NonAbelions, Quantum Computation, and Quantum Hall Effects

work with

Yaroslav Tserkovnyak, Ilya Finkler (Harvard)

Nick Bonesteel, Kerwin Foster (Florida)

+ General Discussions with A. Stern, E. Rezayi, B. Halperin, N. Cooper, V. Gurarie, N. Read, L. Balents, R. dePicciotto, …

Non-Abelian Quantum Hall States

1. Non-Abelian Statistics for Beginners
2. What are the Candidate Quantum Hall States
3. Numerical "Experiments" (some results)
4. Musings about Quantum Computation

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Dictionary:

Non-Abelian Quantum Hall State = A *Quantum Hall* State Whose *Quasiparticle Excitations* are Non-Abelions

Non-Abelion = A Particle Obeying Non-Abelian Statistics

Non-Abelian Statistics = **?**

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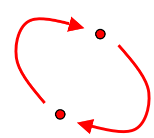
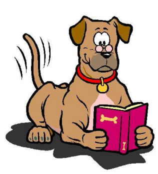
Statistics in Brief:

Statistics:

What happens to a many-particle wavefunction under "exchange" of identical particles.





Dogma:

Exchanging twice should be identity

- Bosons $\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2) = \Psi(\boldsymbol{r}_2,\boldsymbol{r}_1)$
- Fermions $\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\Psi(\boldsymbol{r}_2,\boldsymbol{r}_1)$

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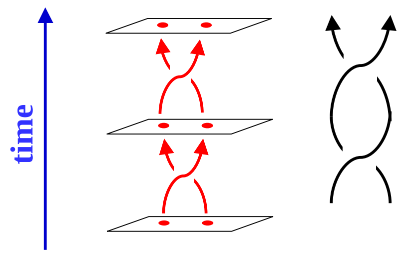
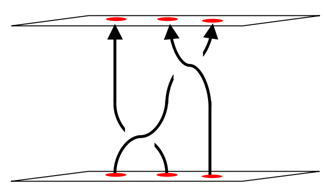
In 2+1 Dimensions: Two Exchanges ≠ Identity



In 3+1 Dimensions: Two Exchanges = Identity

No Knots in World Lines in 3+1 D !



Statistics:

In 3+1 D :

- No Knots in World Lines
- Topologically Different Paths = Different Permutations
- Statistics are Rep of the Permutation Group
- Bosons or Fermions

In 2+1 D :

- Knots in World Lines
- Topologically Different Paths = Different **Braids**
- Statistics are a Rep of the **Braid** Group
- More Possibilities (Anyons + Non-Abelions)

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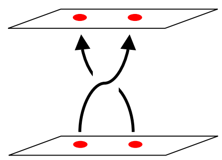
Example: Anyons = Fractional Statistics



W = Topological Winding Number of Braid

$$\Psi_f = e^{iW\alpha}\,\Psi_i$$

α = "Statistical Angle"

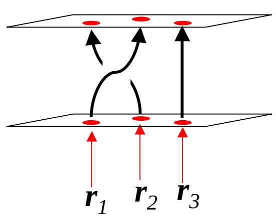
Bosons: $\alpha=0$
Fermions: $\alpha=\pi$
Anyons: other α

Quasiparticle Excitations of "Simple" Fractional Quantum Hall States Really Are Anyons!

Leinass+Myrheim, Wilczek
Laughlin, Halperin, Haldane,
Schrieffer+Arovas+Wilczek, …

No One Has Ever Measured This Cleanly!

What if there is a multiply degenerate ground state?



Suppose 2 Degenerate Orthogonal States $|\psi_1\rangle$, $|\psi_2\rangle$

Vector Represents State

$$\mathbf{\Psi_i} = a_1|\psi_1\rangle + a_2|\psi_2\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

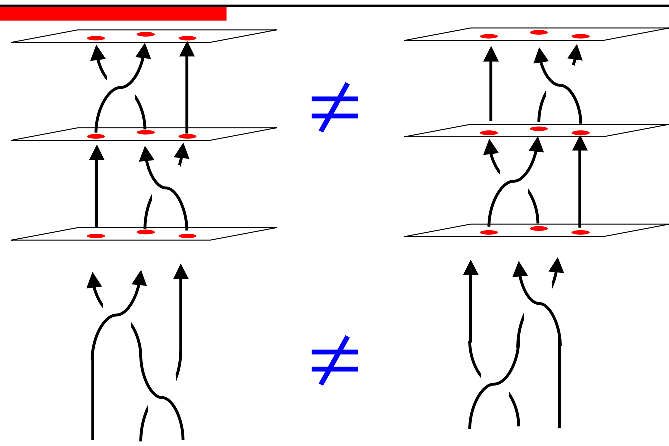
$$\mathbf{\Psi_f} = \tilde{a}_1|\psi_1\rangle + \tilde{a}_2|\psi_2\rangle = \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = U \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Unitary Matrix Represents (Topological) Braid Operation

Matrix Representation Of The Braid Group May Be Non-Abelian!
J. Froelich
Moore+Read

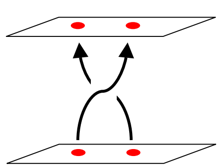
Braid Group is "Non-Abelian" = Non-Commutative



Statistics Are Matrix Representation of Braid Group:
+ Matrices are Non-Abelian → ***NonAbelian Statistics***

(Froelich, Moore+Read)

Example: Anyons = Fractional Statistics



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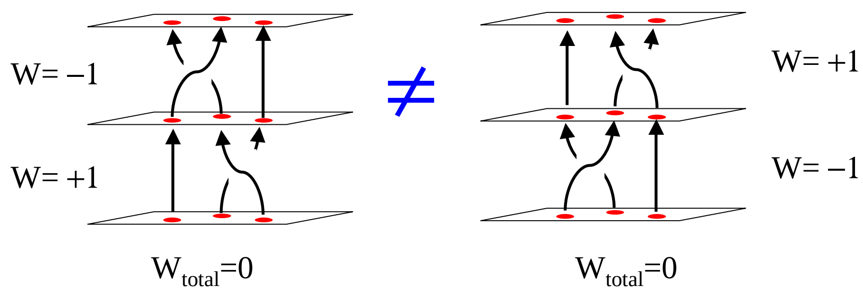
Braid Group is Non-Abelian (Non-Commutative)



•Winding Number is an Abelian Representation of Braid Group
•Fractional Statistics are Abelian

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Non-Abelian Statistics :

•Vector Represents State Within a Degenerate Space

$$|\psi_1\rangle \, , \, |\psi_2\rangle \qquad\qquad \mathbf{\Psi_i} = a_1|\psi_1\rangle + a_2|\psi_2\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

• Unitary (Berry's) Matrix Represents an Adiabatic Braiding Operation

$$\mathbf{\Psi_f} = U\,\mathbf{\Psi_i}$$

• Usually:
    Degenerate Space (Size of Vector/Matrix) is Exponentially Large in Number of Particles

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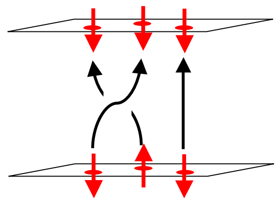
Cartoon Picture of Non-Abelian States:



Possibilities for
Quantum Information/Computation !
Kitaev, Freedman, et al

(More on computation later!)

