

The Universality between few-body nucleon
systems and fermionic atom systems
: Application of Gaussian Expansion Method
to the tetra-neutron system

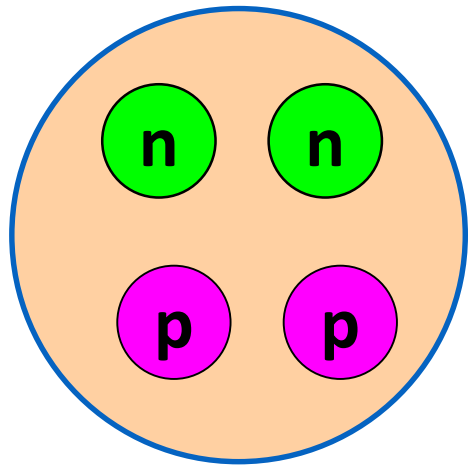
E. Hiyama (RIKEN)

R. Lazauskas (IPHC)

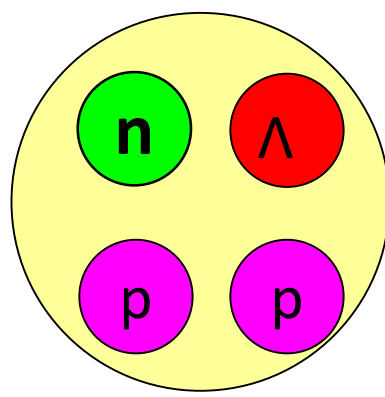
J. Carbonell (CNRS)

M. Kamimura (Kyushu Univ./RIKEN)

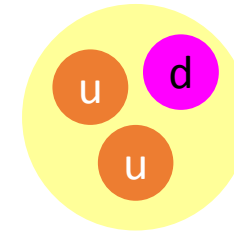
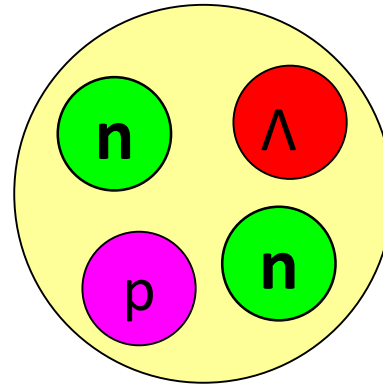
I have been studying the following subjects using my own method called Gaussian Expansion Method.



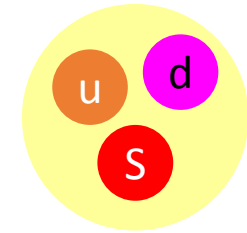
Few-nucleon system



Hypernucleus=Nucleus+hyperon

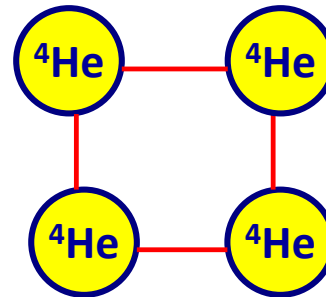
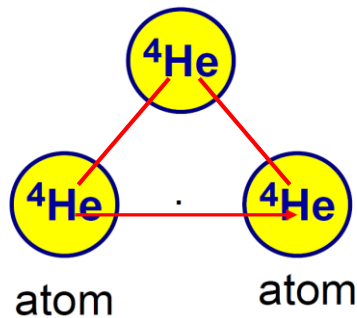


neutron



Λ

~2012, I started to study the structure of three- and four-body ^4He atom systems.



What is Gaussian Expansion Method?

Our few-body calculational method

Gaussian Expansion Method (GEM) , since 1987

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :

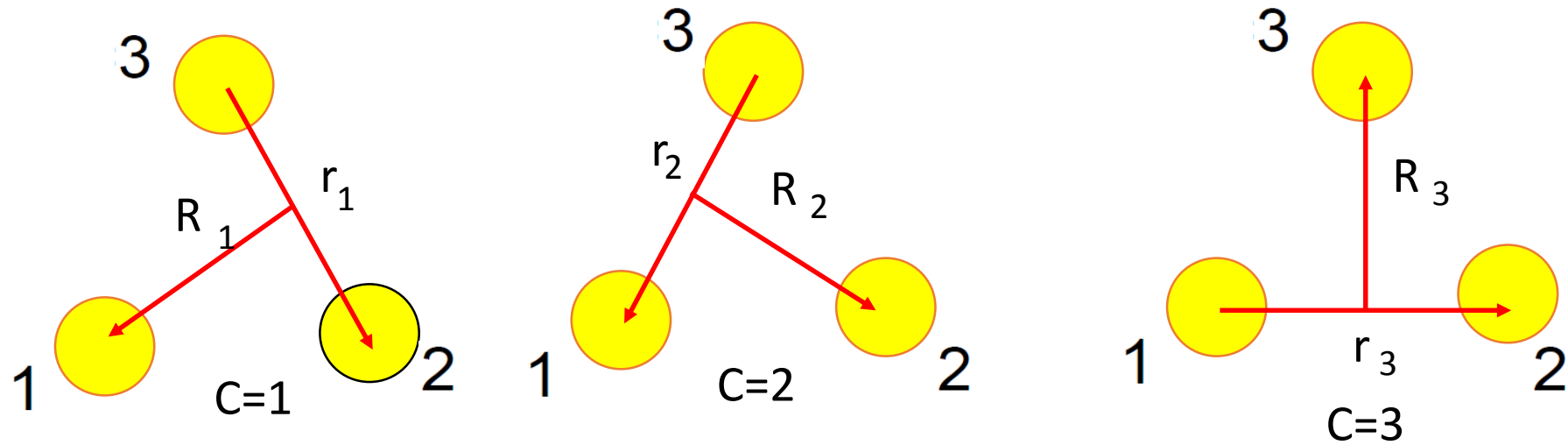
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules ,
3- and 4-nucleon systems,
multi-cluster structure of light nuclei,

Light hypernuclei,
3-quark systems,

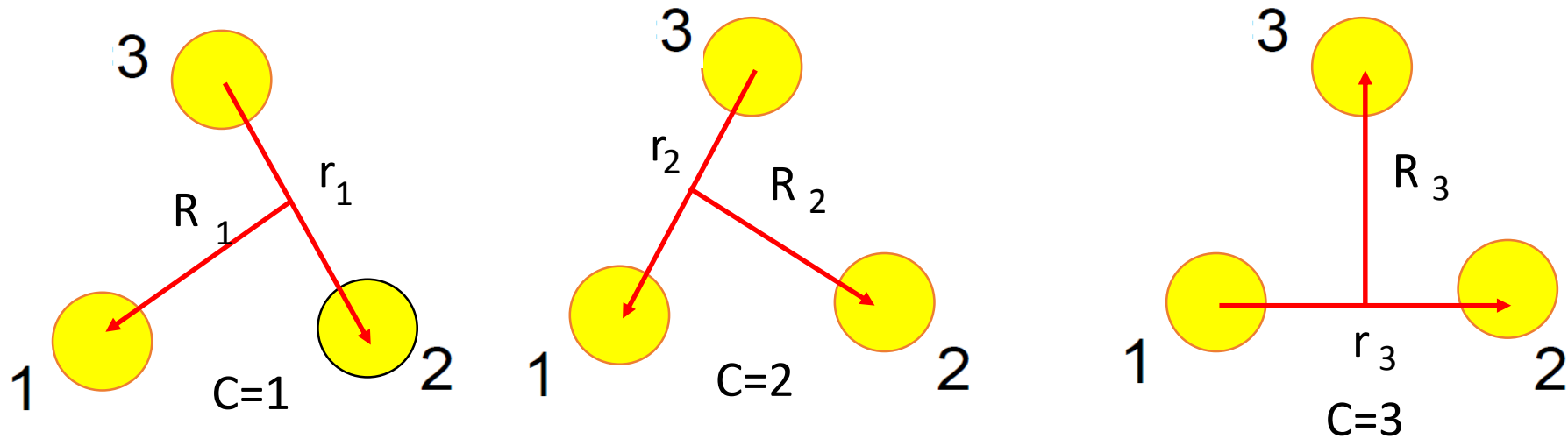
Gaussian Expansion Method (GEM)



$$H = -\frac{\hbar^2}{2\mu_{r_c}} \nabla_{\mathbf{r}_c}^2 - \frac{\hbar^2}{2\mu_R} \nabla_{\mathbf{R}_c}^2 + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3) .$$

$$[H - E] \Psi_{JM} = 0$$

$$\Psi_{JM} = \Phi_{JM}^{(1)}(\mathbf{r}_1, \mathbf{R}_1) + \Phi_{JM}^{(2)}(\mathbf{r}_2, \mathbf{R}_2) + \Phi_{JM}^{(3)}(\mathbf{r}_3, \mathbf{R}_3)$$



$$\Psi_{JM} = \Phi_{JM}^{(1)}(\mathbf{r}_1, \mathbf{R}_1) + \Phi_{JM}^{(2)}(\mathbf{r}_2, \mathbf{R}_2) + \Phi_{JM}^{(3)}(\mathbf{r}_3, \mathbf{R}_3)$$

Basis functions of each Jacobian coordinate

$(c = 1 - 3)$

$$\phi_{nl}^{(c)}(r_c) Y_{lm}(\hat{\mathbf{r}}_c), \quad \psi_{NL}^{(c)}(R_c) Y_{LM}(\widehat{\mathbf{R}}_c)$$

$$\Phi_{JM}^{(c)}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{nl, NL} \mathbf{C}_{NL, lm} \phi_{nl}^{(c)}(r_c) \psi_{NL}^{(c)}(R_c) [Y_l(\hat{\mathbf{r}}_c) \otimes Y_L(\widehat{\mathbf{R}}_c)]_{JM}$$

$(c = 1 - 3)$

Determined by diagonalizing H

Radial part :
Gaussian
function

$$\phi_{nl}(r) = r^l e^{-(r/r_n)^2}$$

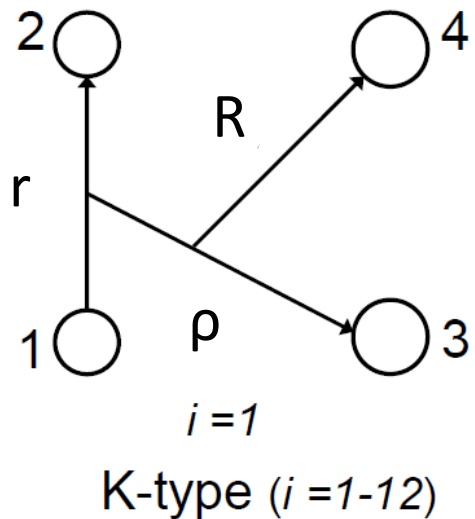
$$\psi_{NL}(R) = R^L e^{-(R/R_N)^2}$$

Gaussian ranges
in **geometric**
progression

$$r_n = r_1 a^{n-1} \quad (n = 1 - n_{\max}),$$

$$R_N = R_1 A^{N-1} \quad (N = 1 - N_{\max})$$

Both the **short-range correlations** and the **exponentially-damped tail** are simultaneously reproduced accurately.



In the case of four-body problem,
We have one more Jabobi coordinate.

Next, by solving eigenstate problem, we get eigenenergy E and unknown coefficients C_n .

$$\left[\begin{array}{c} (H_{i_n}) - E (N_{i_n}) \end{array} \right] \left[\begin{array}{c} C_n \end{array} \right] = 0$$

In principle, we can apply this method to N-body problem.
However,...

By solving eigenstate problem, we get eigenenergy E and unknown coefficients C_n .

$$\left[\begin{array}{c} (H_{i n}) - E (N_{i n}) \end{array} \right] \left[\begin{array}{c} C_n \end{array} \right] = 0$$

The problem:

we need huge memory to calculate N-body systems.

Up to now, it became to calculate five-body problem.

In the case of four-body problem, it was already possible to calculate bound state and resonant state.

I will show you the results.

To show that our method can provide with accurate energies and wavefunction,

- We performed a benchmark test calculation for the ground state of ^4He using AV8 NN potential among 7 different few-body research group.

