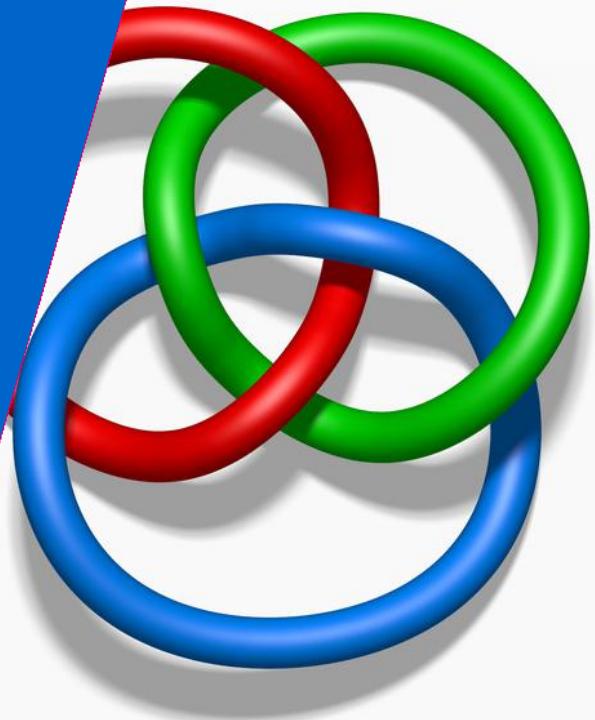


Finite range effects in three-body physics with an off-the-energy-shell T-matrix

15 December 2016, KITP



TU/e

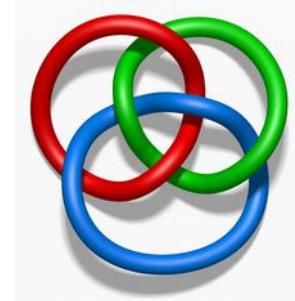
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Where innovation starts

Outline

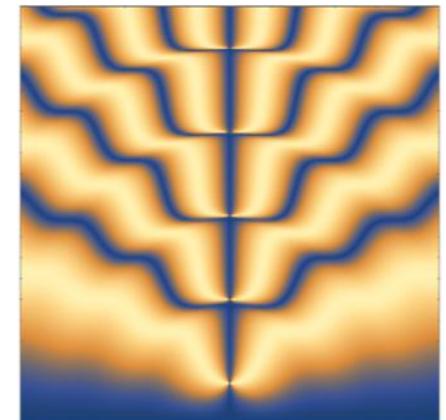
Introduction

- Strongly-interacting systems
- Importance of the interaction range



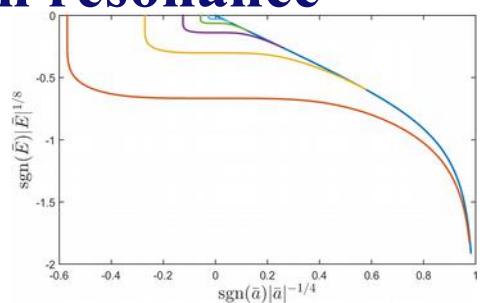
Three-body physics with square well

- Off-the-energy shell interactions
- Shallow vs deep potentials
- More partial waves



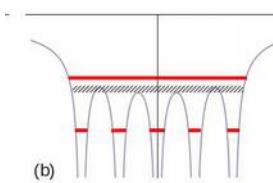
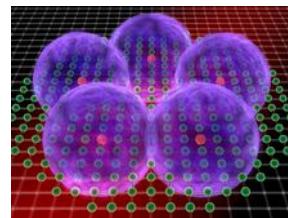
Three-body physics including a Feshbach resonance

- Independent control over width



Strong interactions at different ranges

Particles with different ranges of interaction



Universality

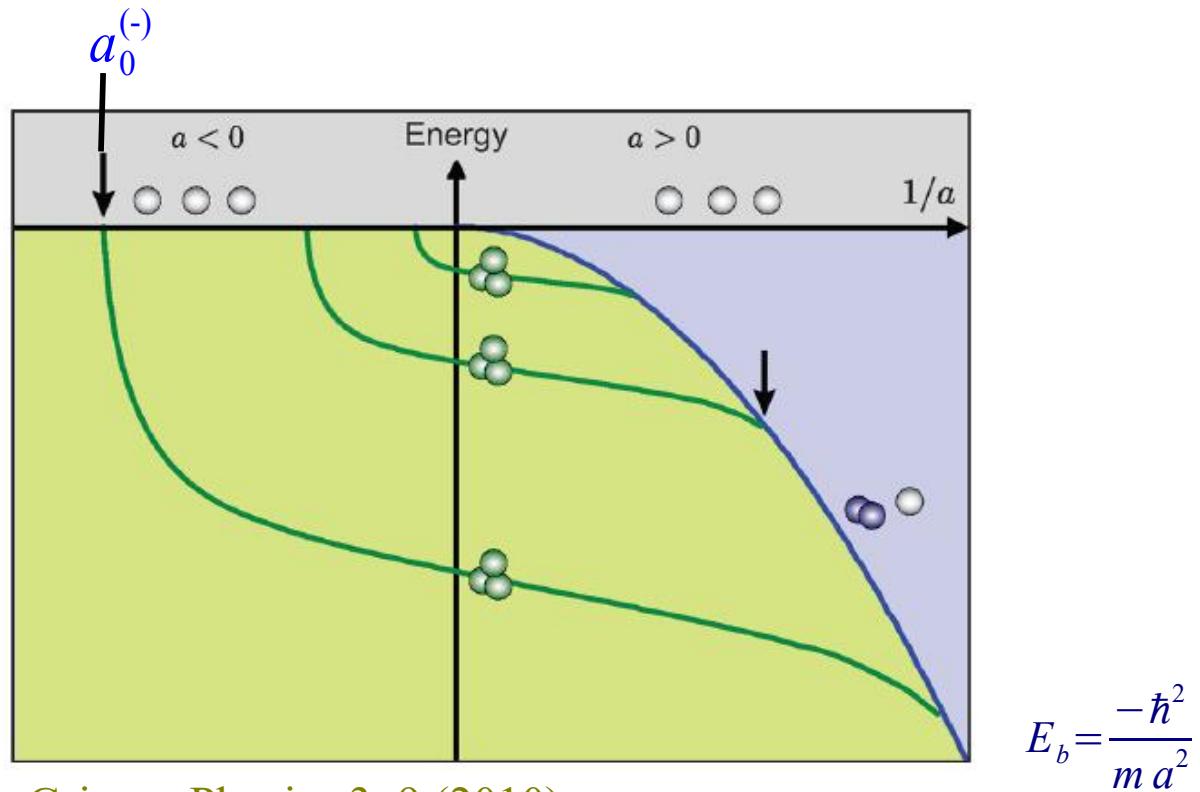
- Universal Few body physics: interactions insensitive to microscopic details interaction
- Efimov spectrum: depends only on two generic two-body parameters

