Cold atoms in the presence of disorder and interactions

Collaboration:

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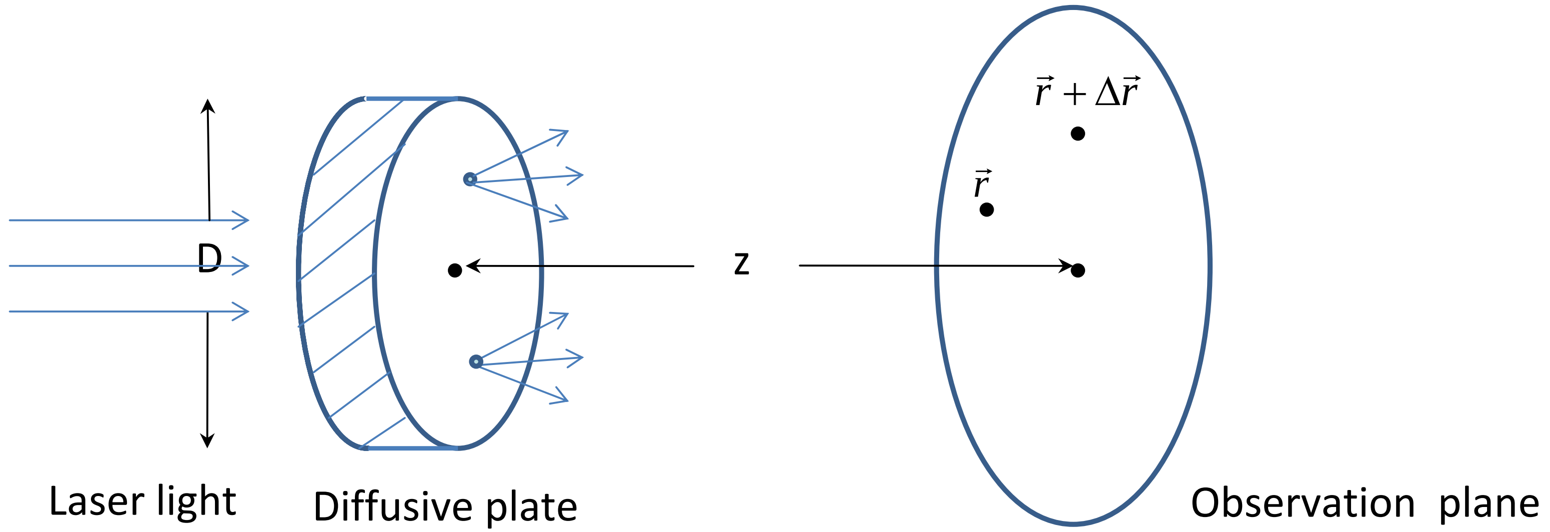
Random Speckle Potential



$I(\vec{r})-$ Light intensity at point $\vec{r}$ in the observation plane

$$\langle I(\vec{r})I(\vec{r}+\Delta\vec{r})\rangle = \langle I(\vec{r})\rangle^2\left[1+4\left(\frac{J_1(\Delta r/R_0)}{\Delta r/R_0}\right)^2\right]$$

$R_0=\lambda z/\pi D$ - correlation radius of the random potential.

$$in\ the\ z-direction, it\ is\ of\ order\ \ \lambda(z/D)^2.$$

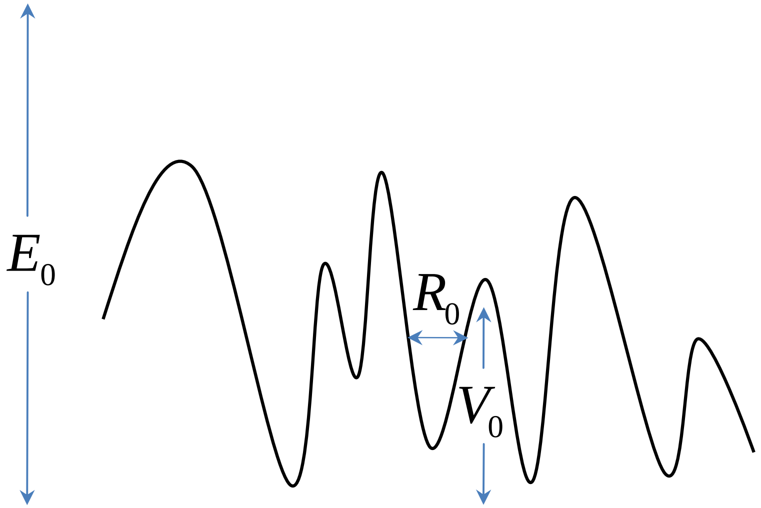
The potential acting on the atoms $V(\vec{r})\propto I(\vec{r})$.

