Particle Entanglement in many-body quantum states

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Entanglement measures for quantum states

mystery quantum state

↓ **entanglement measures**

(topological) order, quantum critical properties, exclusion statistics, ...

How entangled are fqH states?

M. Haque, O.S. Zozulya, KjS

PRL (2007), PRB (2007) (with E.H. Rezayi), PRA (2008)

review on PE: JPhysA (2009)

Entanglement measures for quantum states

fqH state or mystery state in the Lowest Landau Level

↓ **entanglement measures**

???

How entangled are fractional quantum Hall states?

Can we determine the nature of a LLL quantum state based on entanglement measures?

How entangled are fqH states?

Orbital / spatial partitioning (OES, RES)

- reveals edge physics:

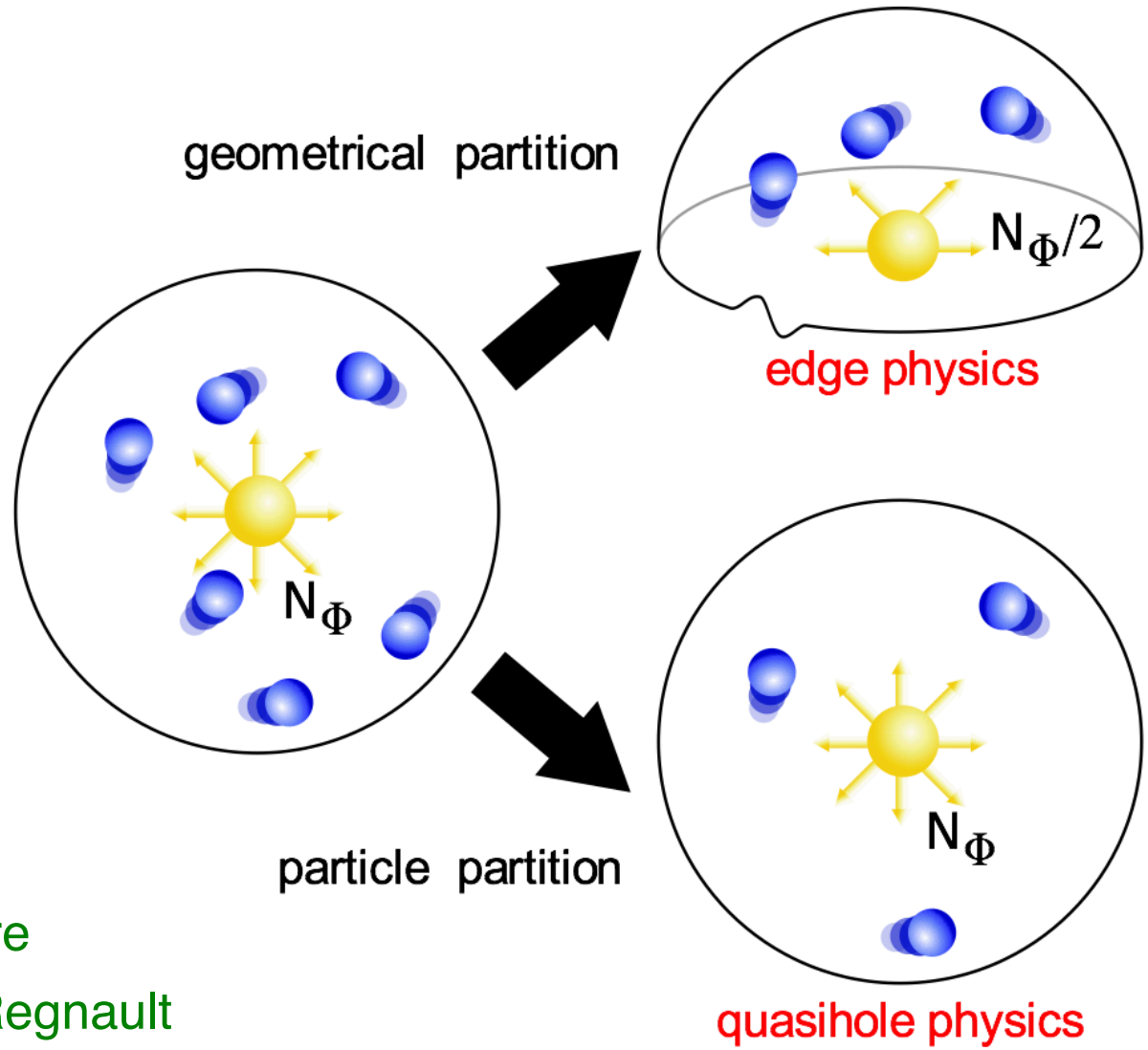
  area law, topological entanglement entropy, ...

Particle partitioning (PES)

- reveals bulk physics:

  state counting, exclusion statistics, condensation, ...

Bipartite entanglement measures



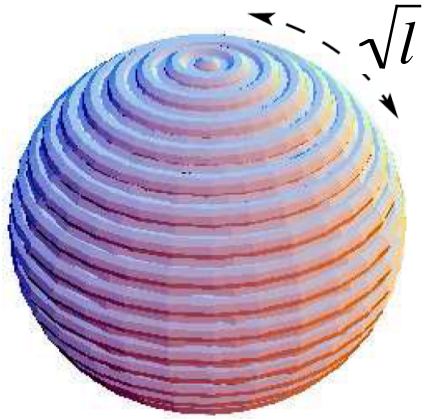
figure

N. Regnault

FqH wave functions on the sphere

- N fermions in spherical geometry, filling factor ν [Haldane '83]
- monopole at center provides magnetic field; total flux

$$N_\phi = \frac{1}{\nu} N - S$$



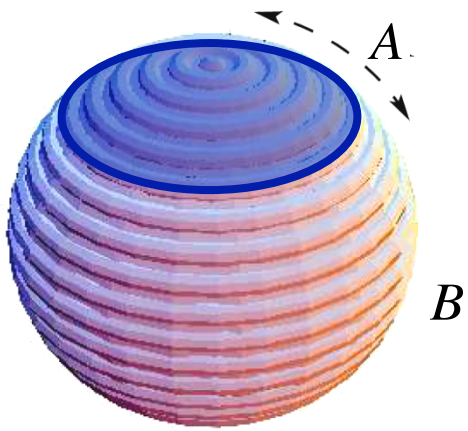
- eigenstates of orbital angular momentum localized on latitude lines → Lowest Landau Level orbitals
- l-th orbital, $l = 0,1,\ldots,N_\phi$ localized at distance $\propto 2\sqrt{l}$ from north pole
- spherical projection onto plane gives standard fqH wavefunctions

