Non-Abelian States in Rotating Atomic Bose Gases

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[NRC, N.K. Wilkin & J.M.F. Gunn, PRL **87**, 120405 (2001);
NRC, cond-mat/0308283]

Overview

- Vortex arrays in ^{4}He and atomic BECs
- Weak interactions $\Rightarrow$ the lowest Landau level
- Vortex Lattices vs Vortex Liquids
- Laughlin, Moore-Read and Read-Rezayi states
- Interaction via a Feshbach Resonance
- Conclusions

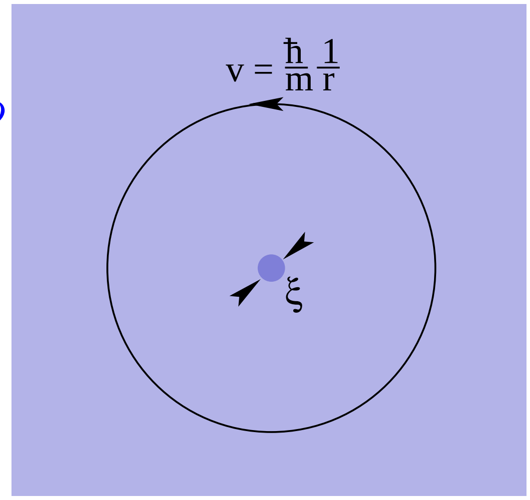
Vortices in Superfluid ^{4}He

[Onsager; Feynman]

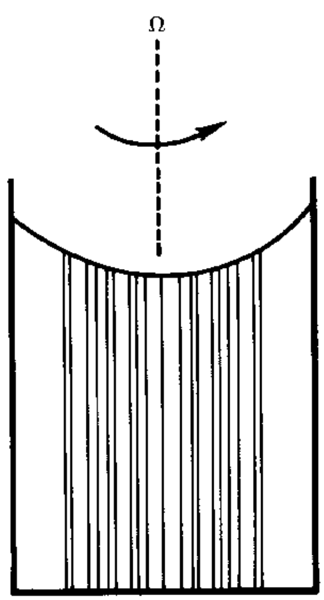
$$\psi_s = \sqrt{\rho_s} e^{i\phi(\vec{r})} \qquad \vec{v}_s = \frac{\hbar}{m}\vec{\nabla}\phi$$

$$\oint \vec{v_s} \cdot d\vec{l} = \frac{h}{m}$$

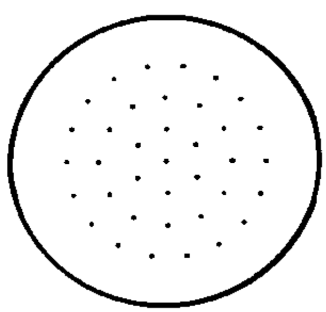
Superfluid density vanishes at the vortex core ($\xi \sim 0.8$Å).

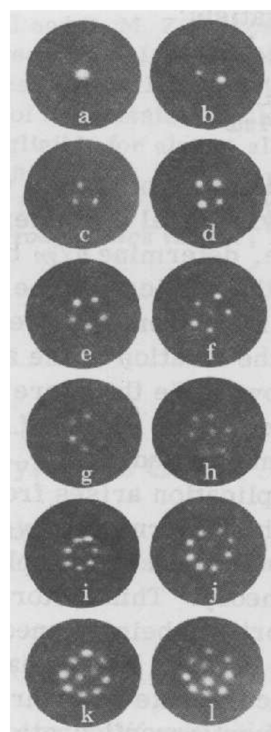


Rotating bucket experiment [Hall & Vinen, 1956]



