August, the 9th, 2018, KITP, 'The dynamics of quantum information'

Applications of time-dependent spin wave theory: chaotic dynamical ferromagnets and many-body Kapitza pendulum

J. Marino
Harvard University

HARVARD
VE RI TAS

MARIE CURIE ACTIONS

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

In this talk: dynamical phase transitions with order parameter

(in quantum spin chains with competing short-range and collective interactions)

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

<u>In this talk</u>: dynamical phase transitions with order parameter

(in quantum spin chains with competing short-range and collective interactions)

<u>quantum quench</u>

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

<u>In this talk</u>: dynamical phase transitions with order parameter

(in quantum spin chains with competing short-range and collective interactions)

<u>quantum quench</u>

$$r_c^{dy} \neq r_c^{eq}$$

dynamical critical point

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

In this talk: dynamical phase transitions with order parameter
(in quantum spin chains with competing short-range and collective interactions)

quantum quench

$$r_c^{dy} \neq r_c^{eq}$$

dynamical critical point

order parameter

$$\langle O(t) \rangle_{t \to \infty} =$$

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

In this talk: dynamical phase transitions with order parameter

(in quantum spin chains with competing short-range and collective interactions)

quantum quench

order parameter

$$r_c^{dy} \neq r_c^{eq}$$

dynamical critical point

$$\langle O(t) \rangle_{t \to \infty} = \mathcal{O} \neq 0 \quad r < r_c^{dy}$$

dynamical ordered phase

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

<u>In this talk</u>: dynamical phase transitions with order parameter

(in quantum spin chains with competing short-range and collective interactions)

<u>quantum quench</u>

$$r_c^{dy} \neq r_c^{eq}$$

dynamical critical point

order parameter

$$\langle O(t) \rangle_{t \to \infty} =$$

Universality in out-of-equilibrium isolated systems:

- thermalization as a renormalization group process (non-linear Luttinger Liquids, interacting Bose gases)
- non-analytical behaviour of the Loschmidt echo (return probability amplitude) for quenches across a critical point ('dynamical phase transition')
- etcetera…

In this talk: dynamical phase transitions with order parameter
(in quantum spin chains with competing short-range and collective interactions)

quantum quench

$$r_c^{dy} \neq r_c^{eq}$$

dynamical critical point

order parameter

$$\langle O(t) \rangle_{t\to\infty} = \quad 0, \quad r > r_c^{dy}$$

dynamical disordered phase

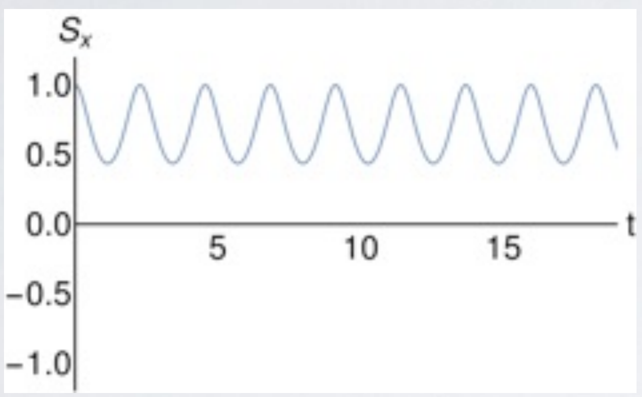
Warm-up: Dynamical phase transition in the Lipkin model

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

$g_0 = 0$ → g, $t > 0$

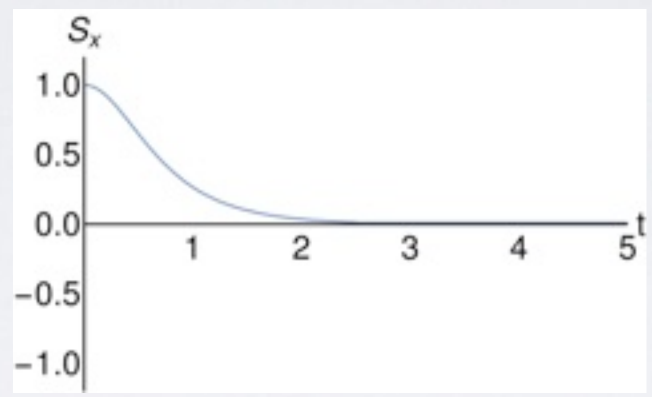
ferromagn. GS

$$S^{(\alpha)} \equiv \sum_i \sigma_i^{(\alpha)}$$



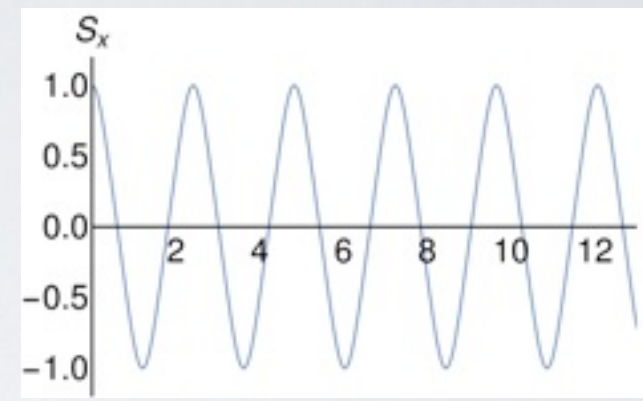
Dy. Ferro. Phase

$g < g_c^{dy}$



Dy. critical point

$g = g_c^{dy}$



Dy. Demagn. Phase

$g > g_c^{dy}$

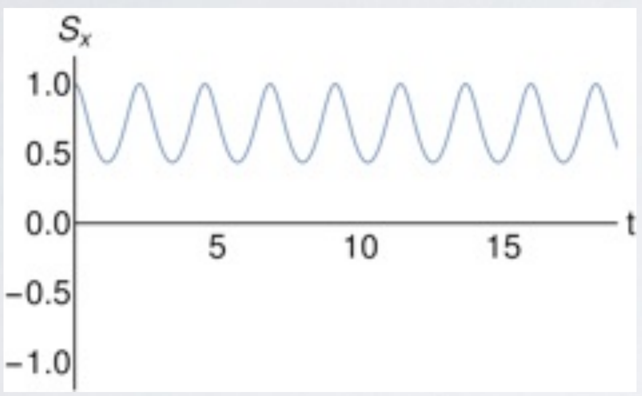
Warm-up: Dynamical phase transition in the Lipkin model

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

$g_0 = 0$ → g, $t > 0$

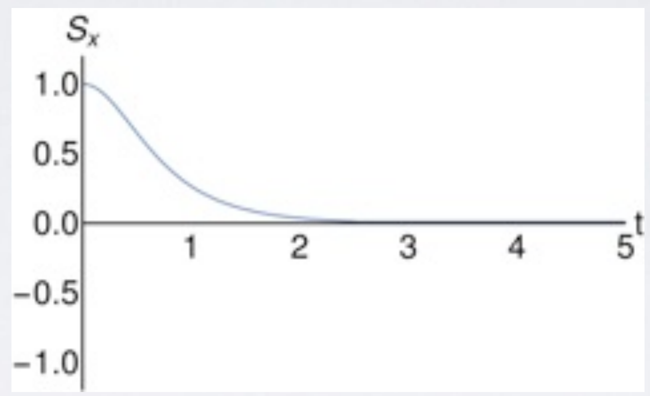
ferromagn. GS

$$S^{(\alpha)} \equiv \sum_i \sigma_i^{(\alpha)}$$



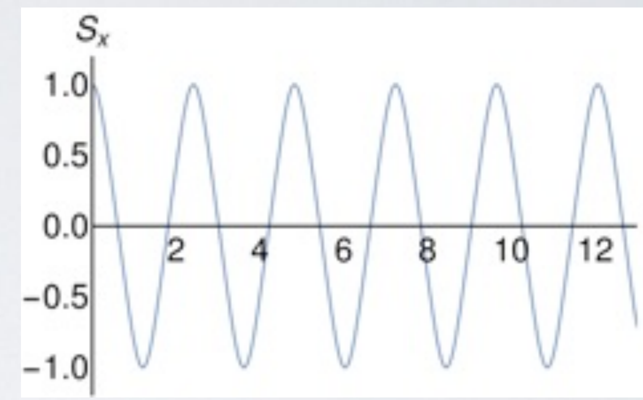
Dy. Ferro. Phase

$g < g_c^{dy}$



Dy. critical point

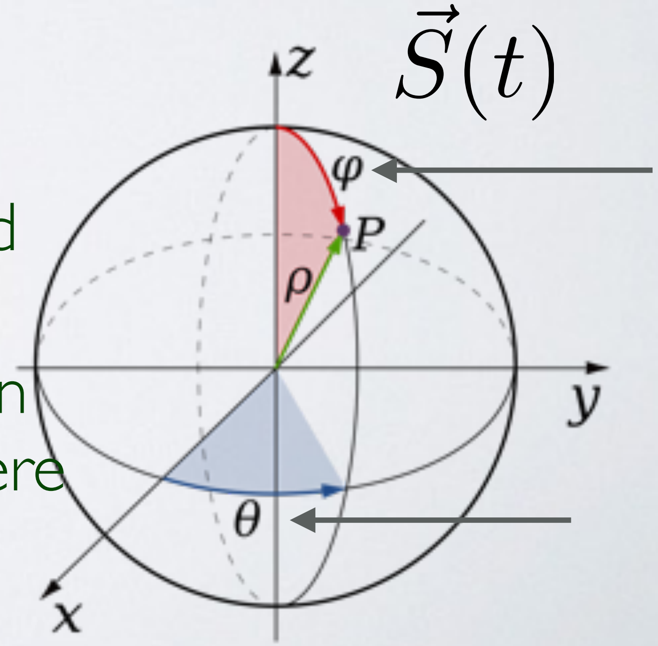
$g = g_c^{dy}$



Dy. Demagn. Phase

$g > g_c^{dy}$

$\vec{S}(t)$



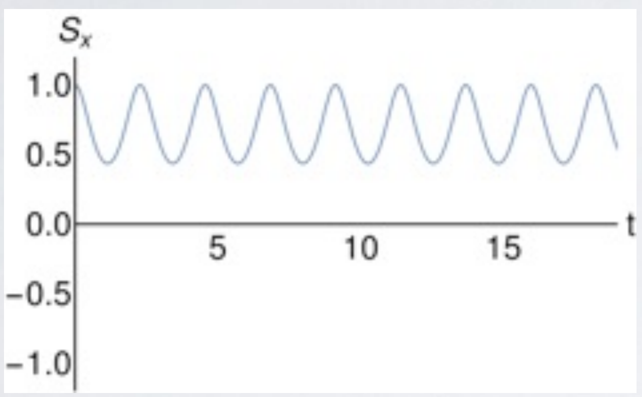
In the thermodynamic limit, mean-field exactly solves the Lipkin model;
just **two classical** equations of motion for the polar angles on the Bloch sphere spanned by $\vec{S}(t)$

Warm-up: Dynamical phase transition in the Lipkin model

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

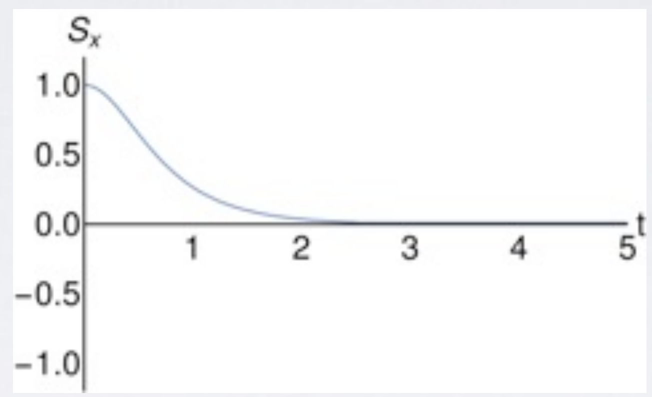
$g_0 = 0$ (ferromagn. GS) → g, $t > 0$

$$S^{(\alpha)} \equiv \sum_i \sigma_i^{(\alpha)}$$



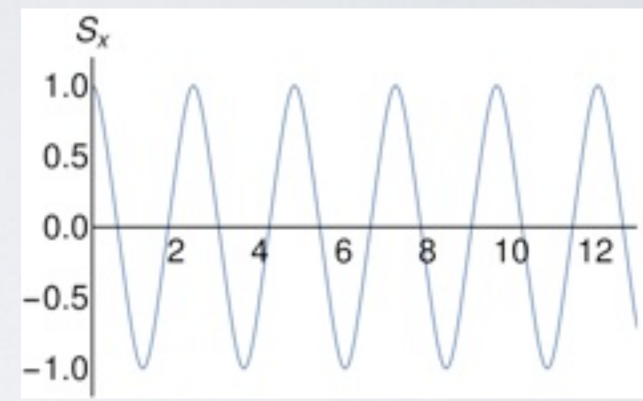
Dy. Ferro. Phase

$g < g_c^{dy}$



Dy. critical point

$g = g_c^{dy}$

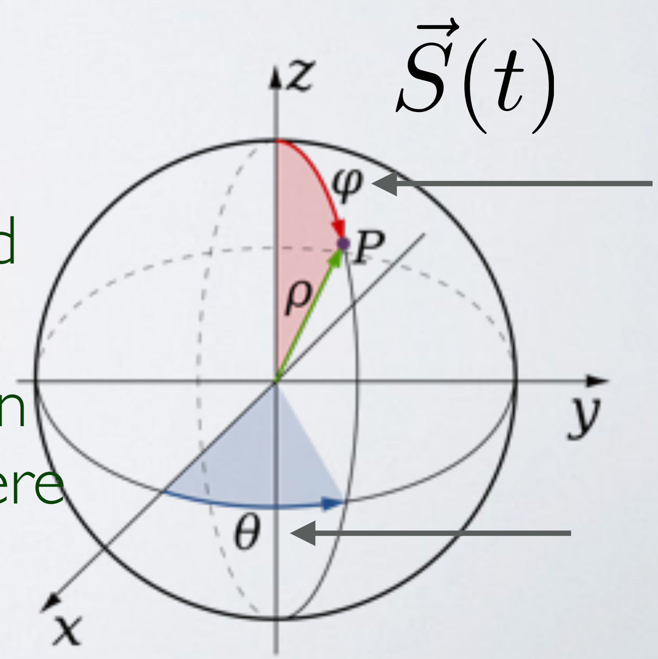


Dy. Demagn. Phase

$g > g_c^{dy}$

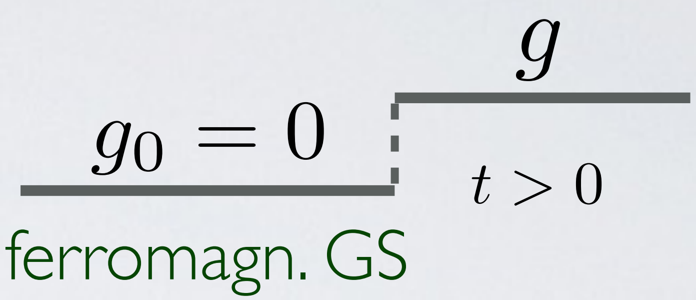
$$m_{(\alpha)} = S_{(\alpha)}/N$$
$$[m_x, m_y] = i m_z/N$$

In the thermodynamic limit, mean-field exactly solves the Lipkin model; just **two classical** equations of motion for the polar angles on the Bloch sphere spanned by $\vec{S}(t)$

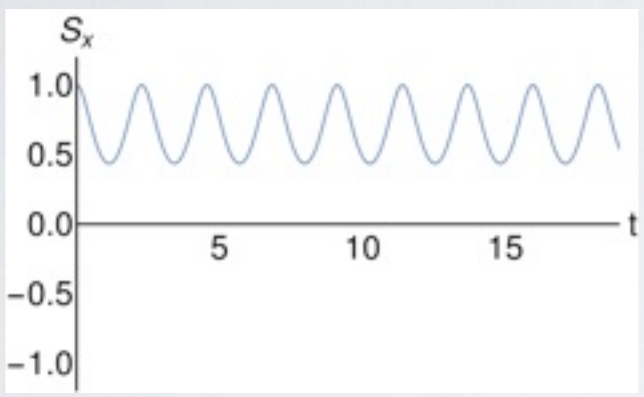


Warm-up: Dynamical phase transition in the Lipkin model

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

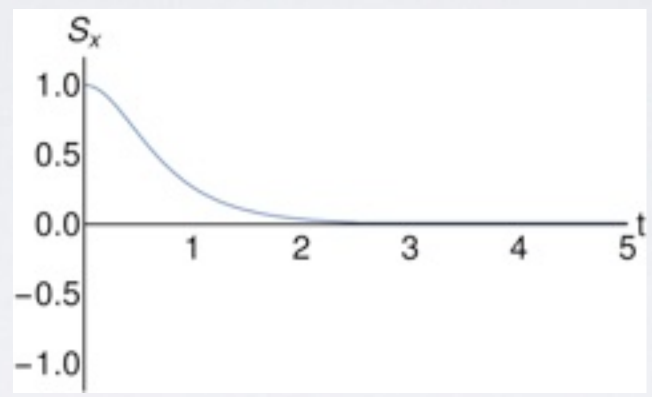


$$S^{(\alpha)} \equiv \sum_i \sigma_i^{(\alpha)}$$



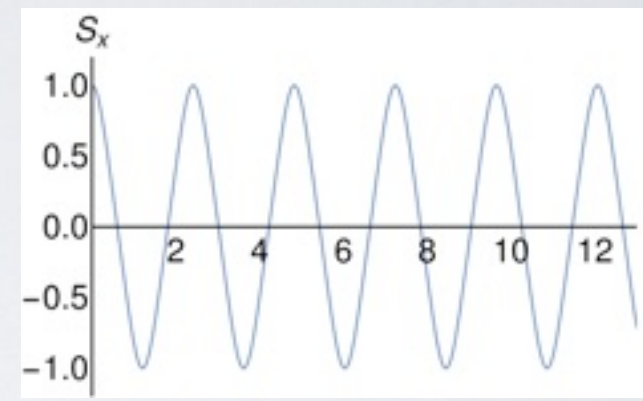
Dy. Ferro. Phase

$g < g_c^{dy}$



Dy. critical point

$g = g_c^{dy}$



Dy. Demagn. Phase

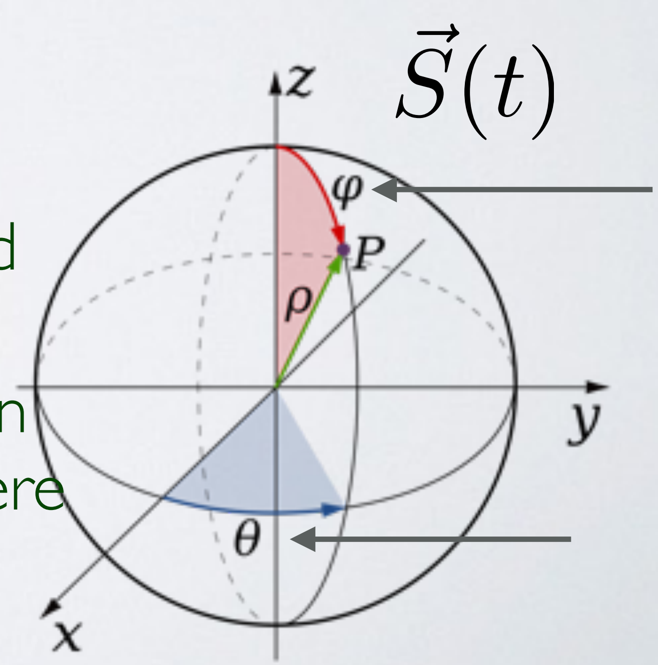
$g > g_c^{dy}$

$$m_{(\alpha)} = S_{(\alpha)}/N$$

$$[m_x, m_y] = \sout{i m_z/N}$$

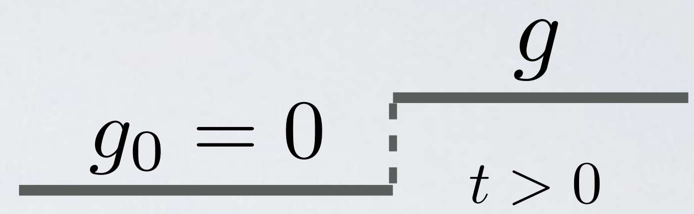
$N \to \infty$

In the thermodynamic limit, mean-field exactly solves the Lipkin model; just **two classical** equations of motion for the polar angles on the Bloch sphere spanned by $\vec{S}(t)$



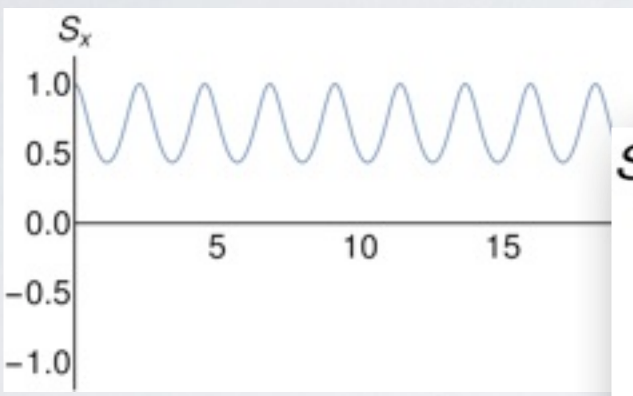
Warm-up: Dynamical phase transition in the Lipkin model

