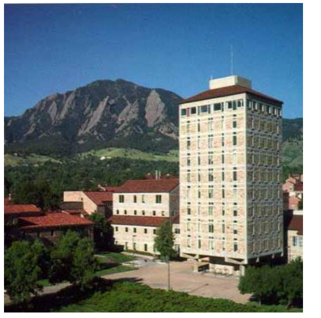
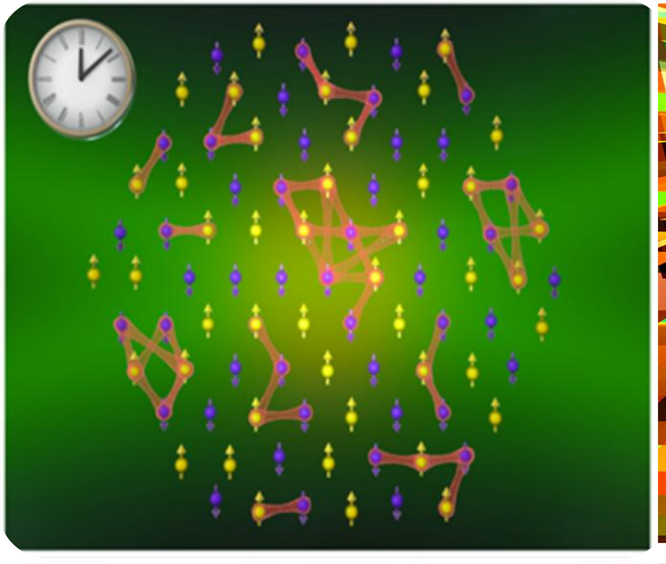
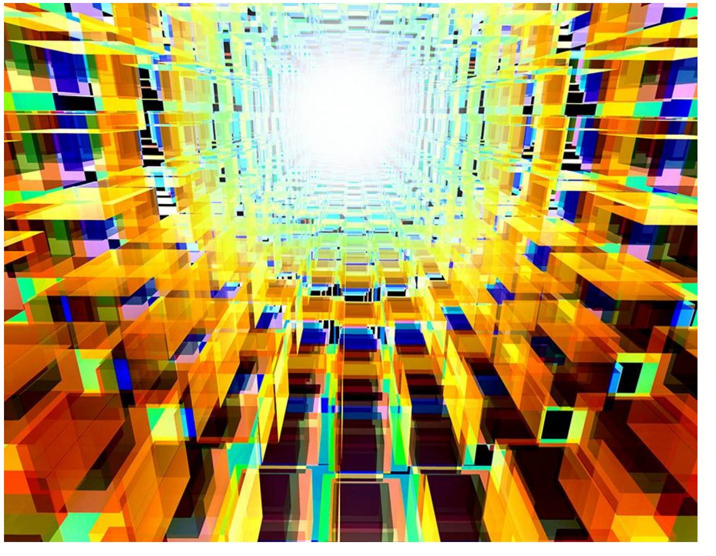
Entanglement Dynamics and Fast Scrambling in a Trapped Ion Magnet

Ana Maria Rey







Novel Approaches to Quantum Dynamics, KITP, Santa Barbara, August 27 (2018)

Theory

R. Lewis-Swan

A. Safavi-Naini

M. Gärttner

M. Wall

M. Foss-Feig

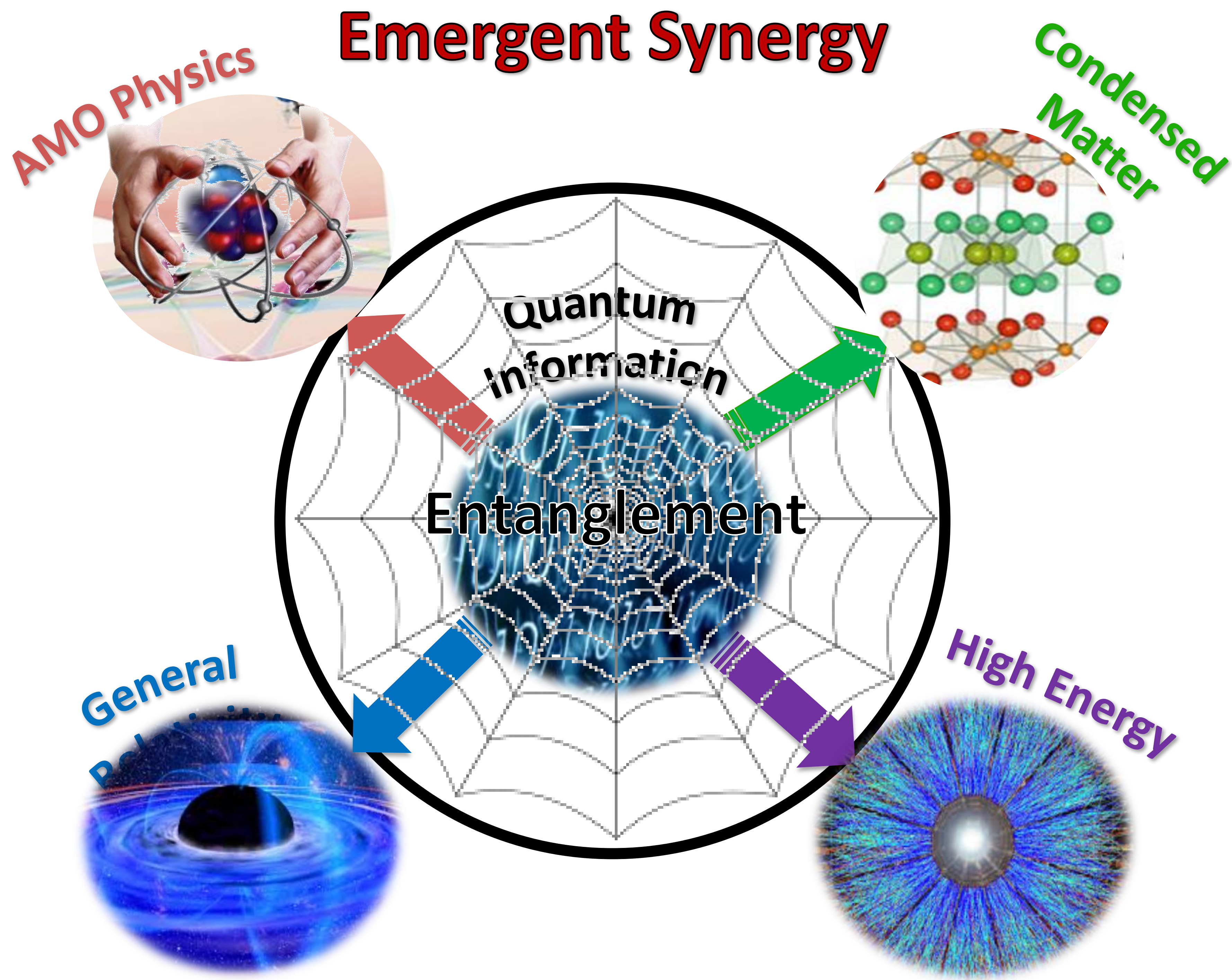
Experiment

J. Bollinger

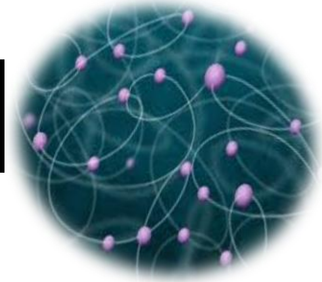
J. Bohnet

K. Gilmore

JILA pfc
an NSF physics frontier center


Emergent Synergy
AMO Physics
Condensed Matter
Quantum Information
Entanglement
General
High Energy

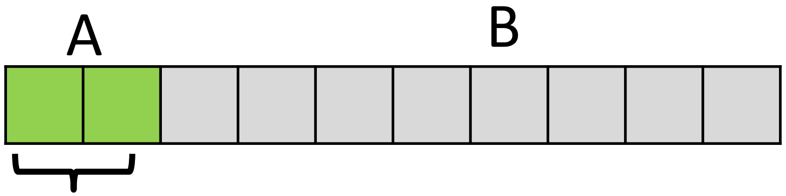
Quantum Thermalization in closed systems



Entanglement

$\hat{\rho}$: Density Matrix

$\mathrm{Tr}(\hat{\rho}^2) = 1$ Purity



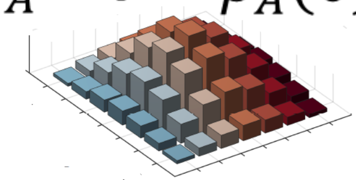
$\hat{\rho}_A = \mathrm{Tr}_B\, \hat{\rho}$ Reduce density Matrix of A

