Planetary Population Synthesis - Comparing Theory and Observation

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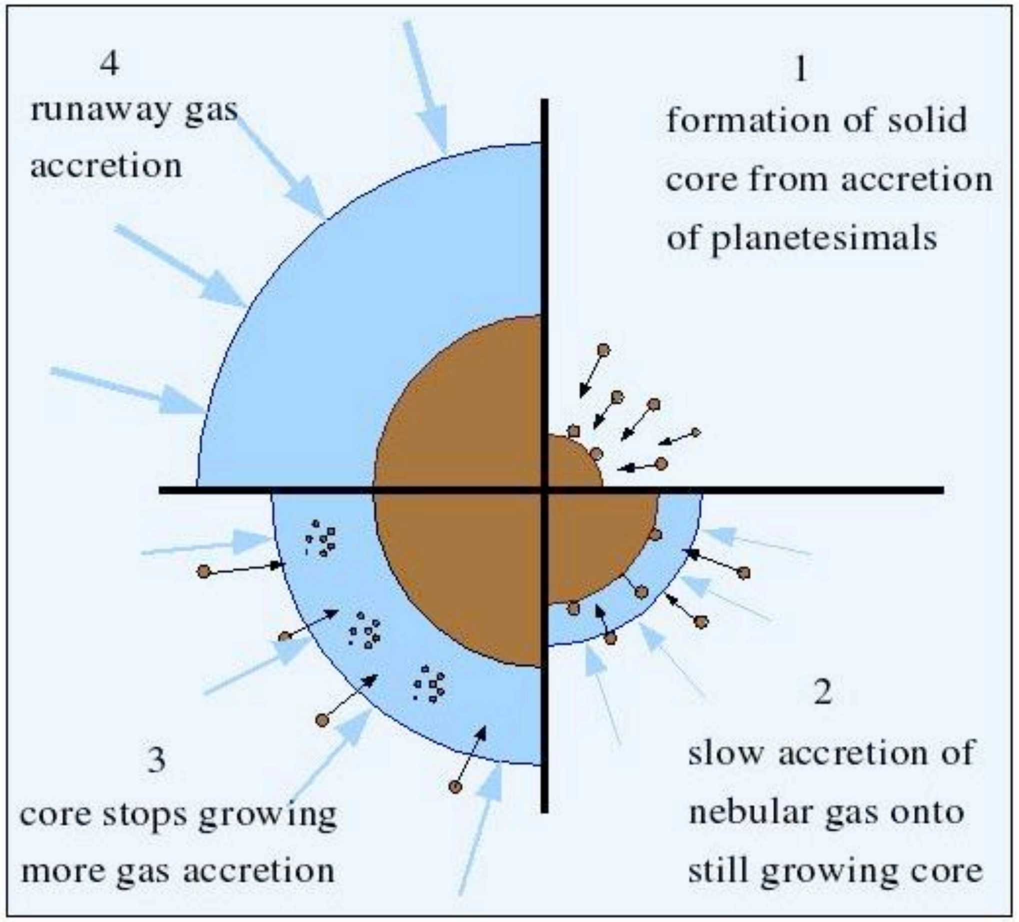
Kavli Institute for Theoretical Physics, 25.03.2010

Core accretion planet formation model

core accretion



Perri & Cameron 1974, Mizuno et al. 1978, Mizuno 1980, Bodenheimer & Pollack 1986, Pollack et al. 1996

follow the concurrent growth of an initially small solid core (ices, rocks) surrounded by a gaseous envelope (H2 & He) in the protoplanetary disk.

- **core growth:**

collisional accretion of background planetesimals

$$\frac{dM}{dt} = F\frac{\Sigma}{h}\pi R^2 \left(1 + \frac{v_{esc}^2}{v_{rel}^2}\right) v_{rel} \quad (= \rho\sigma v)$$

velocity dispersion of planetesimal is key parameter (runaway, oligarchic, orderly)

- **envelope growth:**

1D structure equations (similar to stellar structure)

1) $\frac{dr^3}{dm} = \frac{3}{4\pi\rho}$ mass conservation

2) $\frac{dP}{dm} = -\frac{G(m+M_{core})}{4\pi r^4}$ hydrostatic equilibrium

3) $\frac{dL}{dm} = -\frac{dU}{dt} + \frac{P}{\rho^2}\frac{d\rho}{dt} + \epsilon_{acc}$ energy conservation

4) $\frac{dT}{dP} = \nabla_{ad}$ or ∇_{rad} energy transfer

heating by infalling planetesimals

Accretion of gas: Boundary conditions

- attached case
  - structure goes smoothly to Hill sphere radius
  - low mass, pre-runaway planets
  - boundary conditions: background nebula
  - dM_{XY}/dt given by ability of envelope to radiate away energy (τ_{KH})

$$R_{out} = Min(R_H, R_{acc})$$
$$P(R_{out}) = P_{neb}$$
$$T(R_{out}) \approx T_{neb}$$

- deteached case
  - structure has a free outer radius
  - high mass, post-runaway planets
  - boundary conditions: accretion shock (or circumplanetary disk)
  - disk and gap formation regulate dM_{XY}/dt

$$v_{ff} = \sqrt{2GM\left(\frac{1}{R_{out}} - \frac{1}{R_H}\right)}$$
$$P(R_{out}) \approx P_{ram} = \frac{dM_{xy}/dt}{4\pi R_{out}^2} v_{ff}$$
$$4\pi R_{out}^2 \sigma T(R_{out})^4 \approx \frac{3}{4} L_{shock} + \frac{1}{2} L_{int}$$

- long term evolution: M=cst.
  - Eddington approximation

$$T(R_{out}) = T_{eff}$$
$$P(R_{out}) = 2/3\ g/\kappa$$

Pollack et al. 1996, Papaloizou & Nelson 2005, Stahler et al. 1980, Bodenheimer et al. 2000, Papaloizou & Terquem 1999, Guillot 2005

Jupiter in-situ formation

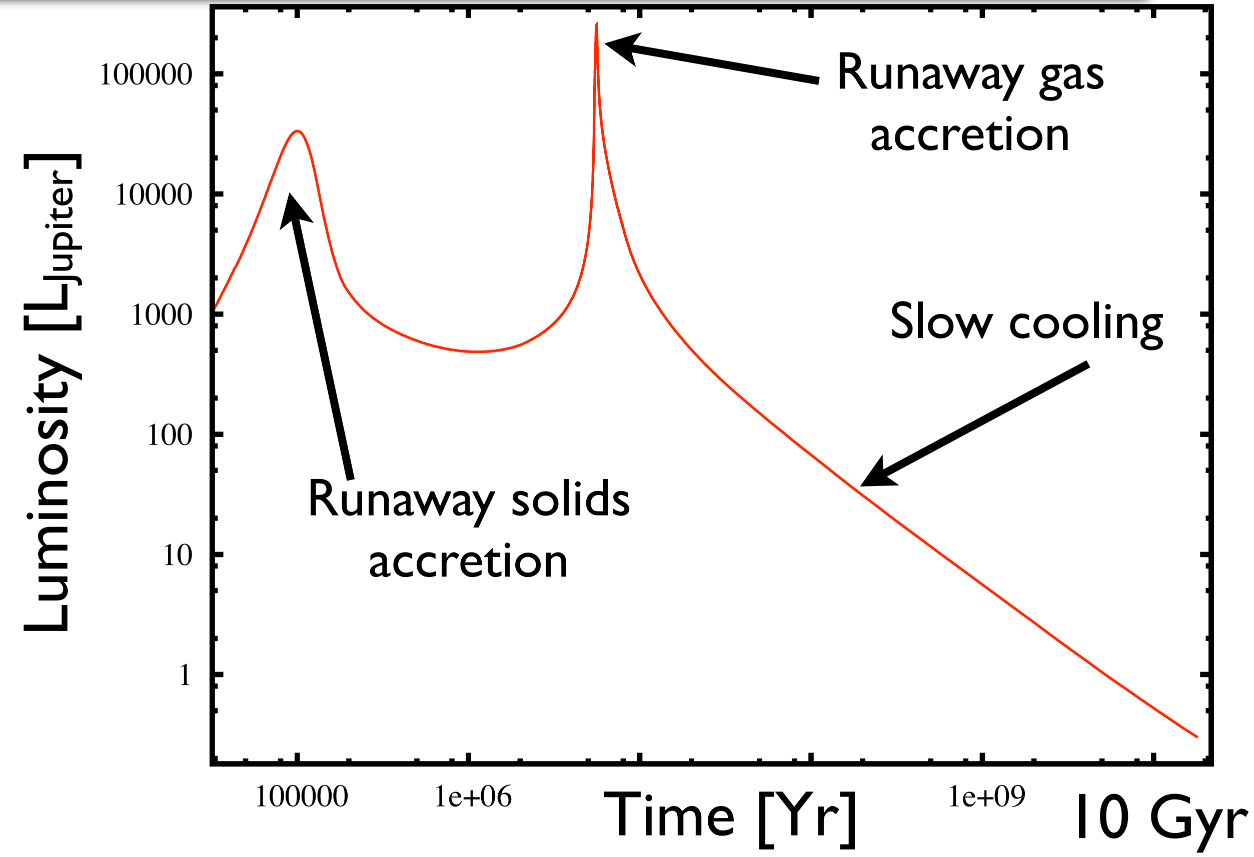
Main model assumptions (as Pollack et al. 1996):

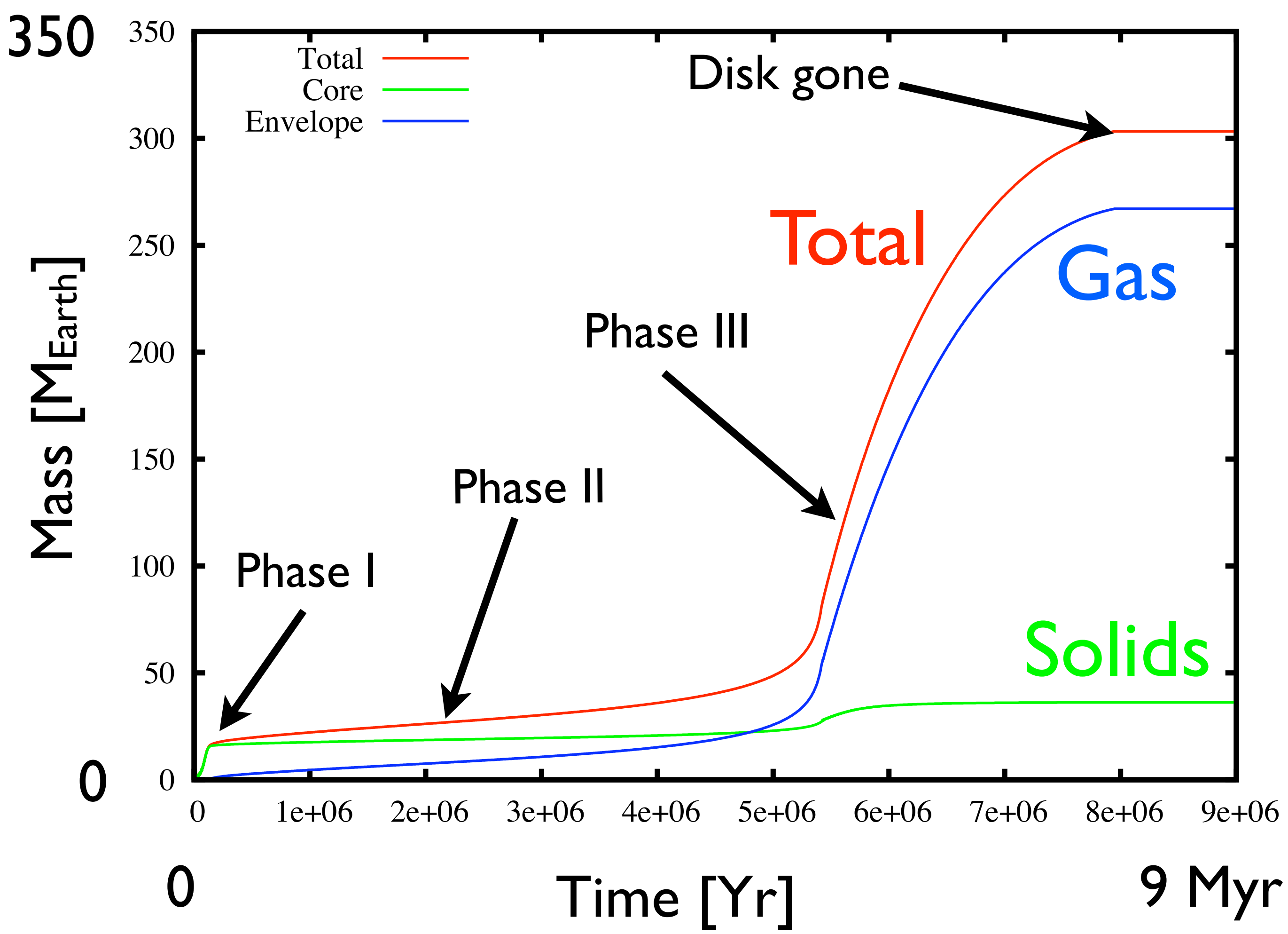
- *in situ formation (no migration)*
- *standard opacities*

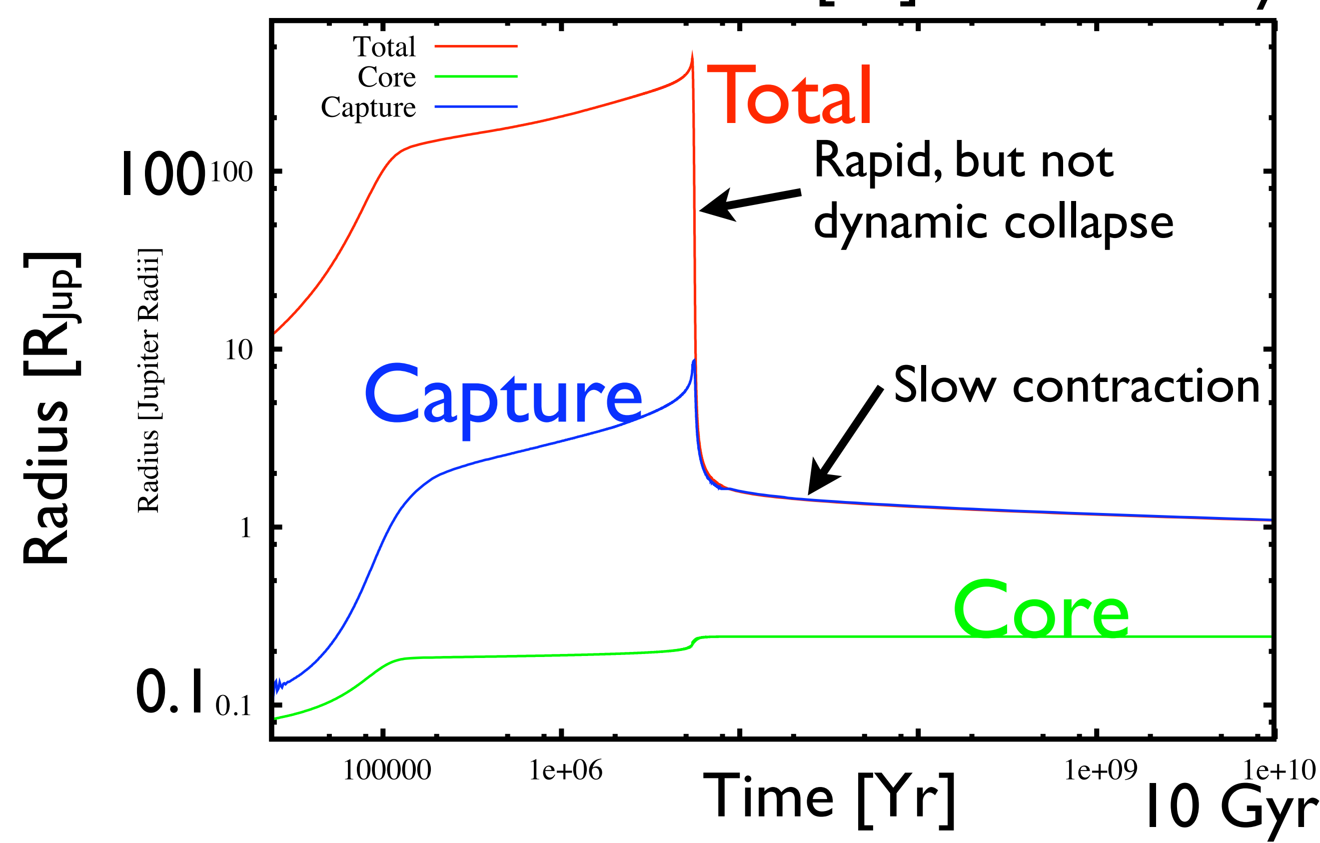
•*Phase I: Rapid build up of a core. Until isolation mass: Emptying planetesimal feeding zone*

•*Phase II: Slow accretion of gas and planetesimals*

•*Phase III: Runaway gas accretion at $M_{core} > M_{crit}$*







Disk evolution and migration

similar timescales of various processes:

$$\tau_{migration} \leq \tau_{formation} \approx \tau_{disk\ evolution}$$

→ *extend model to include in a self consistent way (Alibert, Mordasini, Benz 2004)*

1) type I and type II planetary migration

(Lin & Papaloizou 86; Ward 97; Tanaka et al. 02). Isothermal Type I reduced by constant factor f1 (free parameter). Currently working on eliminating f1 (Dittkrist et al. in prep).

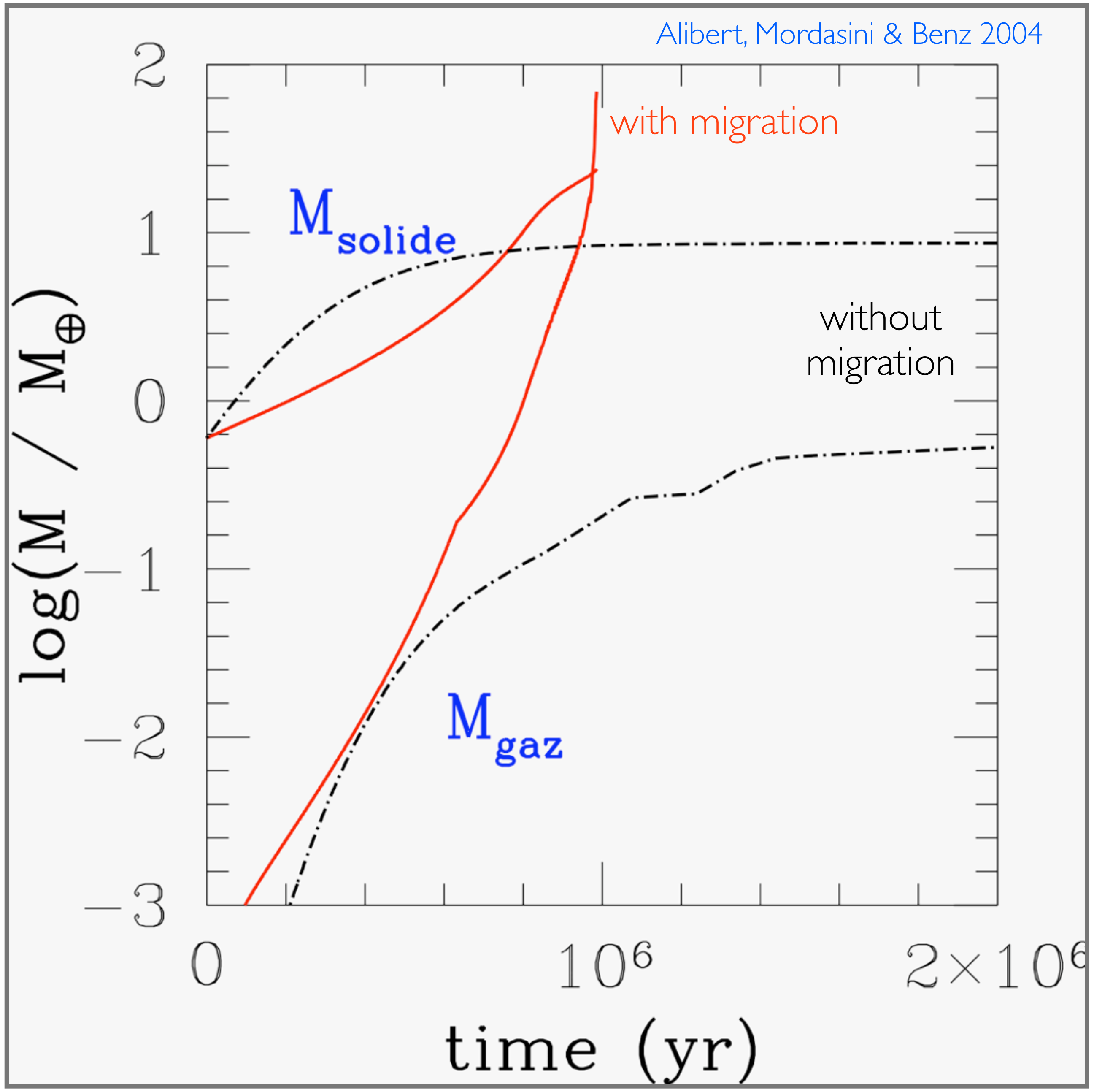
2) disk evolution

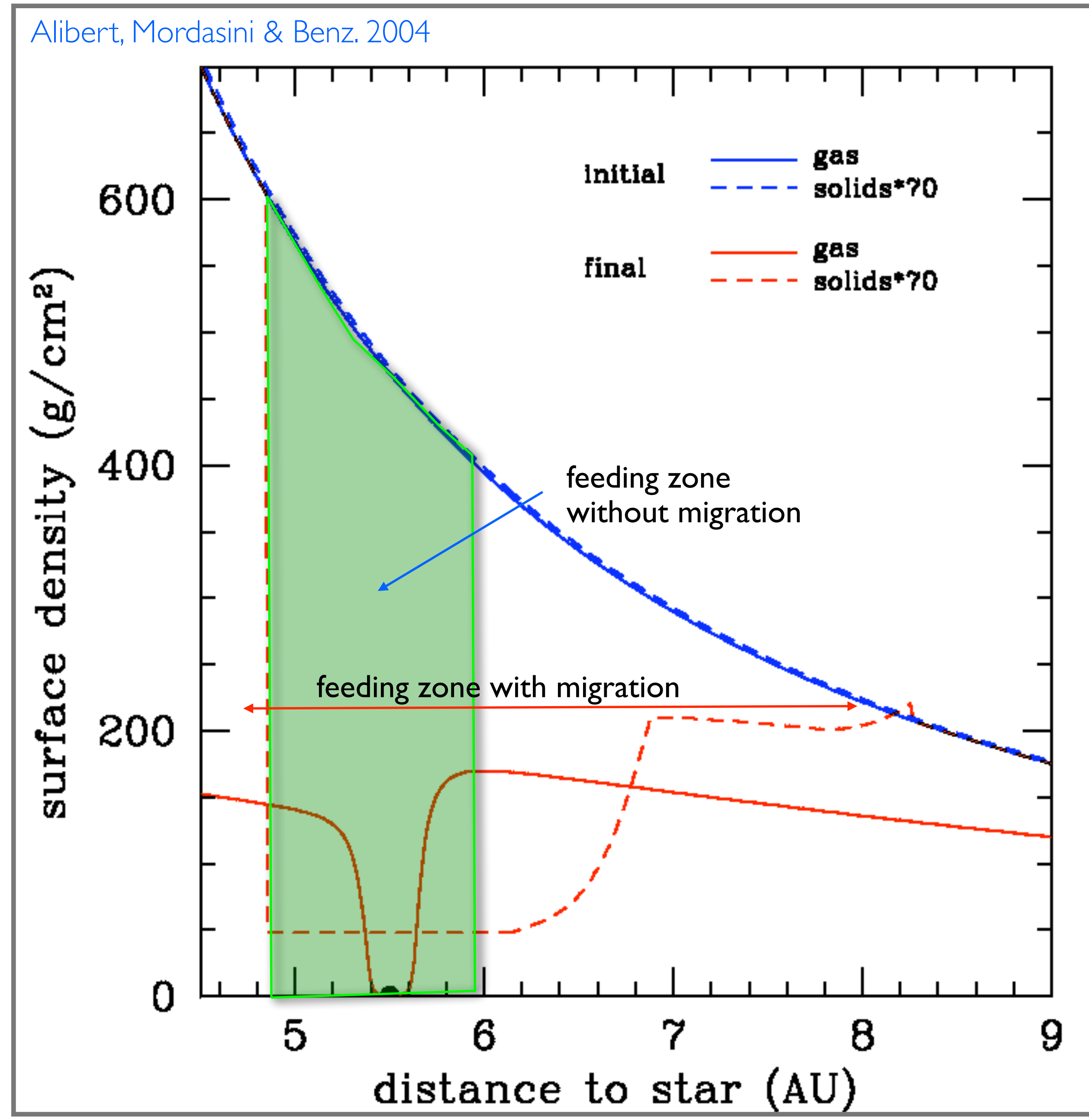
(1+1 D) α-disk with photoevaporation (Papaloizou & Terquem 1999), now also with irradiation from the host star (Fouchet et al. submitted).