difference in scattering length:

$$e^{\pi/s_0} \approx 22.7$$

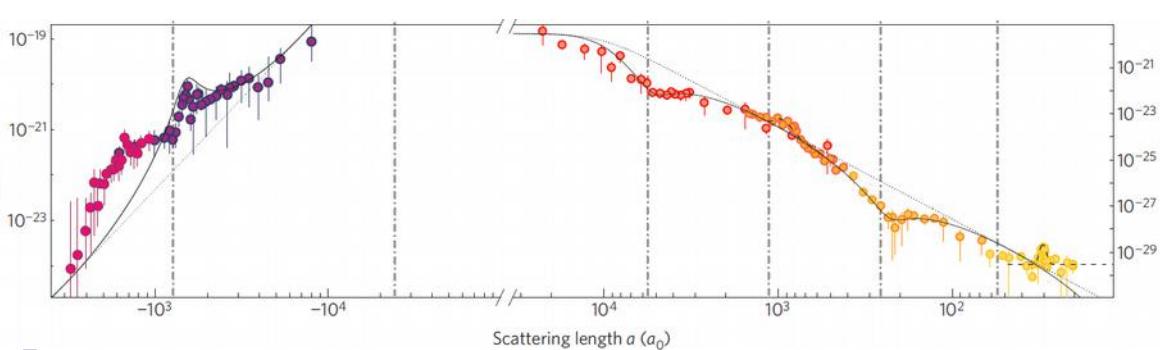
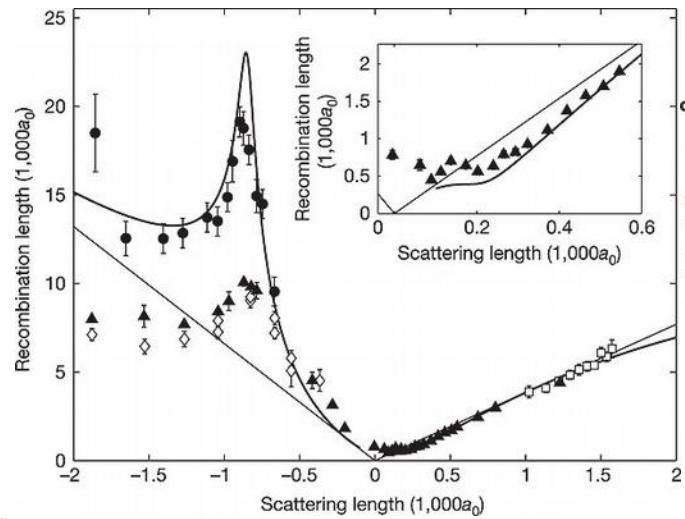
spacing bound states:

$$E_{n+1}/E_n \approx e^{-2\pi} \approx 1/515$$

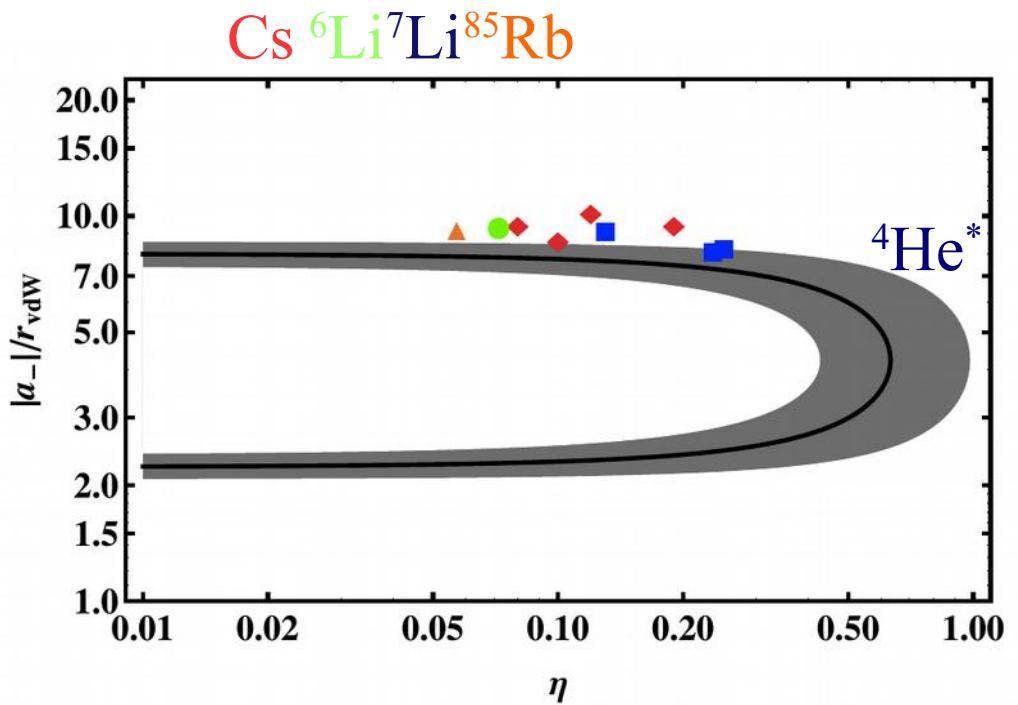


Overview: [F. Ferlaino and R. Grimm, Physics 3, 9 (2010);
C. Greene, Physics Today, march 2010]