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$



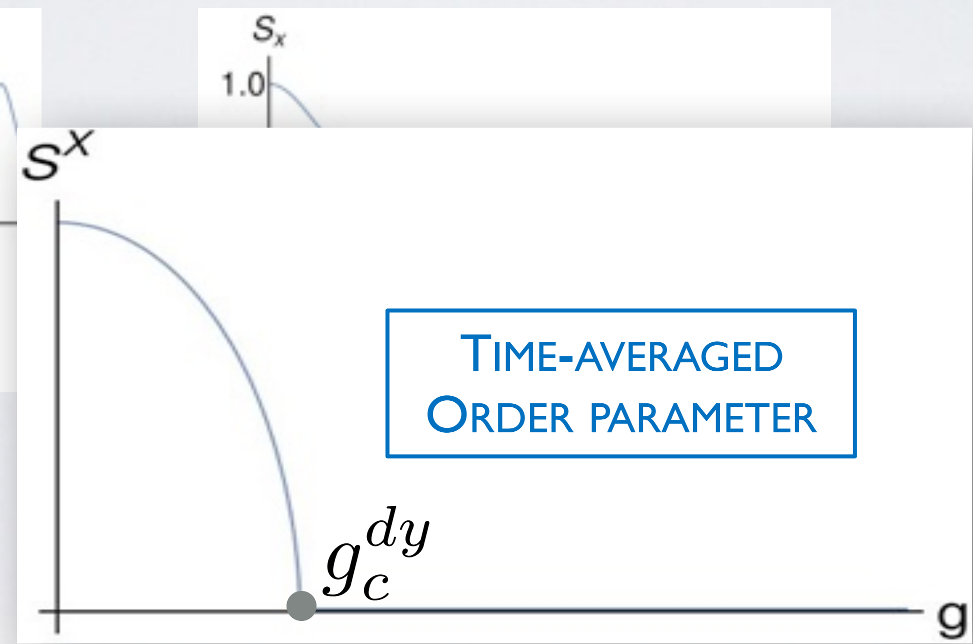
ferromagn. GS

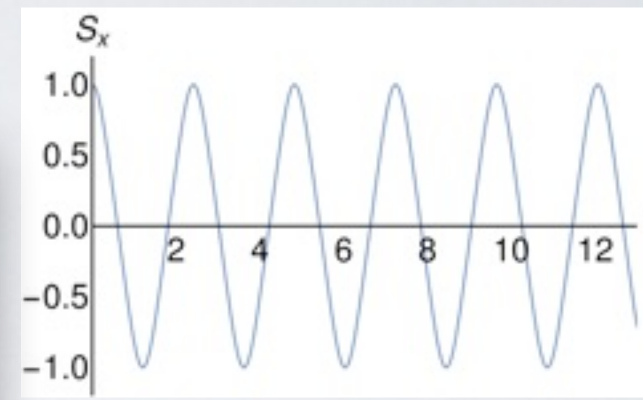
$$S^{(\alpha)} \equiv \sum_i \sigma_i^{(\alpha)}$$



Dy. Ferro. Phase

$g < g_c^{dy}$





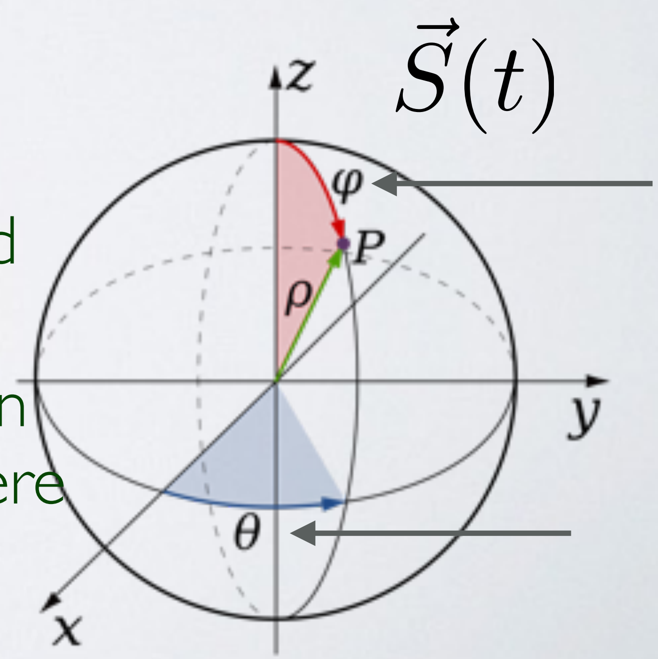
Dy. Demagn. Phase

$g > g_c^{dy}$

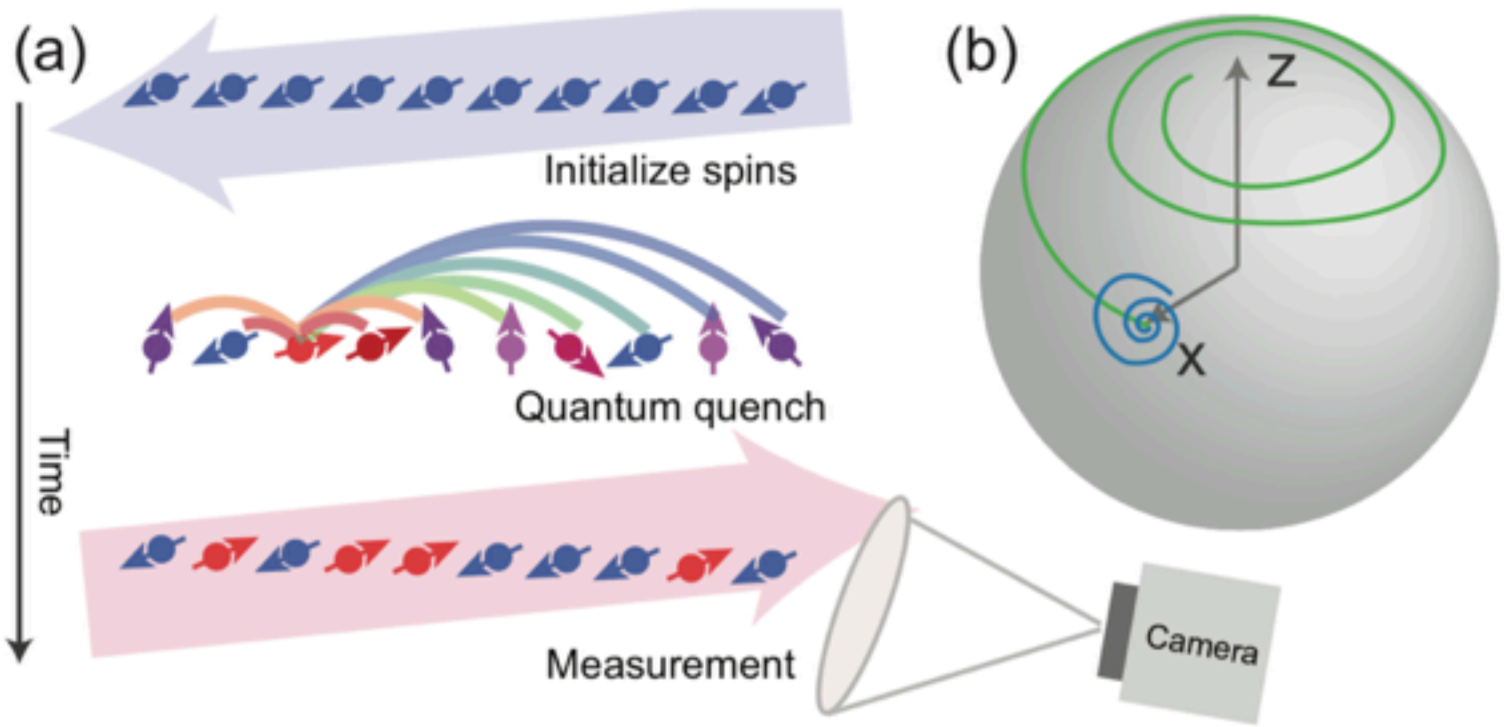
$$m_{(\alpha)} = S_{(\alpha)}/N$$
$$[m_x, m_y] = im_z/N$$

$N \to \infty$

In the thermodynamic limit, mean-field exactly solves the Lipkin model; just **two classical** equations of motion for the polar angles on the Bloch sphere spanned by $\vec{S}(t)$

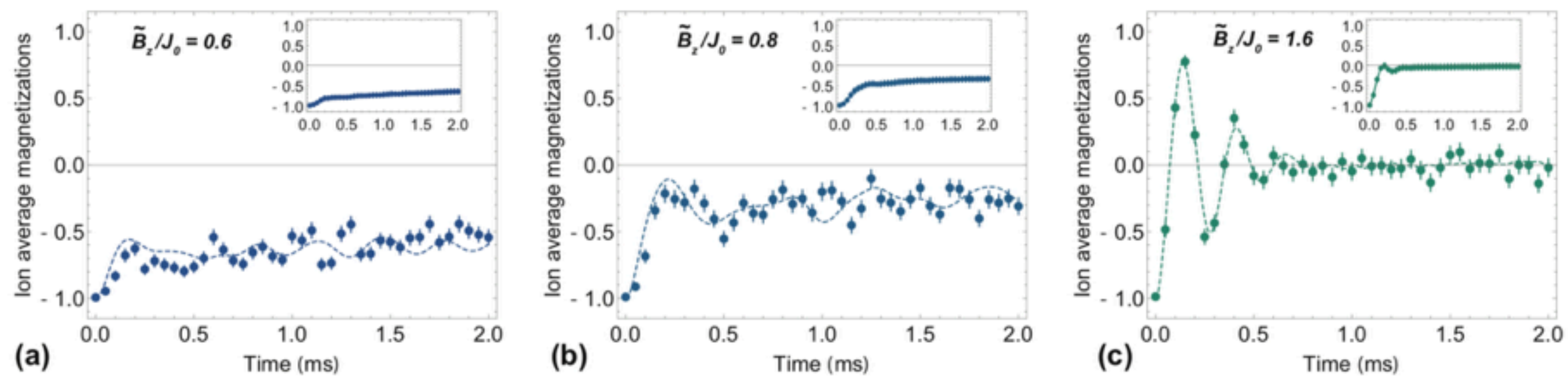


Experiment @ C. Monroe's lab with trapped ion spins (Nature '17)

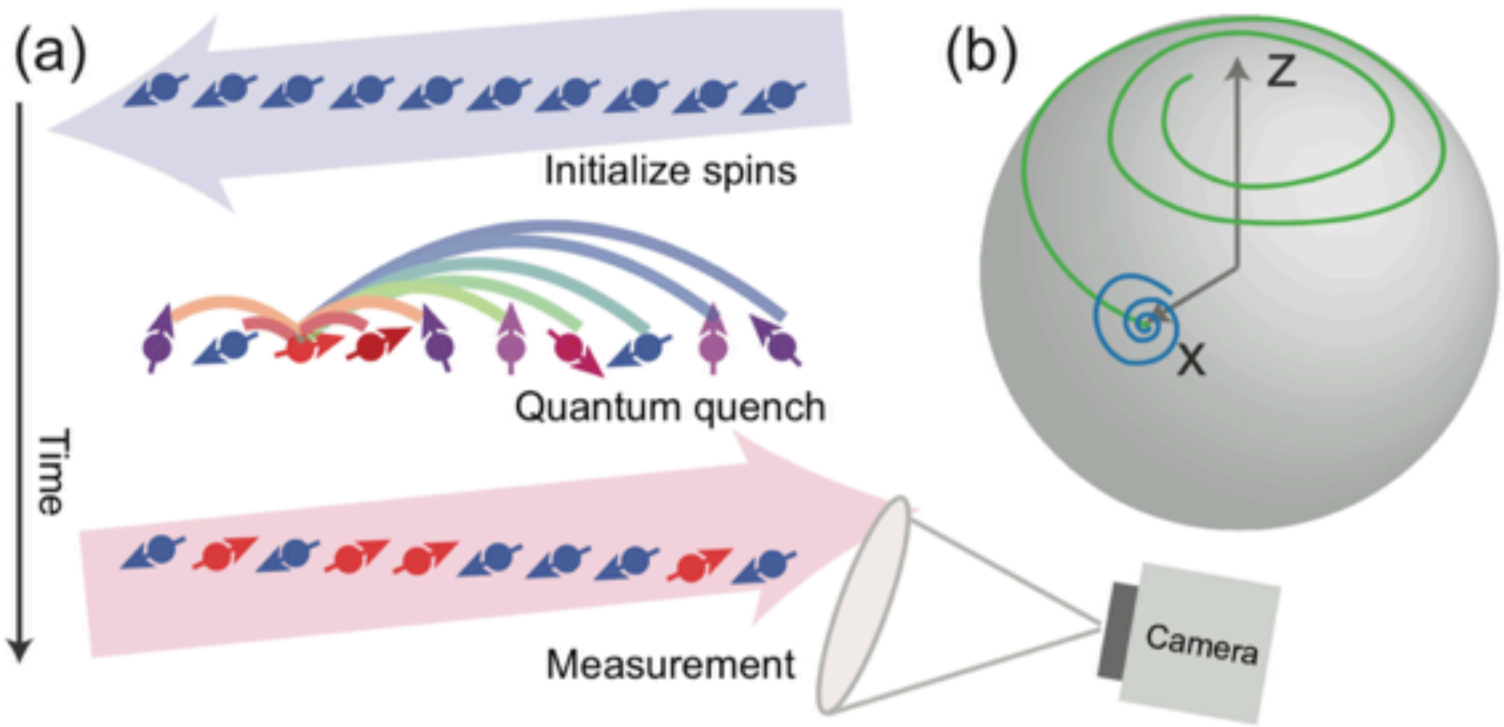


$$J_{ij} \simeq \frac{J_0}{|i-j|^\alpha}$$

$$0.8 < \alpha < 1$$



Experiment @ C. Monroe's lab with trapped ion spins (Nature '17)



$$J_{ij} \simeq \frac{J_0}{|i-j|^{\alpha}}$$

$$0.8 < \alpha < 1$$

DPT with ultracold fermions

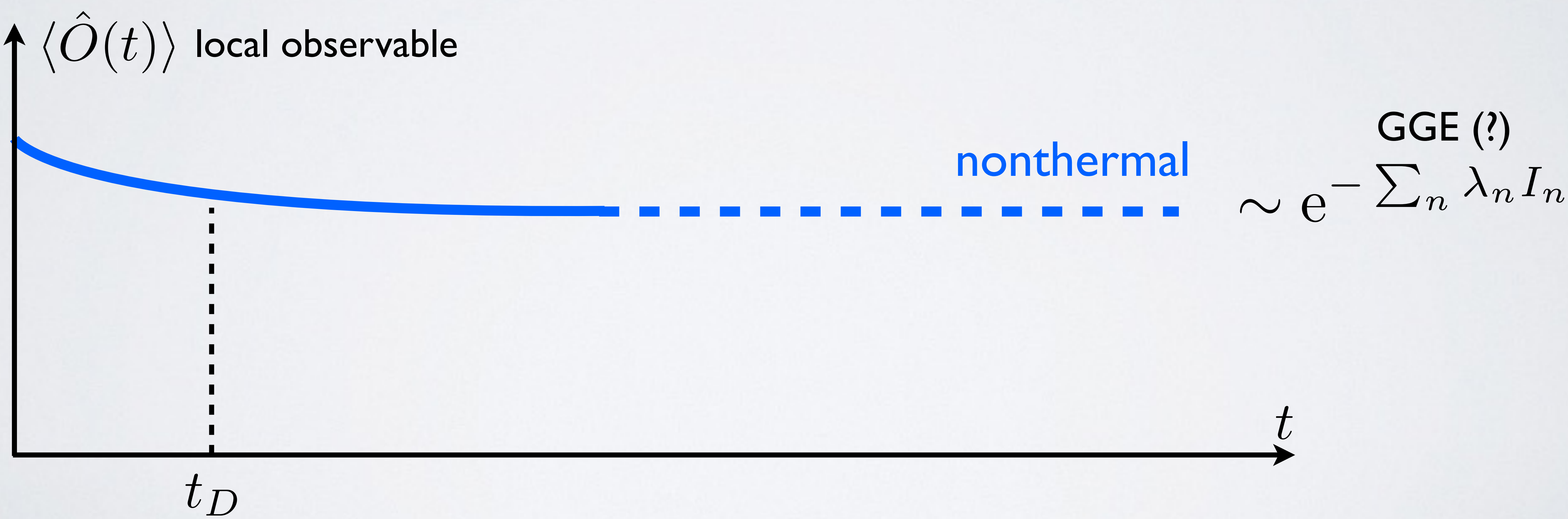
S. Smale, P. He, B. Olsen, K. Jackson, H. Sharum, S. Trotzky, **JM**, A. M. Rey, J. Thywissen, ArXiv 1806.11044 (2018)