$$\langle \delta V(\vec{r})\delta V(\vec{r}+\Delta\vec{r})\rangle = V_0^2\left(\frac{J_1(\Delta r/R_0)}{\Delta r/R_0}\right)^2$$

V_0 – typical magnitude of the random potential.

R_0 – correlation radius, $(\hbar^2/mR_0^2) = \mathrm{E}_0$ – correlation energy.

1. $V_0 \ll E_0$



Consider a particle with energy $E \ll E_0$.

$$k = \sqrt{2mE/\hbar^2} \ll 1/R_0 \;\rightarrow\; kR_0 \ll 1$$ – white noise limit.

In the Born approximation, the scattering crossection (in 3D) on a typical

barrier (or well) is $$\sigma \sim \left(\frac{V_0}{E_0}\right)^2 R_0^2$$

The mean free path $$l \sim \frac{R_0^3}{\sigma} \sim \left(\frac{\hbar^2}{m}\right)^2 \frac{1}{V_0^2 R_0^3}$$

$$kl \gg 1 \quad \rightarrow \quad E \gg V_0 \left(\frac{V_0}{E_0} \right)^3 \equiv \varepsilon_c$$

$\mu -$ chemical potential of the BEC.

$\mu \gg \varepsilon_c$ $-$ coherent weakly disordered BEC.

$\mu \ll \varepsilon_c$ $-$ condensate droplets, or localized bosons.

The critical density $\quad n_c \sim \dfrac{m}{\hbar^2 a} \varepsilon_c \quad$ $(a -$ scattering length $)$

This picture is confirmed by an approach starting with the low density limit and tracing the interaction-induced delocalization.

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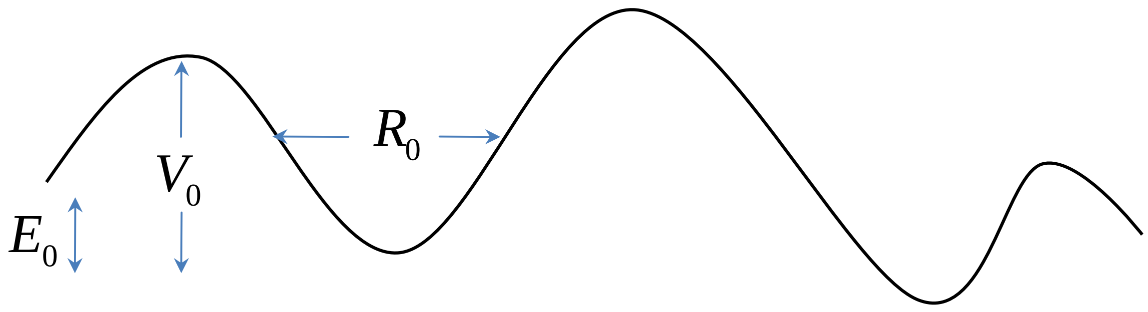
2. $V_0 \gg E_0$



For energy $E \sim V_0$

$$\hbar^2 k^2 / m \sim V_0 \quad \rightarrow \quad kR_0 \sim \sqrt{V_o / E_0} \gg 1$$

On the classical level the problem reduces to that of percolation.

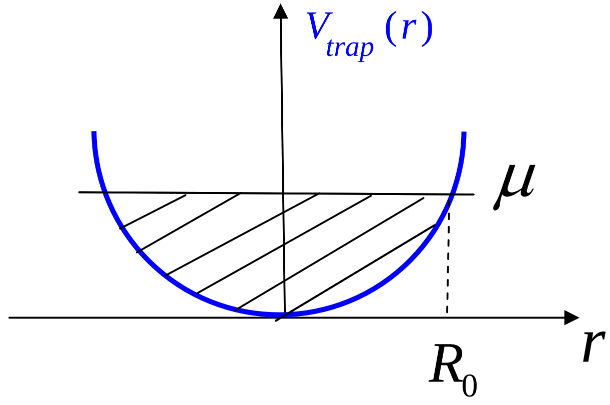
E_p – percolation threshold.

$$E_p^{'} > E_p \; - \; \text{"}quantum\ percolation\text{"}\ threshold.$$

For $\mu < E_p^{'}$ – isolated lakes of condensate.