[Kamada et al., Phys. Rev. C64 \(2001\), 044001](#)

Benchmark-test 4-body calculation : Phys. Rev. C64 (2001), 044001

Benchmark test calculation of a four-nucleon bound state

by 7 groups

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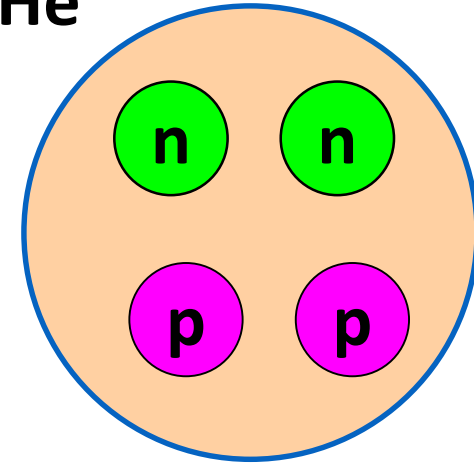
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${}^4\text{He}$



4 nucleon
bound state

Realistic NN
force: AV8'

Benchmark-test calculation of the 4-nucleon bound state

Good agreement among 7 different methods

In the binding energy, r.m.s. radius and wavefunction density

H. KAMADA *et al.*

PHYSICAL REVIEW C **64** 044001

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
GEM	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

very different techniques and the complexity of the nuclear force chosen. Except for NCSM and EIHH, the expectation values of T and V also agree within three digits. The NCSM results are, however, still within 1% and EIHH within 1.5% of the others but note that the EIHH results for T and V are

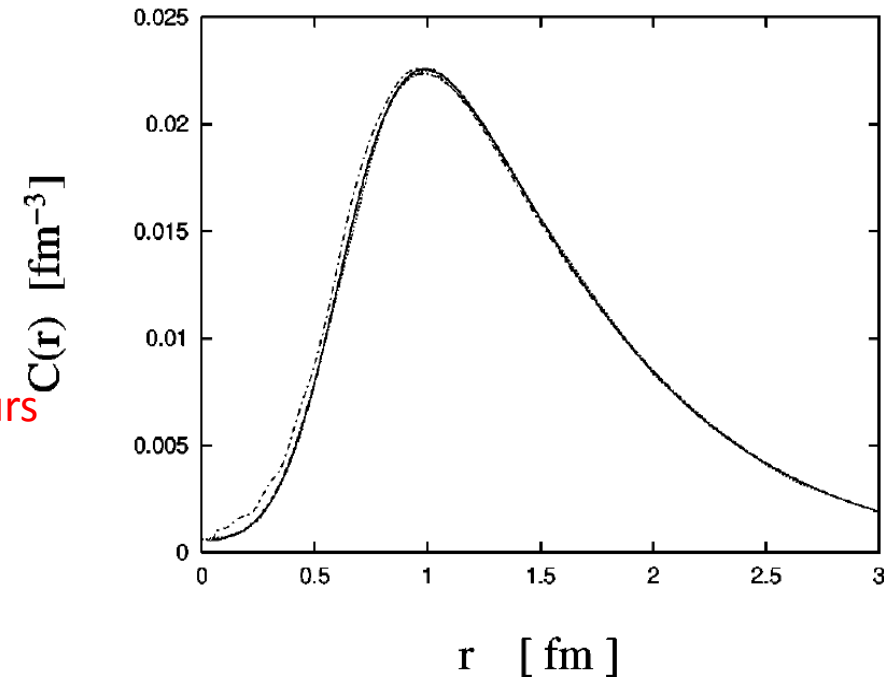
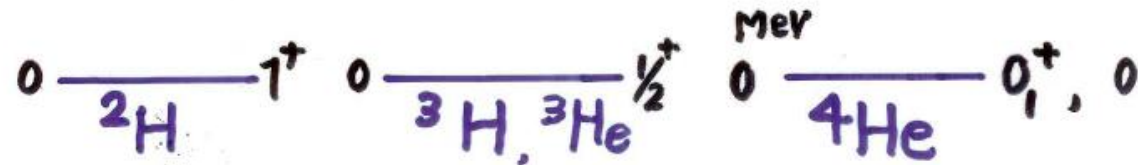


FIG. 1. Correlation functions in the different calculational schemes: EIHH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).

One of the next challenging problems: to solve the **second 0^+** state of ${}^4\text{He}$ using the realistic force.

E_x		J^π	T
24.3	—	1^-	0
23.6	—	1^-	1
23.3	—	2^-	1
21.6	—	2^-	0
21.0	—	0^-	0
<u>20.2</u>	—	<u>0_2^+</u>	0



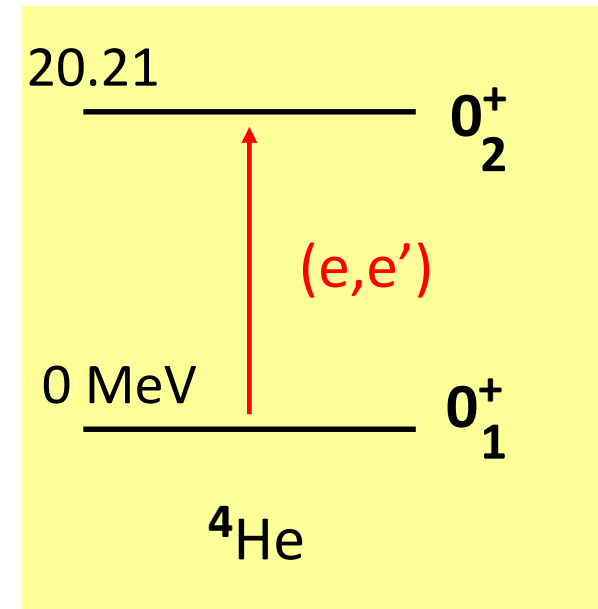
${}^4\text{He}$ is the lightest nucleus that exhibits discrete excited states.

The **second 0^+ state** at $E_x = 20.2$ MeV has the same spin, isospin and parity as the ground state.

Therefore, this excited state should have quite a **different spatial structure** in order to be orthogonal to the ground state.

Important questions to be answered are:

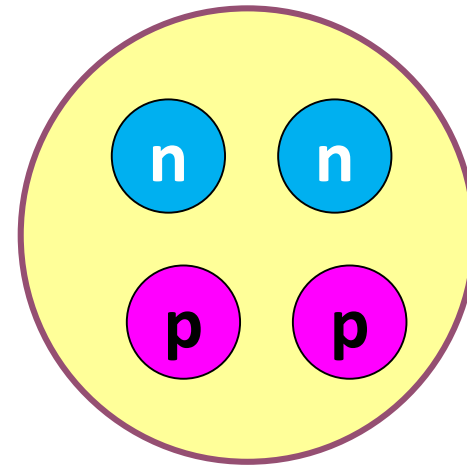
Is it possible to reproduce the energies of the 1st and 2nd 0^+ states simultaneously?



All the questions were clearly answered in our paper:

E. Hiyama, B.F. Gibson and M. Kamimura,
Phys. Rev. C **70** (2004) 031001(R)

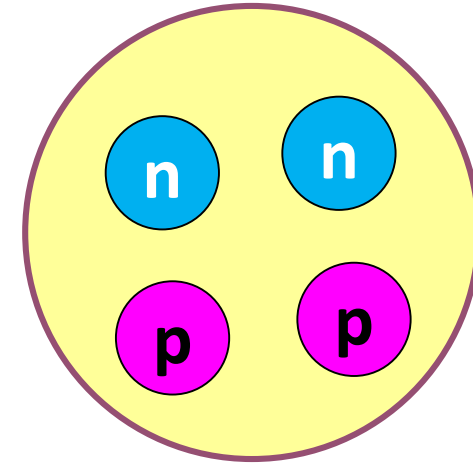
based on 4-body calculation with our method.



In this work, we took

AV8' NN force + Coulomb force

Use of the two-body force alone does **not** reproduce the observed binding energies of ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$.

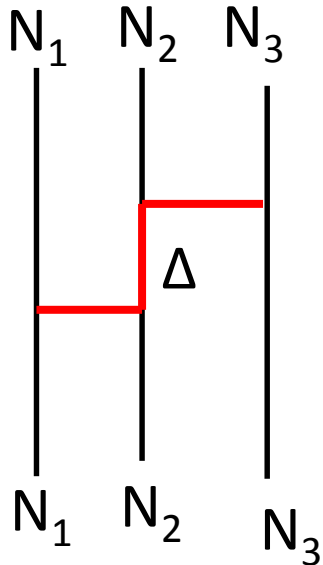


We additionally employ a **phenomenological three-body force** (2-range Gaussians) so as to reproduce those binding energies.



In atomic Physics, there are some discussion on three-body parameter.

Also, in nuclear physics, three-body force is one of important issue.



$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^2 W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$W_1(T = 1/2) = -2.04 \text{ MeV} \quad b_1 = 4.0 \text{ fm}$$

$$W_2(T = 1/2) = +35.0 \text{ MeV} \quad b_2 = 0.75 \text{ fm}$$

Use of

AV8' + Coulomb force + phenomenological 3-body force
well reproduces simultaneously

B.E.(³H) : -8.42 MeV (CAL)
-8.48 (EXP)

B.E.(³He) : -7.74 MeV (CAL)
-7.77 (EXP)

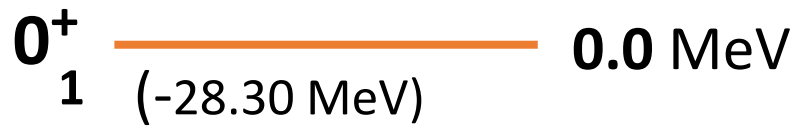
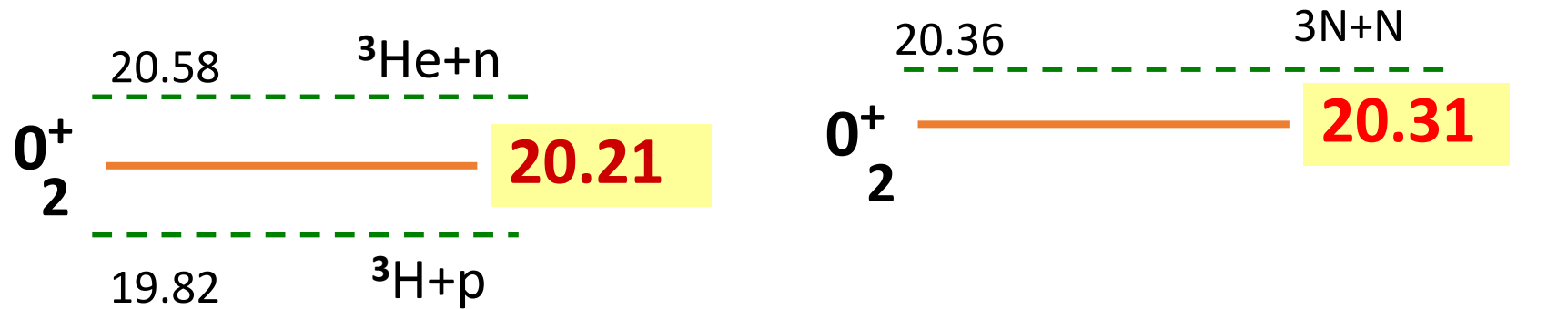
B.E.(⁴He) : **-28.44** MeV (CAL)
-28.30 (EXP)

r.m.s. charge radius of ⁴He (0⁺) : ₁ **1.658** fm (CAL)
1.671 ± 0.014 fm (EXP)

How about the second 0⁺ state?

Answer to question 1) :

Energy of 0_2^+ of ^4He



EXP

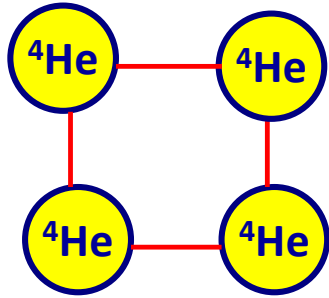
^4He



CAL

^4He

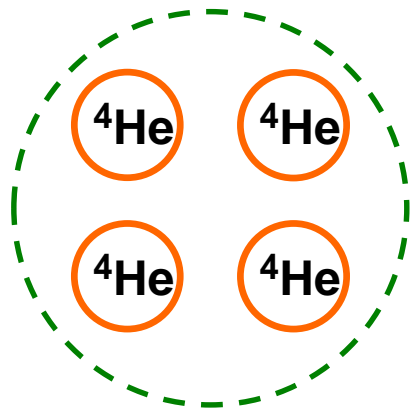
I am nuclear physicist. So, I am interested in finding the universality (similarity) of nuclear physics and atomic physics.



