Exploring the unitary limit: from few nucleons to few atoms

A. Kievsky

INFN, Sezione di Pisa (Italy)

Collaborators

- M. Gattobigio - *INLN & Nice University, Nice (France)*
- M. Viviani - *INFN & Pisa University, Pisa (Italy)*
- E. Garrido - *CSIC, Madrid (Spain)*
- A. Deltuva - *ITPA, Vilnius (Lithuania)*
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Trends in nuclear physics

Inputs from V. Efimov

- The two-nucleon system is close to the unitary limit in both states $S = 0, 1$
- The two scattering lengths are large: $a_0 \approx -23$fm and $a_1 \approx 5.4$fm
- The unitary limit constrains many aspects of the few-nucleon system
- These constrains appear in the theory called pionless
- The Hamiltonian includes a two-body contact term (controlled by a_S) and a three-body contact term (controlled by the ^{3}H binding energy)
- This last term is related to the three-body parameter

Many of these aspects were discussed at the same time in which big efforts were done to construct NN potentials and developed method to solve the few-nucleon problem

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Potential models

- In the eighties and ninties NN potentials describing a large set of NN data appear
- First models: Reid Soft Core, AV14, Bonn A and Bonn B
- These potentials describe or n-p data or p-p data but not both
- The second generation of NN potentials include CSB describing around 5000 data points with a χ^2 per datum close to 1
- Examples are: AV18 and CD Bonn
- Finally potentials from ChPT appear with similar accuracy but consistently describing two and three-boy forces

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There are two aspects

- Efimov physics put in evidence universal aspects of the few-nucleon system
- potential models are very detailed and cover to some extent universal behavior
- are these two descriptions compatible?

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- This is done in trapped atoms experimentally and theoretically
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Exploring the unitary limit with potential models

- Potential models are constructed at the physical point using many data points
- Moving from the physical point the original potential is modified
- For example the strength can be modified
- Equivalent soft potentails can be constructed
- Efimov physics is a zero-range interaction theory
- In this context potentials can be consider a regularization of contact interactions

Universality in the two-body system

scaling and unitary limits

- in the scaling limit the range of the interaction is zero

$$\phi_d = C_d e^{-k_d r}$$

$$\phi_0 = C_0 (r - a)$$

$$\phi_k = C_k(\sin kr - \tan\delta \cos kr)$$

- which is the relation between k_d, a and $\tan\delta$?

orthogonality of the states

$$\int_0^\infty \phi_d \phi_0 = 0 \rightarrow k_d^{-1} = a_B = a$$

$$\int_0^\infty \phi_d \phi_k = 0 \rightarrow k \cot\delta = -1/a_B = -1/a$$

In the scaling limit $a - a_B = 0$

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orthogonality of the states inside a cutoff r_B

$$\int_{r_B}^{\infty} \phi_d \phi_0 = 0 \rightarrow a - a_B = r_B$$

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- r_B takes into account the short range physics
- in nuclear physics, in the triplet state $S = 1$, $E_2 = 2.2245$MeV, therefore $a_B = 4.3$fm and $a_1 = 5.4$fm
- $r_B = 1.1$fm

- in the helium-helium system, $E_2 = 1.3$mK, therefore $a_B = 182\mathrm{a}_0$ and $a = 189\mathrm{a}_0$
- $r_B = 7.2\mathrm{a}_0$

constructing a soft two-body potential

$$V(r) = V_0 e^{-r^2/r_0^2}$$

V_0 and r_0 fixed to describe E_2 and a at the physical point

- The unitary limit is explored by varying V_0
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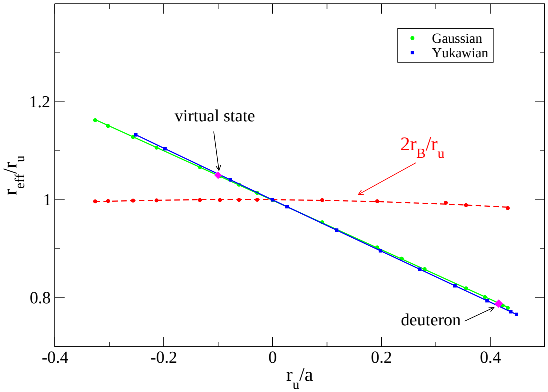
Towards the unitary limit



with r_u the effective range at the unitary limit and $r_B = a - a_B$

The physics around the unitary limit can be studied with a two-parameter potential