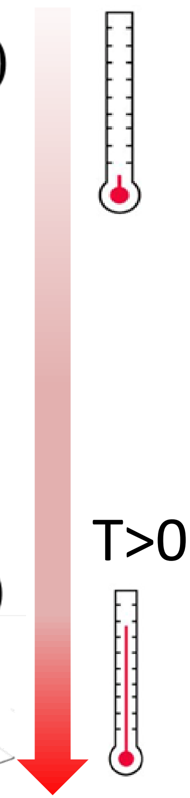
Renyi entropy: Purity of $\hat{\rho}_A$

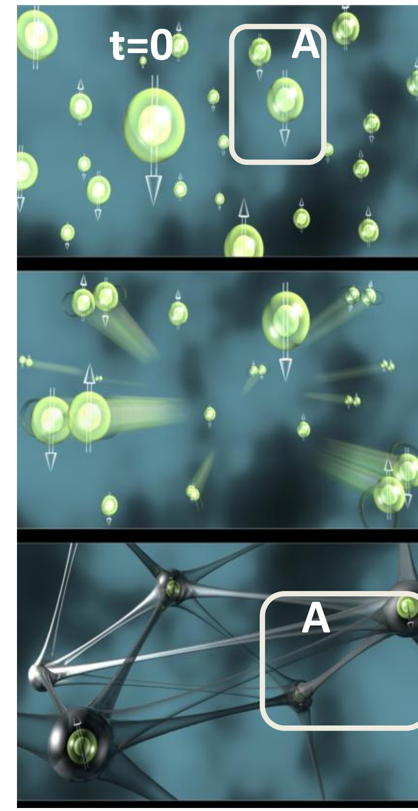
$$S_A = -\log[Tr(\rho_A^2)]$$

$\hat{\rho}(0) = \otimes_i \hat{\rho}_i$ Product state

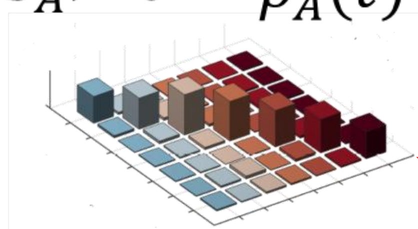
$S_A = 0$ $\hat{\rho}_A(0)$



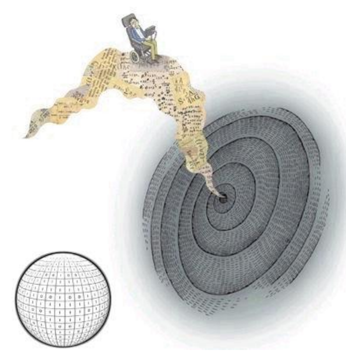




$S_A > 0$ $\hat{\rho}_A(t)$



Entangled state



Apparent loss of information

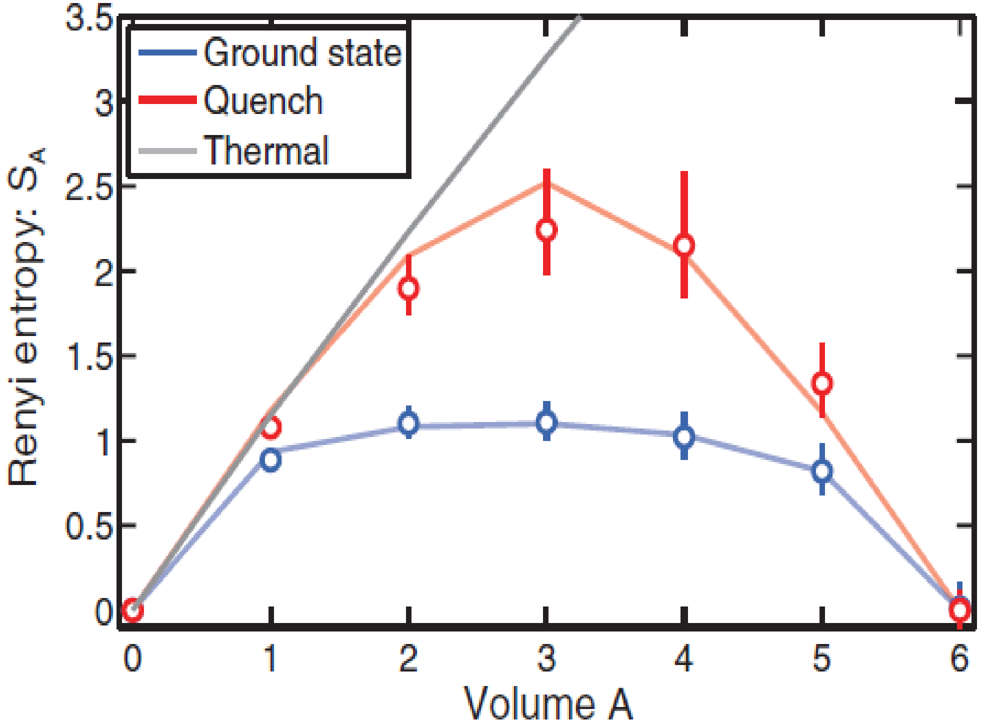
D'Alessio *et al*, Adv. in Phys.(2016)

Scrambling of Quantum Information

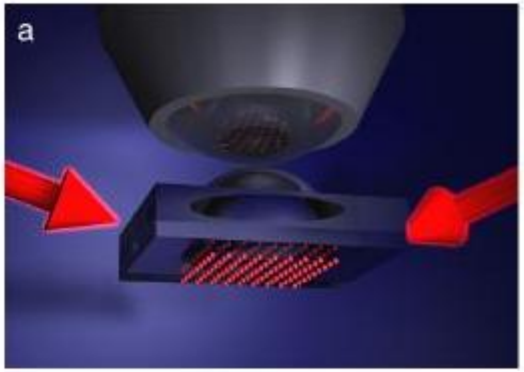
Information not loss but Scrambled

Spread over many-body degrees of freedom, becoming inaccessible to local measurements

Kaufman *et al,* Science(2016)



Greiner group:
quantum gas microscope



- ✓ Single site addressing
- ✓ Only in small systems L=6

But entanglement entropy is hard to measure in large systems

Brydges,…., P. Zoller, R. Blatt, C. F. Roos, arXiv:1806.05747

How to measure scrambling?

Out-of-time-Order-Correlators OTOCs

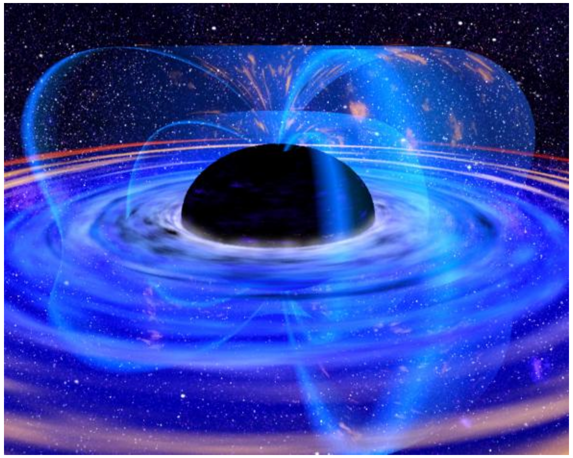
$$F(t) = \langle \widehat{W}^{\dagger}(t)\hat{V}^{\dagger}(0)\widehat{W}(t)\hat{V}(0)\rangle$$

$$F(t) = 1 - C(t) \qquad C(t) = \langle | \ [\widehat{W}(t), \hat{V}(0)] \ |^2 \rangle$$

Measurement of the degree of non-commutativity of $\hat{V}(0)$ and the time evolved version of $\widehat{W}(t)$

[Hayden-Preskill, Sekino-Susskind, Shenker-Stanford ’13, Kitaev ‘14]

OTOCs and Quantum Gravity



- Black holes scramble quantum information as fast as possible
  - Fast scramblers: $C(t) \sim e^{\lambda t}$

$$\lambda \leq 2\pi T$$

- Bound of growth of quantum chaos: λ Lyapunov exponent

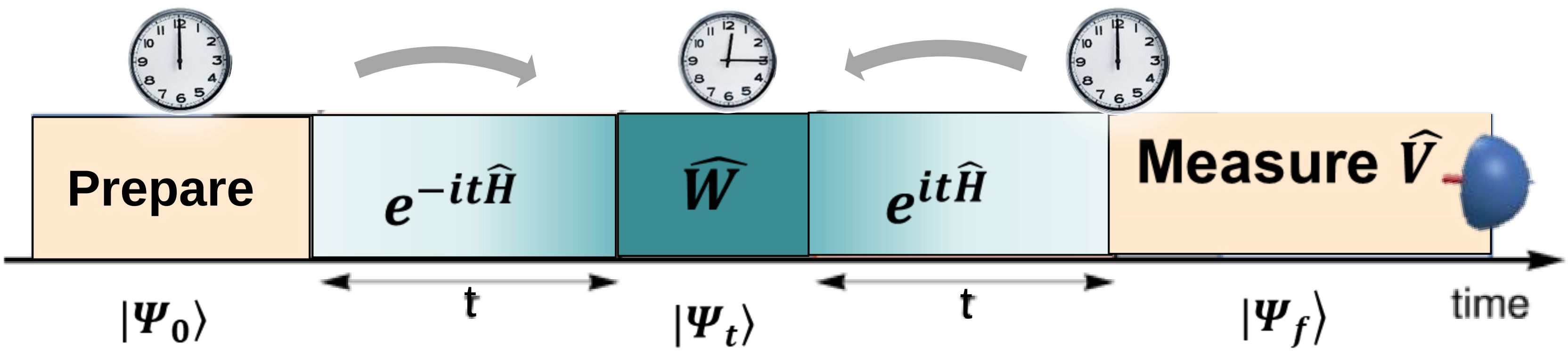
[Maldacena-Shenker-Stanford][Martinis'16]

Can we access OTOCS? [Swingle et al 16]

[Yao et al16]

[Zhu et al16]

Measuring OTOCS



$\widehat{W}$ and $\hat{V}$ Two commuting operators

$$F(t) = \left\langle \Psi_0 \right| e^{it\hat{H}} \; \widehat{W}^\dagger \, e^{-it\hat{H}} \hat{V}^\dagger \;\; e^{it\hat{H}} \widehat{W} \; e^{-it\hat{H}} \left| \Psi_0 \right\rangle \qquad \hat{V}|\Psi_0\rangle = |\Psi_0\rangle$$

$$= \left\langle \Psi_0 \right| \underbrace{e^{it\hat{H}} \; \widehat{W}^\dagger \, e^{-it\hat{H}}}_{\widehat{W}_t^\dagger} \hat{V}^\dagger \;\; \underbrace{e^{it\hat{H}} \widehat{W} \; e^{-it\hat{H}}}_{\widehat{W}_t} \hat{V} \left| \Psi_0 \right\rangle$$

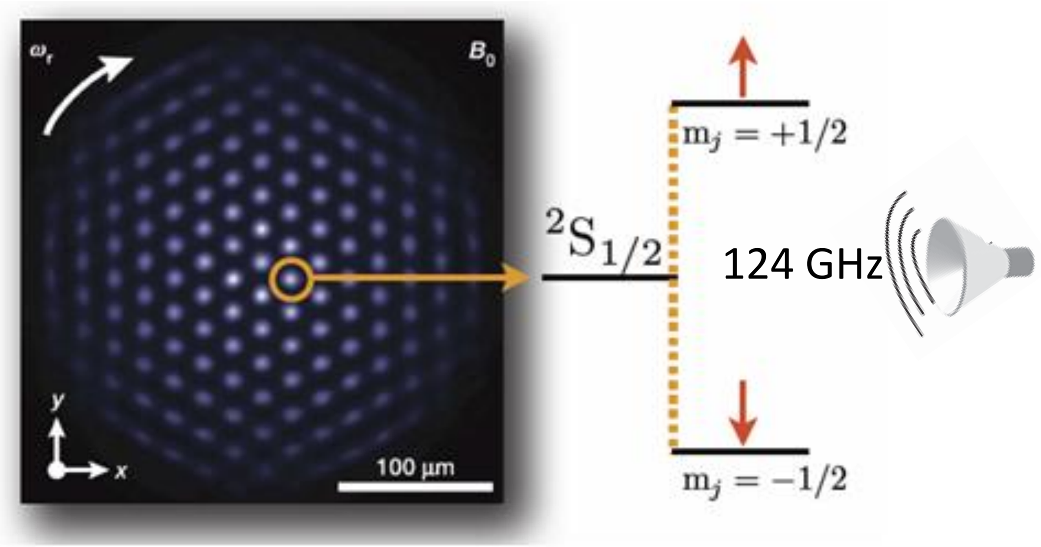
$$= \left\langle \widehat{W}_t^\dagger \;\; \hat{V}^\dagger \widehat{W}_t \hat{V} \right\rangle$$

OTOCS

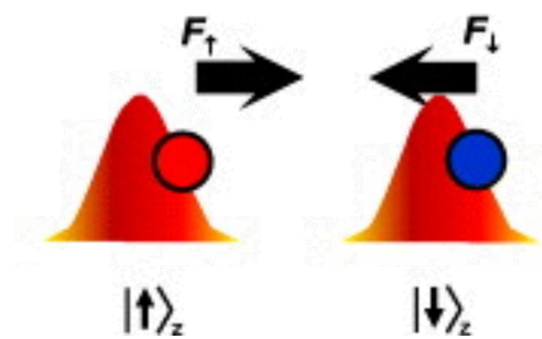
Garttner *et al Nat. Physics (2017)*
Garttner *et al PRL (2018)*

Penning Trap Experiments: $^9Be^+$

- Penning trap: 2D triangular crystals of 20-300 ions
- Two hyperfine states used as spin ½ system