For example, let's think about ${}^4\text{He}$ bosonic atom system and nuclear physics.

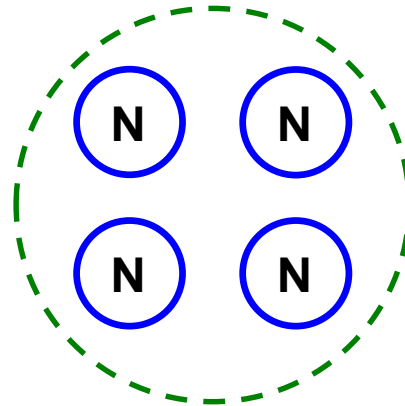
Similarity in energy levels of **atomic** and **nuclear**
4-body systems with large scattering length
(the two lowest-lying states)

Atom



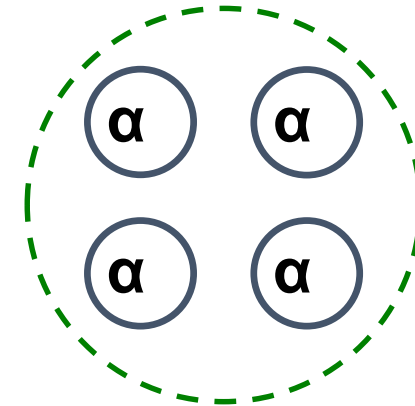
${}^4\text{He}$ - atom tetramer

Nucleus



${}^4\text{He}$ nucleus
(4 - Nucleon)

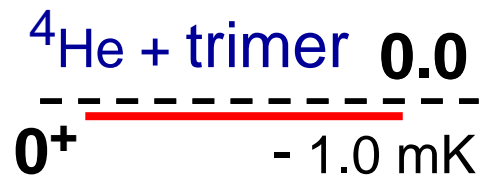
$\alpha = {}^4\text{He}$ nucleus



${}^{16}\text{O}$ nucleus
(4 - α)

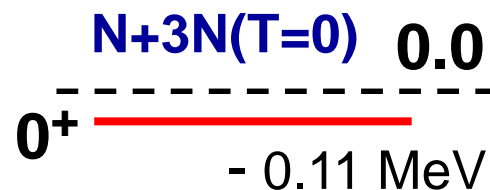
Similarity in energy levels of **atomic** and **nuclear**
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Atom

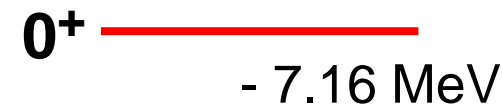
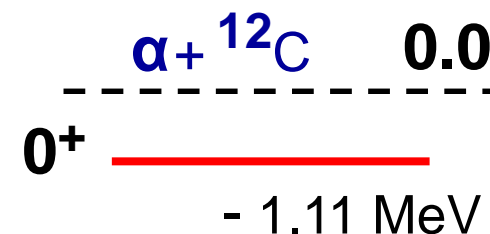


${}^4\text{He}$ -atom tetramer

Nucleus



${}^4\text{He}$ nucleus
 (4-Nucleon)



${}^{16}\text{O}$ nucleus
 (4- α)

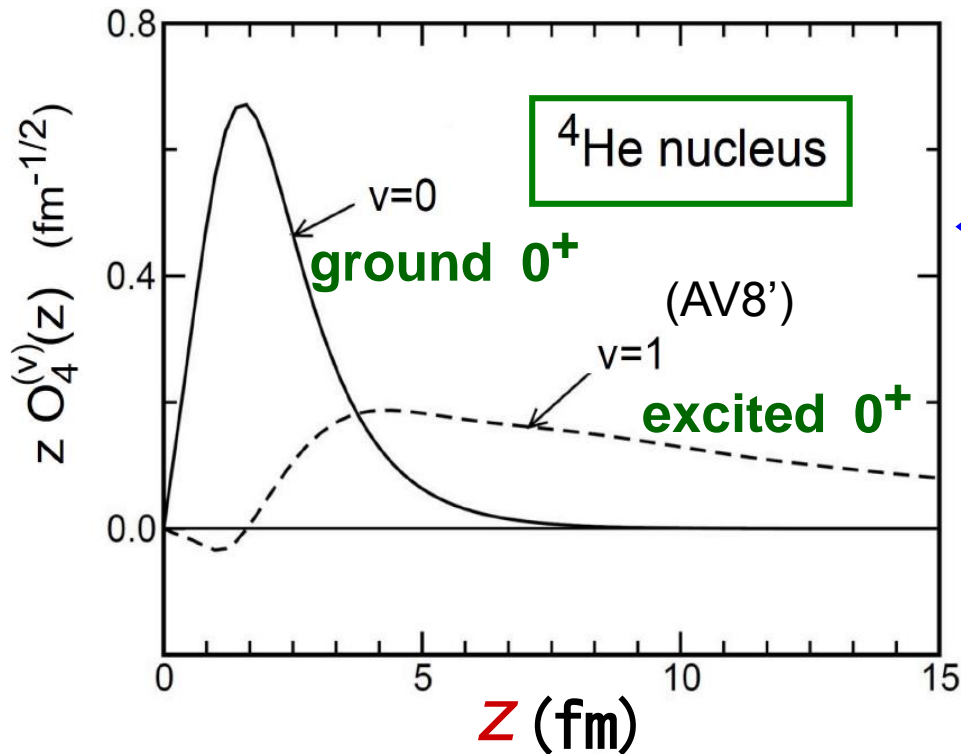
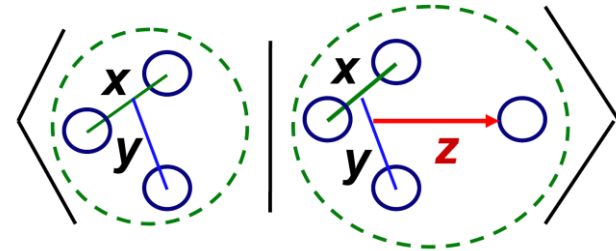
Do we have

Similarity in **wave functions** of **atomic** and **nuclear**
4-body systems with large scattering length ?

Overlap function

$$O_4^{(v)}(\mathbf{z}) = \langle \Psi_3^{(0)} | \Psi_4^{(v)} \rangle_{\mathbf{x}, \mathbf{y}} =$$

(3-body)_{v=0} (4-body)_{v=0,1}



Overlap function
for ${}^4\text{He}$ nucleus

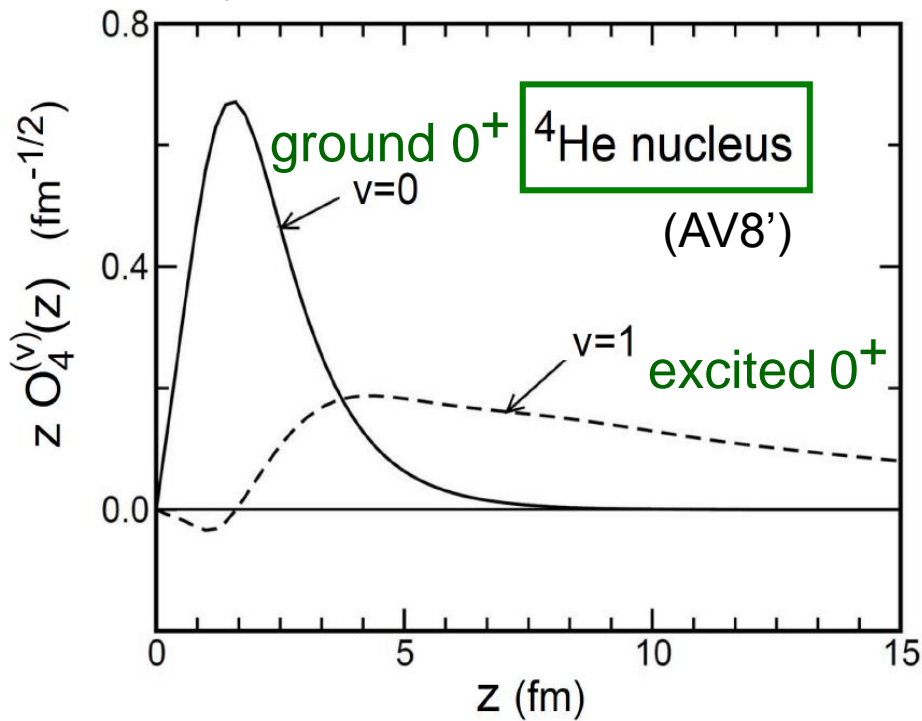
How is in
 ${}^4\text{He}$ -atom tetramer ?

Universality in overlap function

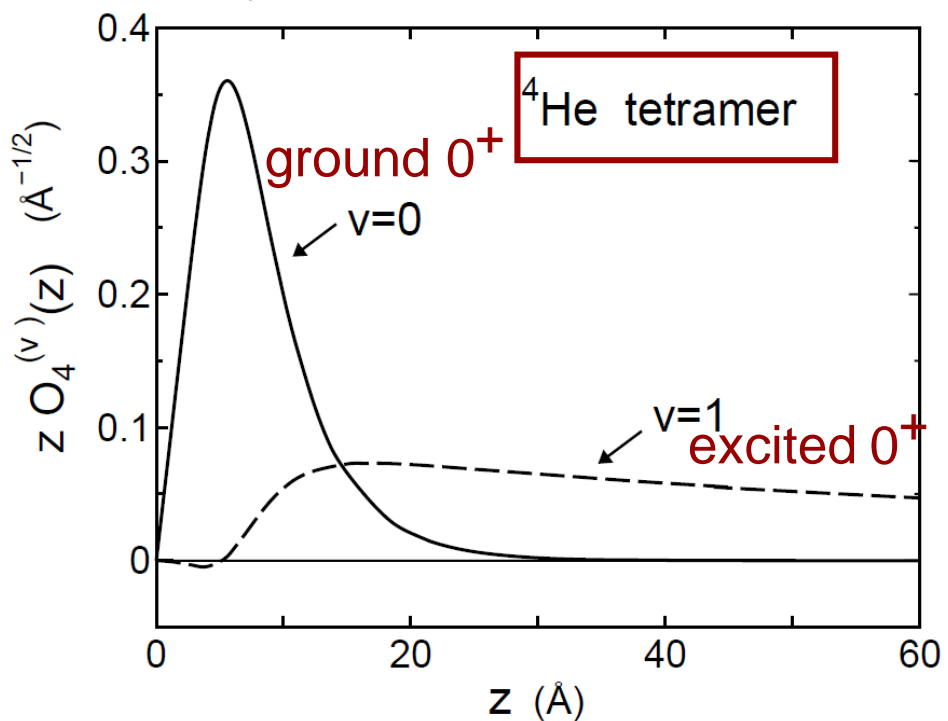
$$O_4^{(v)}(\mathbf{z}) = \langle \Psi_3^{(0)} | \Psi_4^{(v)} \rangle_{\mathbf{x}, \mathbf{y}} = \left(\text{Diagram 1} \right) \left| \left(\text{Diagram 2} \right) \right.$$

(3-body)_{v=0}
(4-body)_{v=0,1}

E. Hiyama *et al.*, PRC 70 (2004)



E. Hiyama & M.K., PRA 85 (2012)

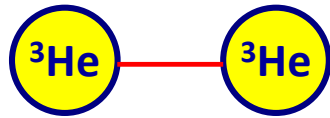


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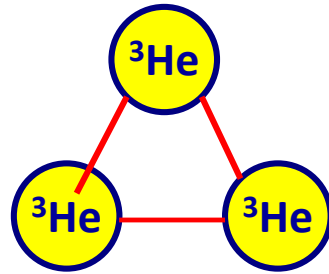
LM2M2 potential

What about fermionic atom systems?