$$H = J \sum_{i,j} \sigma_i^+ \sigma_j^- + \sum_i h_i \sigma_i^z$$

Pre-thermalization

$\{I_n\}_n$ **integrable** $H_0(g_i) \rightarrow H_0(g_f)$

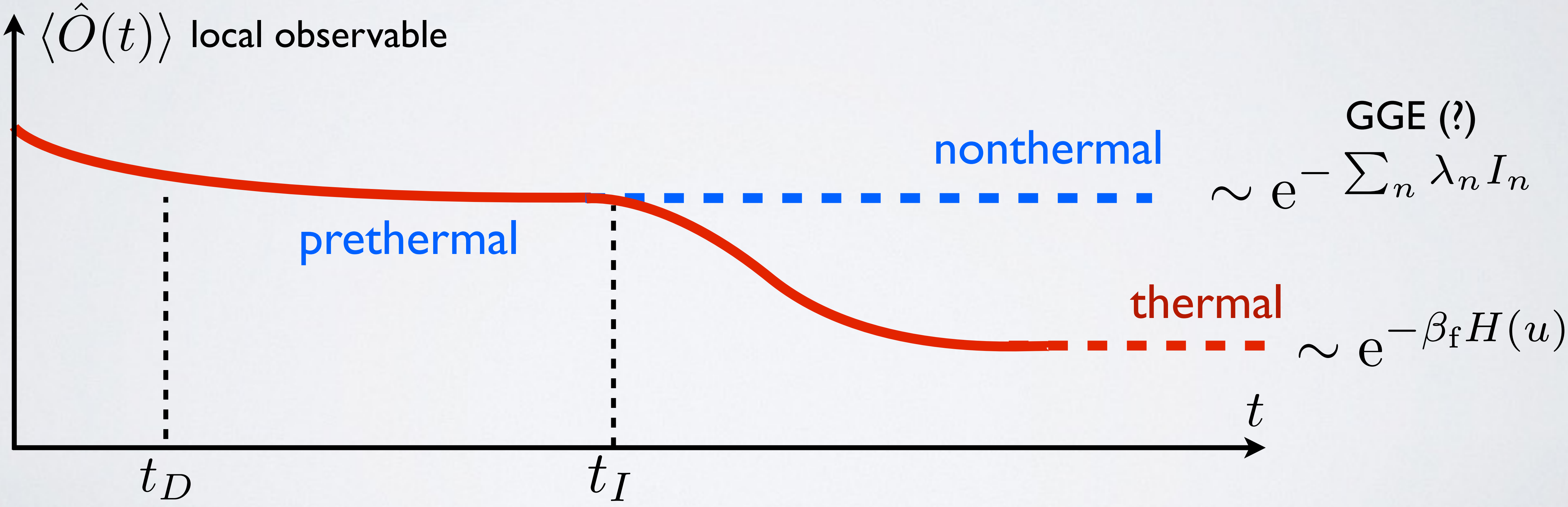
conserved local quantities



Pre-thermalization

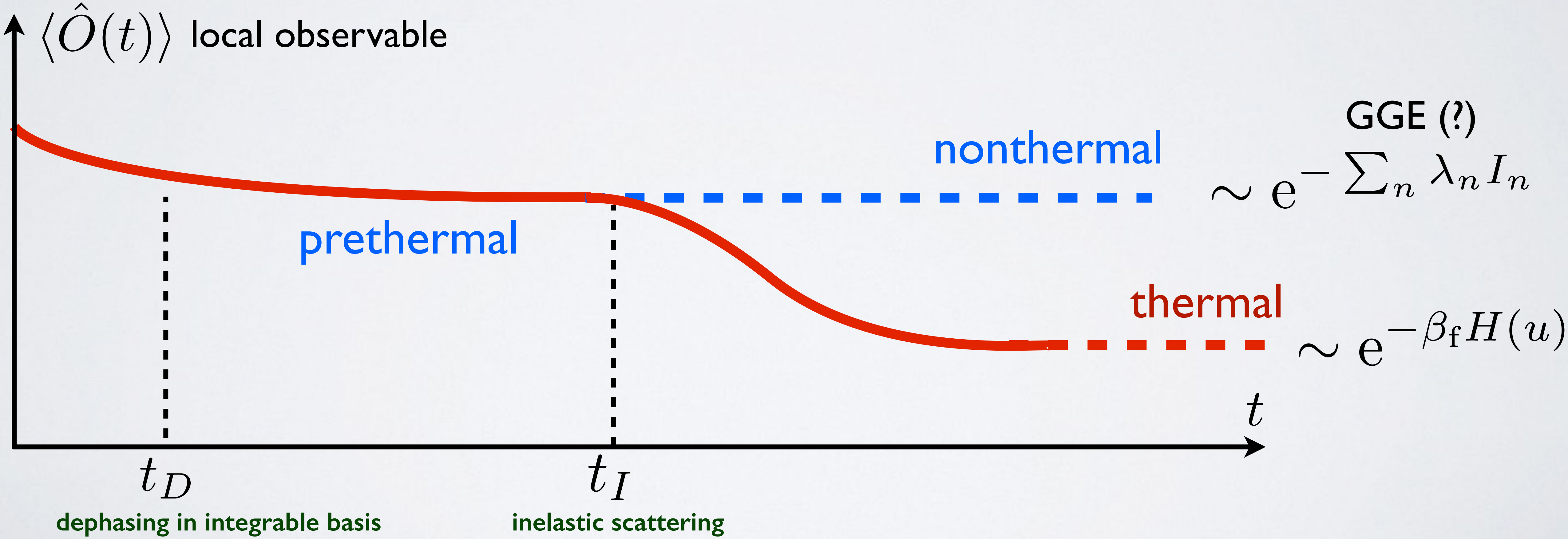
Pre-thermalization

$$H(g_i) \to H(g_f) + \underline{uV}$$



Pre-thermalization

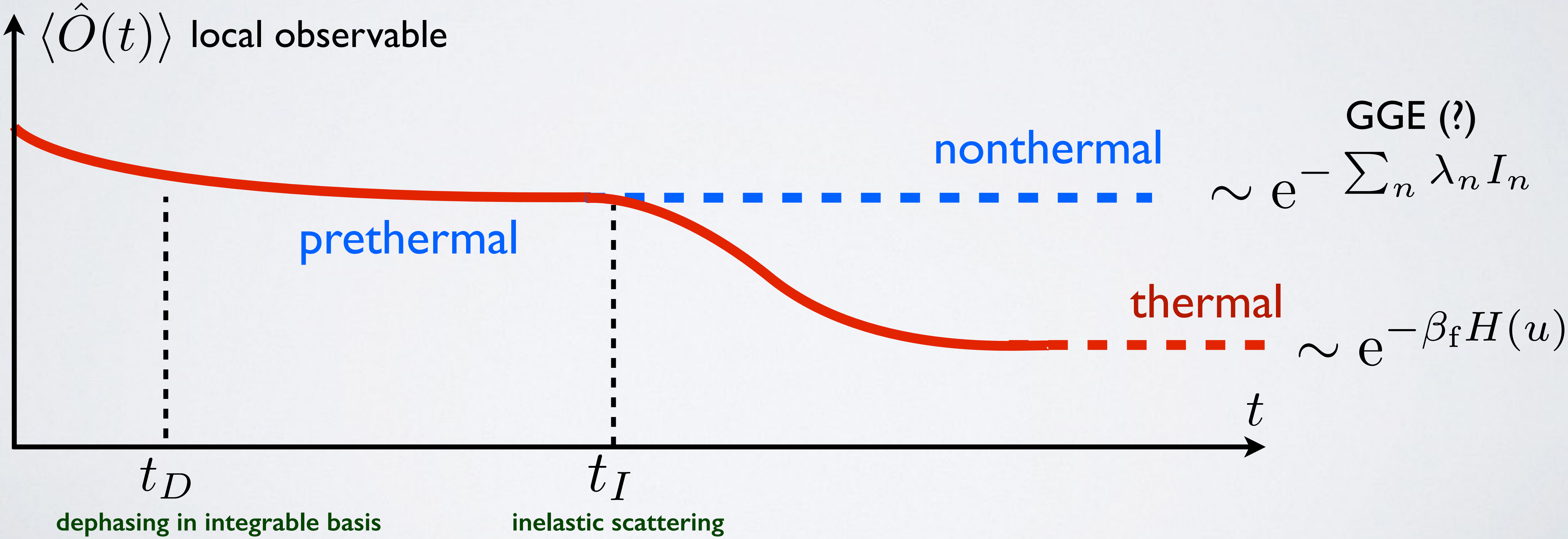
$$H(g_i) \to H(g_f) + \underline{uV}$$



Pre-thermalization

EXPERIMENTS: Schmiedmayer's group 2012-2015; REVIEW: Langen et al. (2016)

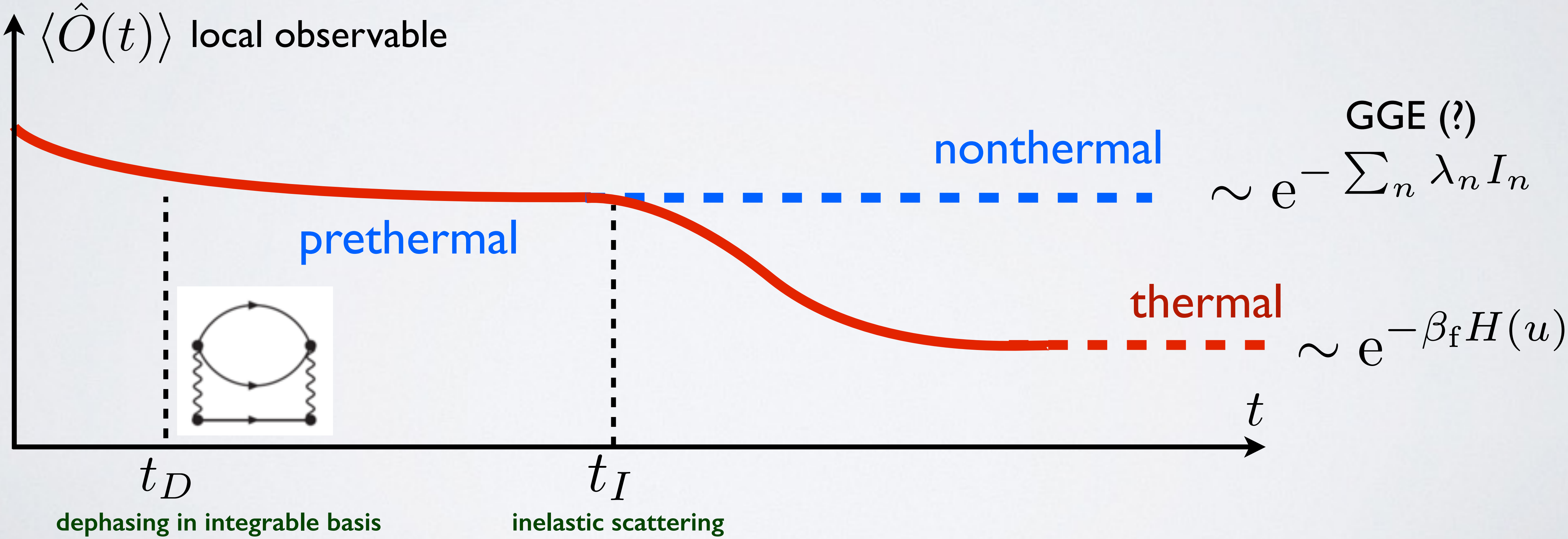
$$H(g_i) \to H(g_f) + \underline{uV}$$



Pre-thermalization

EXPERIMENTS: Schmiedmayer's group 2012-2015; REVIEW: Langen et al. (2016)

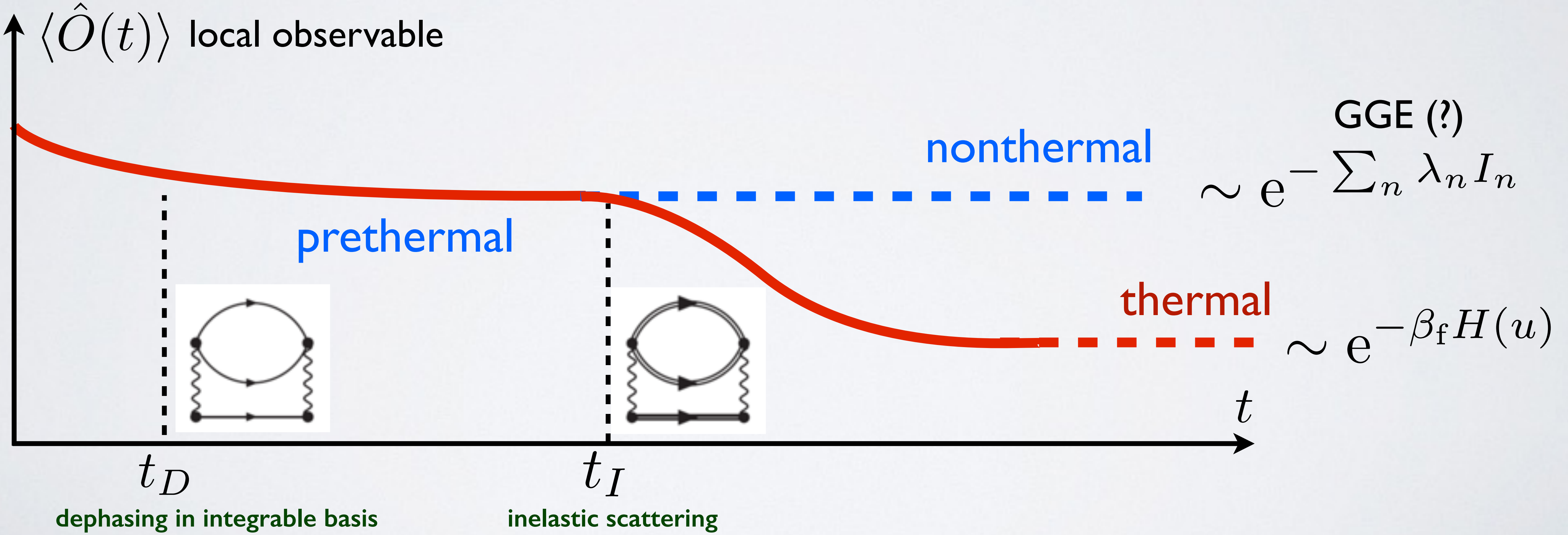
$$H(g_i) \to H(g_f) + \underline{uV}$$



Pre-thermalization

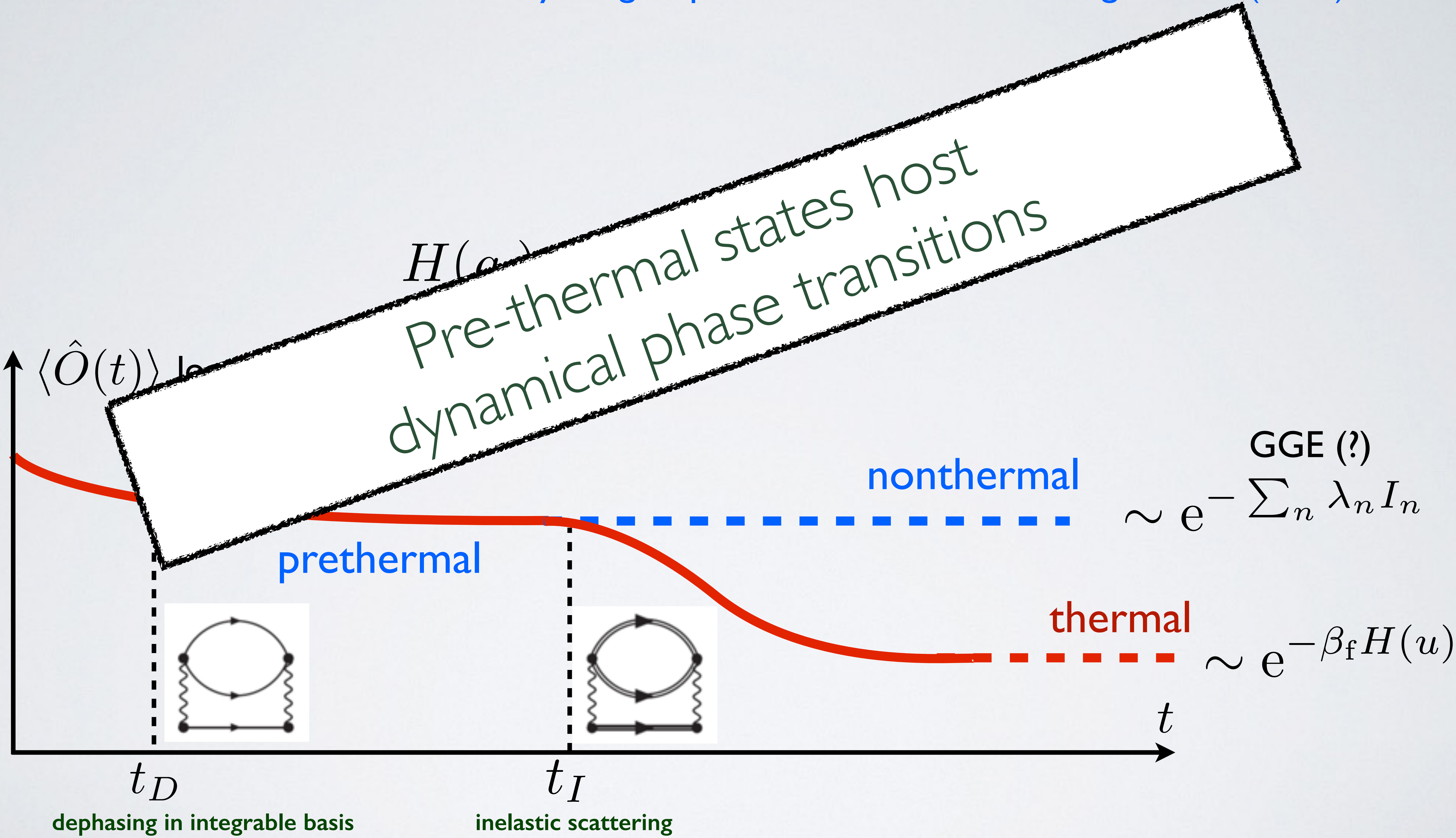
EXPERIMENTS: Schmiedmayer's group 2012-2015; REVIEW: Langen et al. (2016)

$$H(g_i) \to H(g_f) + \underline{uV}$$



Pre-thermalization

EXPERIMENTS: Schmiedmayer's group 2012-2015; REVIEW: Langen et al. (2016)



Message(s) of the talk:

Impact of quantum fluctuations out-of-equilibrium:

1) create novel phases close to dynamical critical point
(chaotic dynamical ferromagnet)

A. Lerose, **JM**, B. Zunkovic, A. Gambassi, A. Silva (PRL 2018)+ ArXiv1807.09797 (2018)

2) can be tamed by a periodic drive
(and stablise a many body Kapitza pendulum)

A. Lerose, **JM**, A. Gambassi, A. Silva, arXiv1803.04490 (2018)

Message(s) of the talk:

Impact of quantum fluctuations out-of-equilibrium:

1) create novel phases close to dynamical critical point
(chaotic dynamical ferromagnet)

A. Lerose, **JM**, B. Zunkovic, A. Gambassi, A. Silva (PRL 2018)+ ArXiv1807.09797 (2018)

2) can be tamed by a periodic drive
(and stablise a many body Kapitza pendulum)

A. Lerose, **JM**, A. Gambassi, A. Silva, arXiv1803.04490 (2018)

Fate of dynamical phase transition in a spin chain with (competing) collective and short range interactions

$t > 0$

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z - J\sum_i \sigma_i^x\sigma_{i+1}^x$$

Fate of dynamical phase transition in a spin chain with (competing) collective and short range interactions

$t > 0$

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z - J\sum_i \sigma_i^x\sigma_{i+1}^x$$

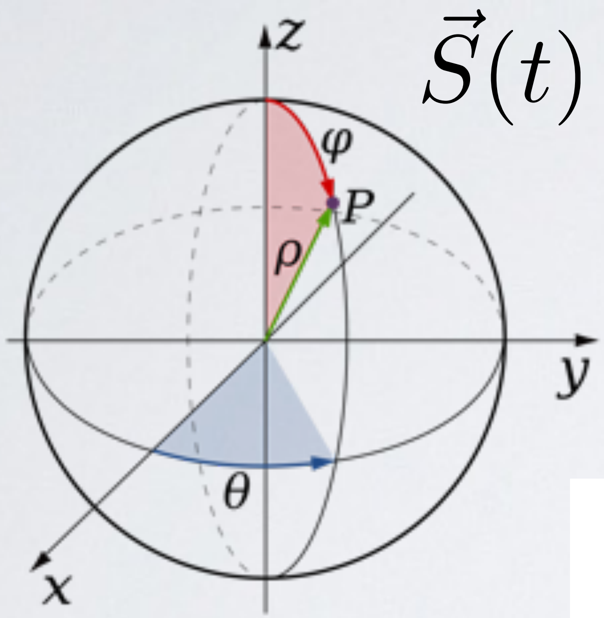
time-dependent spin-wave theory

Time-dependent spin wave theory

$$\vec{S}(t) = \sum_i \vec{\sigma}_i \qquad k = 0 \quad \text{Fourier mode of } \vec{\sigma}_i$$

Time-dependent spin wave theory

$$\vec{S}(t) = \sum_i \vec{\sigma}_i \qquad k = 0 \quad \text{Fourier mode of } \vec{\sigma}_i$$



$\vec{S}(t)$

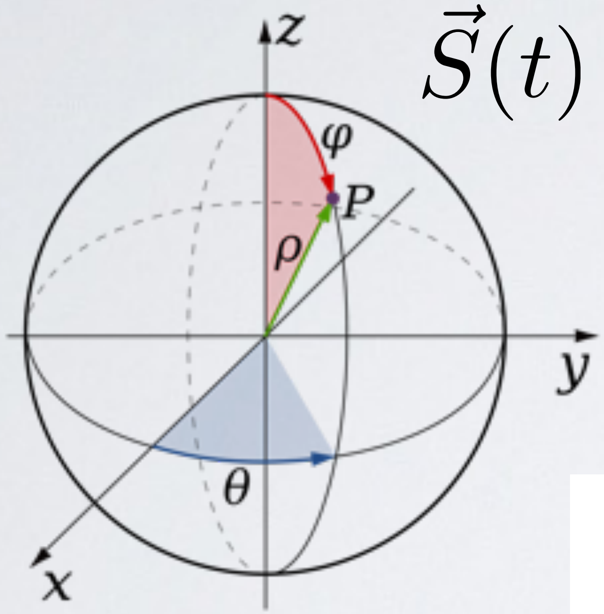
(X,Y,Z): time-dependent reference frame
with Z-axis following the motion of $\vec{S}(t)$

Holstein-Primakoff (lowest order; pre-thermal)

$$\frac{\sigma_i^X}{2} \simeq \sqrt{s}\, q_i, \quad \frac{\sigma_i^Y}{2} \simeq \sqrt{s}\, p_i, \quad \frac{\sigma_i^Z}{2} = s - \frac{q_i^2 + p_i^2 - 1}{2}$$

Time-dependent spin wave theory

$$\vec{S}(t) = \sum_i \vec{\sigma}_i \qquad k = 0 \quad \text{Fourier mode of } \vec{\sigma}_i$$



$\vec{S}(t)$

(X,Y,Z): time-dependent reference frame
with Z-axis following the motion of $\vec{S}(t)$

Holstein-Primakoff (lowest order; pre-thermal)

$$\frac{\sigma_i^X}{2} \simeq \sqrt{s}\, q_i, \quad \frac{\sigma_i^Y}{2} \simeq \sqrt{s}\, p_i, \quad \frac{\sigma_i^Z}{2} = s - \frac{q_i^2 + p_i^2 - 1}{2}$$

spin-wave density $\quad \epsilon(t) \ll 1$

Equations of motion

$$\frac{d}{dt}\theta = +4\bar{\lambda}(1-\epsilon)\sin\theta\cos\phi\sin\phi$$

$$-4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{pp}\right)\sin\theta\cos\phi\sin\phi$$

$$+4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\cos\theta\sin\theta\cos^2\phi,$$

$$\frac{d}{dt}\phi = -2g + 4\bar{\lambda}(1-\epsilon)\cos\theta\cos^2\phi$$

$$-4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qq}\right)\cos\theta\cos^2\phi$$

$$+4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\sin\phi\cos\phi$$

Two-point correlations of spin waves

$$\Delta_k^{qq}(t) \equiv \langle \tilde{q}_k(t)\tilde{q}_{-k}(t)\rangle\,,$$
$$\Delta_k^{pp}(t) \equiv \langle \tilde{p}_k(t)\tilde{p}_{-k}(t)\rangle\,,$$
$$\Delta_k^{qp}(t) \equiv \frac{1}{2}\langle \tilde{q}_k(t)\tilde{p}_{-k}(t) + \tilde{p}_k(t)\tilde{q}_{-k}(t)\rangle$$

$$\frac{d}{dt}\Delta_k^{qq} = +\left(+8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{qq} + \left(+8\bar{\lambda}\cos^2\phi - 8J\cos k\ \sin^2\phi\right)\Delta_k^{qp}$$

$$\frac{d}{dt}\Delta_k^{qp} = +\left(-4\bar{\lambda}\cos^2\phi + 4J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qq} + \left(+4\bar{\lambda}\cos^2\phi - 4J\cos k\ \sin^2\phi\right)\Delta_k^{pp}$$

$$\frac{d}{dt}\Delta_k^{pp} = +\left(-8\bar{\lambda}\cos^2\phi + 8J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qp} + \left(-8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{pp}$$

