Few-Body Physics with Spin-Orbit Coupling

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KITP Program on "Universality in Few-Body Systems"
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What we learn from one-body and two-body physics:

- Highly-symmetric SOC:
  - Ground state degeneracy
  - Enhanced low-E density-of-state
- Spin locked with momentum

} One-body

- Highly-symmetric SOC favors dimer formation
- Coupled relative and center-of-mass motions; Coupled scattering channels

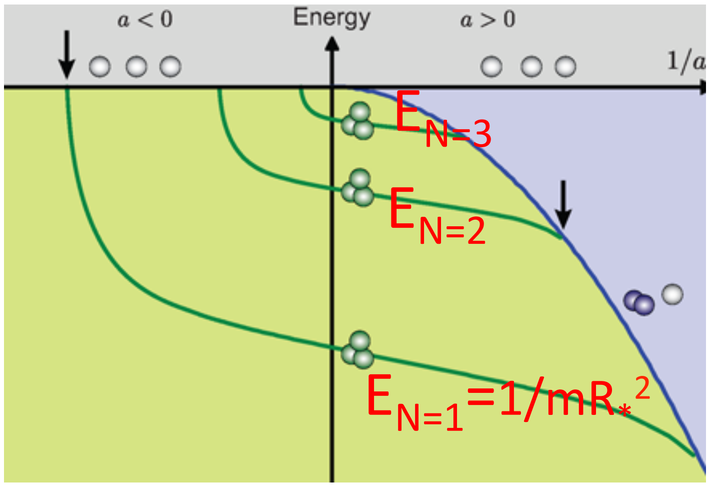
} Two-body

Q: What change will SOC make to three-body physics?

Conceptually new? Practically solvable?

3-body physics without SOC:

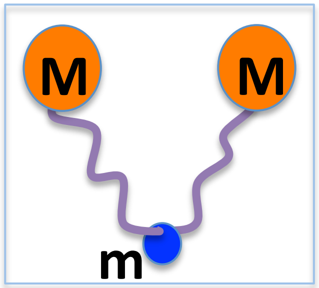
- Three bosons: Efimov spectrum

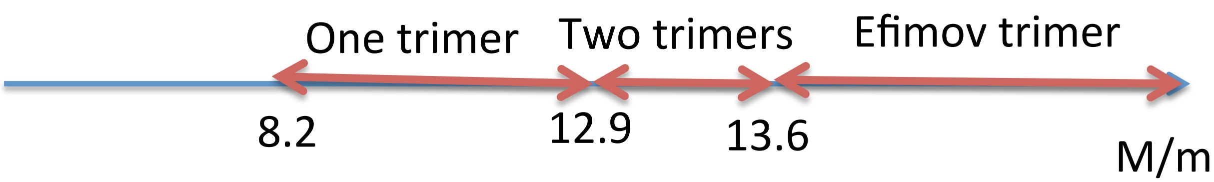


$E_N/E_{N+1}=515$

- induced by $1/R^2$ potential
- short-range physics is important

- Fermion mixtures:





Universal trimer: (a_s>0, M/m=[8.2,13.6])

Kartavtsev & Malykh: J. Phys. B 40, 1429 (2007)

With SOC, we ask:

- ☐ Trimer formation? (universal trimer, Efimov physics…)
- ☐ New universality?
- ☐ Few to Many?

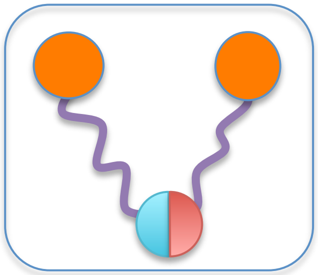
Our strategy:

- ➢ Highly-symmetric SOC (simplest while with great interests)
- ➢ Fermion mixtures

Outline

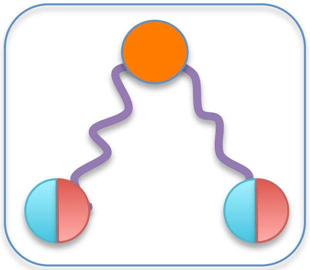
- Universal trimers favored by SOC

  Zheyu Shi, Hui Zhai and XC,
  Phys. Rev. Lett. 112, 013201 (2014);
  Phys. Rev. A 91, 023618 (2015)



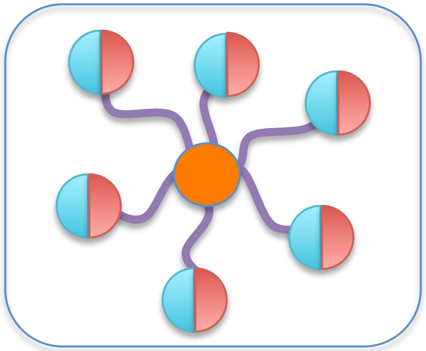
- Universal Borromean binding

  XC and Wei Yi, Phys. Rev. X 4, 031206 (2014)
  Synopsis in Physics: "Knots for All Occasions"



- Three to Many: trimers on top of a Fermi sea

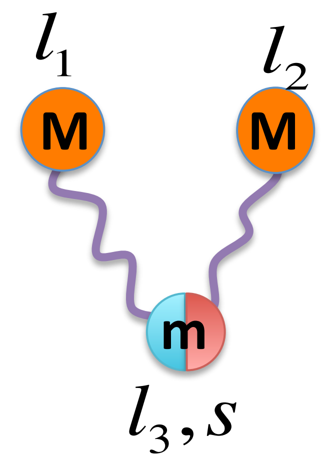
  Xingze Qiu, XC and Wei Yi, arxiv: 1607.03580
  to appear in Phys.Rev.A (R)



Case I: one particle with isotropic SOC

$$\hat{H}_0 = \frac{\mathbf{p}_1{}^2}{2M} + \frac{\mathbf{p}_2{}^2}{2M} + \frac{(\mathbf{p}_3 - \lambda\hat{\sigma})^2}{2m},$$

$$\hat{U} = [g\delta(\mathbf{r}_1 - \mathbf{r}_3) + g\delta(\mathbf{r}_2 - \mathbf{r}_3)]\mathbf{I},$$



$$L = l_1 + l_2 + l_3$$
$$s = 1/2$$

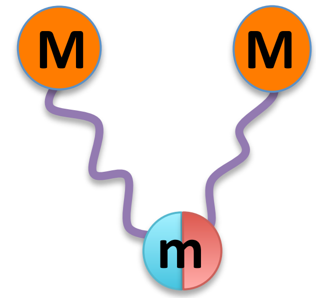
Good numbers:

- total momentum $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3}$
- total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{s}$

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \quad [P_i, P_j] = 0, \quad [P_i, J_j] = i\epsilon_{ijk}P_k$$

We solve three-body bound states with $P = 0,\ J = 1/2, 3/2$

Case I: one particle with isotropic SOC



$$\hat{H}_0 = \frac{{\mathbf{p}_1}^2}{2M} + \frac{{\mathbf{p}_2}^2}{2M} + \frac{(\mathbf{p}_3 - \lambda\hat{\sigma})^2}{2m},$$

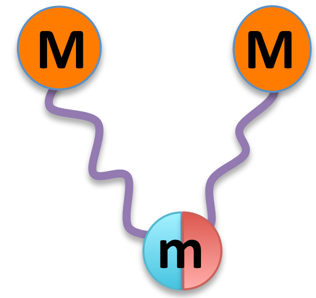
$$\hat{U} = [g\delta(\mathbf{r}_1 - \mathbf{r}_3) + g\delta(\mathbf{r}_2 - \mathbf{r}_3)]\,\mathbf{I},$$

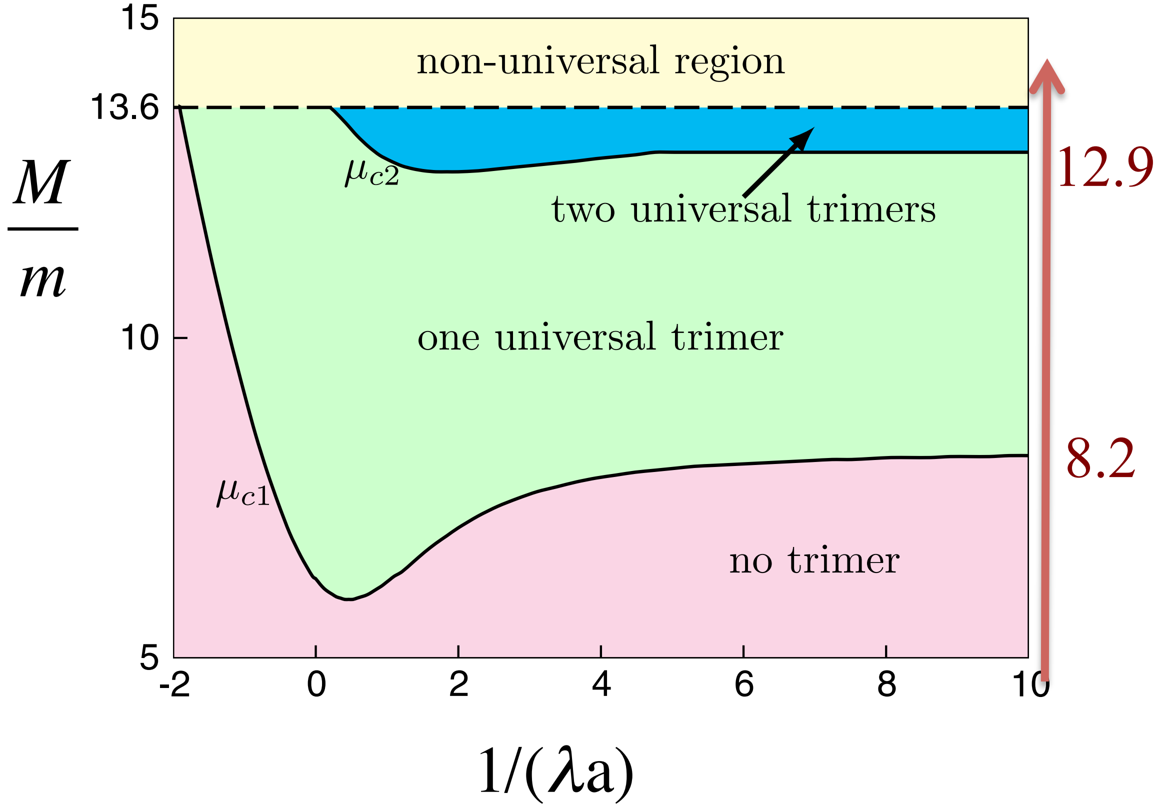
$$|\Psi\rangle = \sum_{\mathbf{p},\mathbf{q},\sigma} \Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q})\hat{\alpha}^\dagger_{\mathbf{q}}\hat{\alpha}^\dagger_{\mathbf{K}_0-\mathbf{p}}\hat{\beta}^\dagger_{\sigma,\mathbf{p}-\mathbf{q}}|0\rangle$$