2^N Size Space
Rotation in spin space *only* from braiding

Topologically Robust !

Complications:
One degree of Freedom per m-particles.
Hard to Identify what Degrees of Freedom are.

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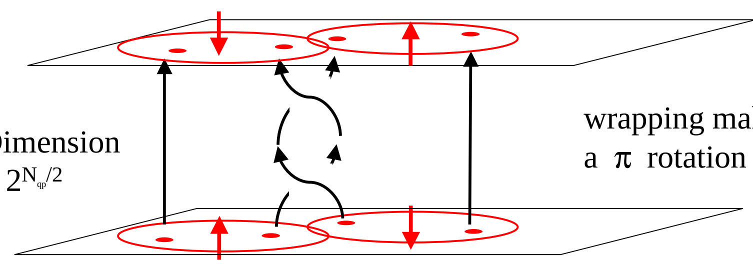
The Best Understood Case:
The Moore-Read Pfaffian / Chiral p-wave 2D superconductor

1 Majorana (1/2 a Fermion) on Each Quasiparticle / Vortex

Dimension
$= 2^{N_{qp}/2}$



wrapping makes
a π rotation

One degree of Freedom per m-particles.
Hard to Identify what Degrees of Freedom are.

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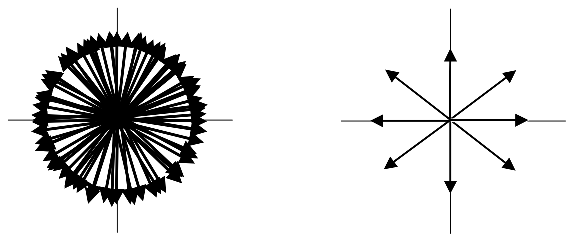
Can it Compute?

Universal Q-Computation ⇔ Approximate any Unitary Transform

The Moore-Read Pfaffian Cannot Do This!

Braid Group Representation is not "Dense"



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Simplest State that Quantum Computes :

k=3 Parfermionic Read-Rezayi State

(Big Brother of Moore-Read Pfaffian)

Moore-Read Pfaffian (AKA: k=2 parafermion state)	Read-Rezayi k=3 Parafermion
• Exact Ground State of a Short Range 3-body interaction	• Exact Ground State of a Short Range 4-body interaction
• Involves Pairing of Electrons	• Involves 3-Electron clusters
• Majorana on each quasiparticle:	• Z_3 parafermion on each qp?
QP-hilbert space dimension $= 2^{N_{qp}/2}$	QP-hilbert space dimension $= \mathrm{Fib}(N_{qp}-2)$
	Horrible "Non-locality"!

How were Non-Abelions "Discovered" ?

Conformal Field Theory Approach - Moore and Read

CAUTION

I am not going to explain this approach in detail

How were Non-Abelions "Discovered" ?

Conformal Field Theory Approach - Moore and Read

•Wavefunctions can be written as correlators of a CFT, ex:

Laughlin Wavefunction: $\Psi_{Laughlin} = \prod_{i<j} (z_i - z_j)^m$

Define a CFT: $\langle \phi(w)\phi(z) \rangle \sim -\log(z-w)$

$$\Psi_{Laughlin} = \prod_{i<j} e^{-m\langle \phi(z_i)\phi(z_j) \rangle} = \left\langle \prod_i e^{i\sqrt{m}\,\phi(z_i)} \right\rangle$$

• More Complex CFT's generate more complex wavefunctions

Can demonstrate **Degeneracy** of Multi-Quasiparticle States
by counting conformal blocks

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Define a CFT: $\langle \phi(w)\phi(z) \rangle \sim -\log(z-w)$

CFT can *Almost* Calculate Braiding Behavior

$= \left\langle \prod_i e^{i\sqrt{m}\,\phi(z_i)} \right\rangle$

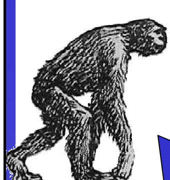
• More Complex CFT's generate more complex wavefunctions

Can demonstrate **Degeneracy** of Multi-Quasiparticle States by counting conformal blocks

The Missing Link

Quantum Hall Wavefunctions

Correlator of a Conformal Field Theory → Conformal Blocks Count Degeneracy



Conjecture: Quasihole Statistics are Given By Braiding Properties of Conformal Field Theory

"Braiding Properties" of a CFT
Nonabelian Chern-Simons Field Theories
Knot Theory /Jones Polynomial /Jones Rep

} Witten 1989

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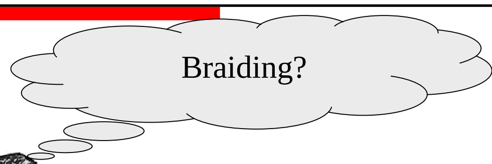
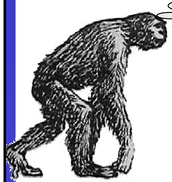
The Missing Link





Conjecture: Quasihole Statistics are Given By Braiding Properties of Conformal Field Theory

Research Direction #1:

Want To Show Numerical Evidence of this Conjecture

Find the Missing Link!

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Assuming the missing link:

k=3 Parfermionic Read-Rezayi State

Related to $SU(2)_k$ Chern-Simons Theory

Theorem by Freedman et al:

$SU(2)_k$ rep of the braid group can quantum compute

Except k=1,2,4

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Non-Abelian Quantum Hall State Candidates:

- $\nu = 5/2;\ 7/2$
  Probably Moore-Read Pfaffian State
  = BCS chiral p-wave paired composite fermions
- $\nu = 12/5;\ 13/5$
  Maybe Read-Rezayi Parafermionic State ?

These States are thought to have Non-Abelian Quasiparticle Excitations!