Equations of motion

$$\frac{d}{dt}\theta = +\,4\bar{\lambda}(1-\epsilon)\sin\theta\cos\phi\sin\phi$$

$$-\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{pp}\right)\sin\theta\cos\phi\sin\phi$$

$$+\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\cos\theta\sin\theta\cos^2\phi,$$

$$\frac{d}{dt}\phi = -\,2g + 4\bar{\lambda}(1-\epsilon)\cos\theta\cos^2\phi$$

$$-\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qq}\right)\cos\theta\cos^2\phi$$

$$+\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\sin\phi\cos\phi$$

$\vec{S}(t) \Longleftrightarrow$ self-generated bath

order parameter — quantum fluctuations

Two-point correlations of spin waves

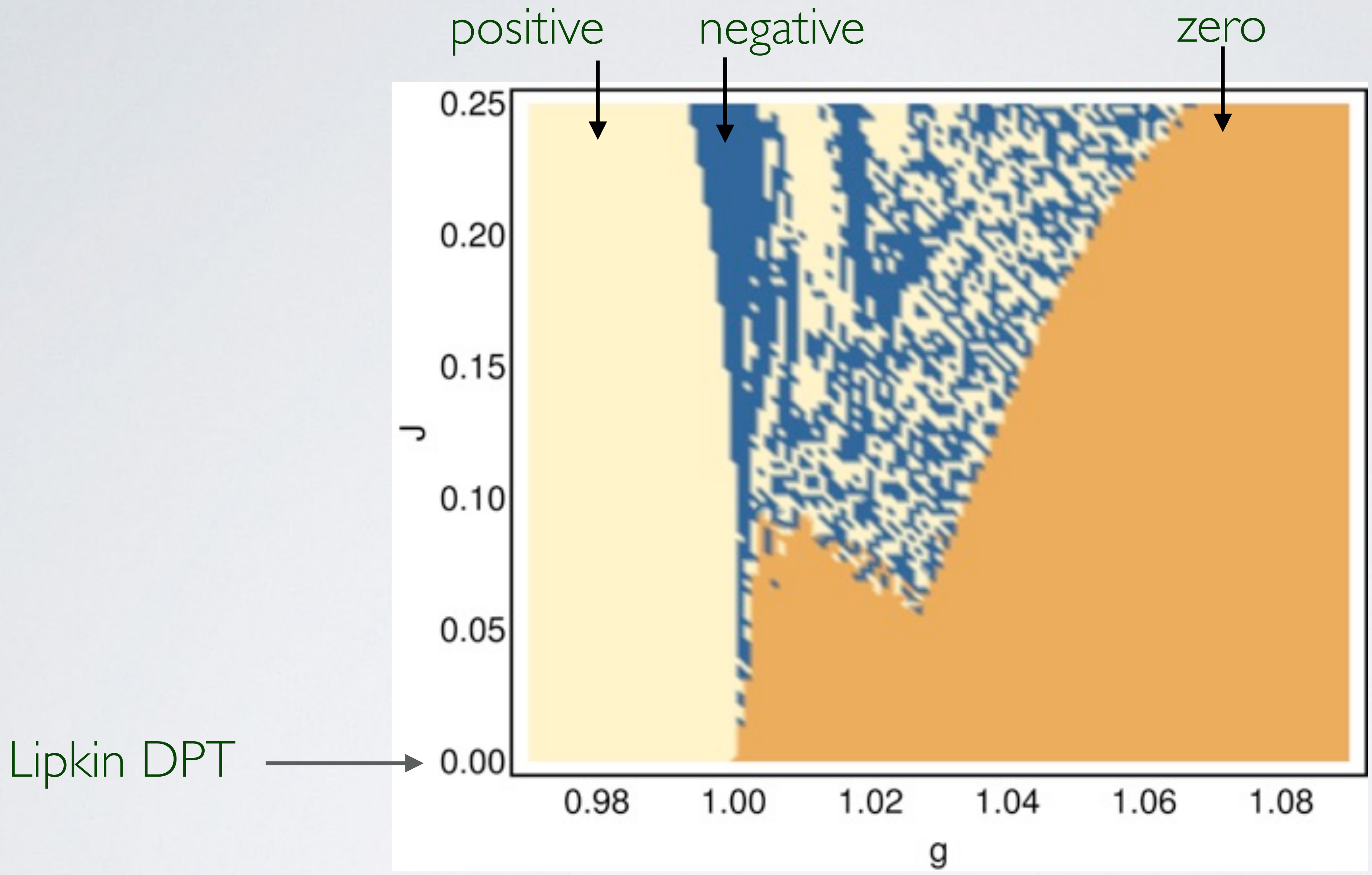
$$\Delta_k^{qq}(t) \equiv \langle \tilde{q}_k(t)\tilde{q}_{-k}(t)\rangle\,,$$
$$\Delta_k^{pp}(t) \equiv \langle \tilde{p}_k(t)\tilde{p}_{-k}(t)\rangle\,,$$
$$\Delta_k^{qp}(t) \equiv \frac{1}{2}\,\langle \tilde{q}_k(t)\tilde{p}_{-k}(t) + \tilde{p}_k(t)\tilde{q}_{-k}(t)\rangle$$

$$\frac{d}{dt}\Delta_k^{qq} = +\left(+\,8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{qq} + \left(+\,8\bar{\lambda}\cos^2\phi - 8J\cos k\ \sin^2\phi\right)\Delta_k^{qp}$$

$$\frac{d}{dt}\Delta_k^{qp} = +\left(-\,4\bar{\lambda}\cos^2\phi + 4J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qq} + \left(+\,4\bar{\lambda}\cos^2\phi - 4J\cos k\ \sin^2\phi\right)\Delta_k^{pp}$$

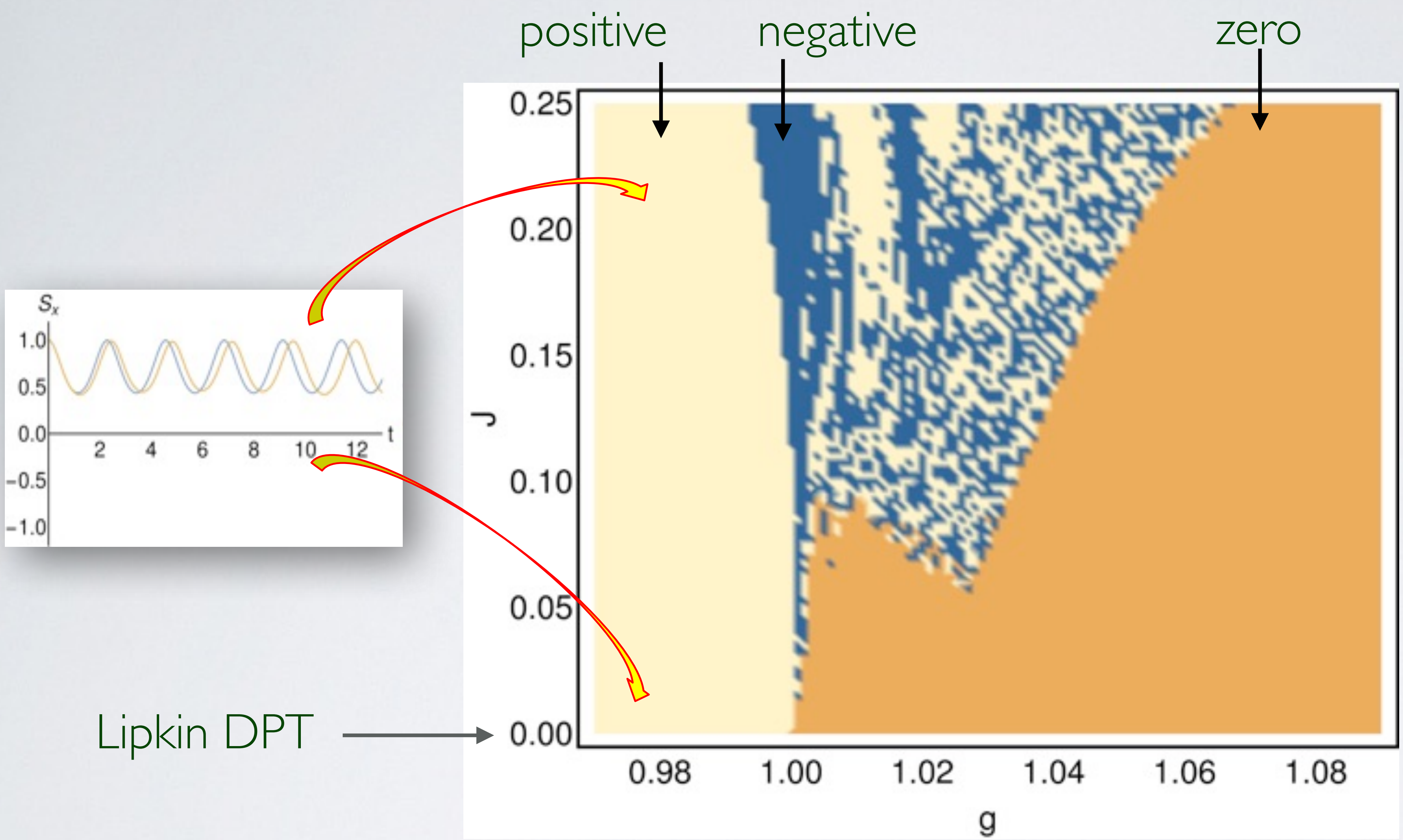
$$\frac{d}{dt}\Delta_k^{pp} = +\left(-\,8\bar{\lambda}\cos^2\phi + 8J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qp} + \left(-\,8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{pp}$$

Dynamical phase diagram including short-range interactions



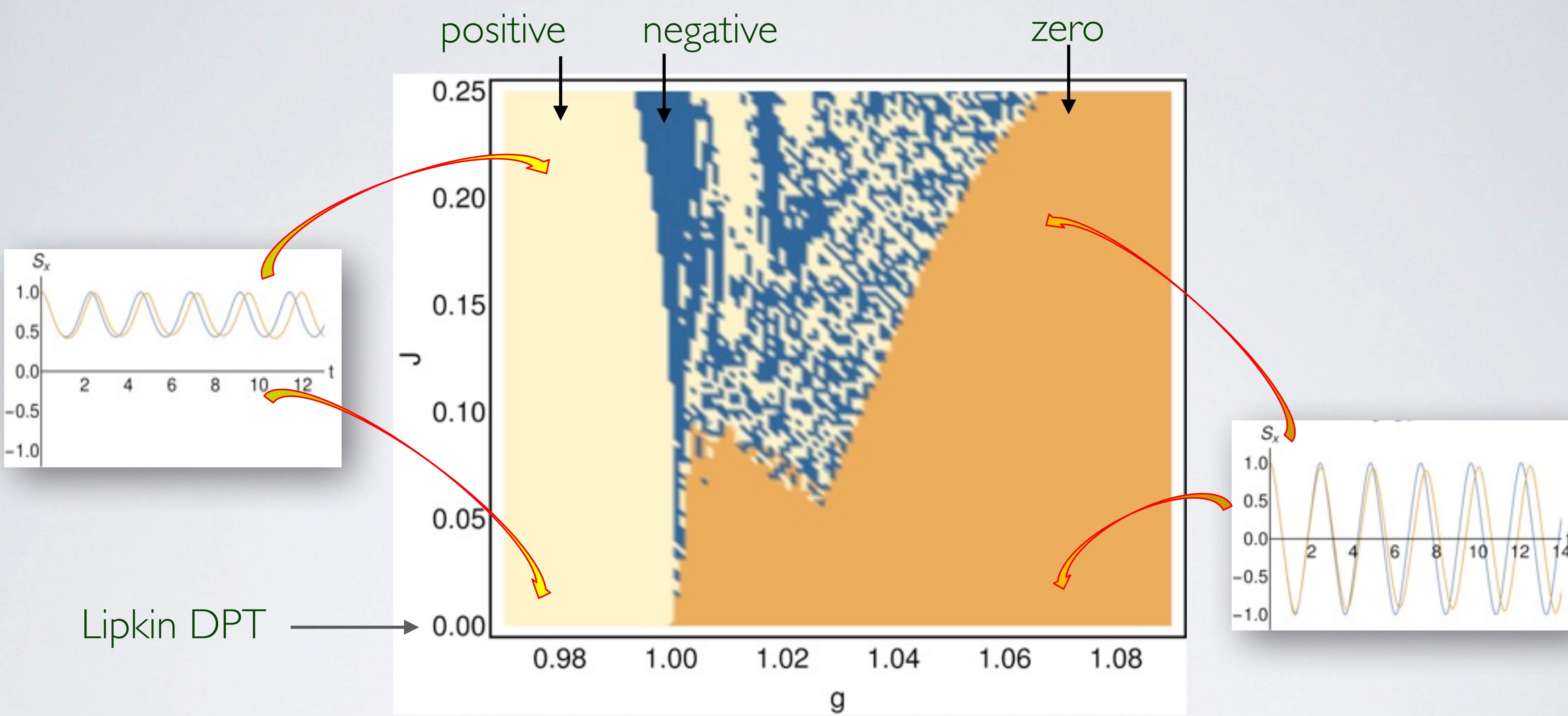
- Dynamical ferromagnetic and paramagnetic phases are robust
- At the dy. critical point quantum fluctuations shape a new ‘mosaic’ phase

Dynamical phase diagram including short-range interactions



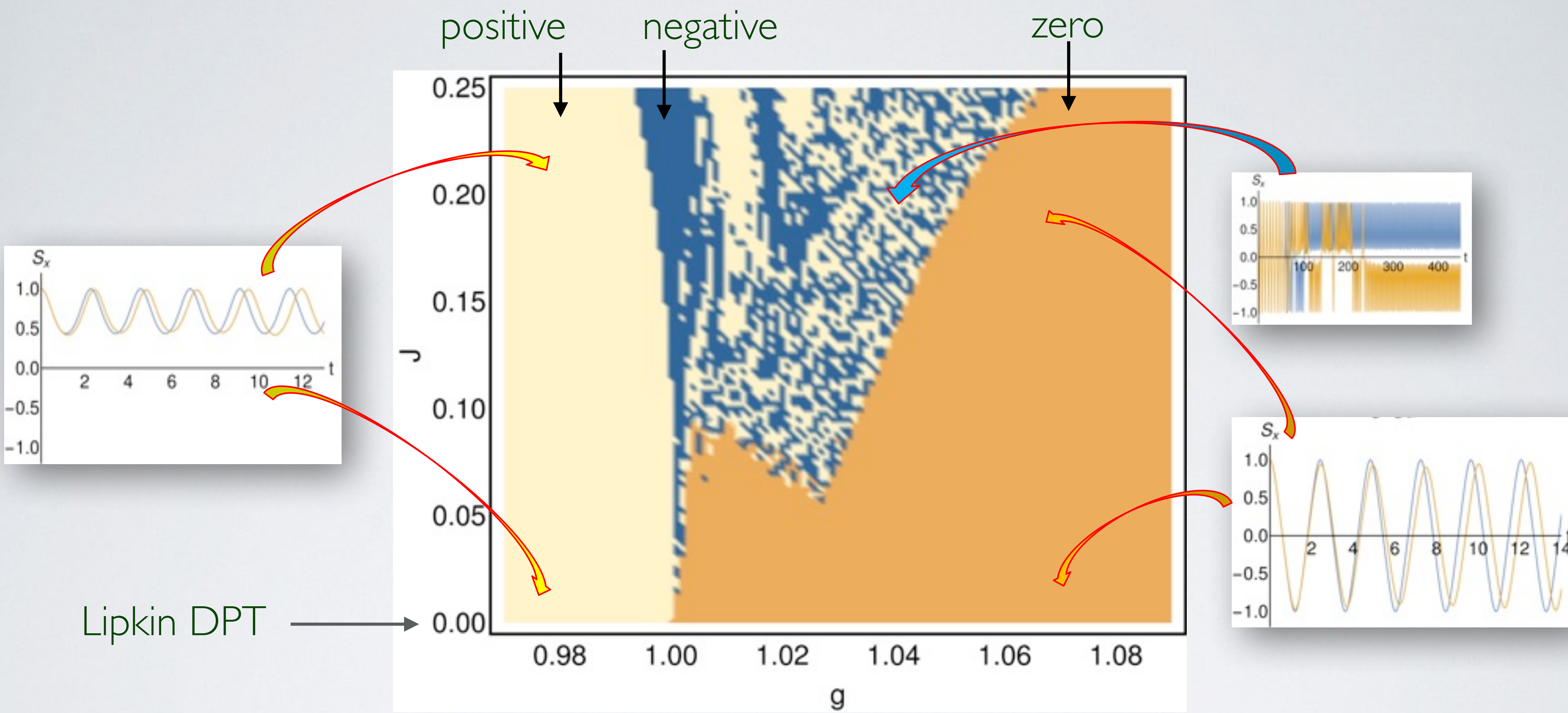
- Dynamical ferromagnetic and paramagnetic phases are robust
- At the dy. critical point quantum fluctuations shape a new 'mosaic' phase

Dynamical phase diagram including short-range interactions



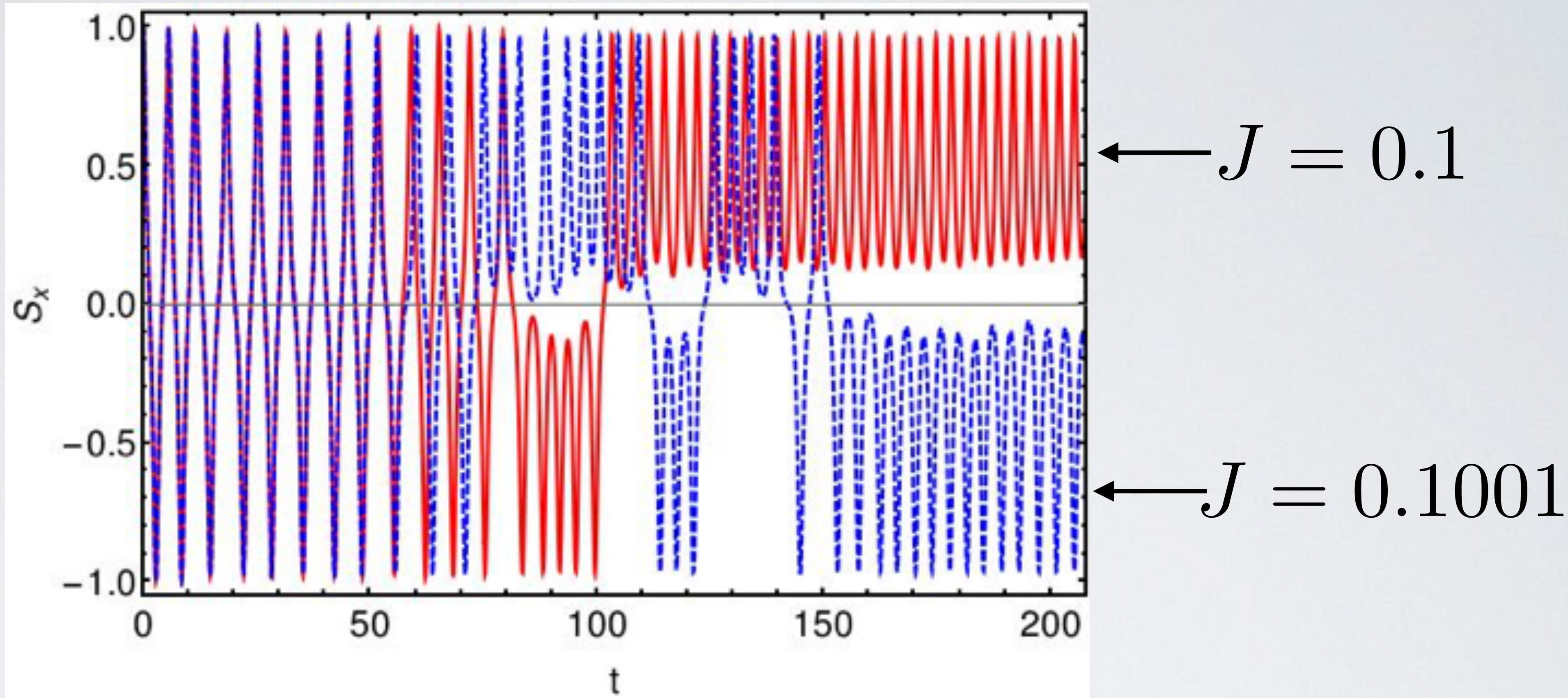
- Dynamical ferromagnetic and paramagnetic phases are robust
- At the dy. critical point quantum fluctuations shape a new 'mosaic' phase