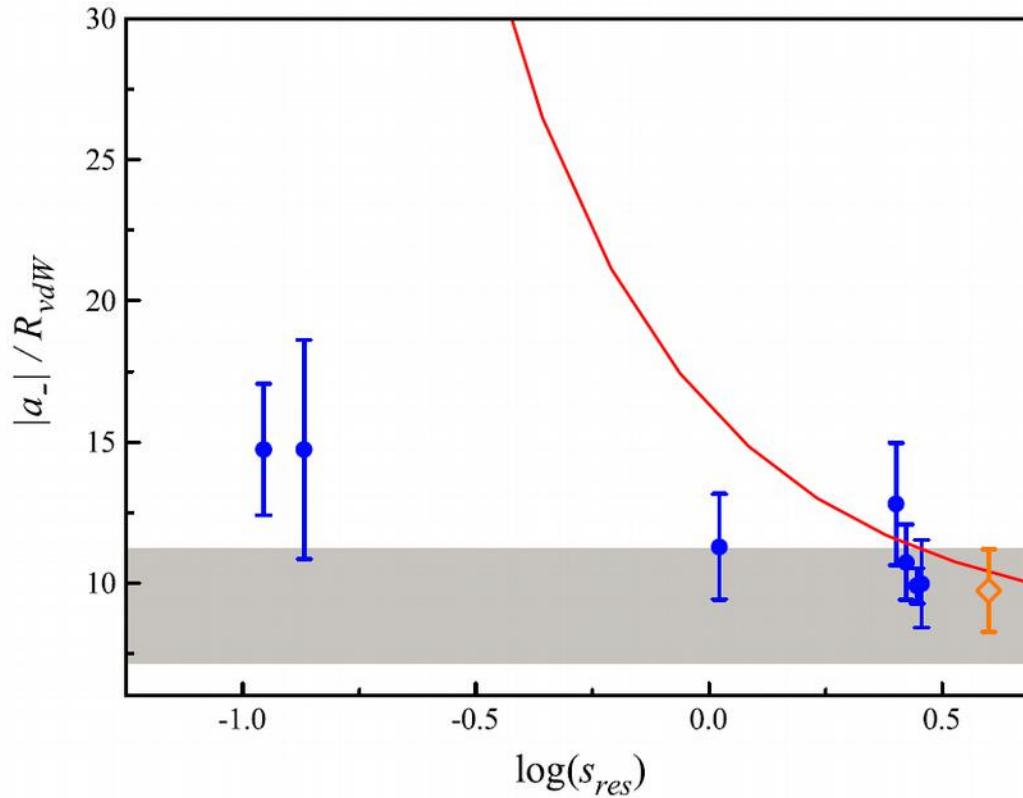
Experimental observations



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Universality for narrow Feshbach resonances?



[S. Roy, M. Landini, A. Trenkwalder, G. Semeghini, G. Spagnolli, A. Simoni, M. Fattori, M. Inguscio, and G. Modugno, Physical Review Letters 111, 053202 (2013)]

Square well

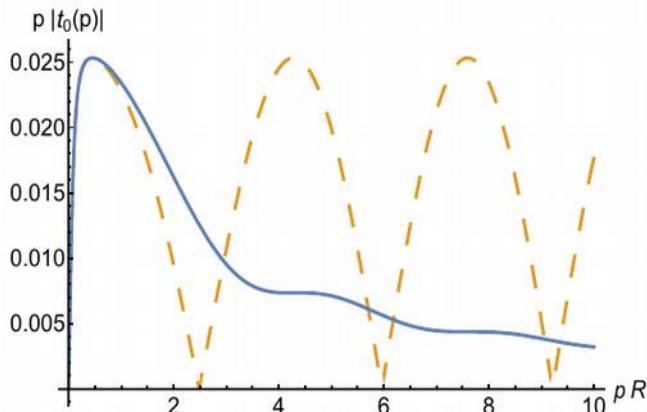
$$V(r) = \begin{cases} -V_0 & \text{if } r \leq R \\ 0 & \text{else} \end{cases}$$

$$t_l = -\frac{iR}{4\pi^2 m \bar{p}} \frac{\bar{p} \hat{j}_{l+1}(\bar{p}) \hat{j}_l(\bar{q}) - \bar{q} \hat{j}_l(\bar{p}) \hat{j}_{l+1}(\bar{q})}{\bar{p} \hat{h}_{l+1}^{(1)}(\bar{p}) \hat{j}_l(\bar{q}) - \bar{q} \hat{h}_l^{(1)}(\bar{p}) \hat{j}_{l+1}(\bar{q})} \quad \bar{p} = \frac{pR}{\hbar}, \quad \bar{q} = \frac{\sqrt{q_0^2 + p^2} R}{\hbar} \text{ with } q_0 = \sqrt{2mV_0}$$



$$\frac{a_0}{R} = 1 - \frac{\tan(\bar{q}_0)}{\bar{q}_0}$$

$$t_l(a, N_{\text{bound}}) \rightarrow t_l(a)$$



$$t_l \approx -\frac{iR}{4\pi^2 m \bar{p}} \frac{\bar{p} \hat{j}_{l+1}(\bar{p}) - \theta_l \hat{j}_l(\bar{p})}{\bar{p} \hat{h}_{l+1}^{(1)}(\bar{p}) - \theta_l \hat{h}_l^{(1)}(\bar{p})}$$

$$\theta_l = \frac{(2l+1)!!^2 a_l}{(2l+1)!!(2l-1)!! a_l - R^{2l+1}} + \frac{1}{2} \bar{p}^2$$

Off-shell wave function

- Two-particle energy not conserved

$$p_E = \sqrt{2mE} R/\hbar$$

$$t(p, p', p_E) = \langle p | \hat{T}(p_E) | p' \rangle = \langle p | V | \psi_{p'} \rangle$$

- Intermediate step: off-shell Schrodinger equation...