simplifications (most important)

- *One embryo per disk, no systems (including Nbody also work in progress)*
- *Formation only until the gas disk disappears: No mass growth/loss after disk dispersal (Terrestrial planets, Ice giants, evaporating planets)*
- *No eccentricity, planets on circular orbits*
- *No particular stopping mechanism, a_{min}=0.1 AU*

Effects of migration





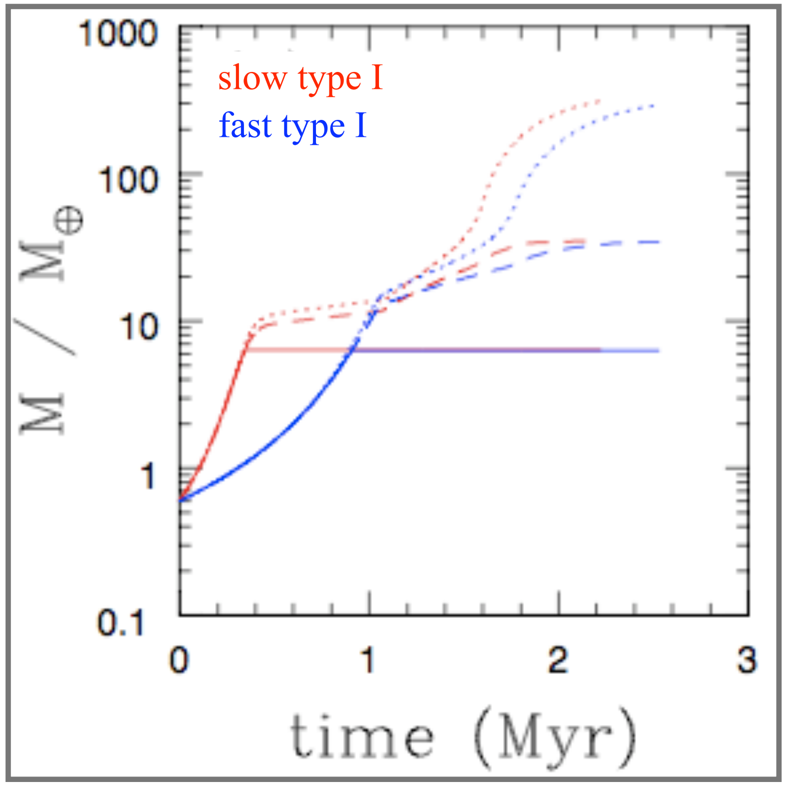
migration greatly speeds up the formation timescale: skip phase 2

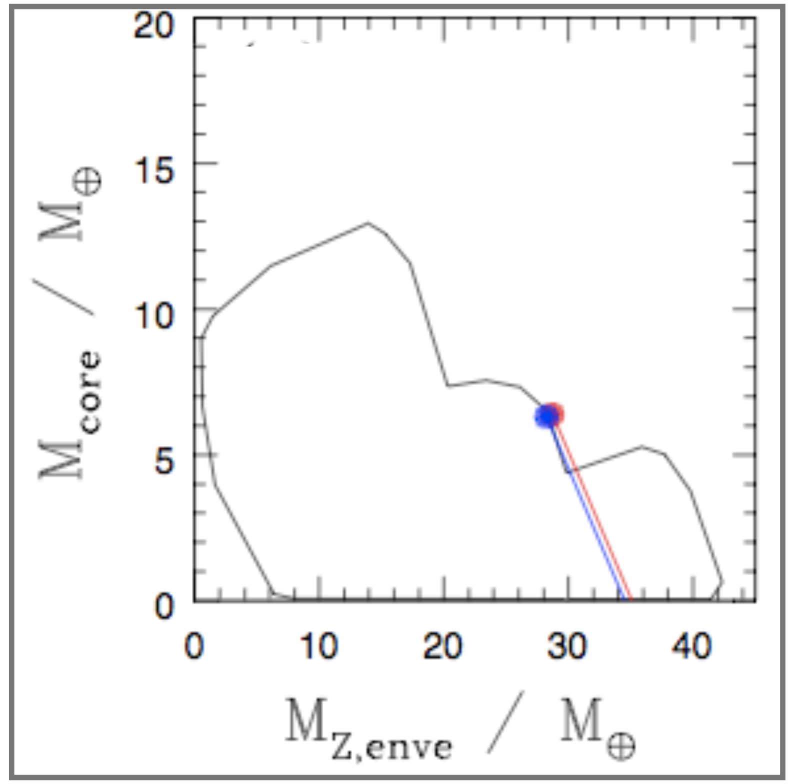
migration prevents the depletion of feeding zone

Models meet *observations*

Jupiter

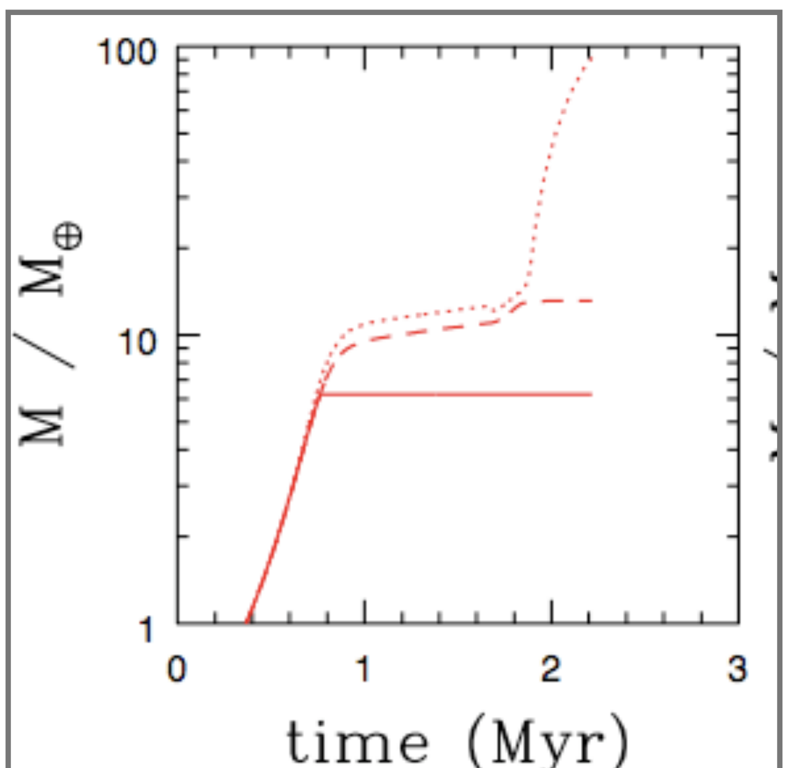
Alibert et al. 2005b

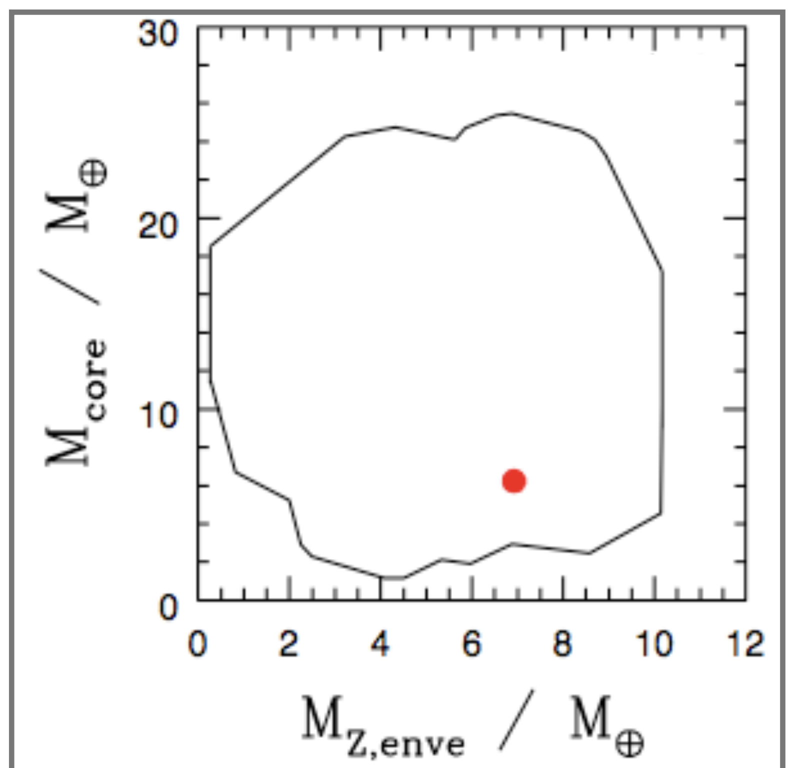




species	measured	computed
Ar	1.8 ± 0.4	2.
Kr	2.4 ± 0.4	2.1
Xe	2.1 ± 0.4	2.6
C	3.7 ± 0.9	2.8
N	3.2 ± 1.2	2.5
S	2.7 ± 0.6	2.1

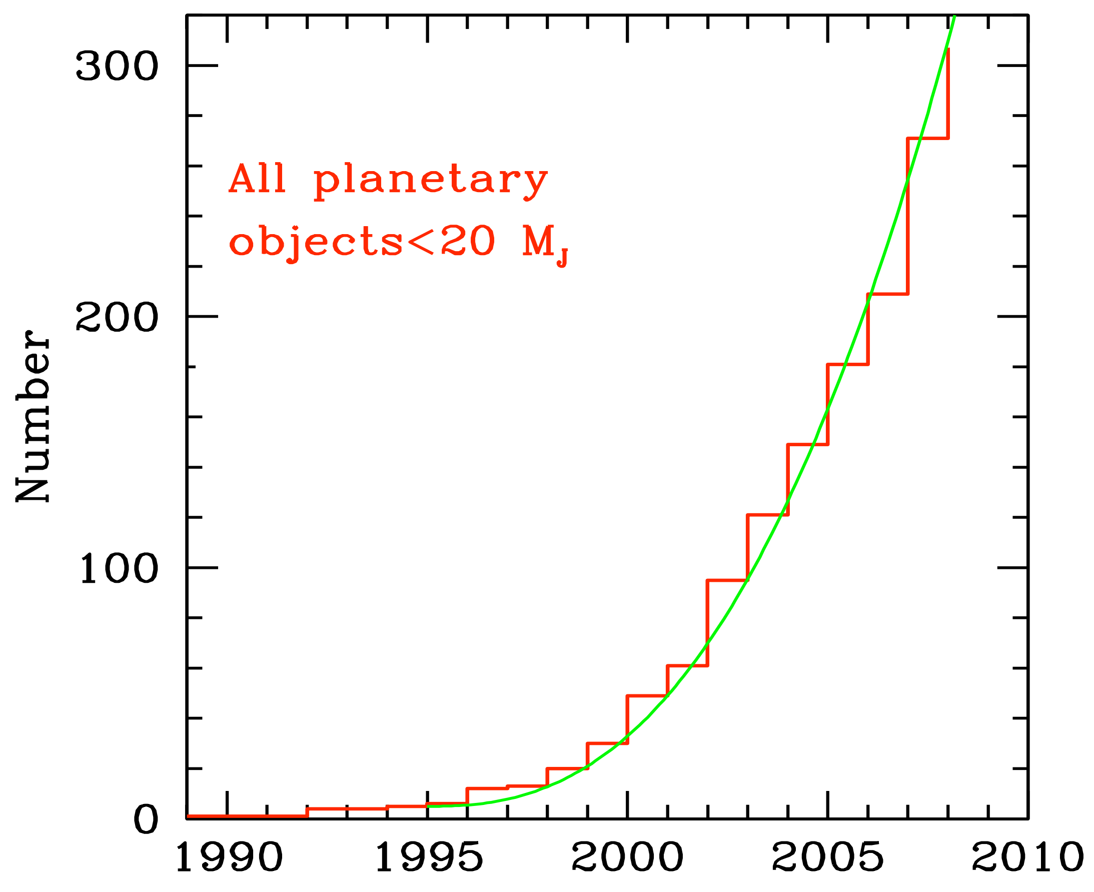
Saturn



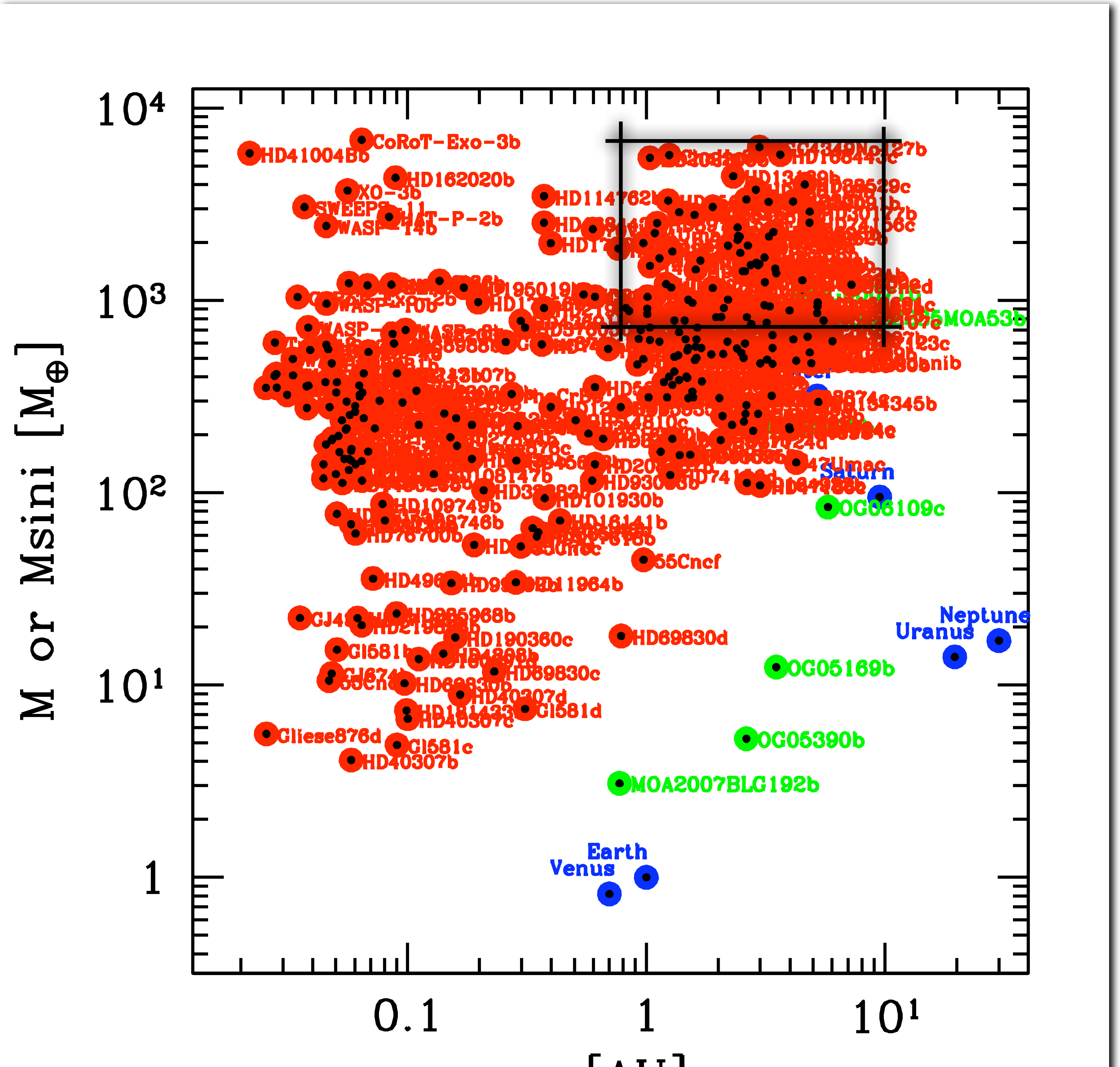


species	measured	computed
Ar		1.7
Kr		1.9
Xe		2.3
C	3.2 ± 0.8	2.4
N	2.4 ± 0.5	2.2
S		1.9

Planetary
Population
Synthesis



Extrasolar planet population synthesis:
Observational Motivation I



J. Schneider's exoplanet.eu

extreme diversity

- super Jupiters
- hot Jupiters
- hot Neptunes
- eccentric planets
- planetary systems

no more single objects, but a population

- distributions of
  - masses
  - semimajor axes
  - host star metallicities
  - eccentricities
- correlations

Reason?

Extrasolar planet population synthesis:
Observational Motivation II

protoplanety disks: diversity too.