Side view


Top view



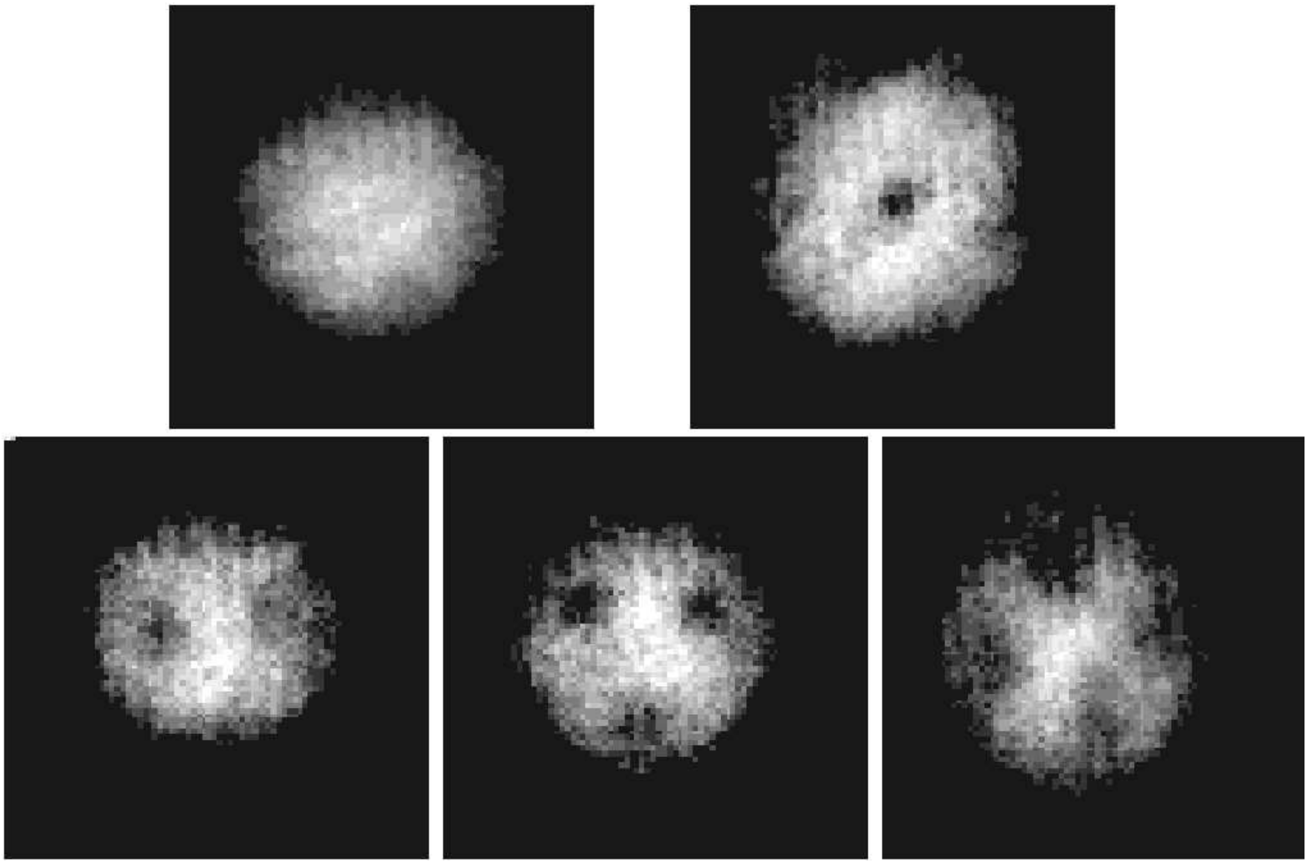
A lattice of vortex lines

[Yarmchuk, Gordon & Packard, 1979]