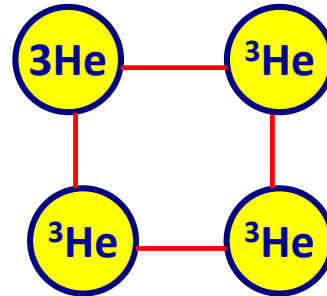
For example, ^3He



unbound



unbound

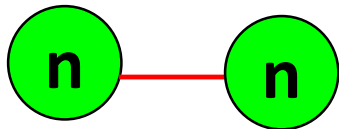


unbound

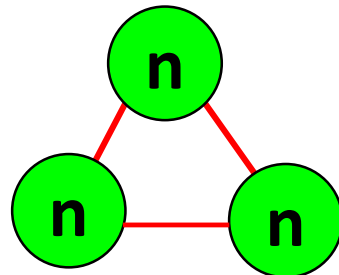
..... 35 ^3He atoms are bound?

R. Guardiola, J. Navarro, PRL.84 (2001) 1144
Diffusion Monte Carlo calculation

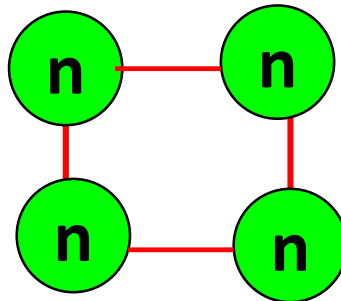
What about nuclear physics?



unbound



May not be bound



?

..... Neutron matter

bound

How many neutrons make bound?

Difference: neutron has isospin. NN interaction is more complicated than atomic system. But, we have similar question.



Candidate Resonant Tetraneutron State Populated by the $^4\text{He}(^8\text{He}, ^8\text{Be})$ Reaction

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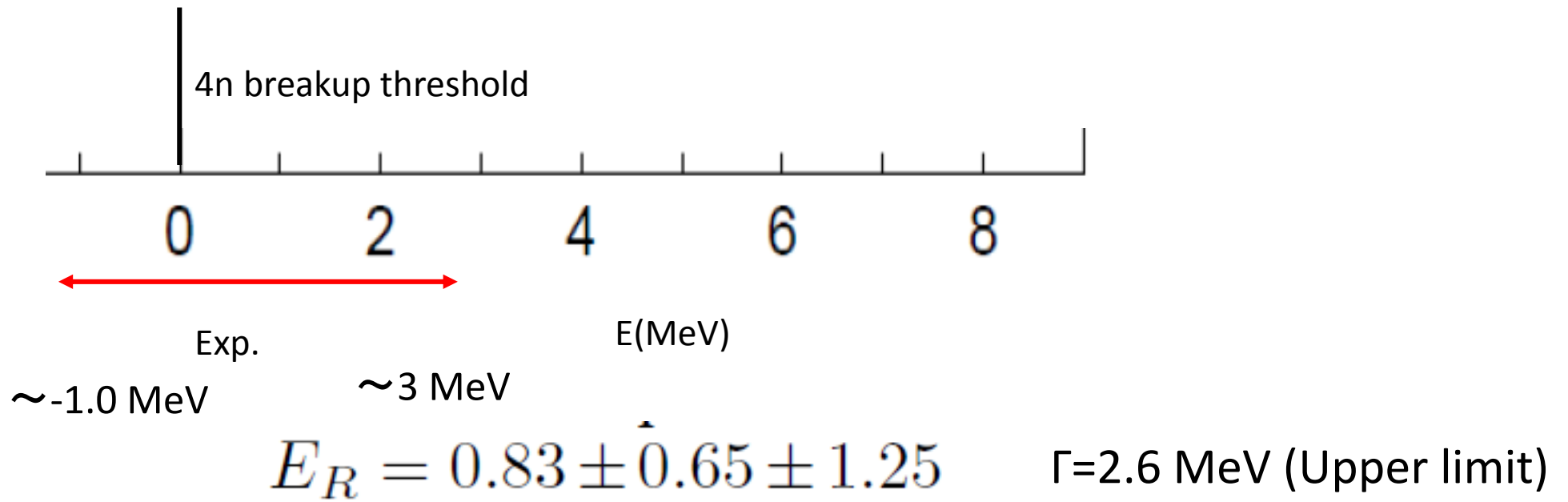
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A candidate resonant tetraneutron state is found in the missing-mass spectrum obtained in the double-charge-exchange reaction $^4\text{He}(^8\text{He}, ^8\text{Be})$ at 186 MeV/u. The energy of the state is $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$ MeV above the threshold of four-neutron decay with a significance level of 4.9σ . Utilizing the large positive Q value of the $(^8\text{He}, ^8\text{Be})$ reaction, an almost recoilless condition of the four-neutron system was achieved so as to obtain a weakly interacting four-neutron system efficiently.

Observation of
Tetra neutron system!



Now, we have new data for tetraneutron system.

Theoretical important issue:

- Can we describe observed 4n system using realistic NN interaction and T=3/2 three-body force?

Motivated by experimental data, we started to study tetra neutron system.

Possibility of generating a 4-neutron resonance with a $T = 3/2$ isospin 3-neutron force

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We consider the theoretical possibility of generating a narrow resonance in the 4-neutron system as suggested by a recent experimental result. To that end, a phenomenological $T = 3/2$ 3-neutron force is introduced, in addition to a realistic NN interaction. We inquire what the strength should be of the $3n$ force to generate such a resonance. The reliability of the 3-neutron force in the $T = 3/2$ channel is examined, by analyzing its consistency with the low-lying $T = 1$ states of ${}^4\text{H}$, ${}^4\text{He}$, and ${}^4\text{Li}$ and the ${}^3\text{H} + n$ scattering. The *ab initio* solution of the $4n$ Schrödinger equation is obtained using the complex scaling method with boundary conditions appropriate to the four-body resonances. We find that to generate narrow $4n$ resonant states a remarkably attractive $3N$ force in the $T = 3/2$ channel is required.

Introduction : historical overview for tetraneutron system

Search for tetraneutron system as a bound or resonant state has been performed for about 50 years...

It was so difficult to confirm existence of tetraneutron system.

And then, recently Shimoura san observed $4n$ system.

Regarding to theoretical calculations,

For example,

S. A. Sofianos et al., J. Phys. G23, 1619 (1997).

N. K. Timofeyuk, J. Phys. G29, L9 (2003).

S.C. Peiper et al., Phys. Rev. Lett. 90, 252501 (2003).

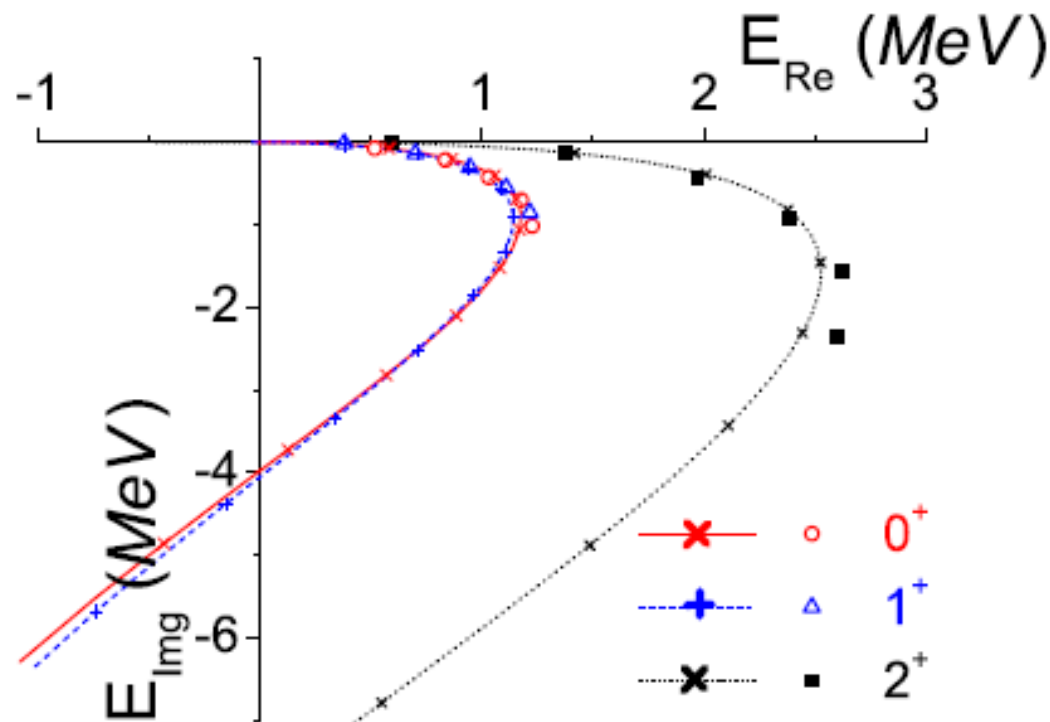
Especially, Peiper et al. suggested that there would be possibility to exist a tetraneutron system as a resonant state at $E_r=2$ MeV using AV18 + IL2 3N force with GFMC.

Unlikely evidence of
tetraneutron system

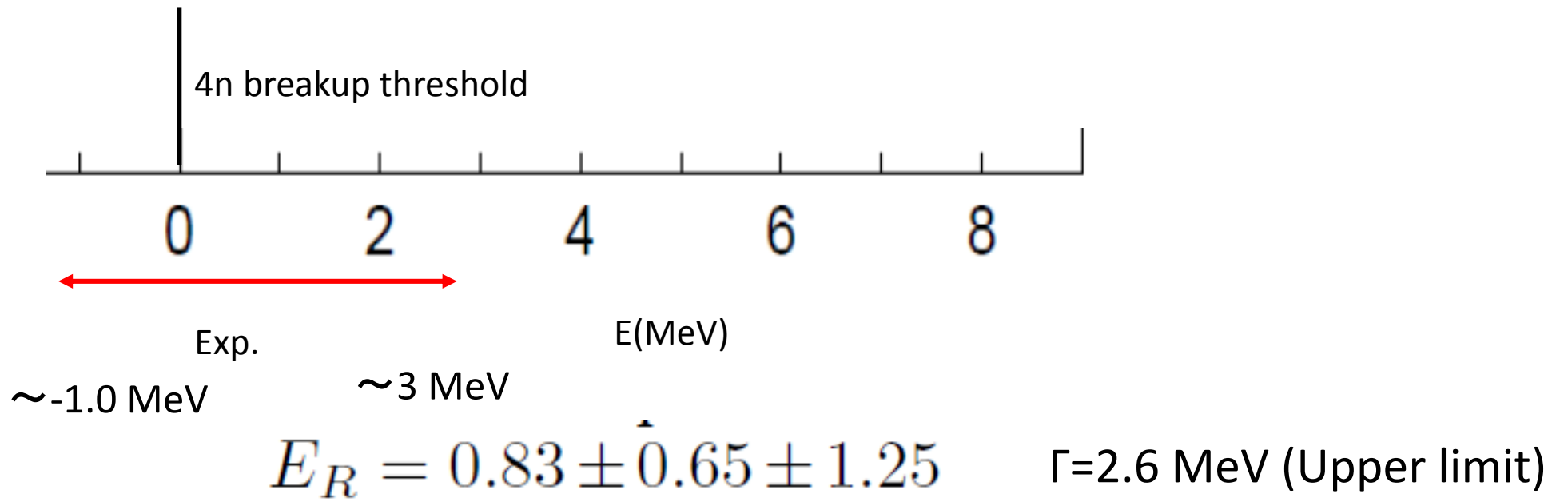
R. Lazauskas, and J. Carbonell, Phys. Rev. C72, 034003 (2005).

Charge-symmetry-breaking Reid93 nn potential +a phenomenological 4N force

$$V_{4n} = -W\rho e^{-\frac{\rho}{\rho_0}}, \quad \text{hyperradius } \rho = \sqrt{x^2 + y^2 + z^2}$$



Since we did not have any observed data at that time, then in this paper it was difficult to tune the strength of W . If $W=0$, energy pole goes to the third quadrant.



Now, we have new data for tetraneutron system.

Theoretical important issue:

- Can we describe observed 4n system theoretically?

$$E_R = 0.83 \pm \overset{\wedge}{0.65} \pm 1.25 \quad \Gamma=2.6 \text{ MeV (Upper limit)}$$

For the study of tetraneutron system

We should consider interaction and method:

NN interaction: realistic NN interaction

Method: They reported the energy of tetraneutron was bound energy region to resonant energy region.

Especially, for the resonant energy region, we should use Complex scaling method.