$$\left[\frac{\hbar^2}{2m} \left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) + z - V(r) \right] \omega_l(r, p, z) = \left(z - \frac{p^2}{2m} \right) j_l(pr)$$

[R. D. Levine, Quantum Mechanics of Molecular Rate Processes (Oxford University Press, 1969)]

- ...for the off-shell wave function

$$\psi_p(\vec{r}) = \frac{1}{2\pi^{3/2} pr} \sum_l (2l+1) i^l \omega_l(r, p, z) P_l(\hat{r} \cdot \hat{p})$$

Off-shell T-matrix

- **Closed-form expression for square well**

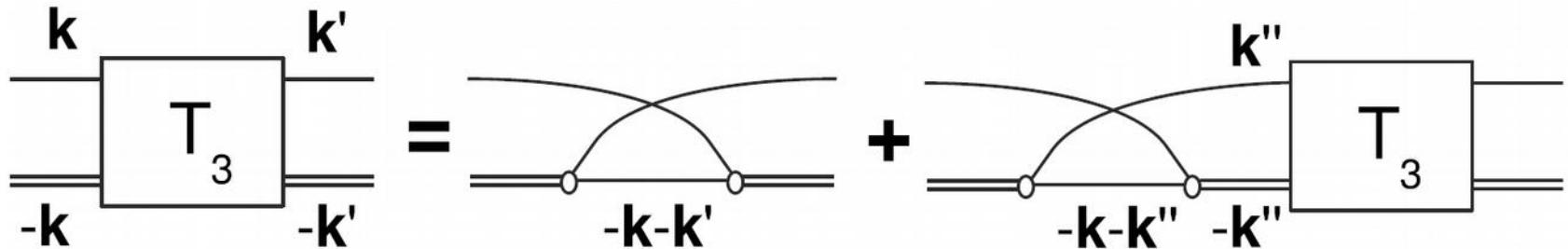
$$t_l(p, p', p_E) = \frac{R}{4\pi^2 \mu p p' \hbar} \frac{q^2 - p_E^2}{q^2 - p^2} [\tau(q; p, p', p_E) - \tau(p; p, p', p_E)]$$

- **with**

$$\begin{aligned}\tau(x; p, p', p_E) &= \frac{p_E^2 - x^2}{p'^2 - x^2} (p' j_{l+1}(p') j_l(x) - x j_l(p') j_{l+1}(x)) \\ &\quad \frac{p j_{l+1}(p) h_l^{(1)}(p_E) - p_E j_l(p) h_{l+1}^{(1)}(p_E)}{x j_{l+1}(x) h_l^{(1)}(p_E) - p_E j_l(x) h_{l+1}^{(1)}(p_E)}\end{aligned}$$

Three-body scattering

- Lippmann-Schwinger for bound states



- Skorniakov-Ter-Martirosian equation

$$t_3(p, p', E) = 2 \int \frac{t_2 \left(\left| \mathbf{p} - \frac{1}{2} \mathbf{p}'' \right|, \left| \mathbf{p}'' - \frac{1}{2} \mathbf{p} \right|, E - \frac{3p''^2}{4m} \right)}{E - \frac{p''^2}{m} - \frac{p^2}{m} - \frac{\mathbf{p}'' \cdot \mathbf{p}}{m}} t_3(p'', p', E) d^3 \mathbf{p}''$$

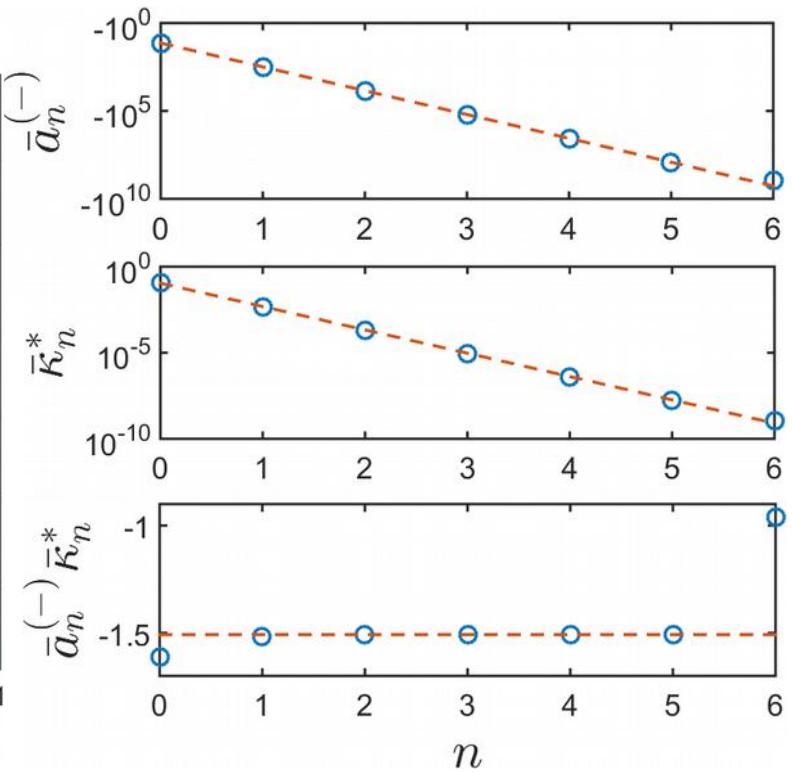
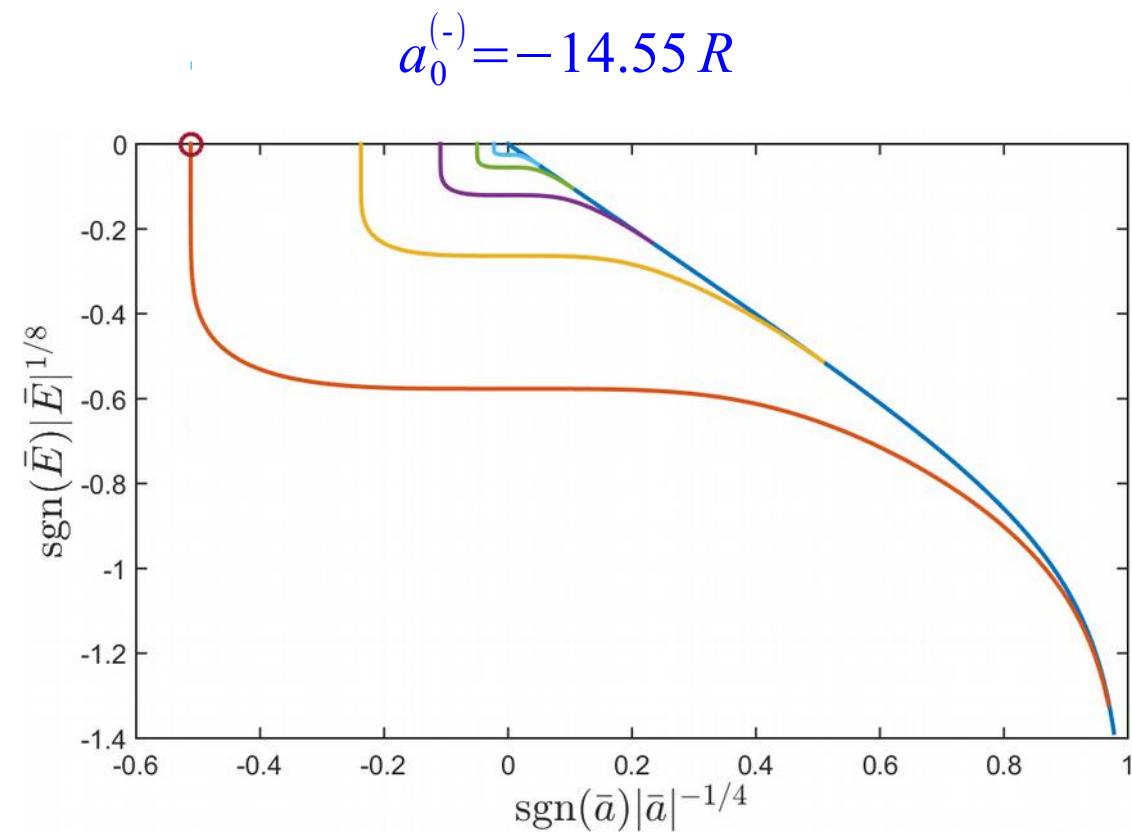
- Discretize kernel

