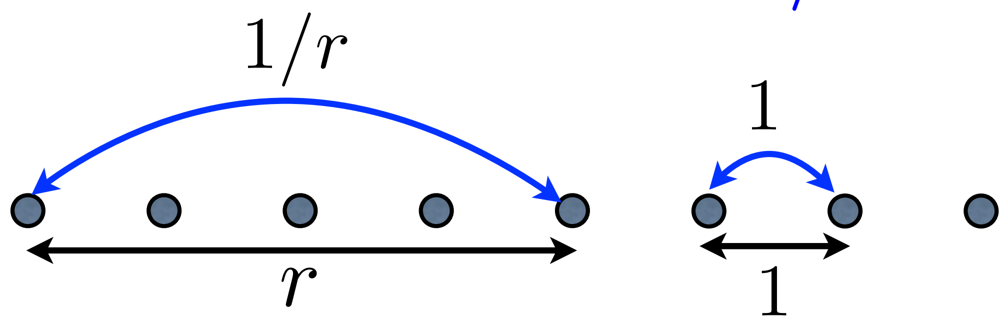
(e.g. $1/r^3$ interactions between dipoles in D = 3 dimensions)

- speed up quantum computing algorithms
- fast preparation of a wide range entangled states (e.g. prepare MERA [e.g. Haah & toric codes] in $T \sim \log^2 L$)

First show how to create GHZ state $|0\ldots0\rangle + |1\ldots1\rangle$ of linear size L in time $T \sim \log L$

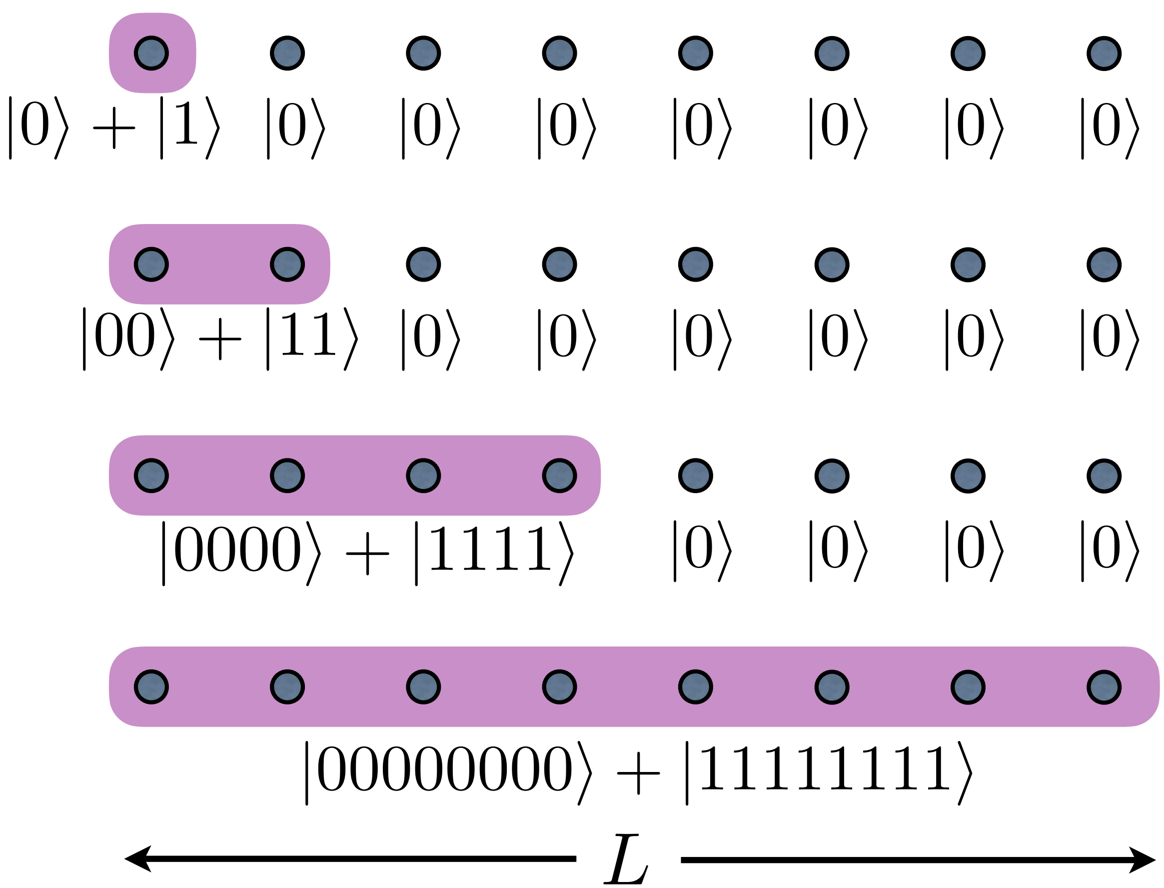
Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

1D with $1/r$ interactions



(realized in trapped ions)