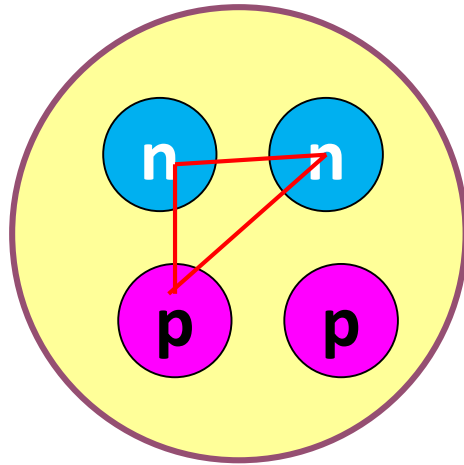
For this purpose, we use AV8 NN interaction (central, LS, Tensor).

The NN potential is applicable for complex scaling method which is one of powerful method to obtain energy and decay width.

Any other missing part in our Hamiltonian?

In 2005, Lazauskas et al. already pointed out that only two-body NN interaction could not find any existence of tetraneutron system.

We need $T=3/2$ three-nucleon force.

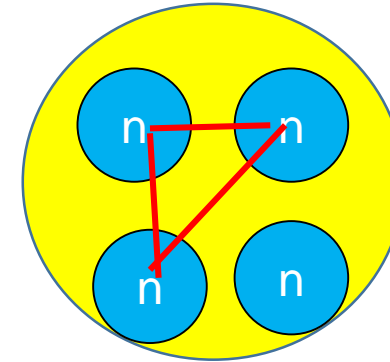


${}^4\text{He}$

$$T_{\text{NNN}} = 1/2$$



Benchmark test, we use $T=1/2$ three-body force.



4n

$$T_{\text{nnn}} = 3/2$$



We need $T=3/2$ three-body force. But, we have no idea how much attraction we need.

$$T_z = 4 \times (-1/2) = -2, \quad T = 2$$

To distinguish proton or neutron, we use isospin.

Proton $\Rightarrow t=1/2, t_z=+1/2$

Neutron $\Rightarrow t=1/2, t_z=-1/2$

$$T_z = 2 \times 1/2 + 2 \times (-1/2) = 0, \quad T = 0$$

a phenomenological three-body force for ${}^4\text{He}$.

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^2 W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$\begin{aligned} W_1(T = 1/2) &= -2.04 \text{ MeV} & b_1 &= 4.0 \text{ fm} \\ W_2(T = 1/2) &= +35.0 \text{ MeV} & b_2 &= 0.75 \text{ fm} \end{aligned}$$



These parameters (W_1, W_2, b_1, b_2) are determined so as to reproduce the binding energies of the ground states of ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$.

For $4n$ system, we need $T=3/2$ three-body force. We use the same potential with $T=1/2$, but, different parameter of W_1 .

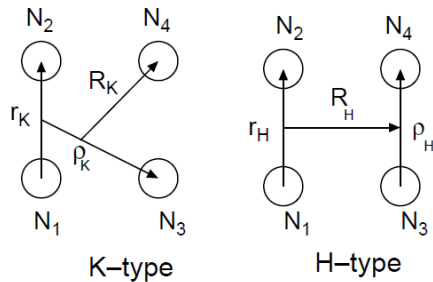
$W_1(T=3/2) = \text{free}$ $b_1 = 4.0 \text{ fm} \Rightarrow W_1$ should be adjusted so as to reproduce the observed $4n$ system

$W_2(T=3/2) = +35 \text{ MeV}$ $b_2 = 0.75$

The observed 4n system was reported from the bound region to resonant region. In order to obtain energy position (E_r) and decay width (Γ), we use complex scaling method.

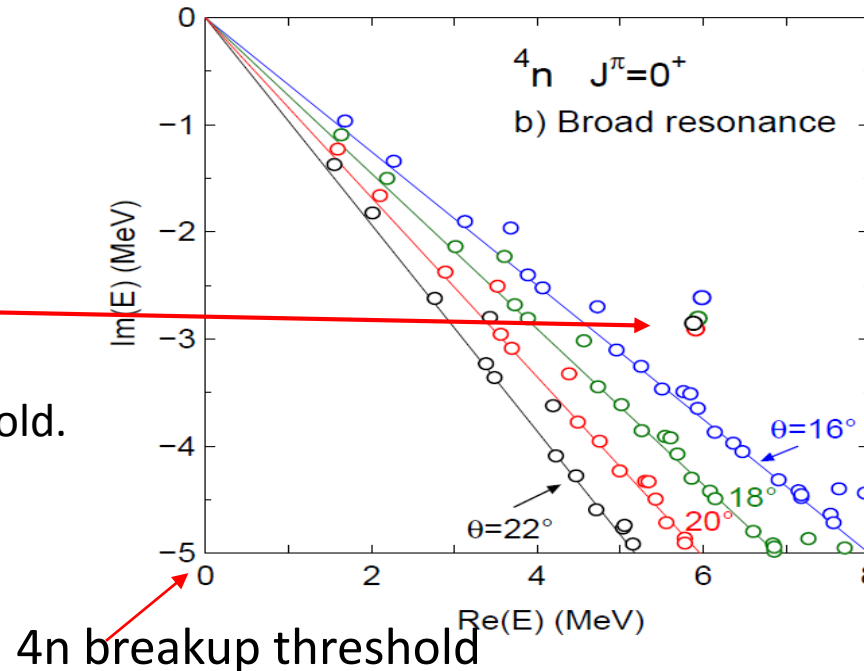
$$[H(\theta) - E(\theta)]\Psi_{JM,TT_z}(\theta) = 0$$

$$\Psi_{JM,TT_z}(\theta) = \sum_{\alpha} C_{\alpha}^{(K)}(\theta)\Phi_{\alpha}^{(K)} + \sum_{\alpha} C_{\alpha}^{(H)}(\theta)\Phi_{\alpha}^{(H)}$$

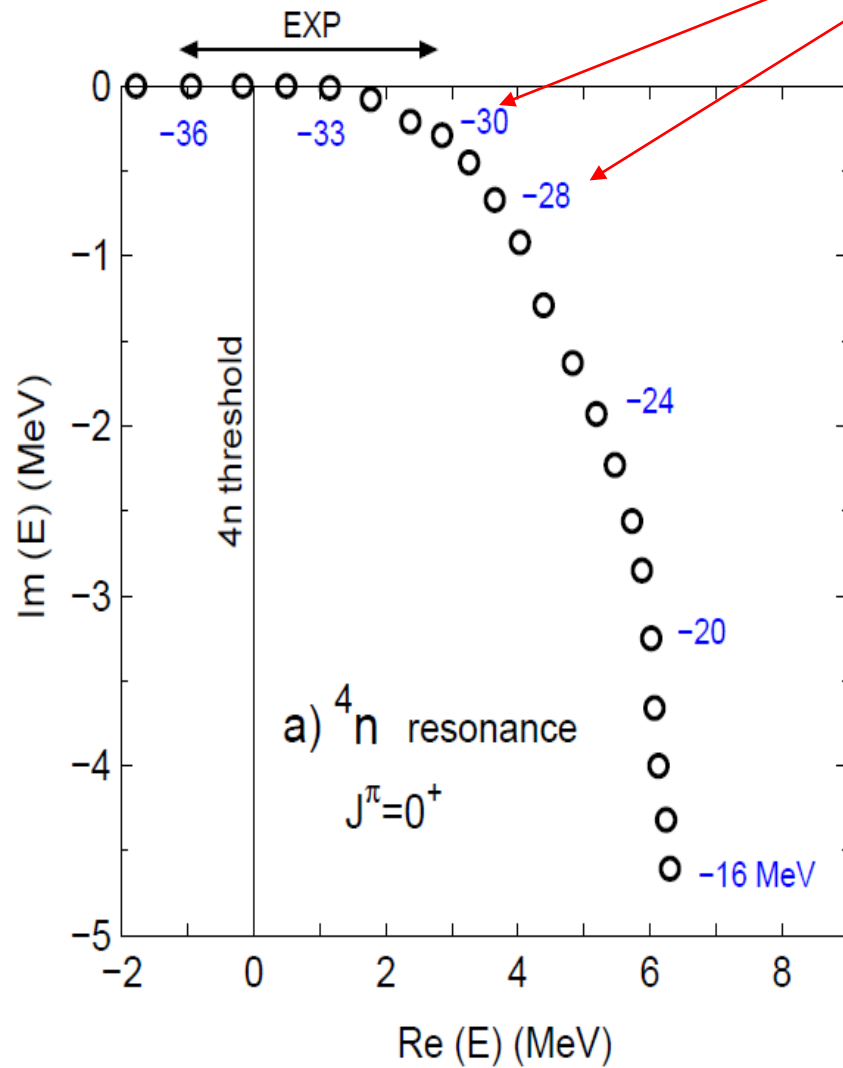


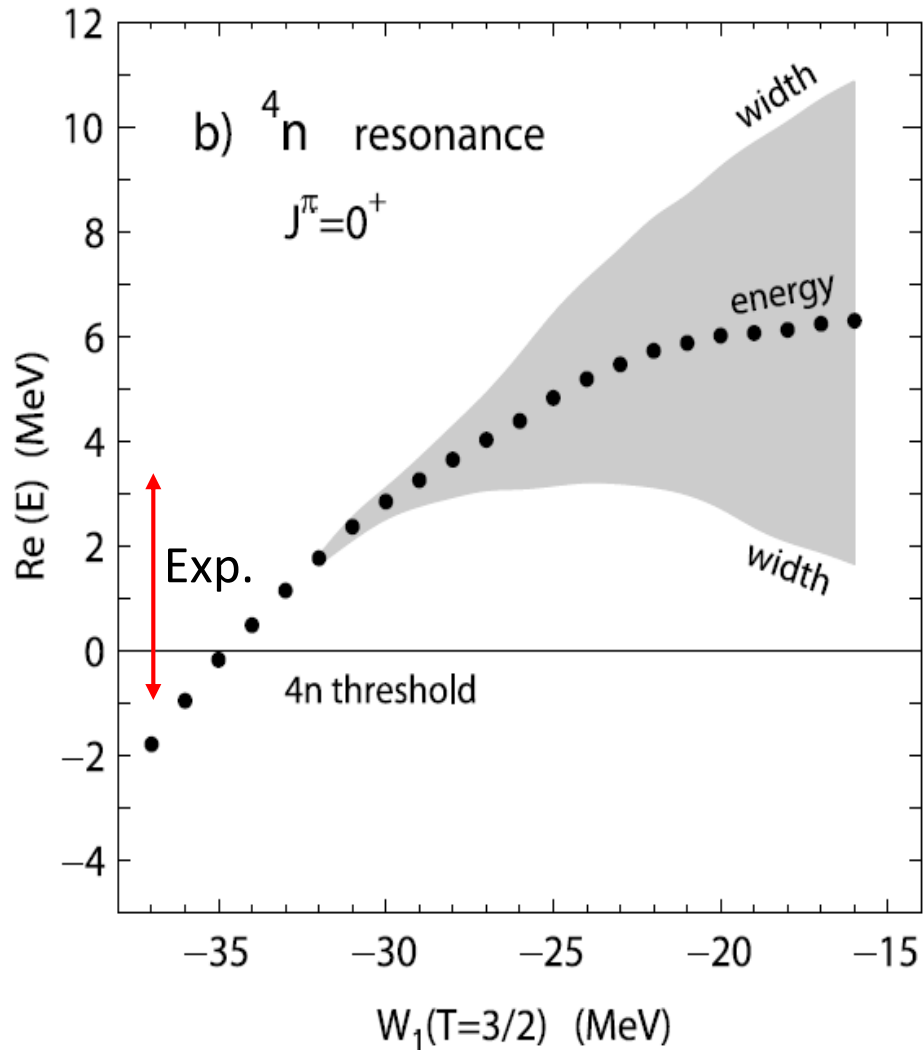
$$r_c \rightarrow r_c e^{i\theta}, \quad R_c \rightarrow R_c e^{i\theta}, \quad \rho_c \rightarrow \rho_c e^{i\theta} \quad (c = K, H)$$

The energy pole is stable with respect to θ .
 $\text{Re}(E)$ corresponds to energy with respect to 4n breakup threshold.
 $\text{Im}(E)$ corresponds to $\Gamma/2$.



energy trajectory of $J=0^+$ state changing W_1





In order to reproduce the data of 4n system,
 We need $W_1(T=3/2) = -36 \text{ MeV} \sim -30 \text{ MeV}$.
 Attraction is 15 times
 Stronger.