$$f_\sigma(\mathbf{p}) = g\textstyle\sum_{\mathbf{q}} \Psi_\sigma(\mathbf{q}, \mathbf{K}_0 - \mathbf{p}, \mathbf{p} - \mathbf{q})$$

$$f_\sigma(\mathbf{k}) = g\sum_{\mathbf{p},\sigma'} G_{\sigma\sigma'}(E; \mathbf{p}, \mathbf{K}_0 - \mathbf{k}, \mathbf{k} - \mathbf{p}) \times [f_{\sigma'}(\mathbf{k}) - f_{\sigma'}(\mathbf{K}_0 - \mathbf{p})]$$

Case I: one particle with isotropic SOC



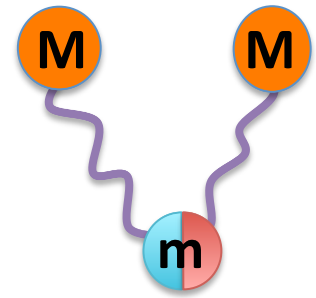


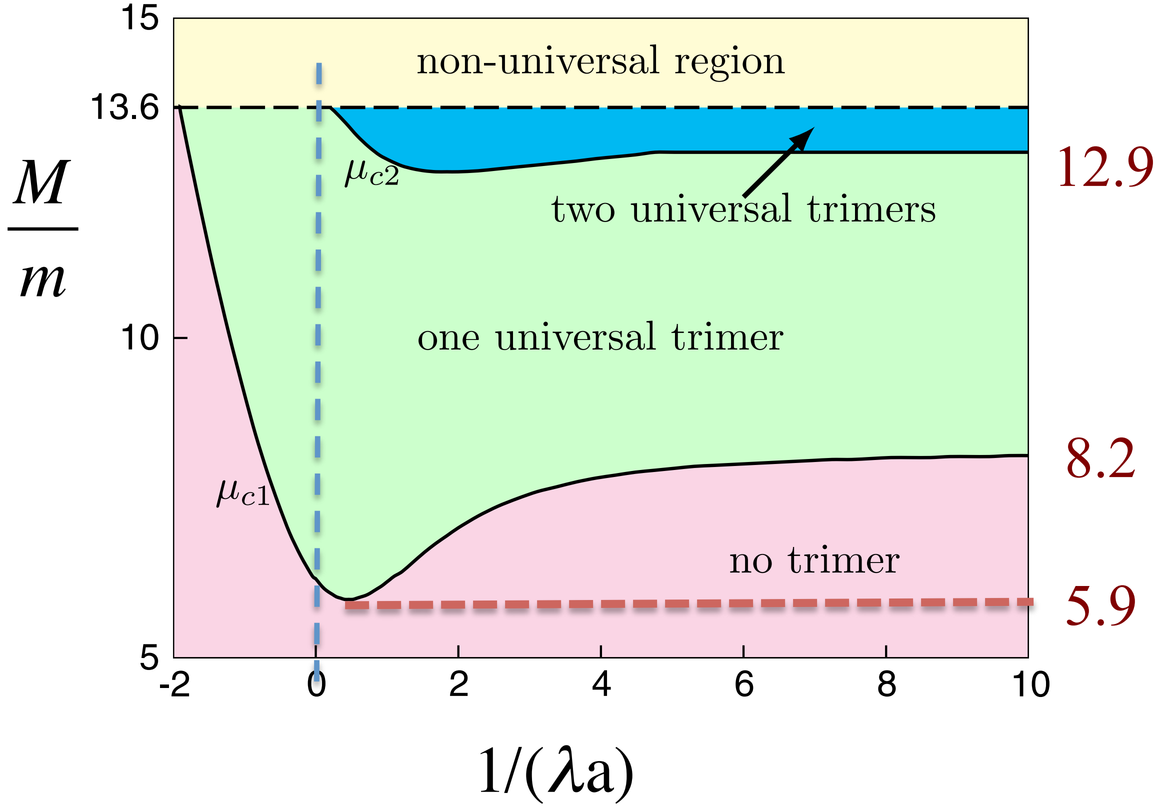
$\lambda a \to 0$:

Kartavtsev-Malykh trimer

J. Phys. B 40, 1429 (2007)

Case I: one particle with isotropic SOC





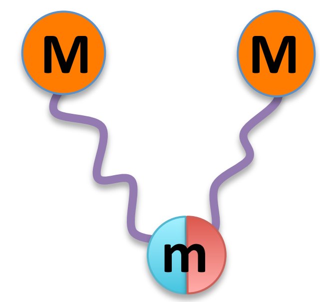
finite λ:

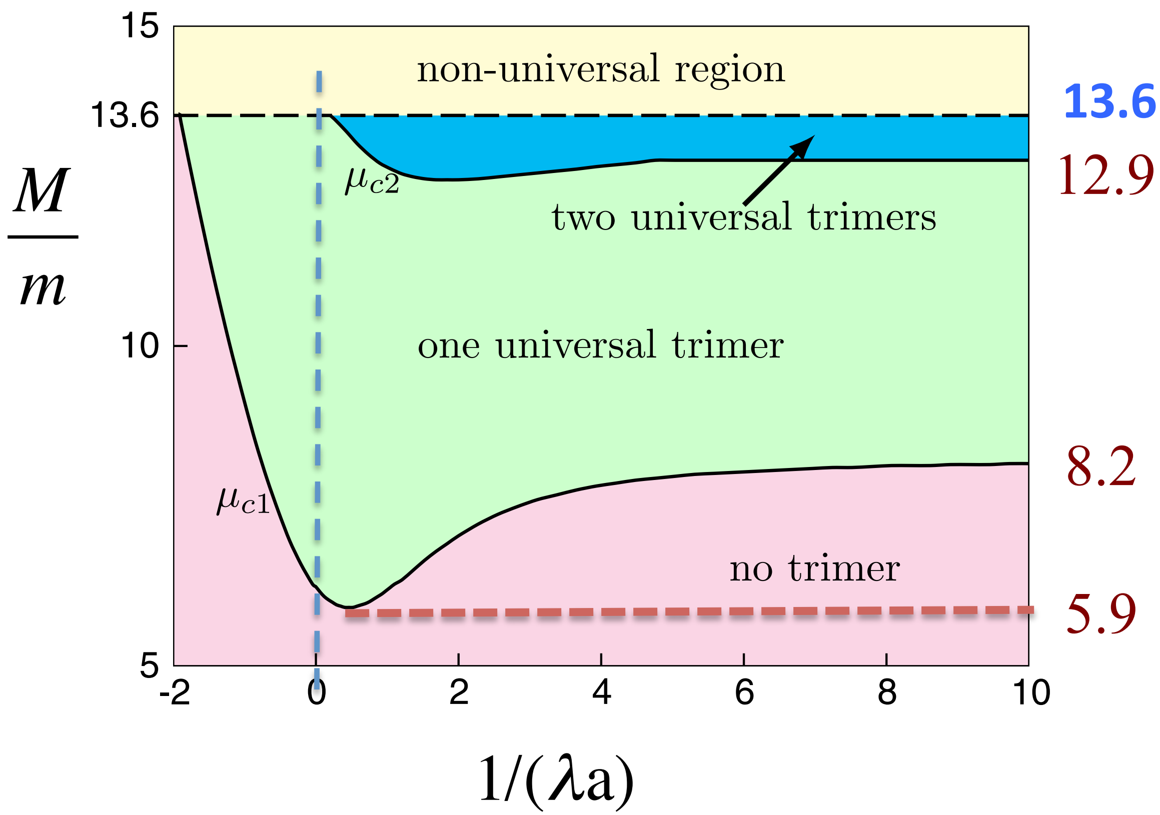
Universal trimer formed at

$a < 0$

$M/m \geq 5.9$

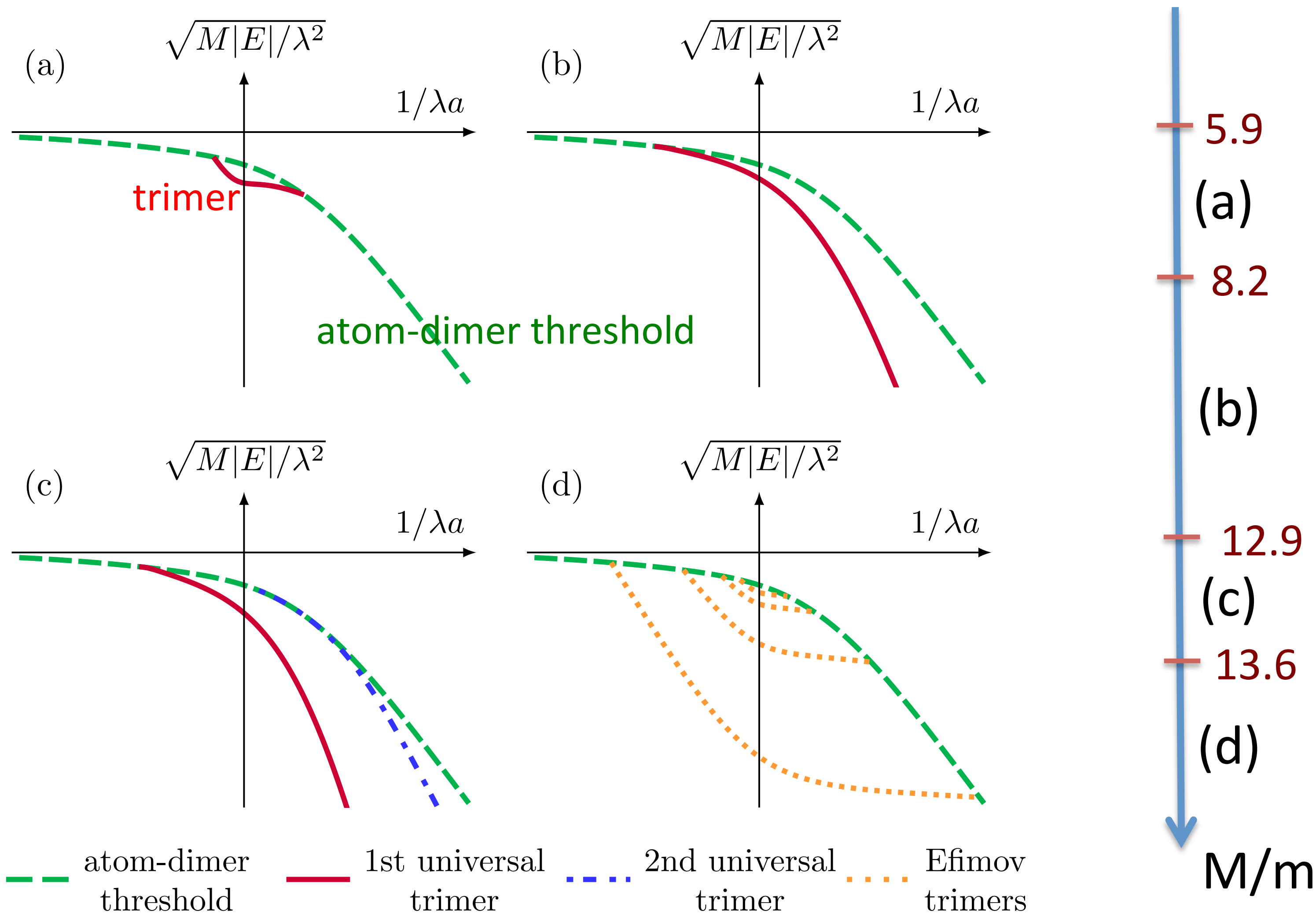
Case I: one particle with isotropic SOC





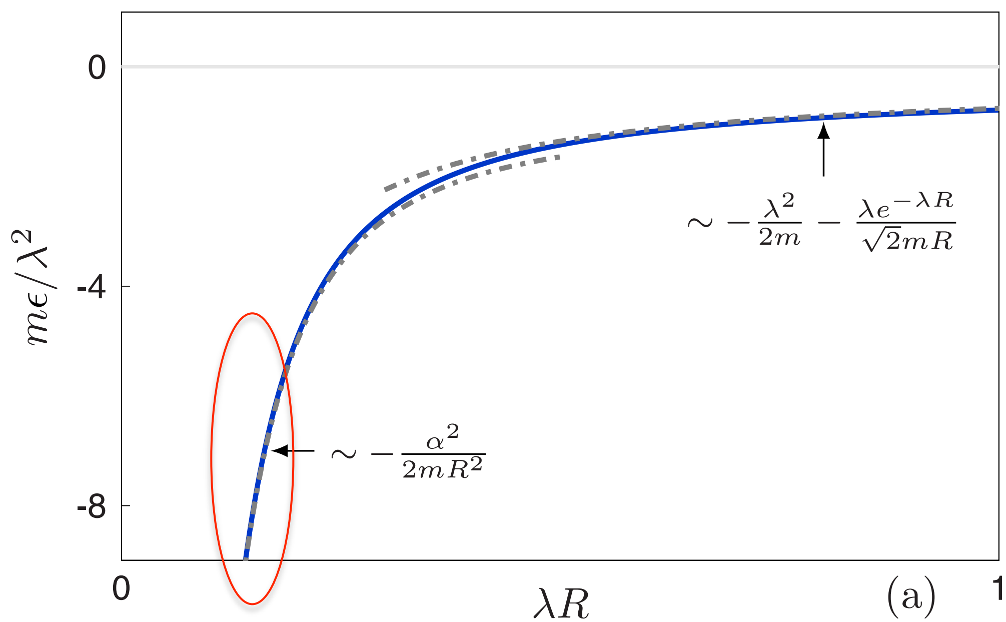
Critical mass ratio for Efimov trimer is *not* changed!

Case I: one particle with isotropic SOC



For Efimov trimer:

- SOC will *not* change critical mass ratio (13.6)



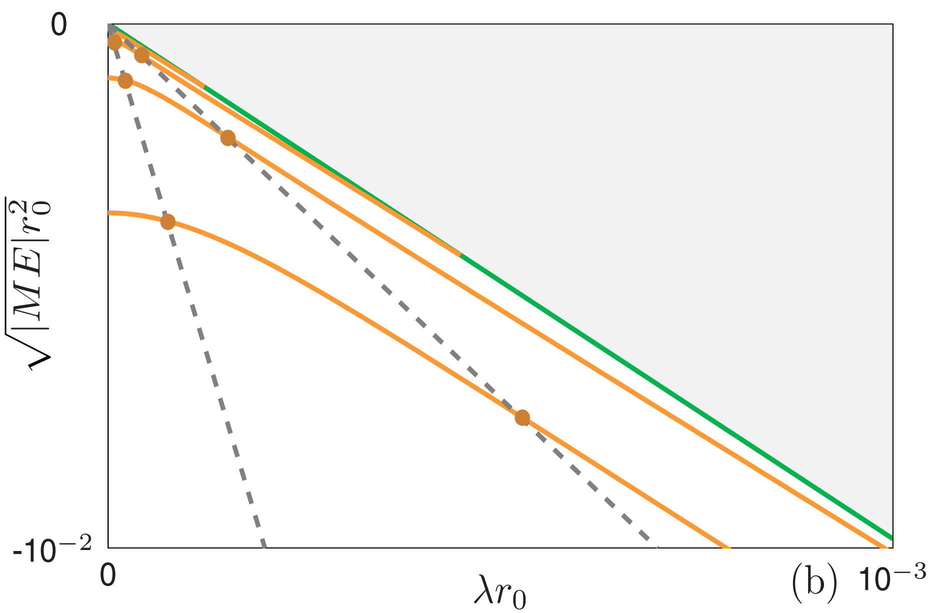
$$p \cdot \sigma << p^2$$

Short-range $1/R^2$ potential not changed

3-body potential in Born-Openheimer limit

For Efimov trimer:

- SOC will *not* change critical mass ratio (13.6)
- New discrete scaling:



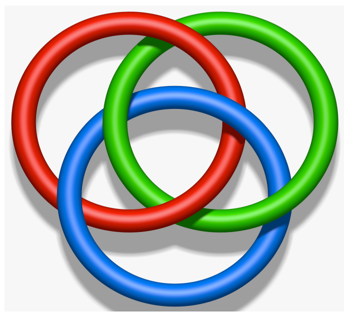
$$\mathbf{R} \to e^{-\pi/s_0}\mathbf{R}, \quad a \to e^{-\pi/s_0}a,$$
$$\lambda \to e^{\pi/s_0}\lambda, \quad E \to e^{2\pi/s_0}E,$$

$$s_0 = \sqrt{\alpha^2\mu - 9/2}$$

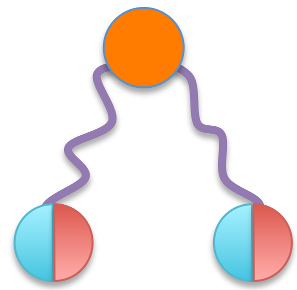
Zheyu Shi, Hui Zhai and XC,
Phys. Rev. Lett. 112, 013201 (2014);
Phys. Rev. A 91, 023618 (2015)

