Five-body Efimov effects and universal pentamer in fermionic mixtures

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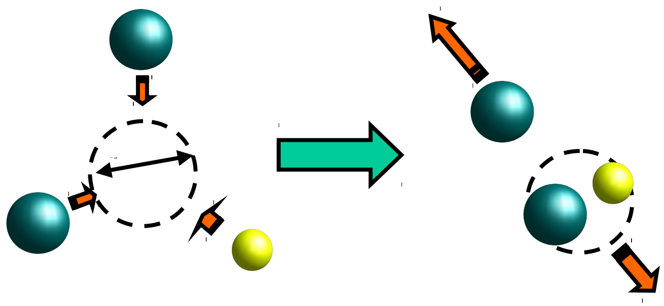
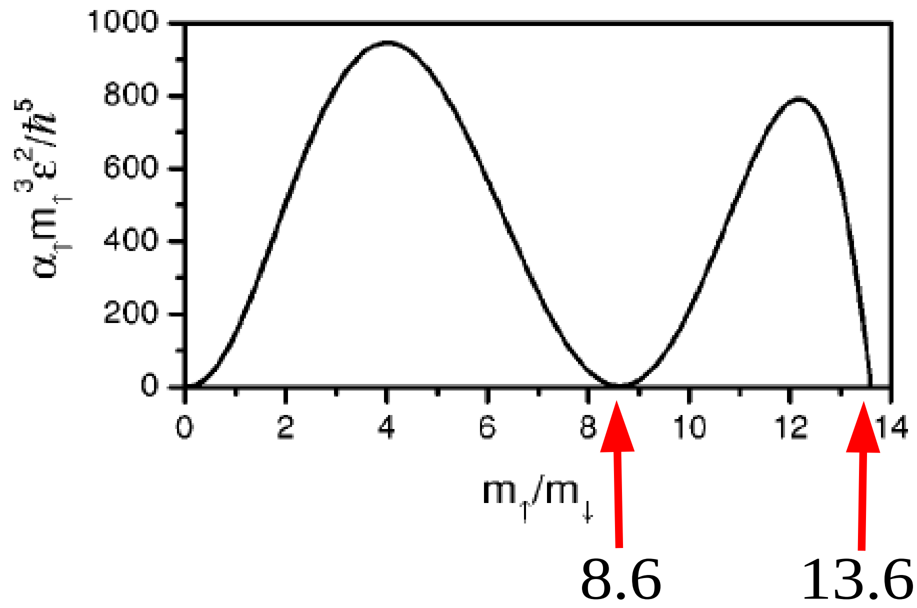
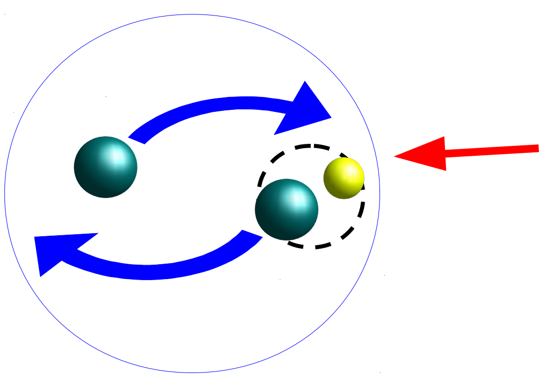
Fermi-Fermi mixtures, magic mass ratios

3-body recombination to a weakly bound level, Petrov (2003)





Emergence of a trimer state for M/m>8.2, Kartavtsev & Malykh (2006)