It should be noted that $W_1(T=1/2) = -2.04 \text{ MeV}$
 to reproduce the observed binding energy
 of ${}^4\text{He}$, ${}^3\text{He}$ and ${}^3\text{H}$.

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^2 W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$W_1(T=3/2) = \text{free} \quad b_1 = 4.0 \text{ fm}$$

$$W_2(T=3/2) = +35 \text{ MeV} \quad b_2 = 0.75 \text{ fm}$$

Question: W_1 value for $T=3/2$ is reasonable?

To check the validity of three-body
 force, we calculate the energies
 of ${}^4\text{H}$, ${}^4\text{He}(T=1)$, ${}^4\text{Li}$.

${}^4\text{H}$

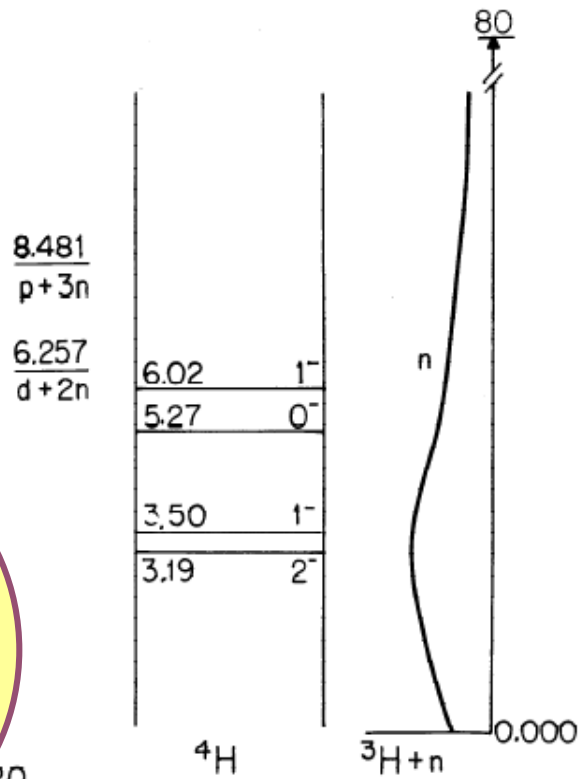
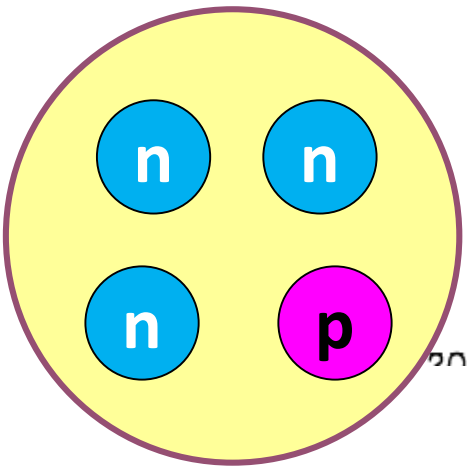


Table 4.1: Energy levels of ${}^4\text{H}$ defined for channel radius $a_n = 4.9$ fm. All energies and widths are in the c.m. system.

E_x (MeV)	J^π	T	Γ (MeV)	Decay	Reactions
g.s. ^a	2^-	1	5.42	$n, {}^3\text{H}$	1, 11
0.31	1^-	1	6.73 ^b	$n, {}^3\text{H}$	11, 12
2.08	0^-	1	8.92	$n, {}^3\text{H}$	
2.83	1^-	1	12.99 ^c	$n, {}^3\text{H}$	11, 12

^a 3.19 MeV above the $n + {}^3\text{H}$ mass.

^b Primarily ${}^3\text{P}_1$.

^c Primarily ${}^1\text{P}_1$.

${}^7.718$
 $n+3p$

${}^5.494$
 $d+2p$

19
 ${}^4\text{Li}$

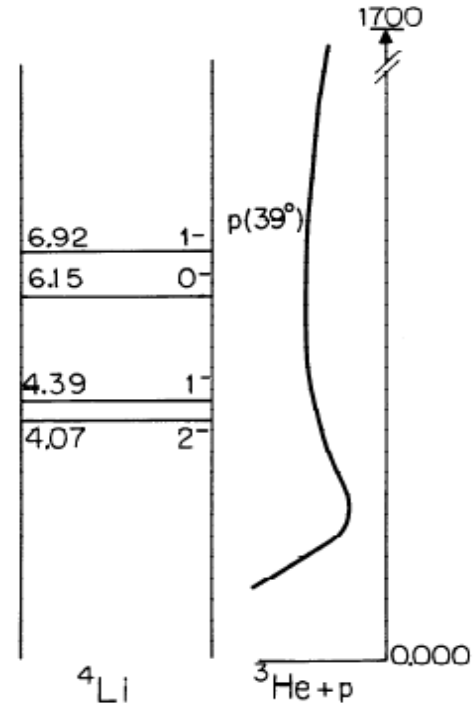


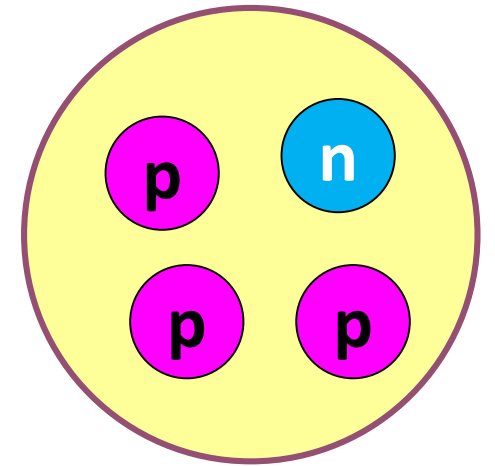
Table 4.24: Energy levels of ${}^4\text{Li}$ defined for channel radius $a_p = 4.9$ fm. All energies and widths are in the c.m. system.

E_x (MeV)	J^π	T	Γ (MeV)	Decay	Reactions
g.s. ^a	2^-	1	6.03	$p, {}^3\text{He}$	3
0.32	1^-	1	7.35 ^b	$p, {}^3\text{He}$	3
2.08	0^-	1	9.35	$p, {}^3\text{He}$	3
2.85	1^-	1	13.51 ^c	$p, {}^3\text{He}$	3

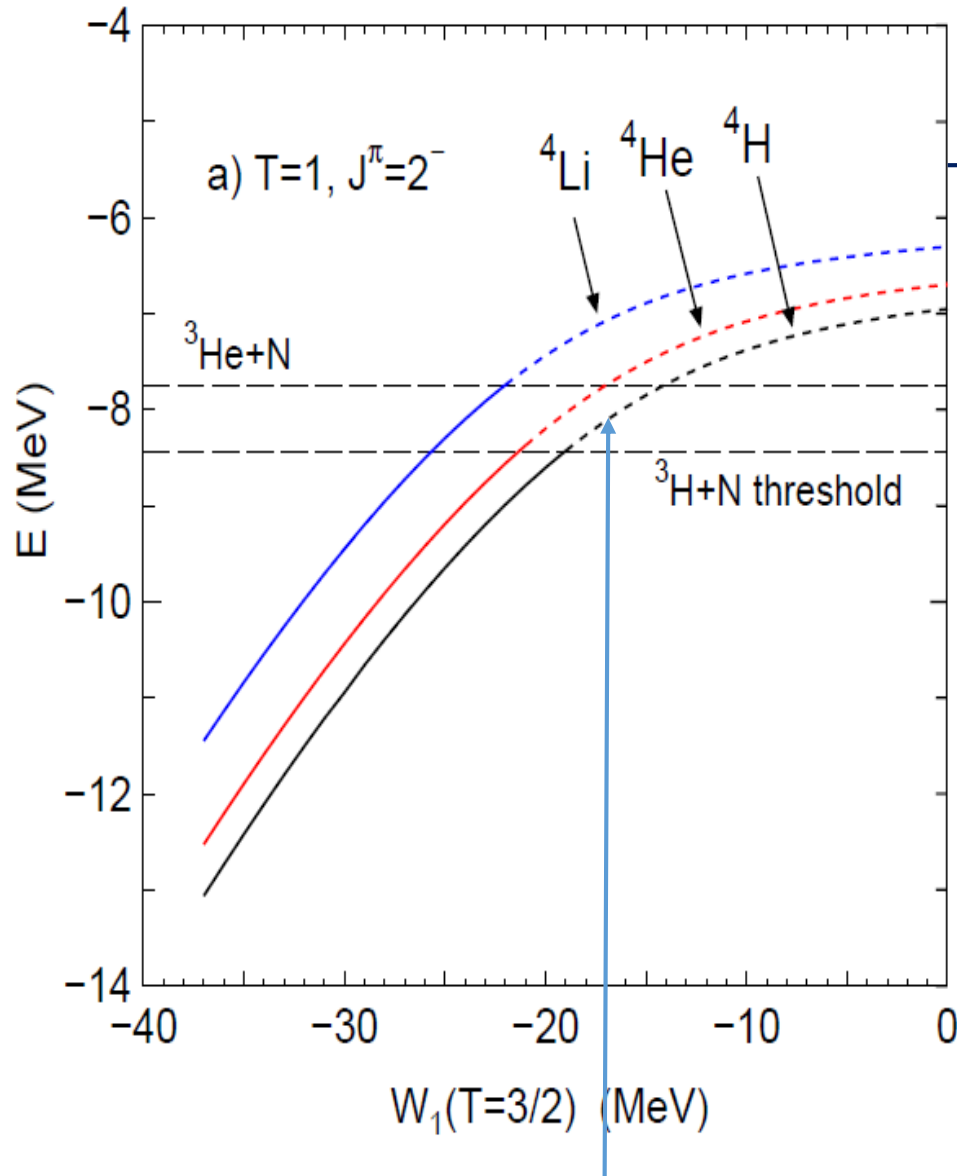
^a 4.07 MeV above the $p + {}^3\text{He}$ mass.

^b Primarily ${}^3\text{P}_1$.

^c Primarily ${}^1\text{P}_1$.



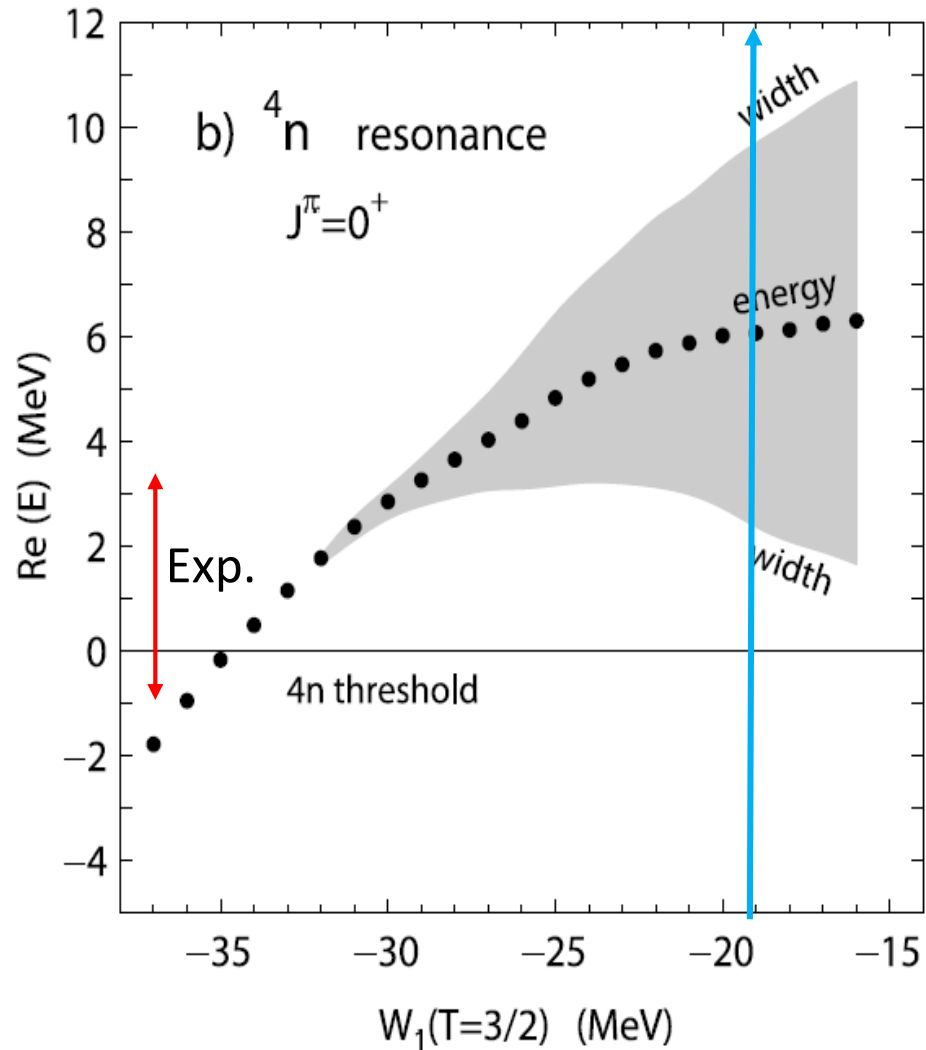
${}^4\text{Li}$



Exp. ${}^4\text{H}$ (-5.29 MeV)

If we use $W_1 = -36 \text{ MeV} \sim -30 \text{ MeV}$ to reproduce the observed data of $4n$, we have strong binding energies of ${}^4\text{H}$, ${}^4\text{He}$ ($T=1$) and ${}^4\text{Li}$. This result is inconsistent with the data of $A=4$ nuclei. The $J=2^-$ state of $A=4$ nuclei should be resonant states.