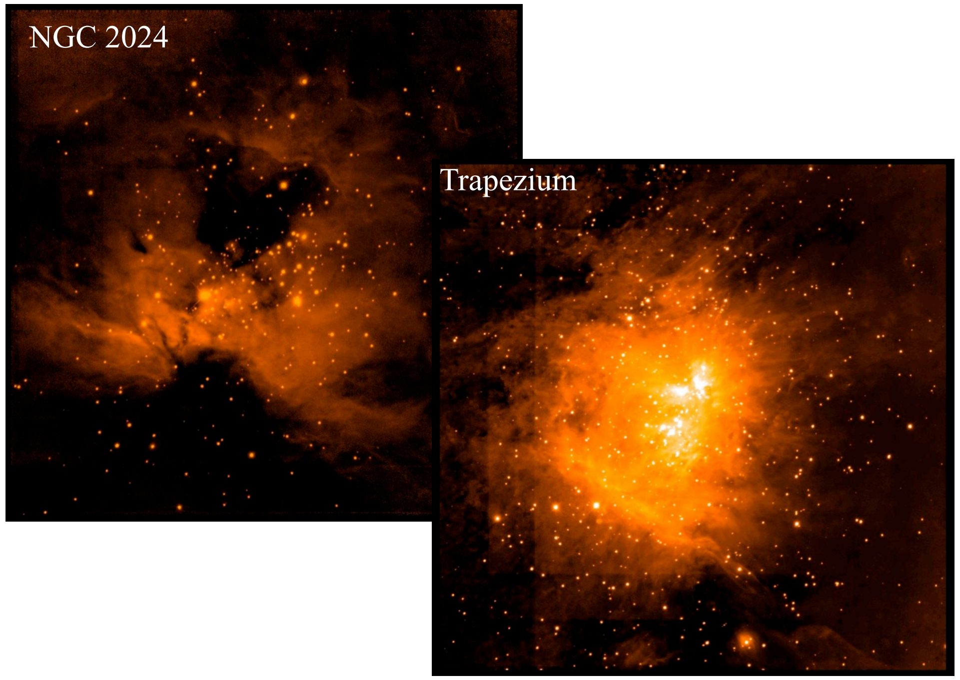
observational determination of distributions of

- disk gas masses
- disk dust masses
- disk lifetimes

diversity of disks (initial conditions)

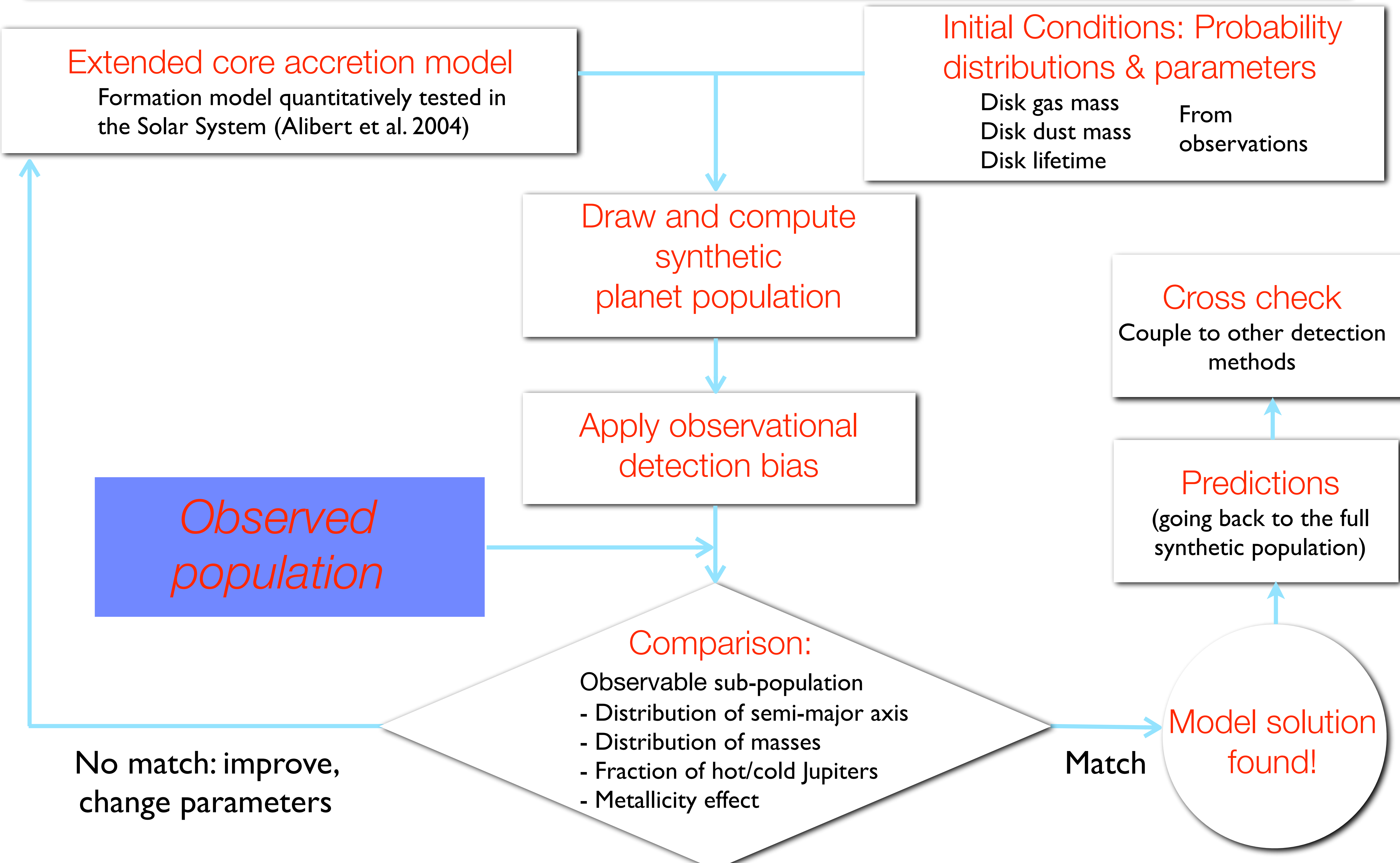
diversity of planets (end products)

reproducible in a theoretical model by population synthesis ?



Extrasolar Planet Population synthesis:
Principle

Mordasini et al. 2009a
Mordasini et al. 2009b



Population Synthesis: Initial conditions

Some can be constrained by observations some from theoretical arguments and some are just "educated" guesses

Four Monte Carlo variables with probability distributions

- Dust-to-gas ratio (solid surface density).
- Initial gas surface density.
- Photoevaporation rate.
- Initial semimajor axis of the small planetary seed put into the disk.

Parameters (fixed for one synthetic population)

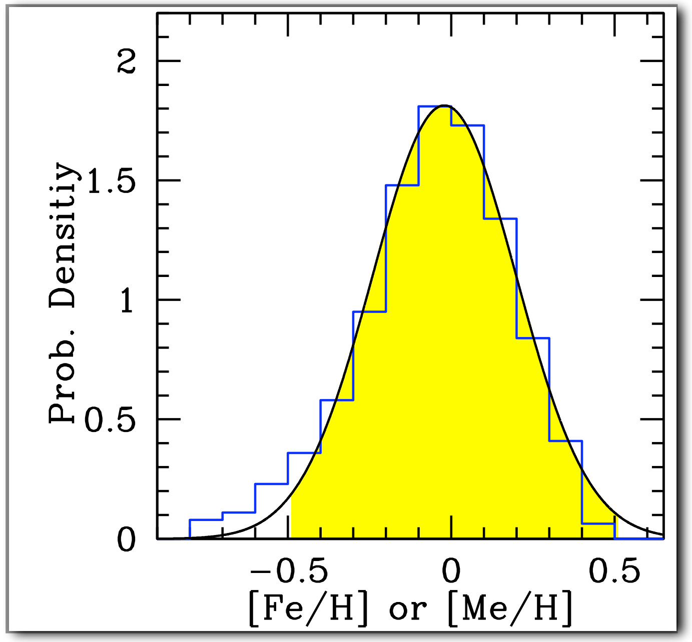
- Type I migration rate reduction factor f_1
- Disk viscosity parameter α (0.007)
- Planetesimal size (R=100 km)
- Initial solid surface distribution ($\propto r^{-3/2}$)
- Stellar mass

Constraints on the initial conditions

1 Metallicity

assume same in star and disk

Stellar [Fe/H] from spectroscopy. Gaussian distribution for [Fe/H] with μ ~0.0, σ~ 0.2. (e.g. Santos et al. 2003)



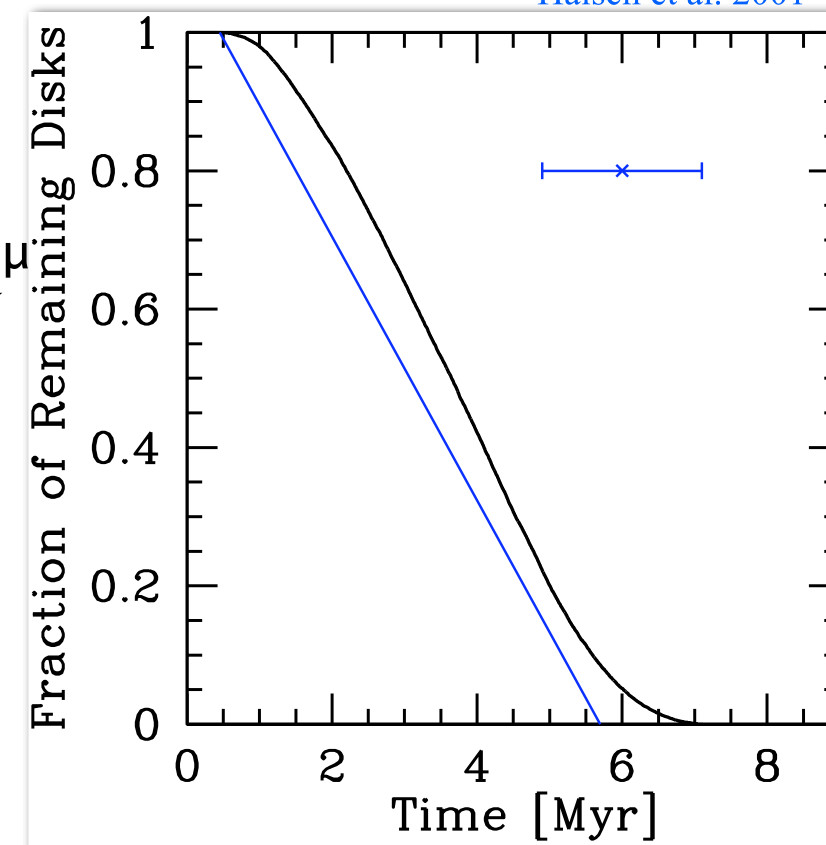
3 Disk lifetime

L-band (3.4 μm) photometry:

- excess caused by μ sized dust @ ~900K

... ok to < 10 AU

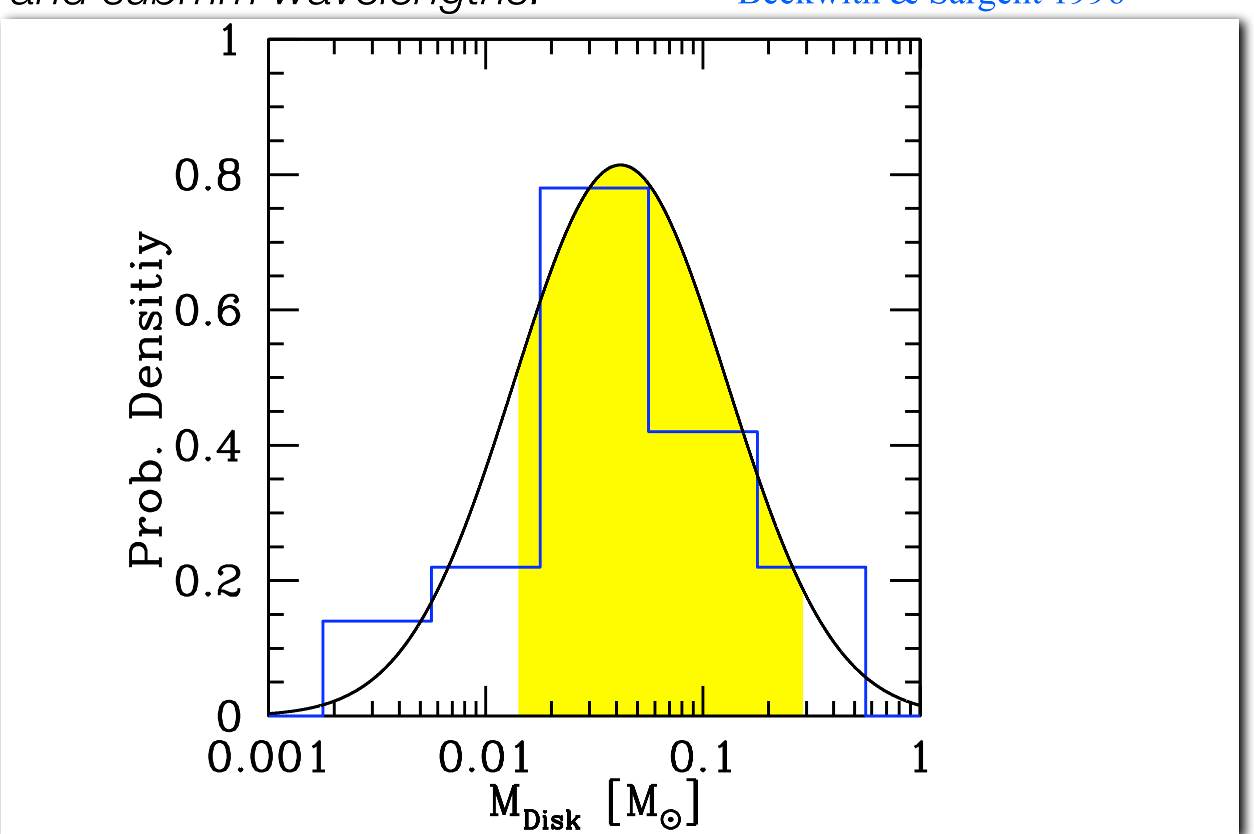
Haisch et al. 2001



2 Disk (gas) masses

Thermal continuum emission from cold dust at mm and submm wavelengths.

Beckwith & Sargent 1996



4 Initial semimajor axis of the seed embryo: not observationally constrained

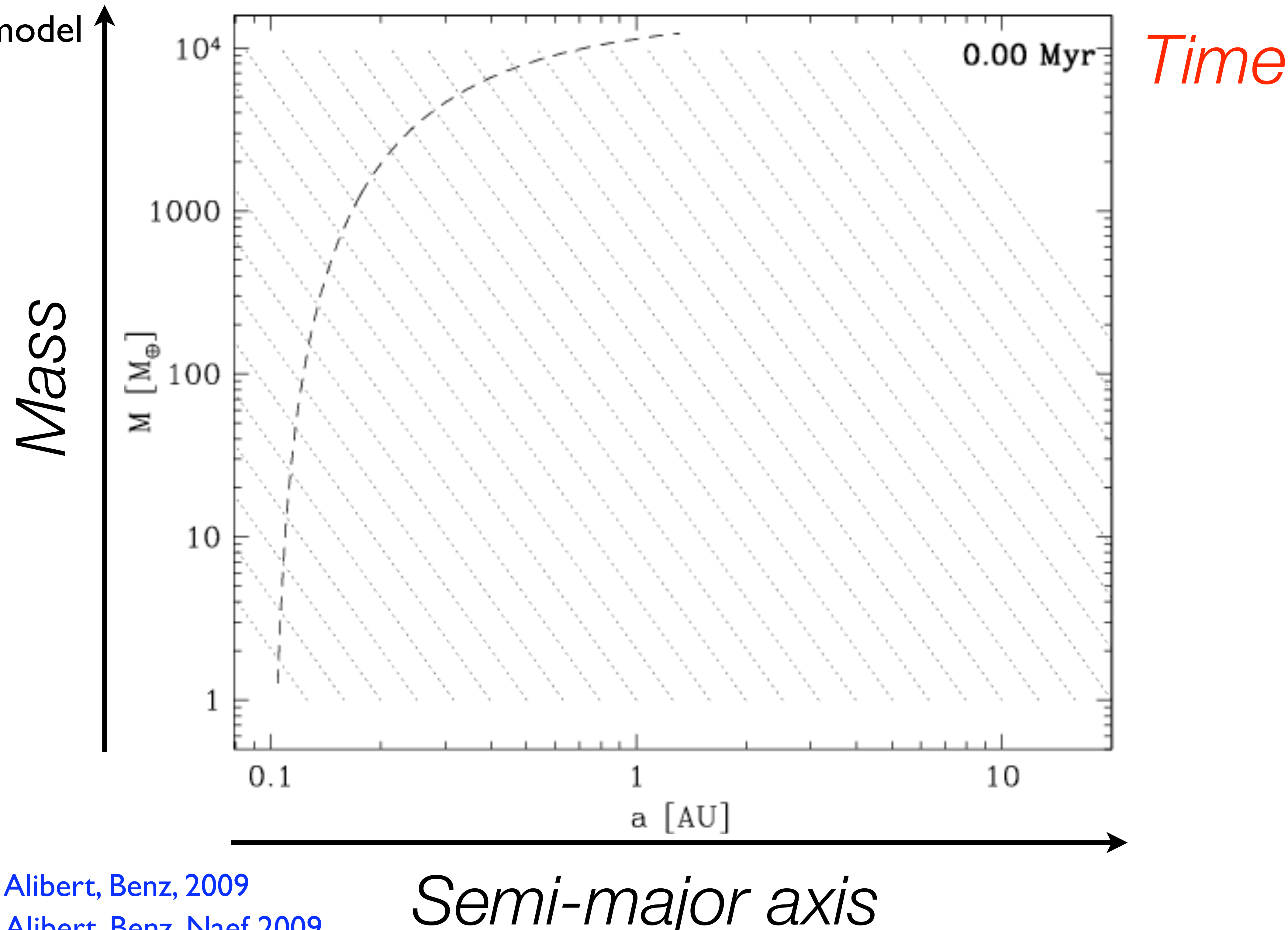
Analytical work (Lissauer & Steward 1992) and numerical simulations (Kokubo & Ida 2000): spacing between bodies $\Delta \propto a$

$$p(a)da \propto \frac{da}{\Delta} \propto \frac{da}{a} = d\log(a) \propto const.$$

i.e. uniform in log(a) (Ida & Lin 2004)

Planetary formation tracks

M_{star}=1 $M_\odot$
Nominal model

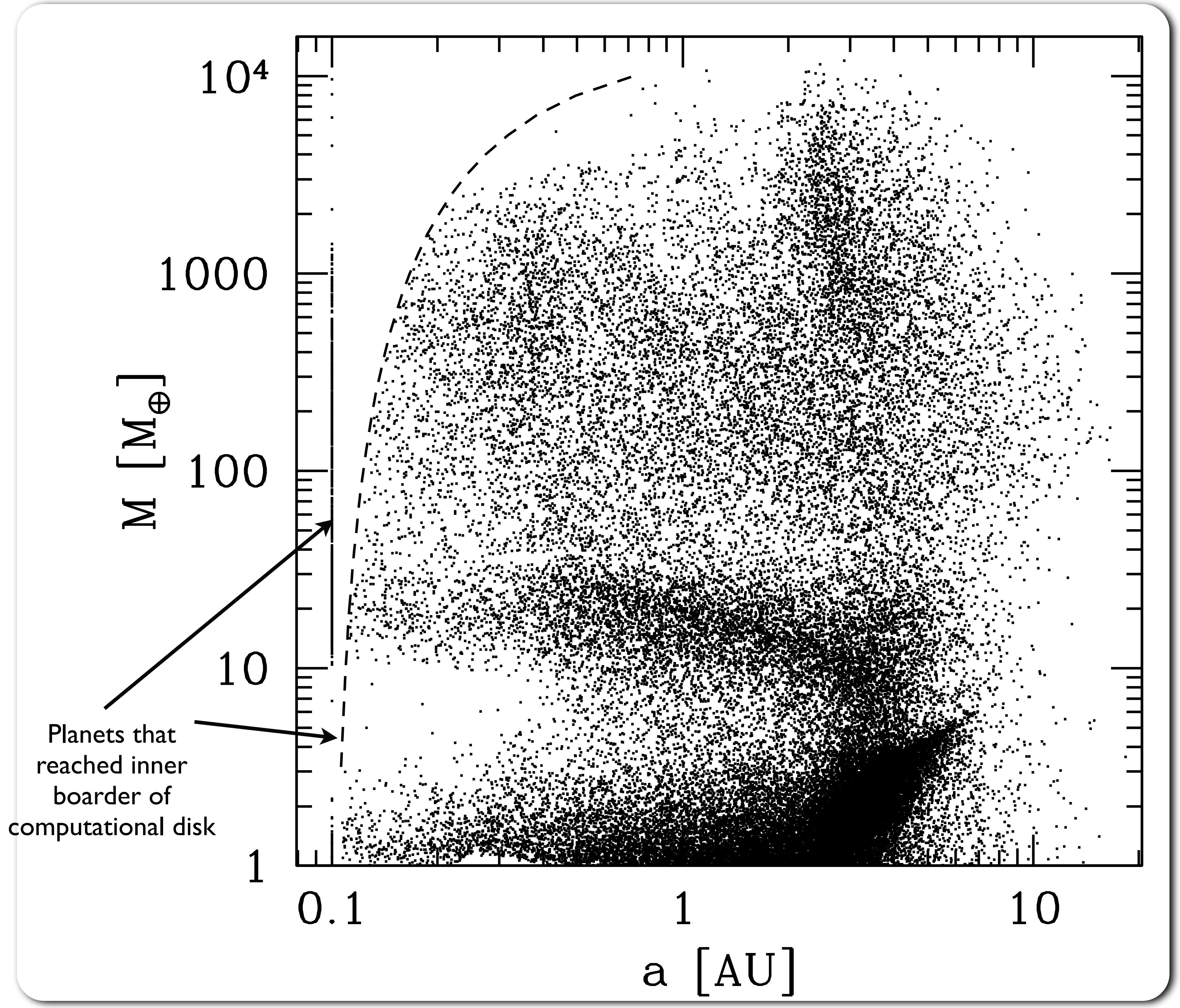


Mordasini, Alibert, Benz, 2009
Mordasini, Alibert, Benz, Naef 2009

Synthetic Population

Nominal Model: alpha= 7x10^{-3}, f$_1$=0.001, M=1 M$_\odot$

Mordasini et al. 2009a

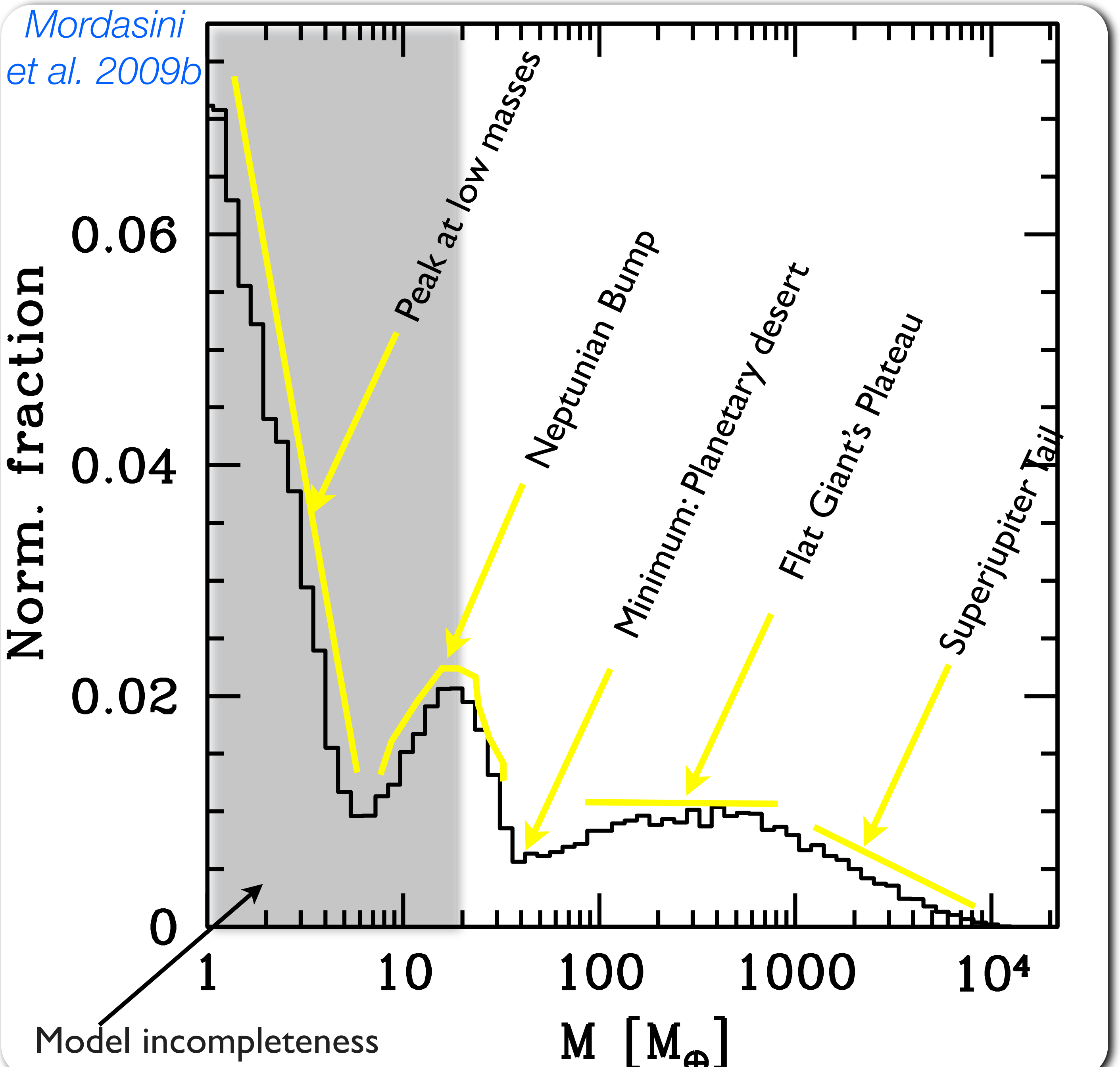


The variation of the initial conditions within the observed limits (protoplanetary disk properties) produces synthetic planets of a very large diversity.