For $\mu > E_p^{'}$ – coherent BEC.

Free expansion of a BEC



$$V_{trap}(r) = \frac{1}{2} m\omega^2 r^2$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V_{trap}\Psi + g|\Psi|^2\,\Psi = \mu\Psi.$$

$$|\Psi^2(r)| = n(r) = \frac{\mu - V_{trap}}{g} = \frac{\mu}{g}(1 - \frac{r^2}{R_0^2}).$$

$$R_0^2 = \frac{2\mu}{m\omega^2}, \quad \mu \gg \hbar\omega \Rightarrow R_0 \gg \sqrt{\frac{\hbar}{m\omega}} \equiv a_0$$

$$\Psi(\vec{r}, t=0) = \sqrt{n(r,0)} = \sqrt{\frac{\mu}{g}(1 - \frac{r^2}{R_0^2})}$$

At t=0 the condensate is released from the trap and it starts expanding according to:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + g\,|\Psi|^2\,\Psi$$

$$\Psi(\vec{r},t) = \sqrt{n(\vec{r},t)}\exp[iS(\vec{r},t)], \qquad \vec{\mathrm{v}} = \frac{\hbar}{m}\nabla S$$

$$\frac{\partial n}{\partial t} + div\, n\vec{\mathrm{v}} = 0, \qquad m\frac{\partial\vec{\mathrm{v}}}{\partial t} + \nabla\left(\frac{1}{2}m\mathrm{v}^2 + gn\right) = 0$$

Solution (self-similar):

$$n(\vec{r},t) = \frac{\mu}{g}\frac{1}{b^3}\left(1 - \frac{r^2}{b^2R_0^2}\right), \qquad \vec{\mathrm{v}}(\vec{r},t) = \frac{\dot{b}}{b}\vec{r}$$

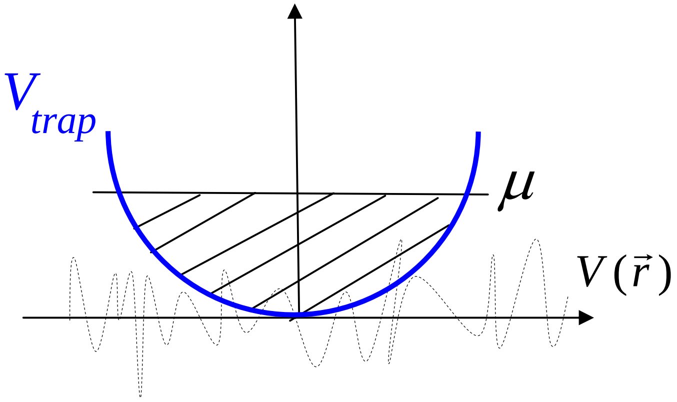
$$\ddot{b} = \omega^2/b^4, \qquad b(0) = 1, \qquad \dot{b}(0) = 0.$$

At $t = \frac{1}{\omega} \equiv t_0$ $\quad \Psi(\vec{r},t) \simeq \sqrt{n(\vec{r},t)}\exp(ir^2/a_0^2) \equiv \Phi(\vec{r}).$

For $t > t_0$ b=ωt $\quad \vec{\mathrm{v}} = \vec{r}/t$ -Linear expansion.

Expansion in the presence of disorder

$For \quad t > t_0$



$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\vec{r})\Psi$$

$\Psi(\vec{r}, t_0) = \Phi(\vec{r})$ - the initial condition.

Reset t_0 back to 0. $\Psi(\vec{r}, t) = \int G(\vec{r}, R, t)\Phi(\vec{R}).$

$$\overline{|\Psi(\vec{r}, t)|^2} \equiv \overline{n}(\vec{r}, t) \simeq \int d\vec{R} \int d^3k \, P_{\varepsilon(k)}(\vec{r}, \vec{R}, t) \, W(\vec{k}, \vec{R}).$$

$P_{\varepsilon(k)}(\vec{r}, \vec{R}, t)$ - Quantum diffusion kernel. $\varepsilon(k) = \hbar^2 k^2 / 2m$