$$t_i = \sum_j K_{ij} t_j$$

$$\det(I - [K_{ij}]) = 0$$

Efimov spectrum – very deep potential

- Finite range: determines three-body parameter



Feshbach resonance

- Resonance from bound state in closed channel

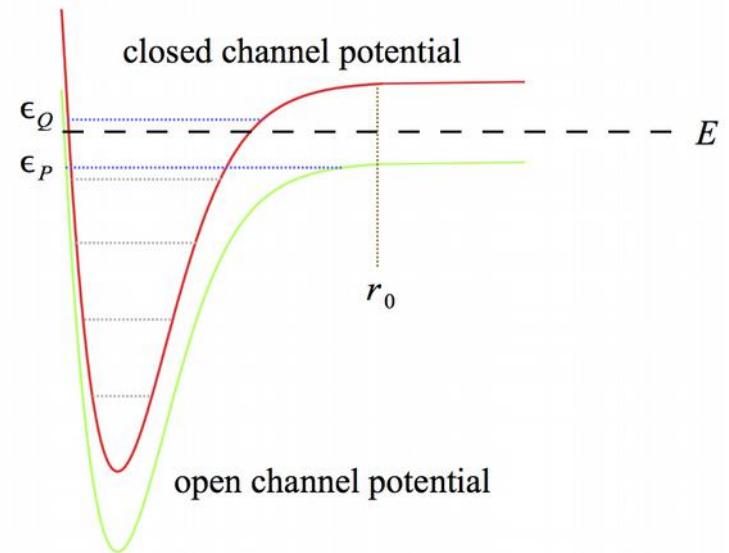
$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

- Add closed channel contribution (s-wave)

$$a = a_{bg}$$

$$t_2 = t(p, p', p_z) + \frac{\langle \psi_p(z^*) | H_{QP} | \phi_Q \rangle \langle \phi_Q | H_{PQ} | \psi_{p'}(z) \rangle}{z - \epsilon_Q - \langle \phi_Q | H_{QP} \frac{1}{z - H_{PP}} H_{PQ} | \phi_Q \rangle}$$

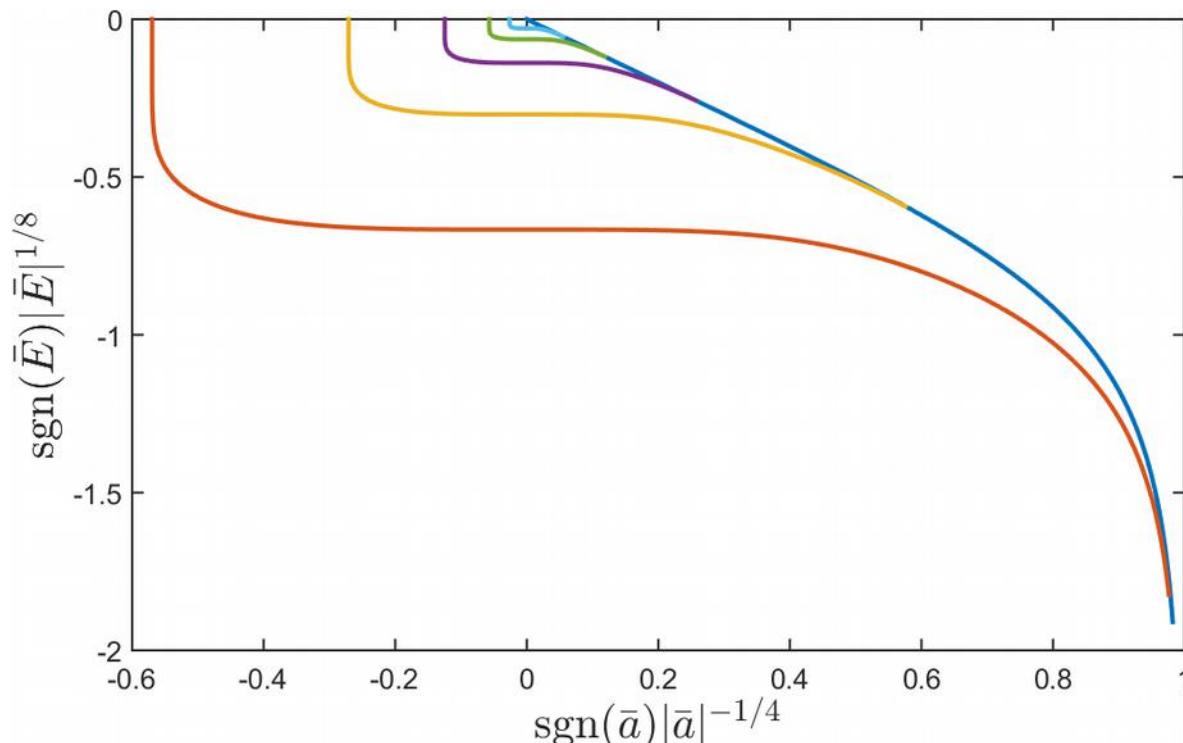
$$R^* = \hbar^2 / (m \delta \mu \Delta B a_{bg})$$



