The Abdus Salam International Centre for Theoretical Physics

Dynamics of Quantum Information,
KITP, Santa Barbara
1 November 2018

NON-ERGODIC EXTENDED PHASE AND CORRELATION-INDUCED LOCALIZATION IN RANDOM MATRIX THEORY.

V.E.Kravtsov
ICTP, Trieste

NJP 17, 12202, (2015),
arXiv: 1805.06472,
arXiv: 1810.01492

Collaboration:
Mohsen Amini, (Isfahan Uni., ICTP)
Soumya Bera (IIT, Bombay)
Emilio Cuevas (Murcia)
Xiaolong Deng (Hannover)
Giuseppe DeTomasi (MPI, Dresden)
Ivan Khaymovich, (MPI, Dresden)
Pavel Nosov (St. Petersburg Uni)
Gora Shlyapnikov (Orsay)

Rosenzweig–Porter RMT

$$\langle H_{nm} \rangle = 0$$

NxN matrix, uncorrelated random entries

Scaling with matrix size

$$\sigma = \frac{\lambda^2}{N^{\gamma}} << 1$$

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

Special diagonal: Rosenzweig-Porter (1960) ensemble

Simplest non-invariant RMT

Localization/delocalization sufficient conditions (for uncorrelated entries)

$$\frac{1}{N}\sum_{n,m=1}^{N}\left\langle |H_{nm}|\right\rangle < \infty, \quad \Rightarrow \text{ localized}$$

Convergence of Anderson's locator expansion

$$\left\langle |H_{nm}|\right\rangle \square \quad \delta \sim N^{-1}$$

$$\frac{1}{N}\sum_{n,m=1}^{N}\left\langle |H_{nm}|^2\right\rangle = \infty \quad \Rightarrow \text{ semicircle}, \quad \Rightarrow \text{ ergodic extended}$$

Mott's criterion V>W

$$V = \sqrt{2S}$$

RP:

$$\left\langle (H_{nn})^2 \right\rangle = W^2 = 1$$

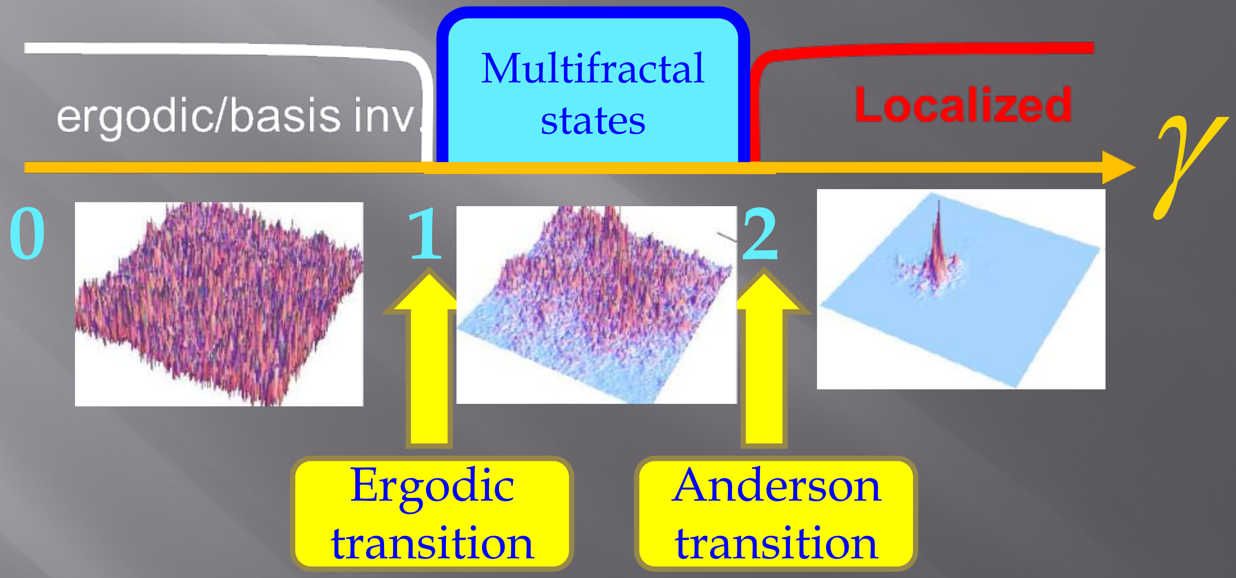
$$\left\langle |H_{n\neq m}|^2 \right\rangle = \frac{\lambda^2}{N^{\gamma}}$$

$$\rho_0 = \frac{\sqrt{2S - E^2}}{\pi S}, \quad \gamma < 1$$

$$S = \frac{1}{N}\sum_{n,m=1}^{N}\left\langle |H_{nm}|^2\right\rangle = \lambda^2 N^{1-\gamma}$$

Ergodic transition

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini,
New J. Phys., v.17, 12202 (2015)