On the contrary, when $W_1 \sim -18 \text{ MeV}$, we have unbound states for $A=4$ nuclei. How about tetra-neutron system?



If $W_1(T=3/2) \sim -18$ MeV, the energy of tetraneutron is ~ -6 MeV and $\Gamma=8$ MeV, which is inconsistent with recent data of tetraneutron.

still 9 times strong attraction

It should be noted that $W_1(T=1/2)=-2.04$ MeV to reproduce the observed binding energy of ${}^4\text{He}$, ${}^3\text{He}$ and ${}^3\text{H}$.

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^2 W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

$$W_1(T=3/2) = \text{free} \quad b_1 = 4.0 \text{ fm}$$

$$W_2(T=3/2) = +35 \text{ MeV} \quad b_2 = 0.75 \text{ fm}$$

How do we consider this inconsistency?

- The $T=3/2$ force is just a phenomenological.

$$V_{ijk}^{3N} = \sum_{T=1/2}^{3/2} \sum_{n=1}^2 W_n(T) e^{-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_n^2} \mathcal{P}_{ijk}(T)$$

Should we consider spin-dependent term in three-body force?
Tensor force, spin-orbit force???

Are there some three-body components to contribute to the energy of tetraneutron system but not to contribute to the energy of 4H , 4He and 4Li ?
There is ambiguity for NN interaction with $T=1$? I have NO idea!

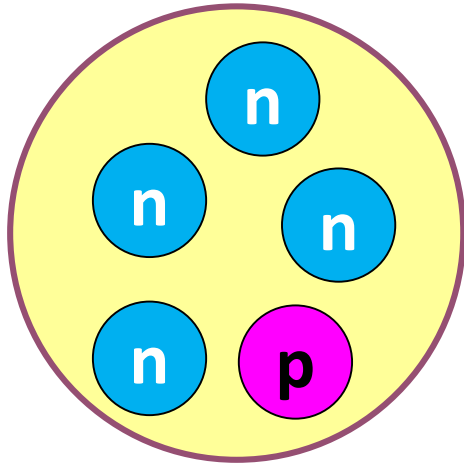
The confirmation experiments for $4n$ will be performed at RIBF.
For examples, S. Shimoura et al., NP1512-SHARAQ10
K. Kisamori and M. Marques et al., NP-1512-SAMURAI34
S. Paschalis et al., NP-1406-SAMURAI19R1

I am waiting for this confirmation.

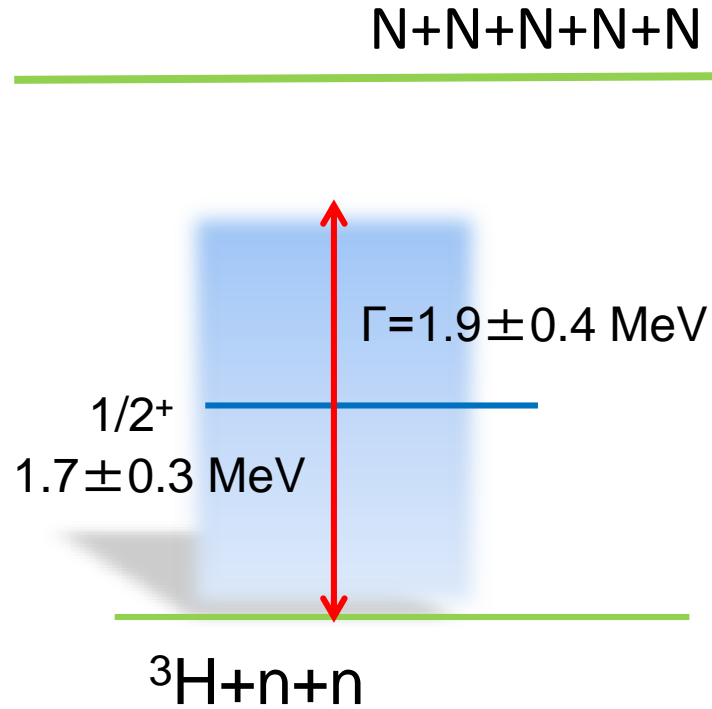
If the experiment for $4n$ is confirmed to have a bound state or resonant state,

...

Another candidate nucleus to check the validity of $T=3/2$ three-body force is ${}^5\text{H}$.

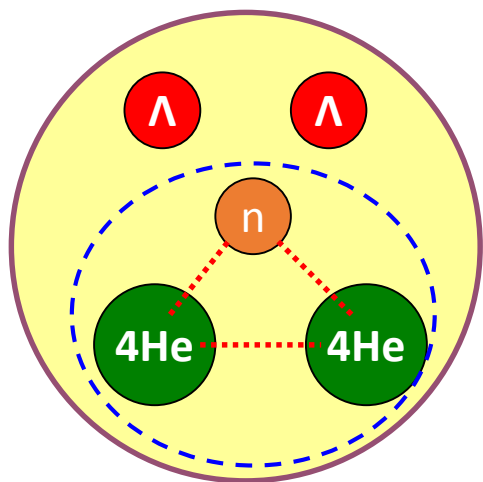


${}^5\text{H}$

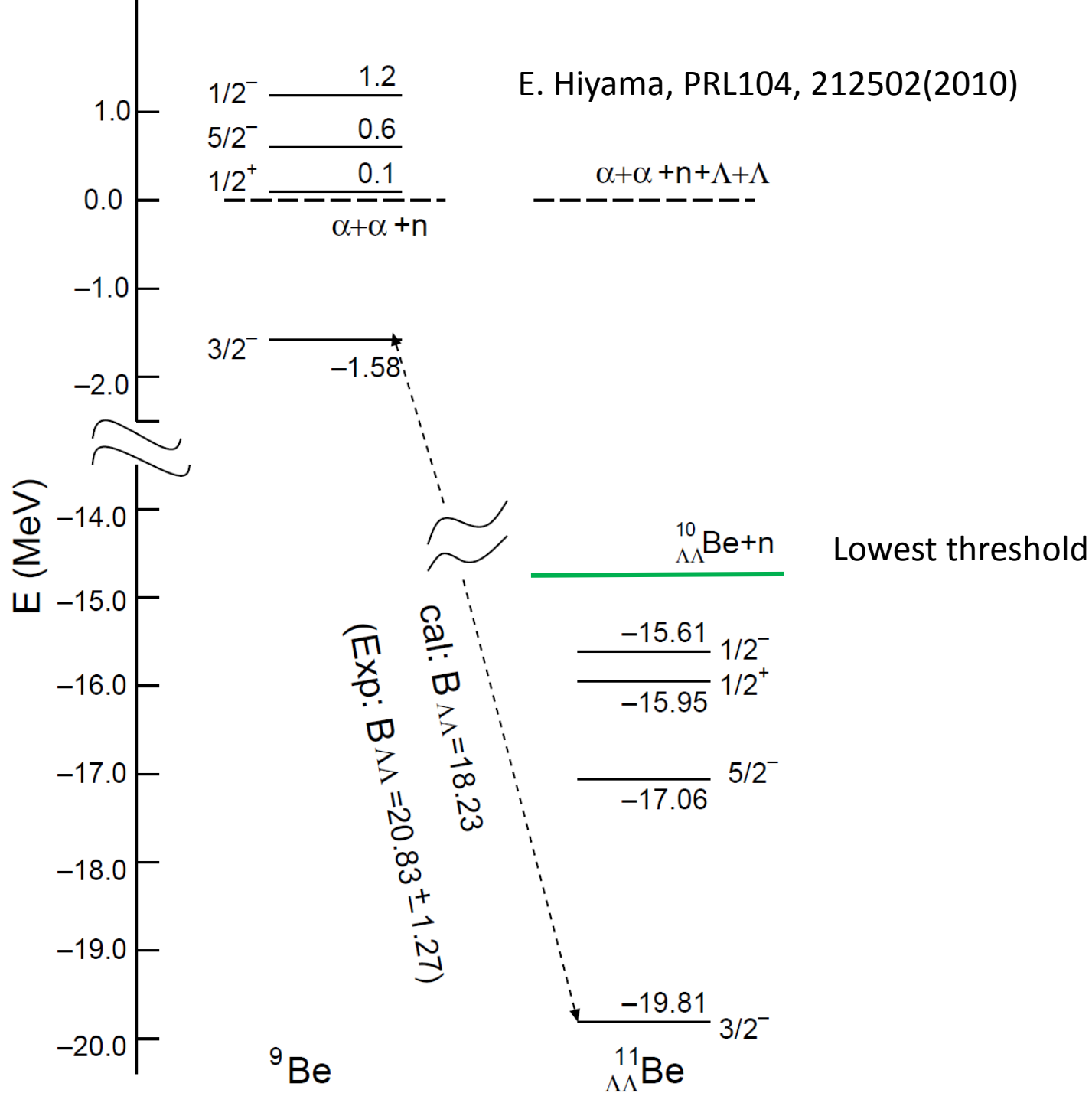


This is resonant state. This is interesting and challenging to develop our method to 5-body problem.

In the case of 5-body problem, I did calculation as follows:

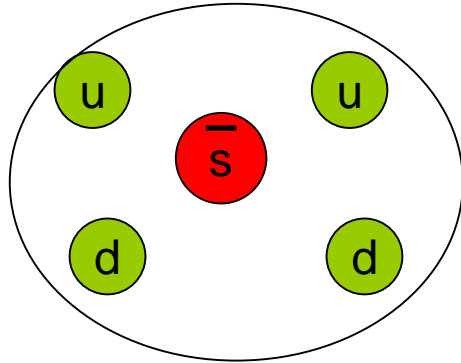


5-body bound state calculation composing of different particles



T.Nakano et al. (LEPS collaboration)

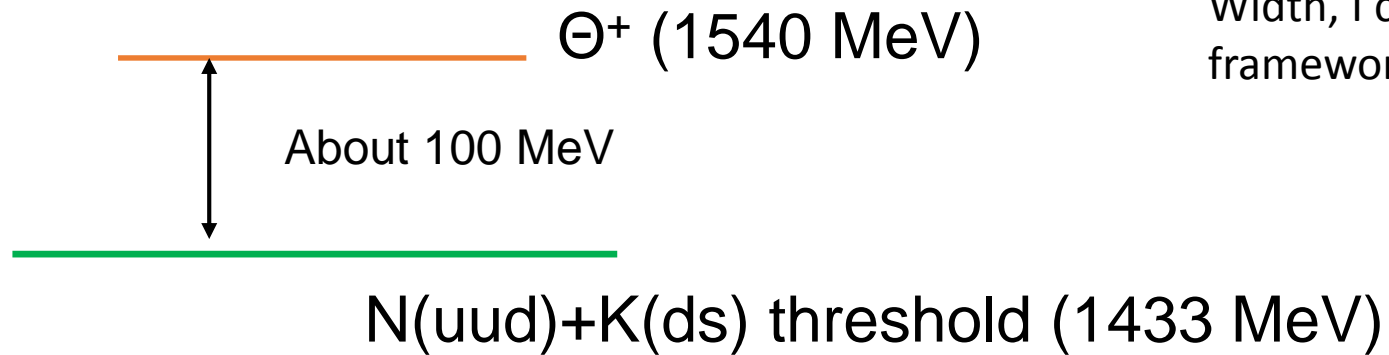
Phys. Rev. Lett. 91 (2003), 012002.