VOLUME 83, NUMBER 17 PHYSICAL REVIEW LETTERS 25 OCTOBER 1999

Exact Quantization of the Even-Denominator Fractional Quantum Hall State at $\nu = 5/2$ Landau Level Filling Factor

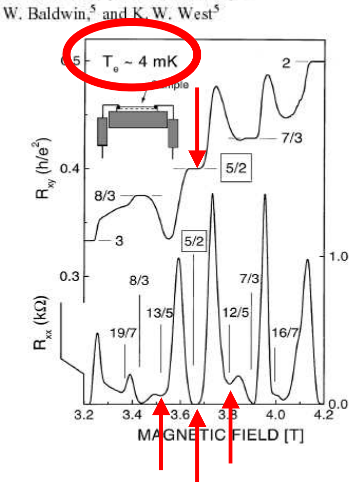
W. Pan,[1,2] J.-S. Xia,[2,3] V. Shvarts,[2,3] D. E. Adams,[2,3] H. L. Stormer,[4,5] D. C. Tsui,[1] L. N. Pfeiffer,[5] K. W. Baldwin,[5] and K. W. West[5]

Mobility = 17 million cm²/V sec

ν = 5/2 is a FQHE state

ν = 13/5 and 12/5

Look Like they might be forming



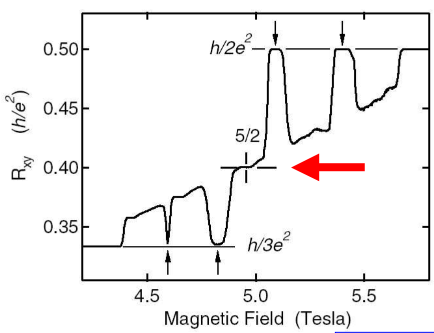
VOLUME 88, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 2002

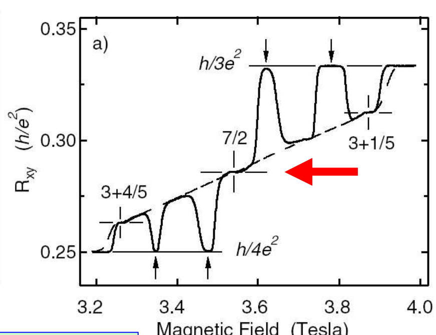
Insulating and Fractional Quantum Hall States in the First Excited Landau Level

J. P. Eisenstein,[1] K. B. Cooper,[1] L. N. Pfeiffer,[2] and K. W. West[2]

Mobility = 31 million cm²/Vsec.

T = 15 mK.





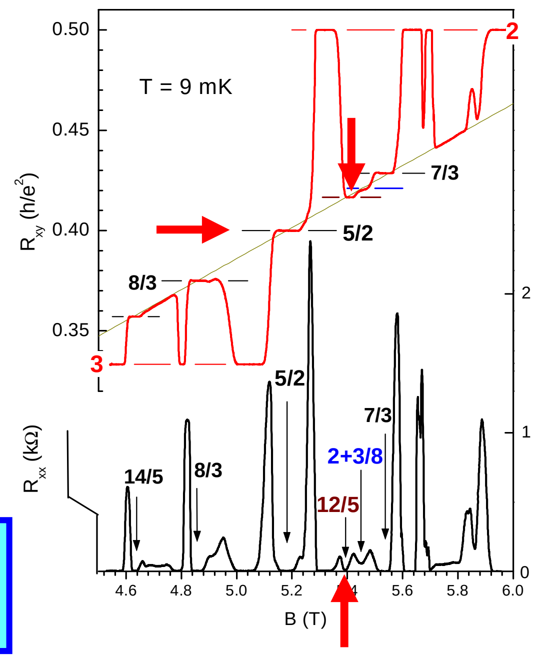
Yes: Pfaffian
No: Parafermion

Pan, Tsui, Stormer, Pfeiffer, and West, (unpublished).

Mobility

31 million cm^2/ Vsec

T = 9 mK



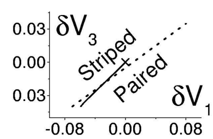
NEEDS TO BE VERY COLD AND VERY CLEAN

Numerical Evidence from Small System Diagonalization

5/2, 7/2 (Rezayi and Haldane, Morf)

•Very Close to a transition from Moore-Read (Pfaffian) State to Stripe State

•Observation of quantization is very good evidence for Moore-Read (no other good candidate)



12/5, 13/5 (Read and Rezayi)

• Parafermion State seems favored compared to Hierarchy FQHE state

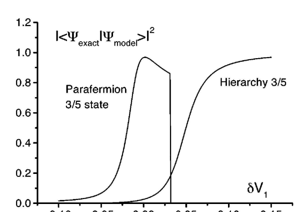


FIG. 3. Overlap squared of the two reference states, the $\nu = 3/5$ parafermion state ($N = 15$) and the $\nu = 3/5$ hierarchy state ($N = 12$), with the state obtained by diagonalizing the $\mathcal{N} = 1$ Coulomb potential with an added δV_1 component.

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  Probably Moore-Read Pfaffian State
  = BCS chiral p-wave paired composite fermions
- ν = 12/5; 13/5
  Maybe Read-Rezayi Parafermionic State ?

These States are thought to have Non-Abelian Quasiparticle Excitations!

Also discussion of ν = 3/8, 4/11, …

Also Good Candidates in Rotating "2D" Bose Condensates
(Cooper, Wilkin, Gunn)

Rotating Bose Condensates : (Overhead Stolen From Nigel Cooper)

Formulation of the Problem

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{1}{2} m \omega_0^2 |\vec{r}_i|^2 \right] + \eta \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j)$$

Bosons in Trap (ω_0) w/ Short Range Interactions

Rotating Frame: $H_\Omega = H - \vec{\Omega} \cdot \vec{L}$ $\quad [\eta = \frac{4\pi\hbar^2 a_s}{m}]$

One-Body Terms

$$H_\Omega^{(1)} = \frac{|\vec{p}|^2}{2m} + \frac{1}{2} m \omega_0^2 |\vec{r}|^2 - \vec{\Omega} \cdot \vec{r} \times \vec{p}$$

$$= \frac{|\vec{p} - m\vec{\Omega} \times \vec{r}|^2}{2m} + \frac{1}{2} m (\omega_\perp^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2} m \omega_\parallel^2 z^2$$

$$q^* \vec{B}^* = 2m\vec{\Omega}$$

In rotating frame:
(1) Rotation takes the place of magnetic field
(2) Confinement out of plane is much stronger than in plane (ie, 2D)

+ Weak Interactions [Wilkin, Gunn & Smith (1998)]

$\eta \bar{n} \ll \hbar\omega_\perp, \hbar\omega_\parallel \Rightarrow$ lowest Landau level & 2D

Maps to Bose-Quantum Hall System

Theoretical Status for Rotating Bosons

1. If you tune to a Feshbach Resonance, you get (exactly) the Bose analogue of the Moore-Read Pfaffian *(Cooper)*

2. From Exact Diagonalizations, we *believe* an entire set of Read-Rezayi Non-Abelian Parafermionic states occur at filling fractions *(Cooper, Wilkin, Gunn)*

$$\nu = k/2 \quad \text{for } k = 2,3,\ldots 11,12$$

(Z_k parafermionic theory)

3. ***BUT*** it is hard to extract convincing information from exact diag for Bose systems (Hilbert space is larger than for fermions).