Orbital partitioning

- orbital partitioning

  A-block — orbitals $l = 0, \ldots l_A - 1$

  B-block — orbitals $l = l_A, \ldots N_\phi$

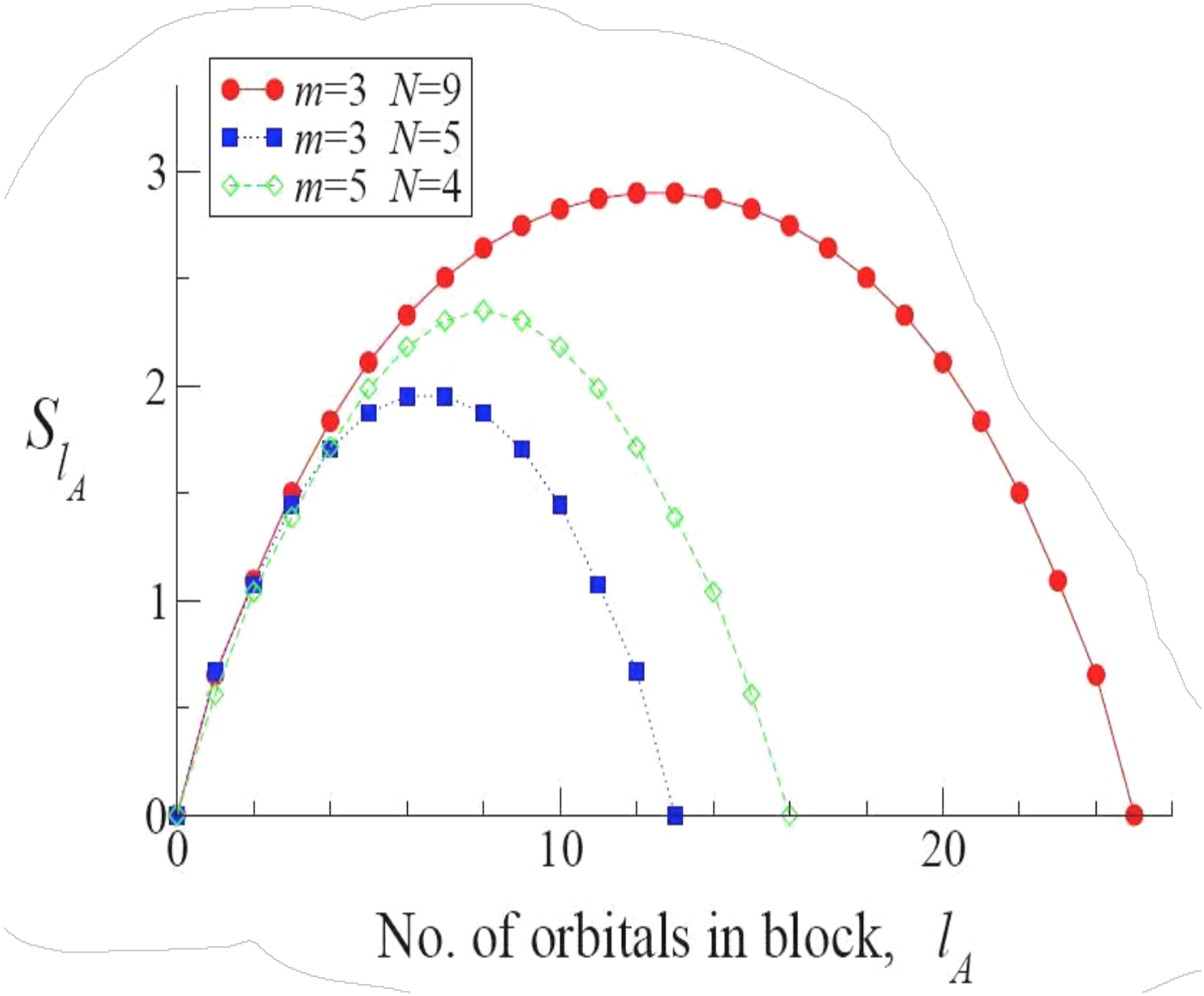


- boundary between orbitals $l_A - 1$ and l_A located

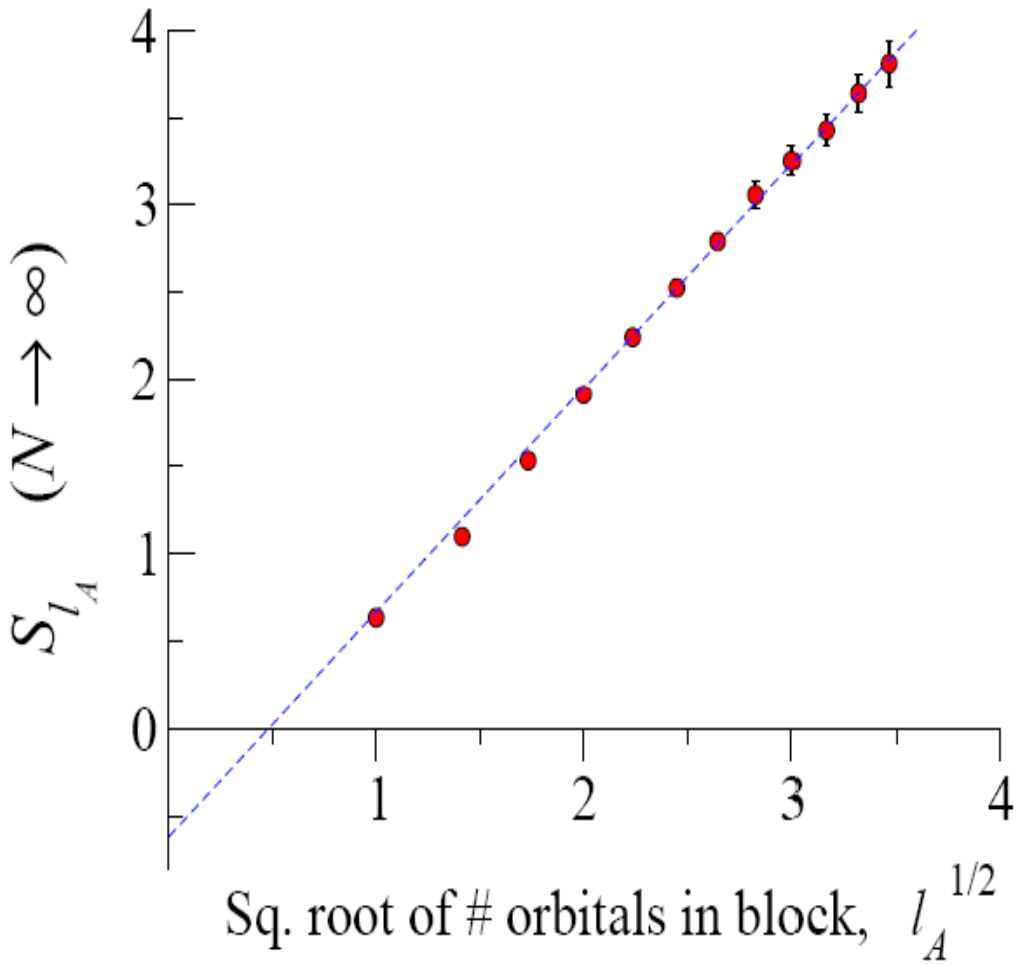
  at $r = 2\sqrt{l_A}$; expect asymptotic behavior

$$S_{l_A} \cong -\gamma + \alpha\sqrt{l_A}$$

ν=1/3 Laughlin — orbital entropies



ν =1/3 Laughlin — extracting γ



best fit to

$$S_{l_A} \cong -\gamma + \alpha\sqrt{l_A}$$

$$\longrightarrow \quad \gamma = 0.51 \pm 0.14$$