$W(\vec{k}, \vec{R})$ - Wigner function corresponding to $\Phi(\vec{R}).$

In the long time, large distance limit

$$\overline{n}(\vec{r},t) \simeq \int \frac{d^3k}{(2\pi)^3} |\tilde{\Phi}(\vec{k})|^2 \, P_{\varepsilon(k)}(\vec{r},t).$$

$\tilde{\Phi}(\vec{k})$ – Fourier transform of $\Phi(\vec{R})$.

$$For \;\; \varepsilon >> \varepsilon_c, \quad P_\varepsilon(\vec{r},t) = \frac{1}{(4\pi D_\varepsilon t)^{3/2}} e^{-r^2/4D_\varepsilon t}.$$

$$For \;\; \varepsilon < \varepsilon_c, \quad P_\varepsilon(\vec{r},t \to \infty) = \frac{1}{4\pi r} e^{-r/\xi}.$$

$$\xi(\varepsilon) \sim (\varepsilon_c - \varepsilon)^{-\nu}.$$

Anisotropic 2D Diffusive Expansion of Ultracold Atoms in a Disordered Potential

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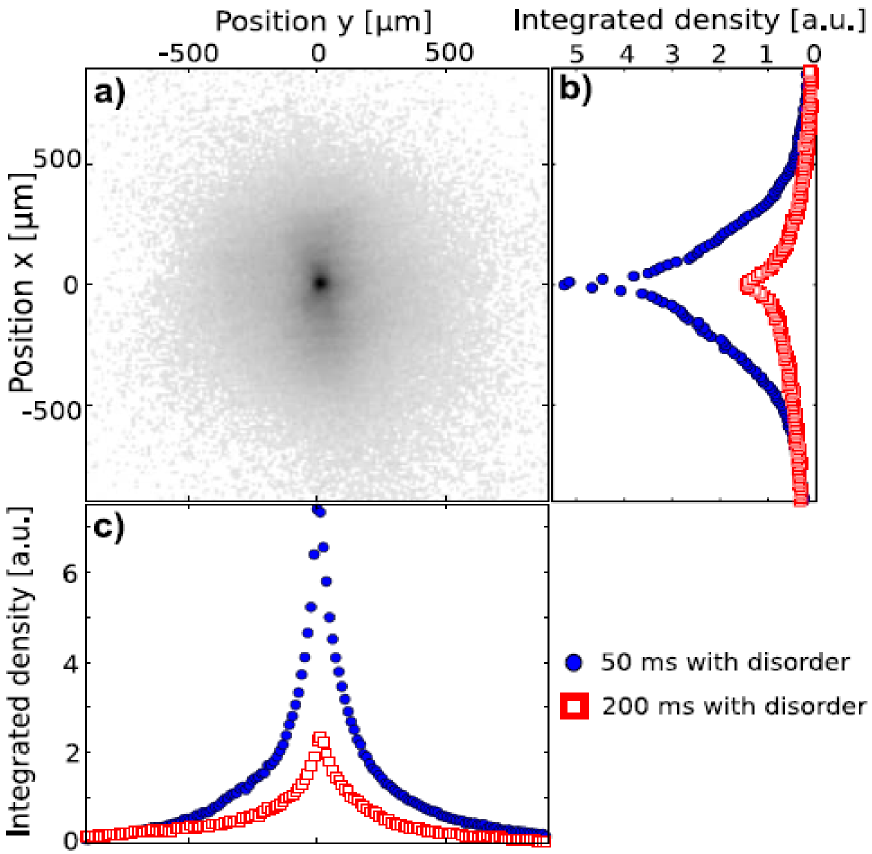


FIG. 2: Atomic column density after planar expansion of an ultra-cold gas in an anisotropic speckle potential. a: Image after 50 ms of expansion. b,c: Integrated density along the two major axes. The plain dots (open squares) correspond to 50 ms (200 ms) of expansion.

$$\bar{n}(\vec{r}, t \to \infty) = f\left(\frac{\varepsilon_c}{\mu}\right)\frac{N}{r^3}\left(\frac{l}{r}\right)^{1/\nu}.$$

$$f(x) \sim x^{3/2} \quad (x << 1),$$

$$f(x) \sim const \quad (x \gg 1)$$

$$\frac{N_{loc}}{N} \sim \left(\frac{\varepsilon_c}{\mu}\right)^{3/2} \quad (\text{for } \varepsilon_c << \mu)$$