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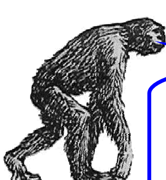

What Happens When You Braid Quasiparticles Around Each Other?

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SKIP THE NUMERICAL WORK

(insert numerical story at end, if there is time)

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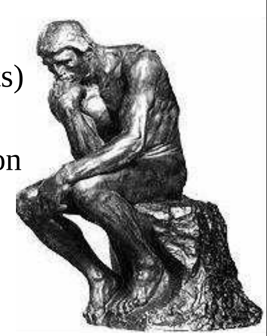
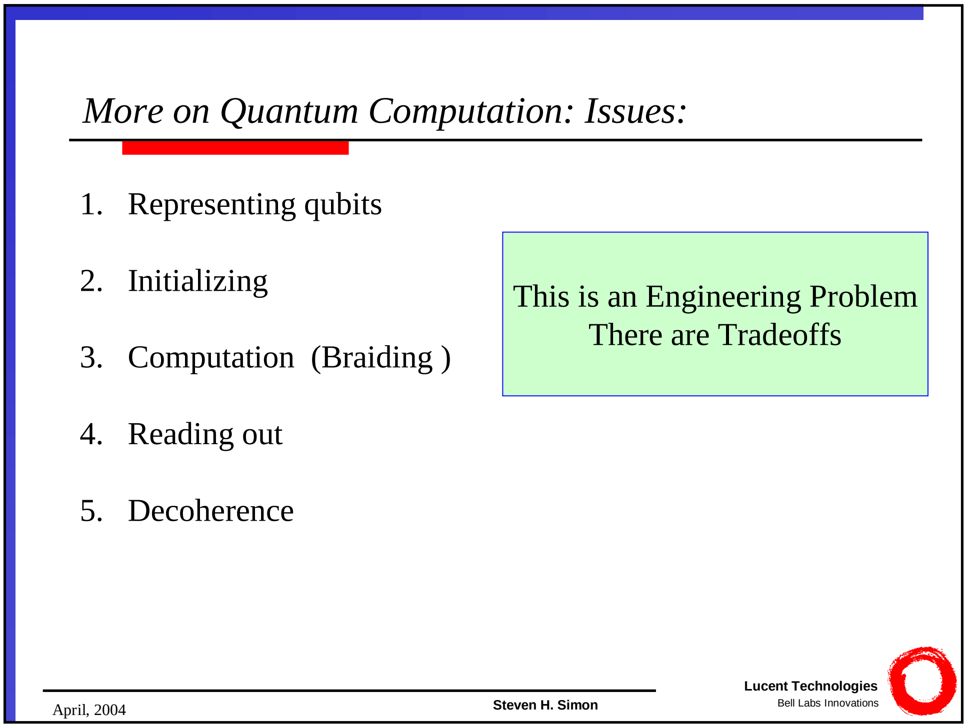
↑ Topological




More on Quantum Computation: Issues:
1. Representing qubits
2. Initializing
3. Computation (Braiding)
4. Reading out
5. Decoherence
This is an Engineering Problem
There are Tradeoffs
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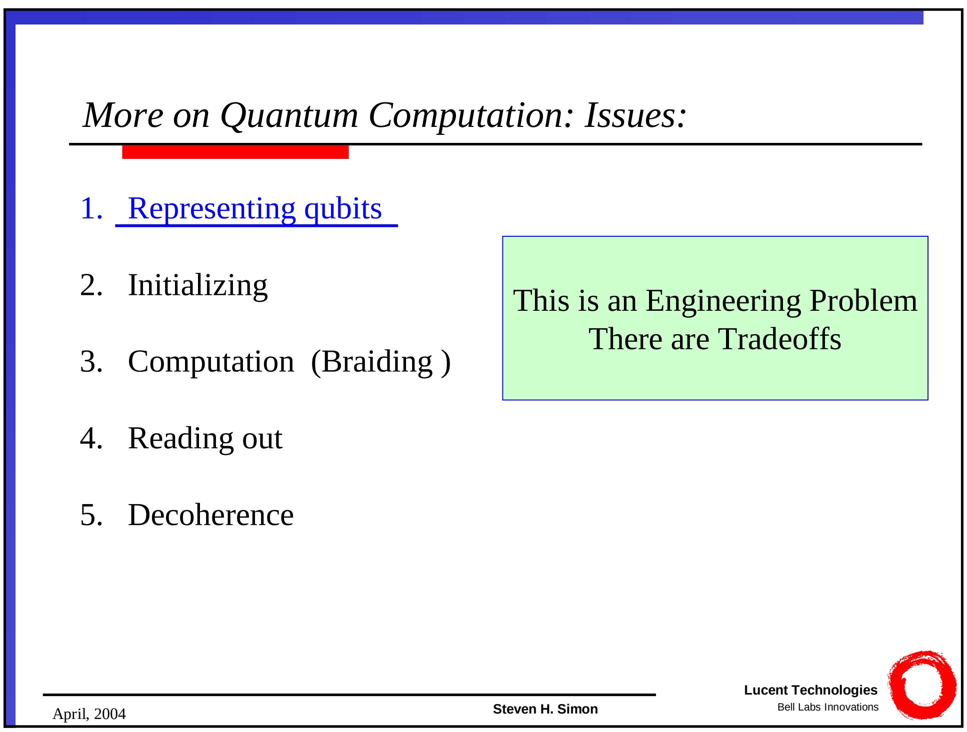
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Representing a qubit

For k=3 Read-Rezayi Parafermionic State

Hilbert space grows as

$$\mathrm{Fib}(N_{qp} - 2) \sim (2 + \mathrm{Sqrt}(5))^{N_{qp}/3} \sim (1.6)^{N_{qp}} < 2^{N_{qp}}$$

Need at least 2qps to make a qubit (But, might represent 2 qubits w/ 3 qps)

- 2 qps can make a qubit, but:
  - Cannot move it around easily (Only nearest neighbor operations)
  - Need to “borrow” other qps to do operations on qubit (B_2 is trivial)

- With 3,4 qps reps of qubit :
  - can move qubits around freely(4 qps can “looks like the qp-vacuum” to outsiders)
  - can do single qubit operations easily. (B_3 and B_4 are sufficiently nontrivial)

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Initialization:

Locality Principle: Local operations are undetectable far away unless already entangled with far away quasiparticles.