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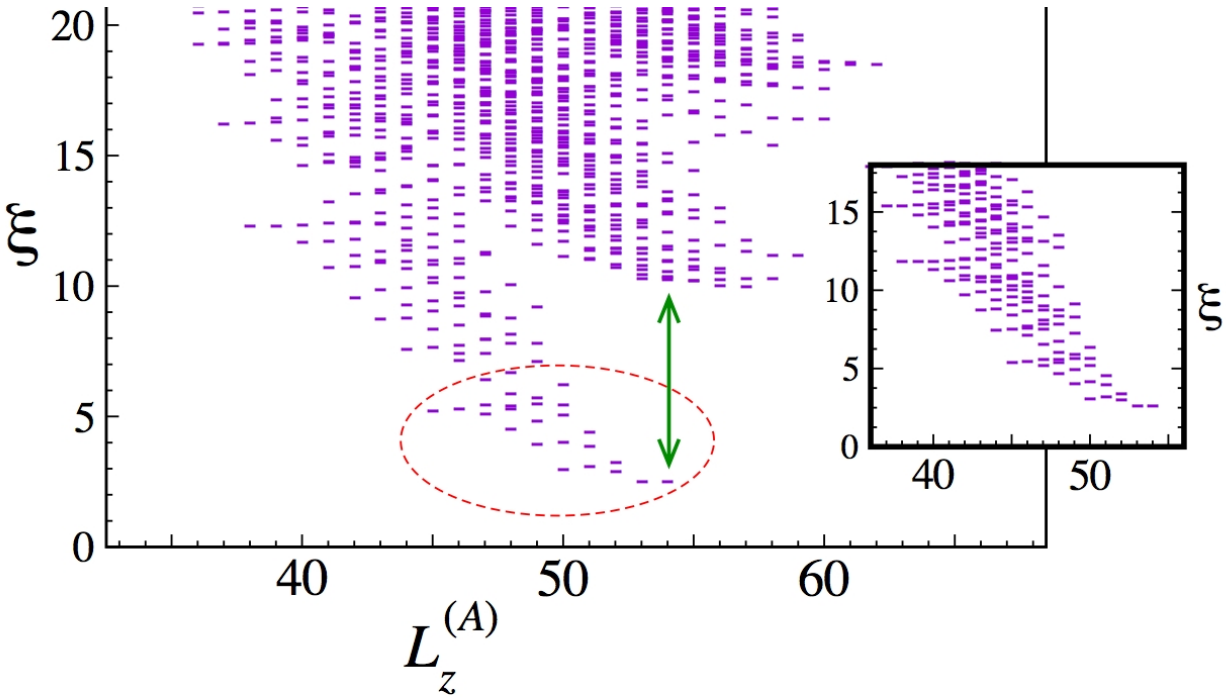
from TQFT / CFT

$$\gamma_3 = \ln\sqrt{3} = 0.55$$

Entanglement spectrum (OES)



Li, Haldane (2008)

Regnault, Bernevig, Haldane (2009)

Zozulya, Haque, Regnault (2009)

- entanglement spectrum mirrors edge spectrum
- entanglement gap for non-model states

Particle partitioning

- particle partitioning

  A-block — n_A particles

  B-block — $n_B = (N\text{-}n_A)$ particles

$$\rho_{n_A}(\vec{x}, \vec{y}) = \int dz_{n_A+1} \cdots dz_N \psi^*(x_1, \cdots, x_{n_A}; z_{n_A+1}, \cdots, z_N) \times \psi(y_1, \cdots, y_{n_A}; z_{n_A+1}, \cdots, z_N)$$