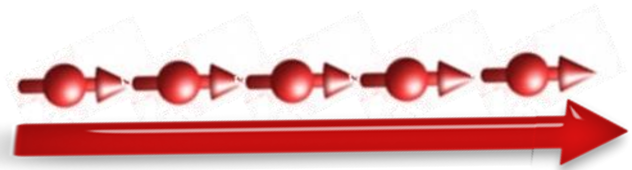
Single qubit gates
99.9 fidelity



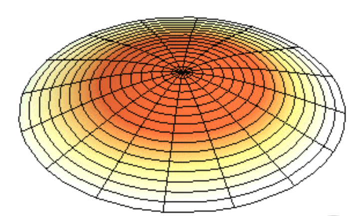
Center of Mass

- Spin–spin interactions generated by lasers by exciting phonons

$$\hat{H}_{zz} = \frac{J_0}{\delta}(\hat{S}_z)^2 \qquad \hat{S}^{x,y,z} = \frac{1}{2}\sum_i \hat{\sigma}_i^{x,y,z}$$



Collective spin

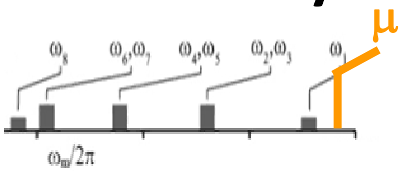


$$\delta = \omega_1 - \mu$$

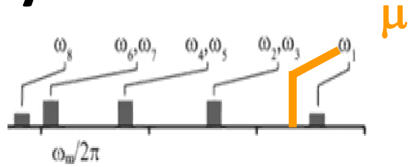
OTOC Measurements

Prepare $|\Psi_0\rangle = |\rightarrow \cdots \rightarrow\rangle$

1) Invert many-body time evolution. $\delta = \omega_1 - \mu$



$$J \sim \frac{J_0}{\delta}$$



$$J \sim -\frac{J_0}{\delta}$$

2) Measure

a) Fidelity: FOTOCs

$\widehat{W} = \widehat{R}_x(\phi)$

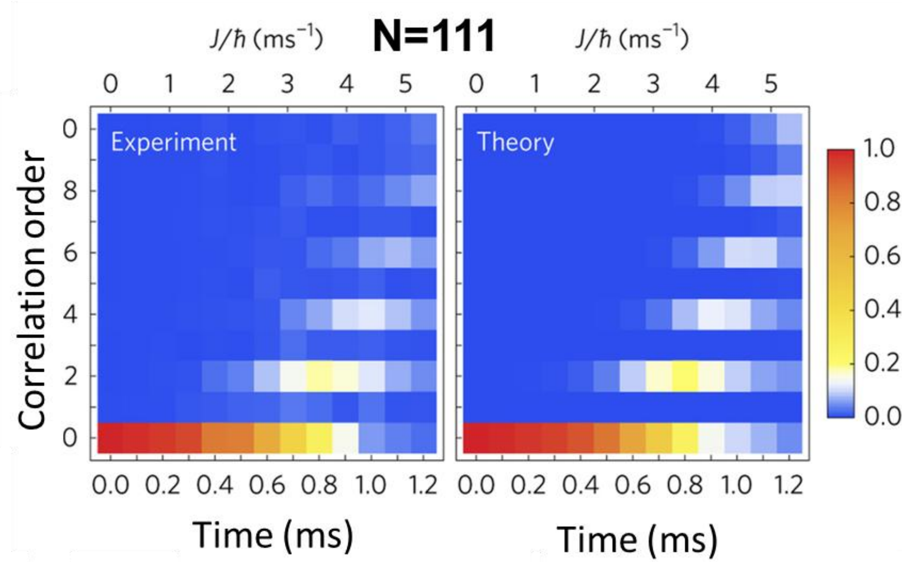
$\widehat{V} = |\Psi_0$

b) Magnetization

$\widehat{W} = \widehat{R}_x(\phi)$

$\widehat{V} = 2/N\widehat{S}_x$

We Measured OTOCs for the first time



Up to m=8 significant correlations!!

Garttner *et al* Nature Physics(2017)

But… no chaos in the Ising Model

Can we simulate fast scrambling and analogs of black holes with trapped ions?

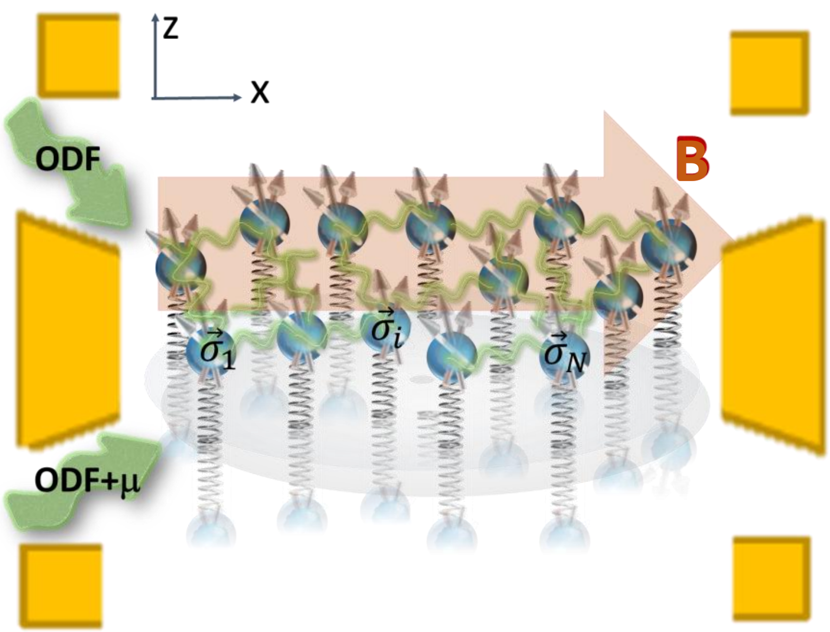
- Add Transverse Field
- Involve Phonons

Dicke Model

$$\hat{H} = \delta \hat{a}^{\dagger}\hat{a} + \frac{2g}{\sqrt{N}}(\hat{a}^{\dagger}+\hat{a})\hat{S}_z - B\hat{S}_x$$

Phonons | Spin-Phonon | Transverse Field

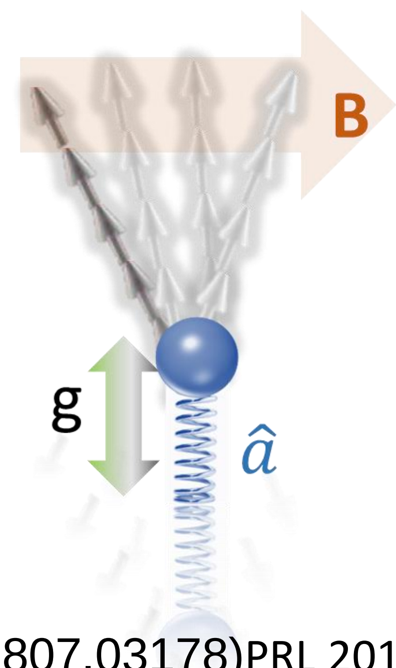
$\hat{a}^{\dagger}$: CM phonons creation operator



$\hat{a}^{\dagger}$: CM phonons creation operator

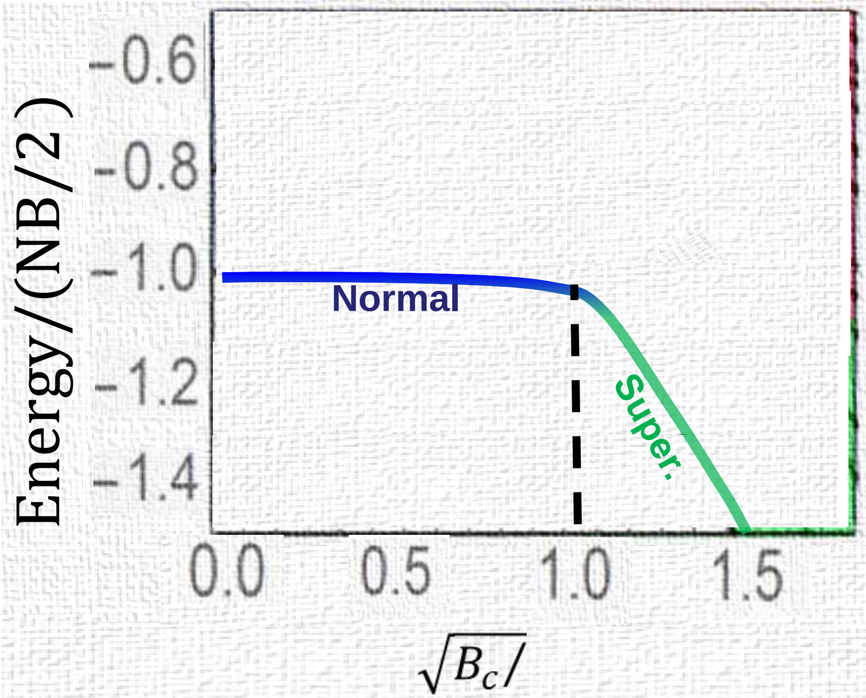
~

Only CM



Safavi-Naini et al (arXiv:1807.03178)PRL 2018

Dicke Model (R.H. Dicke 1953)

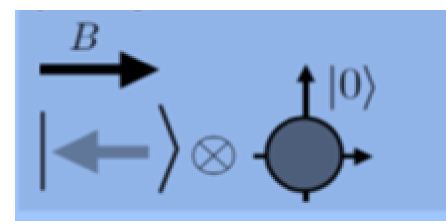


$$\hat{H} = \delta \hat{a}^{\dagger}\hat{a} + \frac{2g}{\sqrt{N}}(\hat{a}^{\dagger}+\hat{a})\hat{S}_z - B\hat{S}_x$$

- Quantum Phase Transition at $B_c = {}^{4g^2}/_{\delta}$:

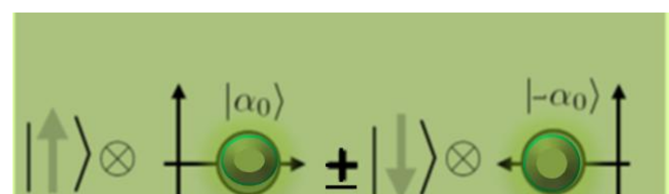
Normal ($B>B_c$): $\hat{H} \sim \delta \hat{a}^{\dagger}\hat{a} - B\hat{S}_x$