Case II: two particles with Rashba SOC

Universal Borromean binding



Physical system



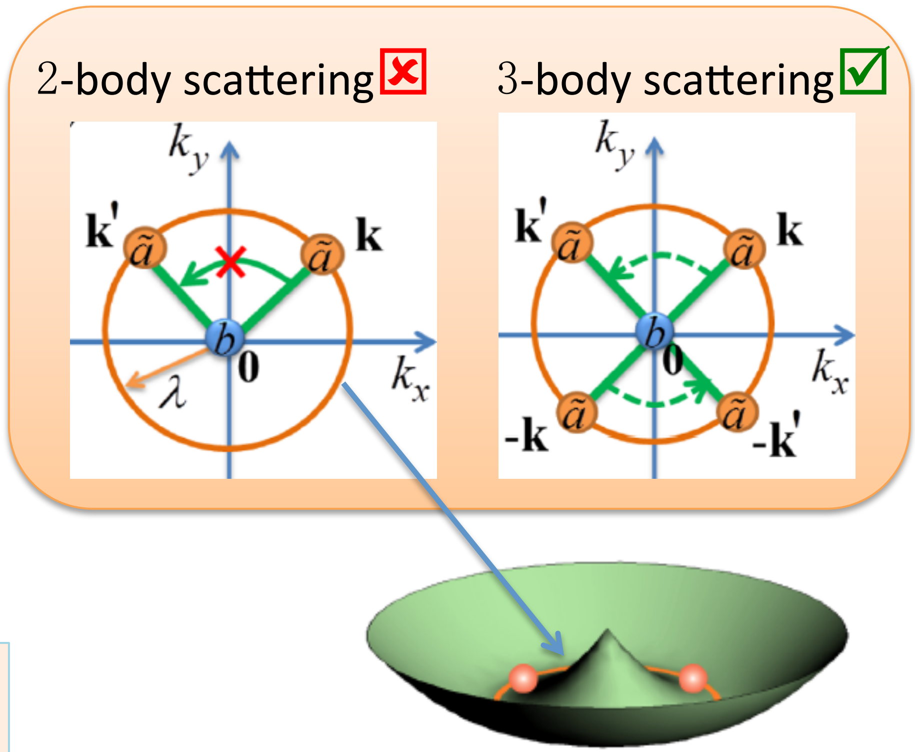
XC and Wei Yi, Phys. Rev. X 4, 031206 (2014)
Synopsis in Physics: "Knots for All Occasions"

Case II: two particles with Rashba SOC

Borromean ring

coat of arms of Borromeo family
(Northern Italy, 14th)

Borromean binding



U(1) ground state degeneracy under Rashba SOC

- Facilitated by single-particle spectral symmetry
- Universal against short-range interaction detail

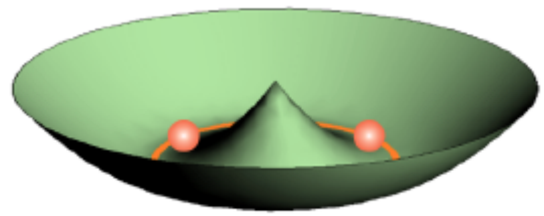
Case II: two particles with Rashba SOC

$$H = \sum_{\mathbf{k},\alpha=\uparrow,\downarrow} \frac{\mathbf{k}^2}{2m_a} a^\dagger_{\mathbf{k},\alpha} a_{\mathbf{k},\alpha} + \sum_{\mathbf{k}} \frac{\mathbf{k}^2}{2m_b} + \frac{\lambda}{m_a} \sum_{\mathbf{k}} [(k_x - ik_y) a^\dagger_{\mathbf{k},\uparrow} a_{\mathbf{k},\downarrow} + \mathrm{H.c.}]$$

Rashba SOC

$$+ \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{Q}} a^\dagger_{\mathbf{k},\uparrow} b^\dagger_{\mathbf{Q}-\mathbf{k}} b_{\mathbf{Q}-\mathbf{k}'} a_{\mathbf{k}',\uparrow}$$

Spin-selective interaction



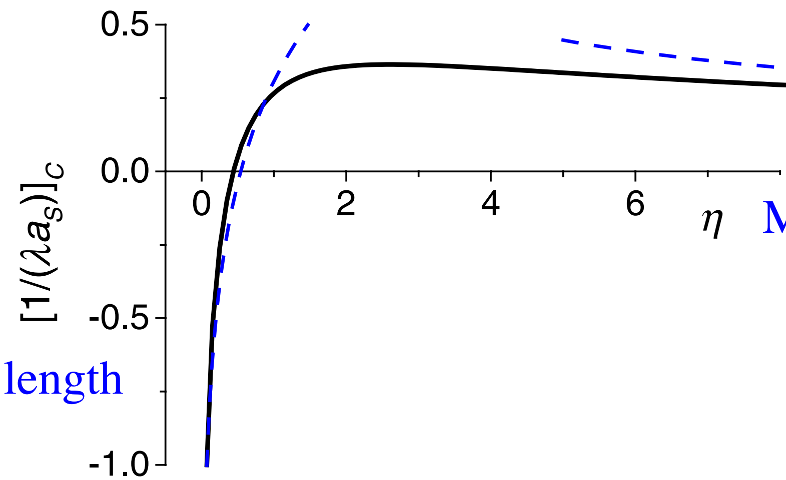
$$H\psi^{(N)} = E_N \psi^{(N)}$$

$$|\Psi^{(2)}\rangle = \sum_{\mathbf{k},\sigma=\pm} \Psi^{(2)}(\mathbf{Q}-\mathbf{k};\mathbf{k}\sigma) b^\dagger_{\mathbf{Q}-\mathbf{k}} a^\dagger_{\mathbf{k}\sigma} |0\rangle$$

$$|\Psi^{(3)}\rangle = \sum_{\mathbf{k}\sigma} \sum_{\mathbf{q}\xi} \Psi^{(3)}(-\mathbf{k}-\mathbf{q};\mathbf{k}\sigma;\mathbf{q}\xi) b^\dagger_{-\mathbf{k}-\mathbf{q}} a^\dagger_{\mathbf{k}\sigma} a^\dagger_{\mathbf{q}\xi} |0\rangle$$

Results:

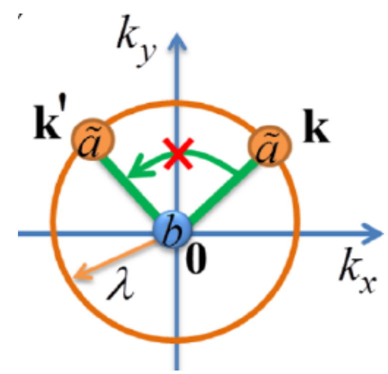
1) dimer formation:



Mass ratio between a and b

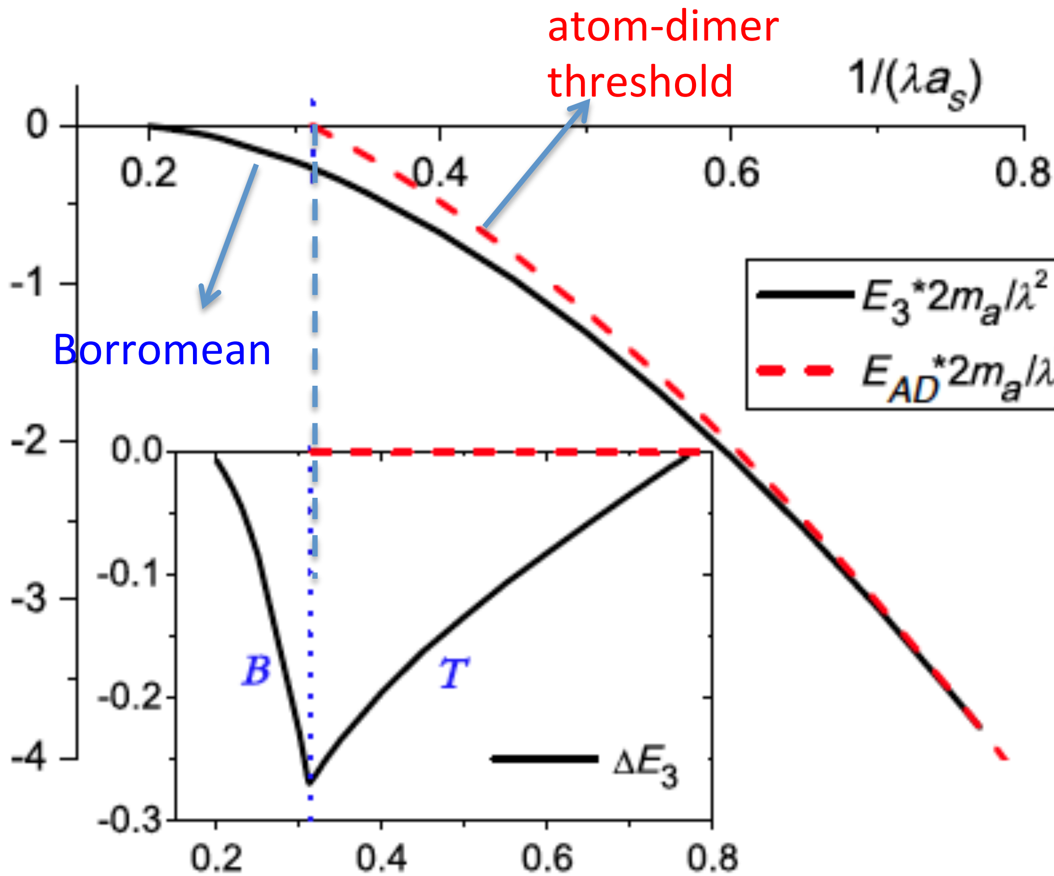
Critical scattering length
to support a dimer