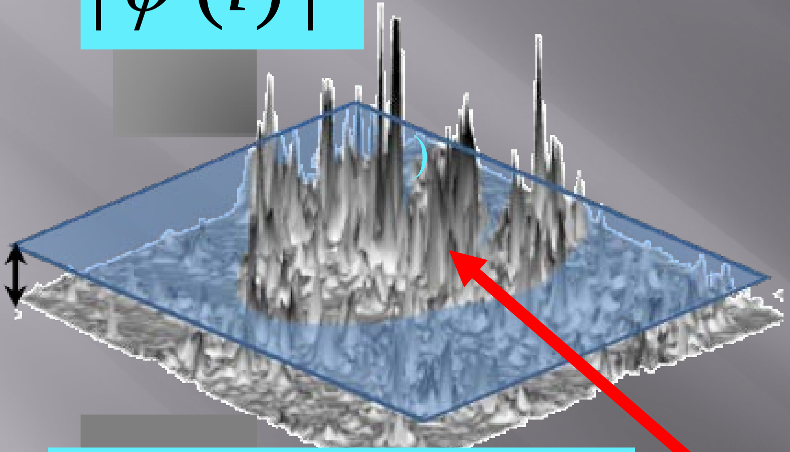
Crash course on multifractality

$$|\psi(i)|^2$$

$$N^{-\alpha}$$

$$|\psi_i|^2 \leq 1$$

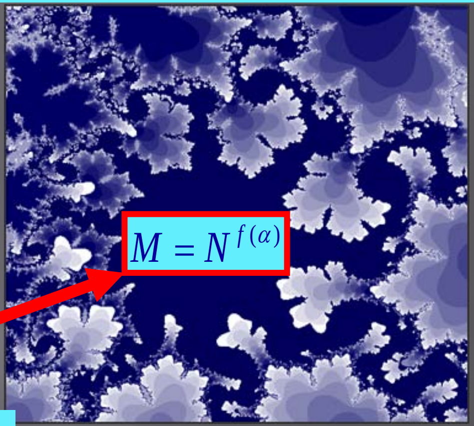
$$\alpha \geq 0$$



N sites {i} in the sample

M sites in this region

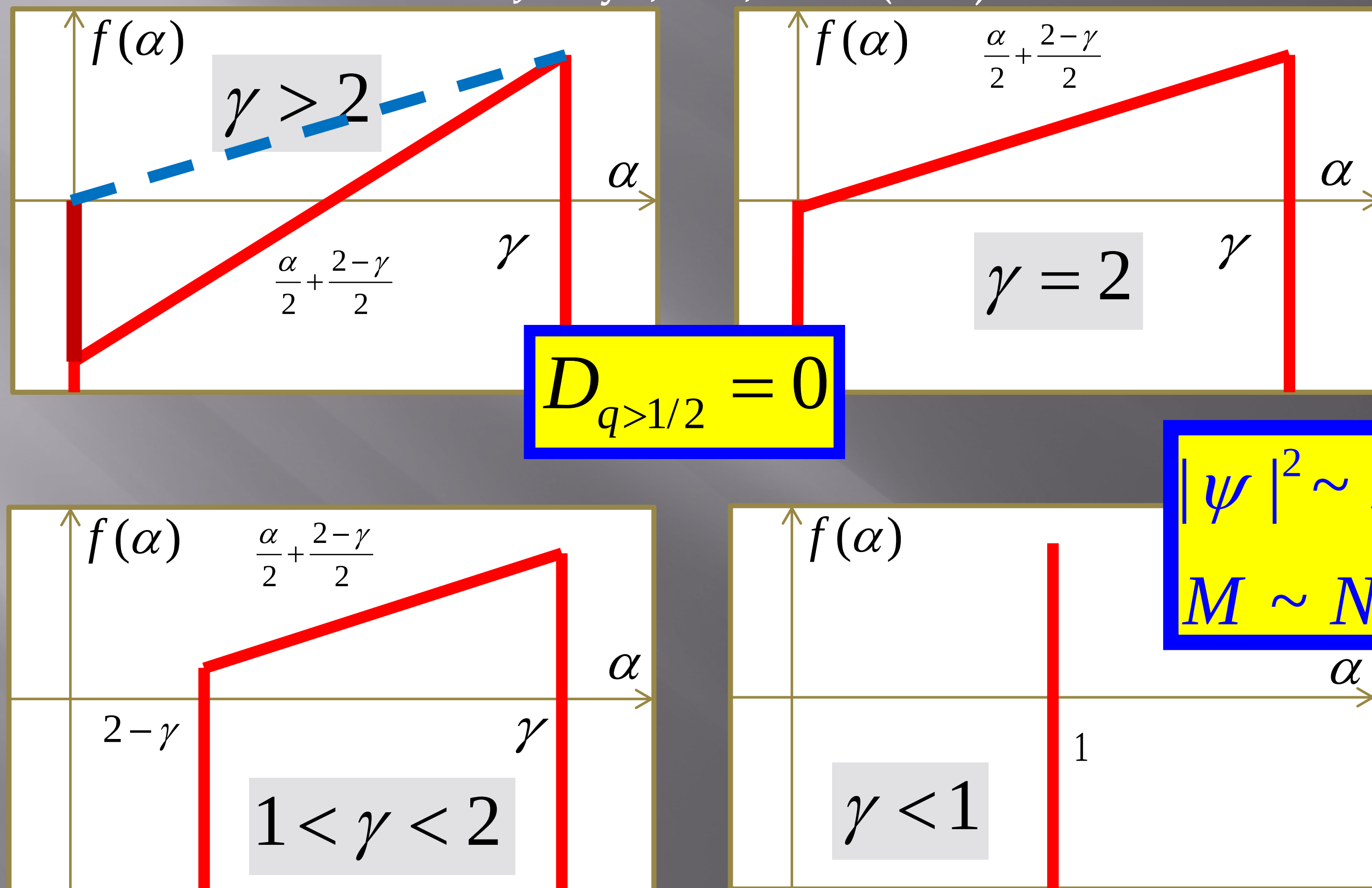
Map of the regions with amplitude larger than the chosen level



$$\sum_i |\psi(i)|^{2q} = \frac{c_q}{N^{D_q(q-1)}}$$

Multifractality spectrum f(α)

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini,
New J. Phys., v.17, 12202 (2015)



$$D_{q>1/2} = 0$$

$$|\psi|^2 \sim N^{-\alpha}$$

$$M \sim N^{f(\alpha)}$$

$$D_{q>1/2} = 2 - \gamma$$

Fractal dimension of wavefunction support set

$$D_q = 1$$

Existence of multifractal phase and ergodic transition in RP RMT is suggested (with the physical standard of rigor) in:

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini, New J. Phys., v.17, 12202 (2015)

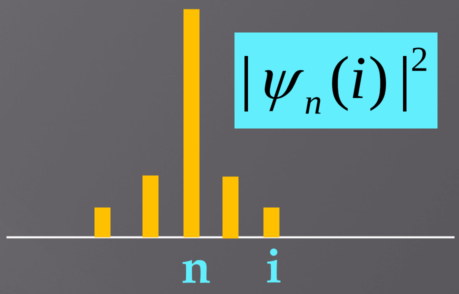
and rigorously proven (on the level of a math theorem):

Per von Soosten and S. Warzel, Electron J. Probab. 23, 1 (2018).

arXiv: 1709.10313 [math-ph)]

Ansatz for random wave functions of Rosenzweig-Porter RMT

$$|\psi_n(i)|^2 = \frac{|H_{ni}|^2}{(E_n - E_i)^2 + \Gamma(N)^2}$$



$$\delta(N) = (\rho_0 N)^{-1}$$

$$\Gamma(N) = \begin{cases} \delta(N) N^D & \text{extended states} \\ \sqrt{\langle |H_{n\neq m}|^2 \rangle}, & \text{localized states} \end{cases}$$

$$\rho_0 = p(E) \sim 1, \quad (\gamma > 1)$$

Semi-circle

$$\rho_0 = \frac{\sqrt{2S - E^2}}{\pi S}, \quad \gamma < 1$$

$$S = \sum_{n,m=1}^{N} \langle |H_{nm}|^2 \rangle = \lambda^2 N^{1-\gamma}$$

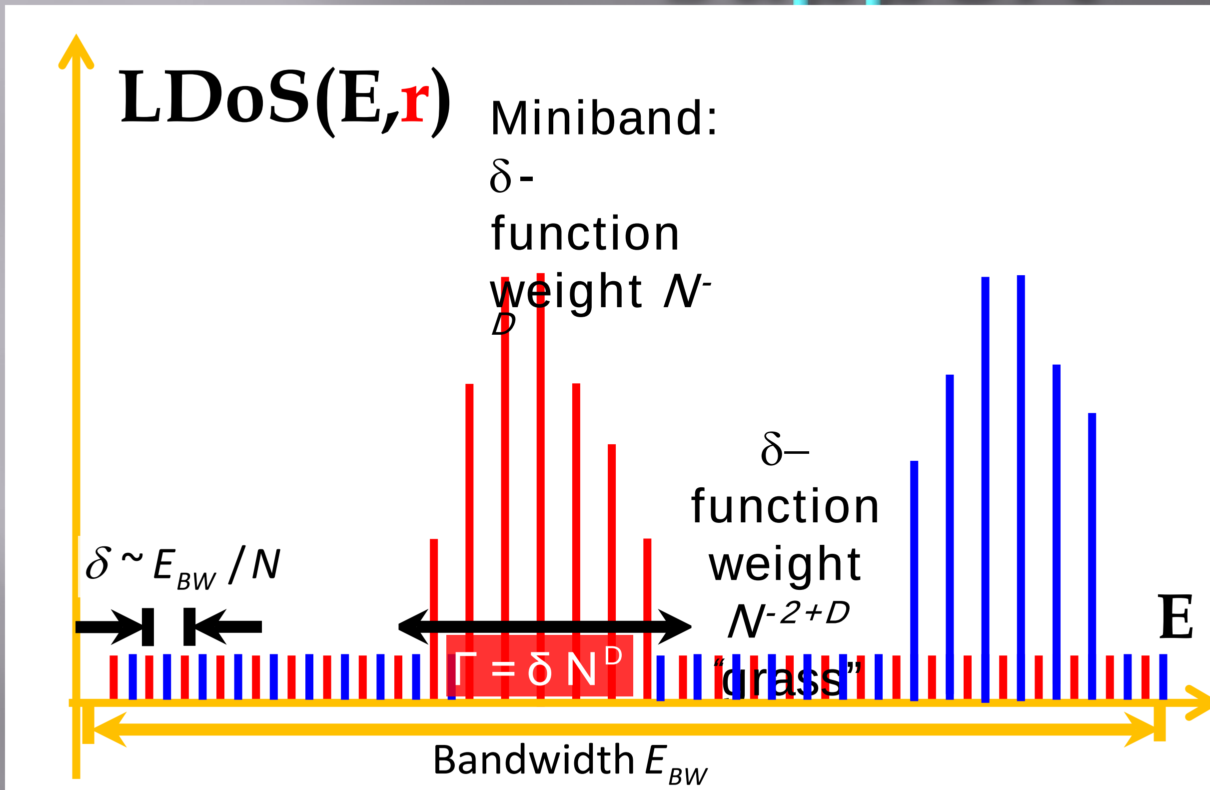
$$\Gamma(N) = \begin{cases} N^{\frac{1-\gamma}{2}}, & (\gamma < 1) \\ N^{-(\gamma-1)}, & (1 < \gamma < 2) \\ N^{-\gamma/2}, & (\gamma > 2) \end{cases}$$