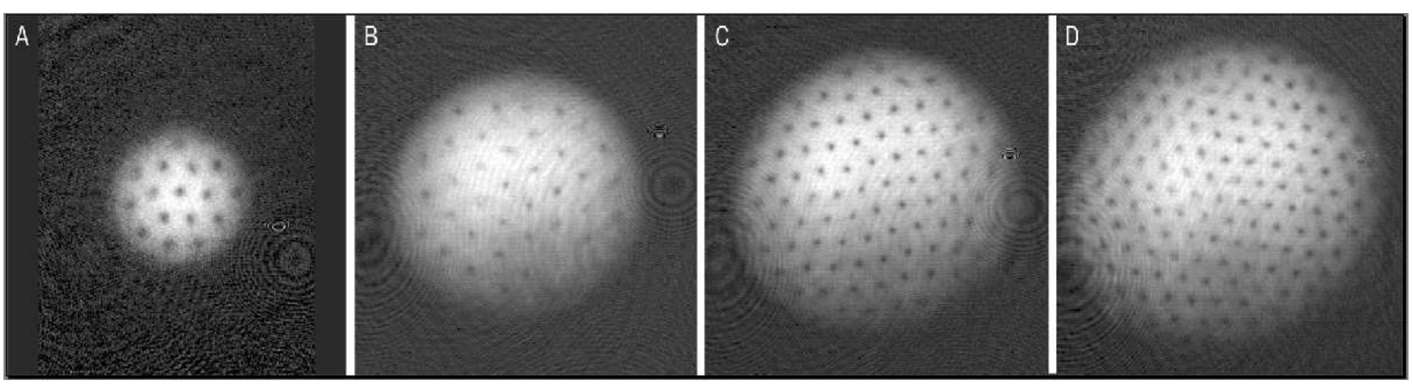
$$n_V = \frac{2m\Omega}{h}$$

Vortices in Atomic Bose Condensates

^{87}Rb [Madison *et al.*[ENS], PRL **84**, 806 (2000)]



^{23}Na [Abo-Shaeer *et al.*[MIT], Science **476**, 476 (2001)]



What's special about Atomic BECs?

- Imaging
- Dynamics
- Phase imprinting
- Multiple components
- ...

▷ *Weakly interacting*

▷ *Tunable interactions*

s-wave scattering length, $a_s \sim 5\text{nm}$

mean particle separation, $\bar{a} \sim 100\text{nm}$

$\Rightarrow$healing length large $\xi = \frac{1}{\sqrt{\bar{n} a_s}} \sim 500\text{nm}$

Vortex cores are large.

Are there novel *uncondensed* many-vortex states?

[Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

Formulation of the Problem

$$H = \sum_{i=1}^{N}\left[\frac{p_i^2}{2m} + \frac{1}{2}m\omega_0^2|\vec{r_i}|^2\right] + \eta\sum_{i<j}\delta(\vec{r_i}-\vec{r_j})$$

Rotating Frame: $H_\Omega = H - \vec{\Omega}\cdot\vec{L}$ $\quad [\eta = \frac{4\pi\hbar^2 a_s}{m}]$

One-Body Terms

$$H_\Omega^{(1)} = \frac{|\vec{p}|^2}{2m} + \frac{1}{2}m\omega_0^2|\vec{r}|^2 - \vec{\Omega}\cdot\vec{r}\times\vec{p}$$

$$= \frac{|\vec{p}-m\vec{\Omega}\times\vec{r}|^2}{2m} + \frac{1}{2}m(\omega_\perp^2-\Omega^2)(x^2+y^2) + \frac{1}{2}m\omega_\parallel^2 z^2$$

$$\boxed{q^*\vec{B}^* = 2m\vec{\Omega}}$$

$+$ Weak Interactions [Wilkin, Gunn & Smith (1998)]

$\eta\bar{n} \ll \hbar\omega_\perp, \hbar\omega_\parallel \Rightarrow$ lowest Landau level & 2D

$$\langle x,y|m\rangle \propto z^m e^{-|z|^2/2}$$

$[z \equiv (x+iy)/\ell_\perp]$

See Ho, PRL **87**, 060403 (2001) for LLL in 3D ($\eta\bar{n} \gtrsim \hbar\omega_\parallel$).