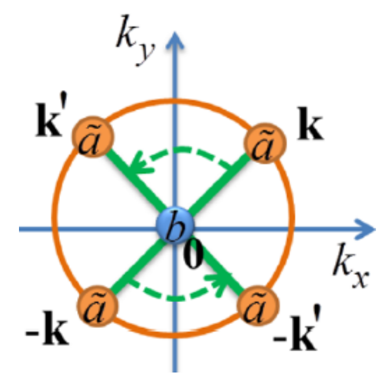
- For most region of mass ratio, dimer formation requires a>0



Results:

2) Trimer formation:

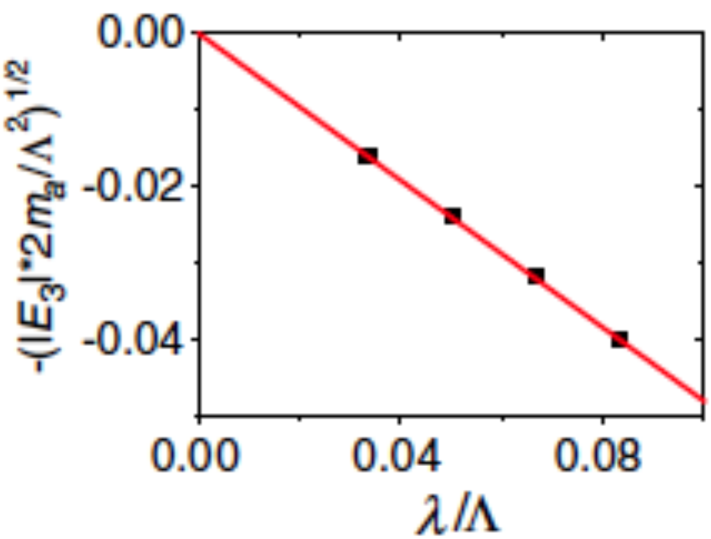




Trimer appears before Dimer --- Borromean

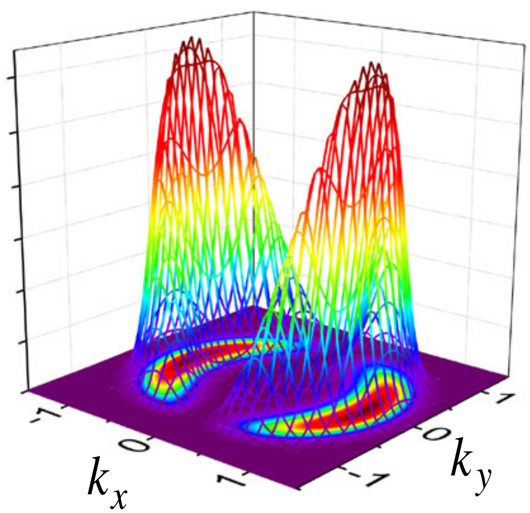
Results:

3) Universal feature of Borromean binding:



Universal binding energy

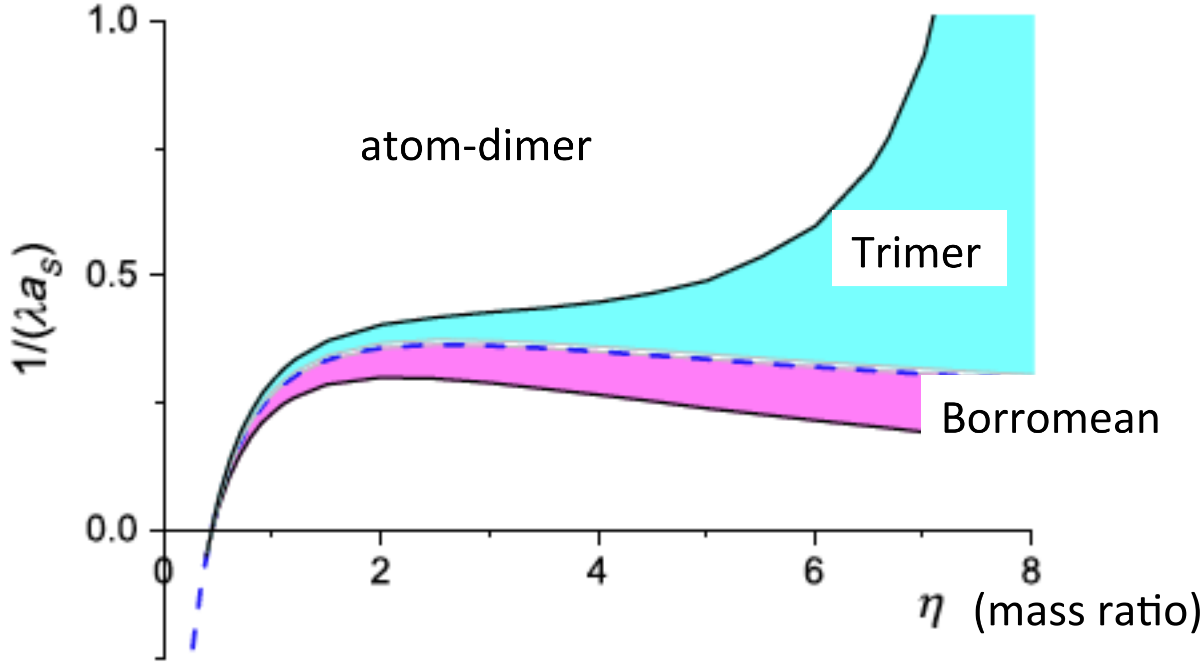
$$E_3(a_s, \lambda_{soc})$$



Wave-function in k-space:
most weight lies around
degenerate circle

Results:

4) robustness of Borromean binding:



Ground state phase diagram

**Borromean exists in a wide range of mass ratio $\eta \geq 0.4$ **

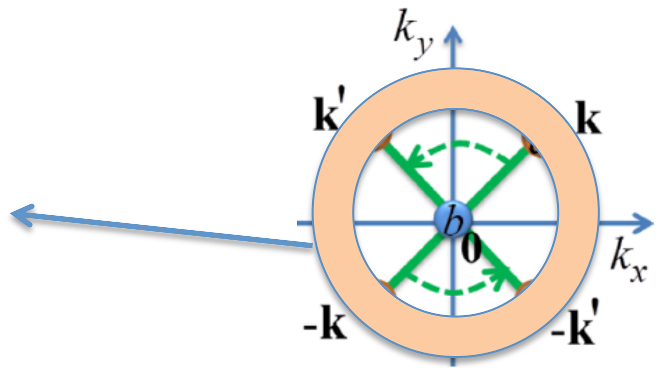
(without SOC, $\eta \geq 8.2$ to support trimer (not Borromean))

Few to Many

Stability of symmetry-facilitated trimer in a many-body environment?

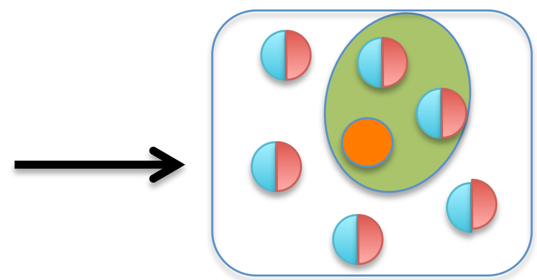
Xingze Qiu, XC and Wei Yi, arxiv: 1607.03580
to appear in Phys.Rev.A (R)

Impurity (b) interacting with spin-orbit coupled Fermi sea (a):

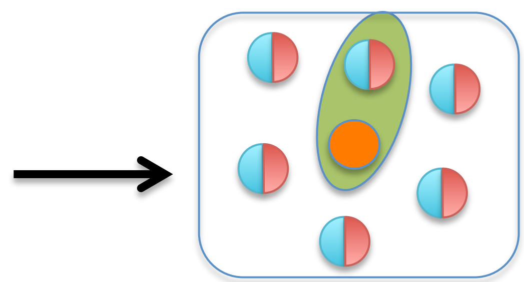


Three possible states:

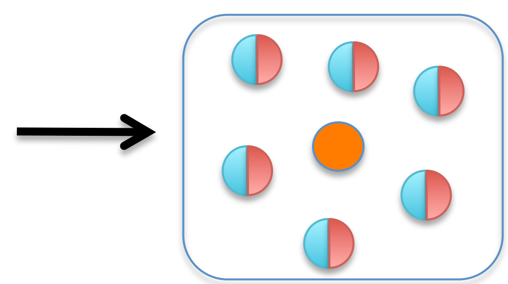
$$|T_0\rangle = \sum_{\mathbf{k}\lambda\mathbf{k}'\beta}{}' \phi^{\lambda\beta}_{\mathbf{k}\mathbf{k}'} b^\dagger_{-\mathbf{k}-\mathbf{k}'} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} |\mathrm{FS}\rangle_{N-2} + \sum_{\substack{\mathbf{k}\lambda\mathbf{k}'\beta \\ \mathbf{k}''\gamma\mathbf{q}\nu}}{}' \phi^{\lambda\beta\gamma\nu}_{\mathbf{k}\mathbf{k}'\mathbf{k}''\mathbf{q}} b^\dagger_{\mathbf{q}-\mathbf{k}-\mathbf{k}'-\mathbf{k}''} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} a^\dagger_{\mathbf{k}''\gamma} a_{\mathbf{q}\nu} |\mathrm{FS}\rangle_{N-2}$$