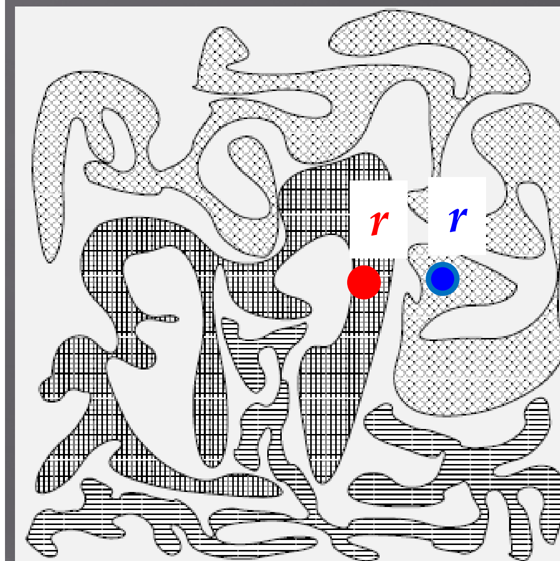
$\Gamma \to \infty$

$\Gamma \to 0, \ \Gamma/\delta \to \infty$

$\Gamma \to 0, \ \Gamma/\delta \to 0$

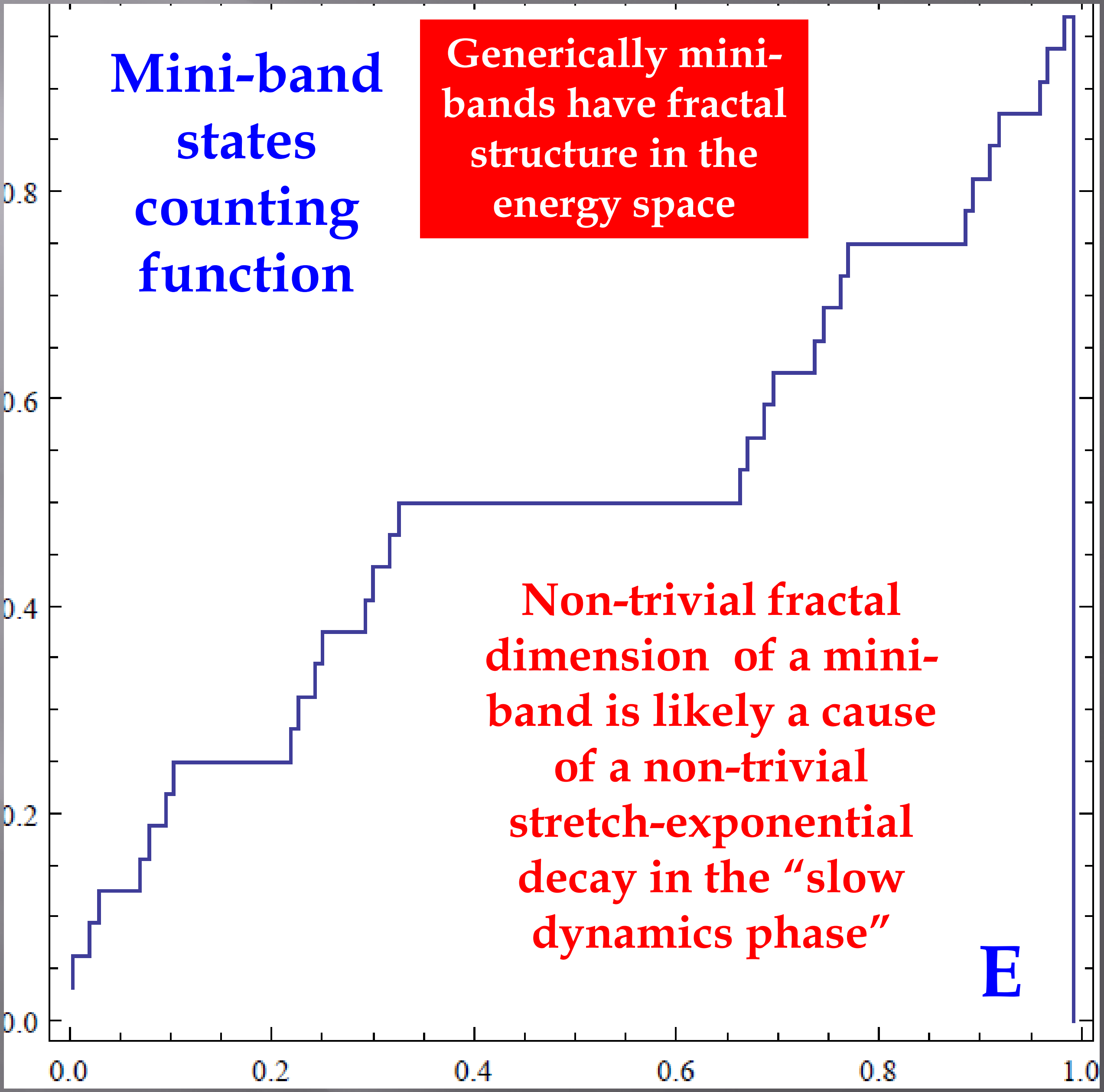
Minibands and fractal support





States living on the same fractal support form a mini-band

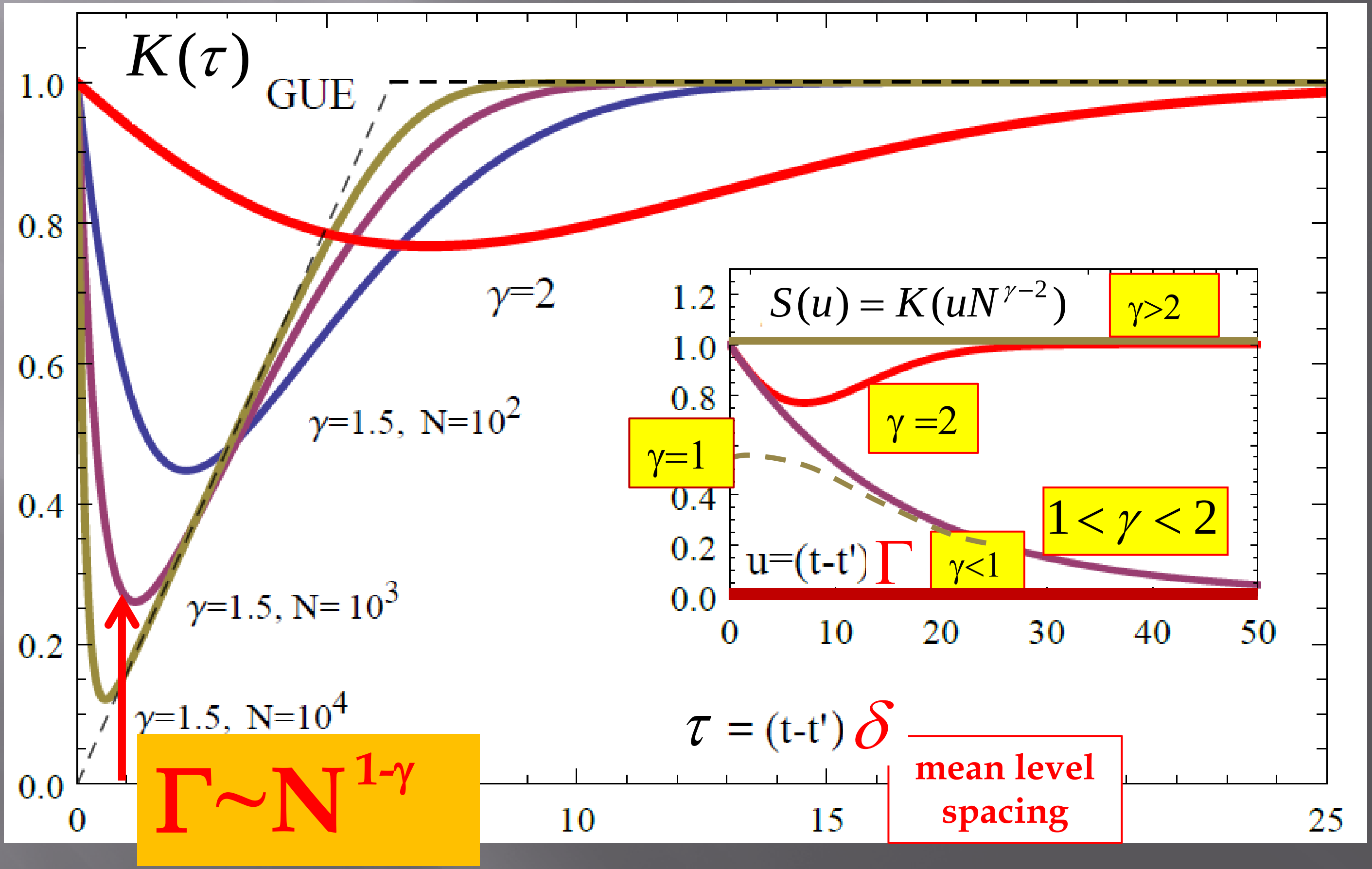
Typically only one mini-band is seen in the local spectrum; global spectrum is a unification of all N^{1-D} mini-bands



Mini-band states counting function
Generically mini-bands have fractal structure in the energy space
Non-trivial fractal dimension of a mini-band is likely a cause of a non-trivial stretch-exponential decay in the "slow dynamics phase"
E
0.0
0.2
0.4
0.6
0.8
1.0