Gross-Pitaevskii Mean-Field Theory

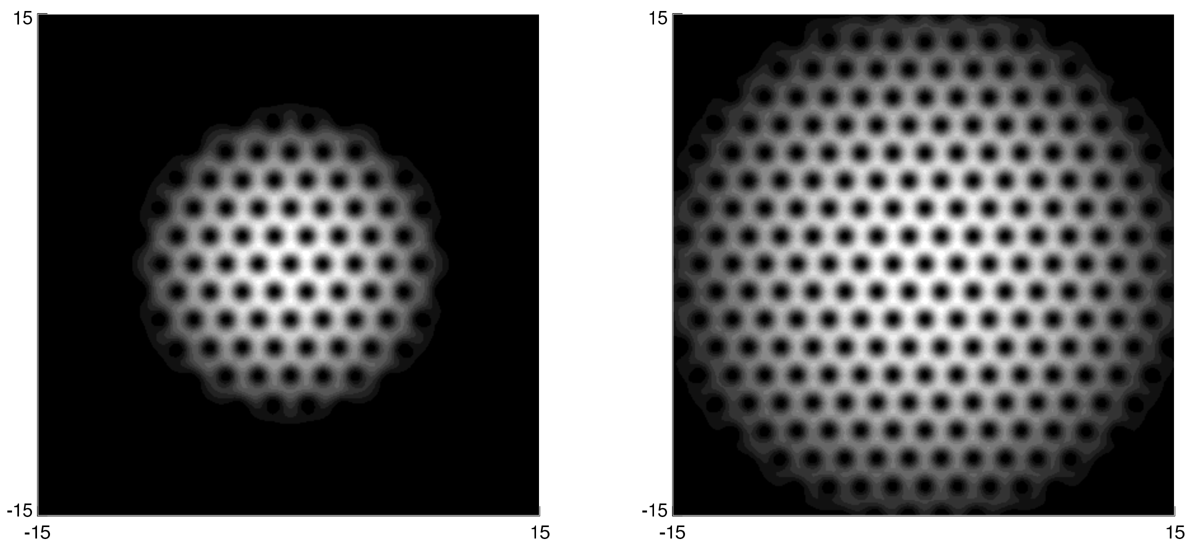
[Butts & Rohksar, Nature **397**, 327 (1999)]

$$\Psi(\{\vec{r_i}\}) = \prod_i \psi(\vec{r_i})$$

Minimise the expectation value of the energy

$$\psi(x,y) \propto \prod_V (z - Z_V) \quad \left[e^{-|z|^2/2}\right]$$

Numerical results at large angular momentum:



[NRC, S. Komineas & N. Read, cond-mat/0404112

Exact Groundstates

Conserved angular momentum: $L \equiv \sum_i m_i$

$L \leq N$ Analytic results

[Smith & Wilkin, PRA **62**, 061602 (2000); Hussein & Vorov PRA **65**, 035603 (2002).]

$$\Psi[L = N] \;\propto \prod_i (z_i - Z_c)$$

$L \geq N(N-1)$ Laughlin state (+ quasiholes)

[Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

$$\Psi[L = N(N-1)] \propto \prod_{i<j} (z_i - z_j)^2$$

$N < L < N(N-1)$ Numerical exact diagonalisations

$\Rightarrow$ "Vortex Liquids" [Wilkin & Gunn, PRL **84**, 6 (2000)]

Closely related to FQH liquids

$$\psi_{\mathrm{B}}(\{\vec{r}_i\}) = \mathcal{P} \prod_{i<j} (z_i - z_j) \psi_{\mathrm{CF}}(\{\vec{r}_i\})$$

[NRC & Wilkin, PRB **60**, R16279 (1999); Regnault & Jolicoeur, PRL 2003]

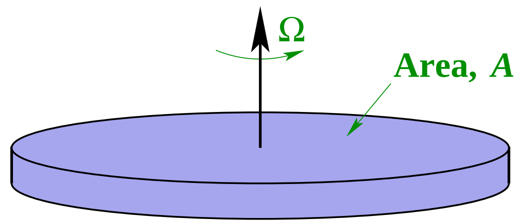
Vortex Lattice vs. Vortex Liquid

[NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

The Filling Fraction

FQHE: $\nu = n_e \frac{h}{eB}$

Here: $\nu = n_{2d} \, \frac{h}{q^* B^*}$



Number density of vortices: $n_V = \frac{2m\Omega}{h} = \frac{q^* B^*}{h}$

$$\boxed{\nu = \frac{n_{2d}}{n_V} = \frac{N}{N_V}}$$

What is the nature of the states for large ν?