No phonons, Paramagnetic



Superradiant ($B<B_c$): $\hat{H} = \delta \hat{b}^{\dagger}\hat{b} + \frac{B_c}{N}(\hat{S}_z)^2$

$$\hat{b} = \left(\hat{a} - \frac{2g}{\delta\sqrt{N}}\hat{S}_z\right)$$

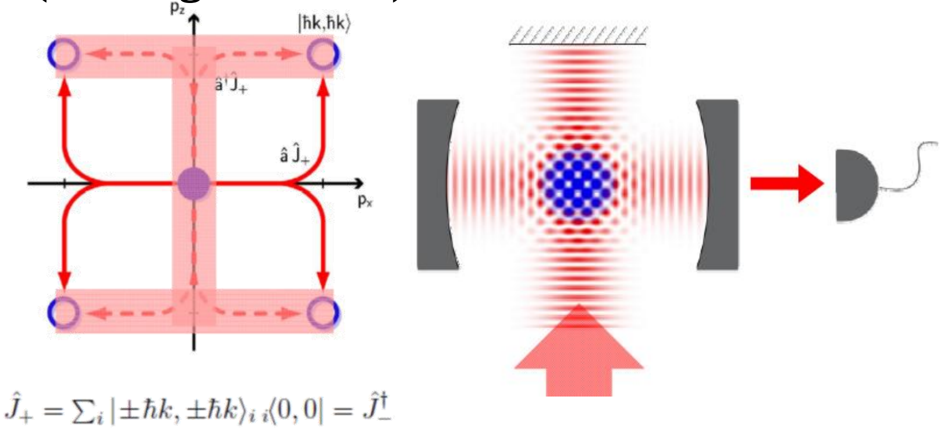


Macroscopic population, Ferromagnetic

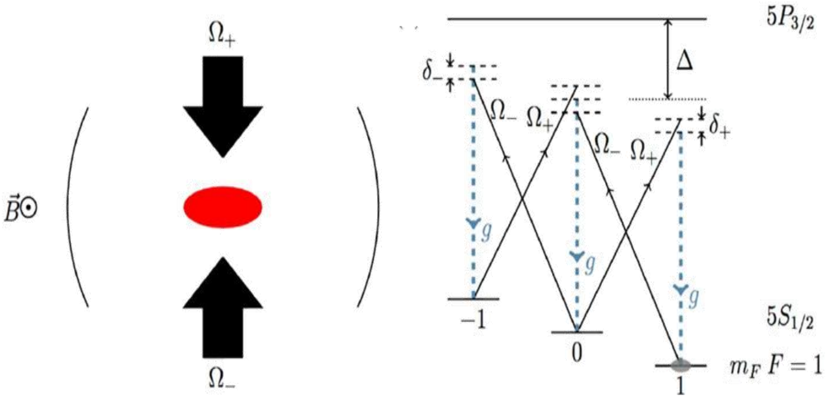
Dicke Model (R.H. Dicke 1953)

Renew interest in cold-atoms

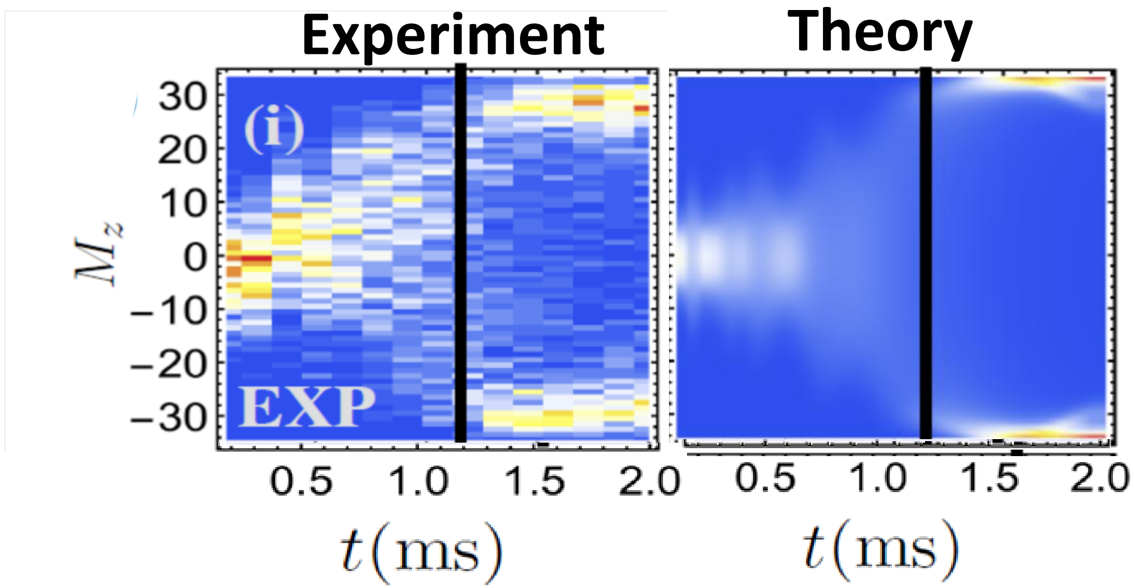
T. Esslinger group 2010:
Effective Dicke Model in a BEC
(self-organization)



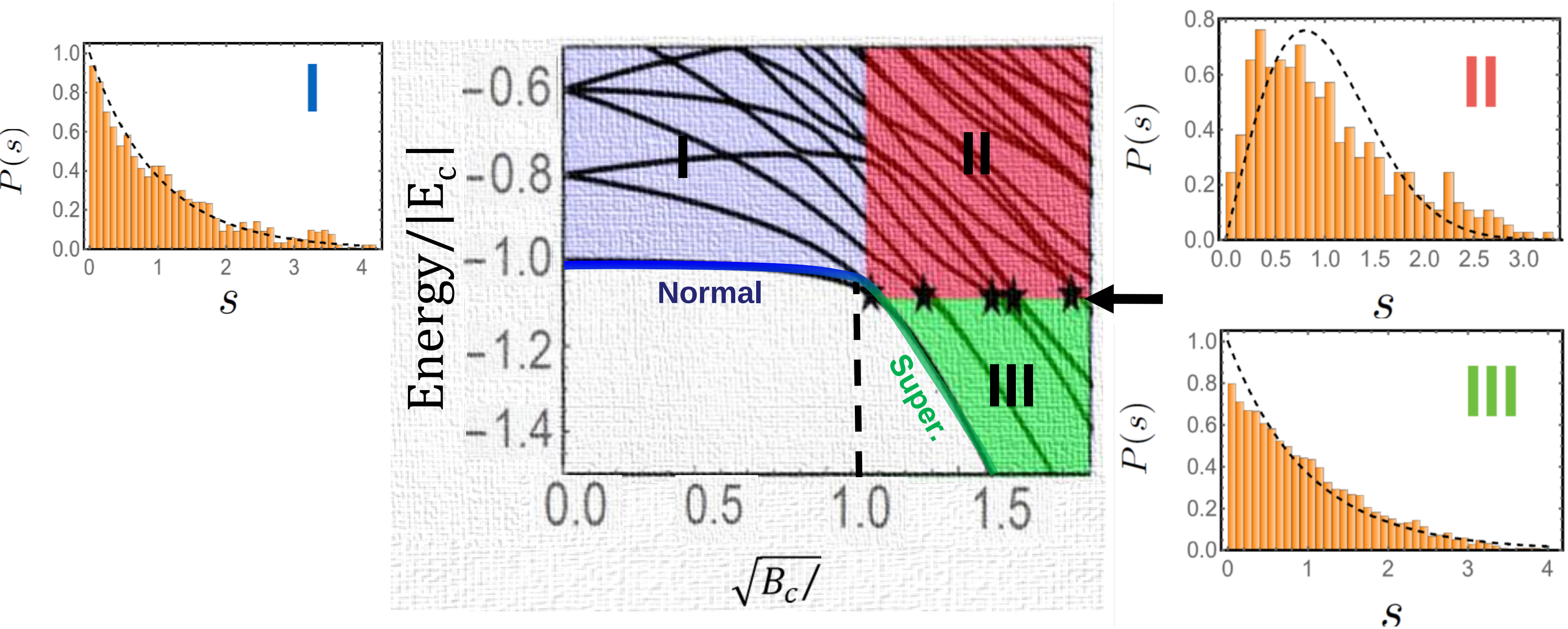
M. Barret group 2017:
S=1 Dicke Model in cavity QED
with Rb



J. Bollinger group 2018:
PRL (2018)
Trapped Ions



Dicke Model: Rich Physics



- Excited State Phase Transition at E_c =-NB/2 and B>B_c
  Singularity in the energy level structure

- B>B_c **Poissonian Integrable**

- **E>Ec: Wignger-Dyson: Chaos**
  E<Ec: mixture (Wigner Dyson/ Poissonian)

Emary&Brandes, PRE (2003)
Brandes,PRE (2013)
Altland & Haake, PRL (2012)

Dicke Model: Rich Physics

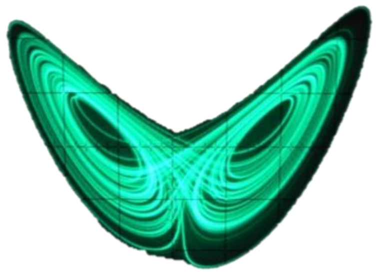
Connection to classical Chaos

- Solve mean field equations for $\vec{x} = \{\langle \hat{S}_x \rangle, \langle \hat{S}_y \rangle, \langle \hat{S}_z \rangle, \langle \hat{a} \rangle_R, \langle \hat{a} \rangle_{Im}\}$

  $|\vec{x}(t) - \vec{x}(0)| = \Delta x(t) = \Delta x(0) e^{\lambda_L t}$

  λ_L: Lyapunov Exponent

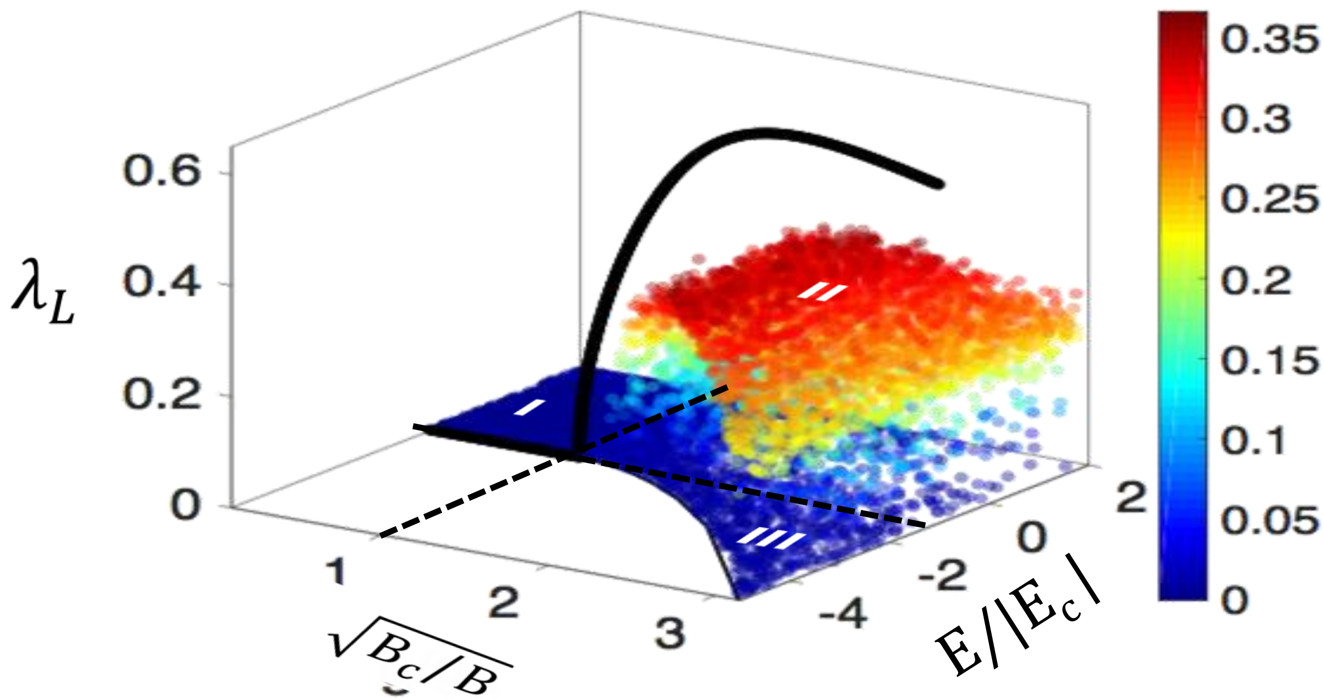
  $\lambda_L > 0$: Signature of classical chaos