Dynamical phase diagram including short-range interactions

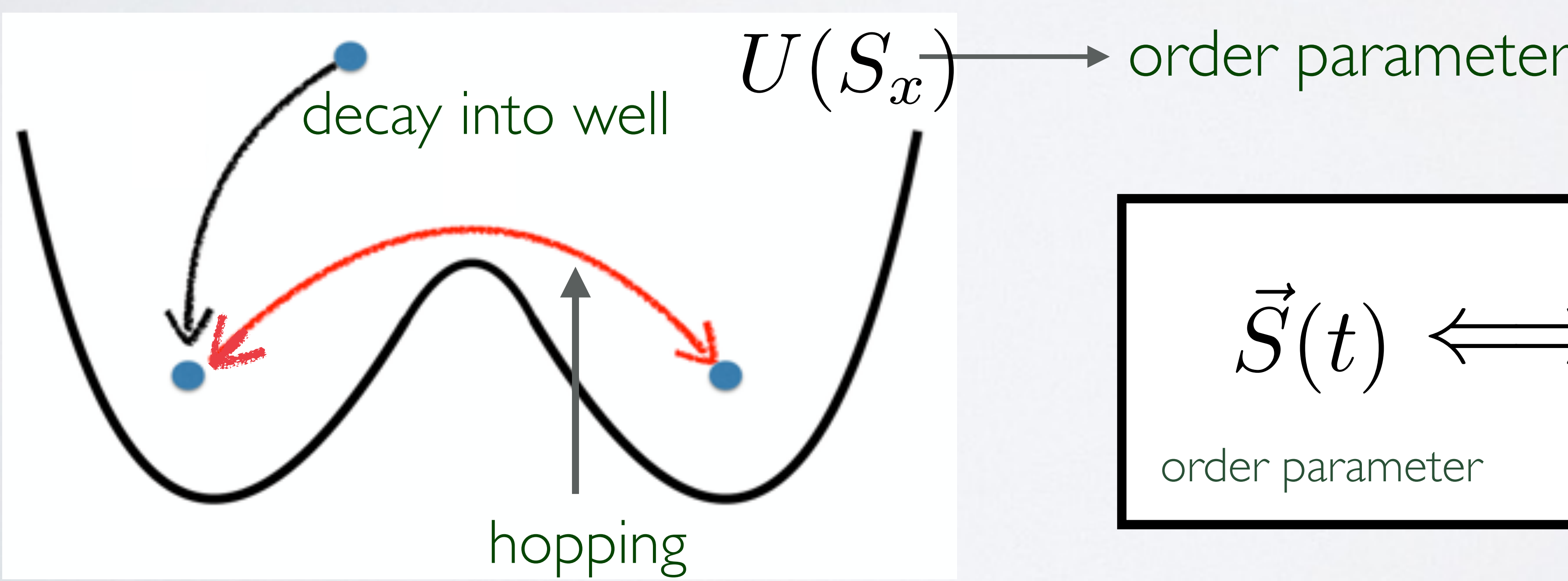


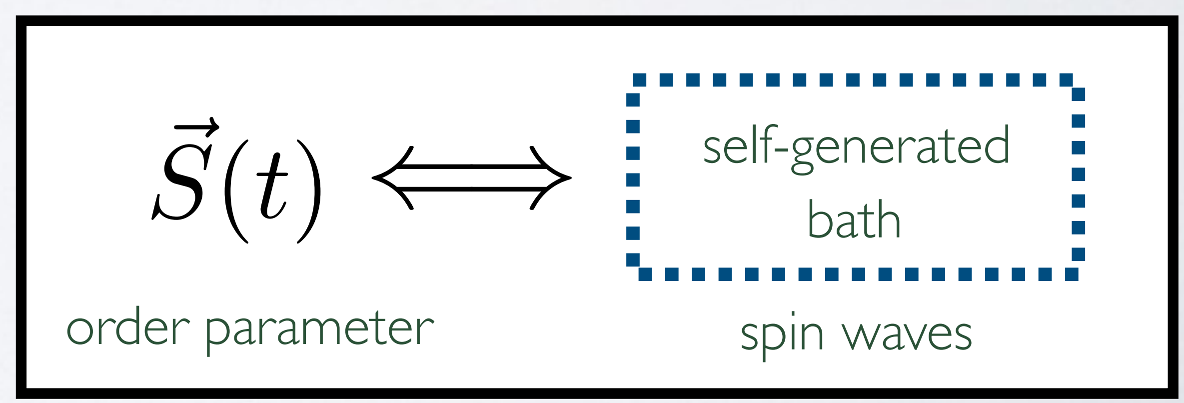
- Dynamical ferromagnetic and paramagnetic phases are robust
- At the dy. critical point quantum fluctuations shape a new ‘mosaic’ phase

Chaotic dynamical ferromagnetic phase

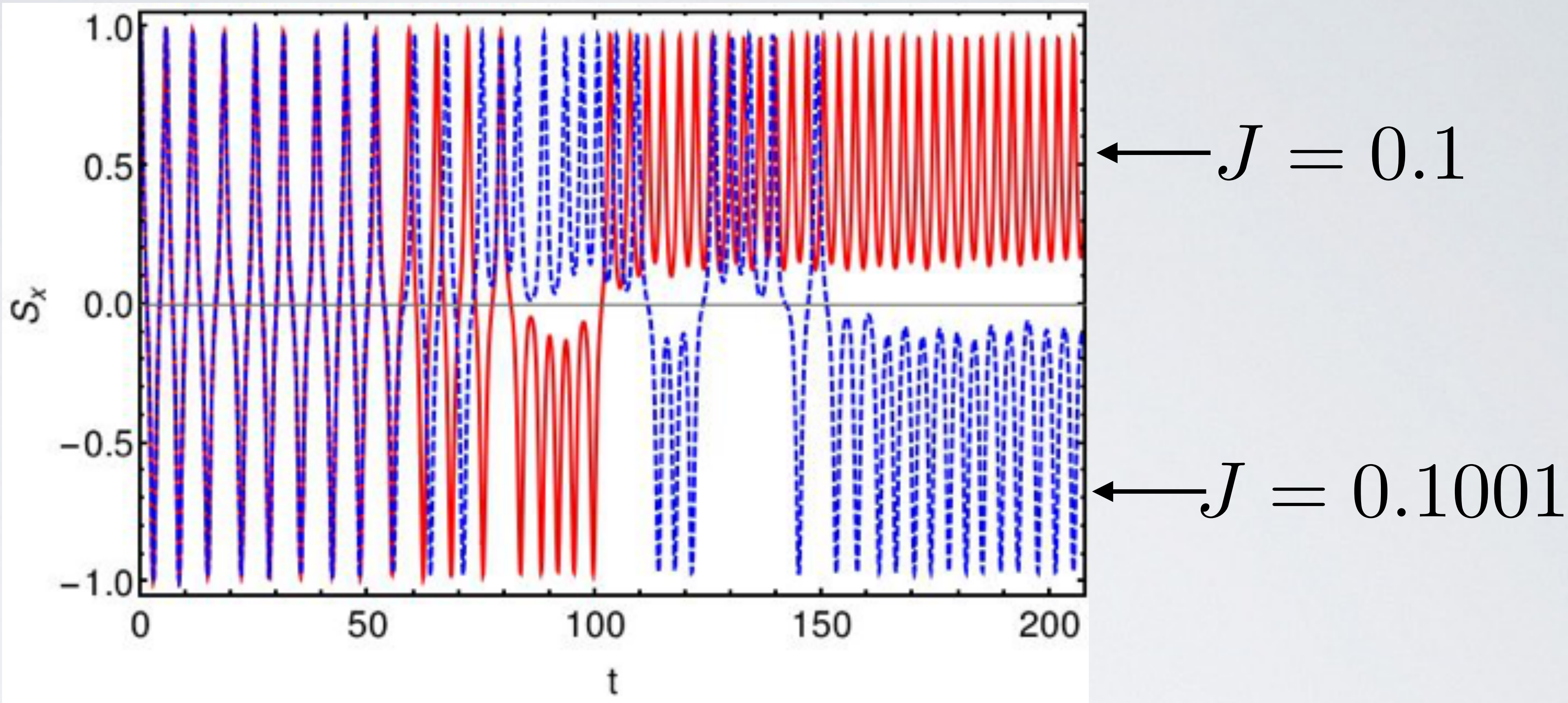


Double well potential for collective spin

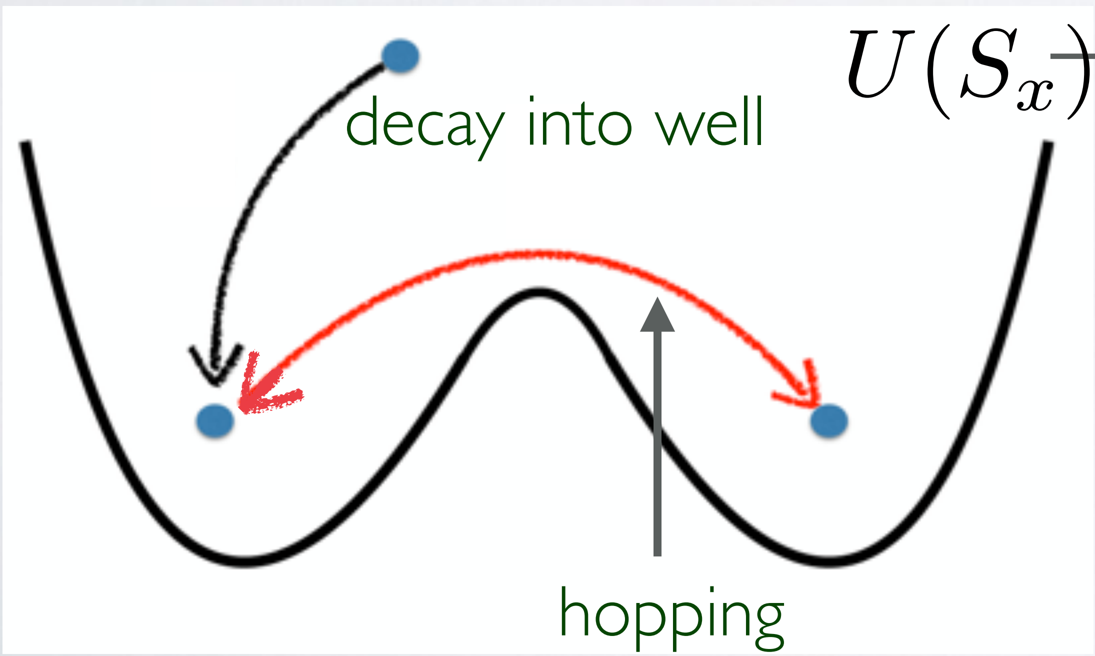




Chaotic dynamical ferromagnetic phase



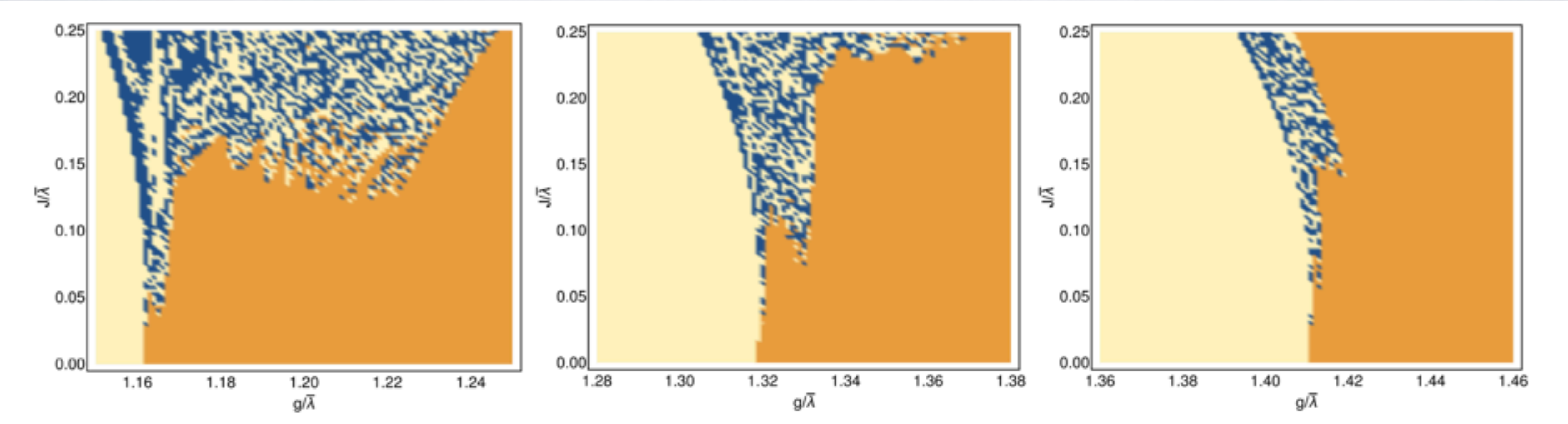
Double well potential for collective spin



Ramps

$$g(t) = \begin{cases} g_0, & \text{if } t \le 0; \\ g_0 + (g - g_0)\frac{t}{\tau}, & \text{if } 0 \le t \le \tau; \\ g, & \text{if } t \ge \tau. \end{cases}$$

$\bar{\lambda}\tau = 0.7$ $\bar{\lambda}\tau = 1$ $\bar{\lambda}\tau = 1.15$



Position of critical point shifts
from non-equilibrium (fast ramp) to equilibrium value (slow ramp)

Chaotic dynamical phase: a general phenomenon

1) Anisotropic Lipkin model + anisotropic short range interactions

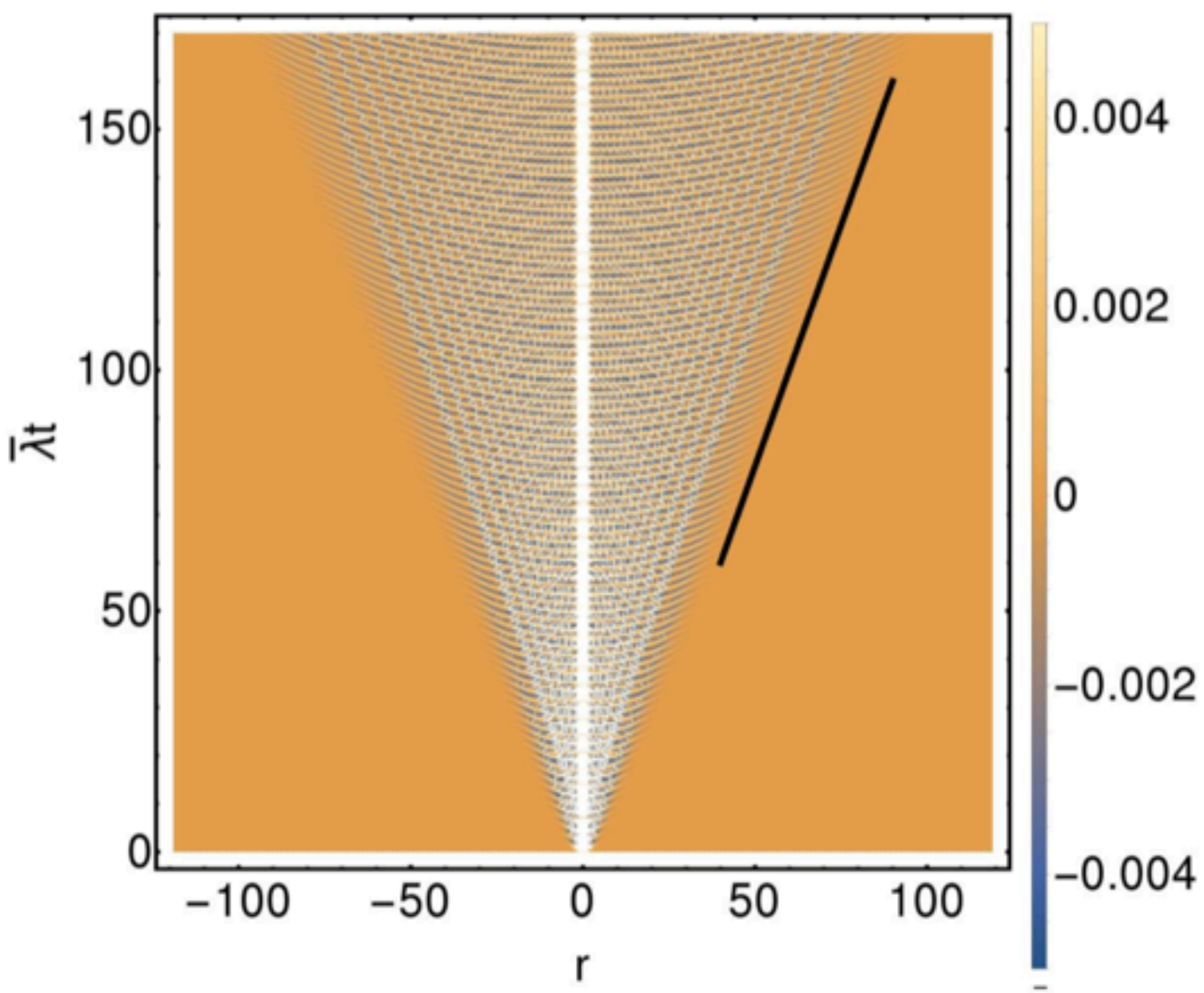
$$
\begin{aligned}
H_{XYZ} = & -\frac{\lambda}{N}\sum_{i,j}\left(\sigma_i^x\sigma_j^x + \alpha_y\sigma_i^y\sigma_j^y + \alpha_z\sigma_i^z\sigma_j^z\right) \\
& - g\sum_i \sigma_i^z \\
& - J\sum_i \left(\sigma_i^x\sigma_{i+1}^x + \alpha_y\sigma_i^y\sigma_{i+1}^y + \alpha_z\sigma_i^z\sigma_{i+1}^z\right)
\end{aligned}
$$

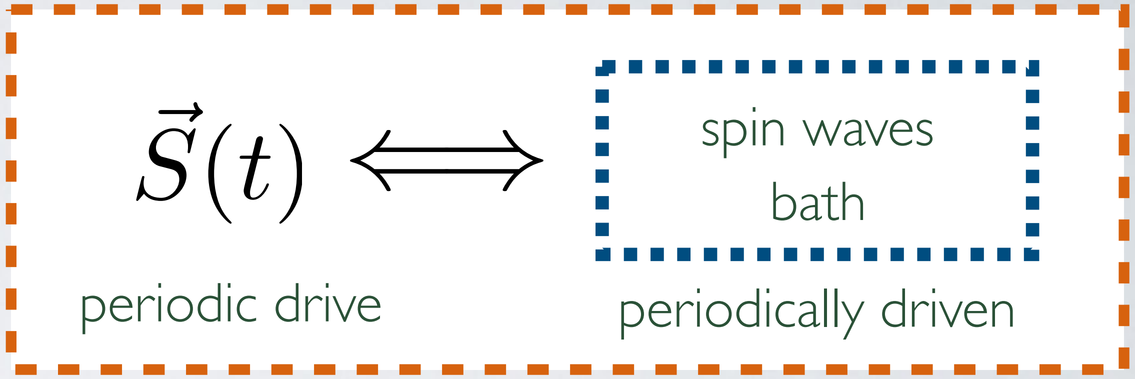
2) Lipkin model + second nearest neighbour interactions (or long-range interactions)

$$
H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g\sum_i\sigma_i^z - J\sum_{i,r} v(r)\sigma_i^x\sigma_{i+r}^x
$$