Quantum uncertainty of vortex position $\sim a_{2d}$

[Haldane & Wu, PRL **55**, 2887 (1985)]

Lindemann criterion: quantum melting for $a_{2d} > \# a_V$

$$\nu = \left(\frac{a_V}{a_{2d}} \right)^2 < \nu_c$$

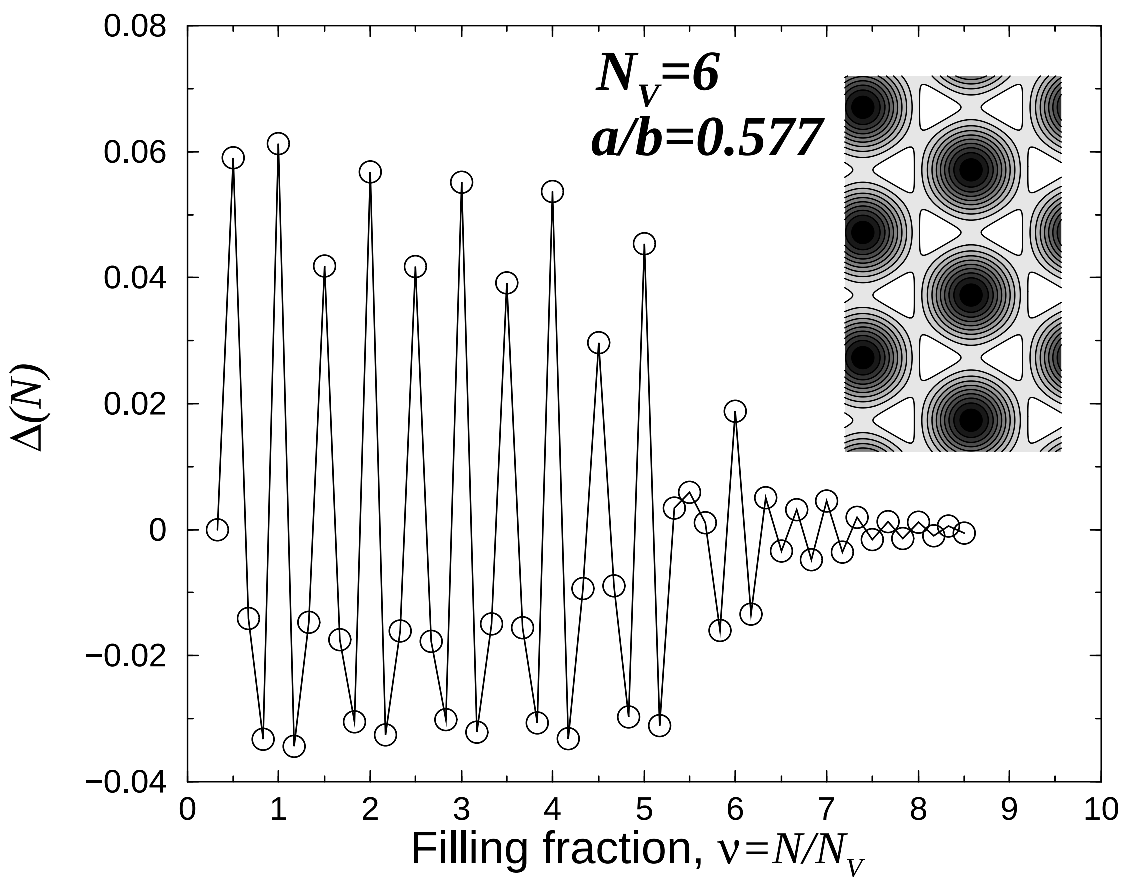
[See also Sinova, Hanna & MacDonald, PRL **89**, 1030403 (2002)]

Exact diagonalisations on a torus

[NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

Charged Excitation Gaps

$$\Delta(N) \equiv N\left[\frac{E(N+1)}{N+1}+\frac{E(N-1)}{N-1}-2\frac{E(N)}{N}\right]$$
$$\rightarrow [E(N+1)-E(N)]-[E(N)-E(N-1)]$$



Transition to vortex lattice at $\nu \sim 6$.

Incompressible liquids at $\nu = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 5$

Read-Rezayi "Parafermion" states

[Read & Rezayi, PRB **59**, 8084 (1999)]

Incompressible states, whose quasiparticle excitations obey non-abelian exchange statistics.