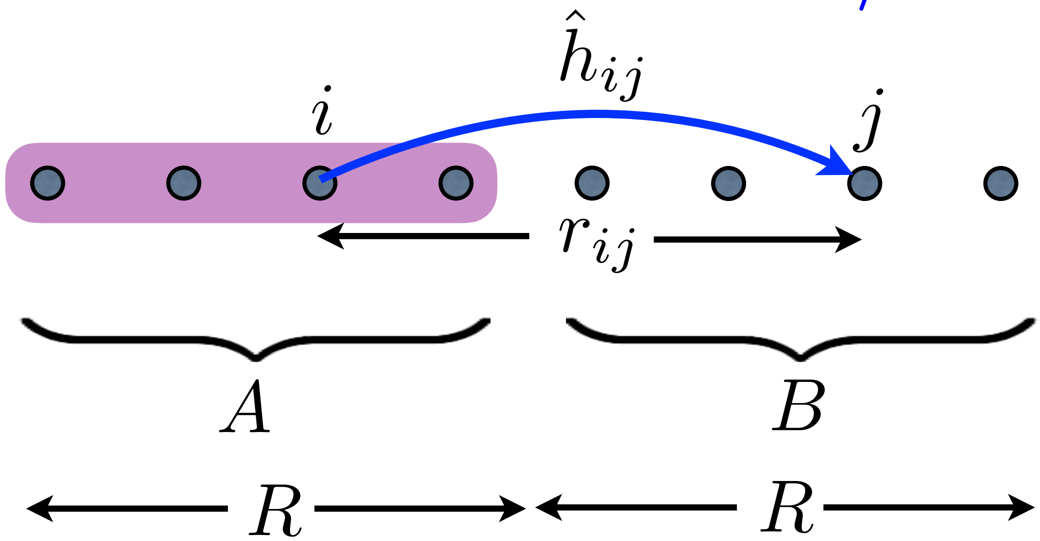
- use individual addressing to turn individual interactions on and off



- # of doubling steps $\sim \log L$
- remains to show that each doubling step takes constant time

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

1D with $1/r$ interactions



Need: controlled-NOT with **any** qubit in A as control & **every** qubit in B as target.

$$(|0\rangle + |1\rangle)_A |0\rangle_B \to |00\rangle + |11\rangle$$

$\hat{I}$ = identity

$$\hat{h}_{ij} = r_{ij}^{-1}(|0\rangle\langle 0|_i \otimes \hat{I}_j + |1\rangle\langle 1|_i \otimes \hat{X}_j)$$

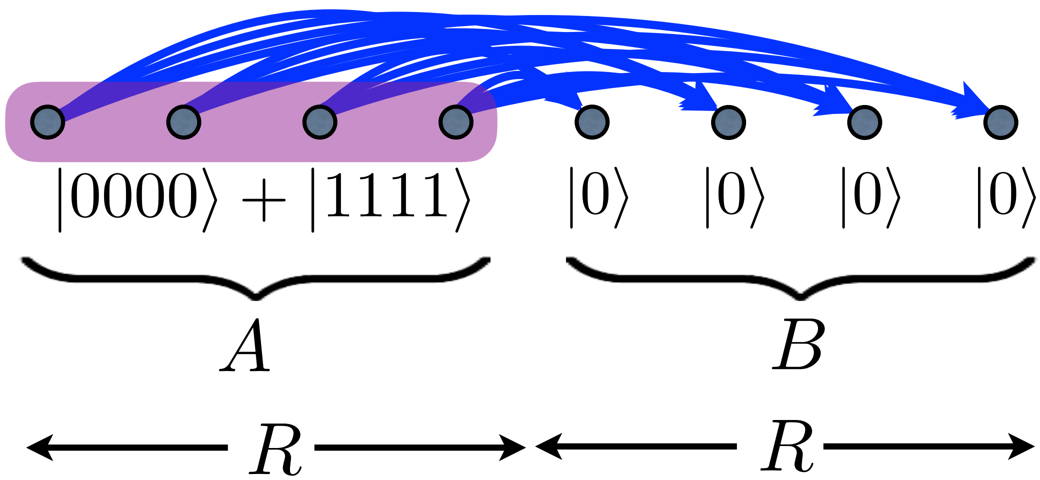
controlled (by qubit i) X rotation of qubit j

$t = \pi r_{ij}/2 \;\Rightarrow$ controlled-NOT

$$\hat{H} = \sum_{i\in A, j\in B} \hat{h}_{ij}$$ (all commute)

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

1D with $1/r$ interactions



Need: controlled-NOT with **any** qubit in A as control & **every** qubit in B as target.

$(|0\rangle + |1\rangle)_A|0\rangle_B \rightarrow |00\rangle + |11\rangle$

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controlled (by qubit i) X rotation of qubit j