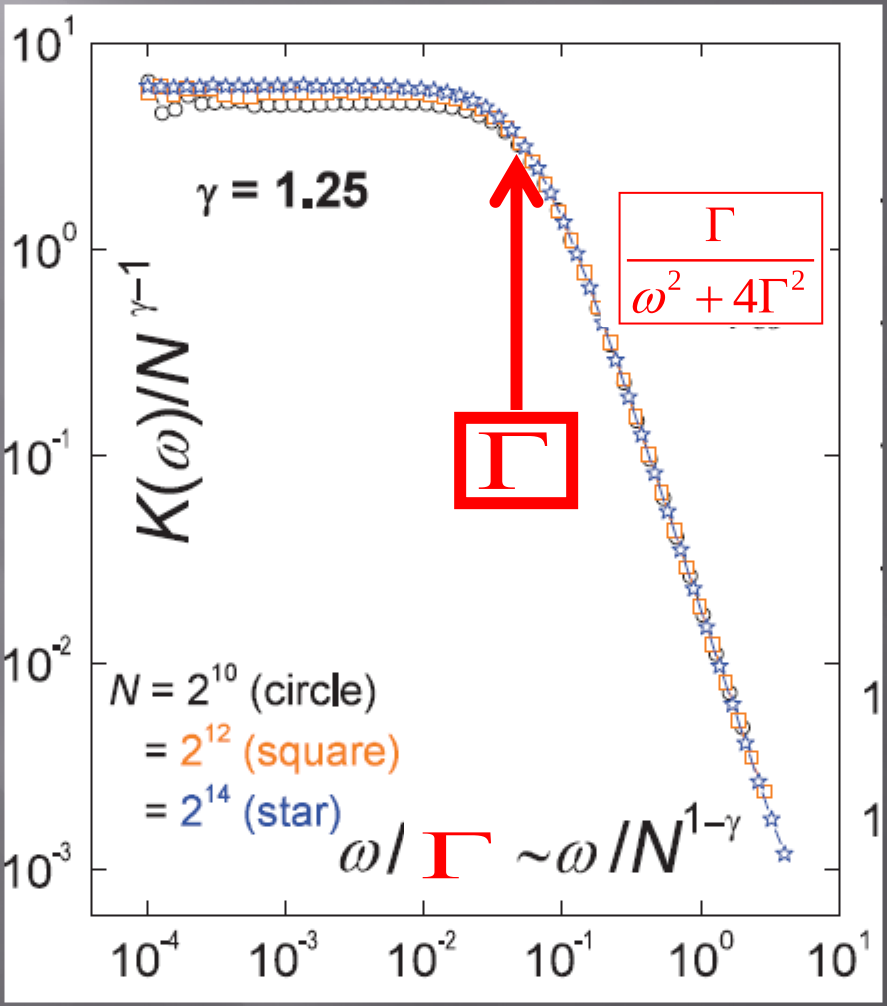
Spectral form-factor and the 'hybrid' level statistics

$$K(\tau) = FT_{\omega}\langle \rho(E)\rho(E+\omega)\rangle$$



How to detect the new scale Γ?

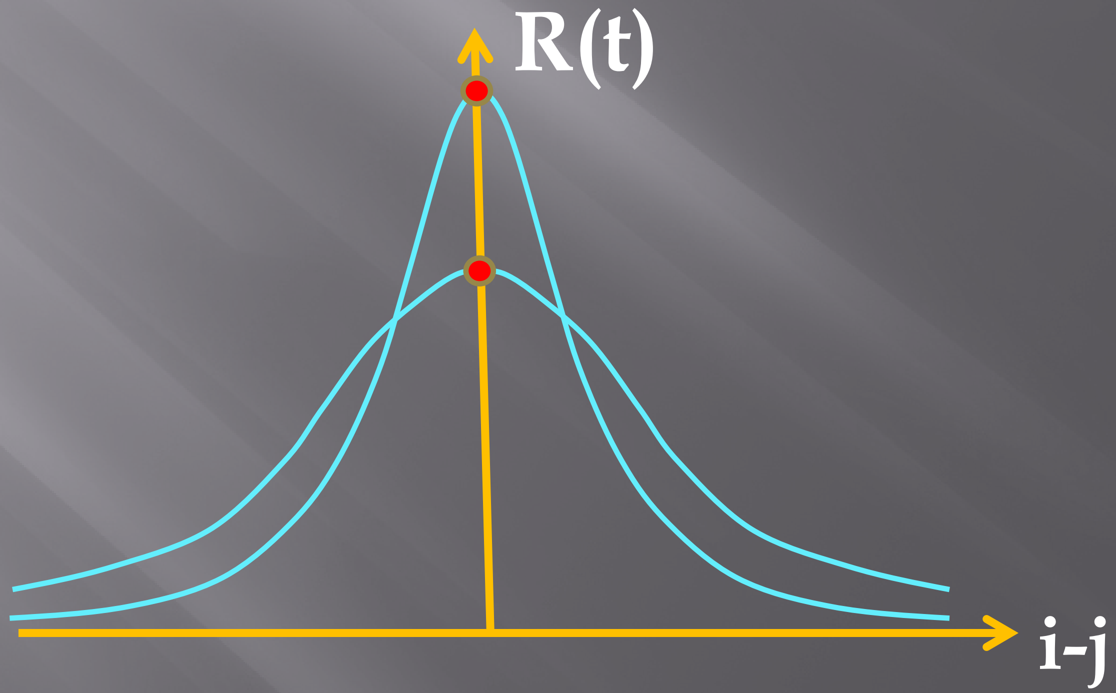
$$K(\omega)=\sum_{\alpha,\beta}|\psi_\alpha(i)|^2|\psi_\beta(i)|^2\delta(\omega-E_\alpha+E_\beta)$$



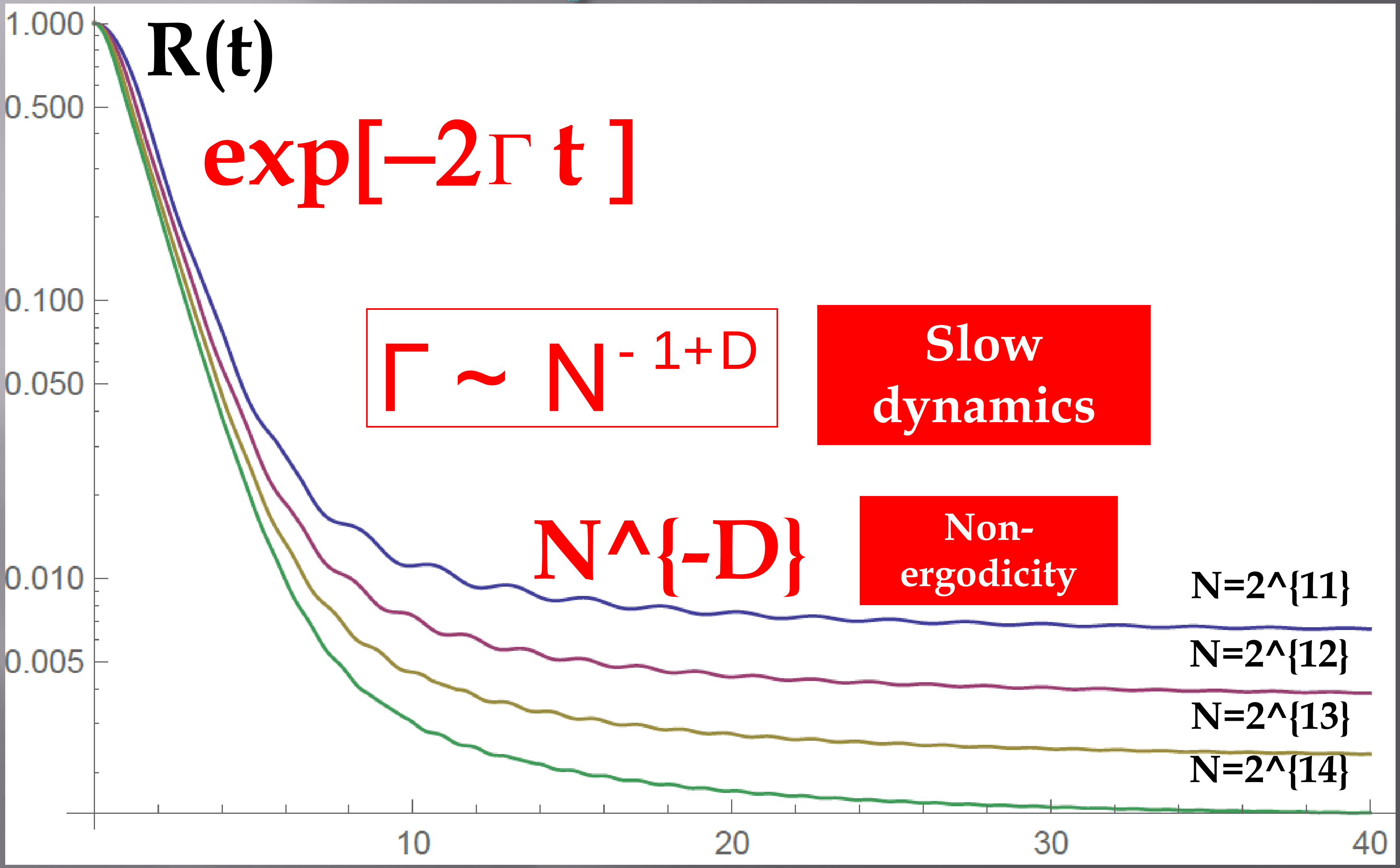
More informative Fourier transform: Survival Probability