- Create States that have same quantum numbers as the qp-vacuum:
  1. Adiabatic flux insertion (Laughlin creation of quasiholes) creates k (=3) quasiparticles
  2. Pulling qh-qp pair from the qp-vacuum

- If you are very good at measuring your state, you can generate random qubits locally until you find one in the initial state you like.

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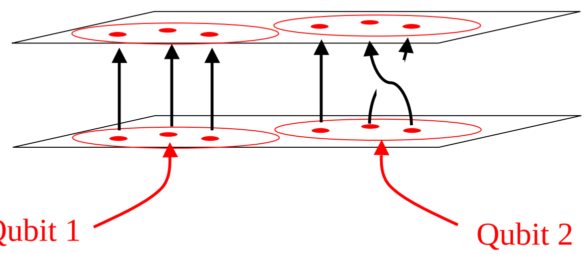
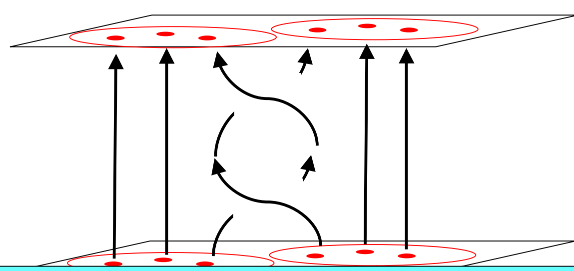
Computation by Braiding (Ex: k=3 parafermions)

Operation on qubit 2 only



Computation by Braiding (Ex: k=3 parafermions)

A two qubit gate:



Note: It may take ***Many Braid Operations*** to approximate even something simple like a single qubit rotation or a CNOT gate.

Kitaev-Solovay Approximation Theorem assures us it only gets log harder to approximate answer more closely (but prefactor can be big)

Computation by Braiding

There is no escaping the need to move around quasiparticles in complicated ways.

(1) Rotating Bose : perhaps easier?
(2) Quantum Hall : the edge of what is conceivable

How much more insane than other Q-computation proposals?

Note: You can work with Blobs of Quasiparticles, so long as you can move them without dropping any

"Blobological" quantum computation

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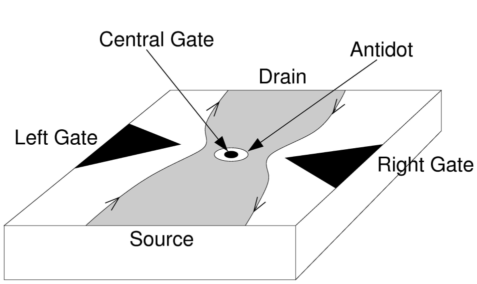
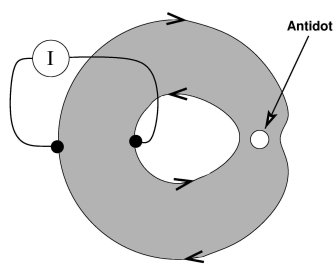
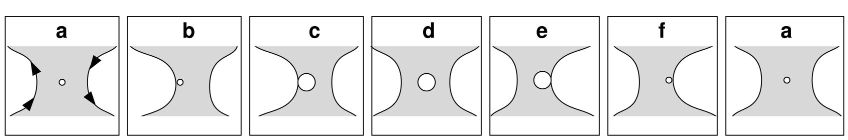
Preliminary Experiment to Manipulate Quasiparticles *(Simon '99)*

Quantum Hall Quasiparticle Pump







Experiment Currently Being Pursued by Marcus et al

More on Quantum Computation: Issues:

1. Representing qubits
2. Initializing
3. Computation (Braiding)
4. Reading out
5. Decoherence

This is an Engineering Problem
There are Tradeoffs

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Measurements on NonAbelian States 1

When 2 qp's are moved close together, the degenerate states will split. Details of splitting depend on state of the qubit

Example: Pfaffian w/ Majorana Quasiparticles

Energy splits: ex:

Spin up increases in energy
Spin down "decreases"

In Principle, could Measure force between Quasiparticles.

In practice this is almost inconceivably hard.

Measurements on NonAbelian States 2

Mutual Annihilation: Only groups of particles with the same quantum numbers as the qp-vacuum can annihilate.

Inverse of state initialization problem:
- try to annihilate qp-qh pairs
- try inverse Laughlin flux insertion

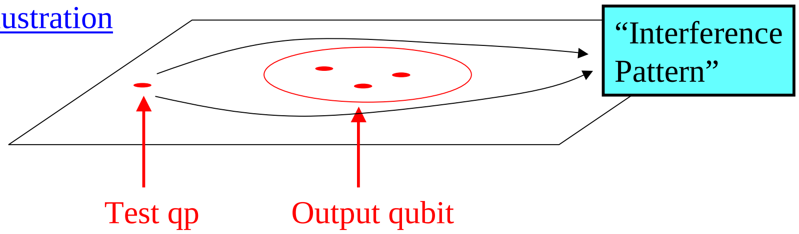
"Failed Attempts" to Annihilate qps leave neutral bound states (charge density distribution and excitation spectrum different from the qp-vacuum).

In practice this is very very hard.