$t = \pi r_{ij}/2 \;\Rightarrow$ controlled-NOT

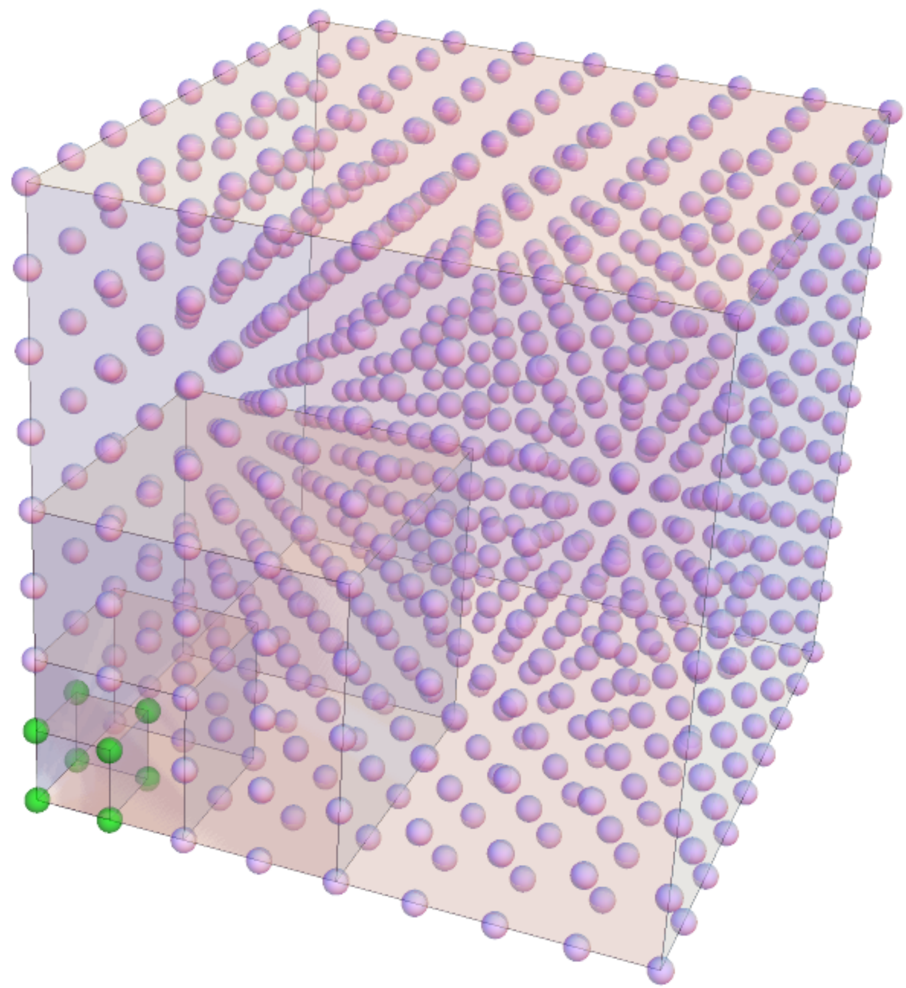
$$\hat{H} = \sum_{i\in A, j\in B} \hat{h}_{ij}$$ (all commute)

$$\text{rotation rate} > (\text{number of controls}) \times (\text{weakest coupling})$$
$$= R \times \frac{1}{2R} = \frac{1}{2}$$

time to double independent of R

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

3D with $1/r^3$ interactions



Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

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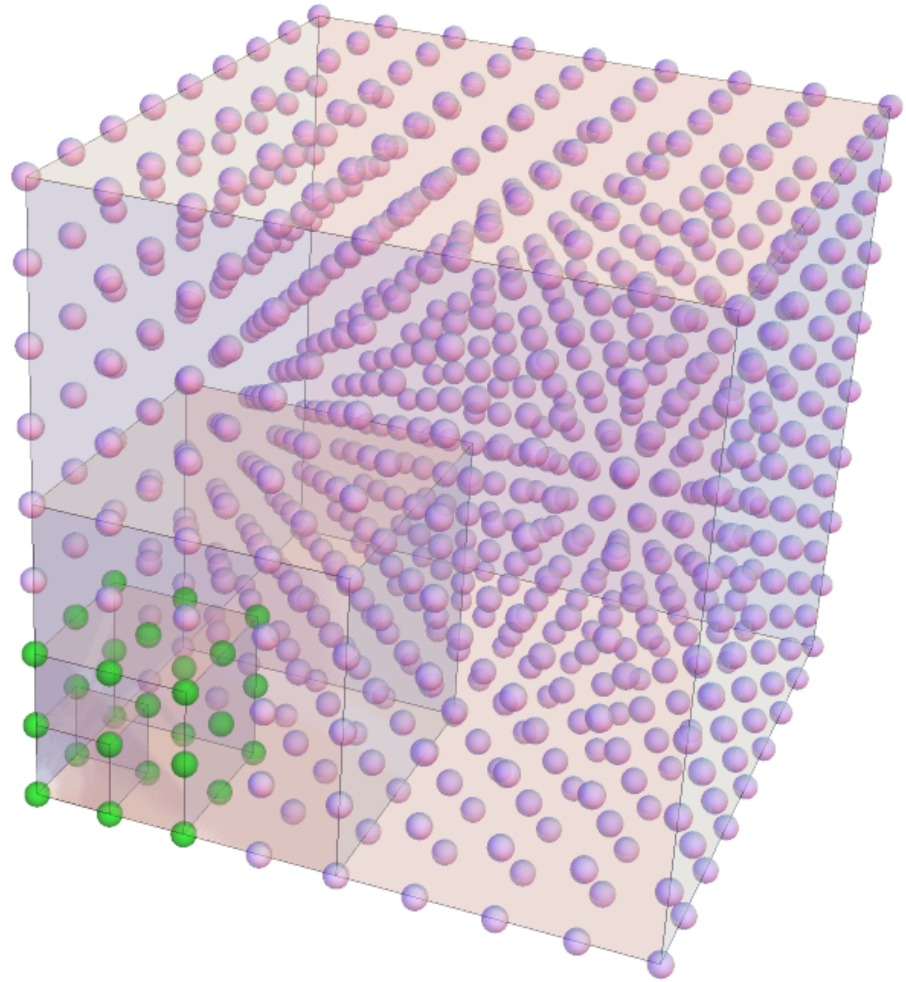