$$R(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2$$

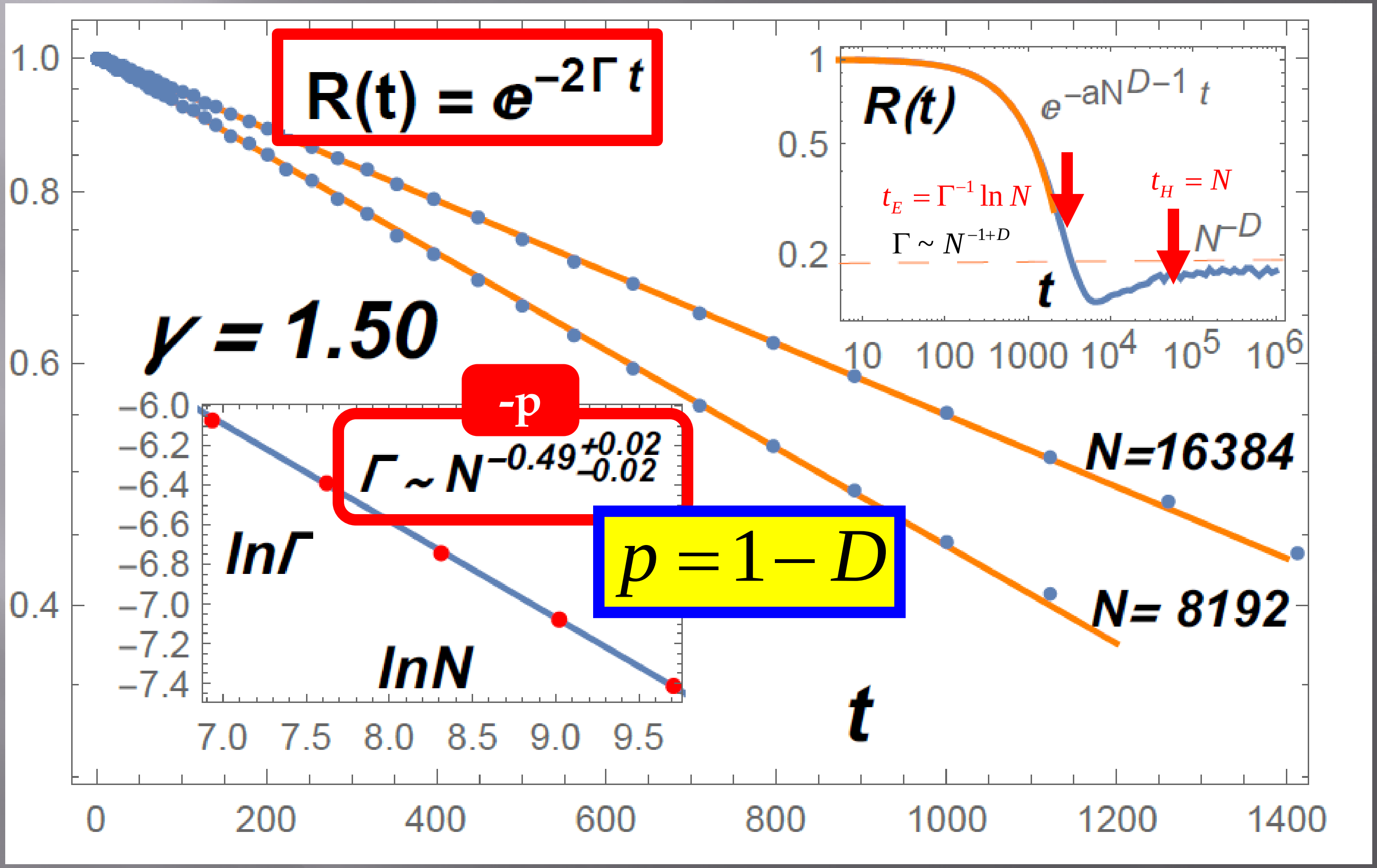
$$= \sum_{\alpha,\beta} |\psi_\alpha(i)|^2 |\psi_\beta(i)|^2 \exp\left[i(E_\alpha - E_\beta)t \right]$$



Survival Probability in MF phase



Survival probability in NEE phase



What about correlated/deterministic hopping terms?

Exactly soluble: all states are localized (γ>2) or critically localized (γ<=2)