'Self periodically driven' light-cone

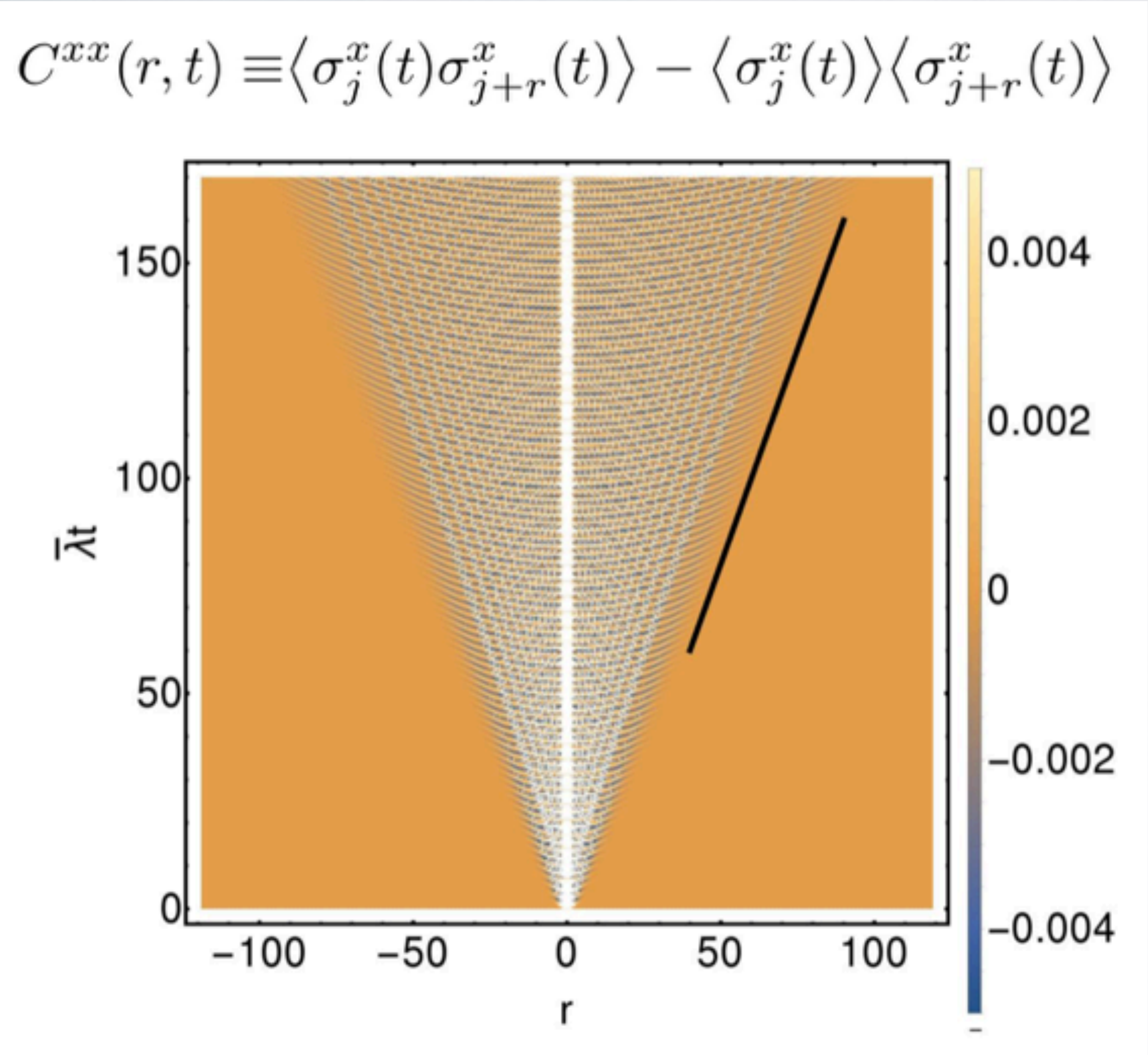
$$C^{xx}(r,t) \equiv \langle \sigma_j^x(t)\sigma_{j+r}^x(t)\rangle - \langle \sigma_j^x(t)\rangle\langle \sigma_{j+r}^x(t)\rangle$$

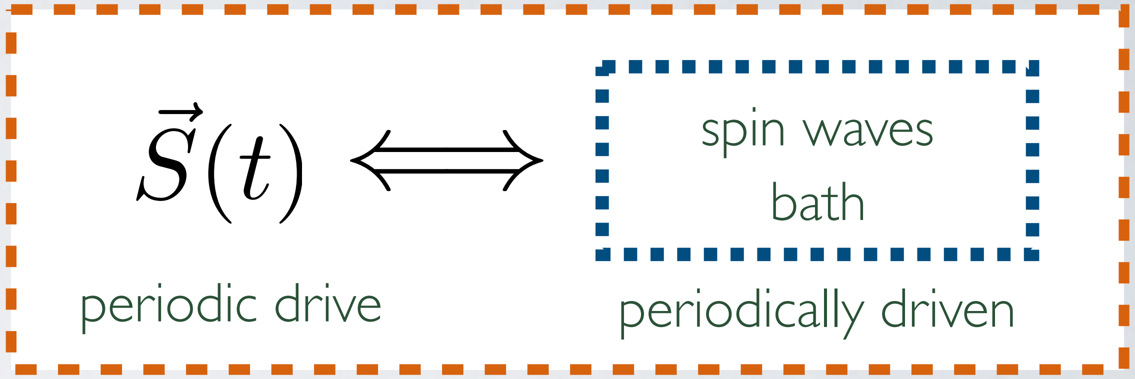




$$g/\bar{\lambda} = 3, J/\bar{\lambda} = 0.25, N = 240$$

'Self periodically driven' light-cone



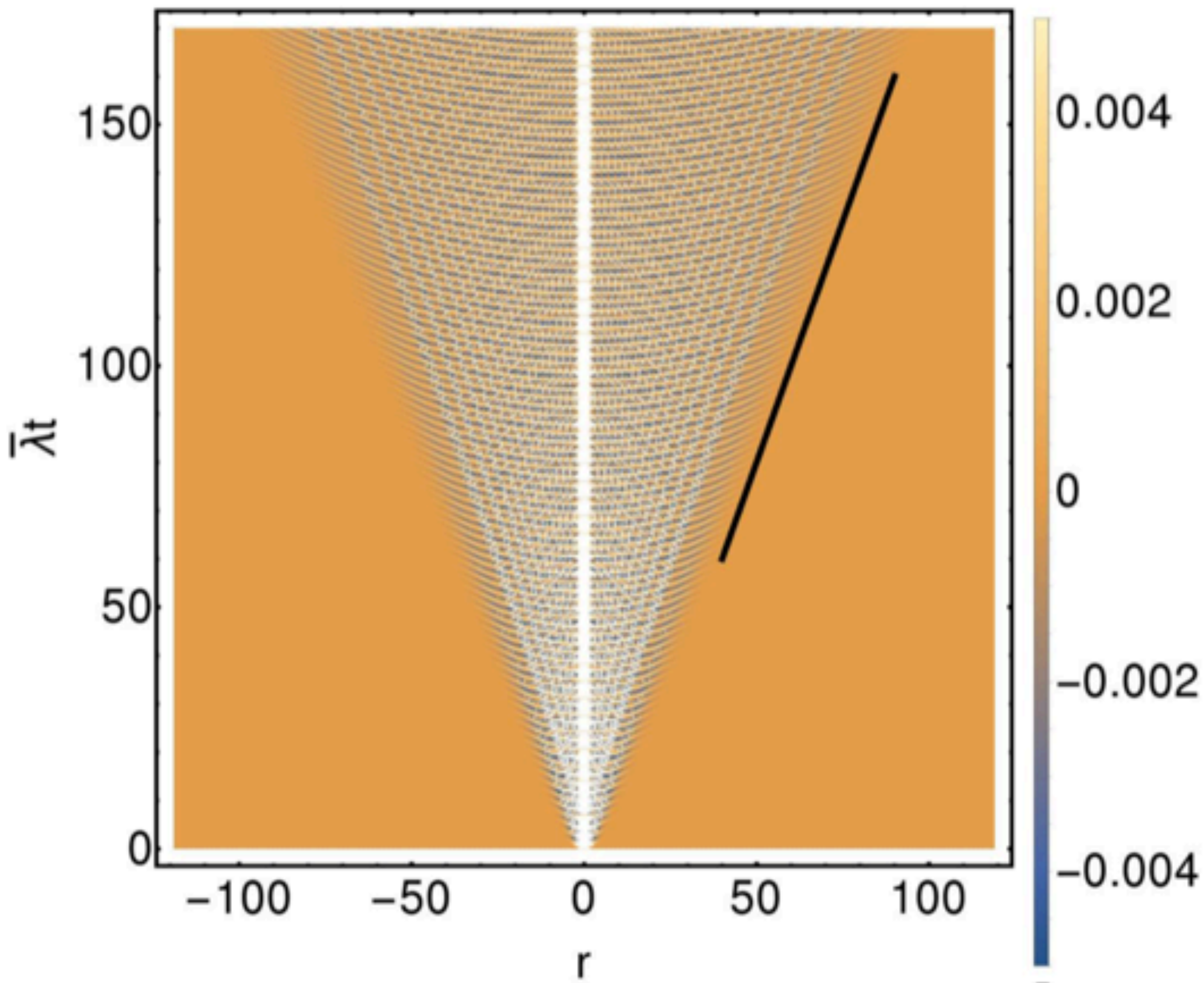


$$\phi(t) \simeq 2gt \quad \theta(t) \simeq \pi/2$$

$$g/\bar{\lambda} = 3, J/\bar{\lambda} = 0.25, N = 240$$

'Self periodically driven' light-cone

$$C^{xx}(r,t) \equiv \langle \sigma_j^x(t)\sigma_{j+r}^x(t)\rangle - \langle \sigma_j^x(t)\rangle\langle \sigma_{j+r}^x(t)\rangle$$



$\vec{S}(t) \Longleftrightarrow$ spin waves bath

periodic drive — periodically driven

$$\phi(t) \simeq 2gt \quad \theta(t) \simeq \pi/2$$

ham. of spin waves averaged
to lowest order in period of self-drive

$$\omega_k^{(\mathrm{eff})} = 4\sqrt{\bar{\lambda}(\bar{\lambda} - J\cos k)}$$

$$v_{\max} = \max_k \left| \frac{\partial \omega_k^{(\mathrm{eff})}}{\partial k} \right| \sim J$$

$$g/\bar{\lambda} = 3,\, J/\bar{\lambda} = 0.25,\, N = 240$$

Message(s) of the talk:

Impact of quantum fluctuations out-of-equilibrium:

1) create novel phases close to dynamical critical point
(chaotic dynamical ferromagnet)

A. Lerose, **JM**, B. Zunkovic, A. Gambassi, A. Silva (PRL 2018)+ ArXiv1807.09797 (2018)

2) can be tamed by a periodic drive
(and stablise a many body Kapitza pendulum)

A. Lerose, **JM**, A. Gambassi, A. Silva, arXiv1803.04490 (2018)

Message(s) of the talk:

Impact of quantum fluctuations out-of-equilibrium:

1) create novel phases close to dynamical critical point
(chaotic dynamical ferromagnet)

A. Lerose, **JM**, B. Zunkovic, A. Gambassi, A. Silva (PRL 2018)+ ArXiv1807.09797 (2018)

2) can be tamed by a periodic drive
(and stablise a many body Kapitza pendulum)

A. Lerose, **JM**, A. Gambassi, A. Silva, arXiv1803.04490 (2018)

Quantum many body Kapitza phases of periodically driven spin systems

$$H = -\sum_{i<j}^{N} \frac{J}{|i-j|^{\alpha}} \sigma_i^x \sigma_j^x - B \sum_{i=1}^{N} \sigma_i^z$$

$d = 1$

$$B(t) = B_0 + \delta B \cos(\Omega t)$$

Quantum many body Kapitza phases of periodically driven spin systems

$$H = -\sum_{i<j}^{N} \frac{J}{|i-j|^\alpha} \sigma_i^x \sigma_j^x - B \sum_{i=1}^{N} \sigma_i^z$$

$d = 1$

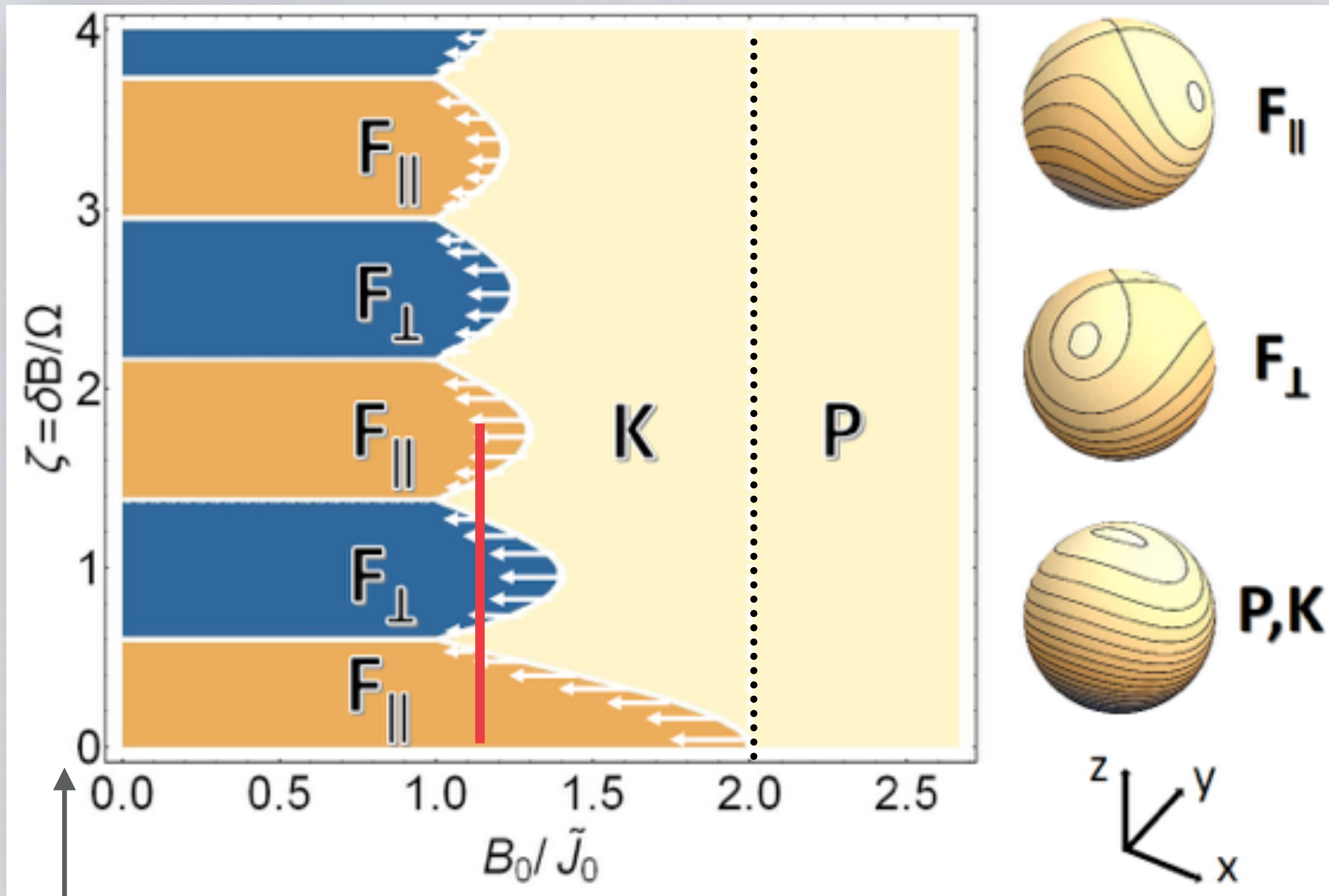
$$B(t) = B_0 + \delta B \cos(\Omega t)$$

$\alpha = 0$ Lipkin-Meshkov-Glick model

NEQ phase diagram under periodic drive (Lipkin)

$$H = -J \sum_{i,j} \sigma_i^x \sigma_j^x - B(t) \sum_i \sigma_i^z$$

$$B(t) = B_0 + \delta B \cos(\Omega t)$$



equilibrium phase diagram

Dynamical Phases

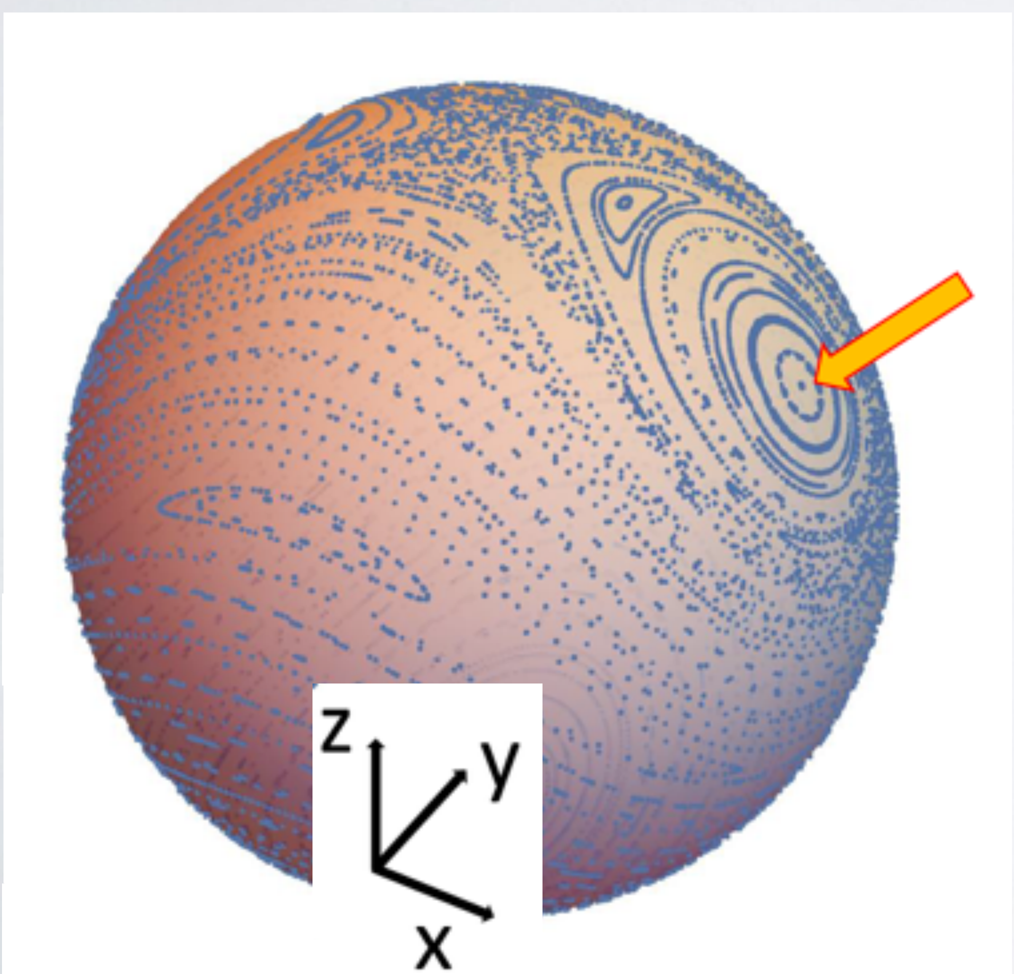
P	paramagnetic
K	Kapitza
$F_{\|\|}, F_\perp$	ferromagnetic

Dynamical ferromagnet
& chaotic trajectories around the unstable paramagnetic point

Stroboscopic trajectories of the order parameter on the Bloch sphere

$$\{\vec{S}(t_n)\}, \quad t_n = 2\pi n/\Omega, \quad n = 0, 1, 2, ...$$

$$\boxed{B_0/J_0 = 1.2, \quad \Omega/J_0 = 5}$$



periodic trajectories (from FM initial states)

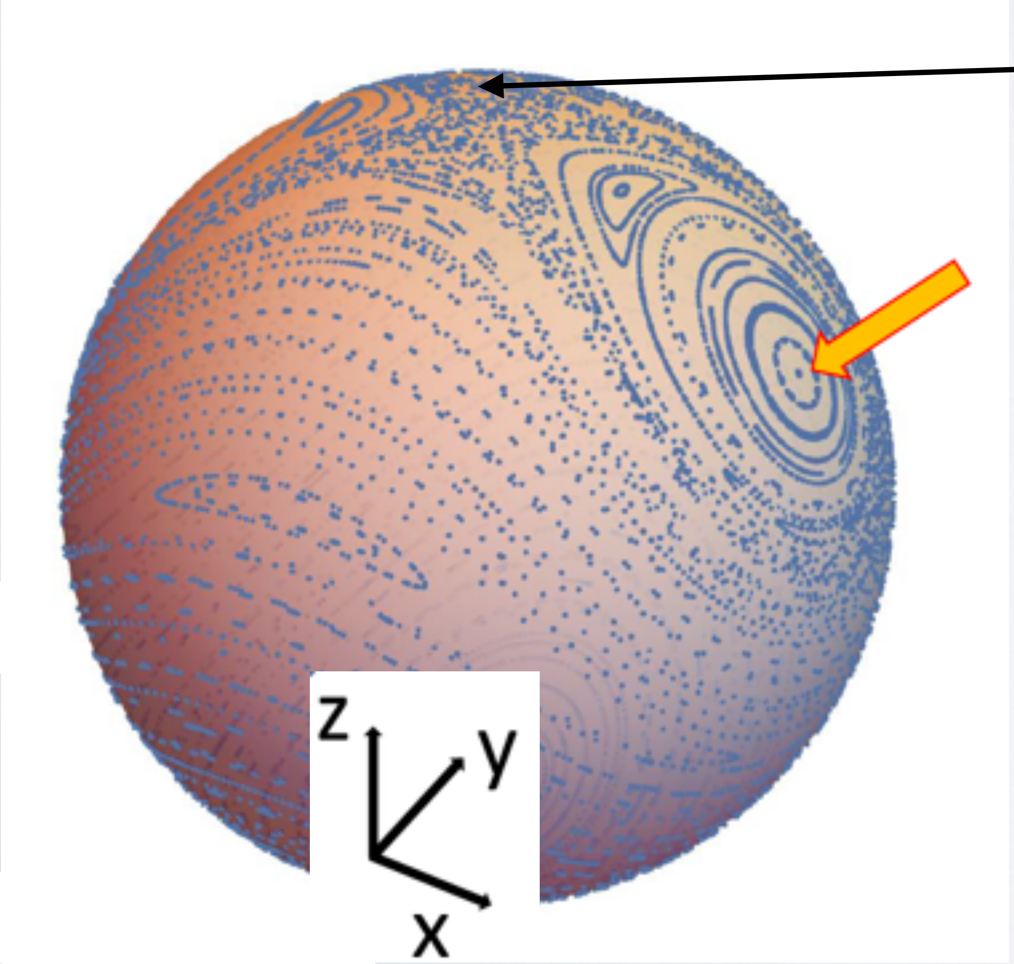
$$\delta B/J_0 = 0.01$$

Dynamical ferromagnet
& chaotic trajectories around the unstable paramagnetic point