A number of clusters can be identified.

- *"Iceline clump"*
- *"Planetary desert"*
- *"Failed cores"*

Planetary Initial Mass Function PIMF



M_{Star}=1 M_{Sun}
Nominal Model

Type	Mass (M_{Earth})	%
(Super)-Earth	< 7	58
Neptunian	7-30	17
Intermediate	30-100	6
Jovian	100-1000	14
Super-Jupiter	> 1000	4

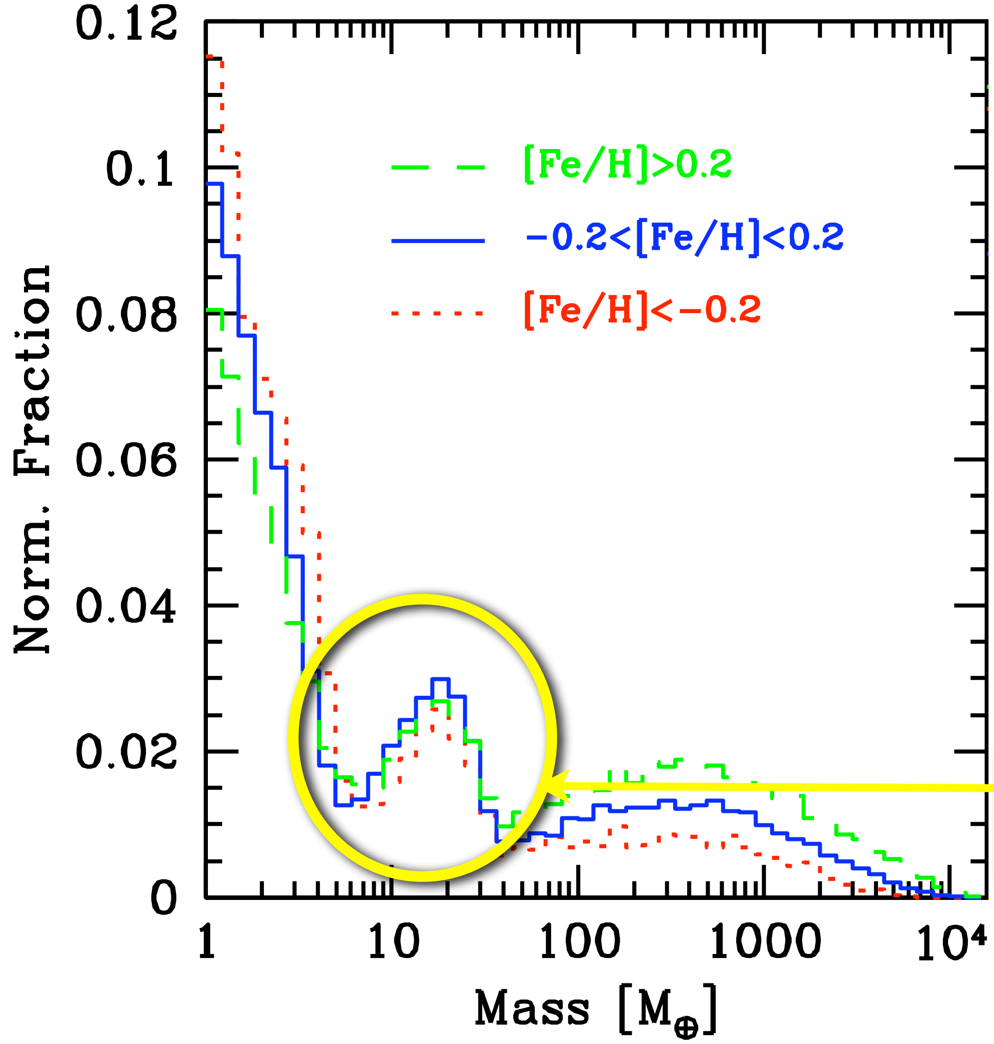
Model predicts that planets with M < 30 M_{Earth} account for over 75% of all planets

- *Complex structure, dominated by low mass planets*
- *Consistent w. non-detection of Jupiters around 90-95% stars.*
- *Maxima at masses similar to Solar System planets.*

Correlations with disk and stellar properties

PIMF: Dependence on disk properties

Metallicity



Fe/H mainly just scales PIMF for giant planets:

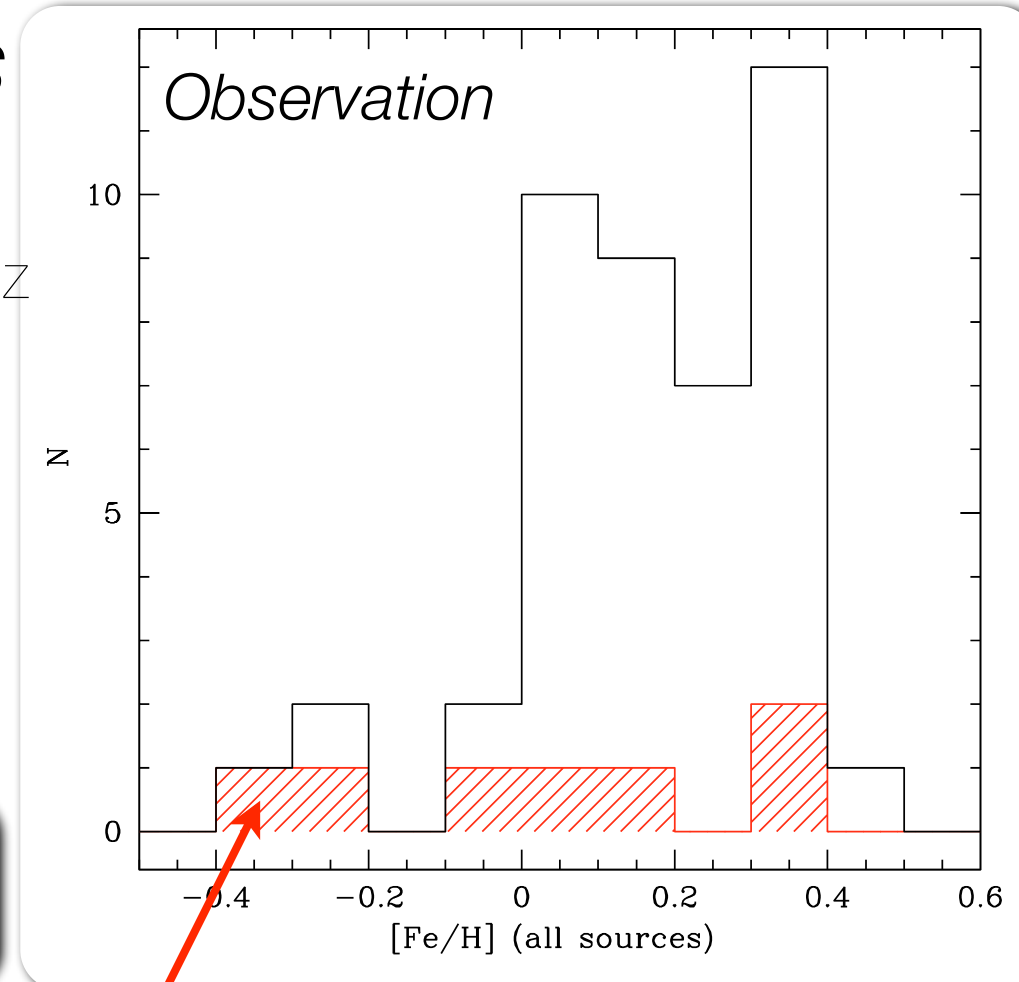
Fe/H: threshold, but final mass not given by Fe/H

- higher number of giants
- but not more massive

Low mass planets

Udry, Mayor, Benz et al. 2006

No metallicity effect for Neptunes

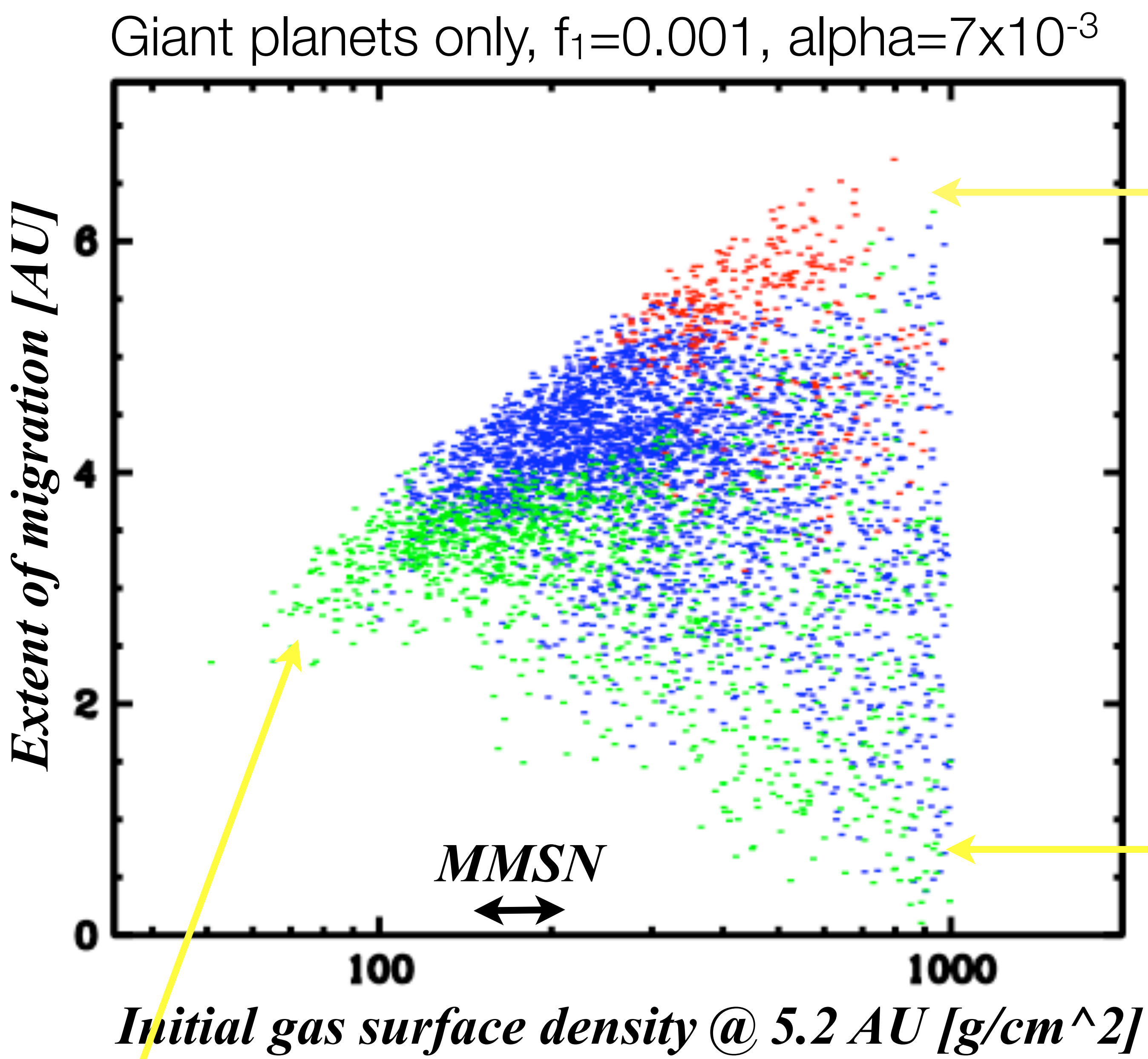


[Fe/H] dist. of Hot Neptunes: flat!

[Fe/H] dist. of all known planets P<20 d

- Metal poor systems produce more small bodies
- Minimum metallicity effect for Super-Earths & Neptunes

Migration: Disk mass influence



High gas mass, low metallicity: max. migration distance: must collect solids: collection effect

Disk metallicity colorcoded

green: Fe/H>0.2

blue: -0.2<Fe/H<0.2

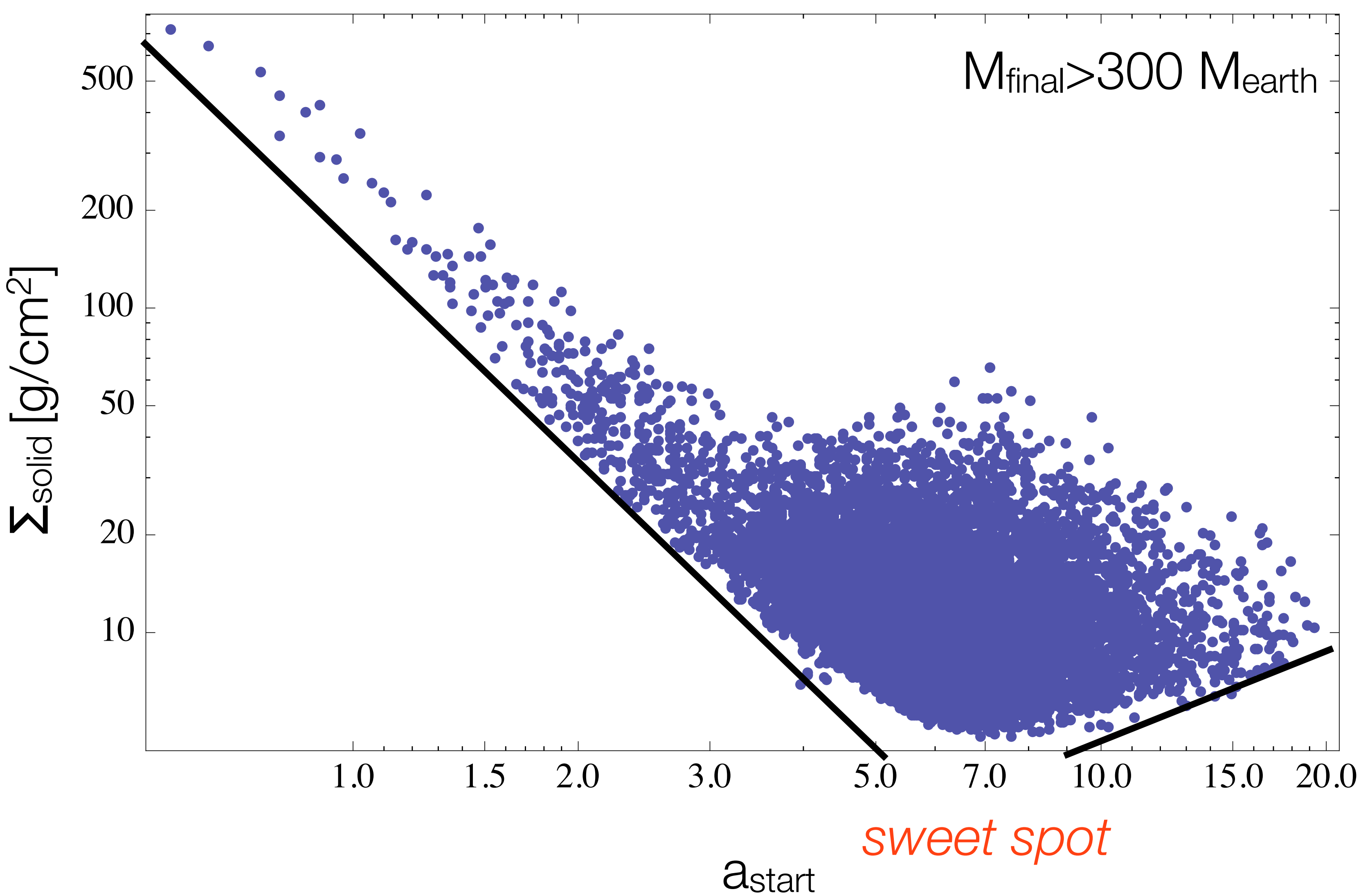
red: -0.2>Fe/H

High gas mass, high metallicity: giant forming inside the iceline. don't migrate much: too massive: braking effect

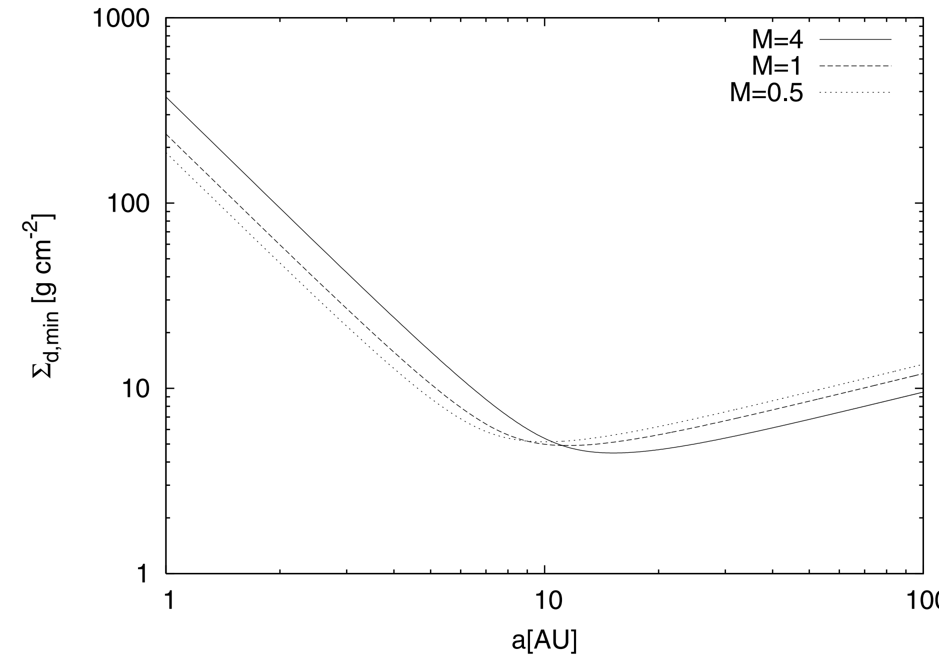
Low gas mass: need high metallicities. Intermediate migration. compensation effect

Indicates migration over roughly 4 AU for Jupiter, consistent with earlier results (Alibert et al. 2005b)