Particle entanglement: bounds

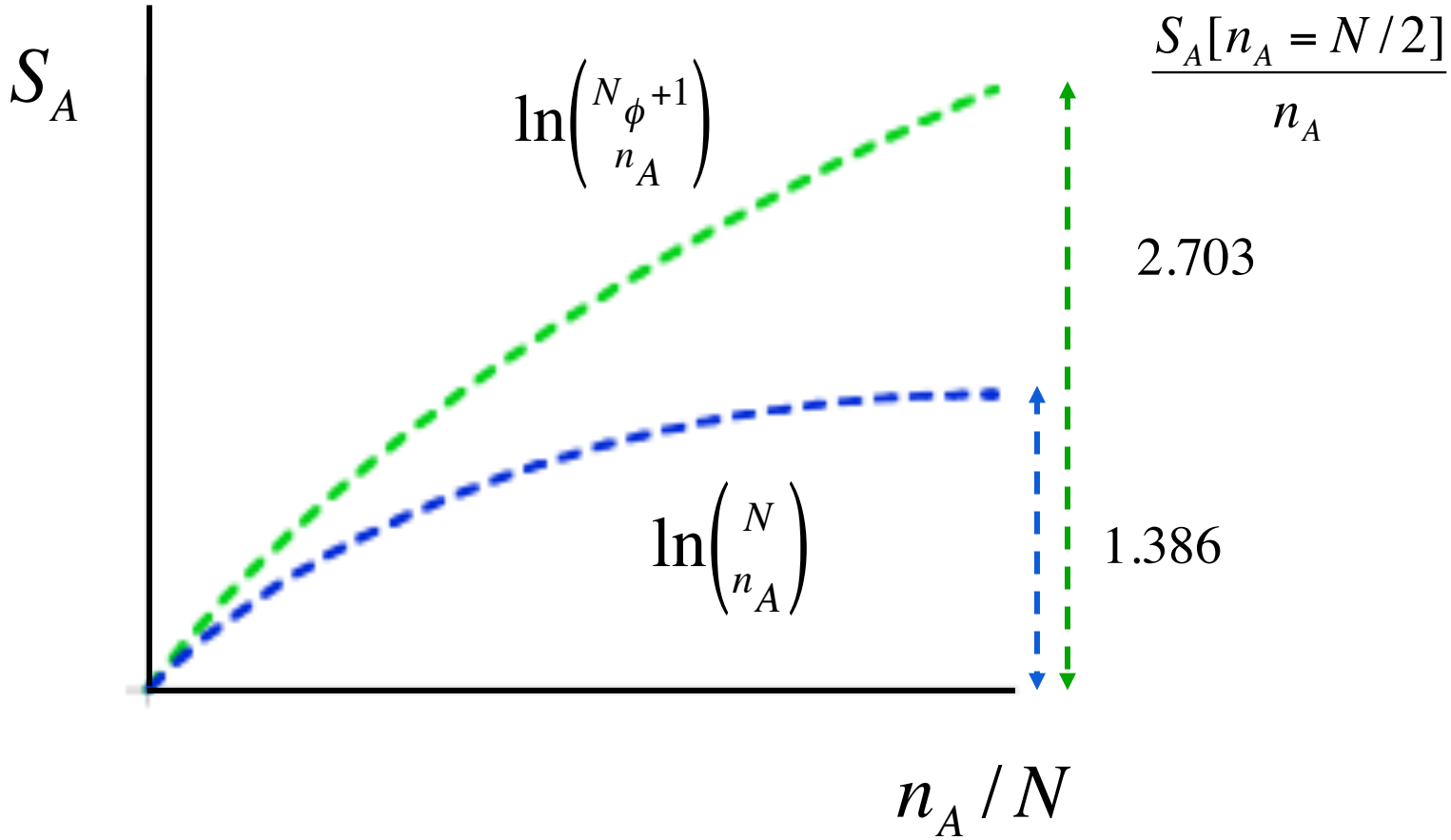
- n out of N bosons in L orbitals

$$0 \le S_n \le \ln\begin{pmatrix} L-1+n \\ n \end{pmatrix}$$

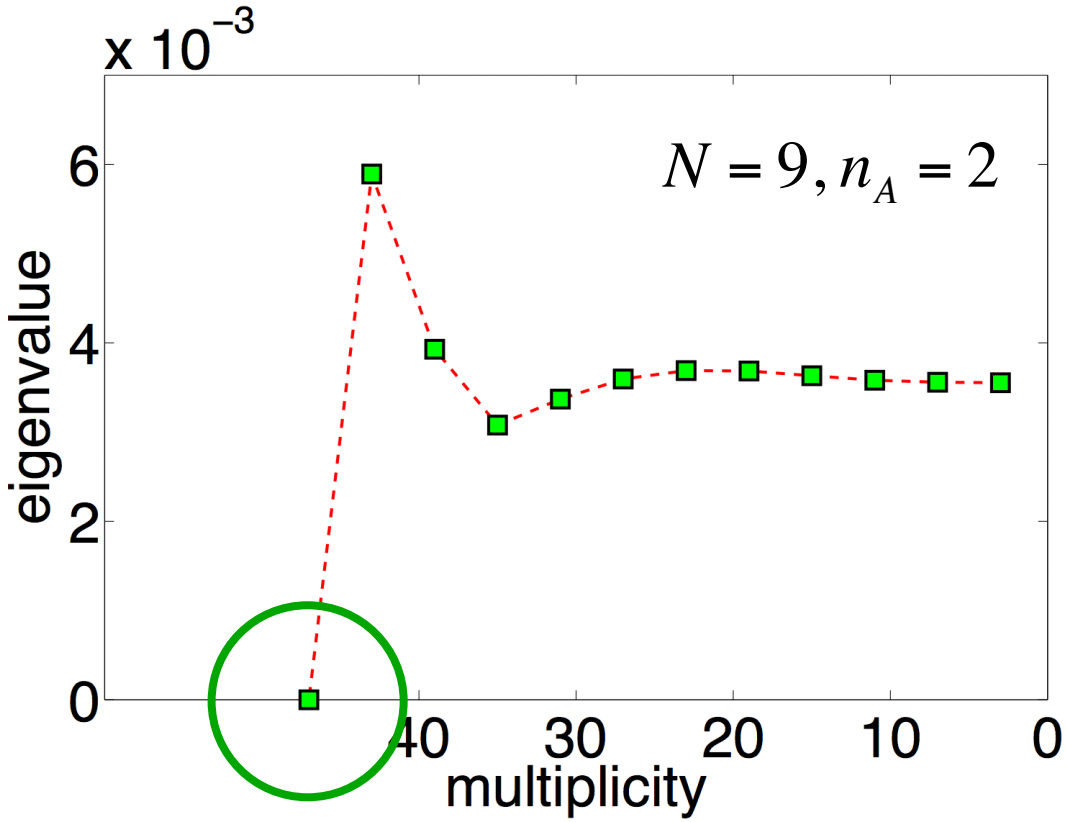
- n out of N fermions in L orbitals

$$\ln\begin{pmatrix} N \\ n \end{pmatrix} \le S_n \le \ln\begin{pmatrix} L \\ n \end{pmatrix}$$

ν=1/3 Laughlin — bounds