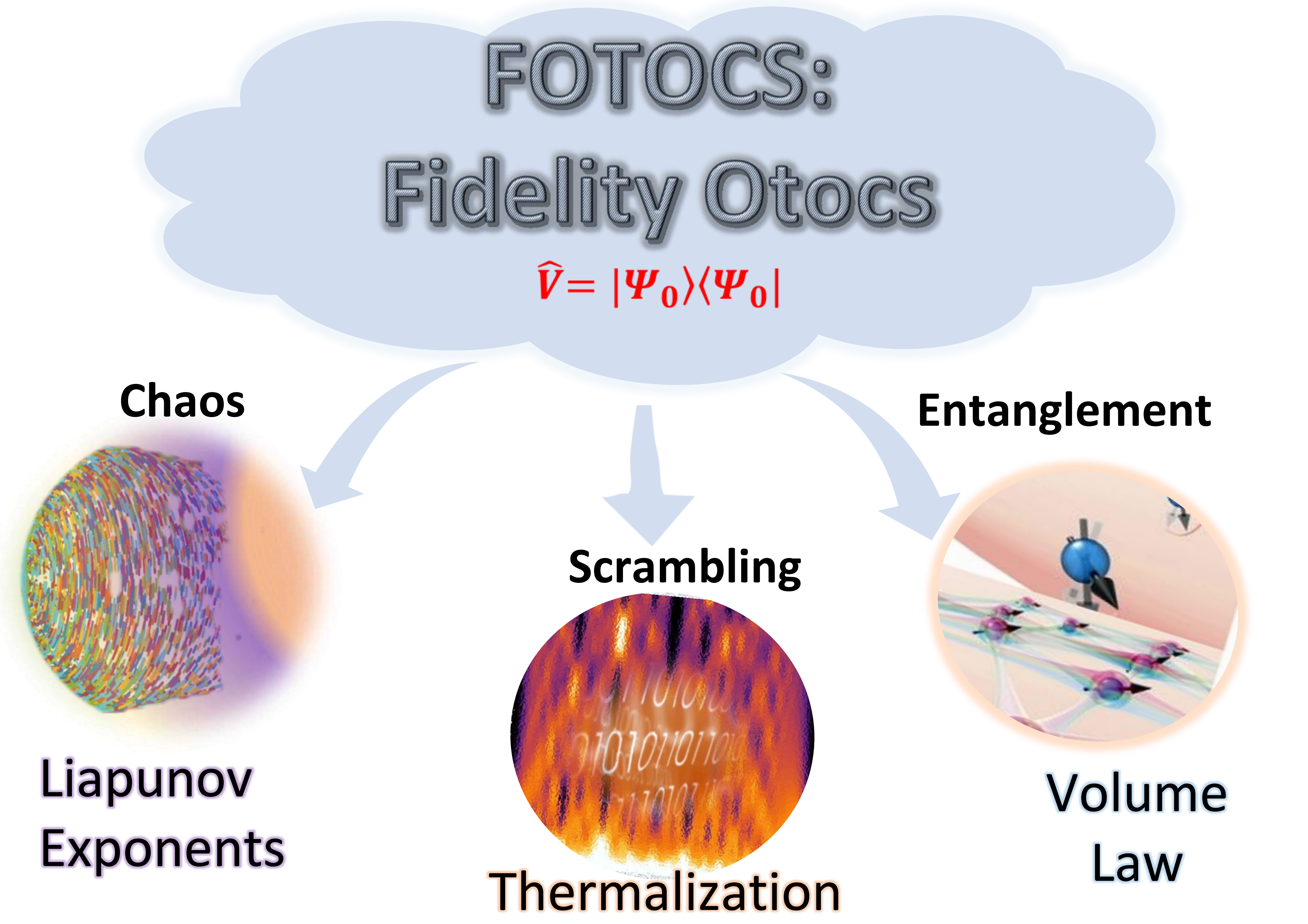


Butterfly effect
Strogatz Book

- State $|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$ $\langle \Psi_0^c | \hat{H} | \Psi_0^c \rangle = E_c$

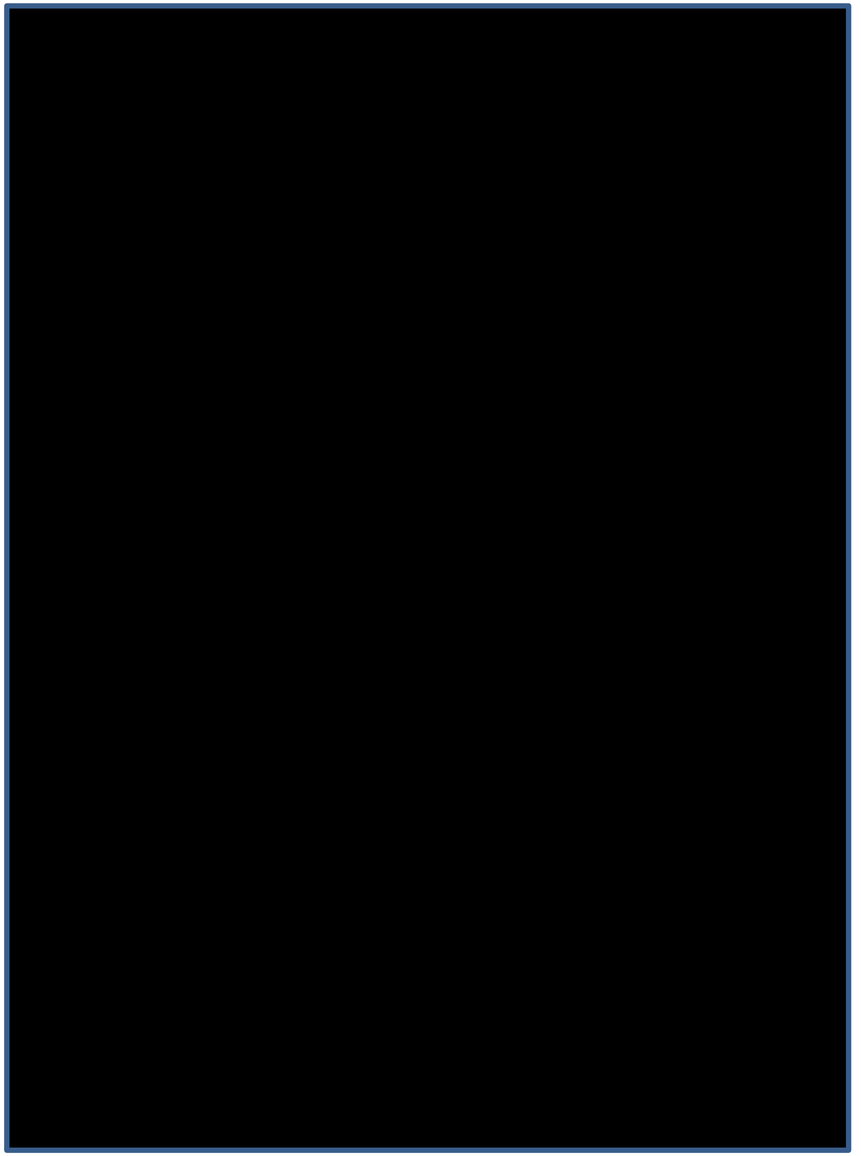
Maximal Exponent




FOTOCS:
Fidelity Otocs
$\hat{V} = |\Psi_0\rangle\langle\Psi_0|$
Chaos
Scrambling
Entanglement
Liapunov Exponents
Thermalization
Volume Law

FOTOCS

Air: Arcimboldo 1566



Water: Arcimboldo 1566

Fotocs and Quantum Chaos

$\widehat{W}=e^{i\delta\phi\widehat{G}}$

$\delta\phi$<<1 Small Perturbation

$\widehat{V}=|\Psi_0\rangle\langle\Psi_0|$

$$F_G(\delta\phi,t)\approx 1-\delta\phi^2\left(\langle\Psi_t|\hat{G}^2|\Psi_t\rangle-\langle\Psi_t|\hat{G}\,|\Psi_t\rangle^2\right)\equiv 1-\delta\phi^2\,\Delta^2 G$$

Great Insight:

- ✓ Provide a semi-classical picture of the scrambling dynamics
  - ➢ Variance can be computed by phase space methods
  - ➢ Compute large systems intractable with numerical methods
- ✓ Connect classical and quantum Liapunov exponents

$\langle\delta G\rangle_c\sim e^{\lambda_L t}$ $\quad 1-F_G(\delta\phi,t)\sim(\delta\phi^2)e^{\lambda_Q t}\equiv\delta\phi^2\,e^{2\lambda_L t}$ $\quad \lambda_Q=2\lambda_L$

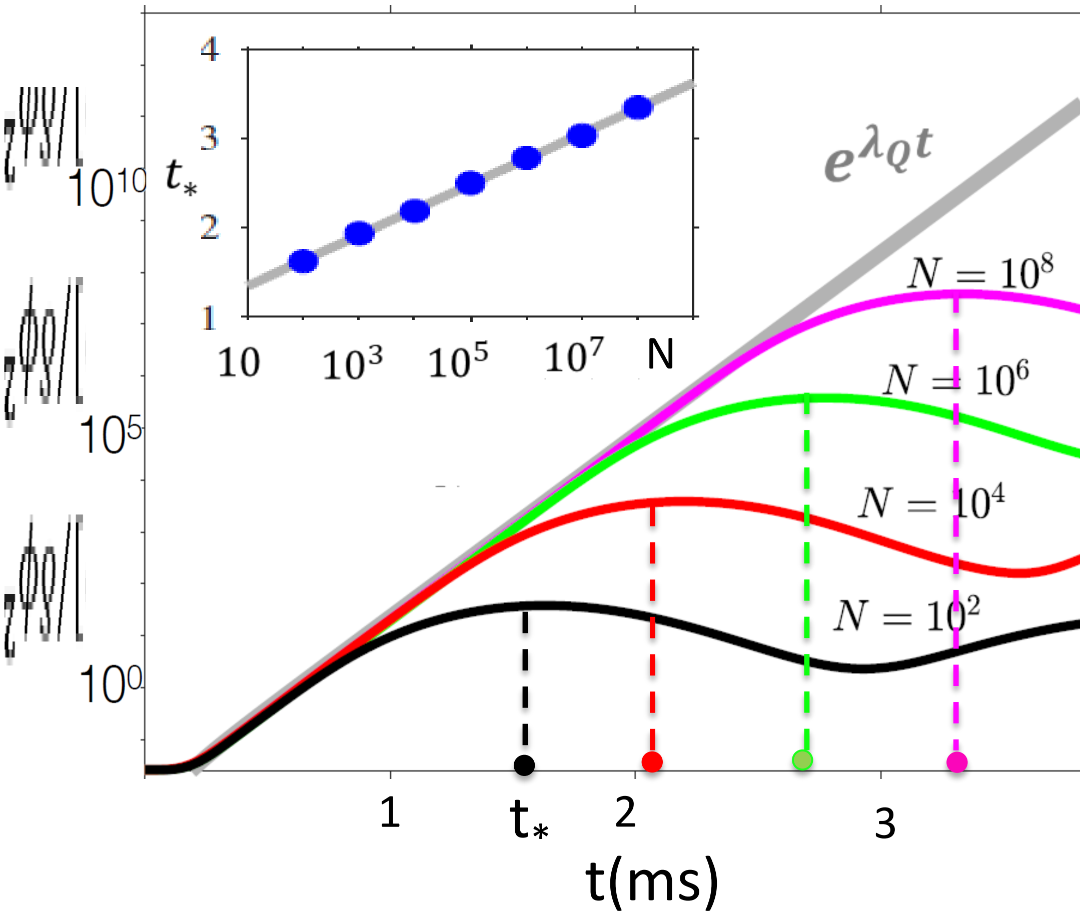
Classical — Quantum

Schmitt et al arXiv:1802.06796
Rozenbaum et al, PRL (2017)