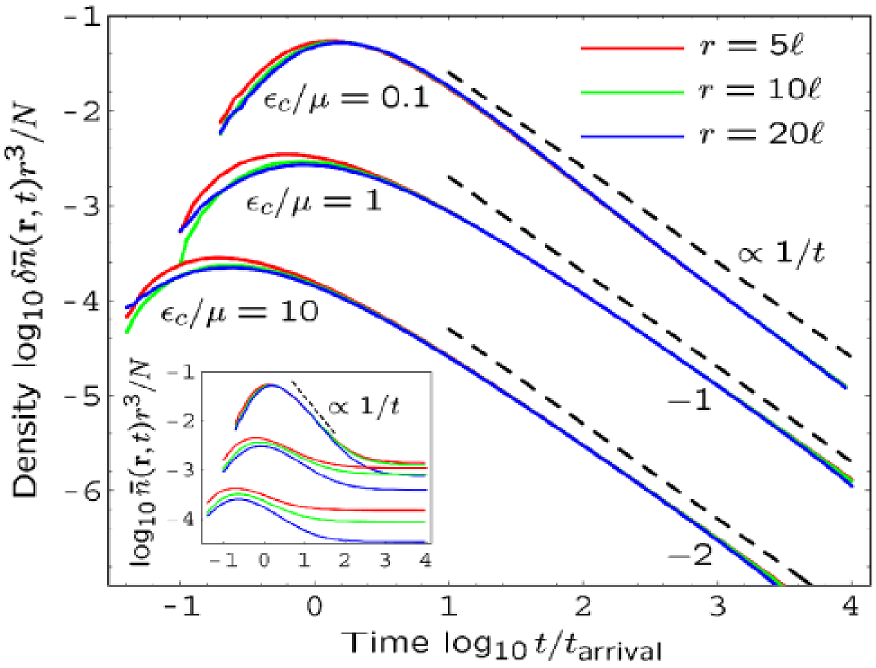


FIG. 2 (color online). Profiles of the dynamic part of the average atomic density obtained from the self-consistent theory of localization (solid lines). The time is given in units of t_{arrival} defined by Eq. (4). The dashed lines show $1/t$ asymptotes. At a given ratio of mobility edge ϵ_c and chemical potential μ, $\delta\bar{n}(\mathbf{r}, t)r^3/N$ for different r (different colors) fall on a universal curve. For clarity, the curves corresponding to $\epsilon_c/\mu = 1$ and 10 are shifted downwards by 1 and 2 units, respectively. The inset shows the complete atomic densities $\bar{n}(\mathbf{r}, t)$ obtained by adding $\delta\bar{n}$ shown in the main plot and $\bar{n}(\mathbf{r}, \infty)$ of Fig. 1.

S. Skipetrov, A. Minguzzi, B. van Tiggelen and B.S.
PRL 100, 165301 (2008).

Two comments about expansion of a Fermi gas, in a disordered potential.

1. The average density

$$\overline{< \hat{n}(\vec{r},t) >} \simeq \int d\vec{R} \int d^3k \, P_{\varepsilon(k)}(\vec{r},\vec{R},t) \, W(\vec{k},\vec{R})$$

$$W = \frac{1}{(2\pi\hbar)^d} \Theta\left(E_F - \frac{\hbar^2 k^2}{2m} - \frac{m\omega^2 R^2}{2} \right)$$

- The shape of cloud, in the long time large distance limit, is the same as for the BEC in the Gross-Pitaevskii approximation.

2. Even in the absence of disorder the density pattern, obtained in a single image, will look noisy and "grainy". A. Legget, Rev. Mod. Phys. 73, 307 (2001).

E. Altman, E. Demler, M. Lukin, PRA 70, 013603 (2004).

* "Atomic Speckles" in the presence of disorder.