ν=1/3 Laughlin - eigenvalue distribution



Laughlin states — improved bound

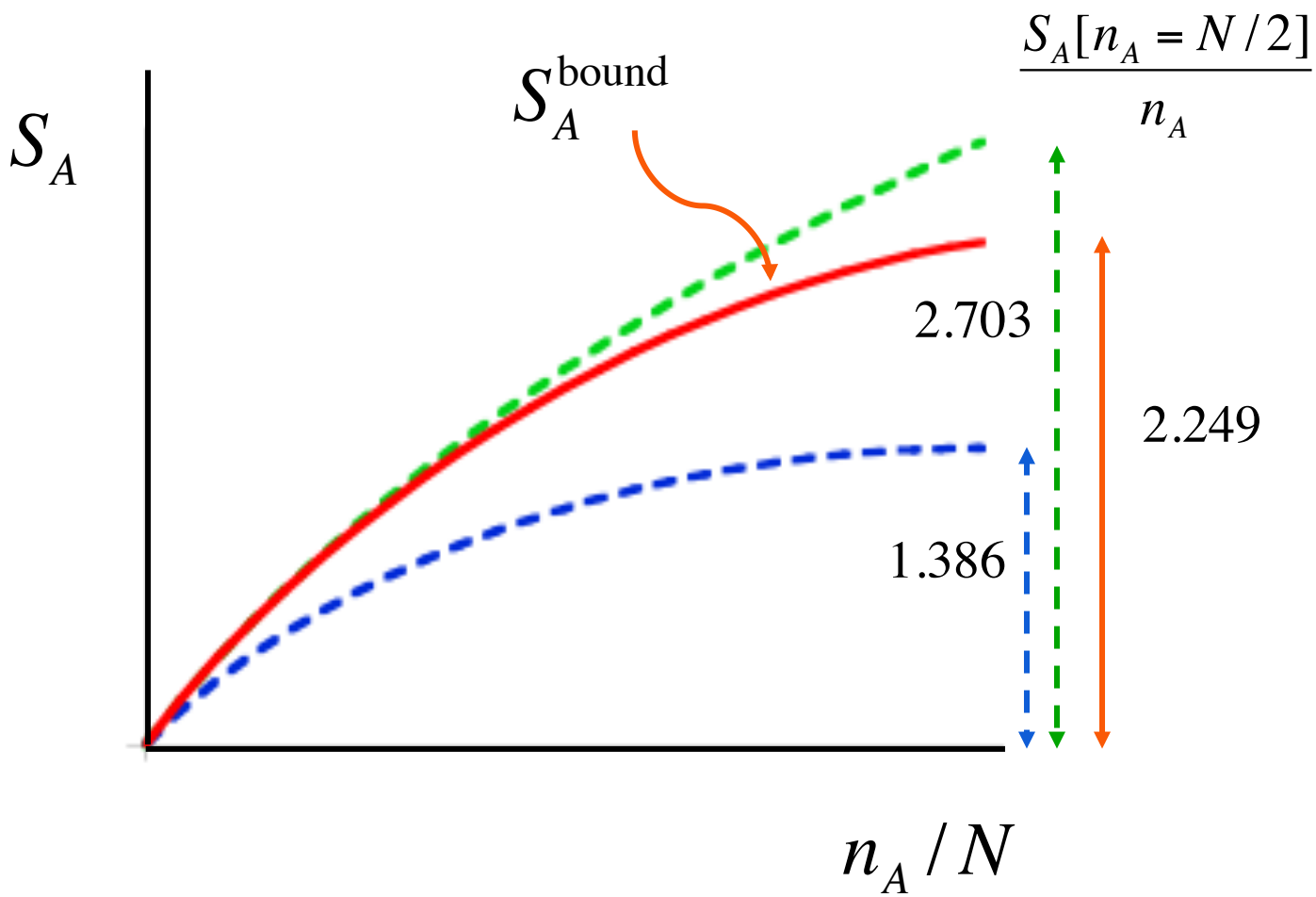
- using quasi-hole counting results [Read-Rezayi,1996] upper bound for the $\nu=1/m$ Laughlin state improved as

$$S_A \leq \ln\binom{N_\phi+1}{n_A} \longrightarrow S_A \leq S_A^{\text{bound}} = \ln\binom{N_\phi+1-(m-1)(n_A-1)}{n_A}$$

- interpretation in terms of exclusion statistics:

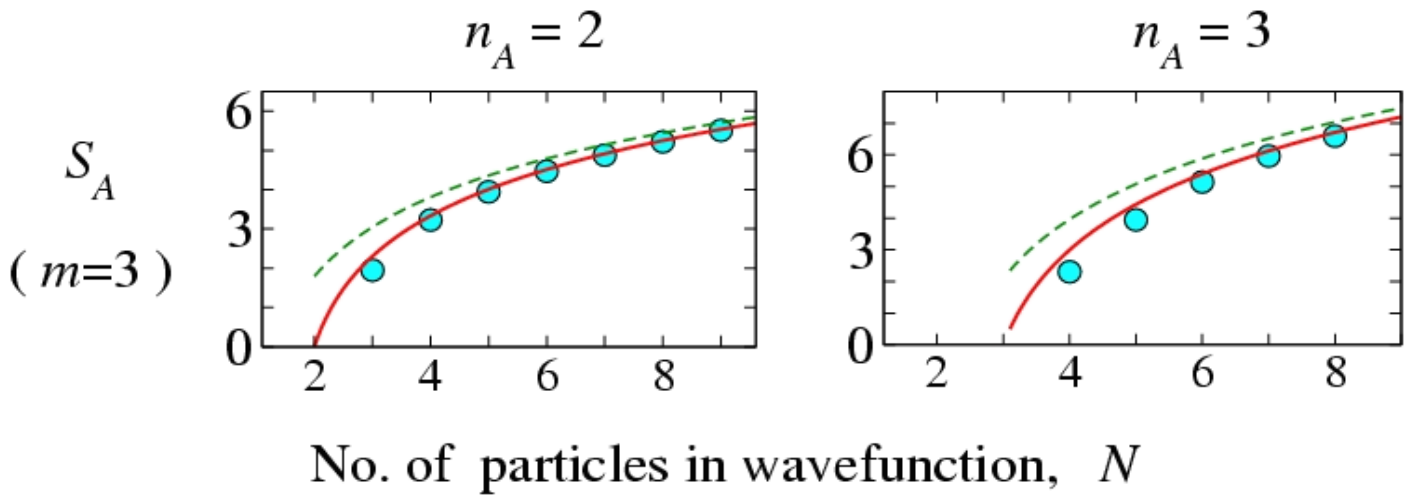
the improved bound is precisely given by the number of ways n_A particles can be put into $N_\phi+1$ orbitals, observing a minimal distance of m between adjacent occupied orbitals

ν=1/3 Laughlin — improved bound



Laughlin states — improved bound

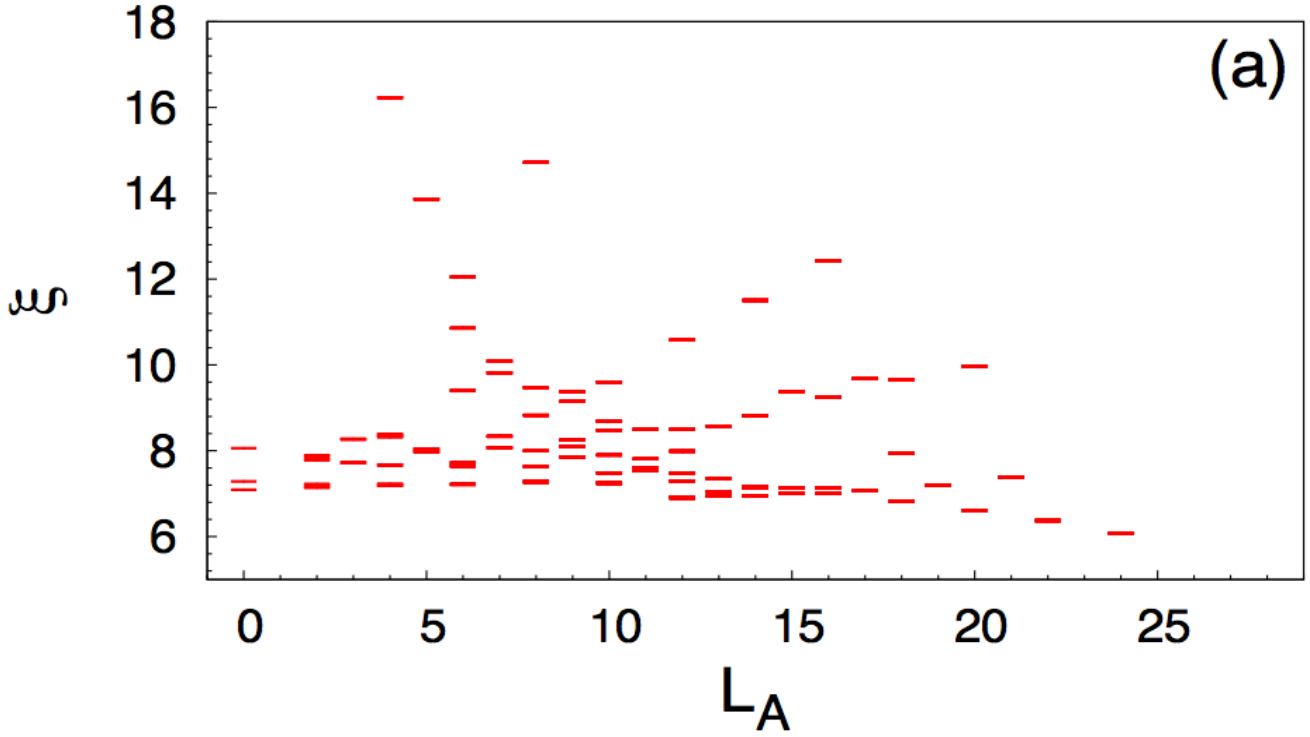
- numerical data-points follow improved bound quite closely



- *1/N* expansion at $n_A << N$

$$\ln\binom{N_\phi + 1}{n_A} - S_A^{\text{bound}} = \frac{1}{N}\frac{m-1}{m} n_A(n_A - 1) + O(\frac{1}{N^2})$$