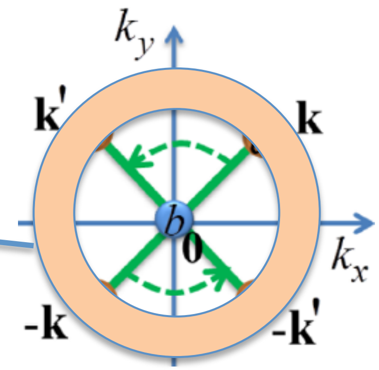
$$|M_{\mathbf{Q}}\rangle = \sum_{\mathbf{k}\lambda}{}' \phi^{\lambda}_{\mathbf{k}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}-\mathbf{k}} a^\dagger_{\mathbf{k}\lambda} |\mathrm{FS}\rangle_{N-1} + \sum_{\mathbf{k}\lambda\mathbf{k}'\beta\mathbf{q}\nu}{}' \phi^{\lambda\beta\nu}_{\mathbf{k}\mathbf{k}'\mathbf{q}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}-\mathbf{k}-\mathbf{k}'+\mathbf{q}} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} a_{\mathbf{q}\nu} |\mathrm{FS}\rangle_{N-1}$$



$$|P_{\mathbf{Q}}\rangle = \left(\phi_{\mathbf{Q}} b^\dagger_{\mathbf{Q}} + \sum_{\mathbf{k}\lambda\mathbf{q}\nu}{}' \phi^{\lambda\nu}_{\mathbf{k}\mathbf{q}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}+\mathbf{q}-\mathbf{k}} a^\dagger_{\mathbf{k}\lambda} a_{\mathbf{q}\nu} \right) |\mathrm{FS}\rangle_N$$



Impurity (b) interacting with spin-orbit coupled Fermi sea (a):



Three possible states:

$$|T_0\rangle = \sum_{\mathbf{k}\lambda\mathbf{k}'\beta}{}' \phi^{\lambda\beta}_{\mathbf{k}\mathbf{k}'} b^\dagger_{-\mathbf{k}-\mathbf{k}'} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} |\mathrm{FS}\rangle_{N-2} + \sum_{\substack{\mathbf{k}\lambda\mathbf{k}'\beta \\ \mathbf{k}''\gamma\mathbf{q}\nu}}{}' \phi^{\lambda\beta\gamma\nu}_{\mathbf{k}\mathbf{k}'\mathbf{k}''\mathbf{q}} b^\dagger_{\mathbf{q}-\mathbf{k}-\mathbf{k}'-\mathbf{k}''} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} a^\dagger_{\mathbf{k}''\gamma} a_{\mathbf{q}\nu} |\mathrm{FS}\rangle_{N-2}$$

$k_F \rightarrow 0$ → trimer

$$|M_{\mathbf{Q}}\rangle = \sum_{\mathbf{k}\lambda}{}' \phi^{\lambda}_{\mathbf{k}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}-\mathbf{k}} a^\dagger_{\mathbf{k}\lambda} |\mathrm{FS}\rangle_{N-1} + \sum_{\mathbf{k}\lambda\mathbf{k}'\beta\mathbf{q}\nu}{}' \phi^{\lambda\beta\nu}_{\mathbf{k}\mathbf{k}'\mathbf{q}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}-\mathbf{k}-\mathbf{k}'+\mathbf{q}} a^\dagger_{\mathbf{k}\lambda} a^\dagger_{\mathbf{k}'\beta} a_{\mathbf{q}\nu} |\mathrm{FS}\rangle_{N-1}$$

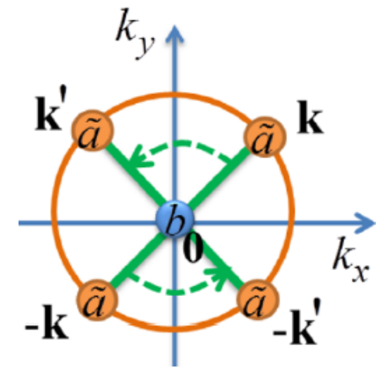
→ dimer

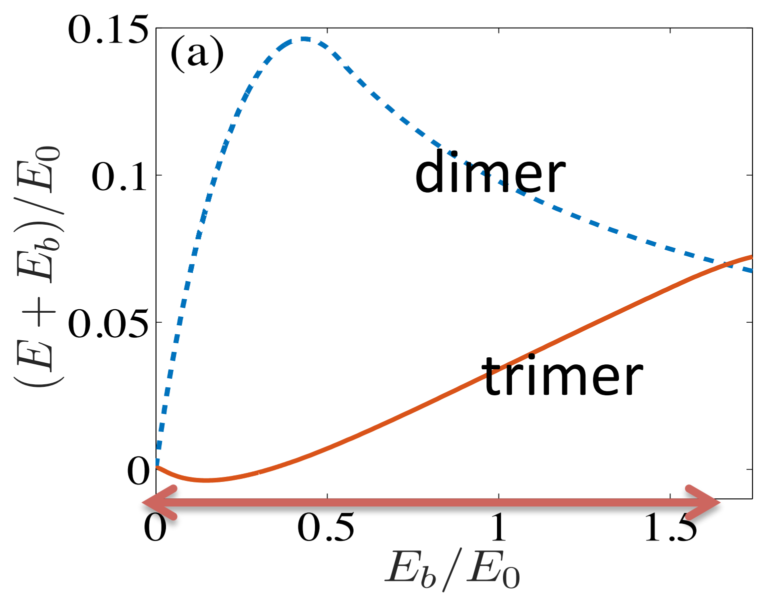
$$|P_{\mathbf{Q}}\rangle = \left(\phi_{\mathbf{Q}} b^\dagger_{\mathbf{Q}} + \sum_{\mathbf{k}\lambda\mathbf{q}\nu}{}' \phi^{\lambda\nu}_{\mathbf{k}\mathbf{q}}(\mathbf{Q}) b^\dagger_{\mathbf{Q}+\mathbf{q}-\mathbf{k}} a^\dagger_{\mathbf{k}\lambda} a_{\mathbf{q}\nu} \right) |\mathrm{FS}\rangle_N$$

→ scattering

Few-body states

We consider 2D case for simplicity.
In few-body sector:





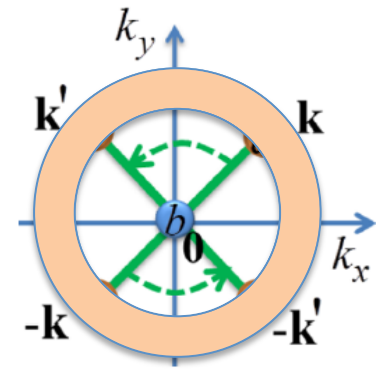
E_b: 2-body binding energy in 2D
E_0: SOC energy