Free expansion of a BEC from a disordered trap

Strongly anisotropic BEC (quasi 1D)

Radial confinement $$V(\rho) = \frac{1}{2} m\omega_\perp^2 \rho^2, \qquad \hbar\omega_\perp << \mu$$

Axial confinement is neglected but there is a potential V(z)

Example: $$V(z) = V_0 \cos k_0 z$$

assume $$k_0 a_\perp << 1, \qquad a_\perp = \sqrt{2\mu / m\omega_\perp^2}$$

In equilibrium $$n_0(\rho, z) = \frac{1}{g}(\mu - V(z) - \frac{1}{2} m\omega_\perp^2 \rho^2)$$

At t=0 all potentials are switched off and the condensate expands according to:

$$\frac{\partial n}{\partial t} + div(n\vec{\mathrm{v}}) = 0, \quad m\frac{\partial \vec{\mathrm{v}}}{\partial t} + \nabla(\frac{1}{2} m\mathrm{v}^2 + gn) = 0 \qquad (\vec{\mathrm{v}} = \frac{\hbar}{m}\nabla\Theta)$$

Initial conditions: $$n = n_0(\rho, z), \qquad \mathrm{v} = 0.$$

The first stage of the expansion, $t < t_0 \simeq 1/\omega_\perp$, is dominated by the nonlinearity: rapid radial expansion and, in addition,

$$v_z(z,t) = \frac{1}{m\omega_\perp}\frac{dV(z)}{dz}\arctan\omega_\perp t \quad \rightarrow \quad \Theta(z) = \frac{\pi}{2\hbar\omega_\perp}V(z).$$

For $|\Theta(z)| << 1$ -only small effects

D. Clement , P. Bouyer, A. Aspect, L. Sanchez- Palencia PRA 77, 033631 (2008).

We consider $|\Theta(z)| > 1$ -large effects, “atomic caustics”

The wave function factorizes as $\Psi(\rho,z,t) = \Phi(\rho,t)\psi(z,t)$.

$|\psi(z,t)|^2$ gives the density at point z, normalized by the radial factor $|\Phi(z,t)|^2$

The function $\psi(z,t)$ obeys:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial z^2}, \quad with\ the\ initial\ condition$$

$$\psi(z,t_0) = \exp(i\Theta(z)).$$

$$\psi(z,t)=\sqrt{\frac{m}{2\pi i\hbar t}}\int dz'\exp[\frac{im}{2\hbar t}(z-z')^2+i\Theta(z')]=\sqrt{\frac{m}{2\pi i\hbar t}}\int dz' e^{i\varphi(z',z,t)}$$

In full analogy with optics:

$$\frac{\partial\varphi(z',z,t)}{\partial z'}=0 \quad - equation\ \ for\ the\ rays\ of\ \ atoms$$

$$\frac{\partial^2\varphi(z',z,t)}{\partial z'^2}=0 \quad - condition\ \ for\ caustics.$$

Example: $\Theta(z)=\Theta_0\cos k_0 z$

$$z=z'-\frac{k_0\Theta_0\hbar}{m}t \qquad\qquad \frac{m}{\hbar k_0^2\Theta_0}\frac{1}{t}=\cos k_0 z'$$

$$t^*=\frac{m}{\hbar k_0^2\Theta_0}$$ -characteristic time for caustic formation.