Set width of resonance to zero

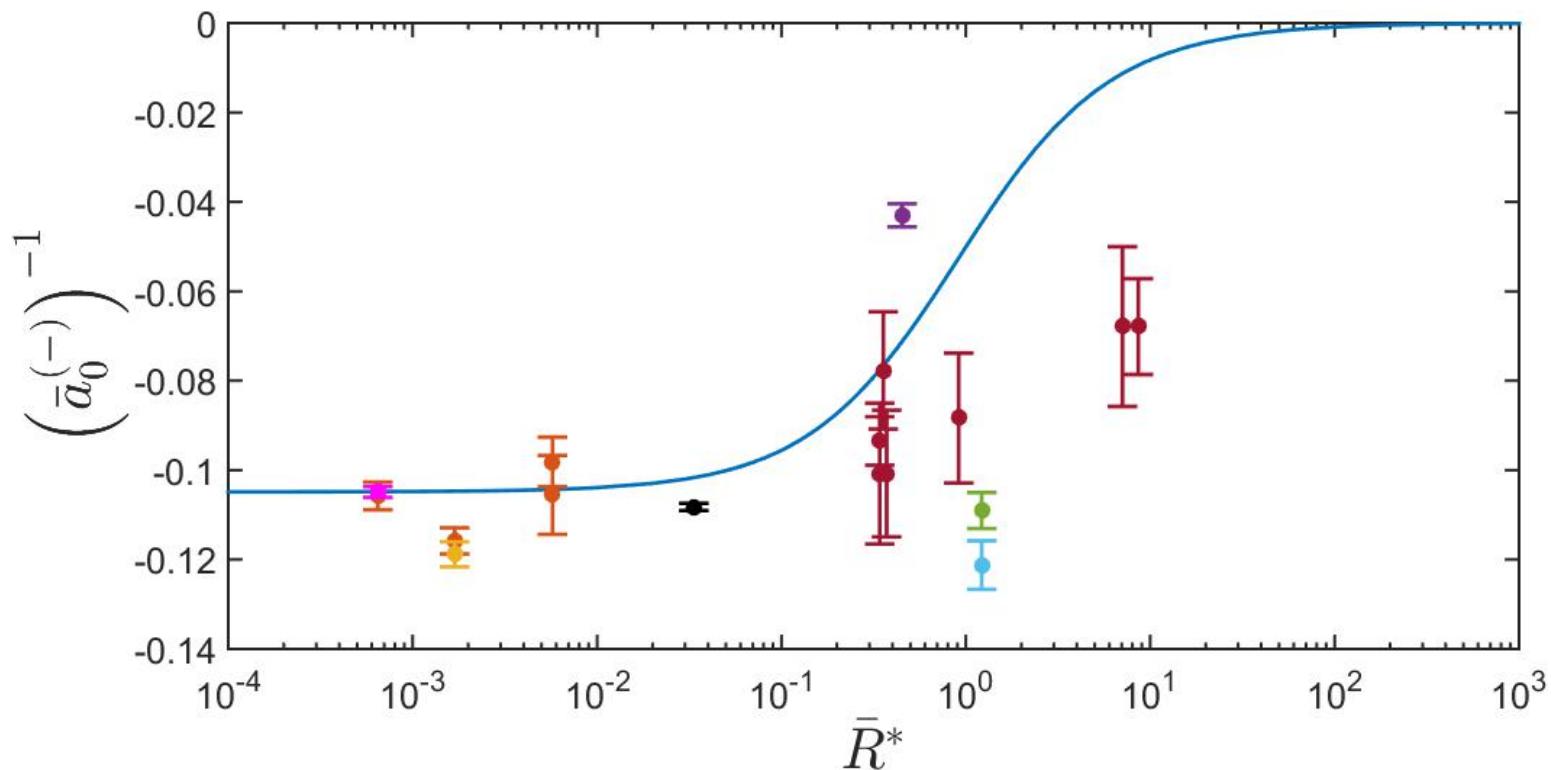
$$R^* = 0$$

$$\frac{a_{bg}}{R} = 1$$



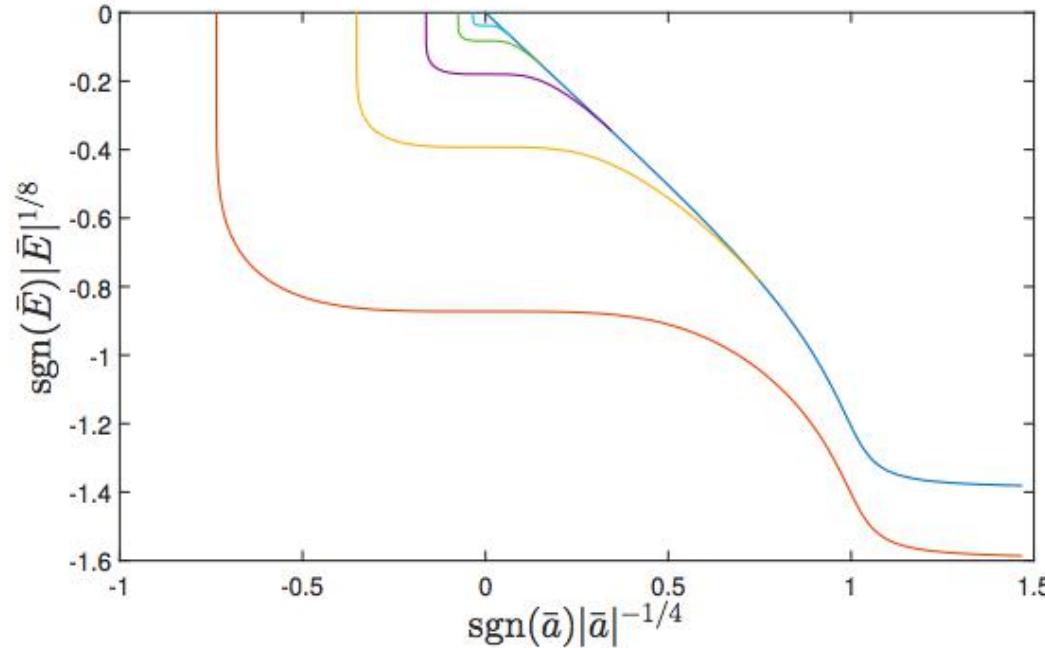
$$a_0^{(-)} = -9.54 R$$

Comparison to experiments with different width



Shallow square-well – Effect on lowest Efimov state