[Exchange of the particles induces unitary transformations within a subspace of degenerate states.]

$$V^{(k)} = \sum_{i_1 < i_2 \ldots < i_{k+1}} \delta(\vec{r}_{i_1} - \vec{r}_{i_2}) \delta(\vec{r}_{i_2} - \vec{r}_{i_3}) \ldots \delta(\vec{r}_{i_k} - \vec{r}_{i_{k+1}})$$

$$\Psi^{(k)}(\{z_i\}) = \mathcal{S} \left[\prod_{i<j \in A}^{N/k} (z_i - z_j)^2 \prod_{l<m \in B}^{N/k} (z_l - z_m)^2 \ldots \right]$$

[Cappelli *et al.*, Nucl. Phys. B **599**, 499 (2001)]

$$\nu^{(k)} = \frac{k}{2}$$

$k = 1$: Laughlin state ($\nu = 1/2$)
$k = 2$: Moore-Read ("Pfaffian") state ($\nu = 1$).

- The dominant sequence of incompressible states
- Large overlaps with the two-body groundstates (up to $\nu = 3$).

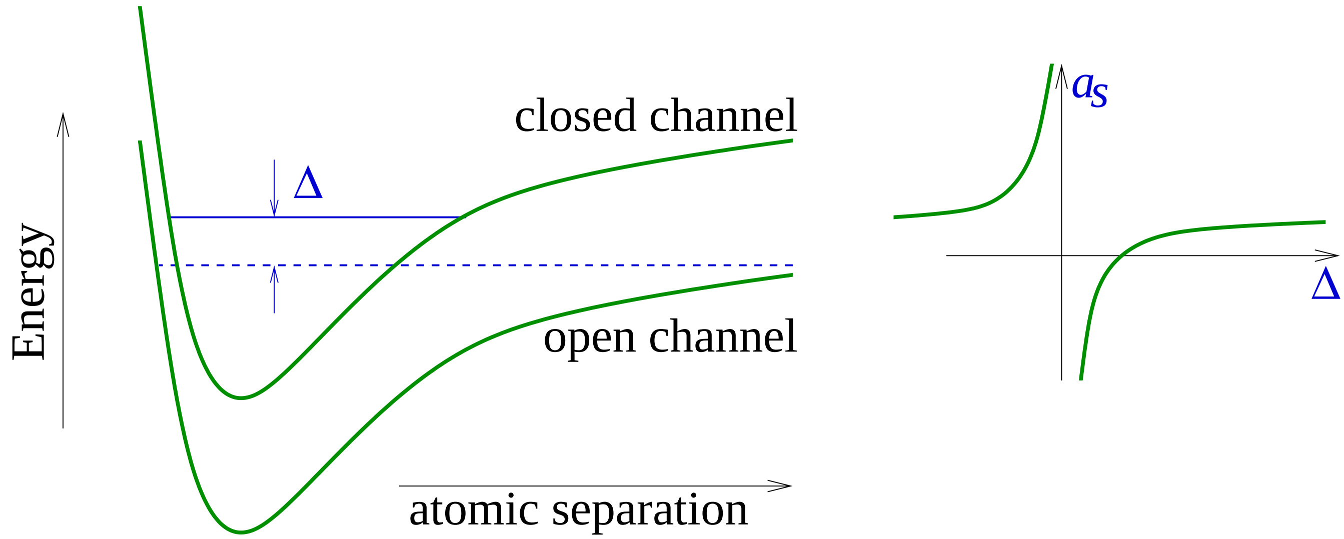
$N_V = 6; a/b = 1/\sqrt{3}$

k	ν	$(K_x, K_y)\times$degeneracy	$\lvert\langle\Psi^{(k)}\lvert\Psi\rangle\rvert$	$\lvert\langle\Psi^{GP}\lvert\Psi\rangle\rvert$
1	1/2 (Laughlin)	(0,0)$\times 2$	1.000	0.555
2	1 (Moore-Read)	(3,3)$\times 1$	0.982	N/W
2	1 (Moore-Read)	(3,0)$\times 1$	0.982	0.408
2	1 (Moore-Read)	(0,3)$\times 1$	0.981	0.493
3	3/2	(0,0)$\times 4$	0.967	0.234
4	2	(0,0)$\times 2$	0.956	0.242
4	2	(3,0)$\times 1$	0.966	N/W
4	2	(0,3)$\times 1$	0.935	N/W
4	2	(3,3)$\times 1$	0.844	0.547
5	5/2	(0,0)$\times 6$	0.955	0.163
6	3	(3,3)$\times 2$	0.960	N/W
6	3	(3,0)$\times 2$	0.944	0.198
6	3	(0,3)$\times 2$	0.744	0.534
6	3	(0,0)$\times 1$	0.852	N/W

$k = 2$ results consistent with earlier results of Haldane (unpublished).

$k = 3$ results have since been confirmed for larger systems by Read & Rezayi (unpublished).

Feshbach Resonance



Δ tuned by applied magnetic field.

$$\hat{H}_F \equiv \Delta \int \hat{m}_{\boldsymbol{r}}^\dagger \hat{m}_{\boldsymbol{r}} \, d^3\boldsymbol{r} + \frac{U_{aa}}{2} \int \hat{a}_{\boldsymbol{r}}^\dagger \hat{a}_{\boldsymbol{r}}^\dagger \hat{a}_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} \, d^3\boldsymbol{r} + \frac{g}{\sqrt{2}} \int \left[\hat{m}_{\boldsymbol{r}}^\dagger \hat{a}_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} + \hat{m}_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}}^\dagger \hat{a}_{\boldsymbol{r}}^\dagger \right] \, d^3\boldsymbol{r}$$