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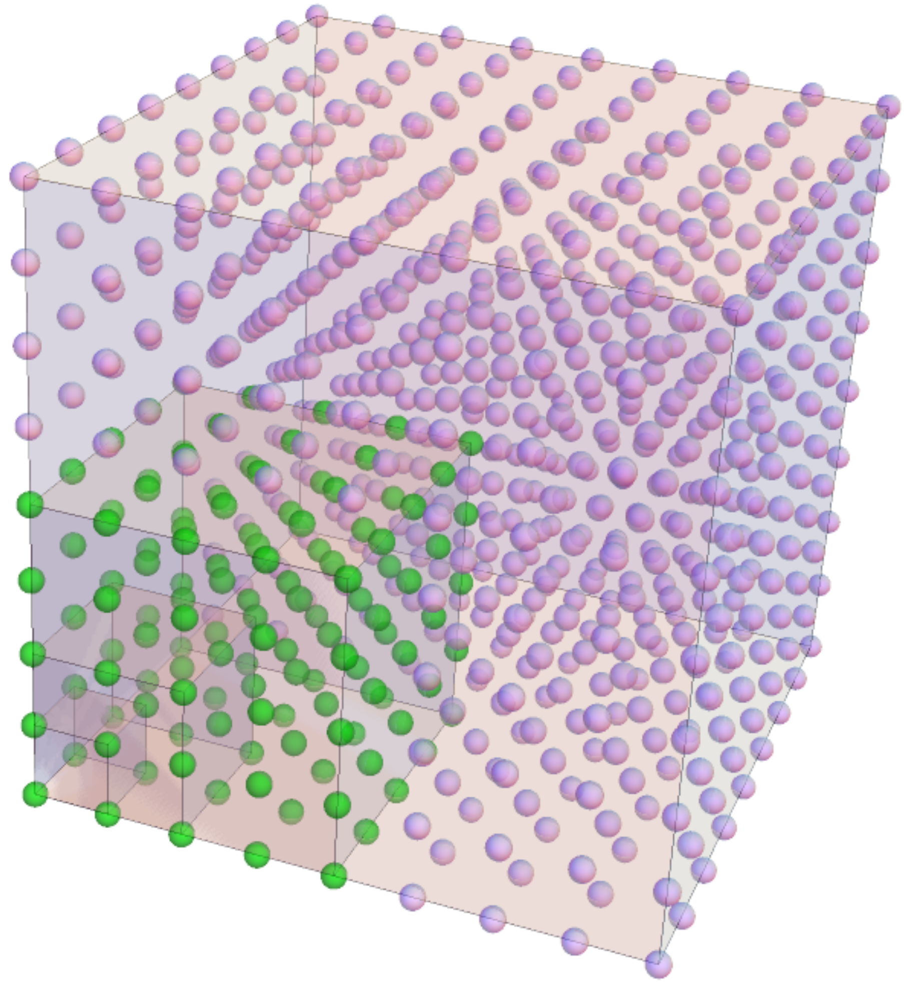