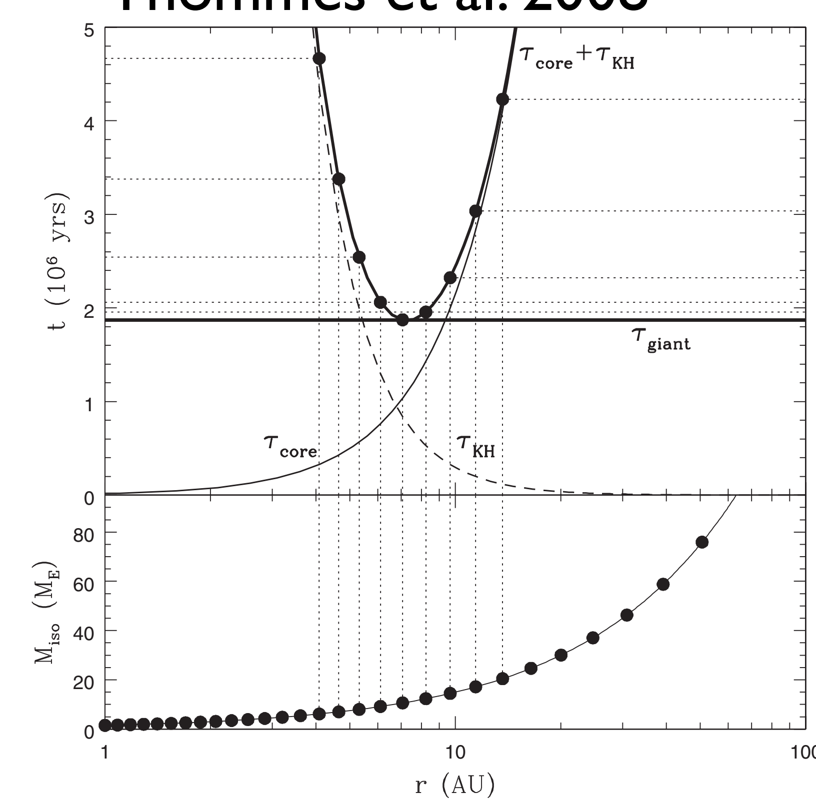
Preconditions for giant planets



Kornet et al. 2006



Thommes et al. 2008



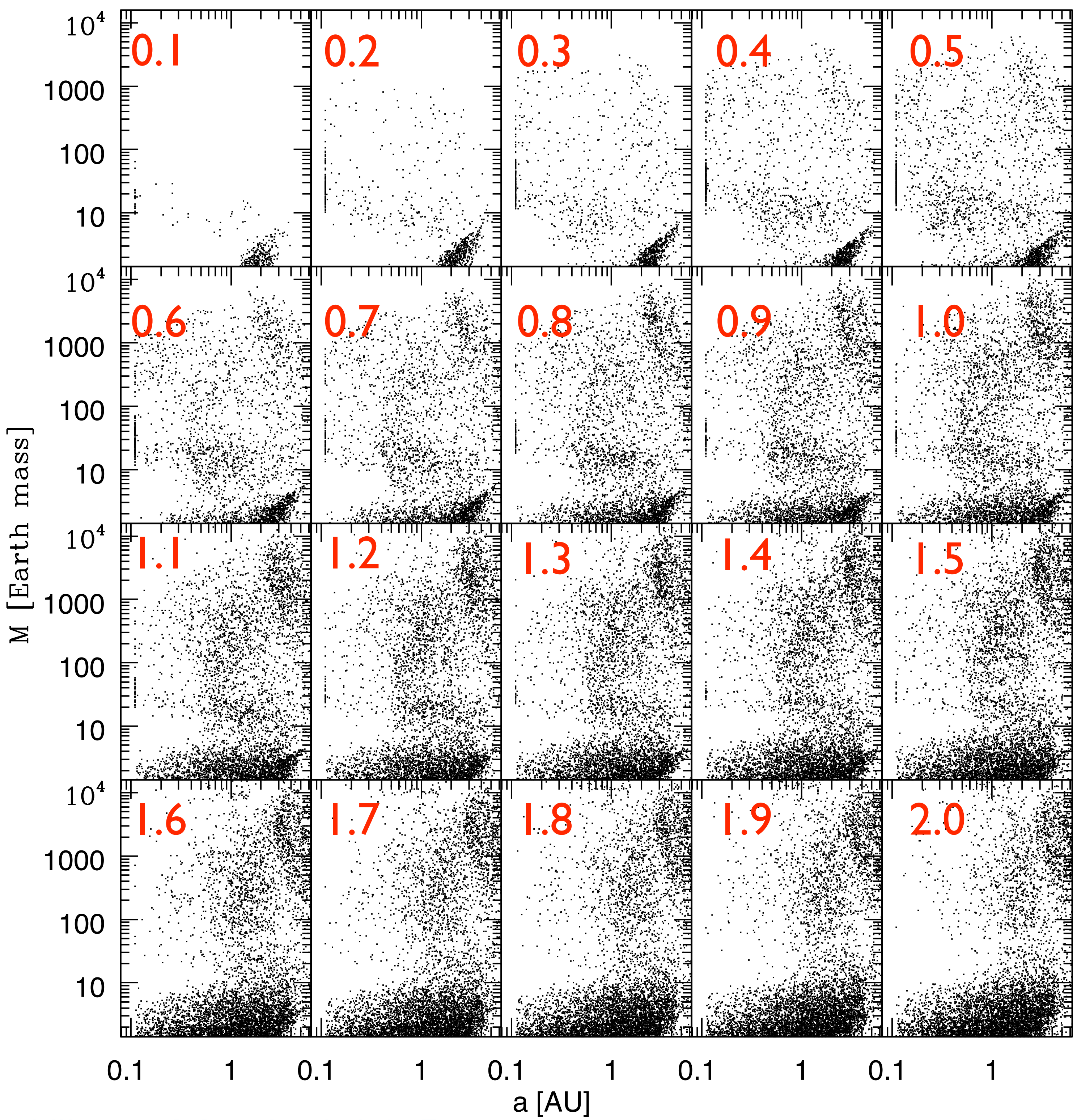
Inside: available mass criterion

-Migration relaxes the condition somewhat

Outside: timescale criterion

-Only long living disk make giants at low Σ_{solid} at large distances

Influences of *stellar mass*



Alibert, Mordasini & Benz. in prep

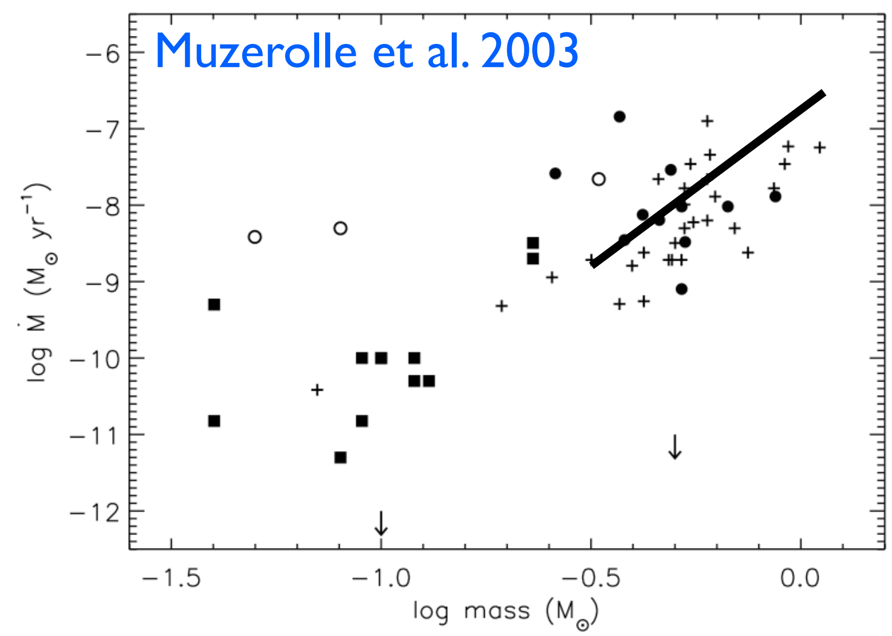
cf. Laughlin et al. 2004, Ida & Lin 2005

Scaled distribution of disk masses:

$$M_{\mathrm{disk}} \propto M_{\mathrm{star}}^{\alpha_{\mathrm{D}}}$$

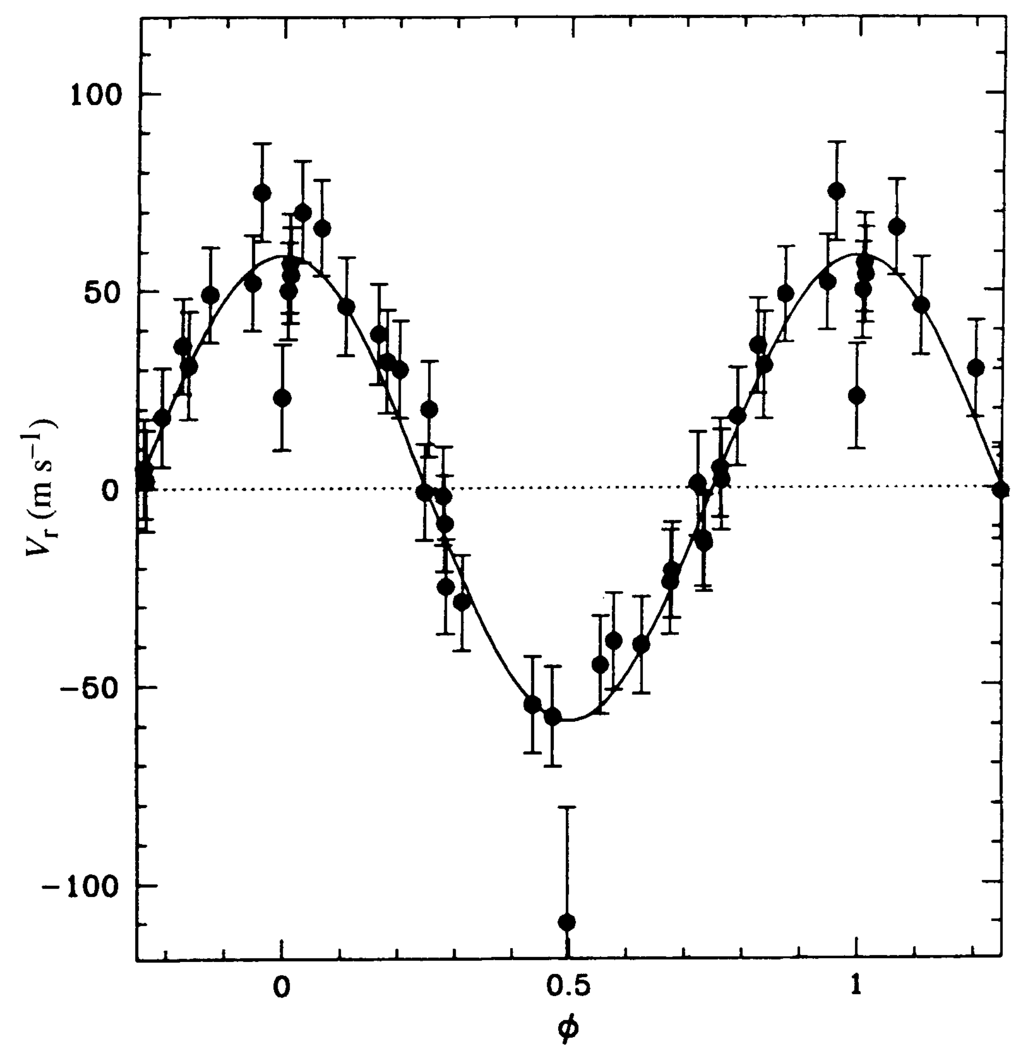
Preferred value: α_D=1.2

Leads (within our disk model) to observed quadratic dependence of disk accretion rate and M_{star} .



Low (high) mass stars lead to the formation of lower (higher) mass planets. M_{star} also influences semi-major axis and metallicity dependence.

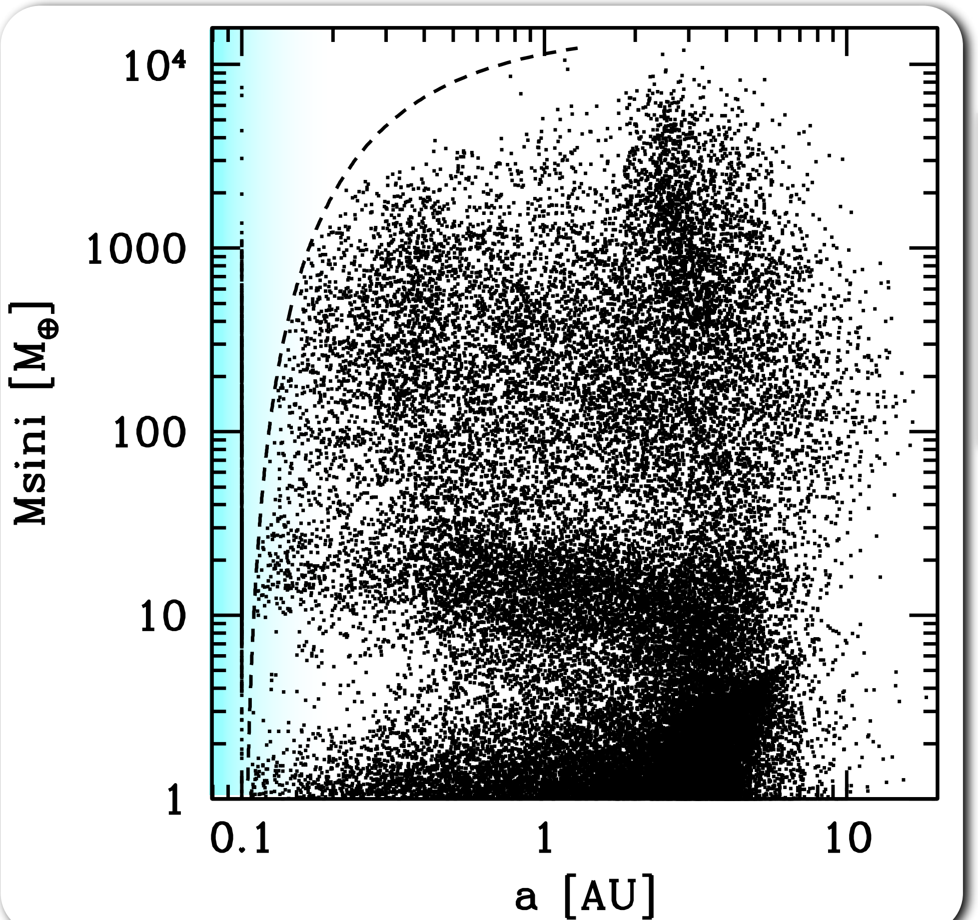
Comparison with observation:
Radial velocity



a-M

best fitting model (α, f_1) is found by statistical comparison with observation

Full synth. population



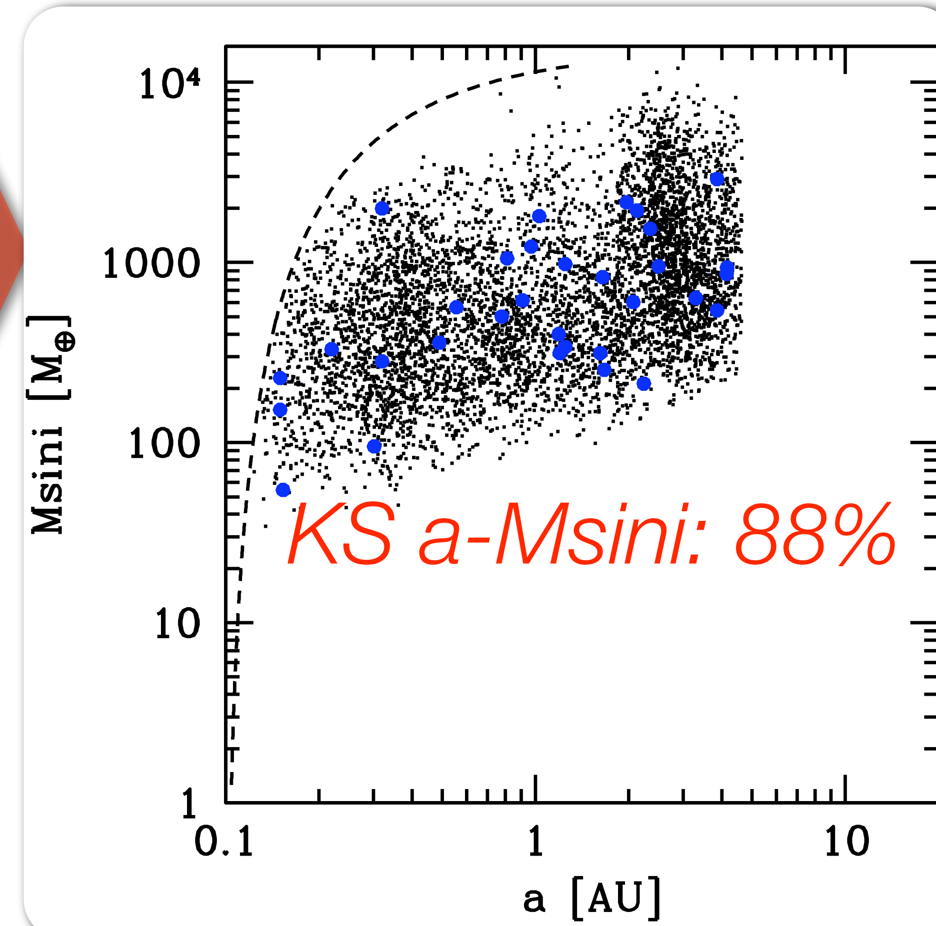
"Observe" 10 yrs at 10 m/s

Tip of the iceberg
Overall detection probability: 9 % (~as obs.)

&

Observ. comp. sample
- $0.7 < M_{star} < 1.3$
- $e < 0.3$
- One planet / star
- Single host stars
- K_{RV}>10 m/s

Detectable sub-population



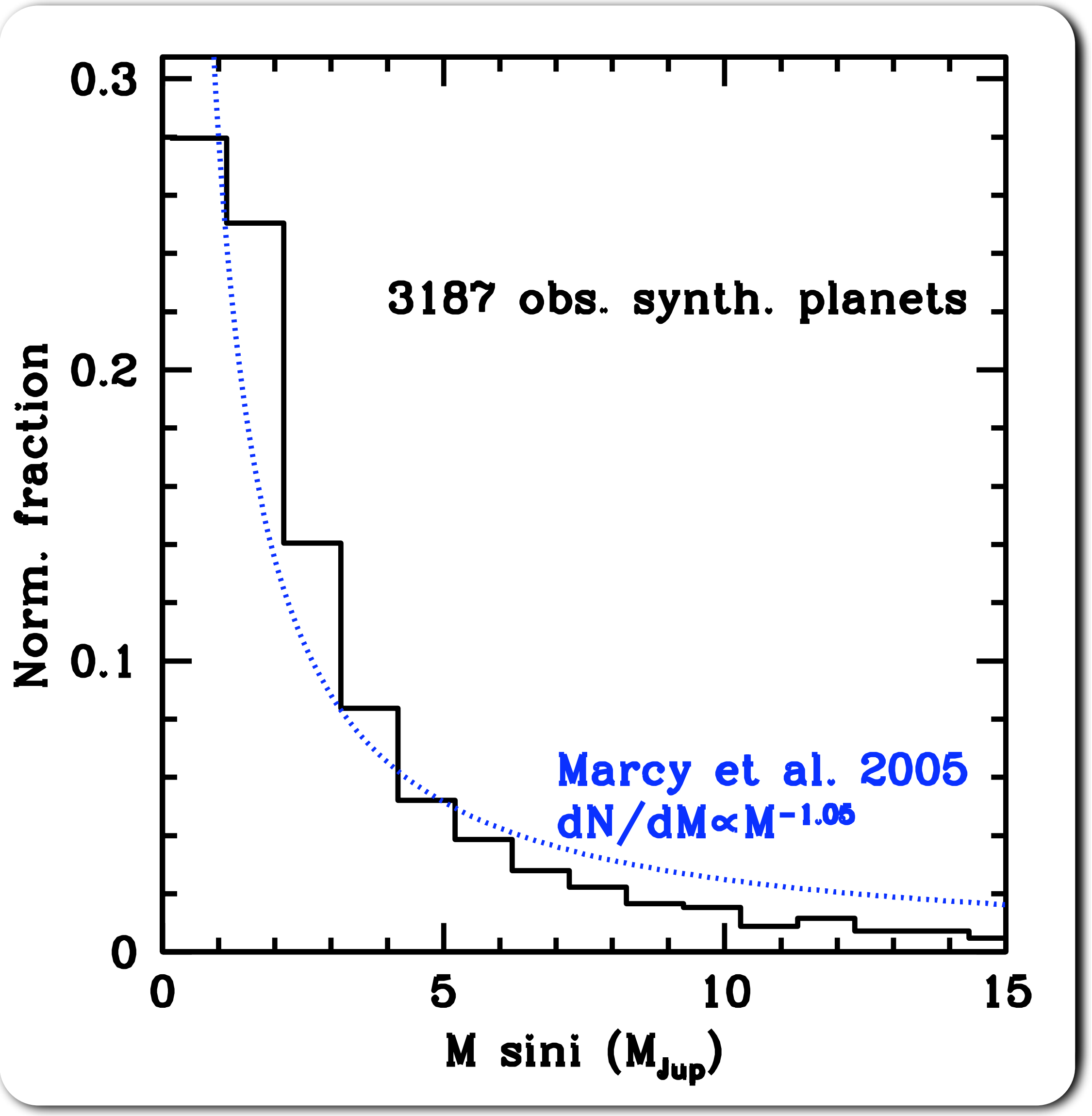
Core accretion is able to reproduce well the mass-distance distribution of the actual extrasolar planet population.