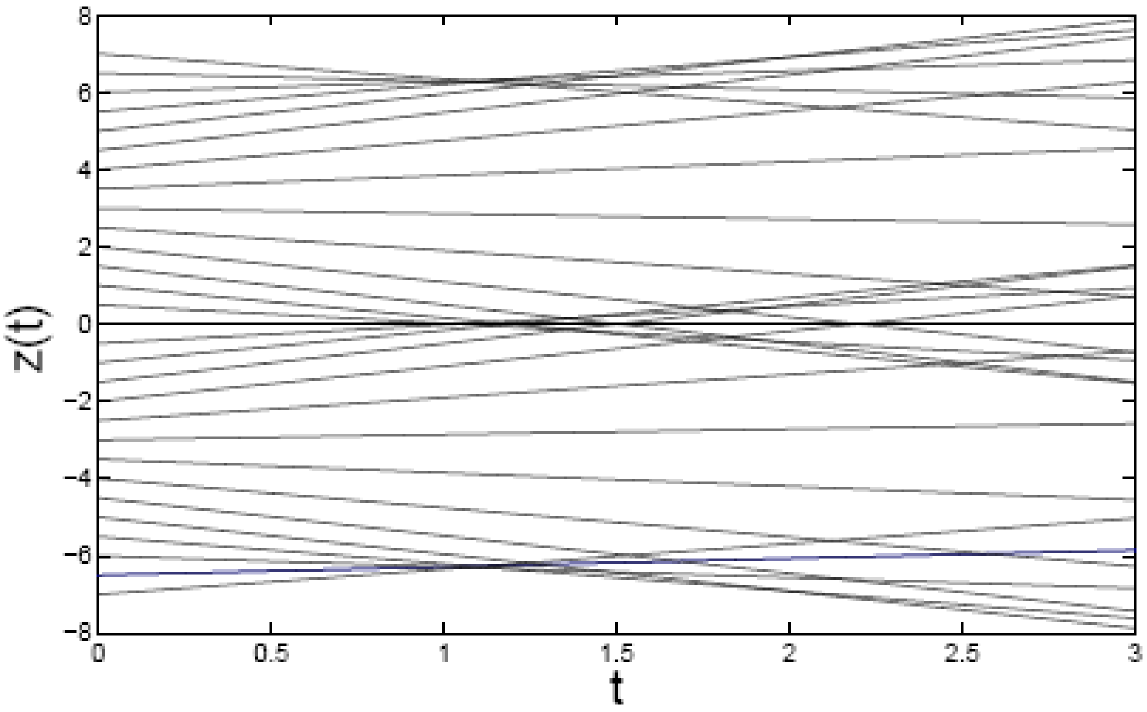
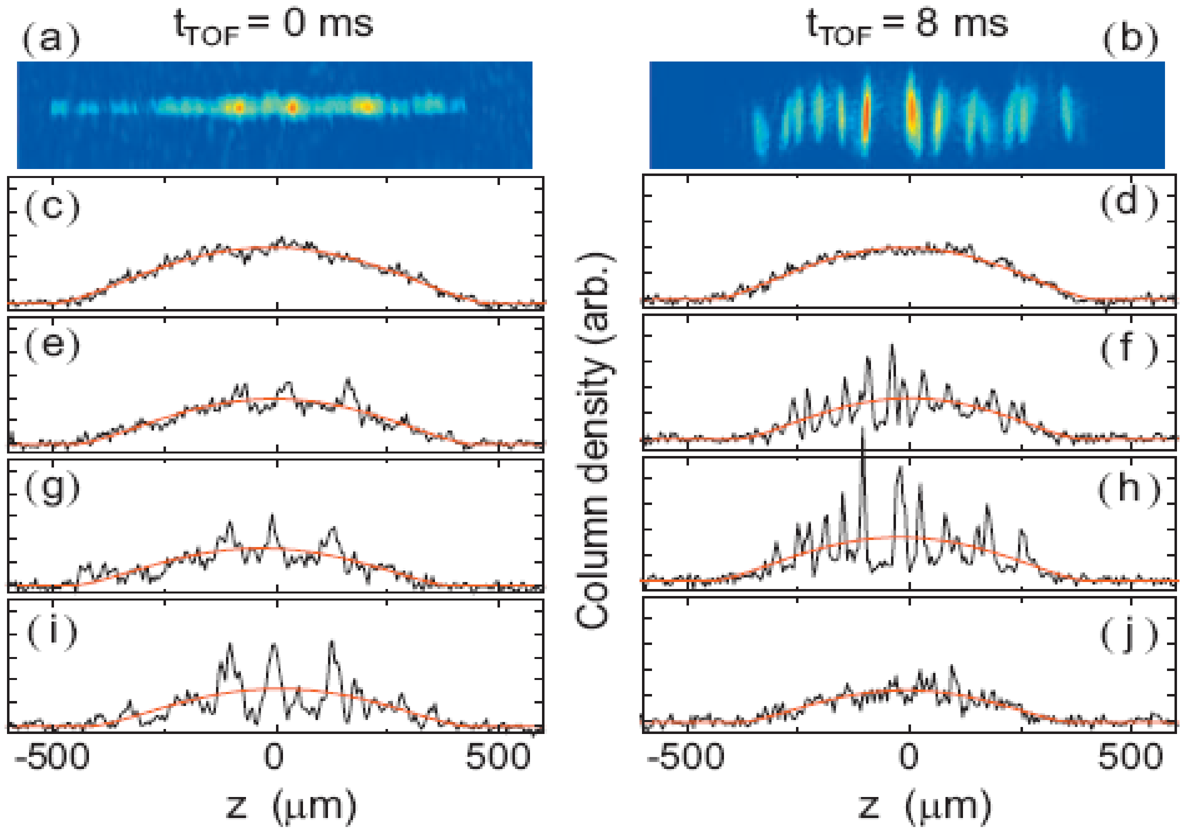


FIG. 1: Paths $z(t)$ followed within the geometrical optics approximation by atoms expanding from a condensate with an initial phase $\theta(z) = \theta_0 \cos(z)$.