Mass: 1540MeV

$\Gamma < 25$ MeV

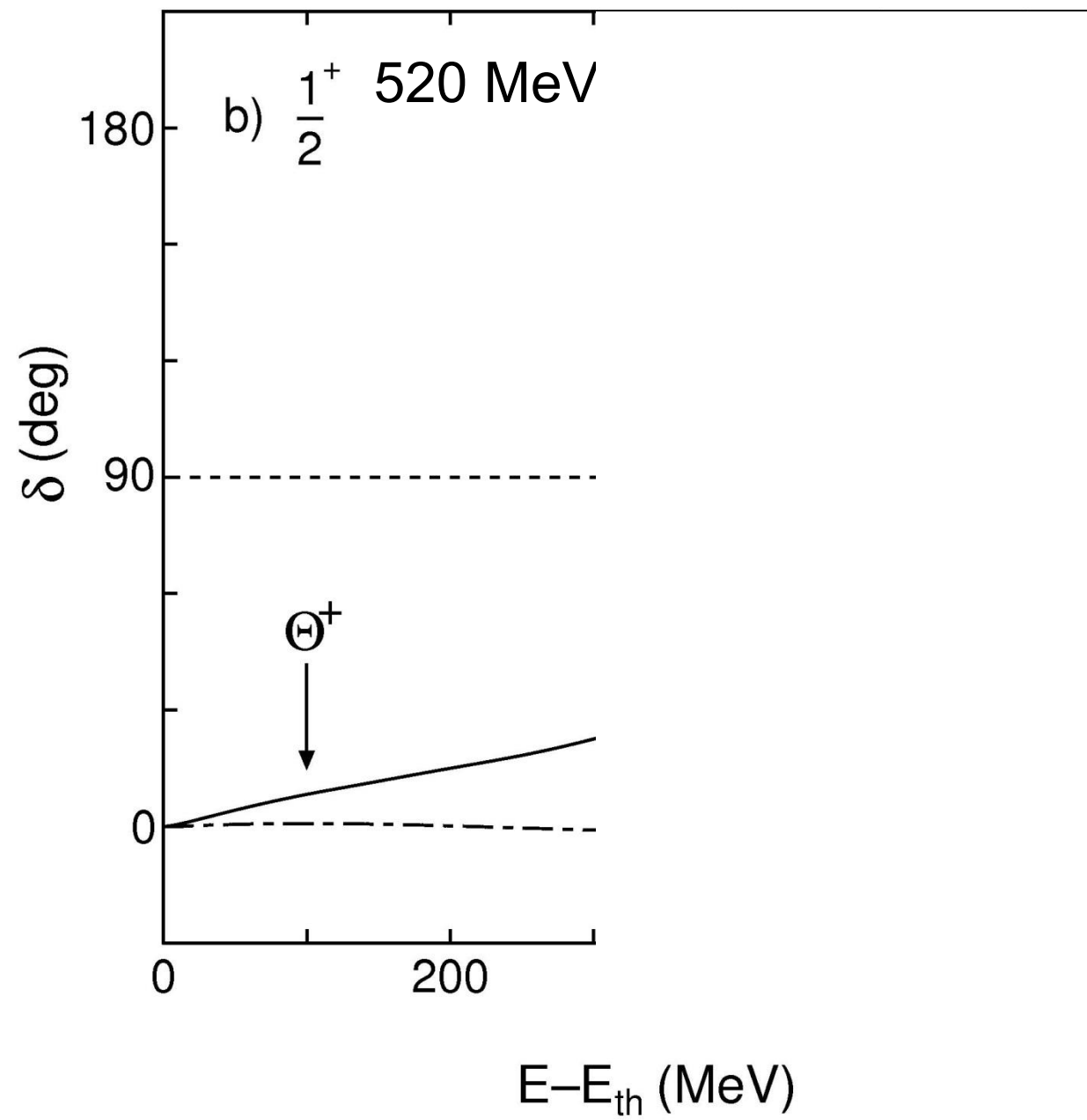
$S = +1$



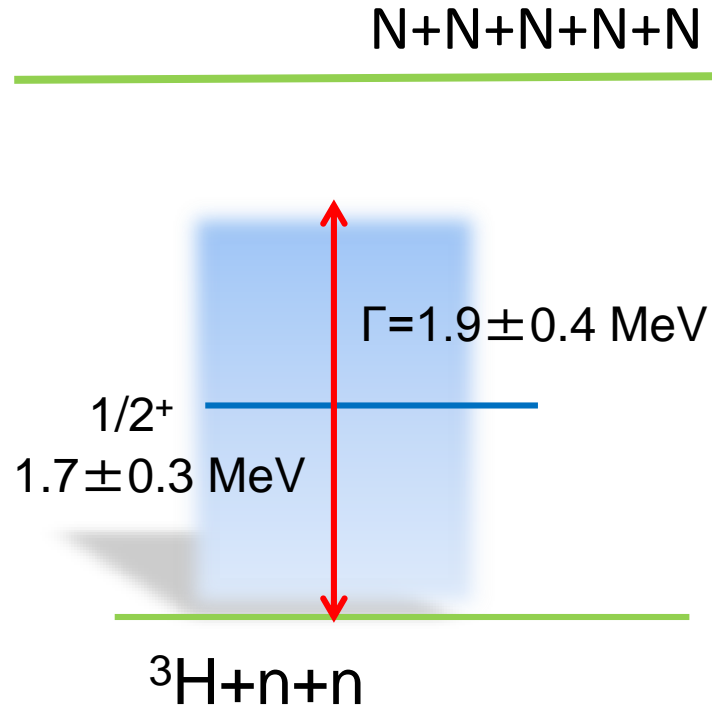
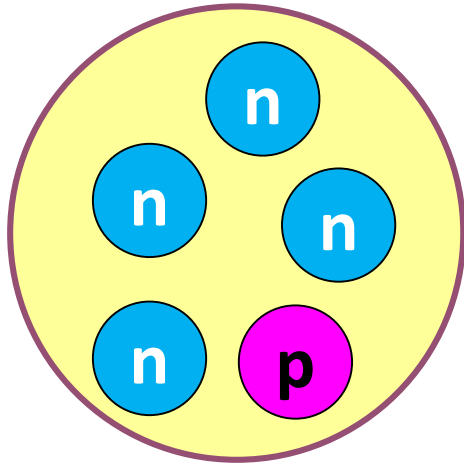
To obtain resonance energy and decay Width, I calculate phase shift within the framework of 5-body problem.

The ground state of N -- 938 MeV

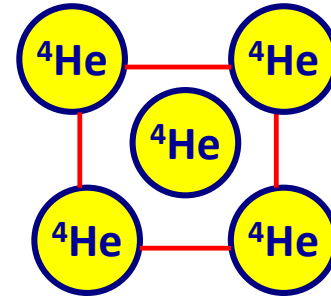
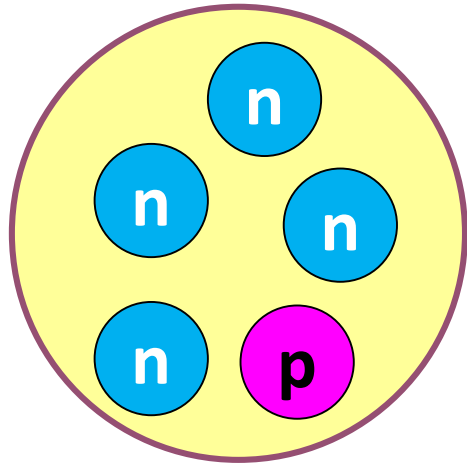
The ground state of K -- 495 MeV



Another candidate nucleus to check the validity of $T=3/2$ three-body force is ${}^5\text{H}$.



Next subject is to calculate this system.
But we should calculate ${}^3\text{H}+\text{n}+\text{n}$ scattering state.
I should use another method...? I am thinking...

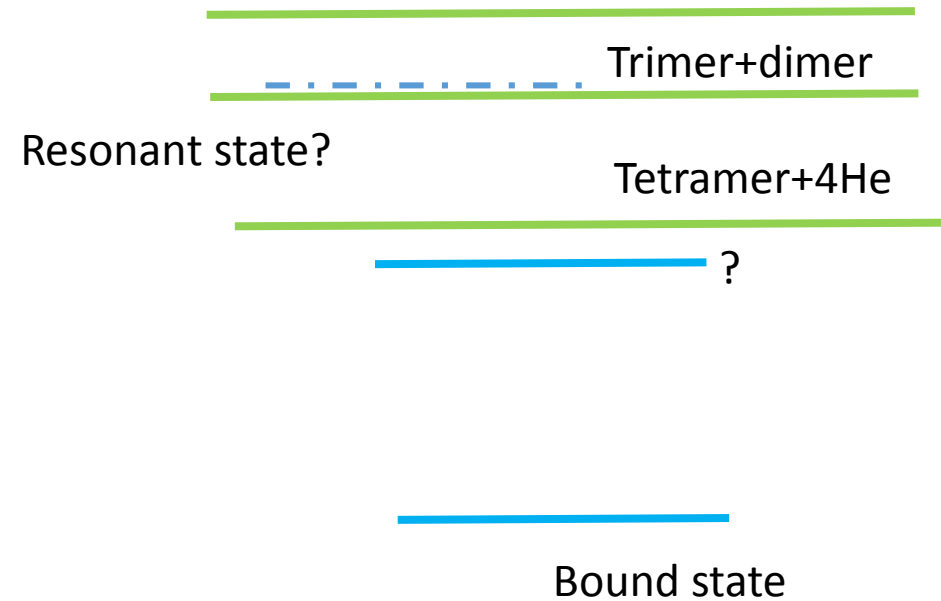
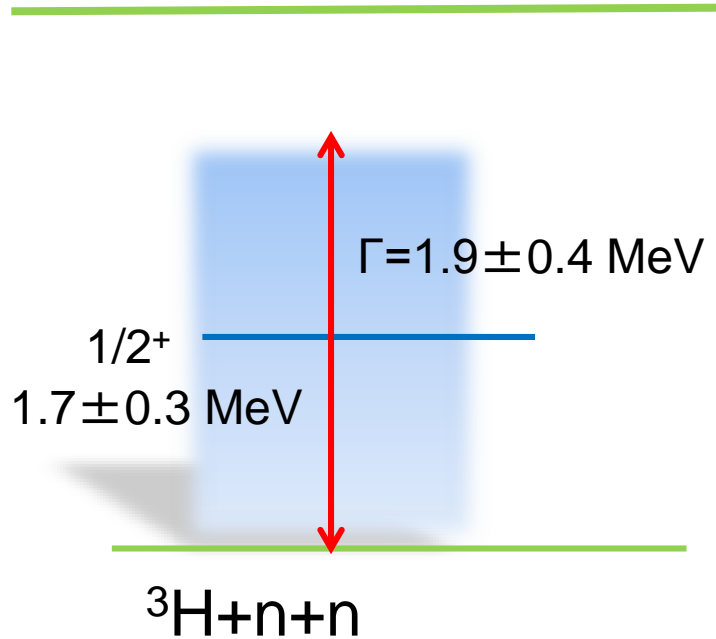


pentamer

Replace by 4He atoms, for instance...

$N+N+N+N+N$

$4\text{He}+4\text{He}+4\text{He}+4\text{He}+4\text{He}$



Discussion (no conclusion)

- I do not find good solution to reproduce the tetra neutron data.
T=3/2 three-nucleon force should be used more realistically?
It would be better to wait for new data.
- The resonant state of 5-body problem is now interesting, especially, 5H.
But, now I am thinking how to calculate the resonant state of 5H.
- Are there any resonant 5-body 4He atom systems?
If so, what is interesting?

Thank you!