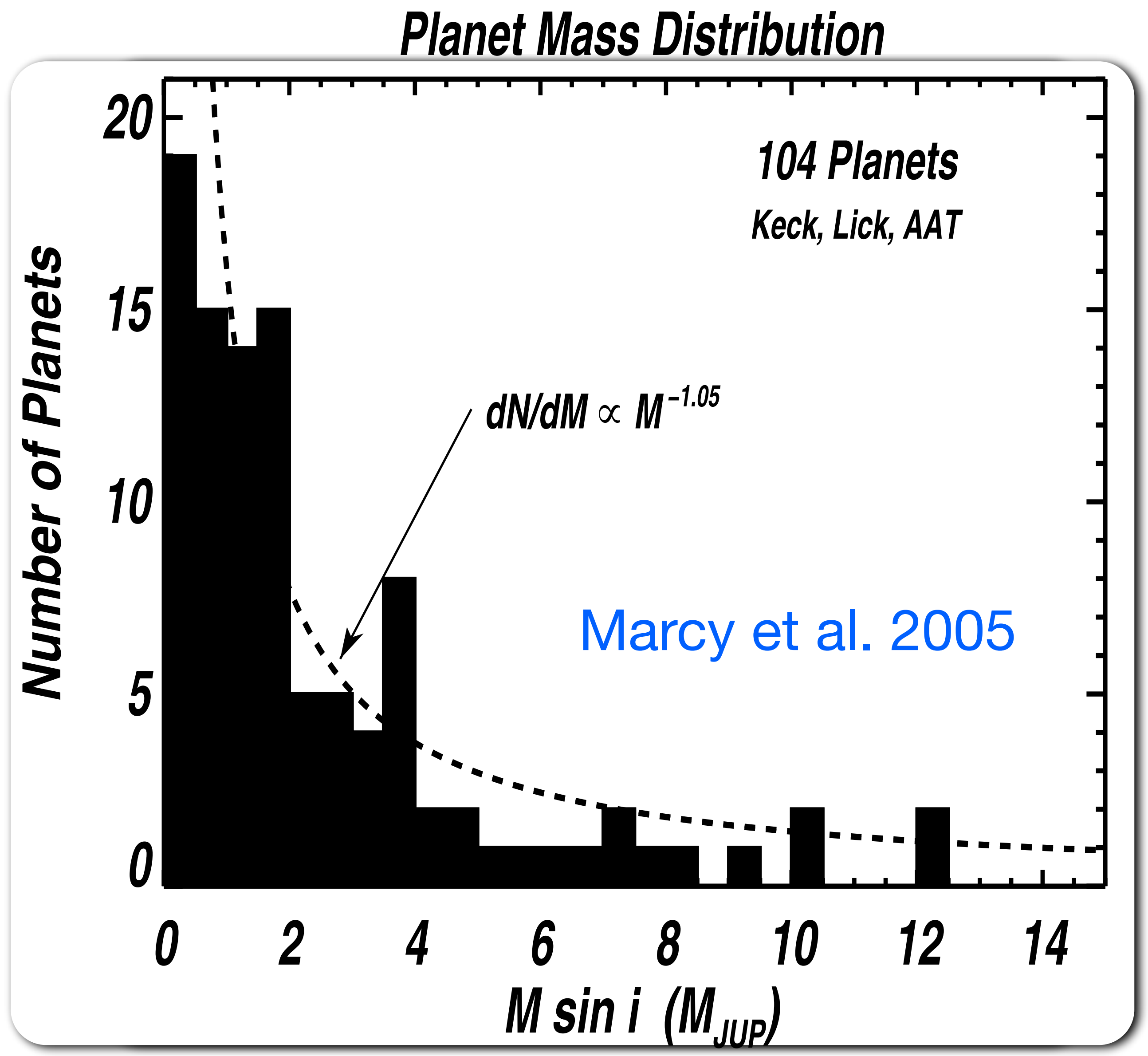
Non-trivial because 1) many constraints at one time 2) few parameters 3) probability distributions fixed 4) cannot force arbitrarily

Mass distribution

Blue lines: Observational comparison sample

Black lines: Detectable synthetic sub-population



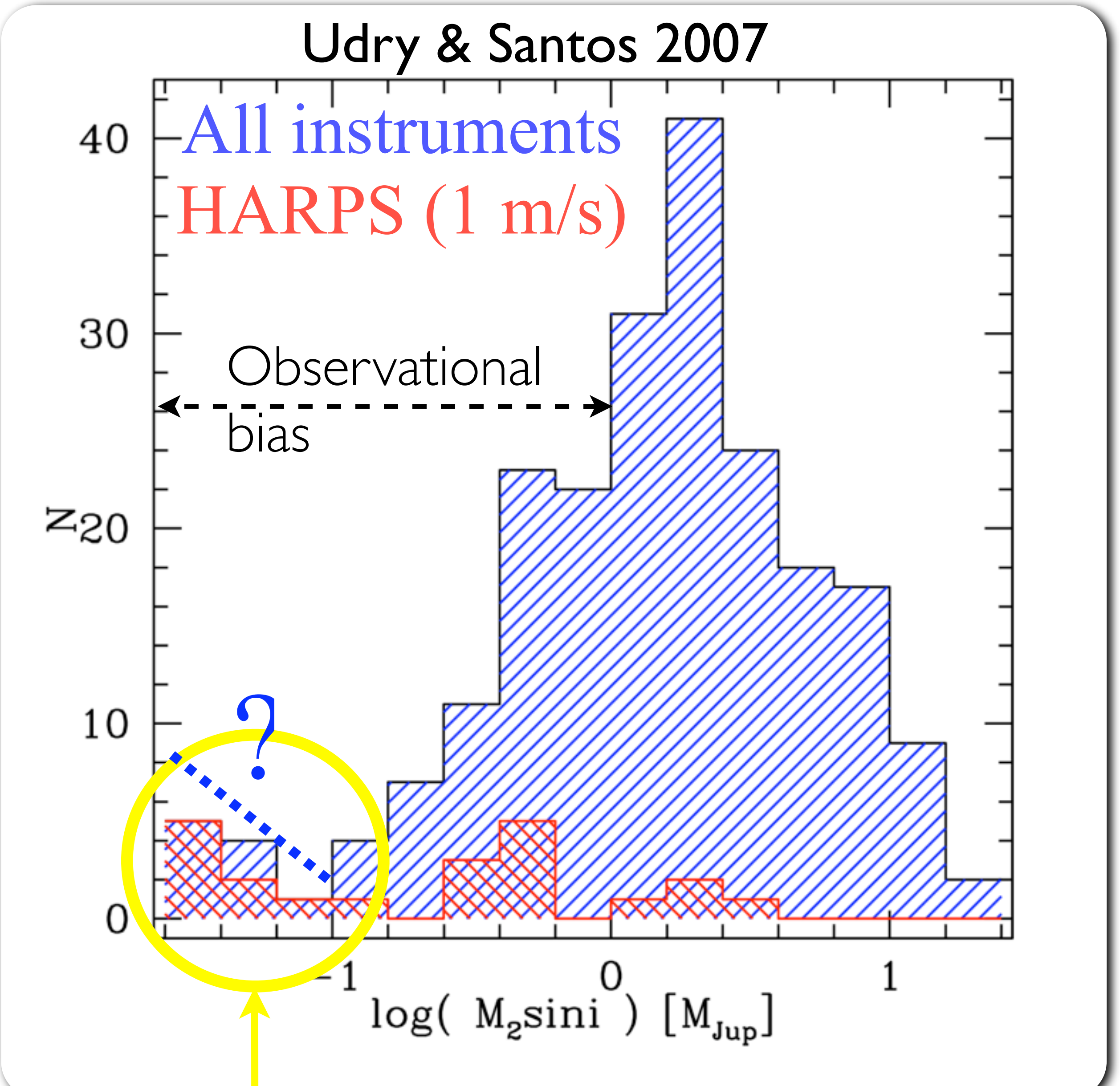


The mass distribution is very well reproduced.

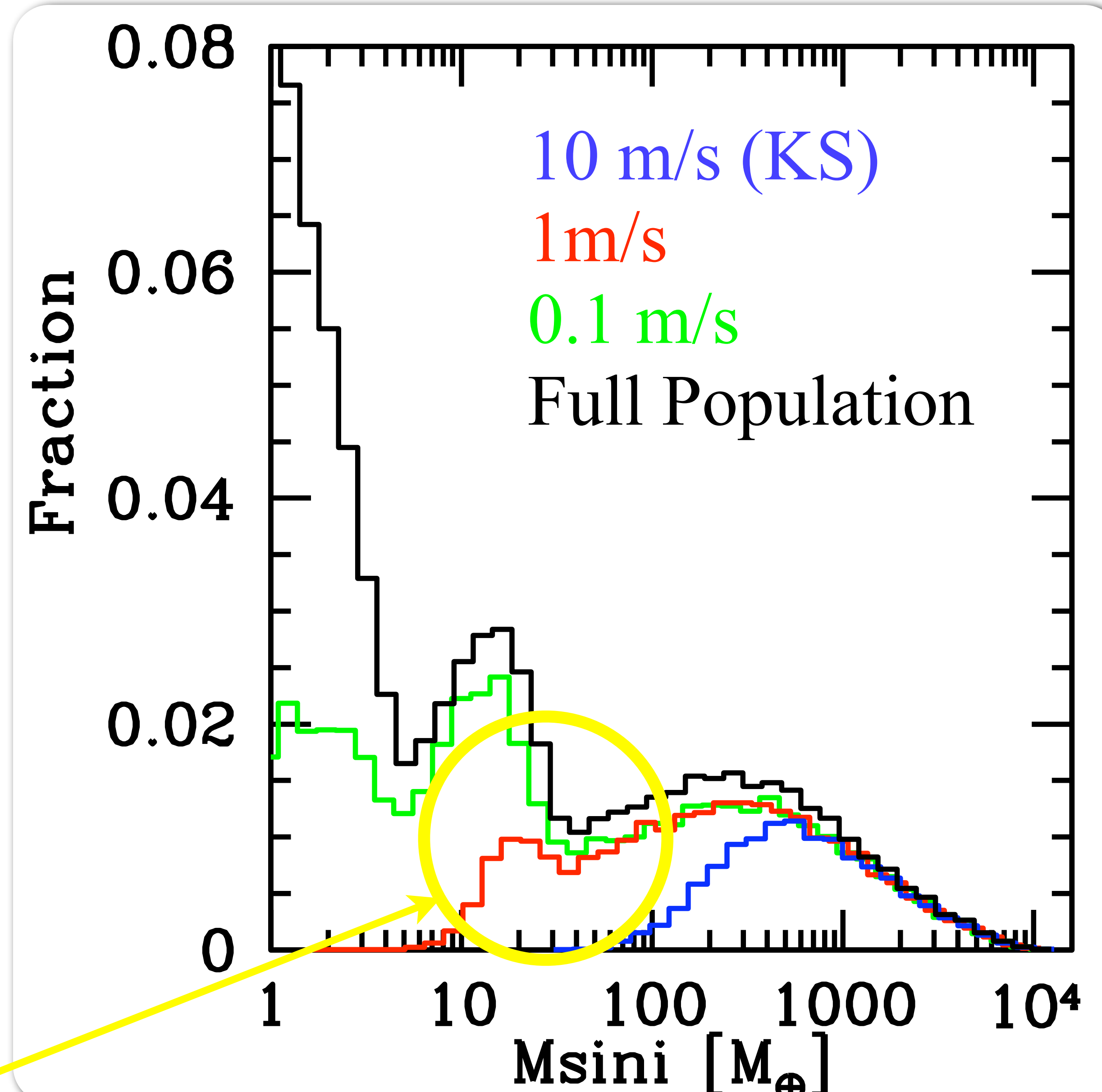
Also studied semimajor axis (KS: 64%) and [Fe/H] (KS 22%) distributions.

Towards the *underlying* mass distribution

Observation



Synthetic

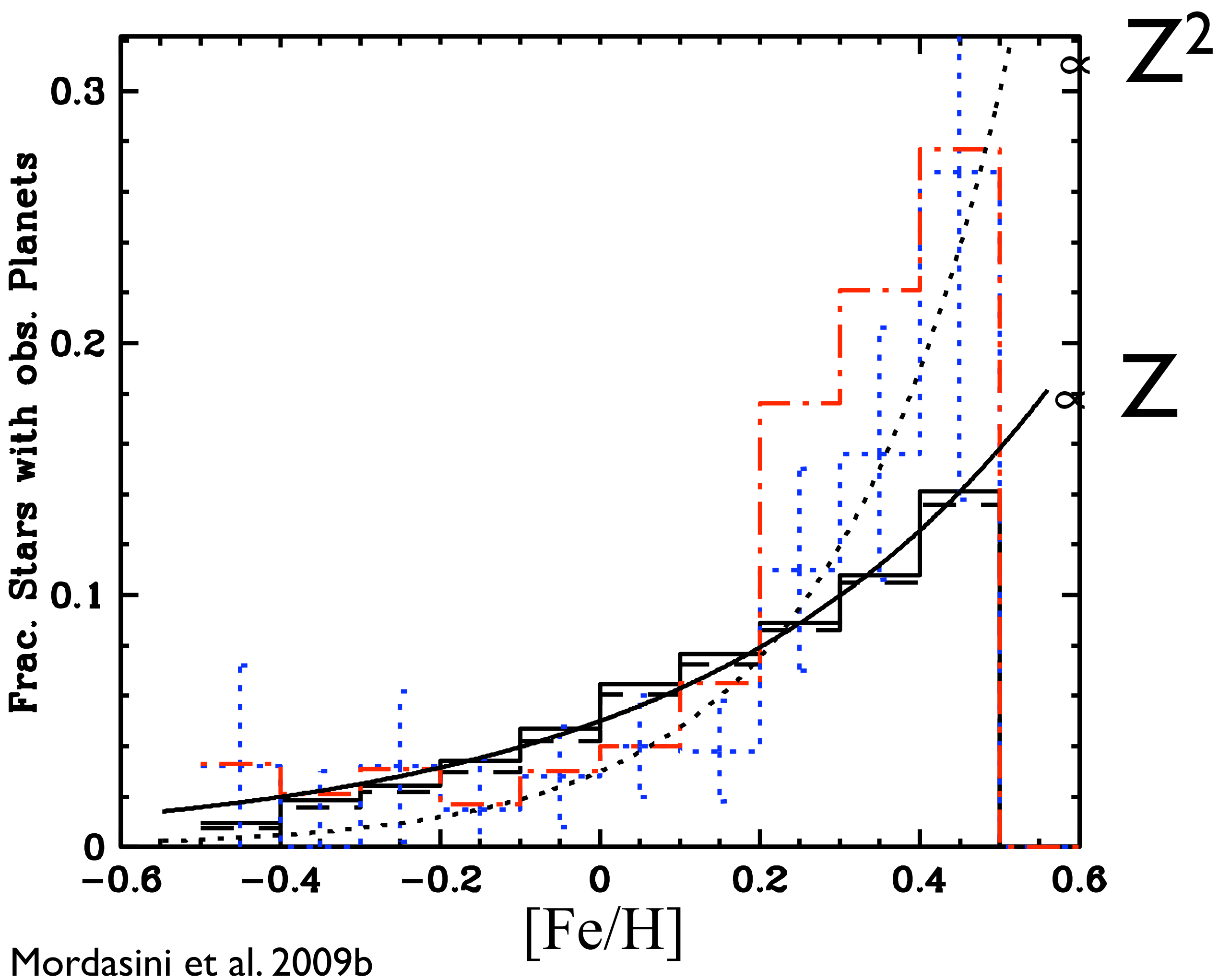


Hints of the Neptunian bump and the minimum at 30 M⊕?

If confirmed, very strong sign of c. a. Dryness of the planetary desert

“Metallicity effect”

Observations show: Extrasolar planets are preferentially found around stars with a high heavy element abundance: “Metallicity effect”



Blue: Observation (Fischer & Valenti 2005)

Red: Observation (Udry & Santos 2007)

Black: Observable synthetic planets

cf. also Santos et al. 2004, 2005

Well reproduced by the synthetic population.

- *Dependence not strong enough: Additional mechanisms? Planetesimal formation?*

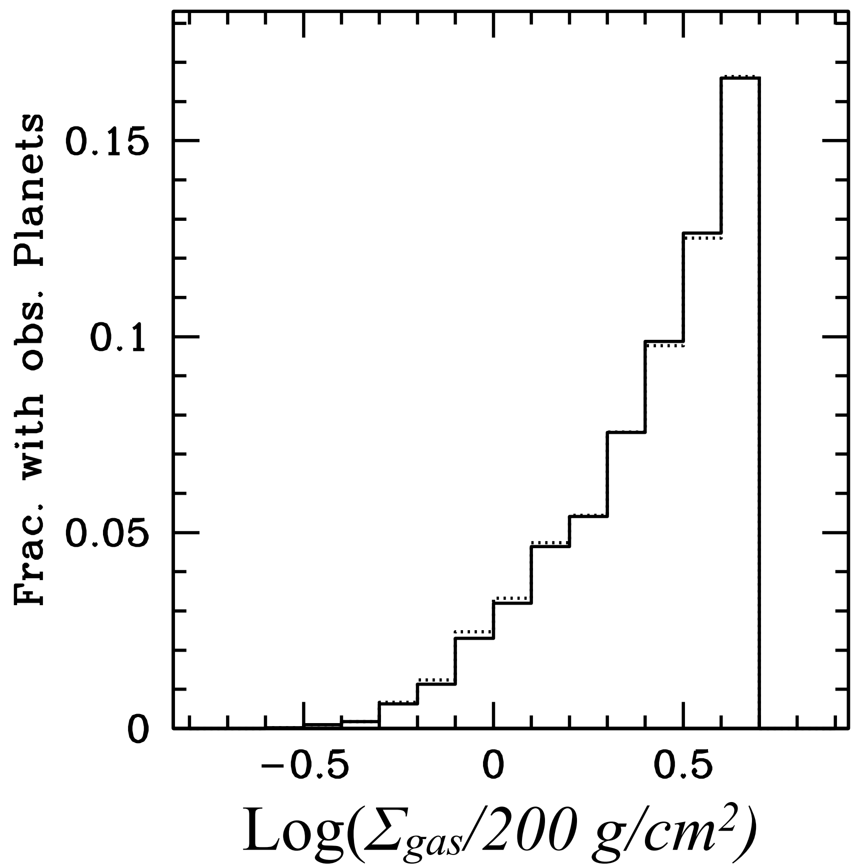
Large metallicity effect on RV detections

- *Metal rich systems tend to produce more massive planets*
- *Radial velocity method favors massive objects*

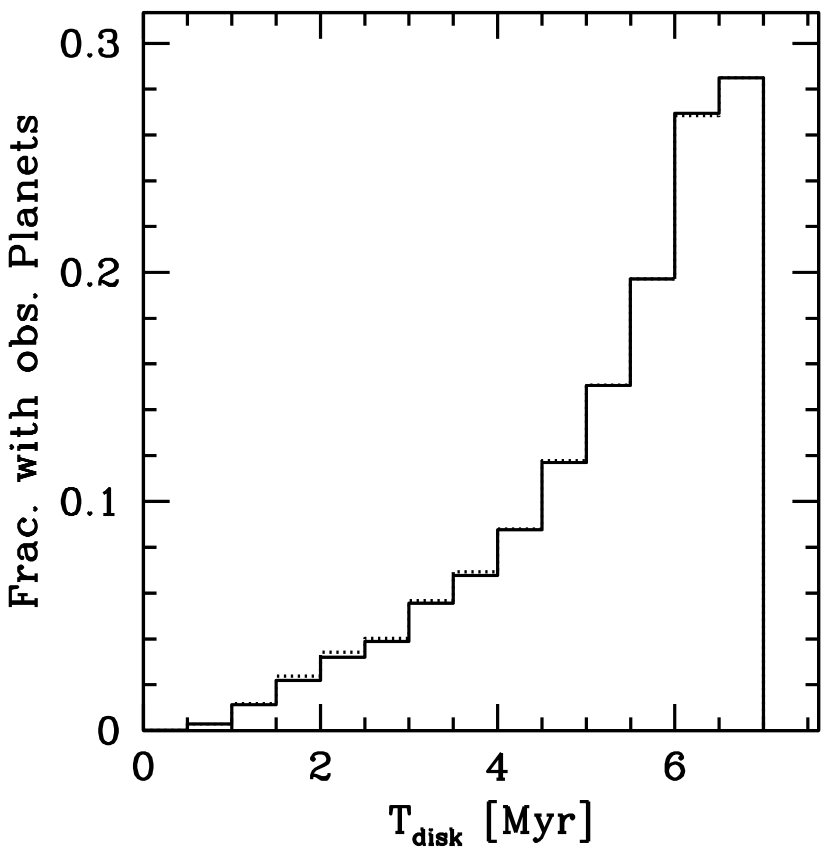
Does not mean there are no planets around low [Fe/H] stars... we just can’t detect them at the moment...

Unobservable (?) *detection probabilities*

Gas surface density



Disk lifetime



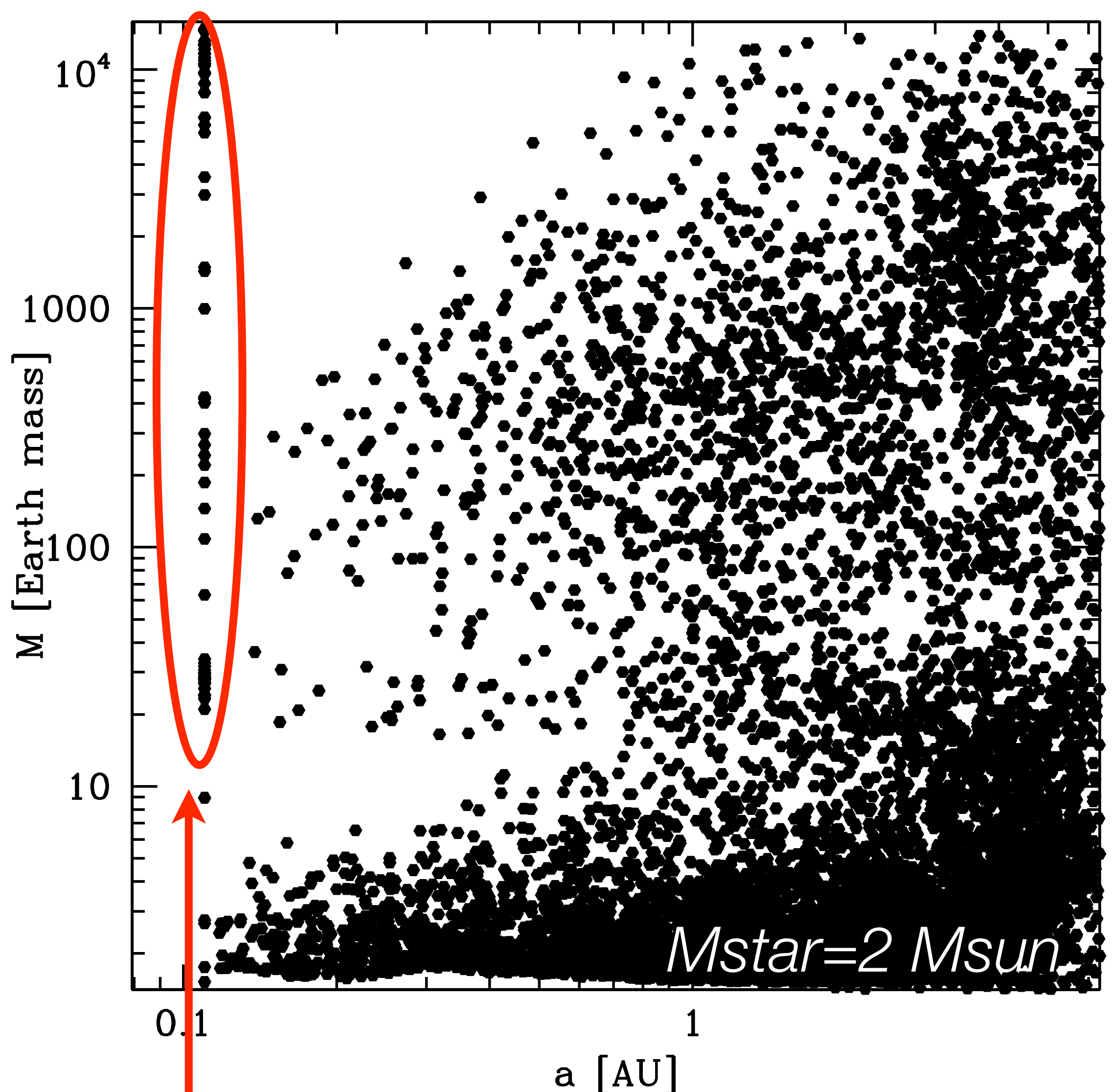
- Formation probability of giant planets increases with increasing disk gas mass and disk lifetime.
- Observation: Solar-type stars with massive planets have lower Li abundances than solar-type stars without massive planets. *Long living disks>stronger breaking of stellar surface layer>differential rotation>internal mixing>stronger Li depletion.* Disk lifetime effect IS observable? Bouvier 2008