Measurements on NonAbelian States 3

Interference Experiments (Mach-Zehnder)

Illustration



Horrible Complication:
- Want to send more than 1 test qp
- Test qp braiding around output qubit can change the state of the system
- Next test qp gives a different result!

unless qubit has quantum numbers of the qp-vacuum

Measurements on NonAbelian States 3

Interference Experiments (Mach-Zehnder)

Propose:

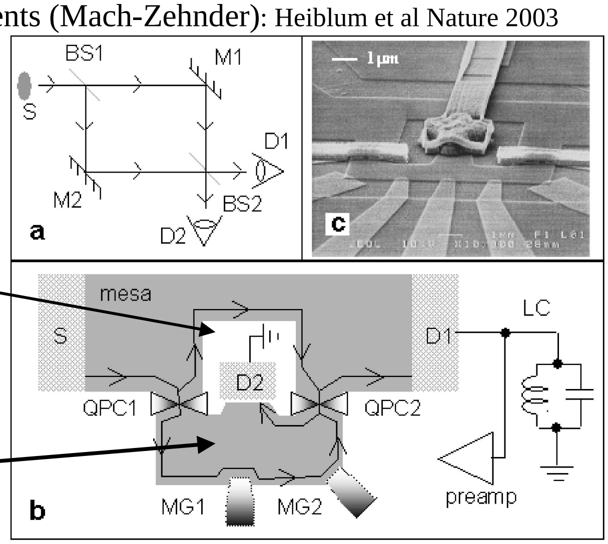
- One of the output states is the qubit in the vacuum state
  (does not change after the test particle goes around it)
  Should show clear and robust intereference pattern
- Other output state of qubit?

unless qubit has quantum numbers of the qp-vacuum

Measurements on NonAbelian States 3

Interference Experiments (Mach-Zehnder): Heiblum et al Nature 2003

Integer QHE interferometer: FQHE has been more difficult



Can put arbitrary qps inside this region

Cannot put arbitrary qps inside this partially isolated region

More on Quantum Computation: Issues:

1. Representing qubits
2. Initializing
3. Computation (Braiding)
4. Reading out
5. Decoherence etc

This is an Engineering Problem
There are Tradeoffs

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Decoherence 1

Process #1 Stray Braids:

• An additional braid occurs that you didnt' intend
Can make this arbitrarily small (keep qps far apart)

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Decoherence 2

Process #2 Local Perturbations (Phonons, Photons, etc)
Do not "Directly" Decohere !!!
.. .but can Make unwanted qp-qh pairs:

- So long as qp-qh pair re-annihilates *without wrapping around any other quasiparticles* this does not decohere
- Free wandering qps are deadly (also kill QHE)

Need to keep system much much colder than the qp-qh gap!
Moore-Read Gap ~ 100 mK
Parafermion 12/5 state ~ << 100mK
Maybe not so hard?

Decoherence 3

Problem #3: Slight Nondegeneracy of Hilbert Space:

- For any finite distance between qp's there is some tunneling (hence splitting) between the states of the degenerate space

[Ex: Hopping of majoranas for the Moore-Read state]
How small is tunneling as a function of the the distance?

- Nonabelian Statistics becomes an approximation valid only over time scales short compared to this tunneling.

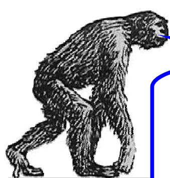
May Need to do computation in a limited time !

Need to keep qp's far apart! ***Even stray trapped qps***

Rotating Bose?

Non-Abelian Quantum Hall States

1. Non-Abelian Statistics for Beginners
2. What are the Candidate States
3. Numerical Experiments (some results)
4. Musings about Quantum Computation



(THE MISSING LINK)

What Happens When You
Braid Quasiparticles
Around Each Other?

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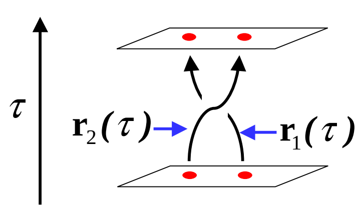
How To Demonstrate Braiding Behavior (Statistics) Theoretically?

Example: Fractional Statistics for Simple Fractional Hall States



W = Winding Number of Braid

$$\Psi_f = e^{iW\alpha}\,\Psi_i$$

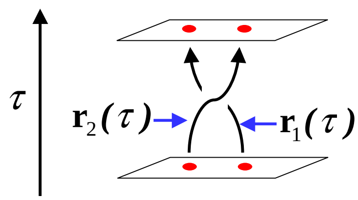
α = "Statistical Angle"

Adiabatically drag one quasiparticle around the other,
and calculate the accumulated (Berry's) phase

$$Phase = \oint d\tau \;\left\langle \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

Where does Berry's Phase Come From? *(correction to Born-Opp)*

Given a Family of Hamiltonians H_τ with (zero energy) Eigenstates Ψ_τ



$$\Phi_\tau = e^{i\theta}\Psi_\tau$$

Solve Time Dependent Shroedinger Eq.

$$i\partial_t \Phi = H\Phi = 0$$

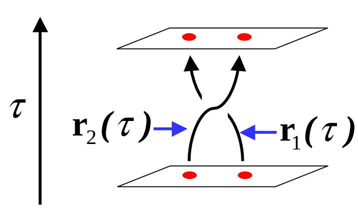
$$(i\partial_t \theta)\,\Psi + i\partial_t \Psi = 0$$

$$\partial_t \theta = \langle \Psi | i\partial_t | \Psi \rangle$$

$$Phase = \oint d\tau \;\; \left\langle \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \left| \frac{i\partial}{\partial \tau} \right| \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

How To Demonstrate Braiding Behavior (Statistics) Theoretically?

Example: Fractional Statistics for Abelian Fractional Hall States



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How To Demonstrate Braiding Behavior (Statistics) Theoretically?

Example: Fractional Statistics for Abelian Fractional Hall States

For Quantum Hall Effect You Know Ψ (Ex, Laughlin)
So You Can Calculate The Berry's Phase
and Hence the Statistics. (Arovas, Schrieffer, Wilczek, 1984)

Even if you know the Ψ you still need to
Integrate over all of the many electron positions to find $\langle\ \rangle$

It is a "scandal" that we have not done something
like this for the Pfaffian – Wilczek 2002

$$Phase = \oint d\tau \ \left\langle \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

•Wavefunction is a Scalar (Abelian Statistics):

$$\Psi_f = U\,\Psi_i \qquad U = \exp\left(-i\oint d\tau\, A(\tau)\right)$$

$$A(\tau) = \left\langle \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

"gauge field"

$$Phase = \oint d\tau \ \left\langle \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \Psi[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

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• Wavefunction is a Vector in a Degenerate subspace (Non-Abelian):

Berry's Matrix

$$\boldsymbol{\Psi}_{\mathbf{i}} = a_1|\psi_1\rangle + a_2|\psi_2\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Path Ordered

$$\boldsymbol{\Psi}_{\mathbf{f}} = U\,\boldsymbol{\Psi}_{\mathbf{i}} \qquad U = P\,\boldsymbol{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

NonAbelian Gauge Field

Wilczek and Zee, 1984

Want To Calculate This

$$\boldsymbol{\Psi}_{\mathbf{f}} = U\,\boldsymbol{\Psi}_{\mathbf{i}} \qquad U = P\,\boldsymbol{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),...] \right\rangle$$

For the Moore-Read Pfaffian:

(a) Have a good guess of the braiding result — Moore-Read; Nayak Wilczek - CFT; Ivanov; Von-Oppen+Stern - BCS

(b) Have simple expressions for trial wavefunction $\psi_\alpha = \sqrt{det[g(z_i - z_j)]}$

ψ_α can be evaluated efficiently numerically : Time ~ $(N_e)^3$

Want To Calculate This

$$\Psi_f = U \Psi_i \qquad U = P\,\mathbf{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau), \mathbf{r}_2(\tau), \ldots] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau), \mathbf{r}_2(\tau), \ldots] \right\rangle$$

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Quantum Monte Carlo !

$$\Psi_f = U \Psi_i \qquad U = P\,\mathbf{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau), \mathbf{r}_2(\tau), \ldots] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau), \mathbf{r}_2(\tau), \ldots] \right\rangle$$

For the Parafermion States:

(a) Know Less About Braiding Behavior (Slingerland+Bais)

(b) Have expressions for trial wavefunctions: $\psi_\alpha = A\,[\,F(z_1,\ldots z_N)\,]$ (antisym) 

ψ_α ***CANNOT*** be evaluated efficiently numerically : Time ~ N_e **!**

Want To Calculate This

$$\Psi_f = U\,\Psi_i \qquad U = P\,\mathbf{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),\ldots] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),\ldots] \right\rangle$$

For the Parafermion States: *(w/ Finkler)*

(a) Know Less About Braiding Behavior

(b) Have expressions for trial wavefunctions

ψ_α ***CANNOT*** be evaluated efficiently numerically : Time ~ N_e **!**

Trick to calculate gauge field $A_{\alpha\beta}(\tau)$ while avoiding ψ_α

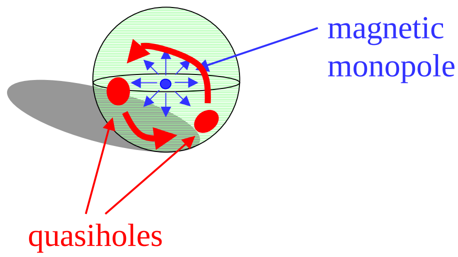
$$\Psi_f = U\,\Psi_i \qquad U = P\,\mathbf{exp}\left(-i\oint d\tau\, A(\tau)\right)$$

$$A_{\alpha\beta}(\tau) = \left\langle \psi_\alpha[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),\ldots] \,\middle|\, \frac{i\partial}{\partial\tau} \,\middle|\, \psi_\beta[\mathbf{r}_1(\tau),\mathbf{r}_2(\tau),\ldots] \right\rangle$$

Our Numerical Experiments *(w/ Tserkovnyak)*:

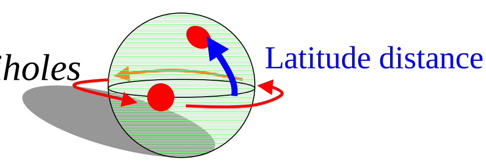
(a) Consider Pfaffian State (and simple Laughlin States) :
up to 64 electrons, up to 6 quasi-holes

(b) Calculations done on a sphere : Bigger $N_e \Rightarrow$ Bigger Sphere

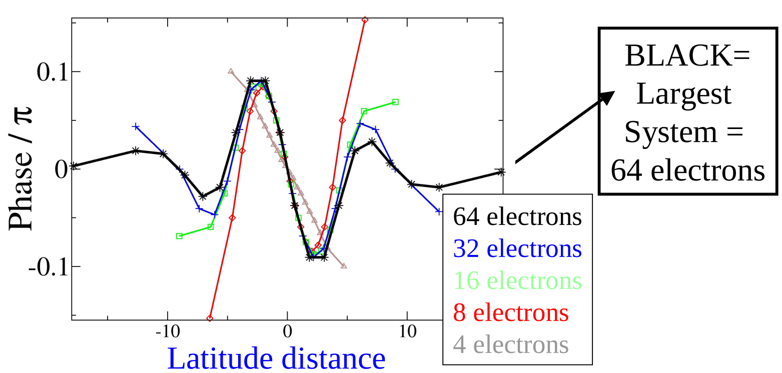


(c) Use quantum monte carlo to find out what happens to Ψ

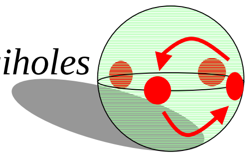
Example: Pfaffian w/2 Quasiholes



(1) Laughlin ν=1/3 State : Phase = ± π / 3 (Confirms old prediction)
(2) Pfaffian :
Non-Degenerate Ground State for 2 quasiholes (Abelian Statistics)
CFT Calculation : Phase = 0 (Moore+Read, Nayak+Gurarie).



Example: Pfaffian w/ 4 Quasiholes



4 such exchanges U_1, U_2, U_3, U_4

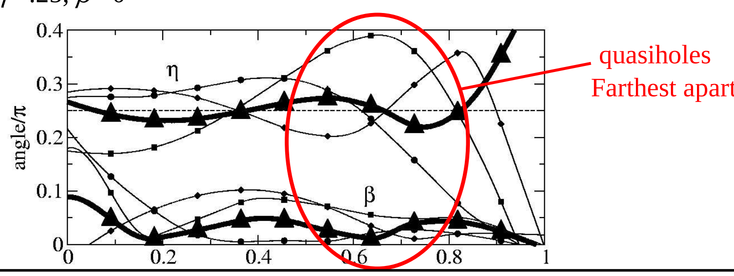
- 2 Dimensional Degenerate Subspace (pick an easy basis)
- CFT Prediction

$$U_1 = U_3 = \tfrac{1}{2}\left[(1+i) + (1-i)\sigma_z\right]$$
$$U_2 = U_4 = \tfrac{1}{2}\left[(1-i) + (-1-i)\sigma_x\right]$$

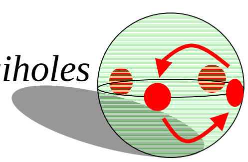
- Parameterize

$$U_1 = e^{i\chi}\begin{pmatrix} e^{i\eta}\cos\beta/2 & ie^{-i\varepsilon/2}\sin\beta/2 \\ ie^{i\varepsilon}\sin\beta/2 & e^{-i\eta}\cos\beta/2 \end{pmatrix}$$

- Predict $\eta = .25$, $\beta = 0$



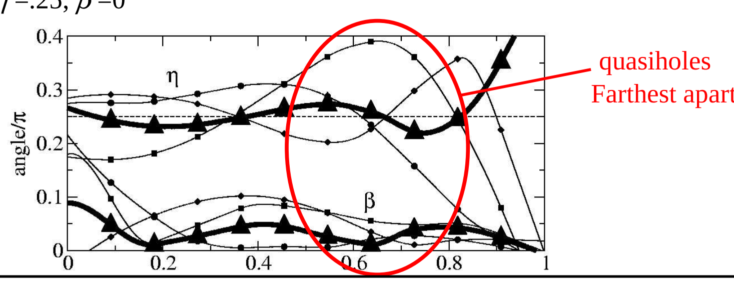
Example: Pfaffian w/ 4 Quasiholes



4 such exchanges U_1, U_2, U_3, U_4

- 2 Dimensional Degenerate Subspace (pick an easy basis)
- CFT Prediction
- Parameterize
- Predict $\eta = .25$, $\beta = 0$

In Agreement With Prior CFT at least for the Pfaffian. Parafermions Coming Soon!