Density at caustics is controlled by the third derivative of the phase and is proportional to $\Theta_0^{1/3}$.

There is also a more singular caustic (cusp), with density proportional to $\Theta_0^{1/2}$.



Y.P. Chen, J. Hitchcock, D. Dries, M. Junker, C. Welford, R. Hulet PRA 77, 033632 (2008).

Random potential with correlation length $R_0 = 15\mu m.$ $\quad \frac{\mu}{\hbar\omega_\perp} = 5.6$

(c), (d) – no disorder.

(e), (f) - $V_0 = 0.3\mu \;\Rightarrow\; \Theta_0 = \frac{\pi}{2\hbar\omega_\perp}V_0 \approx 2.6$

(g), (h) - $V_0 = 0.5\mu \;\Rightarrow\; \Theta_0 = \frac{\pi}{2\hbar\omega_\perp}V_0 \approx 4.4$

(i), (j) - $V_0 = \mu$ (beginning of fragmentation)

1D GAS, MANY BODY CORRELATIONS.

So far we assumed the condition $\mu >> \hbar\omega_\perp \quad \Rightarrow \quad na_\perp^2 a >> 1$

In the opposite case, $na_\perp^2 a < 1,$ the problem becomes strictly one-dimensional: In equilibrium all atoms reside in the ground state, $\chi_0(\rho)$ of the harmonic oscillator. Many body correlations become important.

When the gas is released from the trap, the radial expansion will be governed not by the interaction but by the zero point energy associated with radial motion.

Phase imprinting can be done with the help of a short potential pulse:

$$V(z,t) = -\frac{\hbar}{\tau}\Theta(z), \qquad -\tau < t < 0.$$

(the phase can be deterministic or random function of z)

The radial part of the wave function, at t=0, is $\prod_j \chi_0(\rho_j).$

The axial part is $$\Psi(z_1,....z_N;t=0) = \exp[i\sum_j \Theta(z_j)]\,\Phi_0(z_1,....z_N),$$

where Φ_0 is the ground state, prior to the action of the pulse.

At time t=0, just after the phase has been impressed, the trapping potential is switched off and the gas undergoes radial expansion. The z-dependent part of the manybody wavefunction evolves according to

$$i\hbar\frac{\partial\Psi(z_1,.....z_N;t)}{\partial t}=\left[\sum_{j=1}^{N}\left(-\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial z_j^2}\right]\Psi(z_1,....z_N;t),$$

with the initial condition $\Psi(z_1,....z_N;t=0)=\exp[i\sum_j\Theta(z_j)]\,\Phi_0(z_1,....z_N)$

We are interested in the one-dimensional , z-dependent part of the particle density

$$n_1(z,t)=N\int_0^L|\Psi(z,z_2....z_N;t)|^2\,dz_2....dz_N\equiv\frac{N}{L}F(z,t).$$

In second quantized form

$$F(z,t)=\frac{L}{N}\langle\Psi|\widehat{\psi}^+(z,t)\widehat{\psi}(z,t)|\Psi\rangle.$$

$$F(z,t)=\int\frac{dp}{2\pi}|G_p(z,t)|^2\,n(p).$$

$$n(p)=\frac{L}{N}\int dz\langle\hat{\psi}^+(z)\hat{\psi}(0)\rangle\,e^{-ipz}.$$

$$G_p(z,t)=\sqrt{\frac{m}{2\pi i\hbar t}}\int d\zeta\exp[\frac{im}{2\hbar t}(z-\zeta)^2+i\Theta(\zeta)-ip\zeta].$$

In the mean field approach $n(p)=2\pi\delta(p)$ and $G_p(z,t)$

reduces to $\psi(z,t)$ considered previously (mean field).

In the interacting gas the conditions for caustics are:

$\Theta_0^{1/3}\xi/R_0\gg 1,\quad n_1R_0\Theta_0^{-1/3}\gg 1$ $\quad \xi=\dfrac{1}{\alpha n_1}$ -healing length.

Weak interactions: $\xi\gg n_1^{-1}$ and caustics can be formed.

Strong interactions (Tonks limit): $\xi\approx n_1^{-1}$ No caustics.