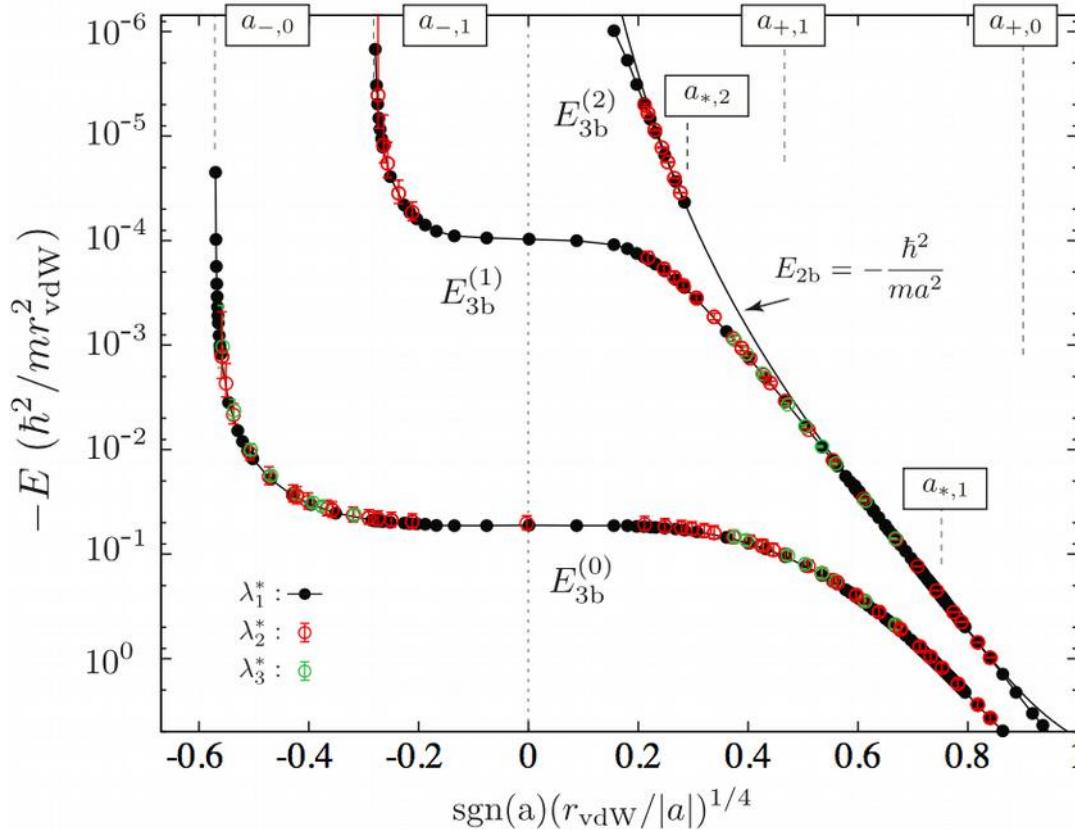
- Square well with only one bound state
- Only s-wave, separable approximation



- Disappears for deeper square well potential
- Consistent with variational argument
[L. W. Bruch and K. Sawada, Phys. Rev. Lett. 30, 25 1973]

Effect of d-wave resonance?