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

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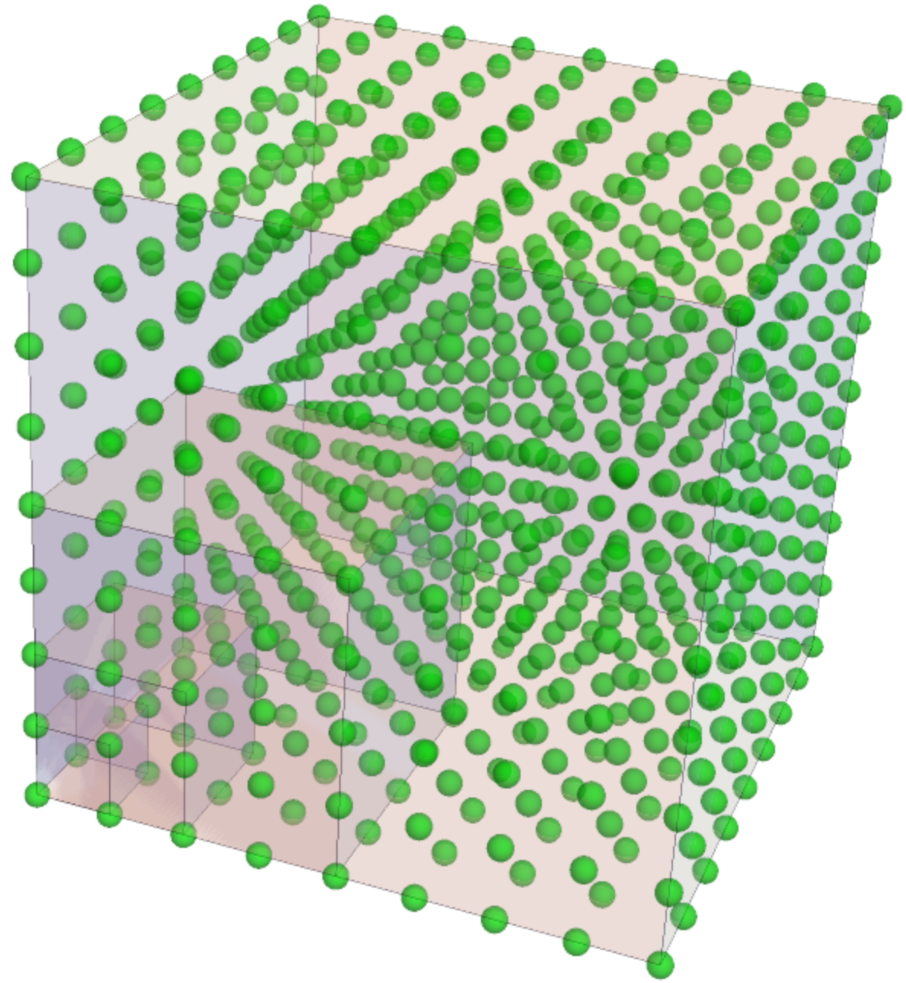
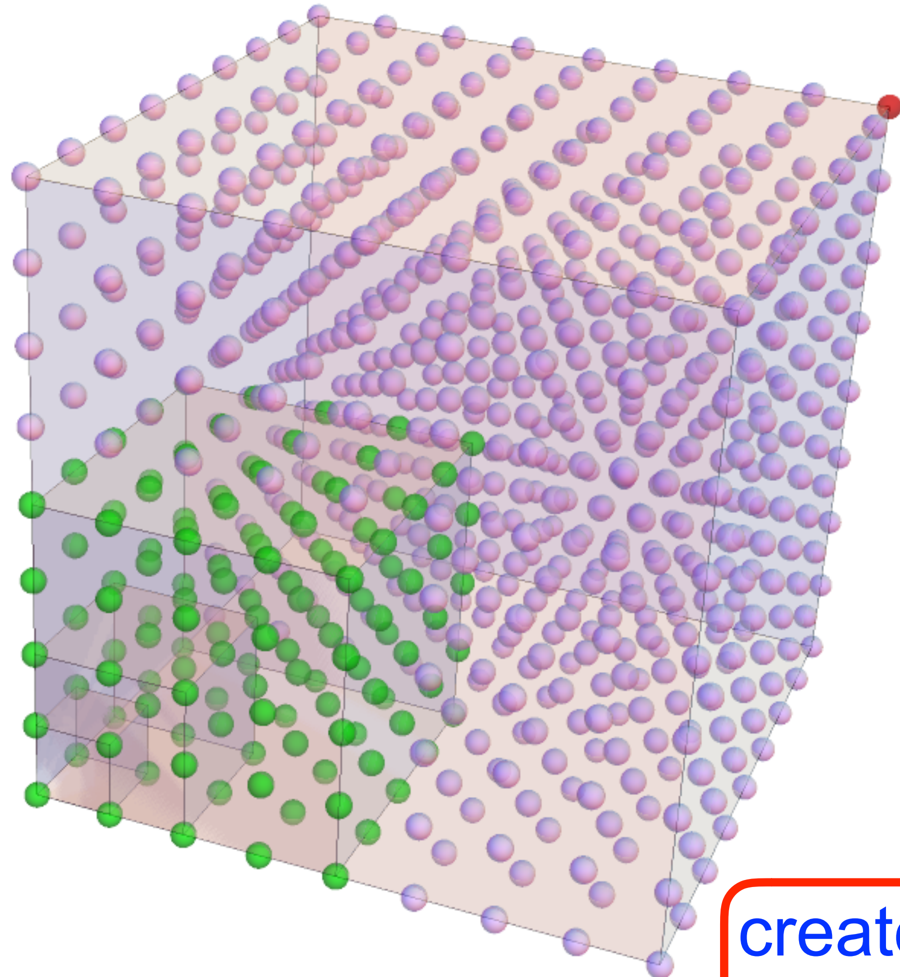


Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

3D with $1/r^3$ interactions



create $|0\ldots0\rangle + |1\ldots1\rangle$
in time $T \sim \log L$

$$\text{rotation rate} > (\text{number of controls}) \times (\text{weakest coupling})$$