Zero-Range vs. Finite-Range Effects

Zero-Range Equations: $E_2 = \hbar^2/ma^2$

$$E_3^n/(\hbar^2/ma^2) = \tan^2\xi$$

$$\kappa_* a = \mathrm{e}^{\pi(n-n_*)/s_0}\mathrm{e}^{-\Delta(\xi)/2s_0}/\cos\xi$$

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V(r) = V_0 e^{-r^2/r_0^2}$

$$E_3^n/E_2 = \tan^2\xi$$

$$\kappa_*^n a_B = \mathrm{e}^{-\widetilde{\Delta}_n(\xi)/2s_0}/\cos\xi$$

$$\widetilde{\Delta}_n(\xi) = s_0 \ln\left(\frac{E_3^n + E_2}{\hbar^2(\kappa_*^n)^2/m}\right)$$

$\widetilde{\Delta}_n(\xi) \rightarrow \Delta(\xi)$ for $n > 0$

Gaussians with variable strength define the same $\widetilde{\Delta}_n(\xi)$

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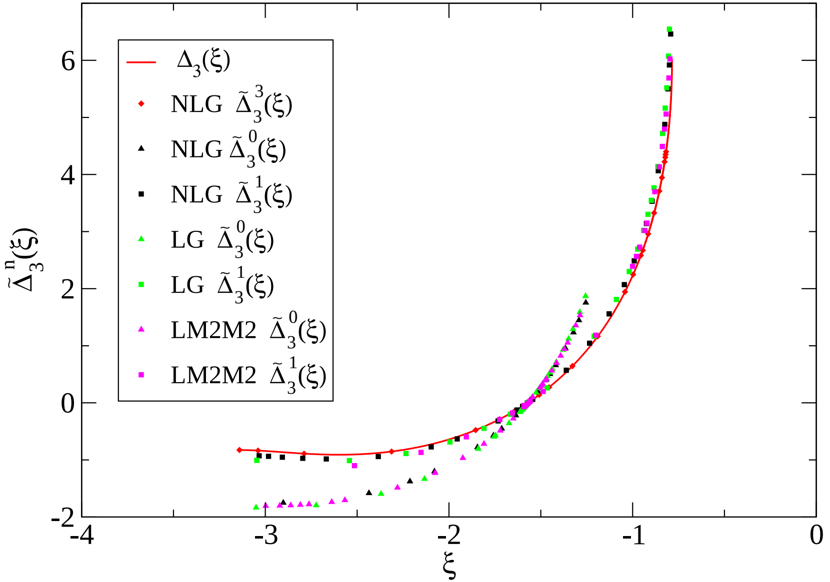

$\Delta_3(\xi)$
NLG $\tilde{\Delta}_3^3(\xi)$
NLG $\tilde{\Delta}_3^0(\xi)$
NLG $\tilde{\Delta}_3^1(\xi)$
LG $\tilde{\Delta}_3^0(\xi)$
LG $\tilde{\Delta}_3^1(\xi)$
LM2M2 $\tilde{\Delta}_3^0(\xi)$
LM2M2 $\tilde{\Delta}_3^1(\xi)$
$\tilde{\Delta}_3^n(\xi)$
ξ
-2
0
2
4
6
-4
-3
-2
-1
0

Using $\widetilde{\Delta}_0(\xi)$

The three-body parameter κ_* for the helium trimer

- The product $\kappa_* a_B$ is a function of ξ ($E_3/E_2 = \tan^2\xi$)
- To determine ξ we use experimental results $E_3 = 126$mK and $E_2 = 1.3$mK. Accordingly $\tan^2\xi = 97.0$
- Using a Gaussian potential with variable strength it is possible to determine the same value of ξ. Therefore

$$[\kappa_* a_B]^{exp} = [\kappa_* a_B]^{gaussian}$$

  and

$$[\kappa_*]^{exp} = [\kappa_* a_B]^{gaussian}/[a_B]^{exp}$$

- application to the helium trimer: $[\kappa_*]^{exp} = 0.044 a_0^{-1}$

The value of a_-

Finite-Range Equations at $\xi = -\pi$

$$E_3^n/(\hbar^2/ma_B^2) = 0$$

$$\kappa_* a_B^- = -e^{-\widetilde{\Delta}(-\pi)/2s_0} = -2.483$$

atomic species with van der Waals tail

The helium trimer as example:

$$a_B^- = -2.483/\kappa_* = -56.4 a_0$$

$$a_- = r_B + a_B^- = -49 a_0$$

$$a_-/\ell \approx -49/5.1 = -9.6$$

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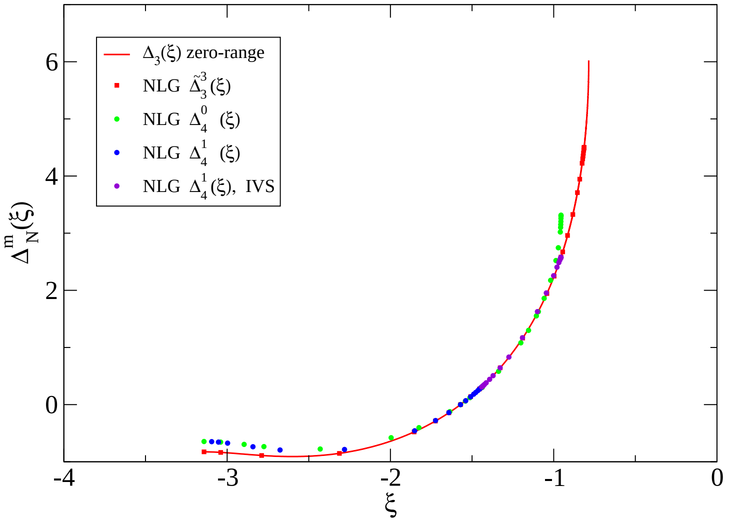
Extension to N=4



Zero-Range Equations for $N = 4$

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V(r)$

$$E_4^{n,m}/E_2 = \tan^2\xi$$

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with $\kappa_*^0/\kappa_*^1 = 4.6003$

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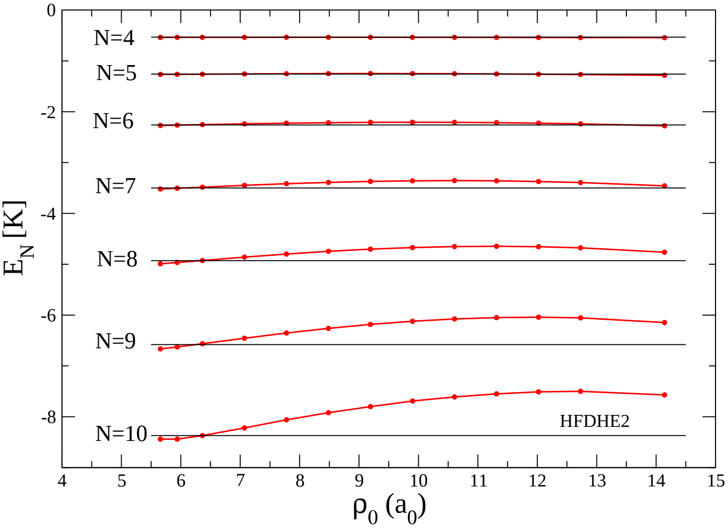
Propagation of universal behavior with N

In collaboration with. A. Polls, B. Julia Diaz and N. Timofeyuk

Saturation properties of helium drops

- We define a soft potential model to describe E_3
- It consists in a two- plus a three-body term $V = V(i,j) + W(i,j,k)$
- $V(i,j) = V_0 e^{-r_{ij}^2/r_0^2}$
- $W(i,j,k) = W_0 e^{-\rho_{ijk}^2/\rho_0^2}$
- W_0 is determined from E_3
- ρ_0 is taken as a parameter
- E/N is calculated for increasing values of N as a function the ρ_0
- the saturation properties are determined from a liquid drop formula:
  $E_N/N = E_v + E_s x + E_c x^2$ with $x = N^{-1/3}$
- In general drops with N around 100 is sufficient to determine E_v and E_s