Yuzbashyan-Shastry (YS) model

$$H_{n\neq m} = g_n g_m^*$$

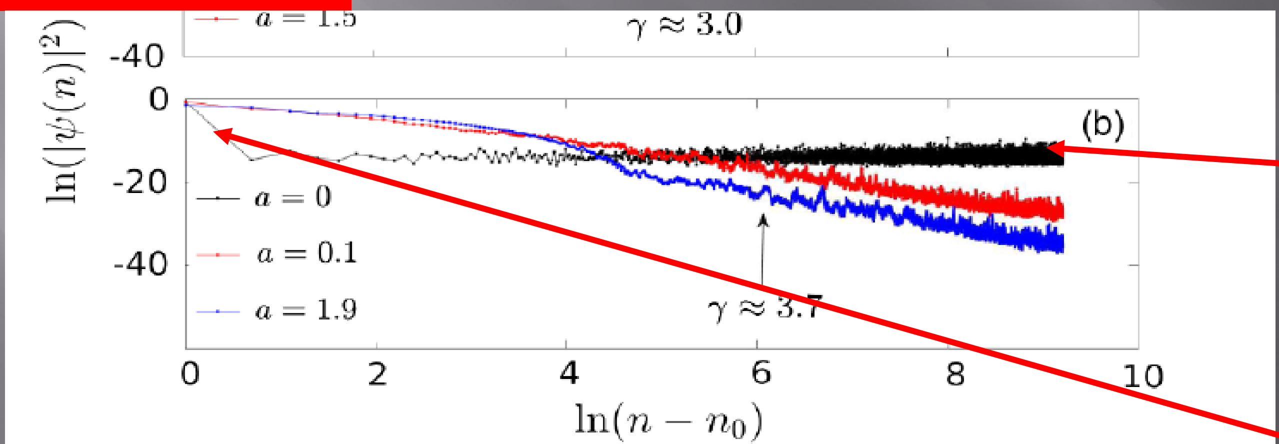
$$g_n = \frac{1}{N^{\gamma/2}}$$

$$\Psi_E(i) = c\,\frac{g_i}{E-\varepsilon_i}$$

$$\sum_{i=0}^{N-1} \frac{g_i^2}{E-\varepsilon_i} = -1$$

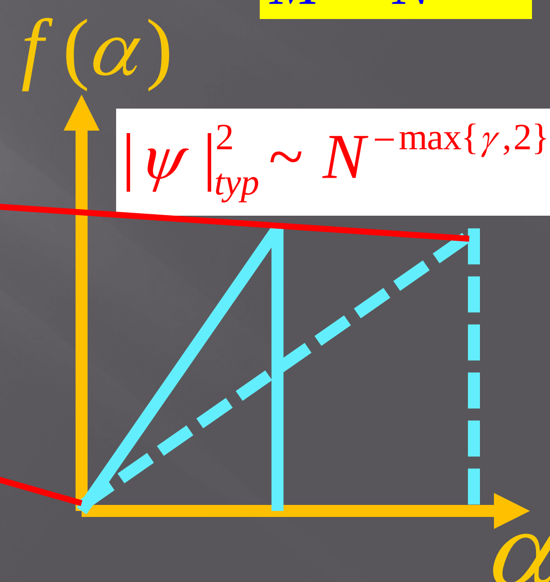
$$|\psi|^2 \sim N^{-\alpha}$$

$$M \sim N^{f(\alpha)}$$

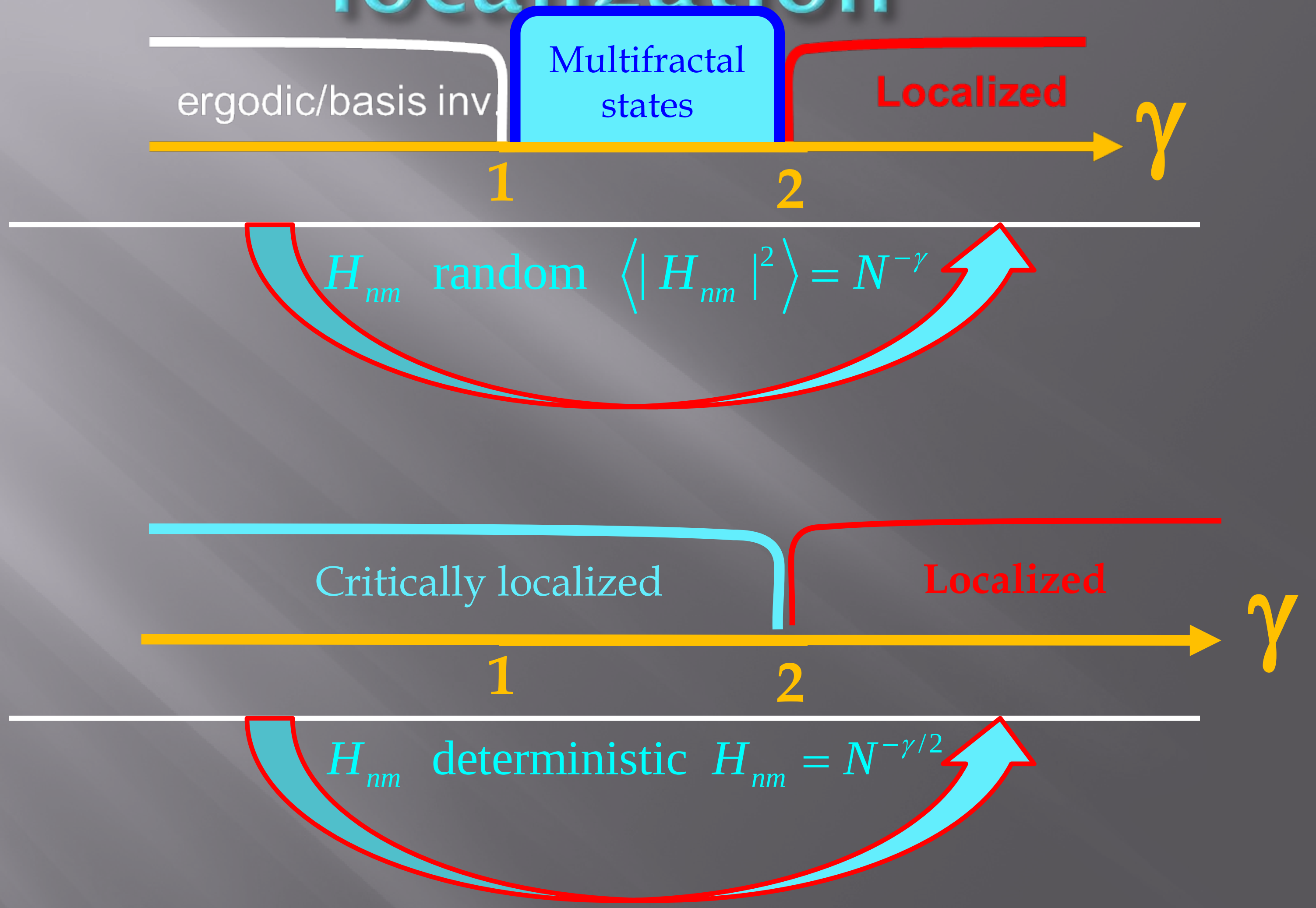


X. Deng, V. E. Kravtsov, G. V. Shlyapnikov, and L. Santos, Phys. Rev. Lett. **120**, 110602 (2018).

$$|\psi|^2_{typ} \sim N^{-\max\{\gamma,2\}}$$



Correlation-induced localization



Why no delocalized states?

Convergence of Anderson's locator expansion

$$\frac{1}{N}\sum_{n,m=1}^{N}\langle |H_{nm}|\rangle < \infty, \quad \Rightarrow \text{ localized}$$

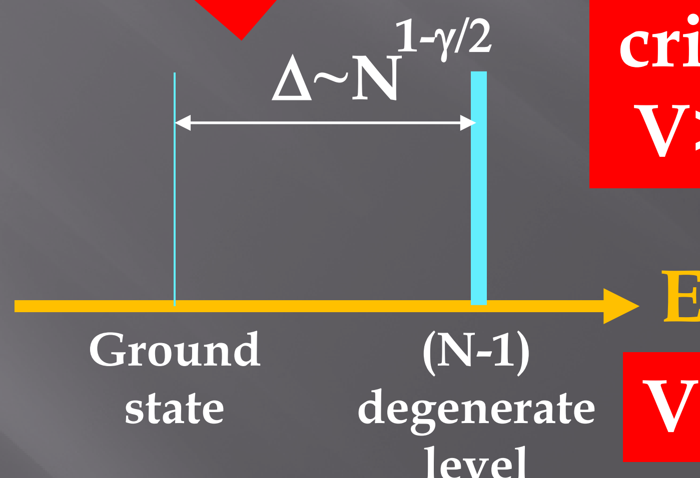
$$\langle |H_{nm}|\rangle \;\square\; \delta \sim N^{-1}$$

$$\frac{1}{N}\sum_{n,m=1}^{N}\langle |H_{nm}|^2\rangle = \infty \quad \Rightarrow$$ …rgodic extended

Not valid for correlated entries

Mott's criterion V>W=1

However, Mott's criterion is valid!



V=0

Not only YS!

No delocalized states also for deterministic power-law hopping!

$$H_{nm} = J\,|n-m|^{-a}$$

Even for a < 1

P. Nosov, I.M.Khaymovich and V.E.K. arXiv: 1810.01492

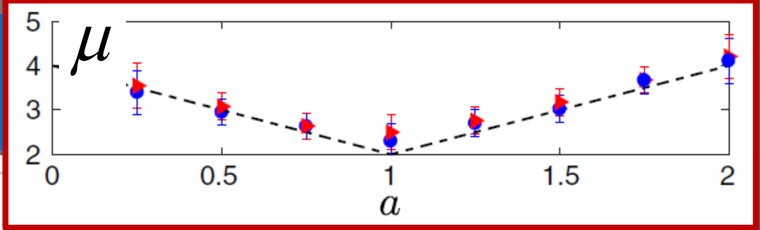
X. Deng, V.E.K. G. Shlyapnikov, L. Santos, Phy. Rev. Lett. **120**, 110602 (2018)