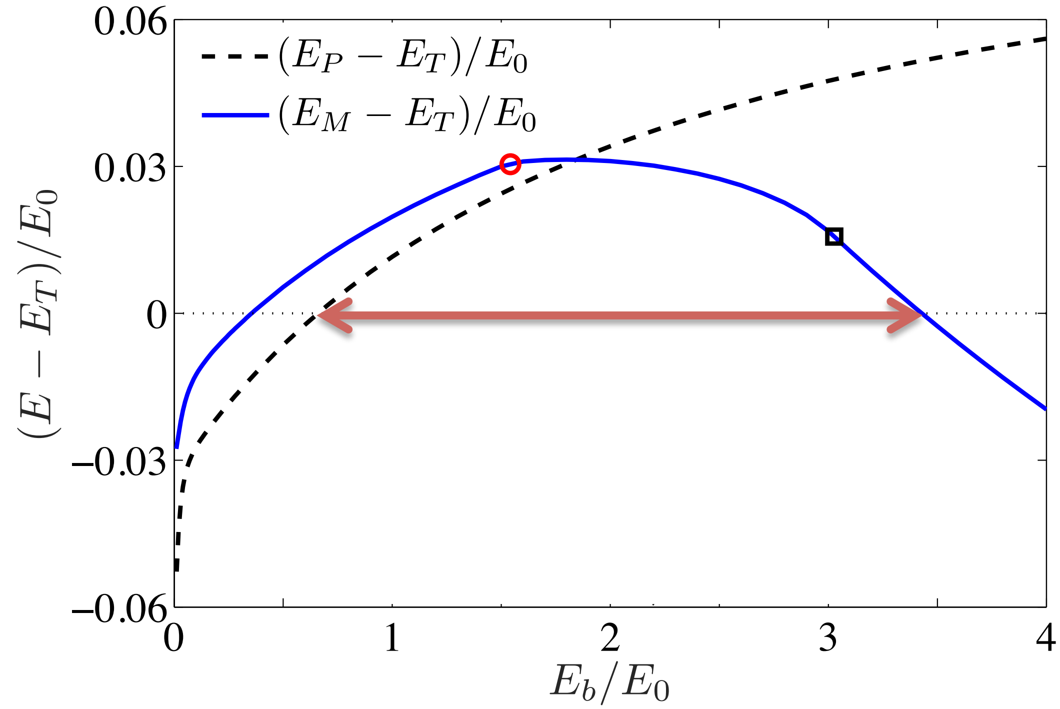
$$E_b \in [0,1.65]E_0$$

Trimer stablization

In the presence of SOC Fermi sea,

$$E_F = E_{th} + 0.05E_0$$

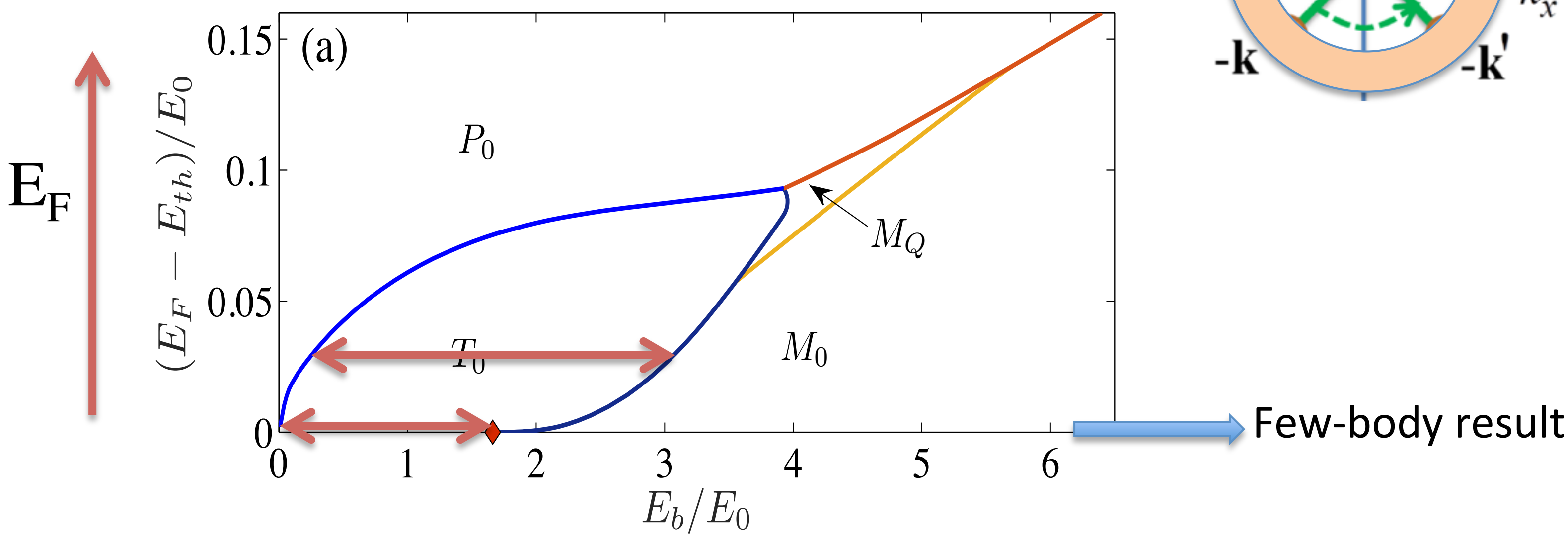


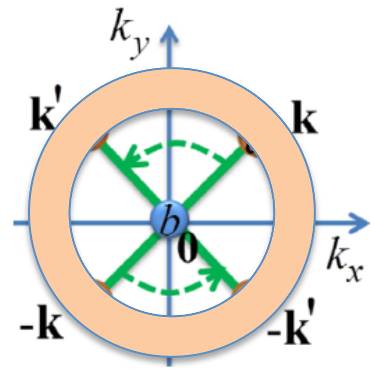


$E_b \in [0.66, 3.43]E_0$ (dressed) trimer stablization

Fermi-sea favors trimer stablization!

Phase diagram as changing E_F

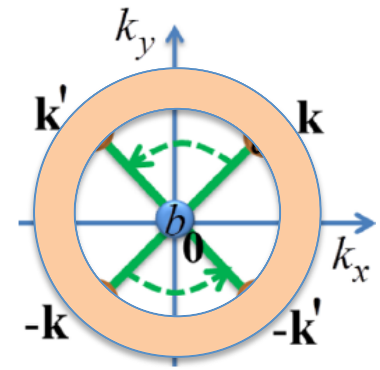


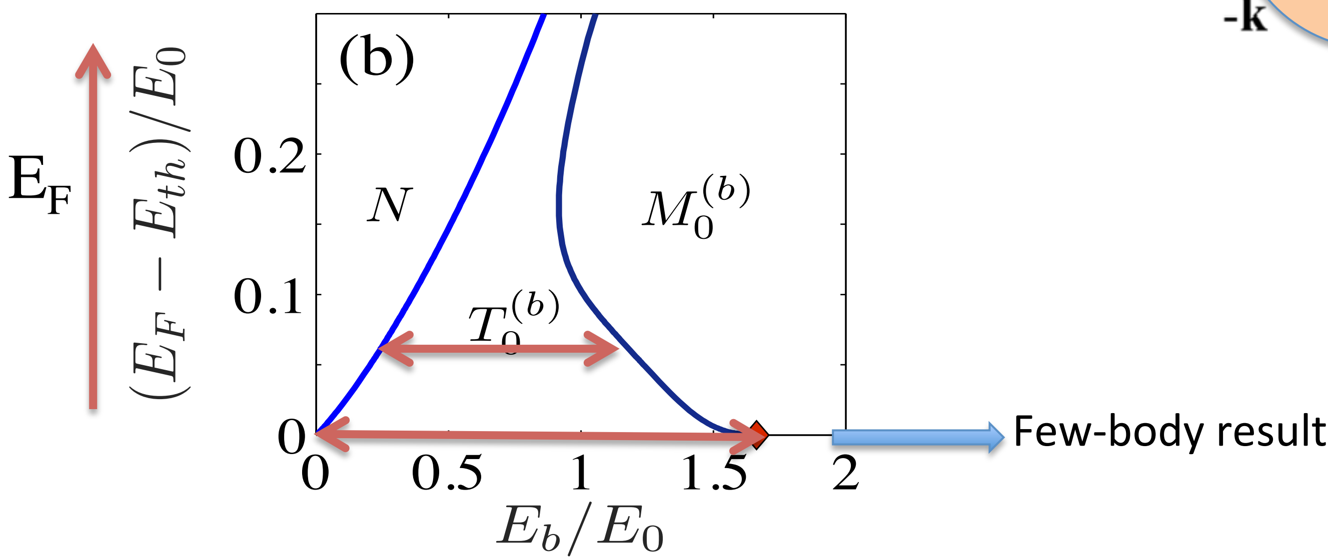


Fermi sea favors trimer against dimer:

- ➢ Pauli-blocking effect (?)
- ➢ Particle-hole fluctuation (?)

Only Pauli-blocking effect :

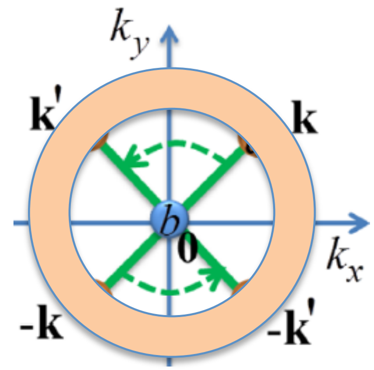


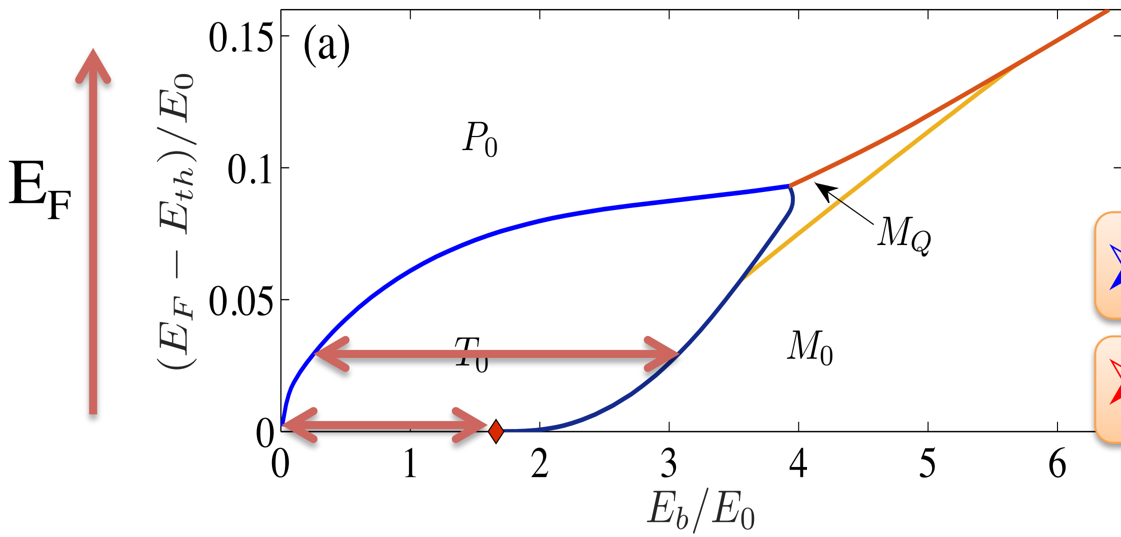


Pauli-blocking disfavors trimer!