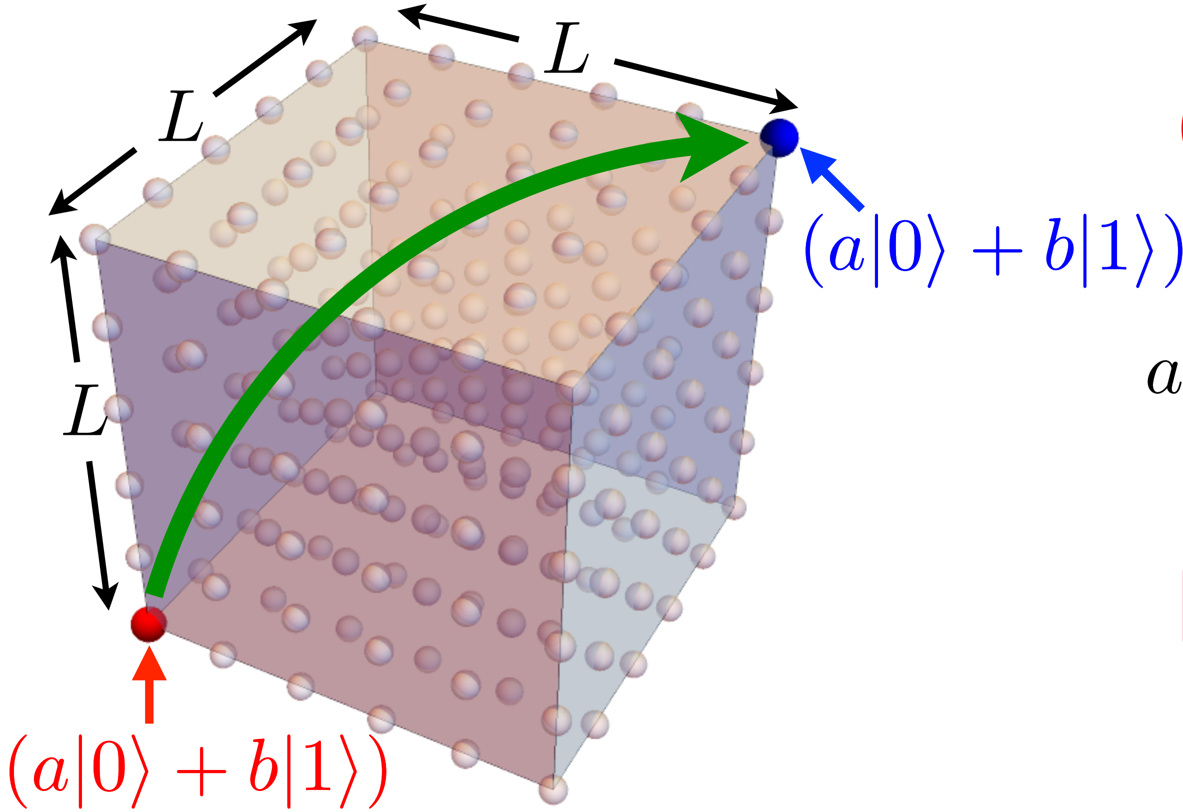
$$\sim R^3 \times \frac{1}{R^3} \sim 1$$