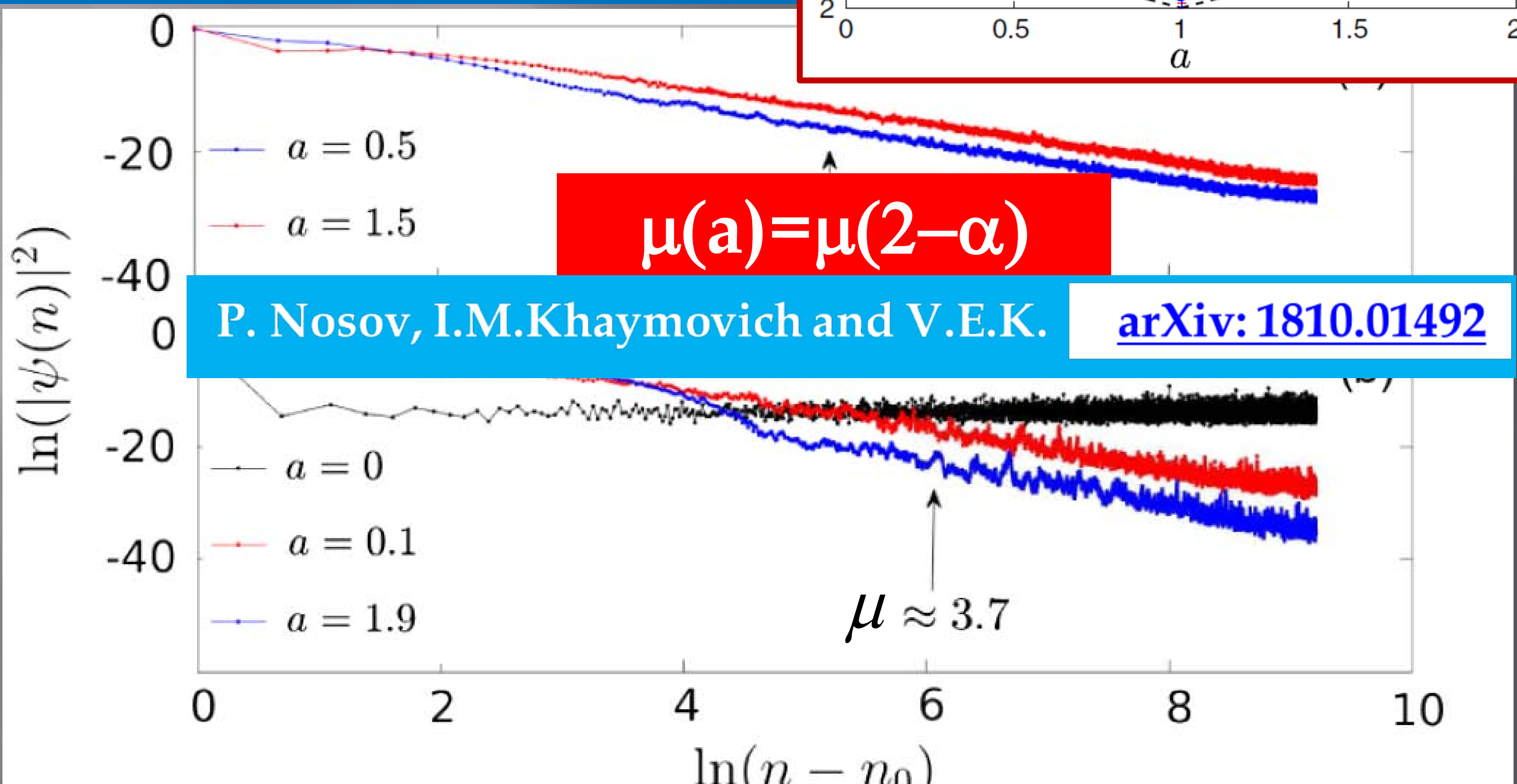
Duality in power-law localization

$$H_{nm} = J\,|n-m|^{-a}$$

$$|\psi_{n_0}(n)|^2_{typ} \sim |n-n_0|^{\mu}$$

X. Deng, V.E.K. G. Shlyapnikov, L. Santos, Phy. Rev. Lett. **120**, 110602 (2018)





μ(a)=μ(2–α)

P. Nosov, I.M.Khaymovich and V.E.K. [arXiv: 1810.01492]

For random off-diagonal terms <H_nm>=0, <|H_nm|^2>~ 1/|n-m|^2a delocalization for a<1

Poof of duality

$$\hat{H} = \hat{\varepsilon} + \hat{j}$$

$$(\hat{j})_{nm} = J\,|n-m|^{-a}$$

$$\psi_E(m) = \sum_{\ell=1}^{N} M_{m-\ell}(E + E_0 - \varepsilon_\ell)\ \psi_E(\ell)$$

$$\hat{M} = (E_0 + \hat{j})^{-1}$$

$$M_n = \frac{N}{J\,|n|^{2-a}}$$

$$FT_p(E_0 + \hat{j}) \Rightarrow 1/FT_p \Rightarrow (FT)_n^{-1}(1/FT_p)$$

$a < 1$

$$|\psi_E(m)|^2 \sim M_m^2 \sim \frac{f(a)}{|m|^{2(2-a)}}$$

$\mu = 2(2-a)$

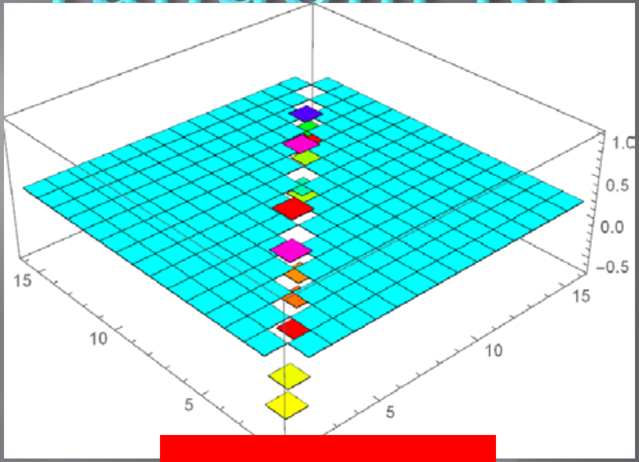
$a > 1$

Perturbation theory

$$|\psi_E(m)|^2 \sim \frac{f(a)}{|m|^{2a}}$$

$\mu = 2a$

Yuzbashyan-Shastry, translation-invariant and fully random RP

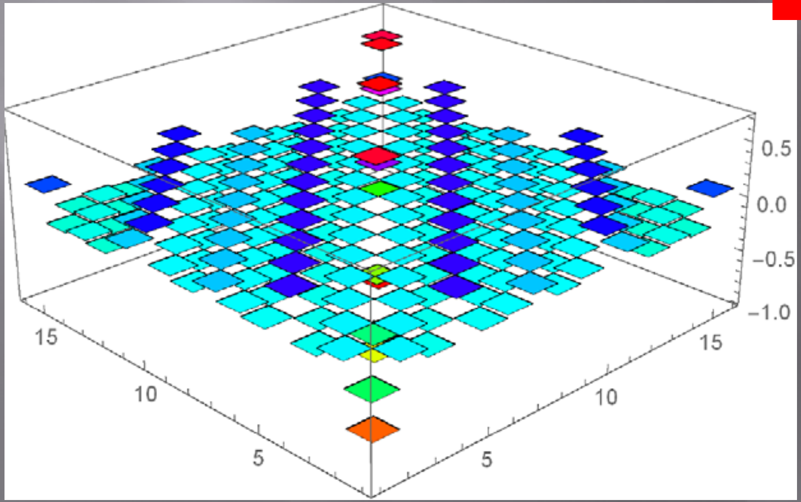


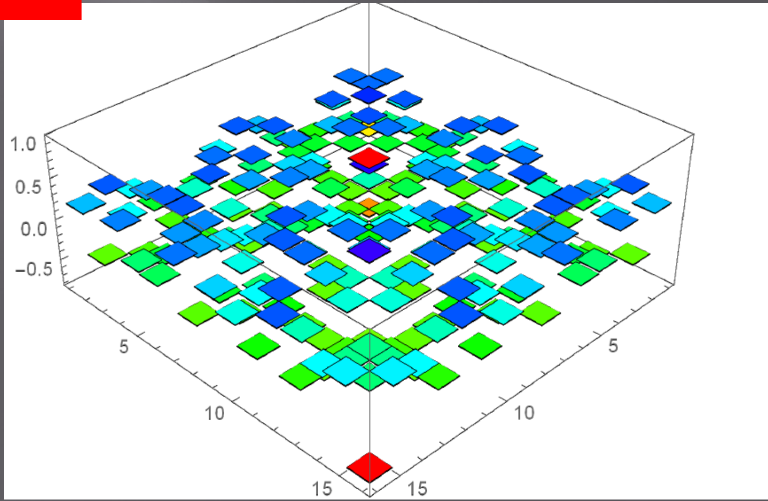
Yuzbashyan-Shastry RMT

Translation-invariant RP

H_nm

Fully random (original) RP





Translation-invariant RP model

$$H_{n\neq m} = H_{n-m}$$

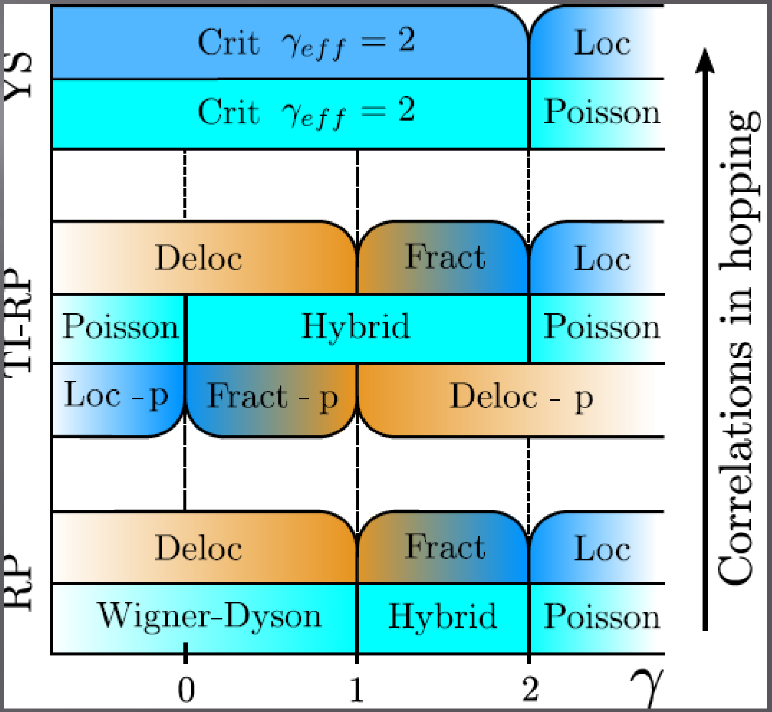
$$\langle H_{n-m} \rangle = 0,$$
$$\langle | H_{n-m} |^2 \rangle = N^{-\gamma}$$