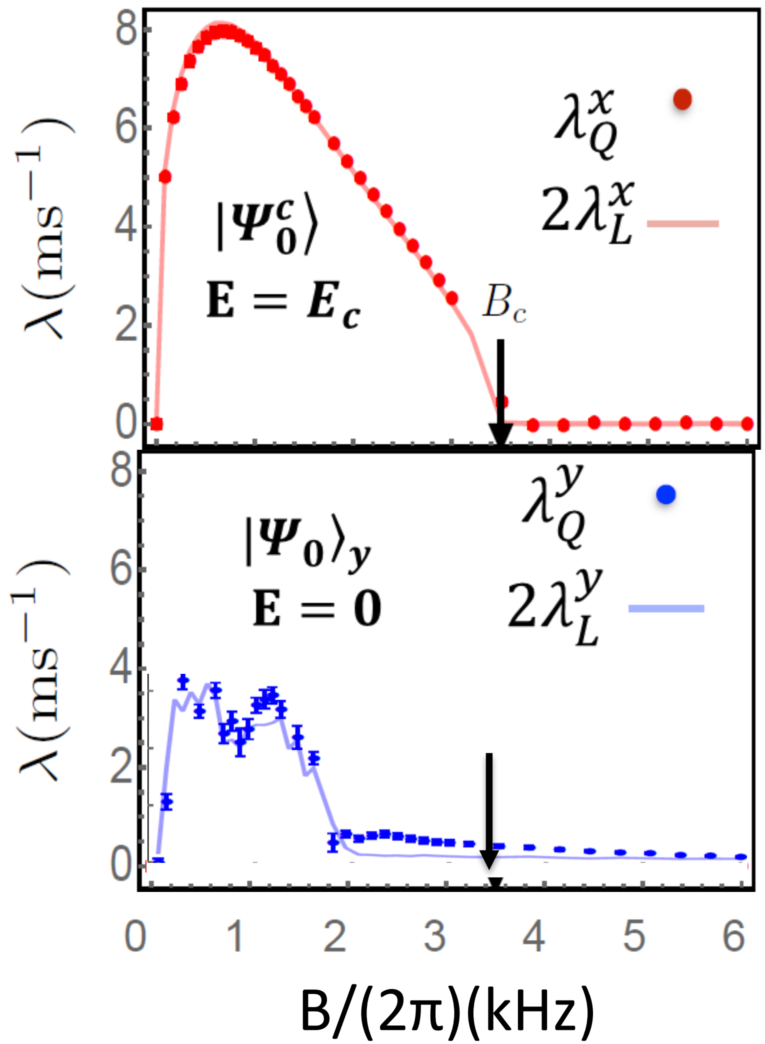
Fotocs and Quantum Chaos

$$\hat{G} = \hat{X} = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$$

$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$$



- Fast scrambling in Dicke M.
- Scrambling time $\lambda_Q t_* \sim \log N$



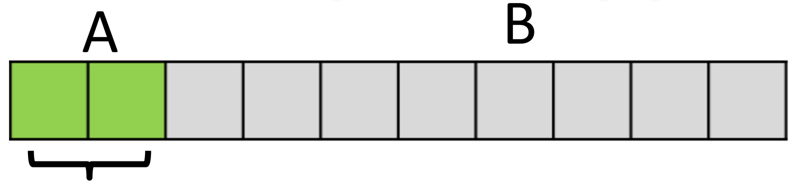
At the critical energy scrambling is faster

$$\lambda_Q = 2\lambda_L$$

FOTOCs & Renyi Entropy

$\hat{\rho}$: Density Matrix

$\mathrm{Tr}(\hat{\rho}^{\,2}) = 1$



$$\hat{\rho}_A = \mathrm{Tr}_B\ \hat{\rho}$$

Reduce density Matrix of A

Renyi entropy: Purity of $\hat{\rho}_A$ $\quad S_A = -\log\left[Tr(\rho_A^2)\right]$

$$e^{-S_A} = \sum_{W \epsilon B} \mathrm{Tr}\left[W_t^\dagger\, \hat{O}\ e^{-\beta H}\, \hat{O}^\dagger W_t\, \hat{O}\, e^{-\beta H}\, \hat{O}^\dagger \right]$$

$$V = \hat{O}\ e^{-\beta H}\, \hat{O}^\dagger$$

H. Zhai: Science Bulletin(2017)

B. Yoshida: JHEP02 (2016)

Sum over complete set of operators acting on B :Exponential 4^B terms

At $\beta \to \infty$ $\quad V = \hat{O}\ |\Psi_g\rangle\langle\Psi_g|\ \hat{O}^\dagger = |\Psi_0\rangle\langle\Psi_0|$ $\quad$ FOTOC

Probing entanglement entropy via randomized measurements: Up to N=20 $\quad$ P. Zoller, R. Blatt, C. F. Roos, arXiv:1806.05747

FOTOCs & Renyi Entropy

$$\hat{\rho} = \sum \varrho^{\,n',n}_{\,m',m} |m'\rangle\langle m| \otimes |n'\rangle\langle n|$$

Spins

Phonons

$$\hat{\boldsymbol{G}}_{S_r}|m_r\rangle = (\boldsymbol{e}_r \cdot \hat{\vec{S}})|m_r\rangle = m_r|m_r\rangle \qquad \hat{\boldsymbol{G}}_n|\boldsymbol{n}\rangle = \hat{\boldsymbol{a}}^\dagger \hat{\boldsymbol{a}}\,|\boldsymbol{n}\rangle = \boldsymbol{n}|\boldsymbol{n}\rangle$$

Spin Renyi Entropy: $S_2(\hat{\rho}_S) = -\log(\mathrm{Tr}[\hat{\rho}_S^2])$: Tracing over phonons

$$\mathrm{Tr}[\hat{\rho}_S^2] = I_0^{\hat{S}_r} + \boldsymbol{I}_{\boldsymbol{0}}^{\hat{\boldsymbol{n}}} - D_{\mathrm{diag}}^{\hat{S}_r,\hat{n}} + C_{\mathrm{off}}^{\hat{S}_r,\hat{n}}$$

Multi-Quantum Intensities. $\hat{V} = |\Psi_0\rangle\langle\Psi_0|$

$$I_0^{\hat{S}_r}(\boldsymbol{t}) = \frac{\boldsymbol{1}}{\boldsymbol{2\pi}} \int_{\boldsymbol{0}}^{\boldsymbol{2\pi}} F_{G_{S_r}}(\phi, t)\,d\phi \qquad I_0^{\hat{n}}(\boldsymbol{t}) = \frac{\boldsymbol{1}}{\boldsymbol{2\pi}} \int_{\boldsymbol{0}}^{\boldsymbol{2\pi}} F_{G_n}(\phi, t)\,d\phi$$

$$\widehat{W}_G = e^{i\phi\hat{G}}$$

Purely-diagonal elements: $D_{\mathrm{diag}}^{\hat{S}_r,\hat{n}} = \sum (\varrho^{\,n,n}_{\,m,m})^2 \sim 1/(N n_{ph})$

Off-diagonal elements:

$$C_{\mathrm{off}}^{\hat{S}_r,\hat{n}} = \sum_{\mathrm{m} \neq m',\, n \neq n'} \varrho^{\,n,n'}_{\,m,m}\, \varrho^{\,n',n}_{\,m',m'} = \sum_{\mathrm{m} \neq m',\, n \neq n'} \varrho^{\,n,n}_{\,m,m'}\, \varrho^{\,n',n'}_{\,m',m} \longrightarrow 0$$

Dephase for $t < t_c \sim \lambda_Q^{-1}$

For $t > t_c$: randomize

FOTOCs & Renyi Entropy

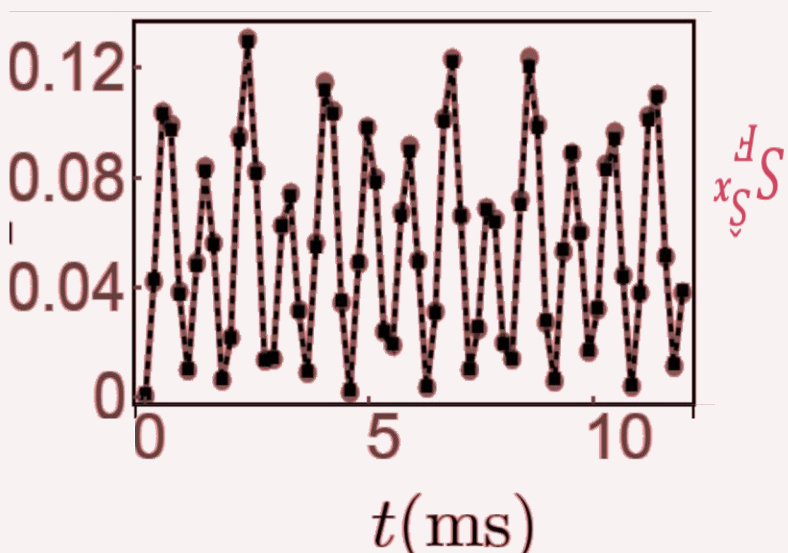
FOTOCS can give use access to Renyi entropy

$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle \quad N=40$$

B>B$_c$: Integrable case

$$S_F^{\hat{S}_x} = -\log\left(I_0^{\hat{S}_x}\right)$$

$|C_{\text{off}}^{\hat{S}_x,\hat{n}}| \ll 0 \quad I_0^{\hat{n}} \approx D_{\text{diag}}^{\hat{S}_r,\hat{n}}$