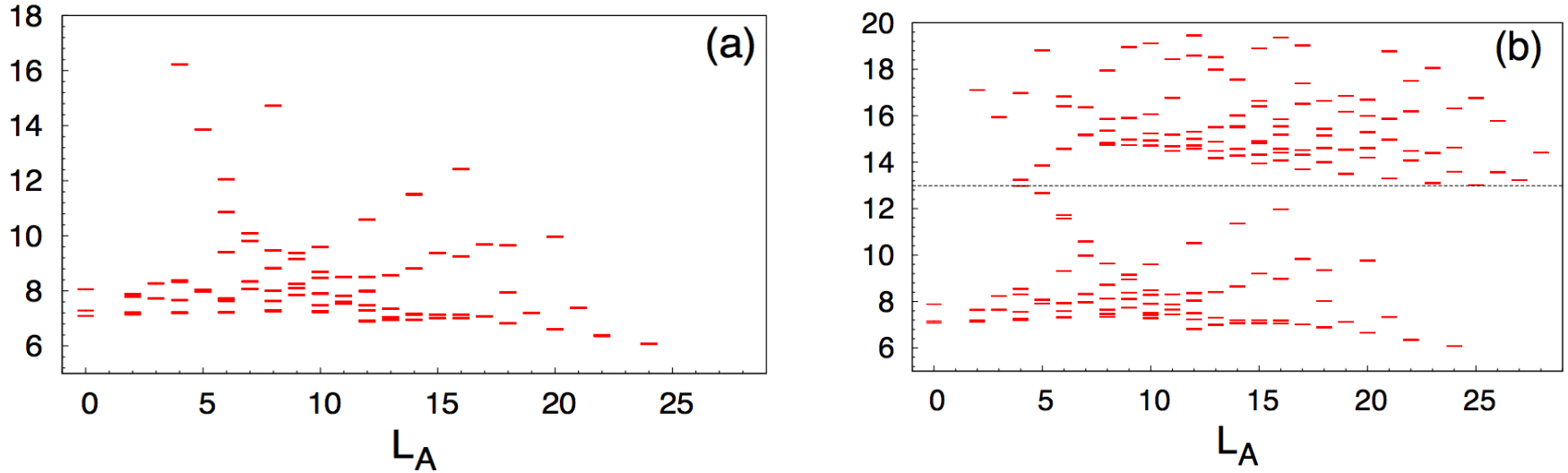
Particle Entanglement Spectrum (PES)



PES for $\nu=1/3$ Laughlin on the sphere, $N=8$, $n_A=4$

Sterdyniak, Regnault, Bernevig (2010)

Particle Entanglement Spectrum (PES)

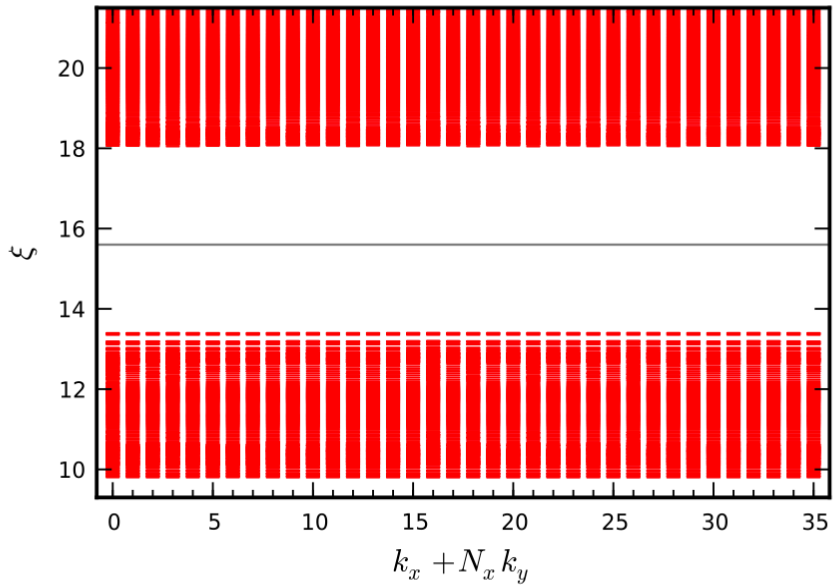


PES for $\nu=1/3$ Laughlin (left) and Coulomb interaction groundstate (right), $N=8$, $n_A=4$

Sterdyniak, Regnault, Bernevig (2010)

Particle Entanglement Spectrum (PES)

Fractional Chern insulators :
Entanglement spectra as a tool



PES for $N = 12$, $N_A = 5$, 2530 states per momentum sector below the gap as expected for a Laughlin state

Wu, Sterdyniak, Repellin, Regnault, Bernevig (2011)