time to double independent of R

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

State transfer over distance L in time $T \sim \log L$

3D with $1/r^3$ interactions



$$(a|0\rangle + b|1\rangle)|0\ldots 0\rangle|0\rangle$$

$$a|00\ldots 00\rangle + b|11\ldots 11\rangle$$

$$|0\rangle|0\ldots 0\rangle(a|0\rangle + b|1\rangle)$$

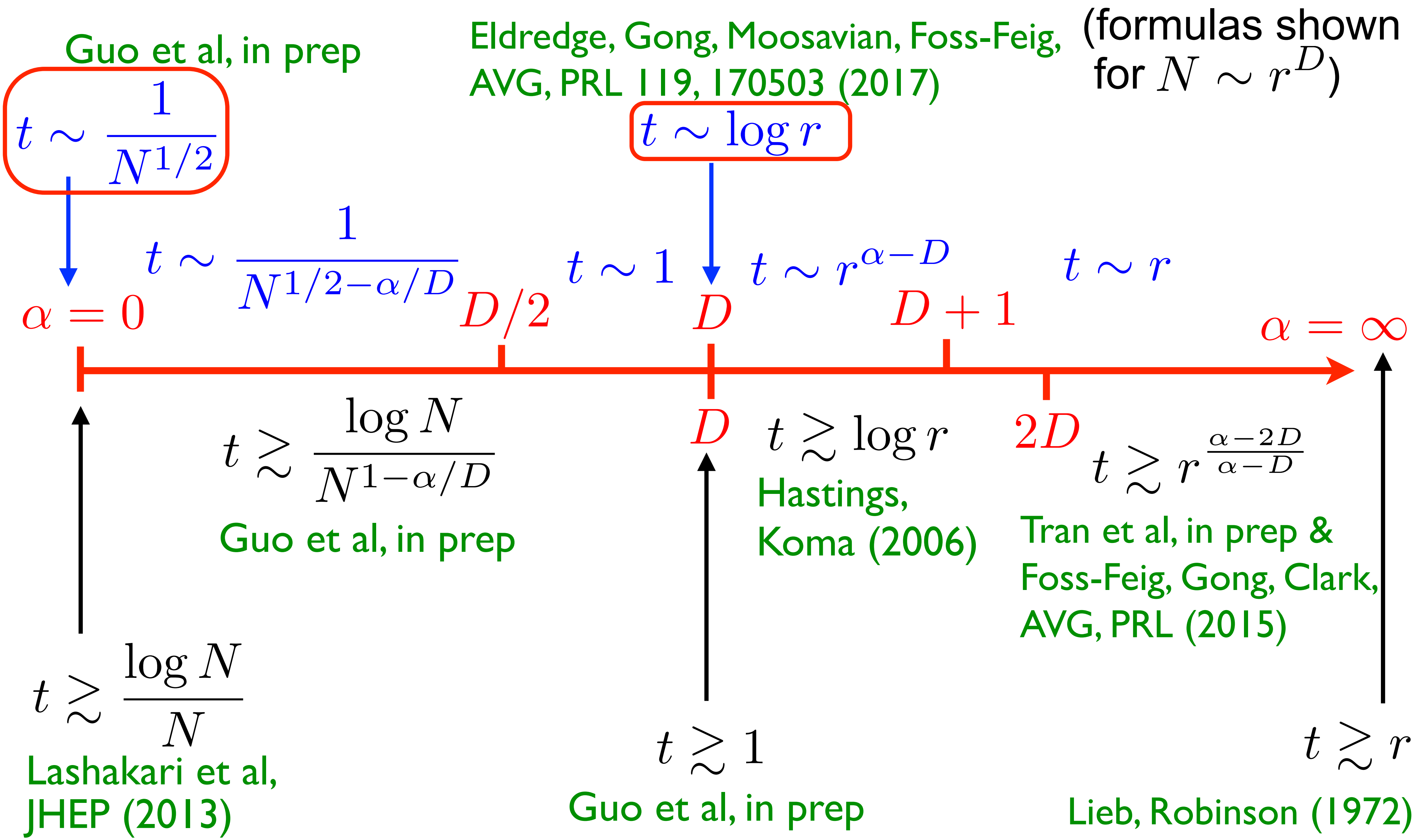
$$T \sim \log L$$

Eldredge, Gong, Young, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

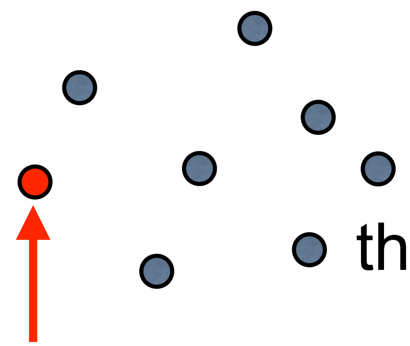
Fastest known protocols

Shortest time t to send quantum info over distance r

$1/r^\alpha$ interactions in D dimensions $\quad N$ = total number of sites



All-to-all case: state transfer in time $t \sim \frac{1}{\sqrt{N}}$



$(a|0\rangle + b|1\rangle)$

the rest in state $|0\rangle$

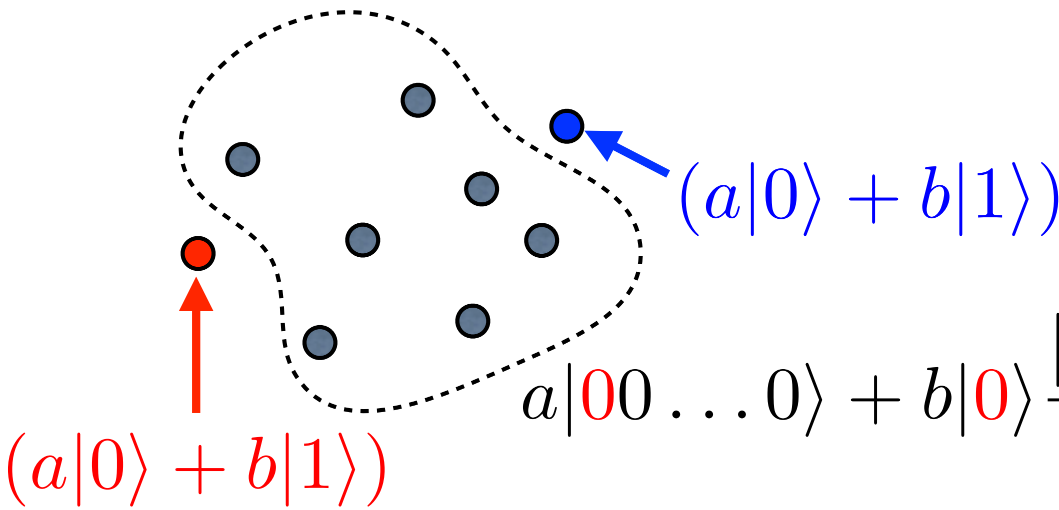
$$H = \sigma_0^- (\sigma_1^+ + \cdots + \sigma_N^+) + \text{h.c.}$$

- $|00\ldots0\rangle$ unchanged
- $|10\ldots0\rangle$ and $|0\rangle \frac{|1\ldots0\rangle + \cdots + |0\ldots1\rangle}{\sqrt{N}}$

form a closed system and are coupled by $\sim \sqrt{N}$

so pi-pulse takes $t \sim \frac{1}{\sqrt{N}}$

All-to-all case: state transfer in time $t \sim \frac{1}{\sqrt{N}}$



$$a|00\ldots0\rangle + b|0\rangle \frac{|1\ldots0\rangle + \cdots + |0\ldots1\rangle}{\sqrt{N}}$$