Stroboscopic trajectories of the order parameter on the Bloch sphere

$$\{\vec{S}(t_n)\}, \quad t_n = 2\pi n/\Omega, \quad n = 0, 1, 2, \dots$$

$$\boxed{B_0/J_0 = 1.2, \quad \Omega/J_0 = 5}$$

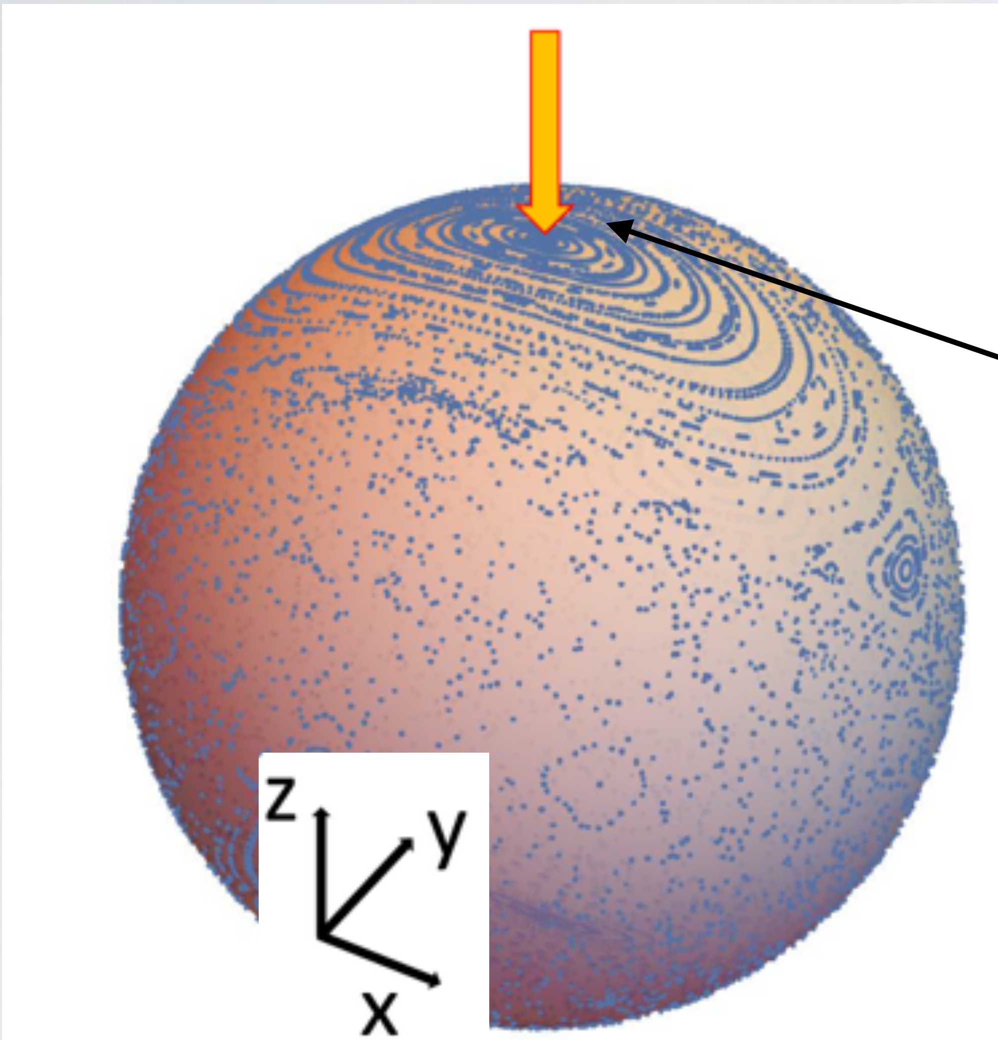


close to the paramagnetic point:
chaotic trajectories

periodic trajectories (from FM initial states)

$$\delta B/J_0 = 0.01$$

Dynamically stabilised (one-body) Kapitza phase

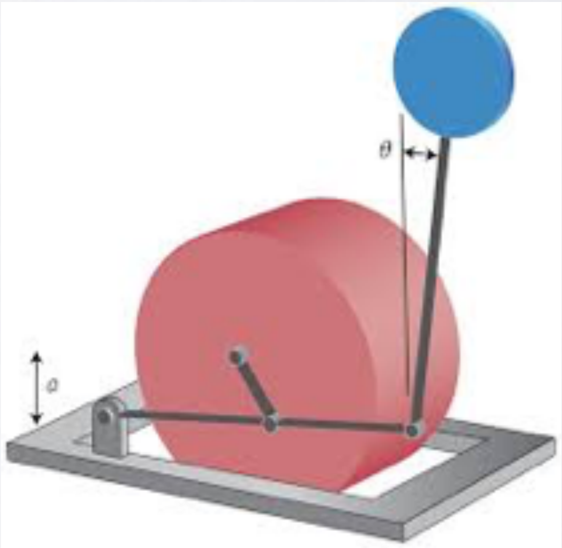


$$B_0/J_0 = 1.2, \quad \Omega/J_0 = 5$$

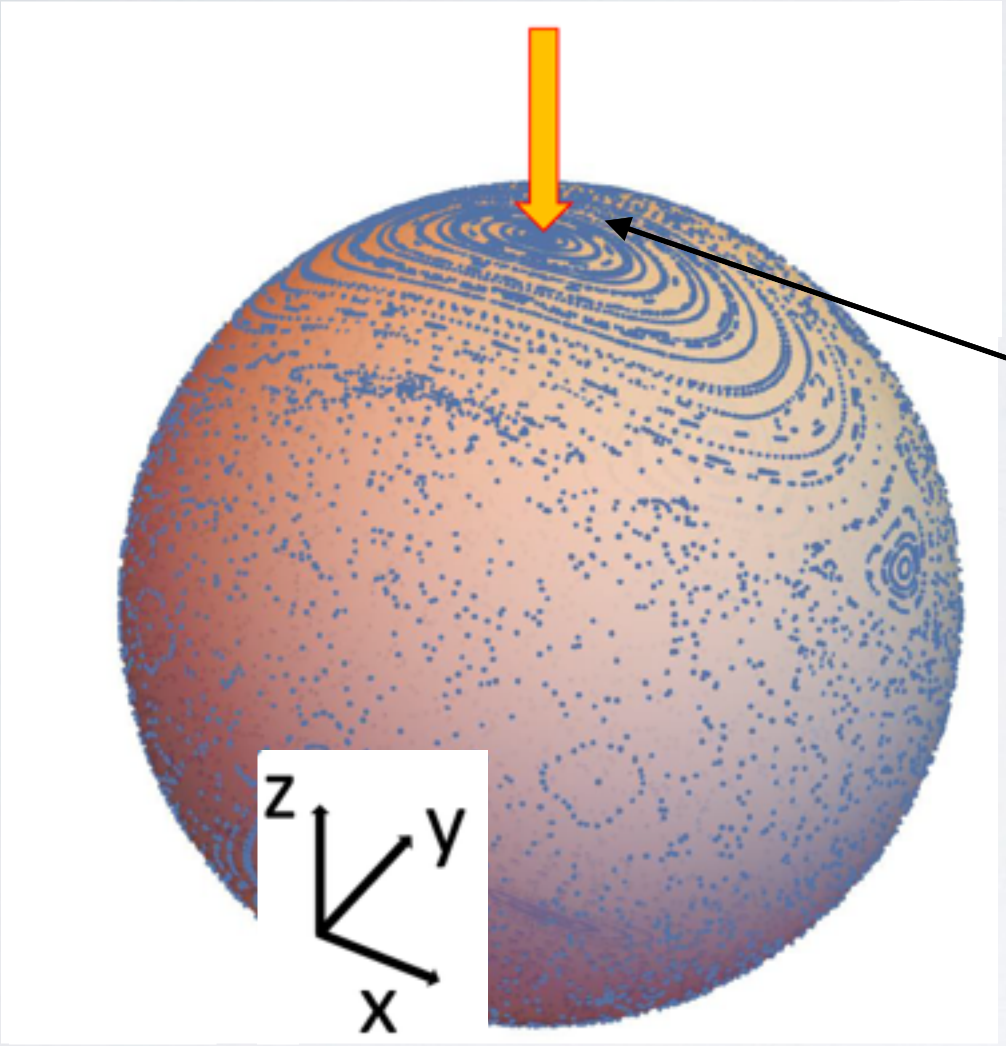
$$\delta B/J_0 = 3.3$$

increasing driving amplitude:
orbits become regular
around unstable paramagn. point
(dynamical stabilization)

Kapitza (1951)



Dynamically stabilised (one-body) Kapitza phase



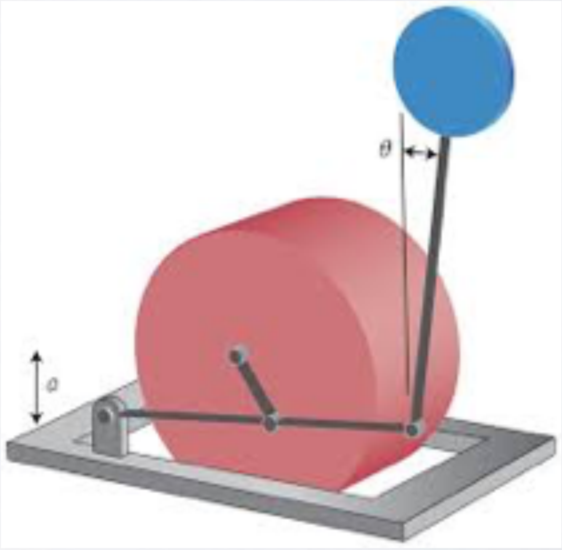
$$B_0/J_0 = 1.2, \quad \Omega/J_0 = 5$$

$$\delta B/J_0 = 3.3$$

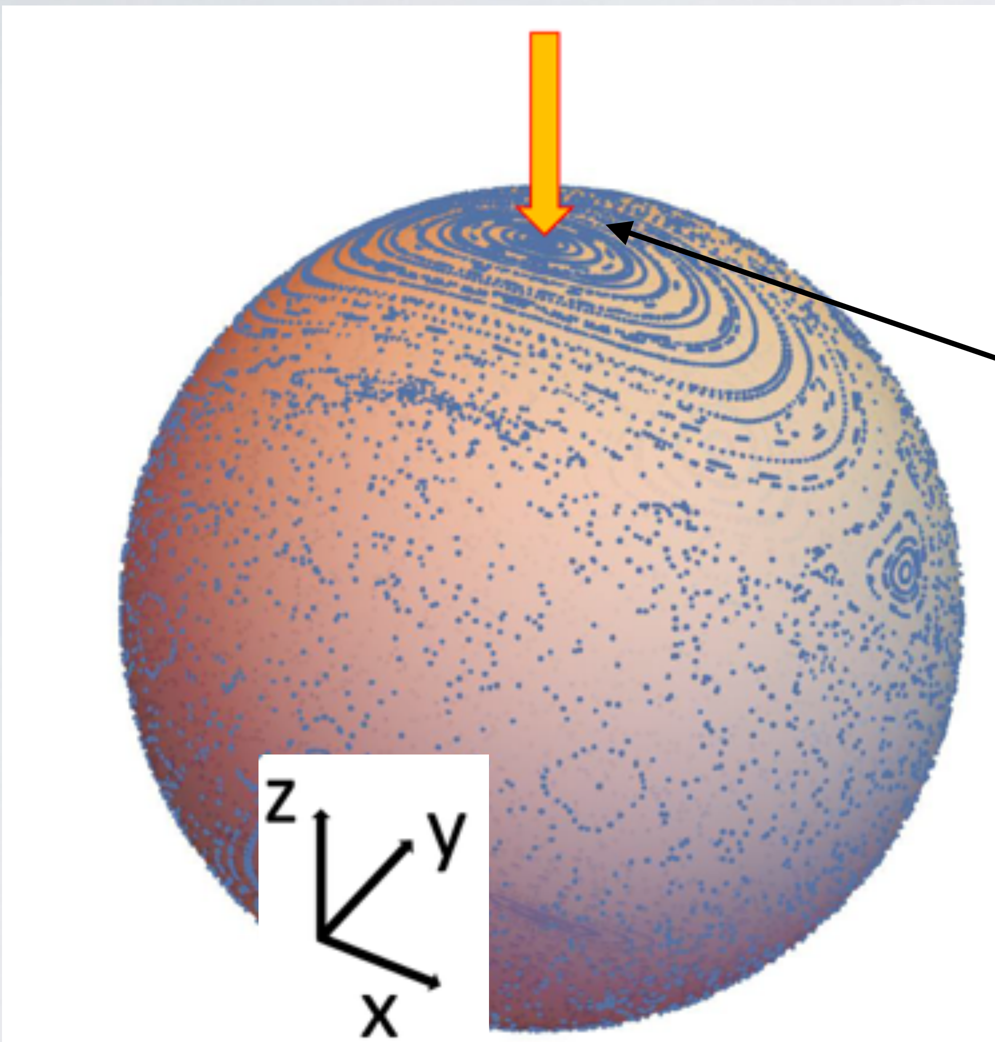
increasing driving amplitude:
orbits become regular
around unstable paramagn. point
(dynamical stabilization)

$$m_{(\alpha)} = S_{(\alpha)}/N$$
$$[m_x, m_y] = i m_z/N$$

Kapitza (1951)



Dynamically stabilised (one-body) Kapitza phase



$$B_0/J_0 = 1.2, \quad \Omega/J_0 = 5$$

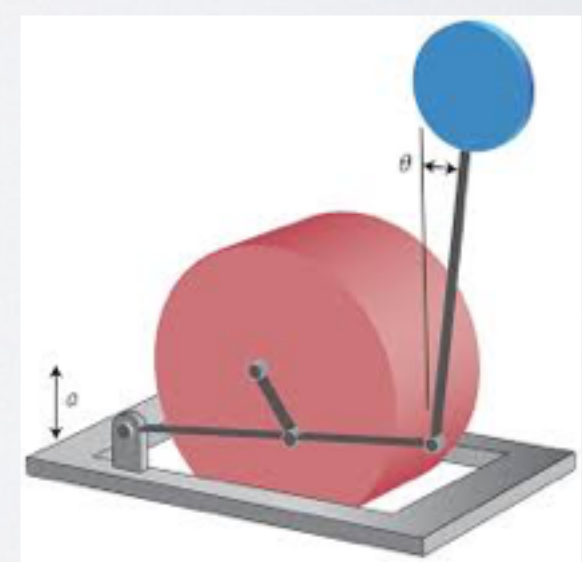
$$\delta B/J_0 = 3.3$$

increasing driving amplitude:
orbits become regular
around unstable paramagn. point
(dynamical stabilization)

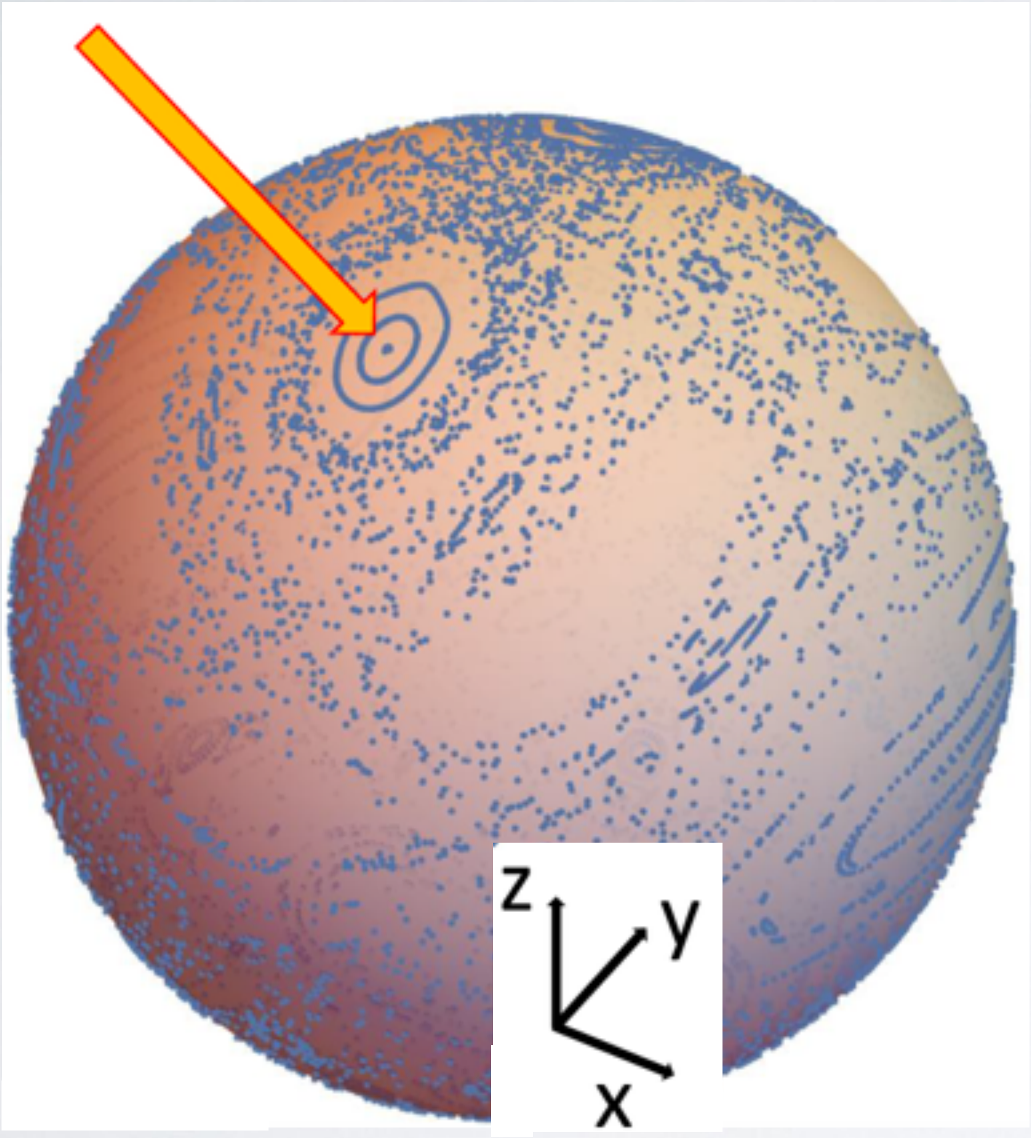
$$m_{(\alpha)} = S_{(\alpha)}/N$$
$$[m_x, m_y] = \cancel{i m_z/N}$$

$N \to \infty$

Kapitza (1951)