Non-Abelian Quantum Hall States

1. Non-Abelian Statistics for Beginners
2. What are the Candidate States
3. Numerical Experiments (a few results)
4. Musings on Quantum Computation
5. Chern-Simons Theory and
   Possible Real Experiments (a few more results)

How Do We Know 5/2 is a Pfaffian?

Need Predictions for Experimental Signatures
(w/ Foster, Bonesteel)

How To Calculate Properties of The Moore-Read Pfaffian?

Moore-Read State is a Chiral p-wave Superconductor
Of Composite Fermions

Grieter, Wen, Wilczek
Green+Read
Ivanov
VonOppen+Stern
…

Another Way of Understanding NonAbelian Properties of Moore-Read

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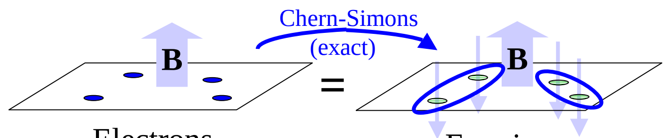
The Moore-Read Pfaffian as a Superconductor of Composite Fermions:



Electrons

Fermions
Bound to Flux

- ν=1/2 @ Mean field level = Fermions in zero magnetic field
- At high T forms a "Composite Fermion" Fermi Liquid (HLR,Jain) (Experiment by Willett et al)
- At low T becomes a (chiral p-wave) superconductor = Pfaffian

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Is the 5/2 State a "Superconductor" ?

How do you tell any state is a superconductor ?

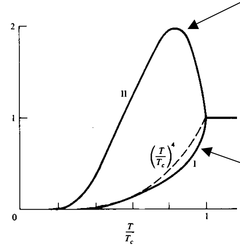
No Resistance = Quantized Hall State

Meissner Effect = Electromagnetic Response Function

$\Rightarrow$ Surface Acoustic Wave, Microwave, Raman Response

Coherence Effects?

NMR Relaxation (Hebel Slichter)
Electromagnetic Absorption



Acoustic Attenuation

Calculations :

Based on Superconducting Composite Fermion Picture Calculate Electromagnetic Response Functions to Make Predictions for ….

⇒ Surface Acoustic Wave, Microwave, Raman Response

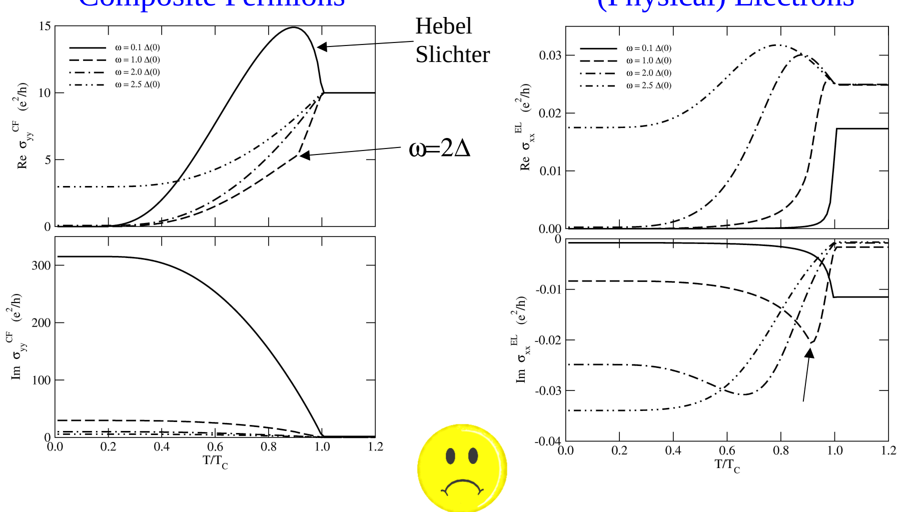
- Step 1:
  Calculate Response Function of Composite Fermions
  "Standard" Superconductivity Calculation
- Step 2:
  Convert Composite Fermion Response To Electron Response (RPA)

$$\sigma_{xx}^{el}(q_x,\omega) \sim \frac{1}{\sigma_{yy}^{CF}(q_x,\omega)}$$

In High B, Longitudinal and Transverse Get Mixed and Inverted

Results $\sigma_{xx}(q=.1k_F\,;\,\Delta=.01\,E_F)$ $\quad\sigma_{xx}^{el} \sim 1/\sigma_{yy}^{CF}$

Composite Fermions (Physical) Electrons



Summary:

- Non-Abelian States are very exotic
- Very Powerful for Quantum Computation?
- Quantum Monte Carlo *(Numerical Expts)* :
  Confirms Conjectures from CFT for Pfaffian
  Parafermions Coming Soon (Hopefully) !
- Mapping to Superconducting Composite Fermions :
  Analysis of Response

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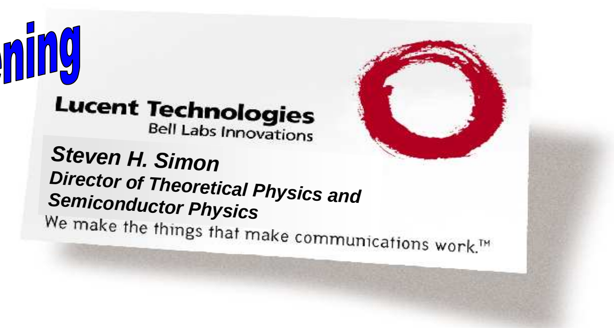
Thanks for Listening



NonAbelions, Quantum Computation, and Quantum Hall Effects

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And Thanks to Other People Too:
V. Gurarie, N. Read, E. Rezayi, A. Stern,
N. Cooper, B. Halperin, L. Balents, …