drops with $N \leq 10$



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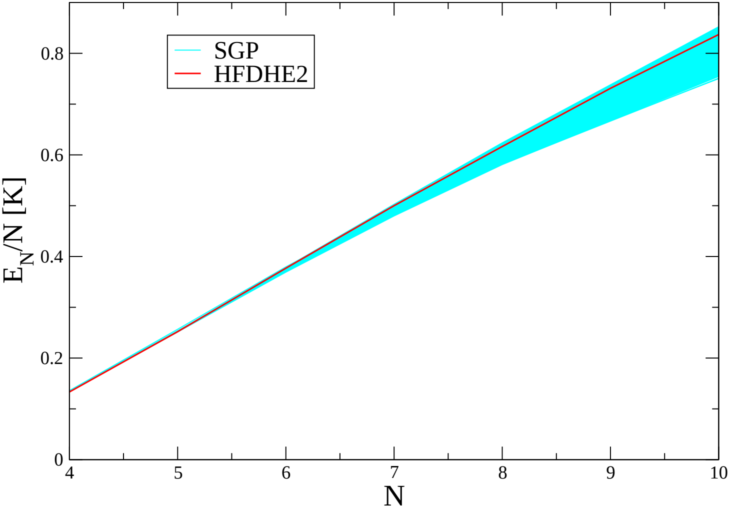
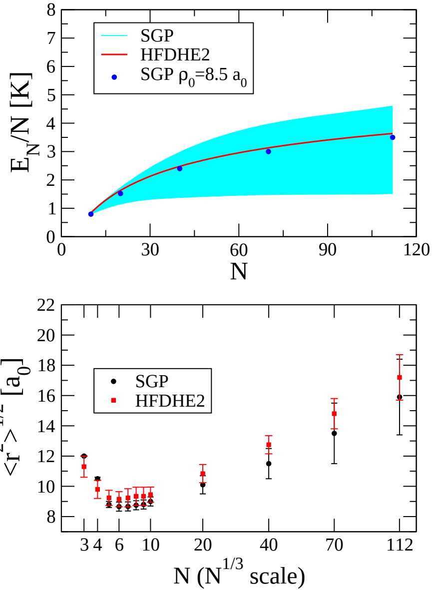
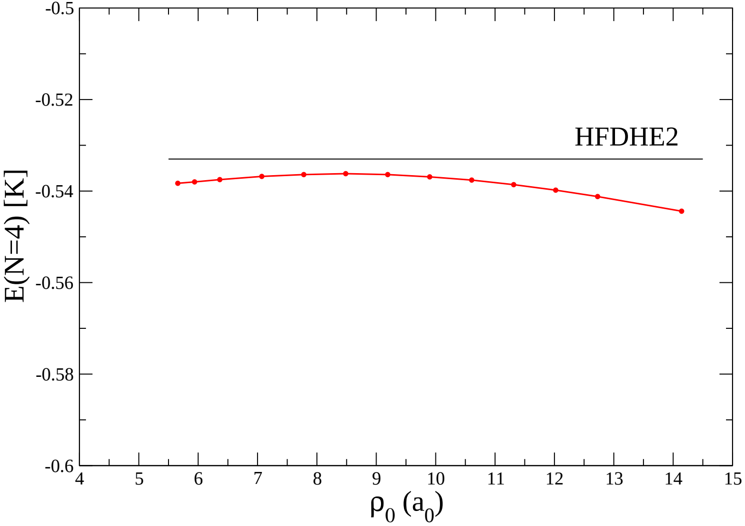


drops with $N \leq 112$



drops with $N = 4$



Propagation of universal behavior with N

Saturation properties of helium drops

- Using the appropriate value of ρ_0
- we obtain (in K):
  $E_N/N = 6.79 - 18.0x + 9.98x^2$
- To be compared to the results of the LM2M2 potential:
  $E_N/N = 7.02 - 18.8x + 11.2x^2$
- The experimental result is 7.12 K
- for the surface tension $t = E_s/4\pi r_0^2(\infty)$
  the experimental value is 0.29 KA^2
  with the gaussian soft potential the result is 0.27 KA^2

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1/2-spin 1/2-isospin fermions close to the unitary limit

The $2N$ system in s-wave

This is a two-channel system with spin $S = 0$ and $S = 1$. For two nucleons the physical values are:

$E_d = -2.2245$ MeV, $a_B = 4.318$ fm

$a_1 = 5.424 \pm 0.003$ fm $\qquad r_1^{eff} = 1.760 \pm 0.005$ fm

$a_0 = -23.740 \pm 0.020$ fm $\qquad r_0^{eff} = 2.77 \pm 0.05$ fm

moving the system to the unitary limit

- The $S = 1$ channel:

  a gaussian $V_1 e^{-r^2/r_1^2}$ with V_1 and r_1 fixed to describe a_1 and a_B (or r_1^{eff}). Then V_1 is varied.