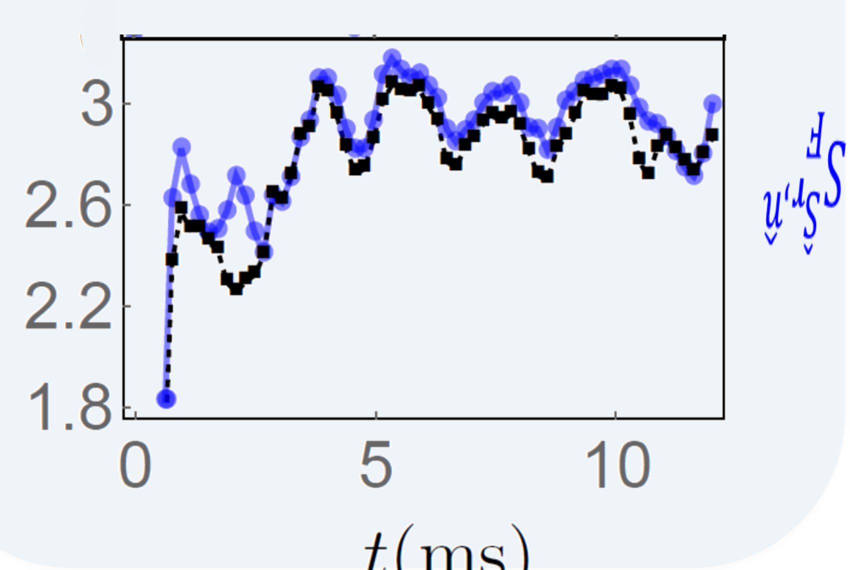
B<B$_c$: Chaotic case

$$S_F^{\hat{S}_r,\hat{n}} = -\log\left(I_0^{\hat{S}_r} + I_0^{\hat{n}}\right)$$

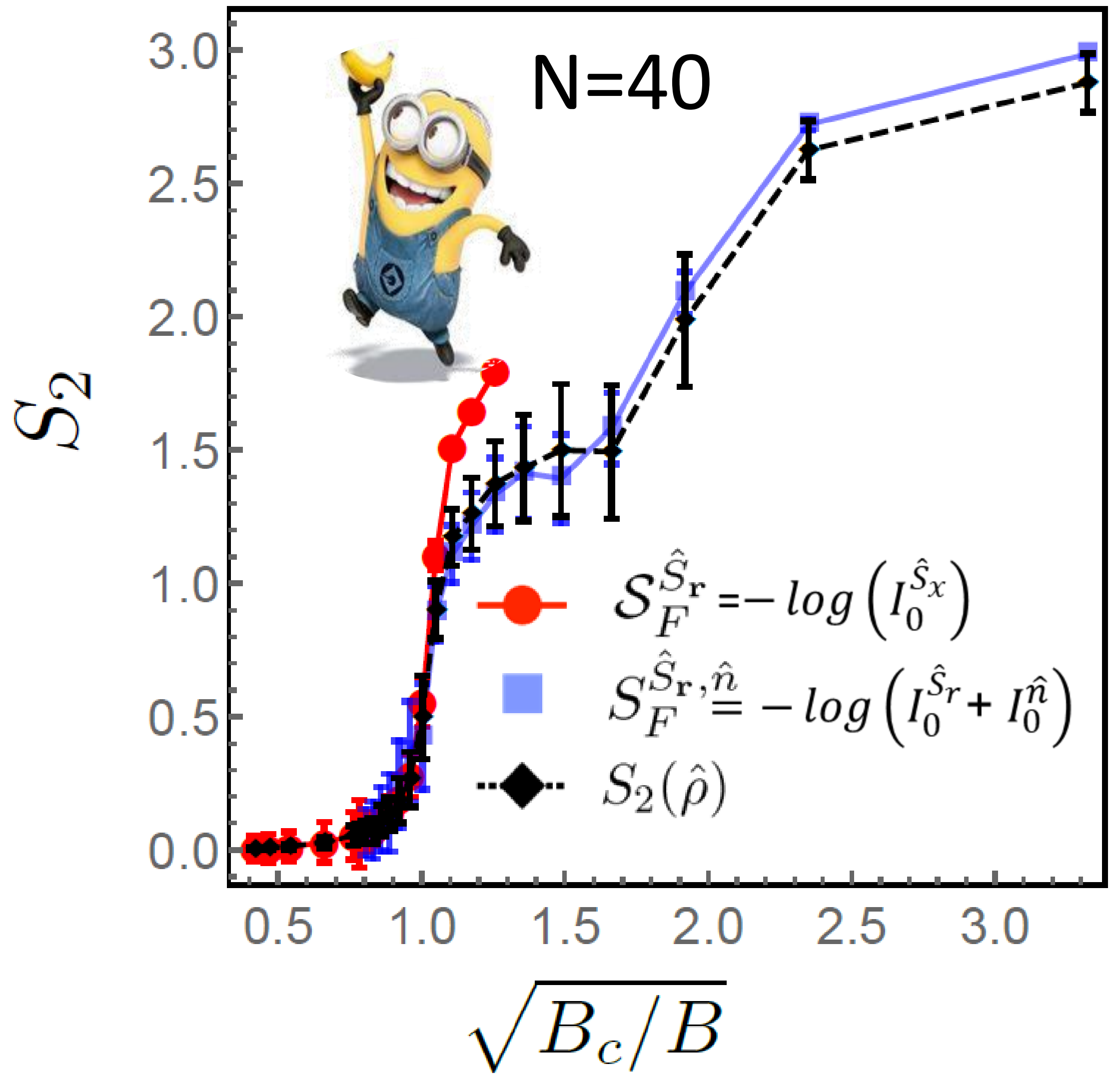
$|C_{\text{off}}^{\hat{S}_x,\hat{n}}(t < \lambda_Q^{-1})| \ll 0$ Initial condition

$|C_{\text{off}}^{\hat{S}_r,\hat{n}}(t > \lambda_Q^{-1})| \ll 0$ Scrambling

$D_{\text{diag}}^{\hat{S}_r,\hat{n}} << 1$

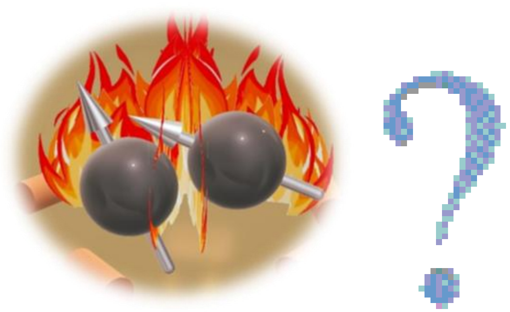


FOTOCs & Renyi Entropy



$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$$

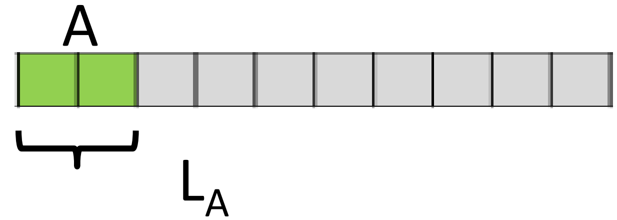
- ❑ Growth of entanglement entropy $B > B_c$
- ❑ System explore large part of Hilbert space: Ergodicity

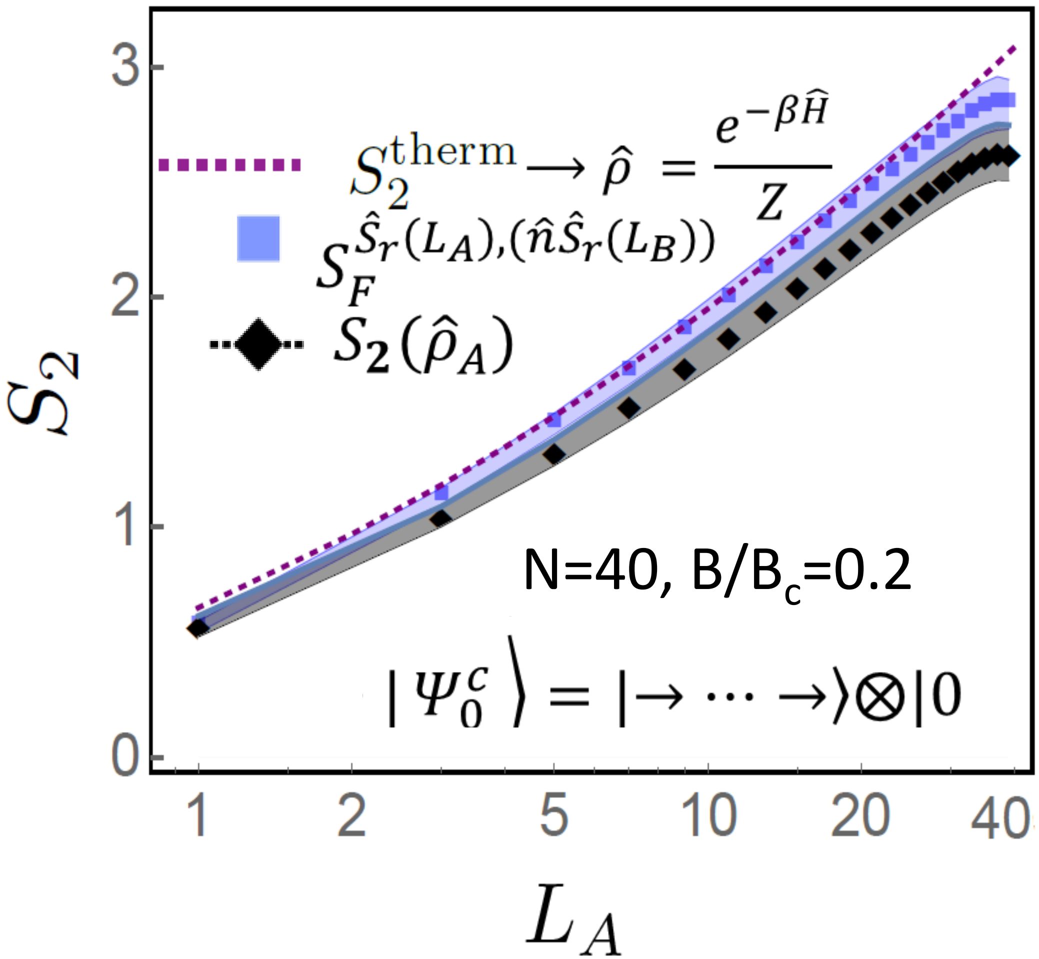


FOTOCs and Thermalization

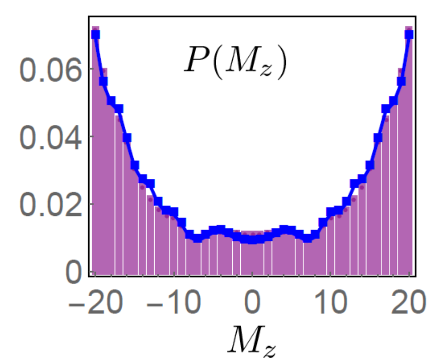
Applying $\widehat{W}_G$ only to part of the spin system

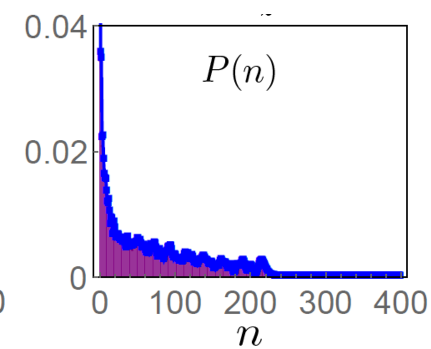
$$S_F^{\hat{S}_r(L_A),(\hat{n}\hat{S}_r(L_B))} = I_0^{\hat{S}_r(L_A)} + I_0^{\hat{n},\hat{S}_r(L_B)}$$





- **Volume law:** $S_2(\hat{\rho}_A) \propto L_A$
- **Thermalization**





Experimental Status

1. Measured FOTOCs (B=0)

    Garttner *et al* Nature Physics(2017)