[Timmermans *et al.*, PRL **83**, 2691 (1999); Holland *et al.*, PRL **86**, 1915 (2001)]

What are the consequences for the correlated groundstates?

[Fischer, Fedichev & Recati, cond-mat/0212419; Bhongale, Milstein & Holland, cond-mat/0305399]

Exact Groundstates

[NRC, cond-mat/0308283]

$$\hat{H} = \hat{H}_K + \hat{H}_F + \hat{H}_I + (\hbar\omega_\perp + \hbar\omega_\parallel/2)\hat{N} + \hbar\omega_\perp \hat{L}$$

$$\begin{aligned}
\hat{H}_K &\equiv \int \hat{a}^\dagger_{\boldsymbol{r}} \left[\hat{h}_a - (\hbar\omega_\perp + \hbar\omega_\parallel/2)\right] \hat{a}_{\boldsymbol{r}} \\
&\quad + \hat{m}^\dagger_{\boldsymbol{r}} \left[\hat{h}_m - (\hbar\omega_\perp + \hbar\omega_\parallel/2)\right] \hat{m}_{\boldsymbol{r}}\, d^3\boldsymbol{r} \\
\hat{H}_F &\equiv \Delta \int \hat{m}^\dagger_{\boldsymbol{r}} \hat{m}_{\boldsymbol{r}}\, d^3\boldsymbol{r} + \frac{U_{aa}}{2} \int \hat{a}^\dagger_{\boldsymbol{r}} \hat{a}^\dagger_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}}\, d^3\boldsymbol{r} \\
&\quad + \frac{g}{\sqrt{2}} \int \left[\hat{m}^\dagger_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} + \hat{m}_{\boldsymbol{r}} \hat{a}^\dagger_{\boldsymbol{r}} \hat{a}^\dagger_{\boldsymbol{r}}\right]\, d^3\boldsymbol{r} \\
\hat{H}_I &\equiv U_{am} \int \hat{m}^\dagger_{\boldsymbol{r}} \hat{a}^\dagger_{\boldsymbol{r}} \hat{a}_{\boldsymbol{r}} \hat{m}_{\boldsymbol{r}}\, d^3\boldsymbol{r} + \frac{U_{mm}}{2} \int \hat{m}^\dagger_{\boldsymbol{r}} \hat{m}^\dagger_{\boldsymbol{r}} \hat{m}_{\boldsymbol{r}} \hat{m}_{\boldsymbol{r}}\, d^3\boldsymbol{r}
\end{aligned}$$

$\hat{H}_K$ and $\hat{H}_I$, are positive semi-definite (for $U_{\alpha\beta} \geq 0$).