- The $S = 0$ channel:

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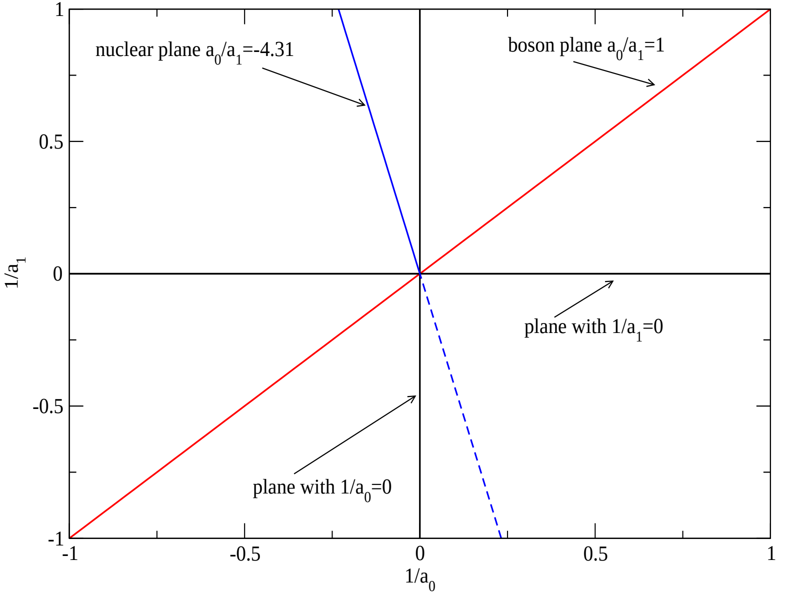

nuclear plane a0/a1=-4.31
boson plane a0/a1=1
plane with 1/a1=0
plane with 1/a0=0
1/a1
1/a0
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Three-body spectrum with spin-isospin symmetry

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V_1(r)$

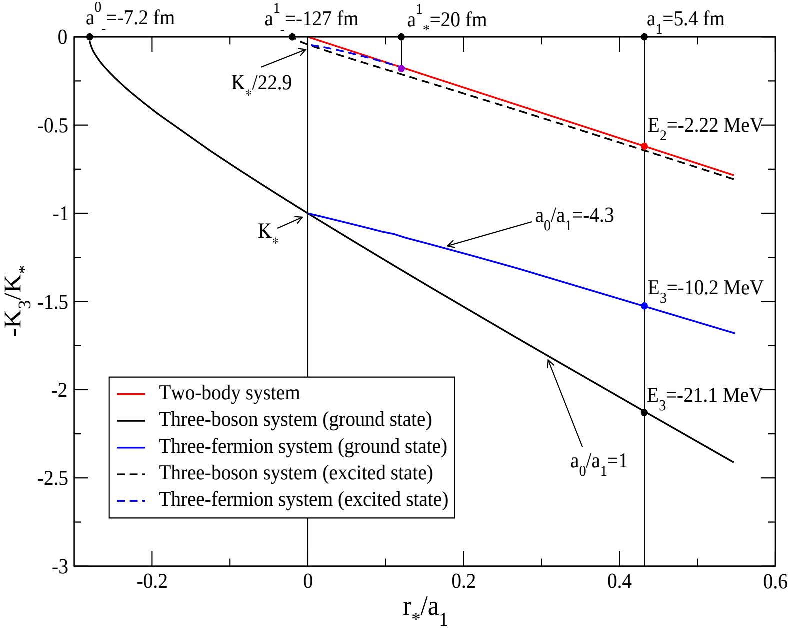
$$E_3^n/E_2 = \tan^2\xi$$

$$\kappa_*^n a_B = \mathrm{e}^{-\widetilde{\Delta}_3(\xi,\phi)/2s_0}/\cos\xi$$

$$\widetilde{\Delta}_n(\xi,\phi) = s_0 \ln\left(\frac{E_3^n + E_2}{\hbar^2(\kappa_*^n)^2/m}\right)$$