- Might be true for Lennard-Jones potential

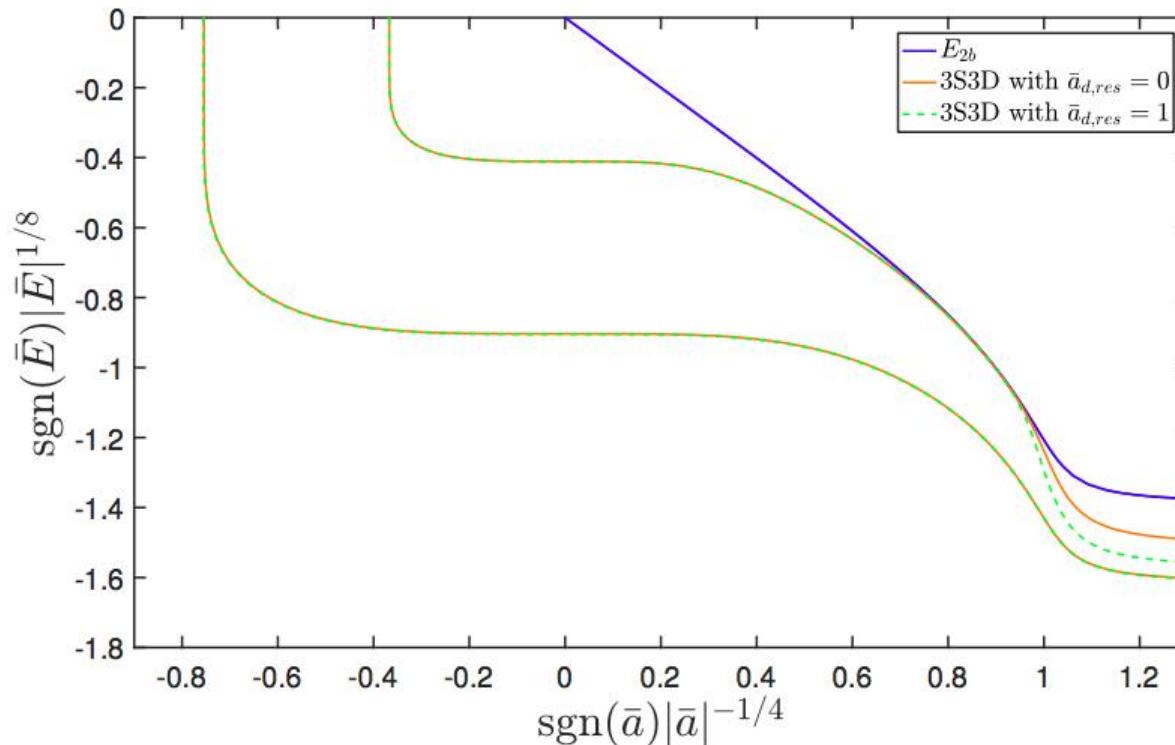


- Always a d-wave resonance present

[Efimov universality for ultracold atoms with positive scattering lengths, Paul M. A. Mestrom, Jia Wang, Chris H. Greene, Jose P. D'Incao, arXiv:1609.02857]

Effect of d-wave resonance? (2)

- Square-well, with expansion $\psi_{nl}(p', p_E) = \sum_{n,l} g_{n,l}(p', E) \tau_{n,l}(E) g_{n,l}(p, E)$
- d-wave can be switched-on/off



- Non-separability of the T-matrix

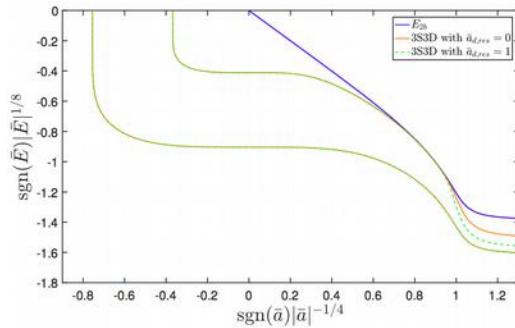
Related work with separable van der Waals potential: [Van der Waals universality in

- Effect also for s-wave: not there in the separable

homonuclear atom-dimer elastic collision, P. Giannakos, Chris H. Greene, arXiv:1608.08276]

Conclusions / outlook

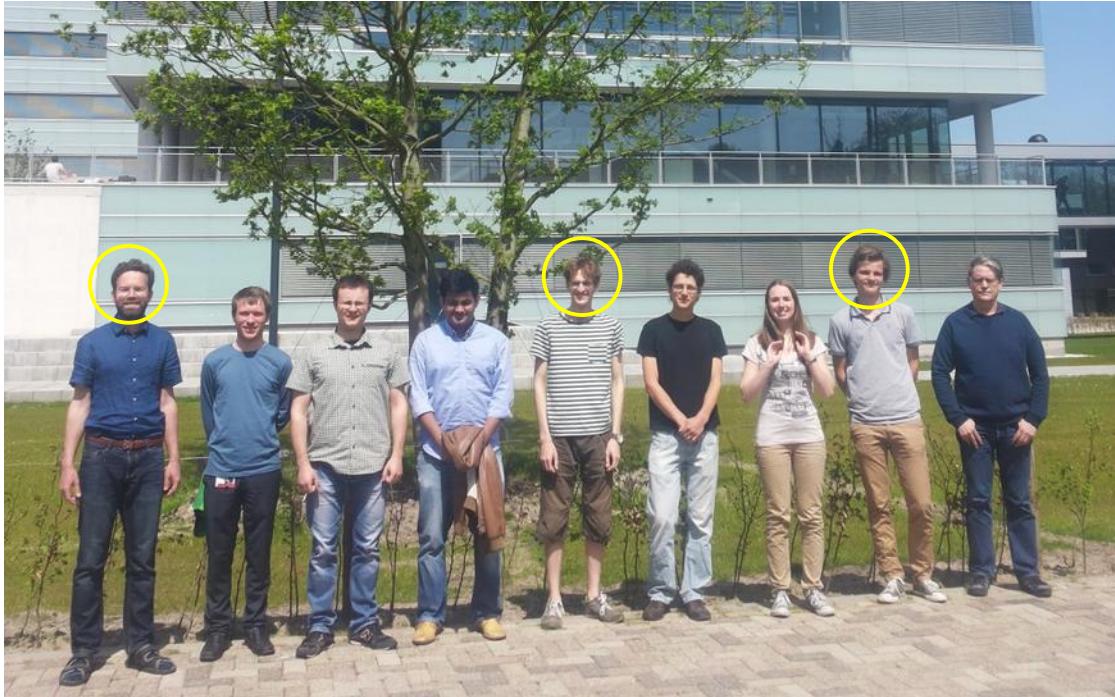
- Finite range effects via off-shell square well
- Off-shell Feshbach resonance
- 3-body parameter consistent with exp.
- Independent control of width Feshbach res.
- Unexpected dependence on a_{bg}
- Effects of other partial waves



Next:

- Off-shell T-matrix for realistic van der Waals pot.

The quantum gases team - Eindhoven



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New PhD and Postdoc
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