P. Nosov, I.M.Khaymovich and V.E.K. **arXiv: 1810.01492**

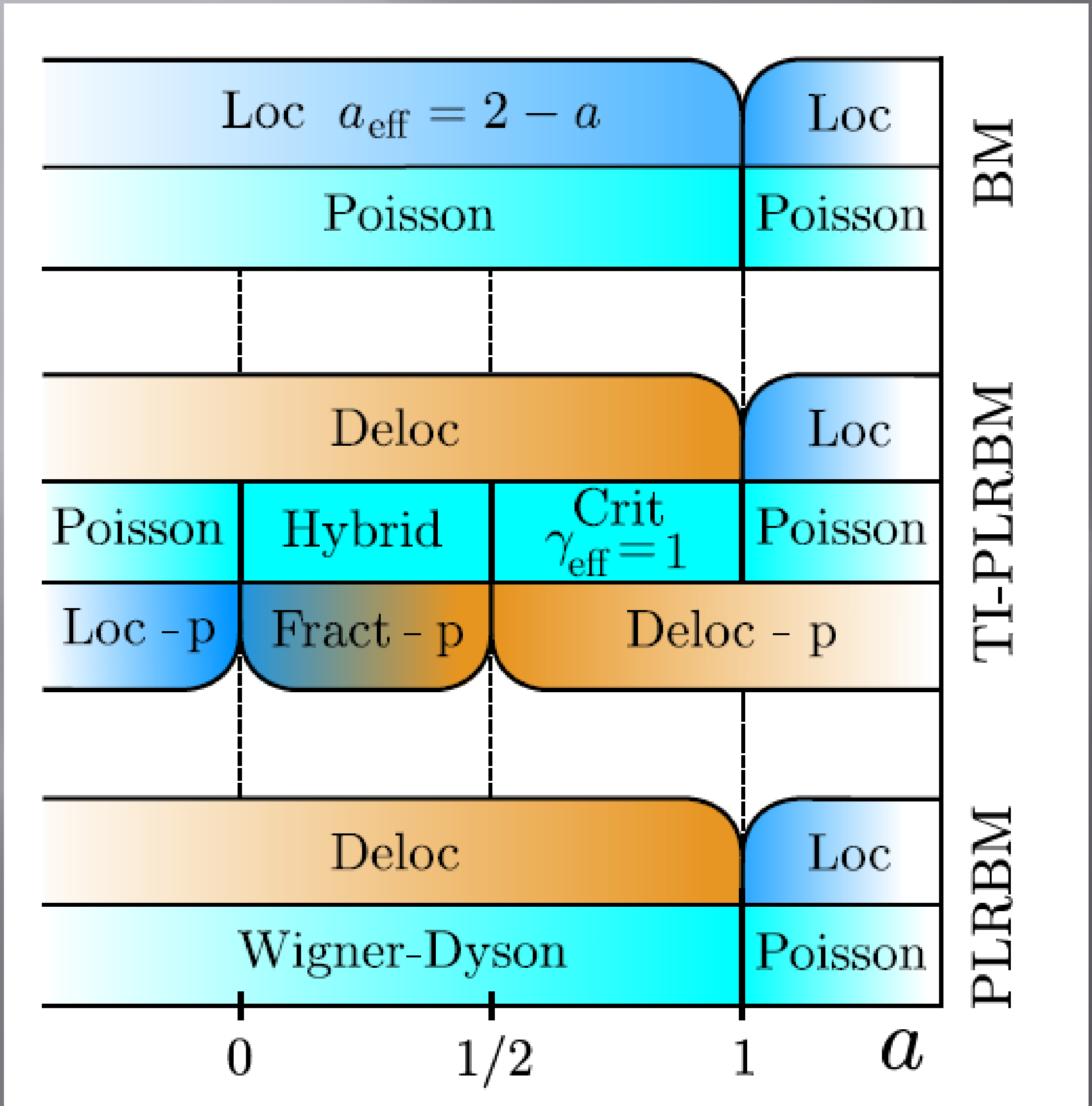
- ▷ Lack of correlations between diagonals destroy localization in the coordinate space for $\gamma<2$.
- ▷ Localization and multifractality in the momentum space

$$\gamma_p = 2 - \gamma$$

- ▷ Poisson and 'hybrid' level statistics in the delocalized phase



Power-law family

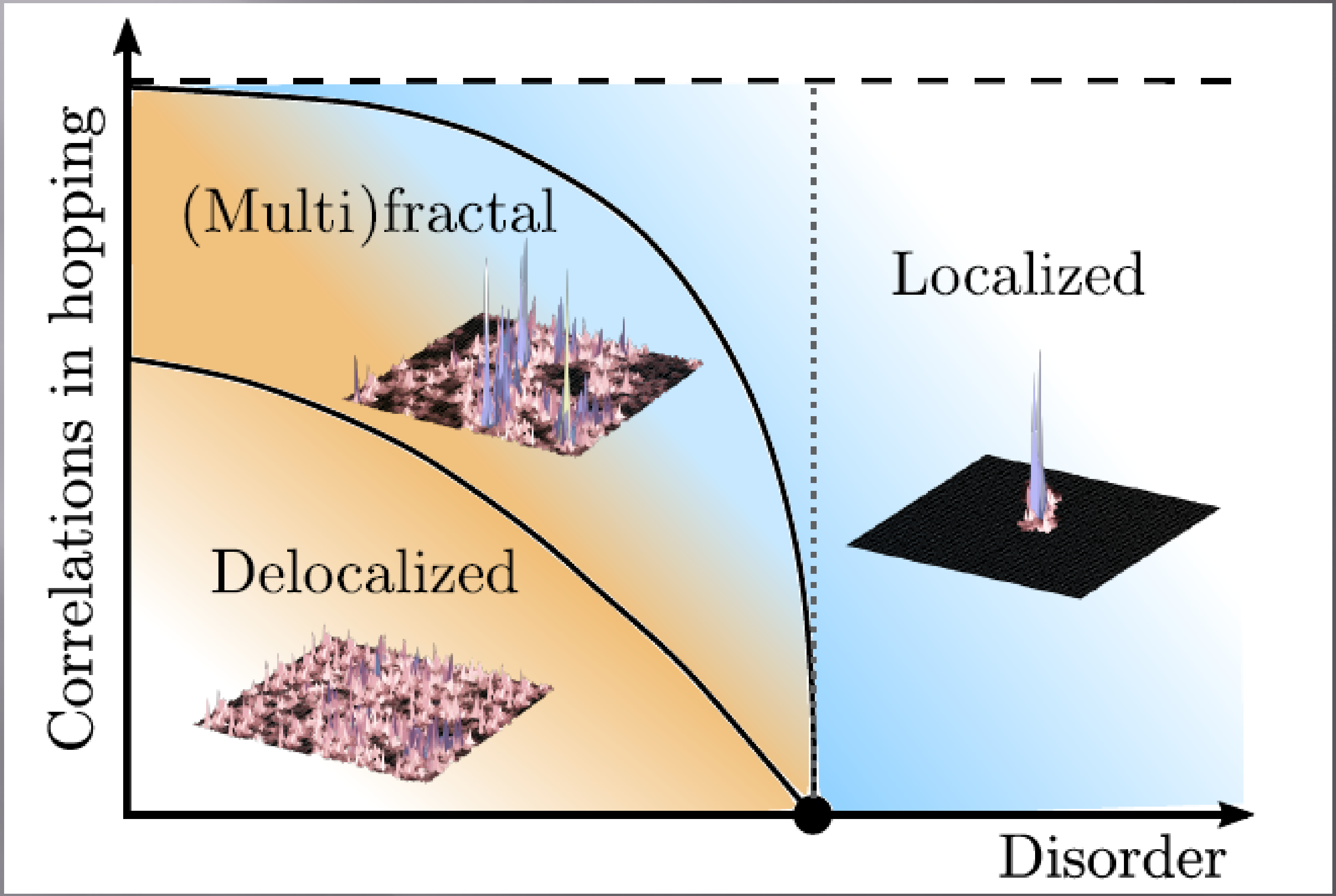


Deterministic PL hopping
H_nm =1/|n-m|^a

Translation-invariant random PL hopping
H_nm=H_(n-m)

Completely random PL hopping
<H_nm>=0,
<|H_{nm}|^2>=
=1/|n-m|^2a

Conclusion



Correlations in hopping
(Multi)fractal
Localized
Delocalized
Disorder

Conclusion

- Two extended phases and ergodic transition in RP RMT
- Ansatz for random wave functions of RP RMT and survival probability
- ‘Hybrid’ level statistics
- Localization in YS exactly solvable model: RP with fully correlated hopping
- Translation-invariant TI-RP: localization and multifractality in the momentum space; Poisson and ‘hybrid’ level statistics in delocalized phase