$$\frac{a_1}{a_0} = \tan\phi$$

For $n > 0$ $\quad \widetilde{\Delta}_n(\xi,\phi) \to \Delta(\xi,\phi)$



$a^0_- = -7.2$ fm
$a^1_- = -127$ fm
$a^1_* = 20$ fm
$a_1 = 5.4$ fm
$K_*/22.9$
K_*
$E_2 = -2.22$ MeV
$E_3 = -10.2$ MeV
$E_3 = -21.1$ MeV
$a_0/a_1 = -4.3$
$a_0/a_1 = 1$
Two-body system
Three-boson system (ground state)
Three-fermion system (ground state)
Three-boson system (excited state)
Three-fermion system (excited state)
$-K_3/K_*$
r_*/a_1
0
-0.5
-1
-1.5
-2
-2.5
-3
-0.2
0
0.2
0.4
0.6

Comments on the two-channel plot

- Studying a three-boson system using finite-range potentials, the first excited state does not dispapear onto the two-body threshold
- In the two-channel system the excited state disappears on the two-body threshold as the ratio a_0/a_1 varies.
- The analysis of the nuclear plane produces a binding energy at the unitary limit of $E_u \approx 3.6$ MeV.
- However at the nuclear point the binding energy of $E_3 \approx 10.2$ MeV is far from the experimental value of 8.5 MeV
- A three-body force has to be included
- using a more realistic potential model and varying the depth, the unitary limit can be reached.
- The value obtained has been $E_u \approx 2.8$ MeV.

Working on the nuclear point

The 2N sector

Low Energy data:

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$a_0 = -23.740 \pm 0.020$ fm $r_0^{eff} = 2.77 \pm 0.05$ fm

Constructing LO 2N potential

Two parameters corresponding to the $l = 0$ partial waves with $S = 0, 1$:

$V_0(r) = -V_0 e^{-r^2/r_0^2}$, $V_1(r) = -V_1 e^{-r^2/r_1^2}$

V_0[MeV]	r_0[fm]	a_0[fm]	r_0^{eff}[fm]	V_1[MeV]	r_1[fm]	a_1[fm]	r_1^{eff}[fm]
53.255	1.40	−23.741	2.094	79.600	1.40	5.309	1.622
42.028	1.57	−23.745	2.360	65.750	1.57	5.423	1.776
40.413	1.60	−23.745	2.407	63.712	1.60	5.447	1.802
37.900	1.65	−23.601	2.487	60.575	1.65	5.482	1.846
33.559	1.75	−23.745	2.644	55.036	1.75	5.548	1.930
30.932	1.82	−23.746	2.756				

Working on the nuclear point

The 3*N* sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	$^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	−12.40	−2.191	−2.175
42.028	1.57	65.750	1.57	−10.83	−2.199	−1.236
40.413	1.60	63.712	1.60	−10.59	−2.197	−1.097
37.900	1.65	60.575	1.65	−10.22	−2.199	−0.860
33.559	1.75	55.036	1.75	−9.584	−2.201	
30.932	1.82	65.750	1.57	−9.715		−0.285
Exp.				−8.482		0.645 ± 0.010

Introducing a Three-Body Force

We choose a simple (two-parameter) form:

$$W(\rho) = W_0 e^{-\rho^2/\rho_0^2}$$

with $\rho^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$

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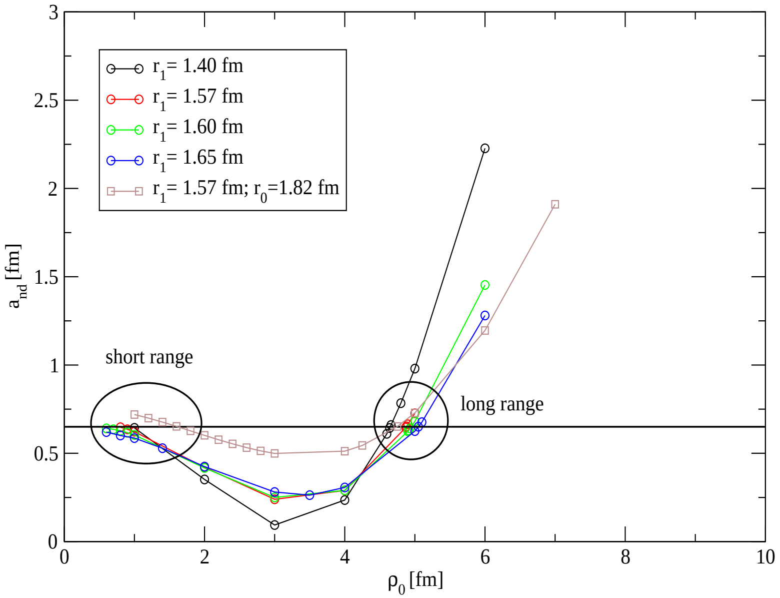
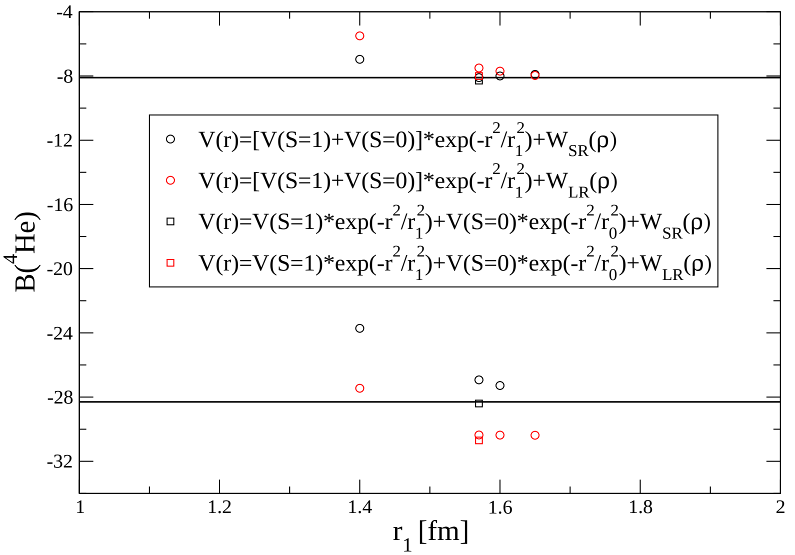
$$V(r)=[V(S=1)+V(S=0)]*\exp(-r^2/r_1^2)+W_0*\exp(-\rho^2/\rho_0^2)$$

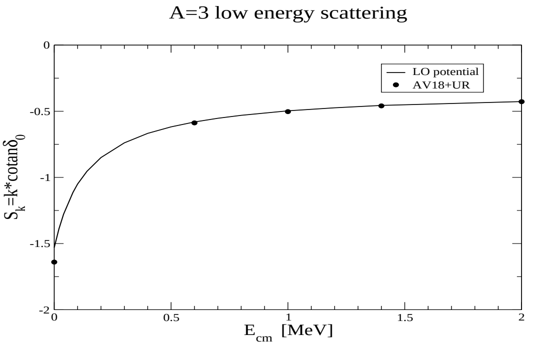


The N=4 ground and excited state



Summary of the LO potential

LO	E_d	$B(^3\mathrm{H})$	$B(^3\mathrm{He})$	$B(^3\mathrm{He}^*)$	$^2a_{nd}$
	-2.225	-8.480	-28.41	-8.29	0.652
Exp.	-2.225	-8.482	-28.296	-8.10	0.645



No bad for a 4-parameter 2*N* potential + 2-parameter 3*N* potential!
next step (in progress)→ ^{6}He and ^{6}Li ground states

Conclusions

- The physics around the unitary limit has been studied using potential with variable depth
- In this region a two-parameter potential captures most of the dynamics
- Finite-range effects have been analyzed through the level functions $\widetilde{\Delta}_n(\xi)$
- The zero-range universal function has been obtained as a limiting case $n > 0$
- A zero-range equation has been proposed for four bosons
- The Efimov plot for three 1/2-spin-isospin fermions has been analyzed
- A detailed study on the nuclear physics point has been performed using gaussian two- and three-body potentials
- Work in progress: Study of universality for the N-boson system

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