$$H = \sigma_0^- (\sigma_1^+ + \cdots + \sigma_N^+) + \text{h.c.}$$

- $|00\ldots0\rangle$ unchanged
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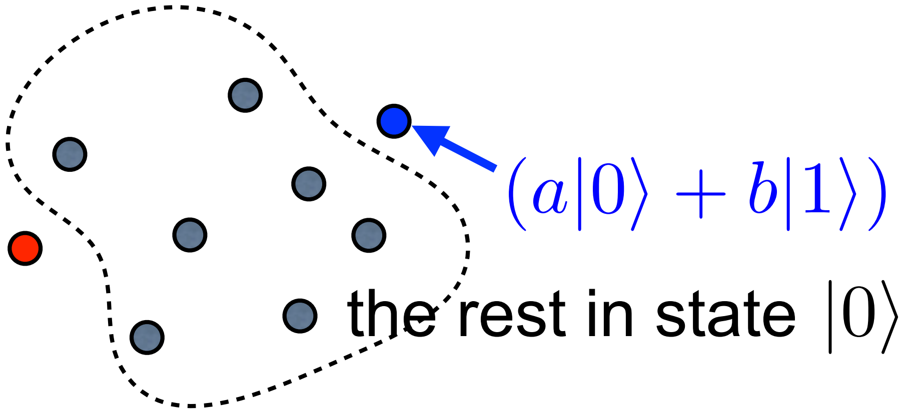
All-to-all case: state transfer in time $t \sim \frac{1}{\sqrt{N}}$



- cannot go faster within 1-excitation subspace
- known general bound:

$$t \gtrsim \frac{\log N}{N}$$

$$H = \sigma_0^- (\sigma_1^+ + \cdots + \sigma_N^+) + \text{h.c.}$$

- $|00\ldots0\rangle$ unchanged
- $|10\ldots0\rangle$ and $|0\rangle \frac{|1\ldots0\rangle + \cdots + |0\ldots1\rangle}{\sqrt{N}}$

form a closed system and are coupled by $\sim \sqrt{N}$

so pi-pulse takes $t \sim \frac{1}{\sqrt{N}}$

Outlook

• tighten both the bounds and the protocols to saturation

• improve understanding of equilibrium and non-equilibrium properties of long-range-interacting many-body systems

• speed up & bound quantum computing, quantum simulation, classical simulation, preparation of entangled states for metrology, etc…

Thank you

Graduate Students

Jeremy Young
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Andrew Guo
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$$$: NSF QIS, NSF Ideas Lab, ARO MURI, ARO, AFOSR, NSF PFC@JQI, ARL CDQI, DoE ASCR Quantum Testbed Pathfinder

Thank you

Minh
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PRL 113, 030602 (2014); PRL 114, 157201 (2015); PRL 119, 050501 (2017)
PRL 119, 170503 (2017); arXiv:1808.05225; Guo et al, in prep.

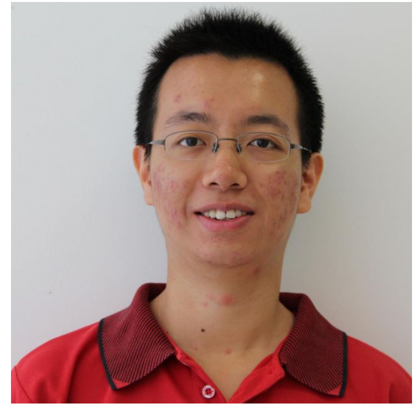
$$$: NSF QIS, NSF Ideas Lab, ARO MURI, ARO, AFOSR, NSF PFC@JQI, ARL CDQI, DoE ASCR Quantum Testbed Pathfinder

Thank you

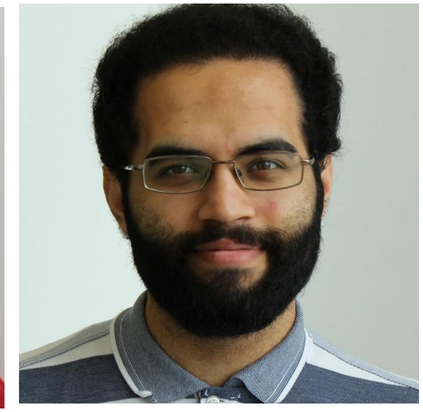
Jeremy Young

Jim Garrison

Andrew Childs



Yuan Su



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PRL 113, 030602 (2014); PRL 114, 157201 (2015); PRL 119, 050501 (2017)
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$$$: NSF QIS, NSF Ideas Lab, ARO MURI, ARO, AFOSR, NSF PFC@JQI, ARL CDQI, DoE ASCR Quantum Testbed Pathfinder

Conclusions

Shortest time t to send quantum info over distance r

$1/r^\alpha$ interactions in D dimensions N = total number of sites

(formulas shown for $N \sim r^D$)

Thank You

Guo et al, in prep

$t \sim \frac{1}{N^{1/2}}$

Eldredge, Gong, Moosavian, Foss-Feig, AVG, PRL 119, 170503 (2017)

$t \sim \log r$

$t \sim \frac{1}{N^{1/2-\alpha/D}}$ $t \sim 1$ $t \sim r^{\alpha - D}$ $t \sim r$

$\alpha = 0$ $D/2$ D $D+1$ $\alpha = \infty$

D $2D$

$t \gtrsim \frac{\log N}{N^{1-\alpha/D}}$

Guo et al, in prep

$t \gtrsim \log r$

Hastings, Koma (2006)

$t \gtrsim r^{\frac{\alpha - 2D}{\alpha - D}}$

Tran et al, arXiv:1808.05225 & Foss-Feig, Gong, Clark, AVG, PRL (2015)

$t \gtrsim \frac{\log N}{N}$

Lashakari et al, JHEP (2013)

$t \gtrsim 1$

Guo et al, in prep

$t \gtrsim r$

Lieb, Robinson (1972)