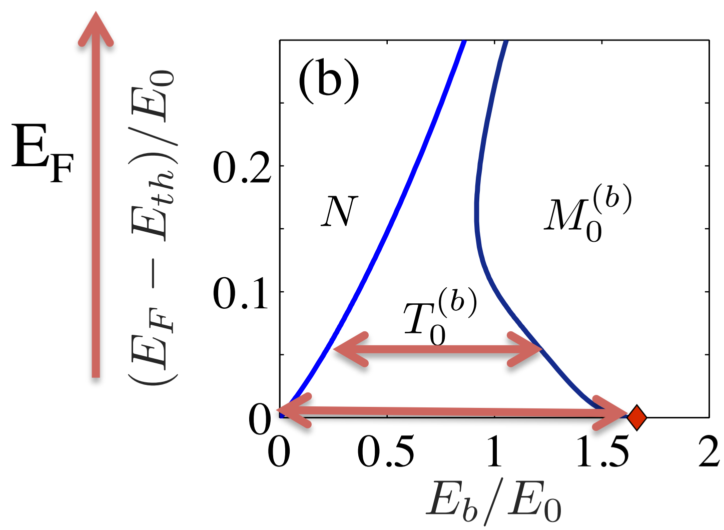
Due to mechanism of trimer formation --- low-energy scattering around the circle

It is the p-h fluctuation that favors trimer!





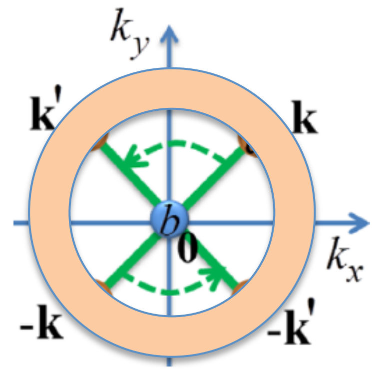
- Pauli-blocking effect
- Particle-hole fluctuation

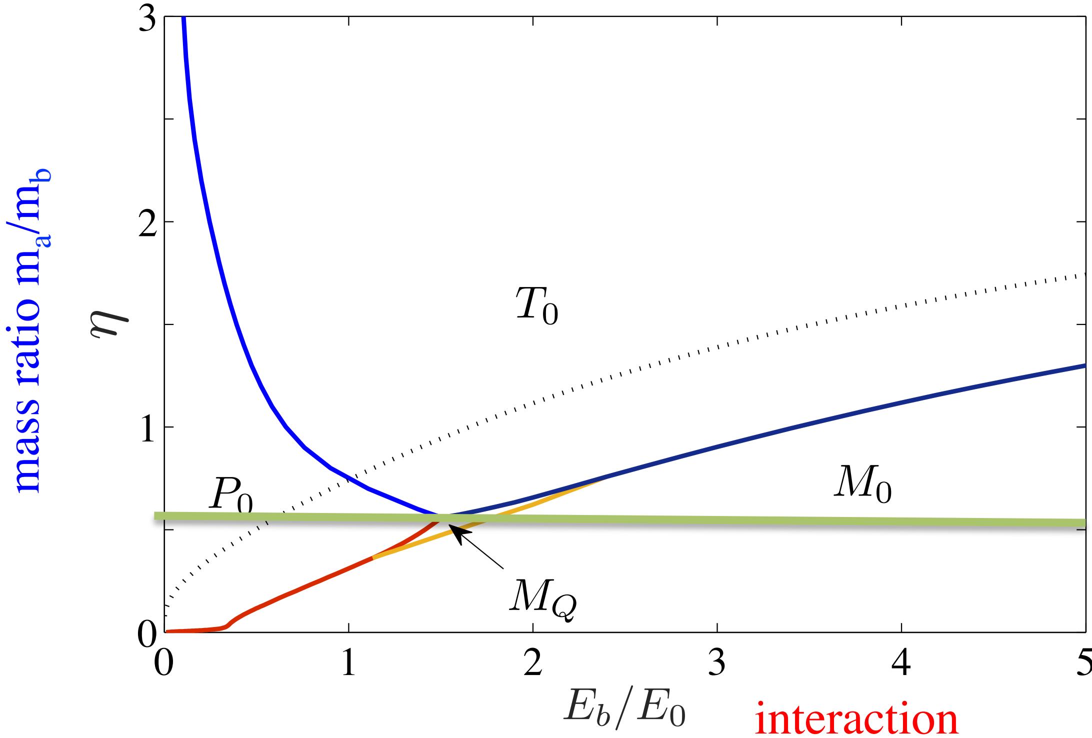


- Pauli-blocking effect

Phase diagram in (interaction, mass ratio) plane:

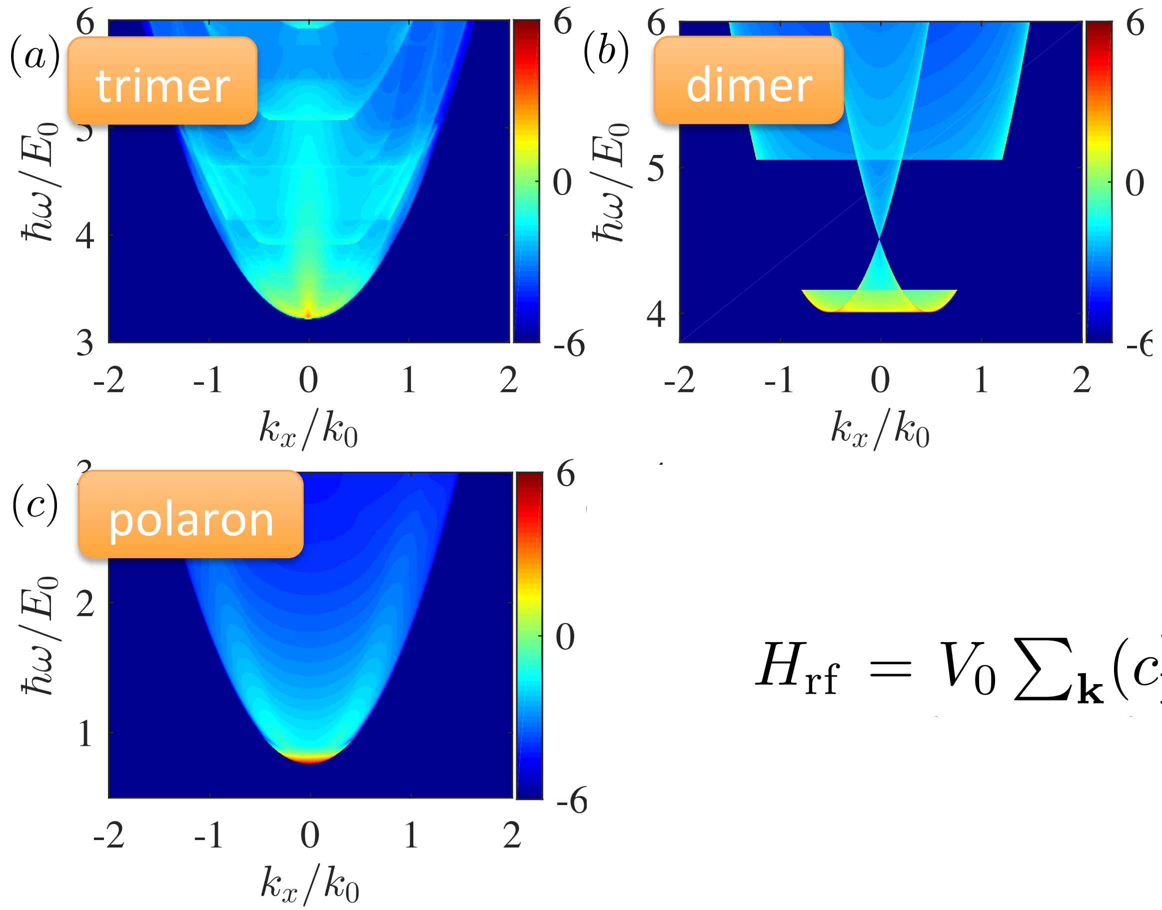
$$E_F = E_{th} + 0.05E_0$$





Trimer stablized for $\eta \geq 0.5$

Detection by rf spectroscopy: $\Gamma(k,\omega)$



$$H_{\rm rf} = V_0 \sum_{\mathbf{k}} (c^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} + H.c.)$$

Summary
--- Three-body physics with SOC

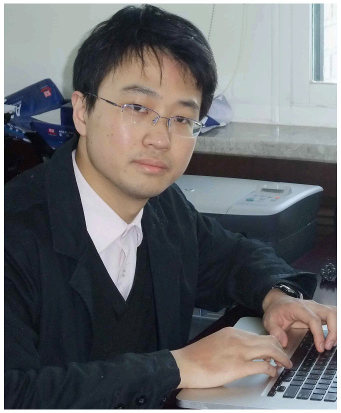
- Universal trimers at smaller mass ratios and negative a_s
- Universal Borromean binding induced by spectral symmetry
- Three-body correlation enhanced by many-body environment

Open questions

- Trimer formation from hyperspherical approach?
- SOC with reduced symmetry? (realistic to expt)
- SOC effect in low-D? (dimensional crossover)
- SOC + high partial waves? (new effect)
- More in "Few-to-Many"? (polaron, BEC…)

SOC --- a new ingredient to few-body physics

Acknowledgement



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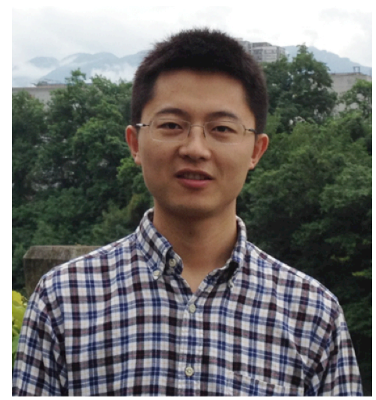

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