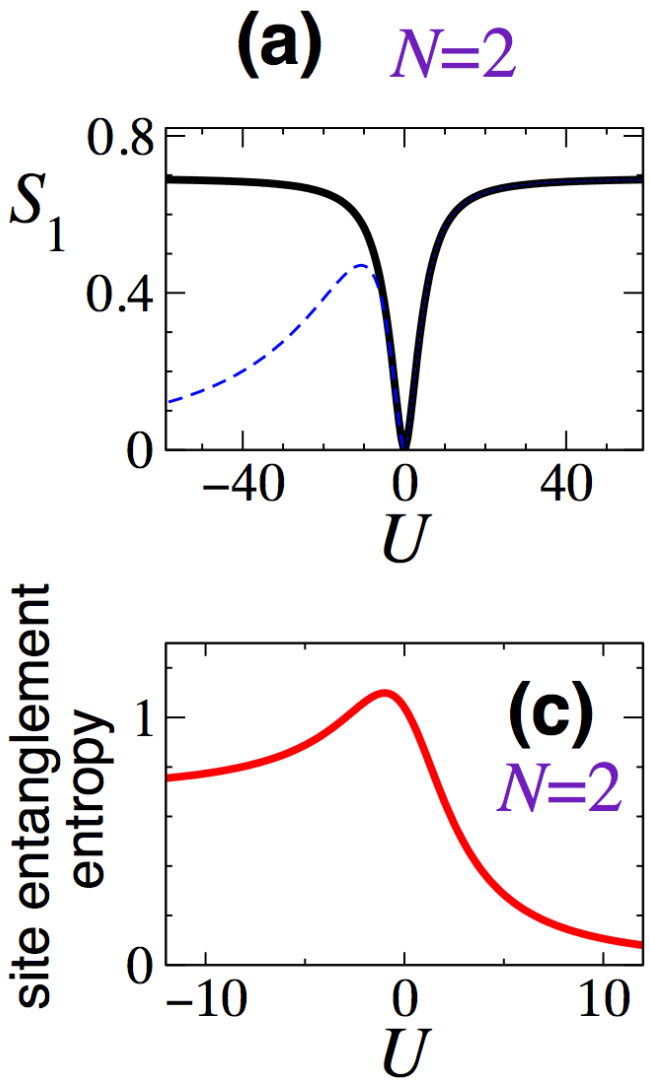
Particle partitioning: lattice bosons

- bounds on entanglement entropy for n out of N bosons in L orbitals

$$0 \leq S_n \leq \ln \binom{L-1+n}{n}$$

- S_n for interacting bosons?

Bose-Hubbard: N=2 particles on 2 sites



***U*<<*o* : cat state**

$$\frac{1}{\sqrt{2}}[(b_1^+)^2+(b_2^+)^2]|0\rangle$$

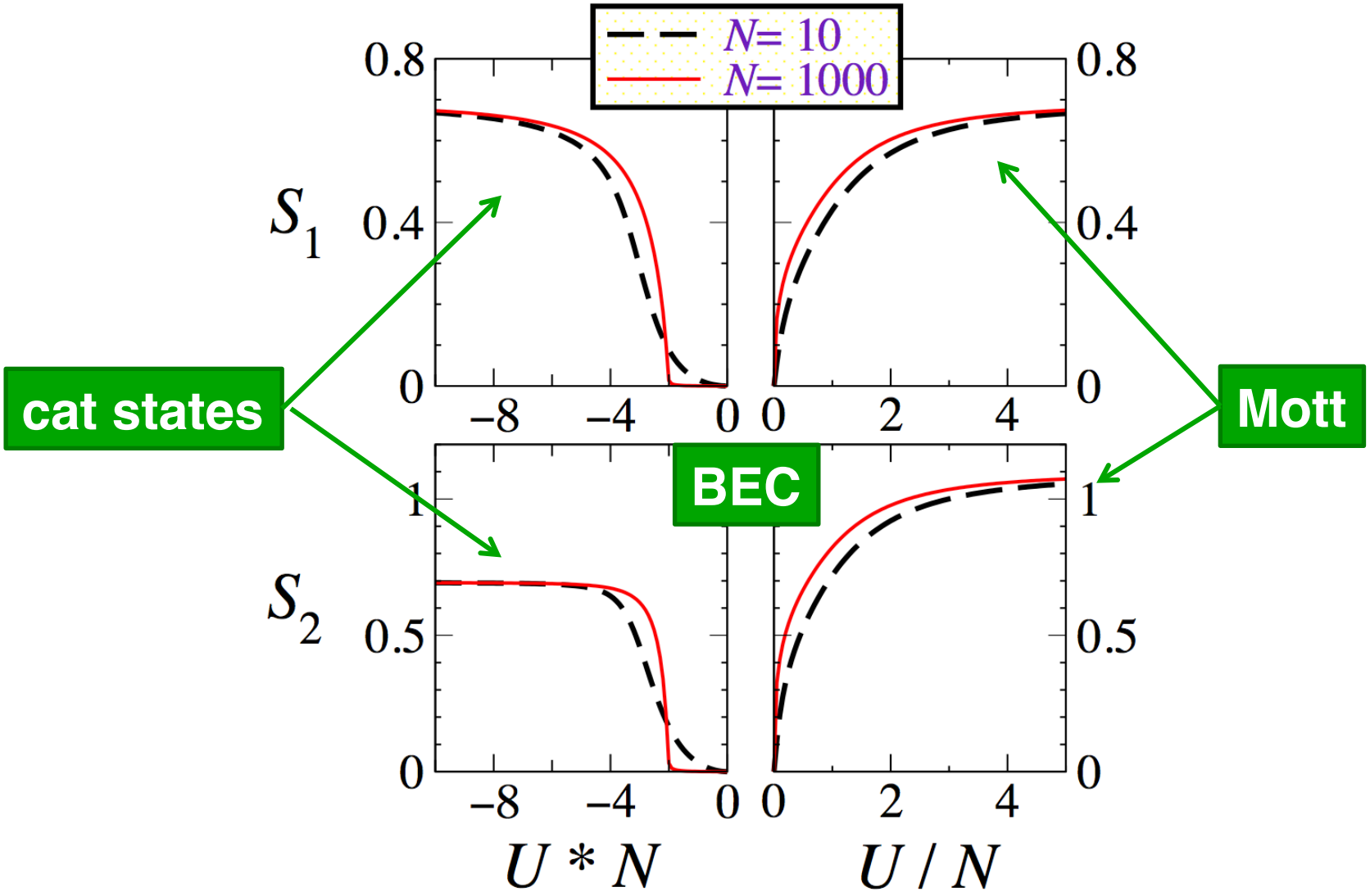
***U*=*o* : BEC**

$$\frac{1}{2}(b_1^+ + b_2^+)^2|0\rangle$$

***U*>>*o* : Mott**

$$b_1^+ b_2^+|0\rangle$$

Bose-Hubbard: *N* particles on 2 sites



Scaling form for interacting 1D bosons

Scaling form for interacting 1D bosons

$$S_n \approx a_0 n \ln N + b$$

un-condensed fraction

Haque, Zozulya, KjS, PRA (2008)

Entanglement measures for quantum states

mystery quantum state

↓ **entanglement measures**

to RES or to PES,

that's the question ...