Disk lifetime and stellar mass

Observation: Shorter disk lifetimes for more massive stars

Kennedy & Kenyon 2009

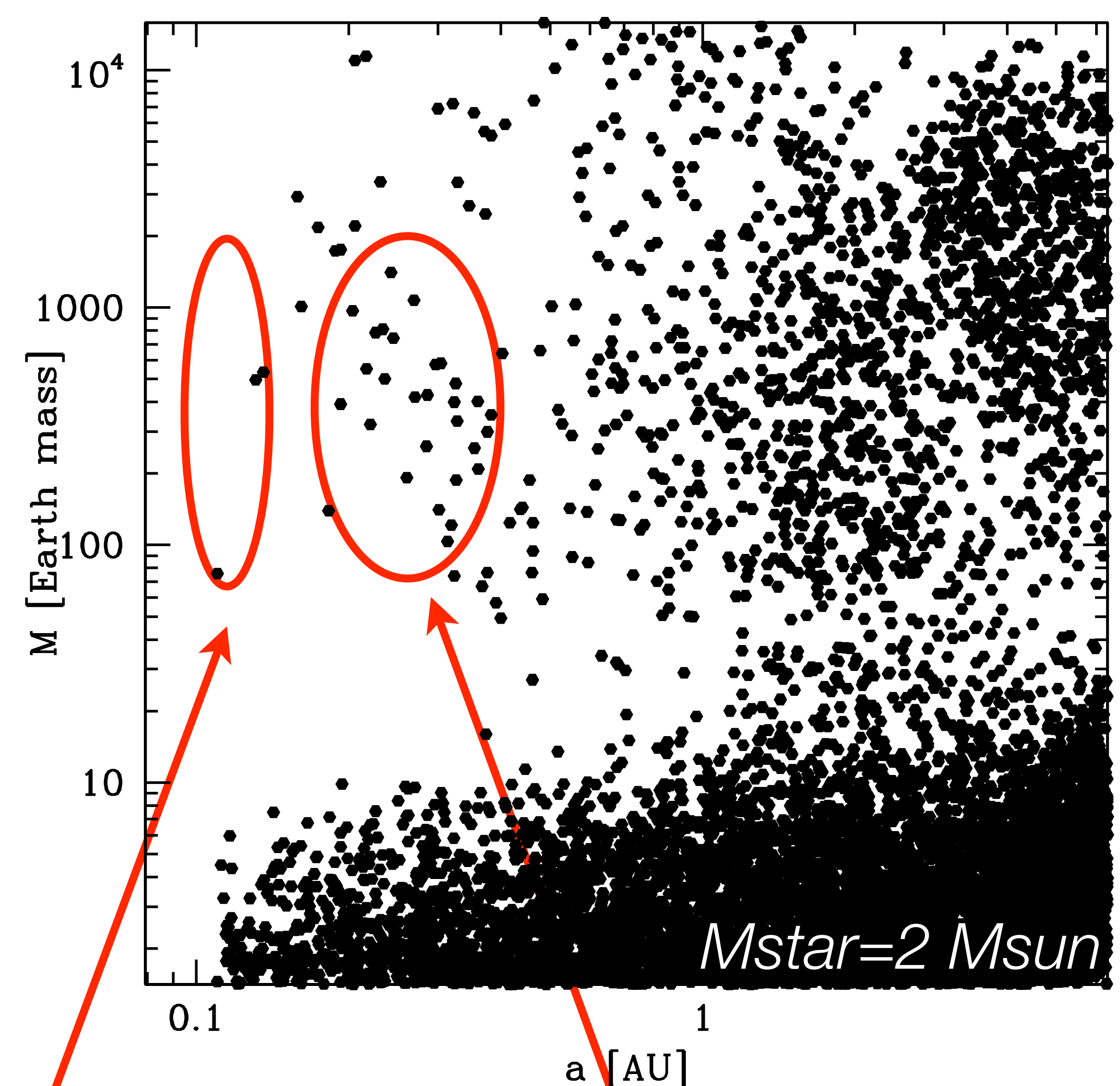
$$T_{\rm disk} \propto M_{\rm star}^{0}$$



many Hot Planets around massive stars

Not observed! Talk J. Johnson

$$T_{\rm disk} \propto M_{\rm star}^{-1/2} \quad \text{for} \quad M > 1.5 M_{\odot}$$



no Hot Planets

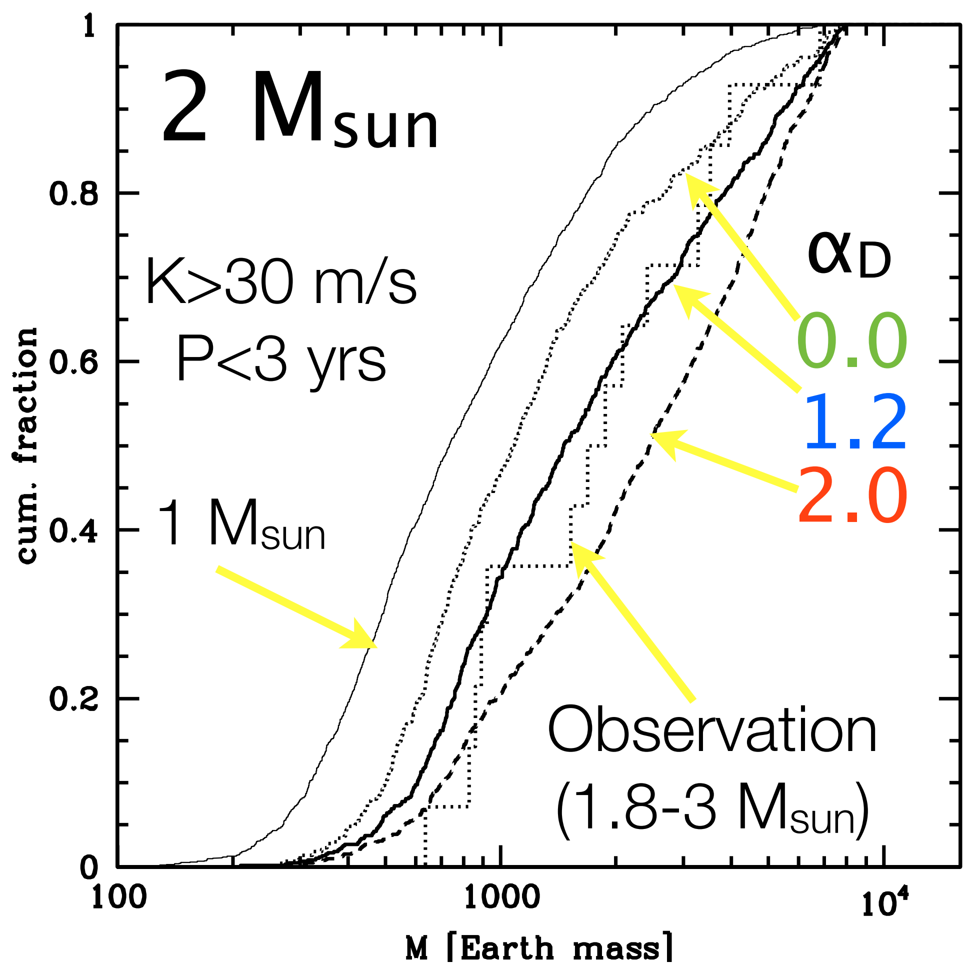
few warm planets

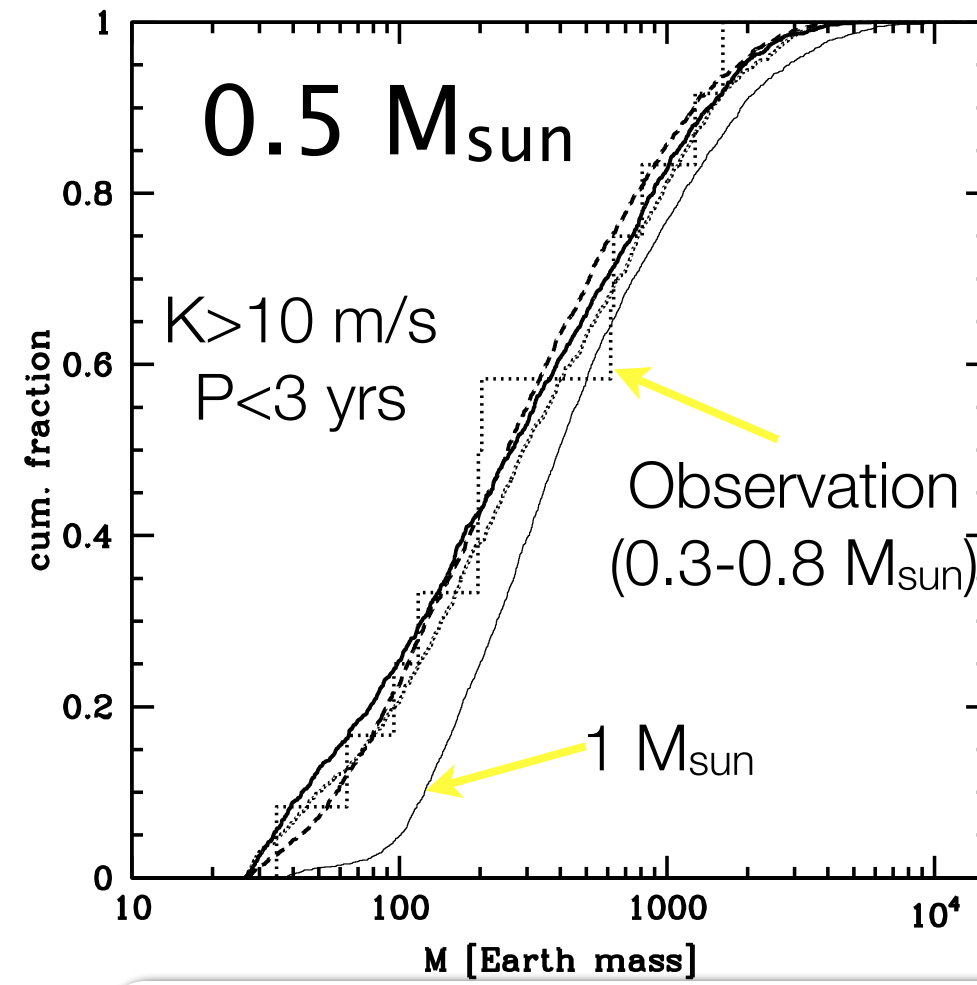
Alibert, Mordasini, Benz, in prep

cf. Currie 2009

Stellar and planetary mass

- Influence of α_D, and comp. with observations ($M_{\rm disk} \propto M_{\rm star}^{\alpha_D}$)





- Ma s (M> tar

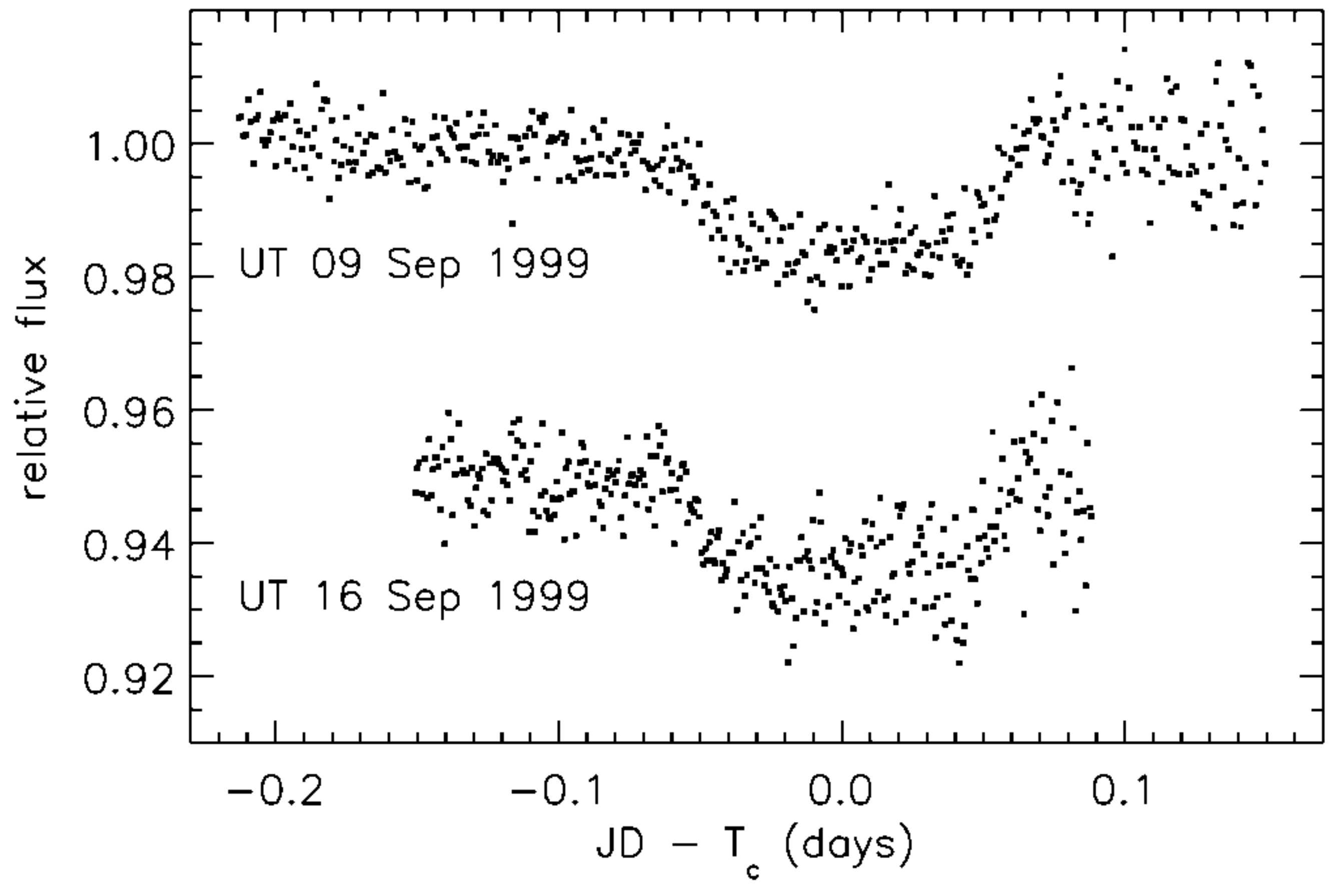
0.3 t s

M 0.8 =0.9 1.3 α_D star

		$M_{\rm planet}$	$a_{\rm planet}$
$M_{\rm star}$	α_D	$M<25M_J$	$M<25M_J$
2.0	1.2	67 %	76 %
2.0	0	10 %	49 %
2.0	2	36 %	0.2 %
1.0		0.2 %	0.3 %

		$M_{\rm planet}$	$a_{\rm planet}$
$M_{\rm star}$	α_D	KS	KS
0.5	1.2	92 %	83 %
0.5	0	74 %	78 %
0.5	2	87%	55 %
1.0		11 %	1 %

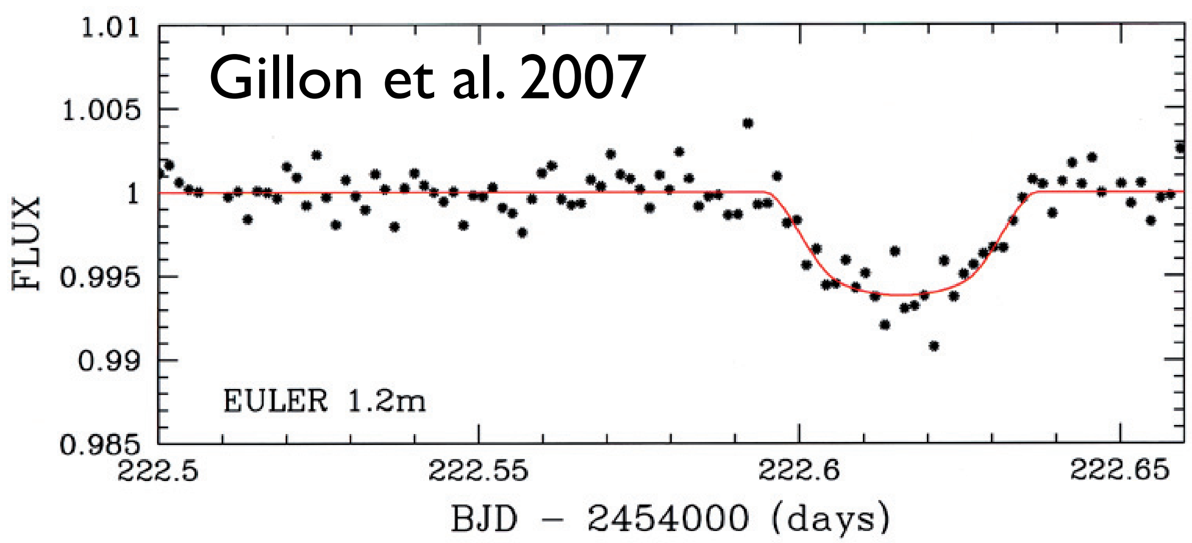
Comparison with observation:
Transits



Individual object: GJ 436b

GJ 436

- *0.44 $M_\odot$*
- *[Fe/H] ~ 0*
- *M2.5 V*
- *10.2 pc*
- *old (> 3 Gyr)*
- *0.024 $L_\odot$*



- M = 23 $M_\oplus$ (1.3 M_{Nept})
- R = 4.2 $R_\oplus$ (1.1 R_{Nept})
- ρ = 1.7 g/cm^3 (ρ_{Nept}=1.6 g/cm^3)
- a = 0.03 AU
- e = 0.16

Butler et al. 2004,
Gillon et al. 2007

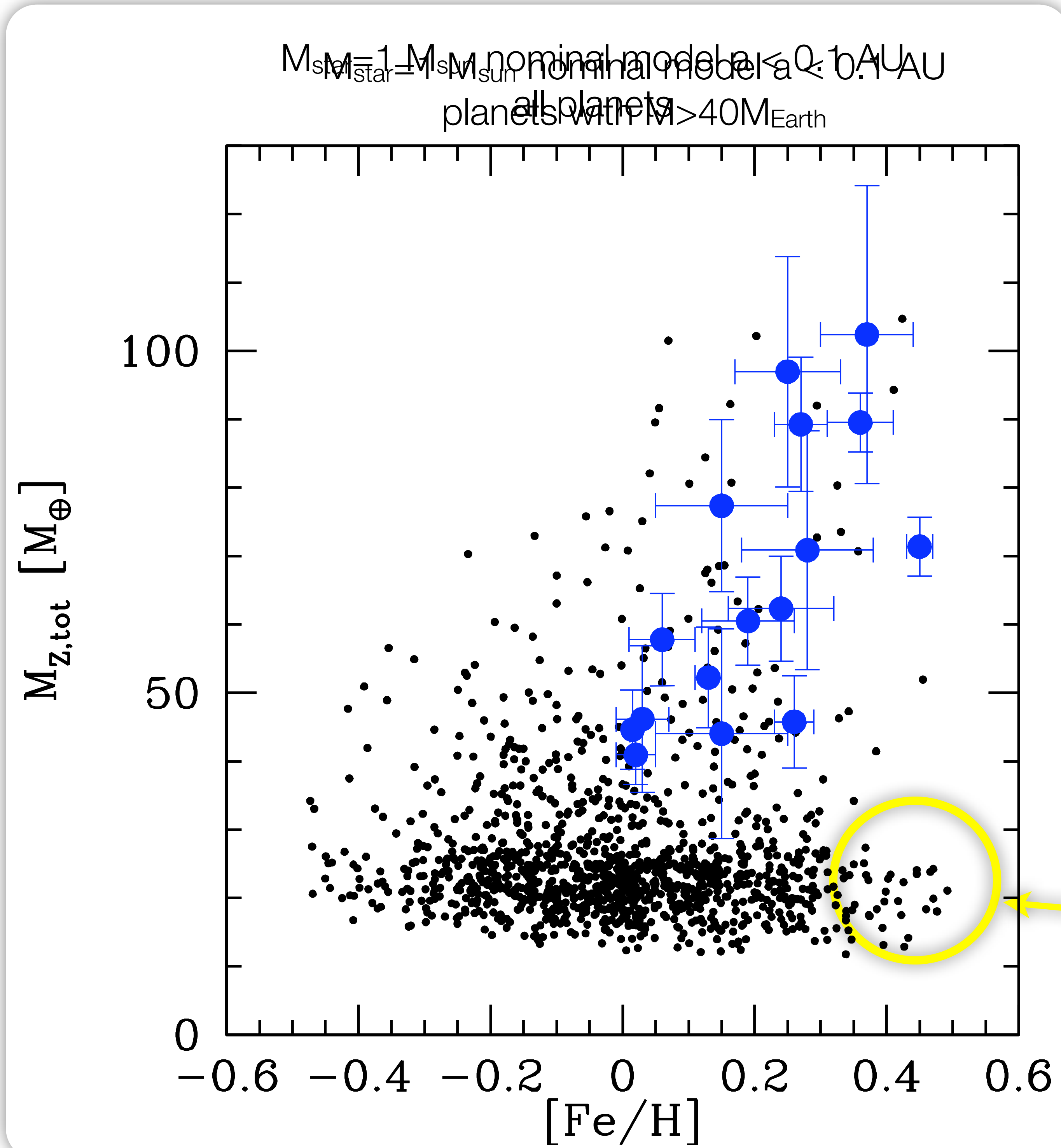
Synt. planets compatible with both observed mass and radius and also a<0.1 AU

Mass fract.	Neptune	GJ 436 b
Rock & Iron	25%	45-70%
Ices	60-70%	17-40%
H-He	5-15%	10-20%

Figueira et al. 2008

- *Reproduced also statistically:*
  Many outcomes similar to GJ 436b
- *Migration through inner system*
  → *More rocks than Neptune*
- *Break partially composition degeneracy*

Transiting *planets and metallicity*



Observations & internal structure models suggest a possible correlation between metallicity and heavy element content of transiting Hot Jupiters *(Guillot et al. 2006, Burrows et al. 2007)*.

The formation model shows a similar correlation between stellar metallicity and heavy element mass for Hot Jupiters.

Detection bias:
- low M_Z planets orbiting high Fe/H stars are low mass planets...