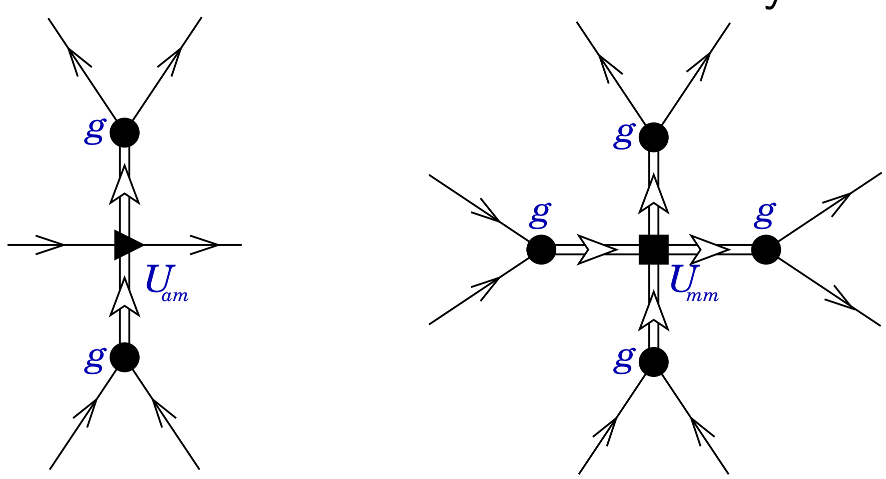
$\hat{H}_F$ is positive semi-definite provided $\frac{g^2}{\Delta} \leq U_{aa}$.

Results

$\frac{g^2}{\Delta} < U_{aa}$: repulsive two-body contact interaction

The Laughlin state for atoms $|\Psi_{\rm L}\rangle_a$ is the exact groundstate at $\nu = 1/2$.

$\frac{g^2}{\Delta} = U_{aa}$: the two-body contact interaction *vanishes*.
effective three- and four-body interactions



The *exact* groundstate at $\nu = 1$ is a strongly correlated atom-molecule mixture formed from the Moore-Read state.

$$|\Psi\rangle = \hat{R}|\Psi_{\rm MR}\rangle_a \qquad \hat{R} \equiv \exp\left(-\frac{g}{\sqrt{2}\Delta}\int \hat{a}_{\boldsymbol{r}}\hat{a}_{\boldsymbol{r}}\hat{m}^\dagger_{\boldsymbol{r}}\, d^3\boldsymbol{r}\right)$$

Experimental Status and Implications

$\mu/(2\hbar\omega_\perp) \lesssim 1 \Rightarrow$LLL
$\nu \gtrsim 500 \Rightarrow$vortex lattice

[Schweikhard *et al.*[JILA], PRL **92**, 040404 (2004)]

It will require further special efforts to access the regime of quantum-melted vortex liquids at $\nu \lesssim 10$.

What would one look for?

– density distribution; [J. Dalibard, K. Schoutens, *et al.*]
– collective excitations; [M. Cazalilla, PRA 2003]
– fractional statistics; [B. Paredes *et al.*, PRL 2001]
– vanishing condensate fraction; [J. Sinova *et al.*, PRL 2003]
– density correlation functions [N. Read & NRC, PRA 2003]

Summary

• In the limit of weak interactions, when the vortex cores overlap strongly, a system of rotating bosons is restricted to states in the lowest Landau level.

• A key parameter characterising a rotating atomic Bose gas at high angular momentum is the filling fraction $\nu \equiv N/N_V$.

• For repulsive contact interactions, numerical studies indicate that strongly-correlated groundstates form at $\nu \lesssim 6$: including the Laughlin state, and the "non-abelian" Moore-Read and Read-Rezayi states.

• For a model describing particles interacting via a nearby Feshbach resonance, the two-body contact interaction can be tuned to zero. The *exact groundstate* at $\nu = 1$ is then a strongly-correlated atom-molecule mixture formed from the Moore-Read state.