2. Benchmarked the Dicke Model

    Safavi-Naini *et al* Phys. Rev. Lett. (2018)

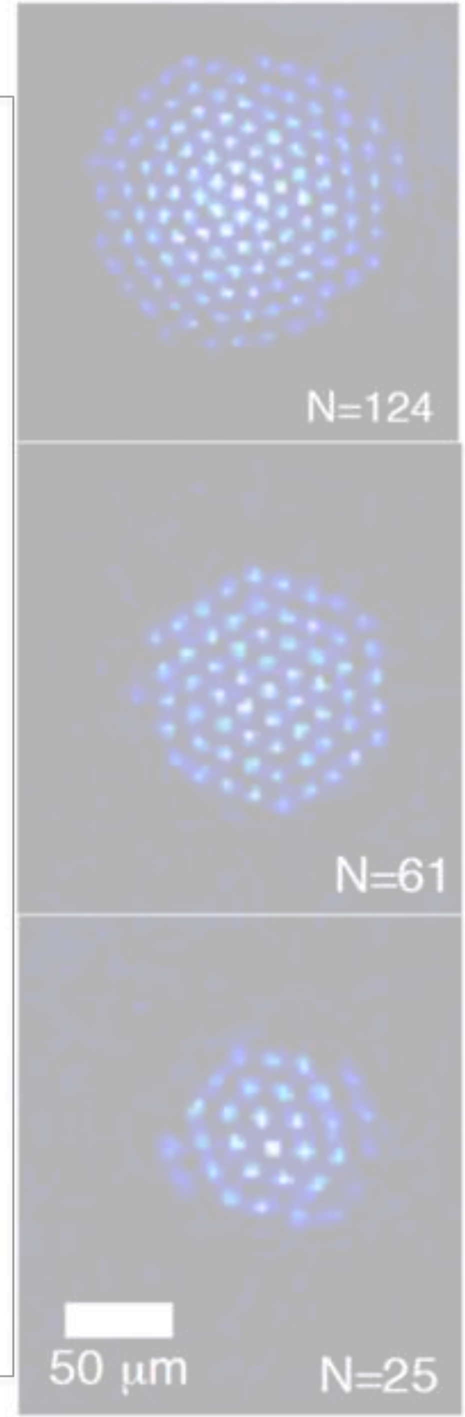
3. Implemented EIT Cooling $(\bar{n} \sim 0)$

    In preparation

4. FOTOCS in Dicke Molel

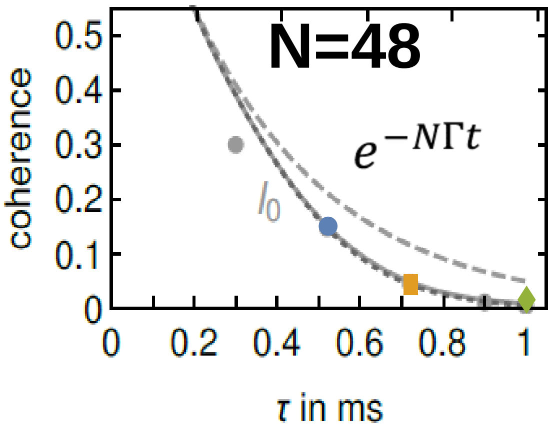
5. Control of Decoherence (Parametric drive)

    Wenchao Ge *et al*: arXiv:1807.00924



Experimental Status

Measured FOTOCs in the Collective Ising model: $\delta \to -\delta$



Garttner et *al Nature Physics (2017)*

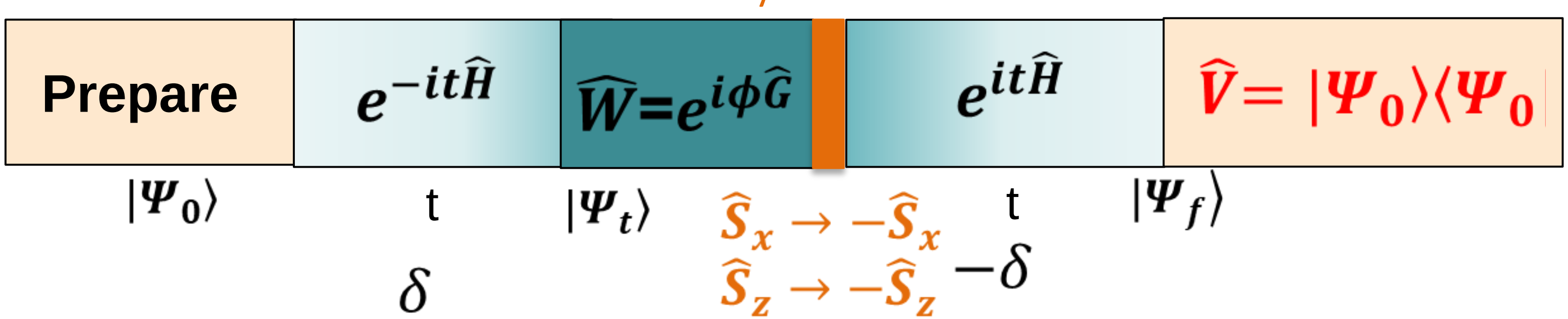
Light scattering

$$I_0(t) = e^{-\Gamma N t} I_0^{pure}$$

Issue: slow measurements
Wanted to decoupled from phonons

- Fotocs in Dicke model:

π_y spin echo



- Need to measure $|-N/2\rangle\langle -N/2| \otimes |0\rangle\langle 0|$
- Possible: Two steps:

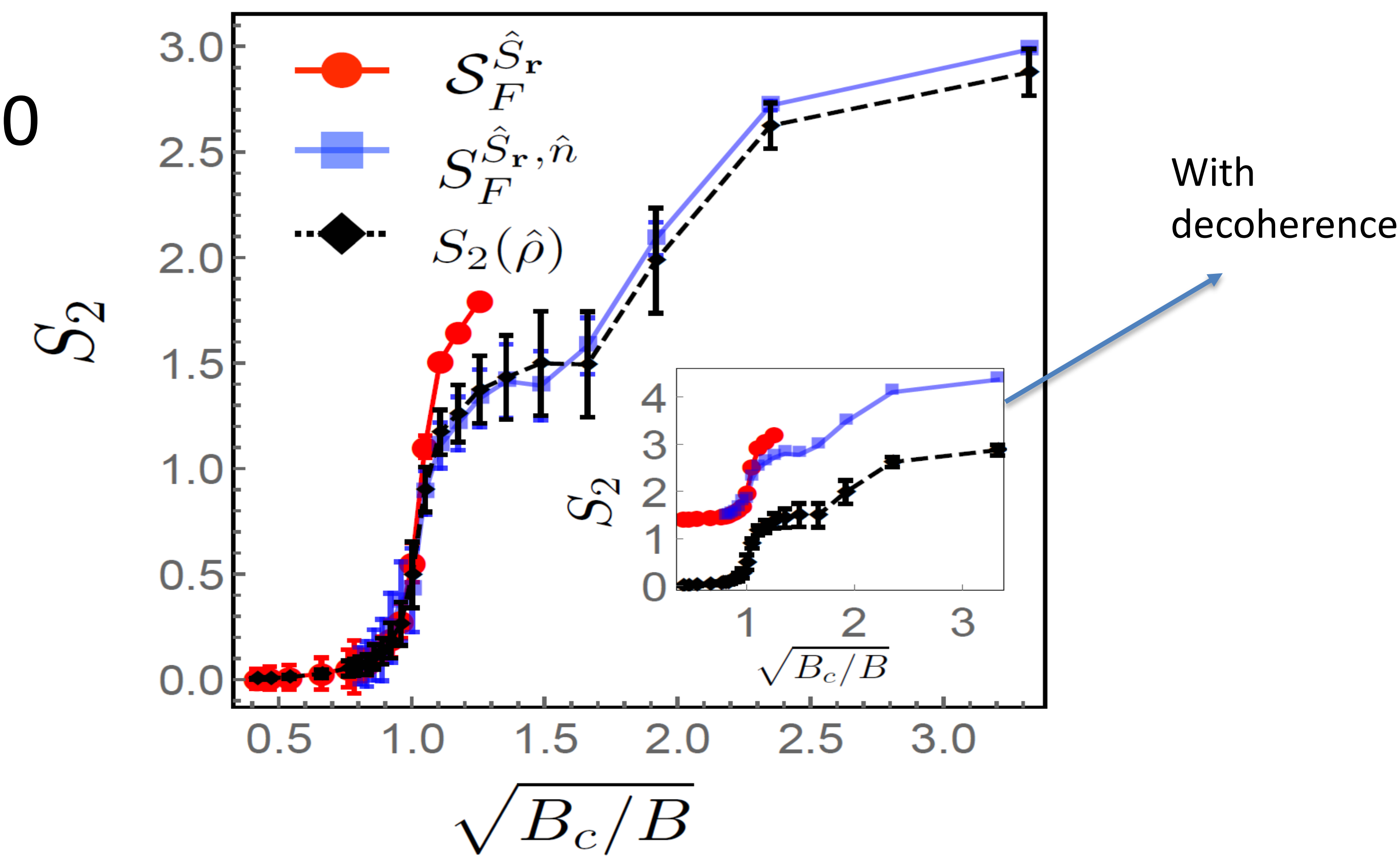
Probability to be dark (no affect motion): DONE

Probability to be in ground motional state (STIRAP): Gebert et al NJP. 18 013037(2016)

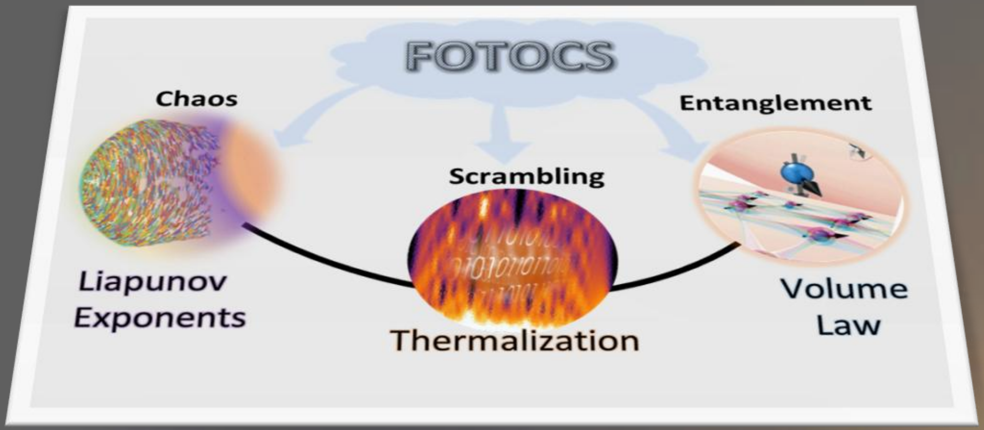
Experimental Status

- Gain: No need to decoupled from phonons (faster dynamics)
- Increase $\frac{B_C}{\Gamma}$ by an order of magnitude (parametric drive, Wenchao Ge *et al:* arXiv:1807.00924)

N=40



Only the beginning: Bright vista ahead



- Bounds on scrambling. Pure states?
- Quantum chaos. (away from semi-classical limit)?
- Error correction / information hiding?
- Design of duals of black hole.
- …..

Thank You!

R. Lewis-Swan *et al*, arXiv:1808.07134