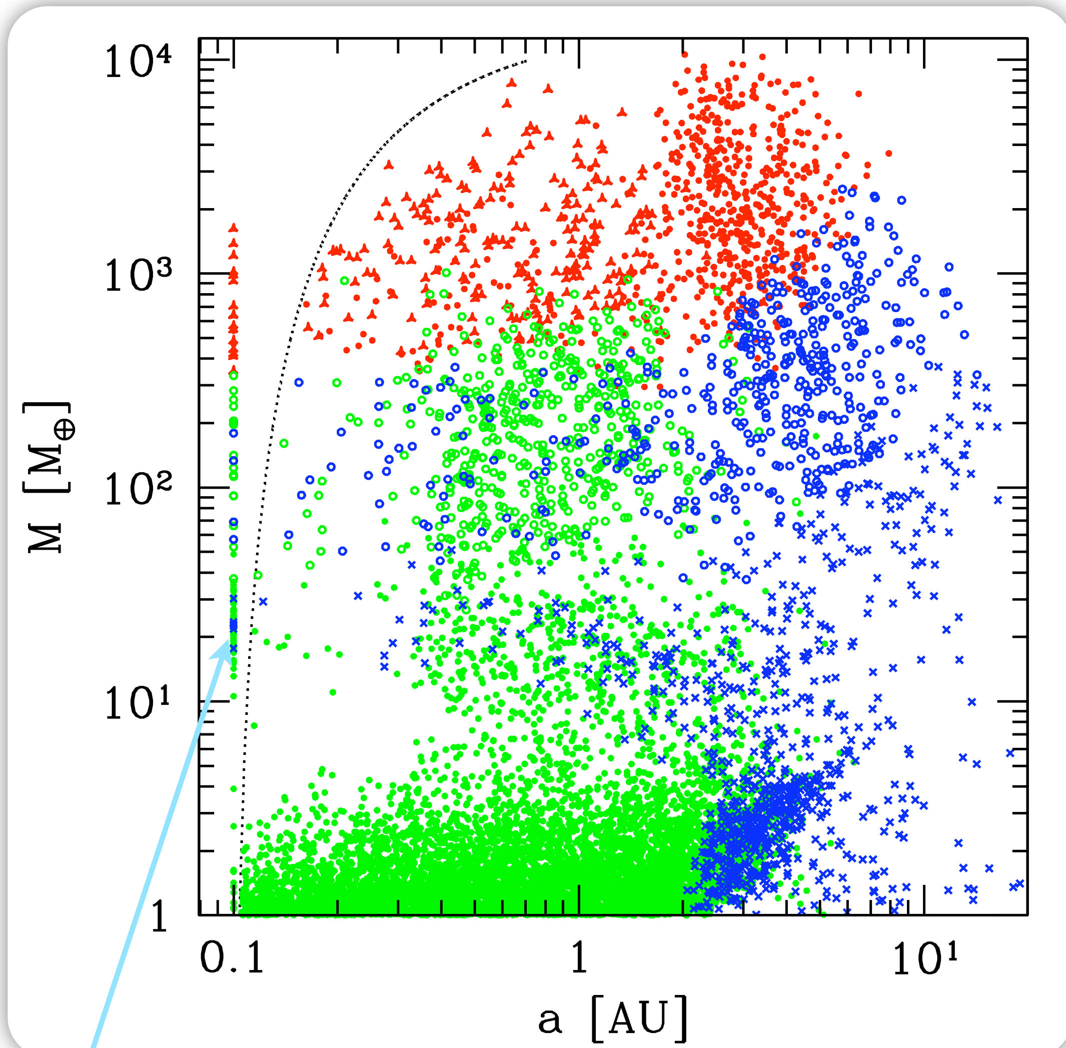
low M_Z planets can exist around high Fe/H stars

Blue Points: Structure models for observed transiting Hot Jupiters (Guillot et al. 2006)

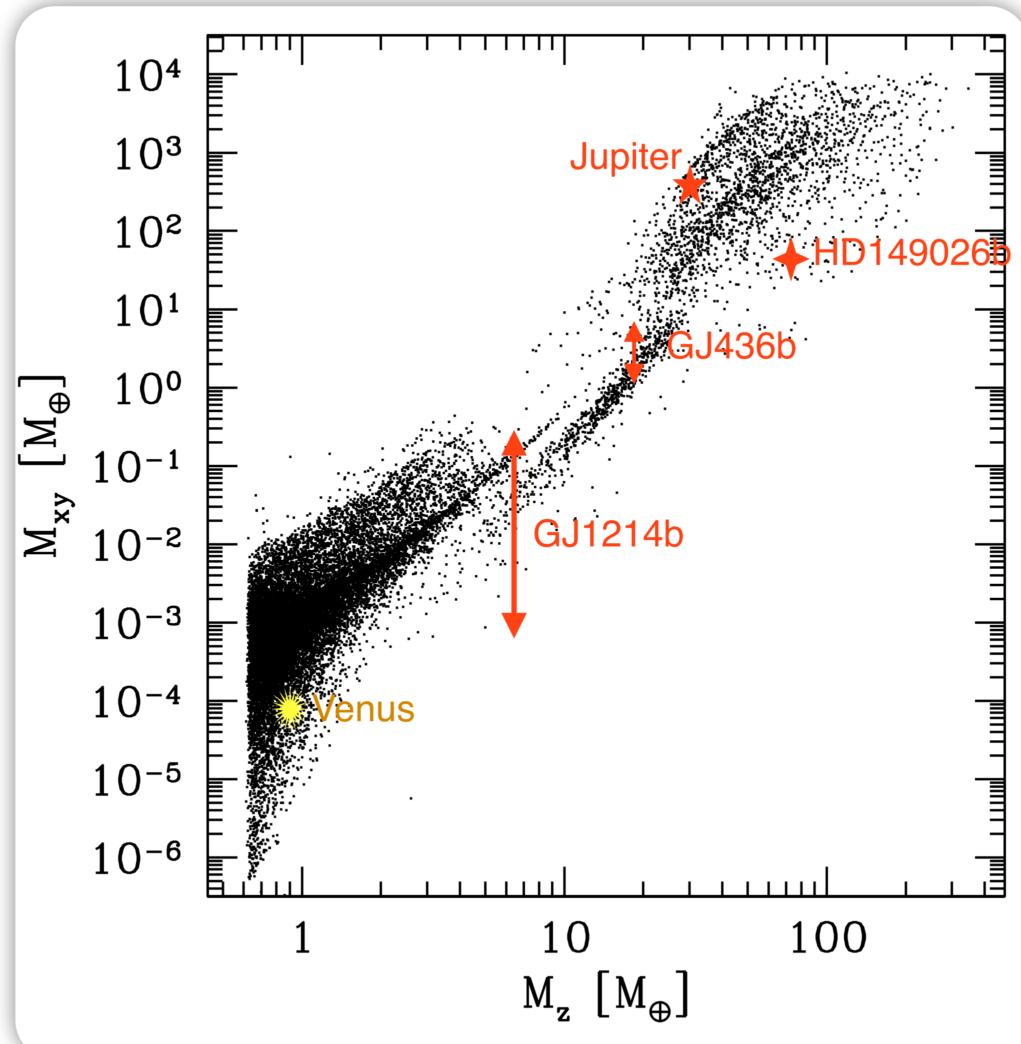
Black points: Synthetic planets inside a<0.1 AU (Mordasini et al. 2009b)

Internal composition

Model gives fractions of rocky and icy material, and of H_2/He in all planets.



M_I=1 M_{sun}
α=7x10^{-3}
f_I=0.001



Some ocean planets

Mainly icy core

Mainly rocky core

$M_{env} / M_{heavy} > 10$

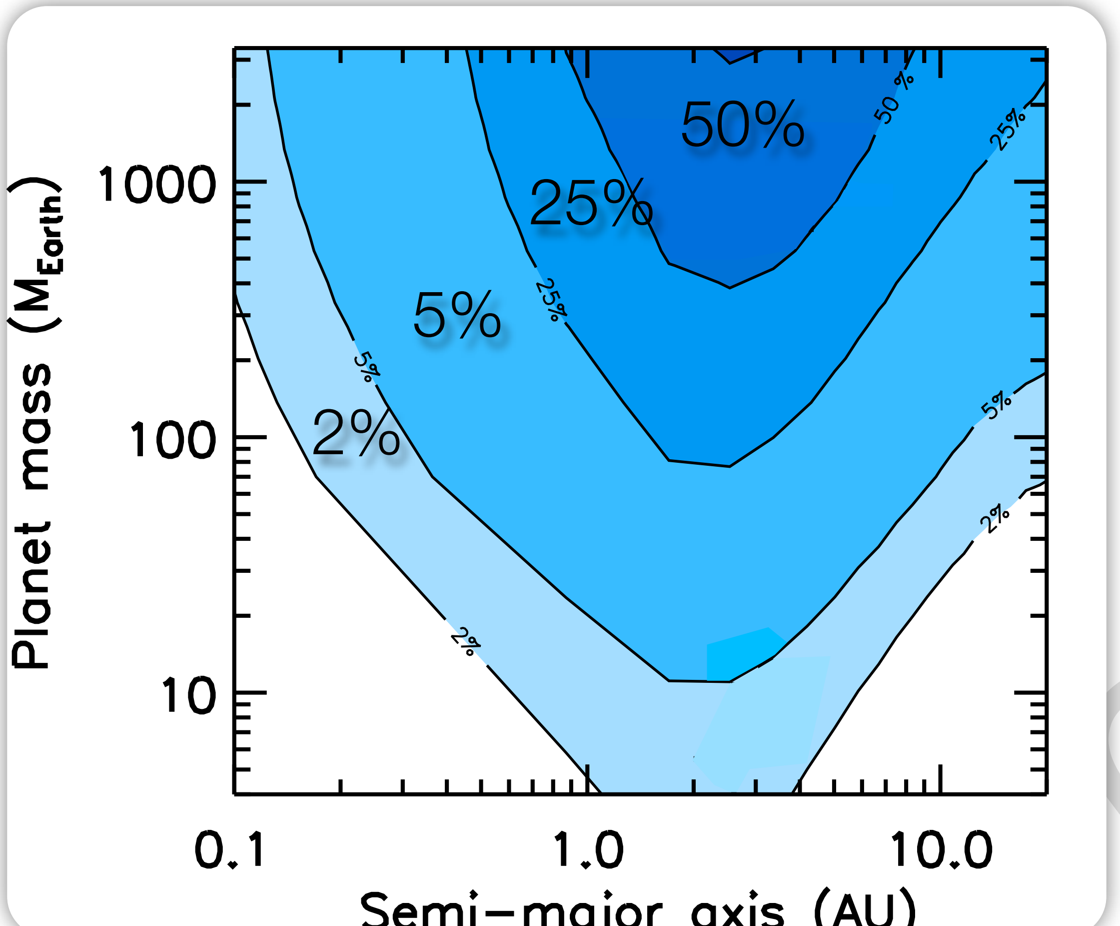
$1 < M_{env} / M_{heavy} < 10$

$M_{env} / M_{heavy} < 1$

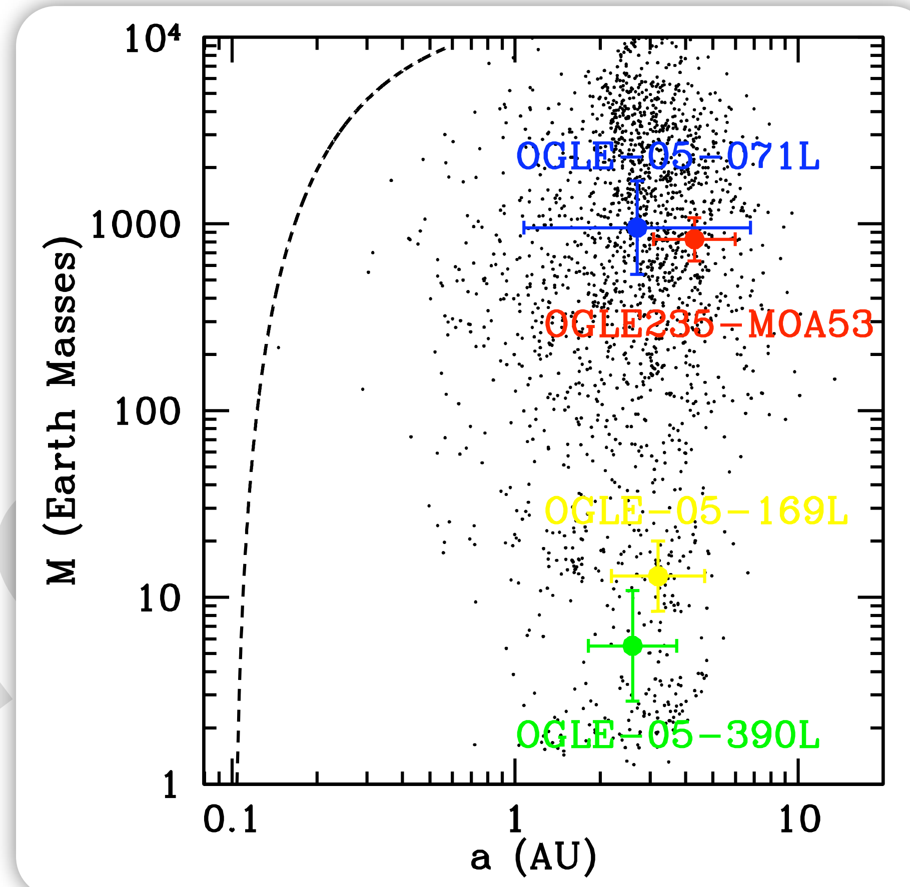
Primordial H_2/He envelopes

- at moment disk goes away
- comparison with individual objects

Other methods: Microlensing



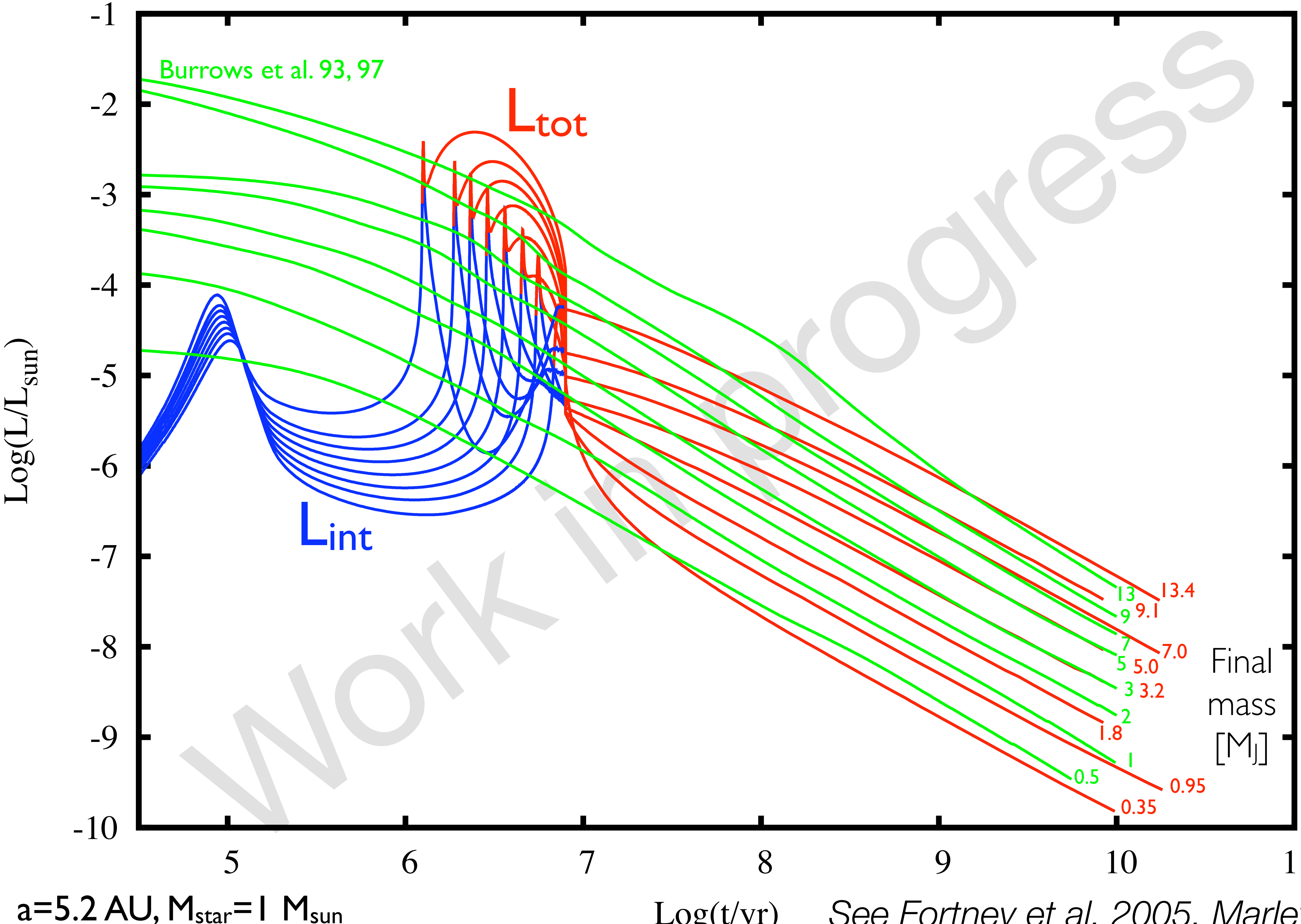
Detection efficiency of the PLANET microlensing network (Cassan, Sumi & Kubas 2008).



Synthetic planets detectable with this bias together with real microlensing discoveries.

Detection of cold failed cores: cross-checking results in a completely different a-M space than used to calibrate model parameters.

Other methods: Direct imaging



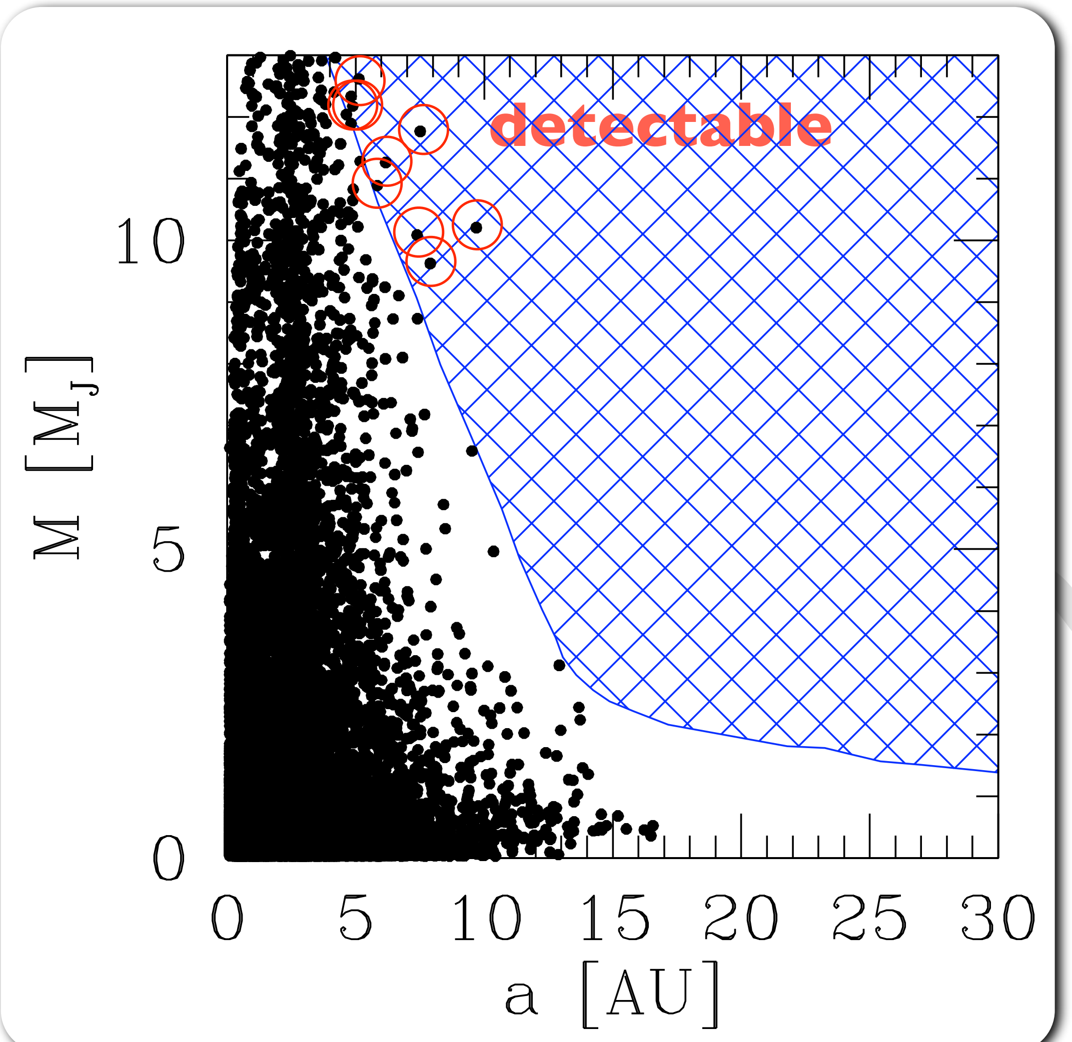
Simplified:
- *Eddington boundary conditions*
- *dL/dR=0*
- *Temporal evolution from energy conservation (L=-dE/dt)*
- *No D burning*

```
Jupiter @ 4.6 Gyr
Mtotal: 303 Mearth
Mheavy: 36 Mearth
R:      1.11 RJ
L(int): 1.17 LJ
T:      129 K
```

$a=5.2$ AU, $M_{star}=1\ M_{sun}$

See Fortney et al. 2005, Marley et al. 2007

Direct imaging



Example synthetic observational bias for direct imaging (Bonavita, Desidera & Gratton 2008) together with synthetic planets.

Instruments like SPHERE can provide new constraint on formation models:

- Outward migration?
- Ejection probabilities?
- Direct grav. collapse model?

★ Estimate detection probabilities & science return

★ Analyze the impact of instrument design

★ Optimize measurement strategy & target selection


Thanks!
HARDY