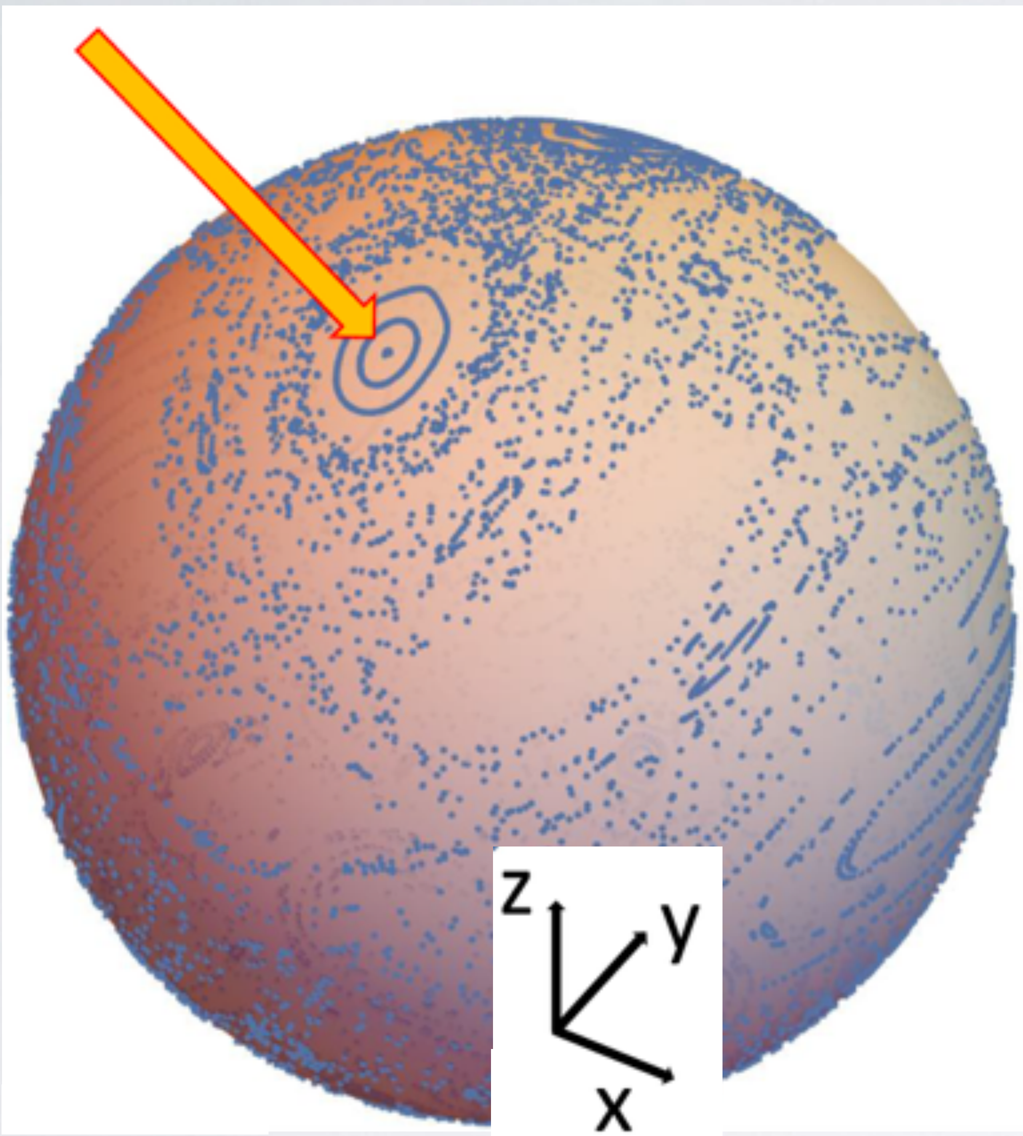
(Dynamically stabilized) Unconventional ferromagnetic phase



$$\boxed{B_0/J_0 = 1.2, \quad \Omega/J_0 = 5}$$

$$\delta B/J = 5$$

(Dynamically stabilized) Unconventional ferromagnetic phase



$$\boxed{B_0/J_0 = 1.2, \quad \Omega/J_0 = 5}$$

$$\delta B/J = 5$$

Effective Floquet hamiltonian

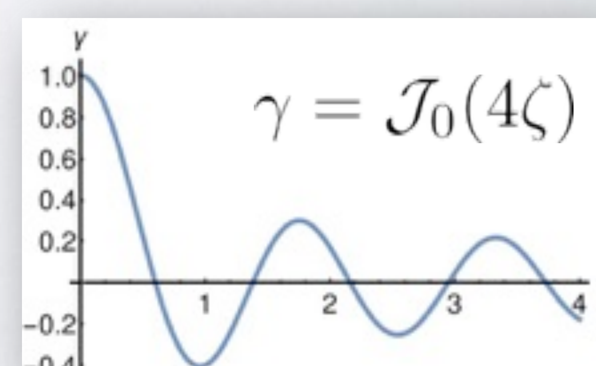
$$\mathcal{H}_{\rm eff} = -J \left[\frac{1+\gamma(\zeta)}{2} S_x^2 + \frac{1-\gamma(\zeta)}{2} S_y^2\right] - B_0\, S_z$$

$\gamma(\zeta) = \mathcal{J}_0(4\zeta)$ Bessel of first kind

$\zeta = \delta B/\Omega$

Order along *y*-direction for:

$J(1+\gamma) < B_0 < J(1-\gamma)$



NEQ phase diagram under periodic drive

Phase diagram persists under quantum fluctuations induced by short-range character of interactions (perturbative deformations)

$$0 < \alpha < 2$$

$$H = -\sum_{i<j}^{N} \frac{J}{|i-j|^{\alpha}} \sigma_i^x \sigma_j^x - B \sum_{i=1}^{N} \sigma_i^z$$

$d = 1$

$$B(t) = B_0 + \delta B \cos(\Omega t)$$

NEQ phase diagram under periodic drive

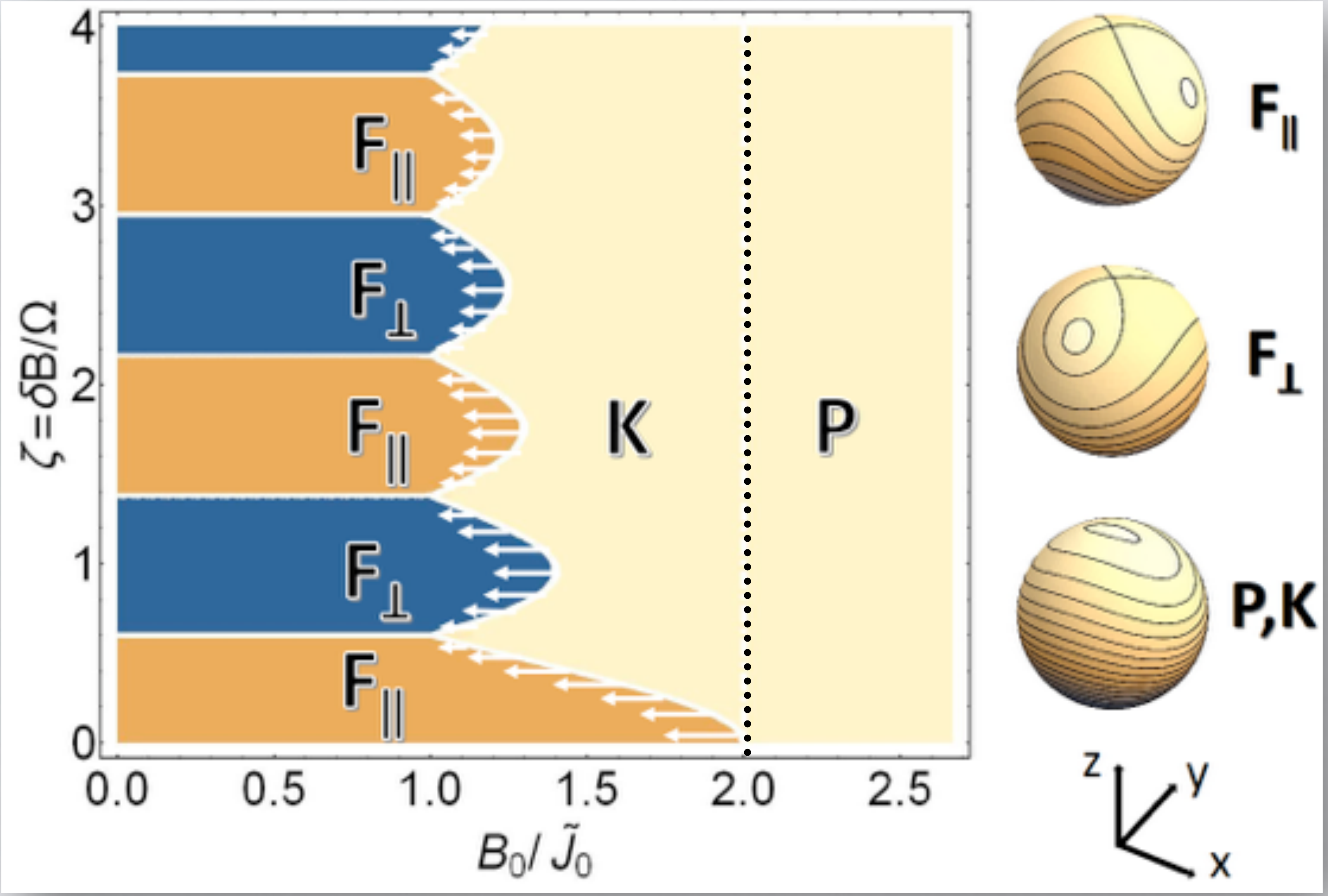
Phase diagram persists under quantum fluctuations induced by short-range character of interactions (perturbative deformations)

$$0 < \alpha < 2$$

$$H = -\sum_{i<j}^{N} \frac{J}{|i-j|^\alpha}\sigma_i^x \sigma_j^x - B\sum_{i=1}^{N}\sigma_i^z$$

$d = 1$

$$B(t) = B_0 + \delta B\,\cos(\Omega t)$$



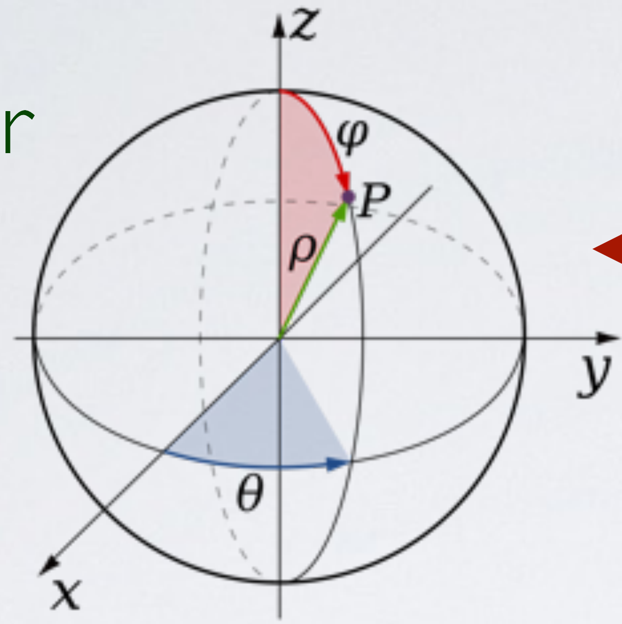
effective coupling

$$\tilde{J}_0 = \sum_{r\neq 0}^{N} J/r^\alpha$$

Time-dependent spin wave theory (reminder)

order parameter

$\vec{S}(t)$



coupled to

spin waves bath (N modes)

order parameter drags an extensive set of quantum harmonic oscillators corresponding to spin wave excitations with finite momentum

expansion is under control till spin-wave density remains small:

$$\epsilon(t) \ll 1$$

Dynamically stabilised (many-body) Kapitza phase

Expand around the unstable paramagn. point θ=0

$$H(t) = \mathcal{E}(t) + 2\sum_k \left[(B(t) - 2\tilde{J}_k)\frac{\tilde{q}_k\tilde{q}_{-k}}{2} + B(t)\frac{\tilde{p}_k\tilde{p}_{-k}}{2} \right]$$

energy of the θ=0 spin coherent configuration

Dynamically stabilised (many-body) Kapitza phase

Expand around the unstable paramagn. point θ=0

$$H(t) = \mathcal{E}(t) + 2\sum_k \left[(B(t) - 2\tilde{J}_k)\frac{\tilde{q}_k\tilde{q}_{-k}}{2} + B(t)\frac{\tilde{p}_k\tilde{p}_{-k}}{2} \right]$$

energy of the θ=0 spin coherent configuration

Static (undriven) model:

$$\omega_k = 2[B_0(B_0 - 2\tilde{J}_k)]^{1/2}$$

Fourier transform of the long-range interaction

$\tilde{J}_k > B_0/2$ band of unstable modes!

Dynamically stabilised (many-body) Kapitza phase

Expand around the unstable paramagn. point θ=0

$$H(t) = \mathcal{E}(t) + 2\sum_k \left[(B(t) - 2\tilde{J}_k) \frac{\tilde{q}_k \tilde{q}_{-k}}{2} + B(t) \frac{\tilde{p}_k \tilde{p}_{-k}}{2} \right]$$

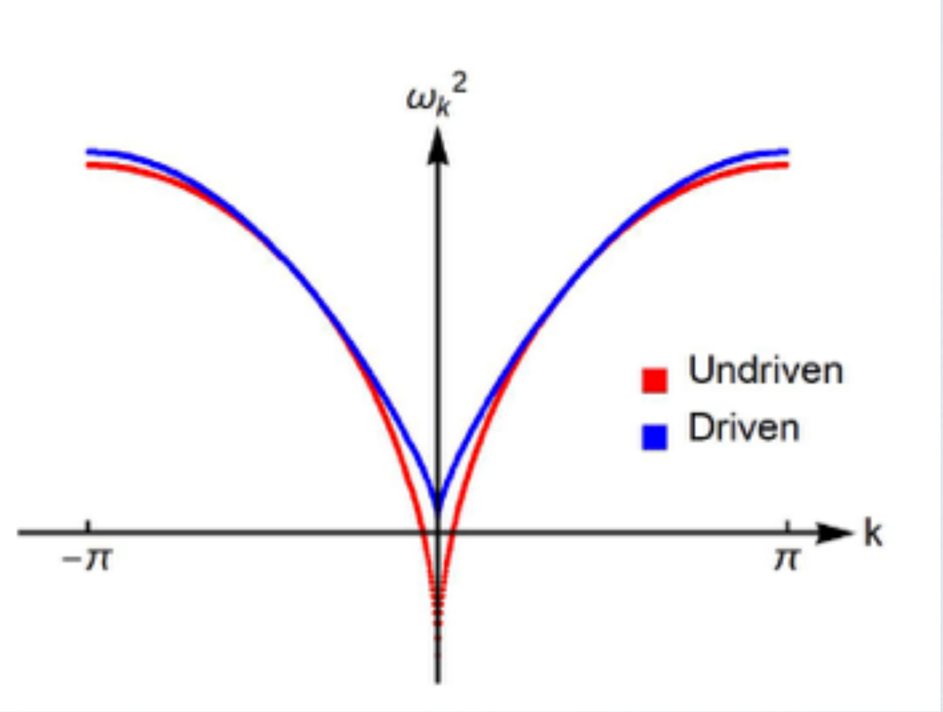
energy of the θ=0 spin coherent configuration

Static (undriven) model:

$$\omega_k = 2[B_0(B_0 - 2\tilde{J}_k)]^{1/2}$$

Fourier transform of the long-range interaction

$\tilde{J}_k > B_0/2$ band of unstable modes!



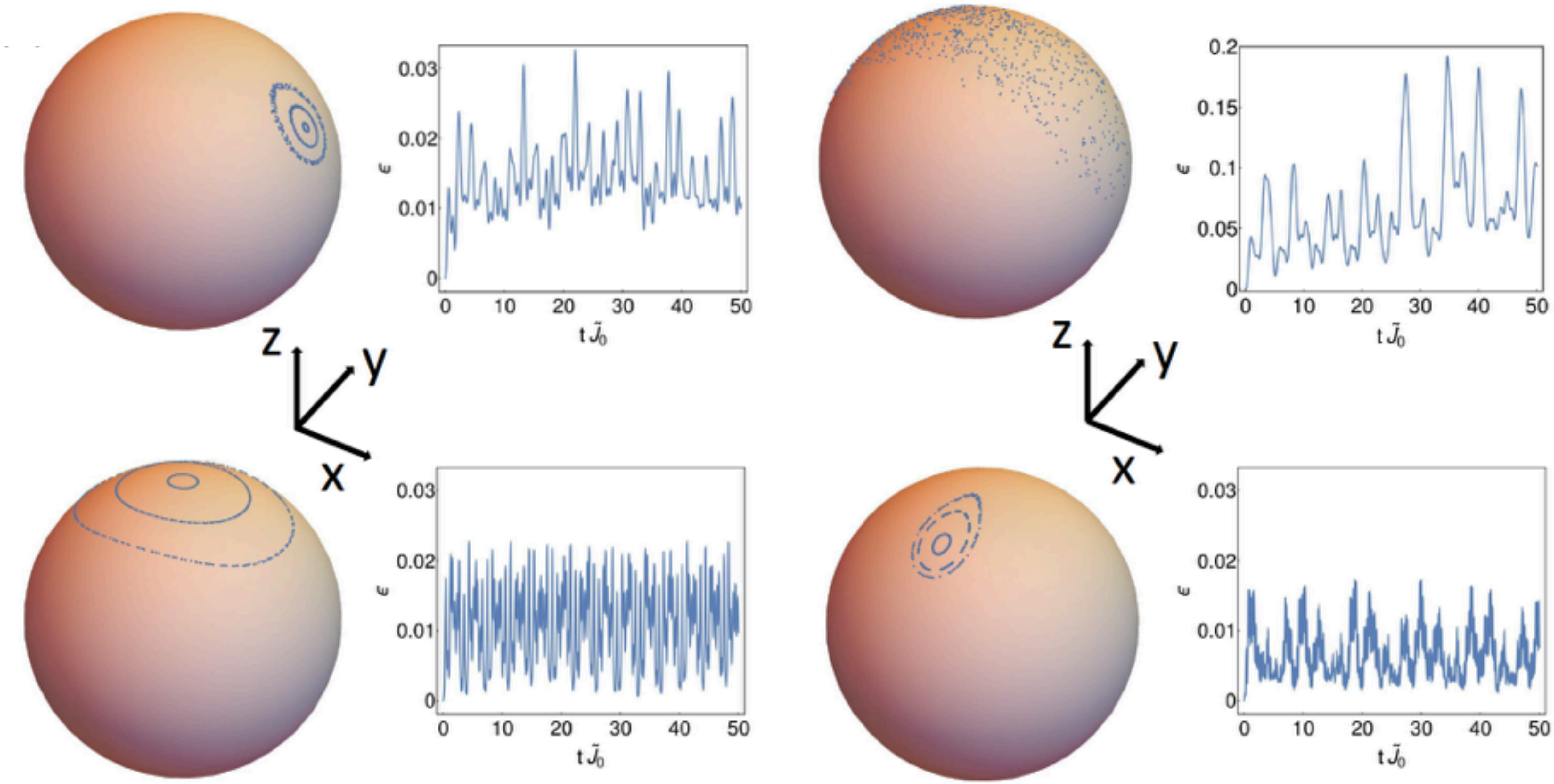
Effective dispersion relation becomes real for selected driving *B(t)*: simultaneous dynamical stabilisation of the whole band of spin waves

Dynamical stabilisation and spin wave density: pre-relaxation

$$B_0/\tilde{J}_0 = 1.2, \quad \Omega/\tilde{J}_0 = 8, \quad \alpha = 1, \quad N = 100$$

Dynamical ferromagnet

Heating (unstable paramagn.)



Many-body Kapitza

Unconventional ferromagnetic order

$\Omega \gg 4\tilde{J}_0$ off-resonance with spin-waves band

$\tau_{(heating)} \propto \exp(\sim \Omega/\tilde{J}_0)$

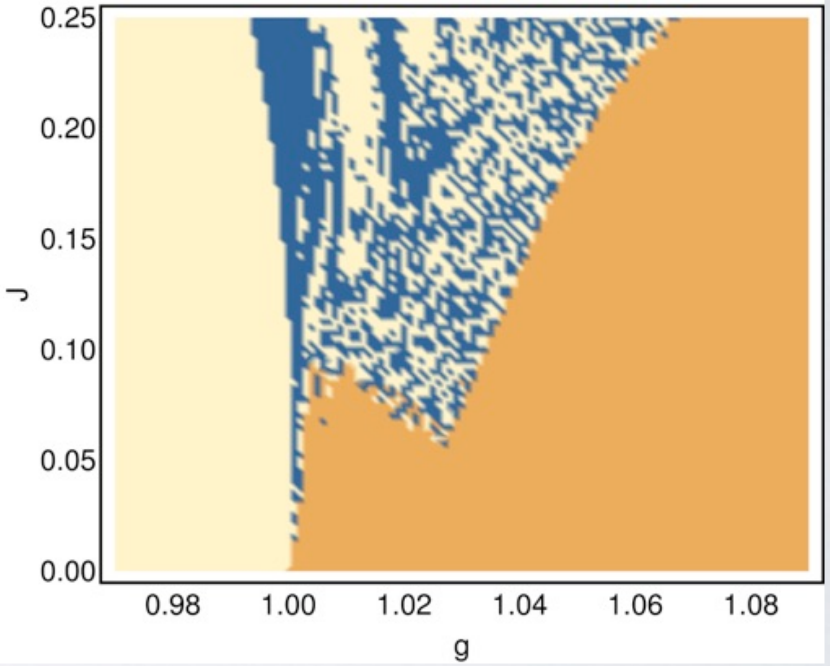
Mori et al. PRL ('16); Abanin et al. PRB ('17)

Summary

Impact of quantum fluctuations out-of-equilibrium:

1) novel phases at a dynamical critical point:
chaotic dynamical ferromagnet

Lerose, **JM**, B. Zunkovic, A. Gambassi, A. Silva (PRL 2018)
+ ArXiv 1807.09797 (2018)



2) can be tamed by a periodic drive
and stablise a many body Kapitza pendulum

A. Lerose, **JM**, A. Gambassi, A. Silva, arXiv1803.04490 (2018)