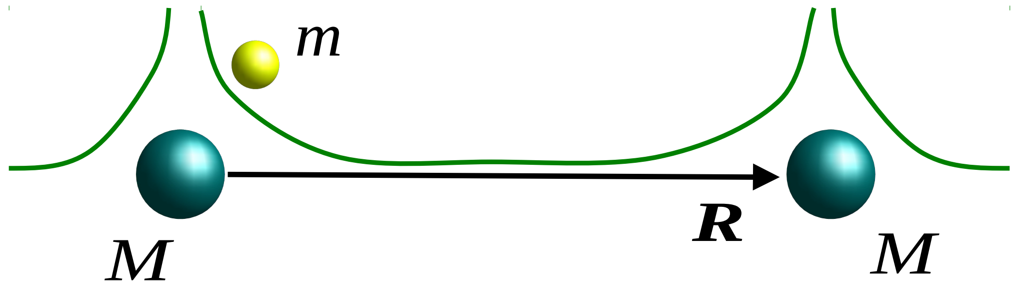


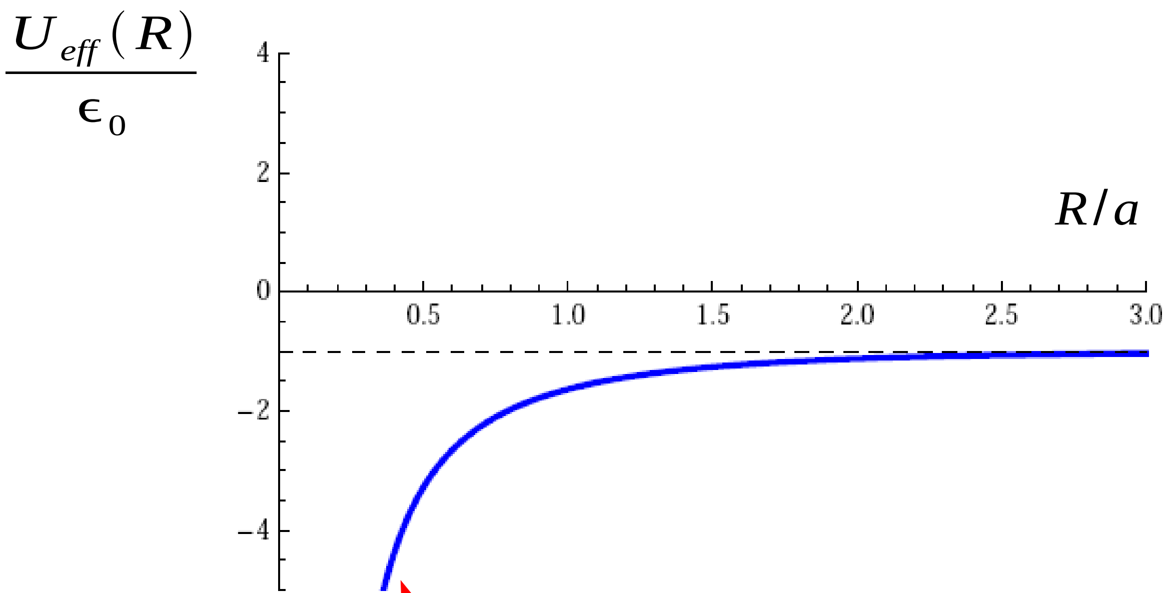
M/m<8.2 p-wave atom-dimer scattering resonance

M/m>8.2 trimer state with L=1

Bound trimer state, NOT EFIMOV

Born-Oppenheimer approximation

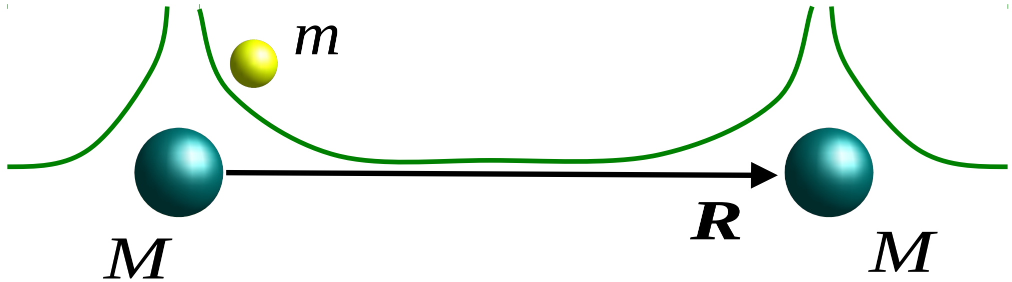




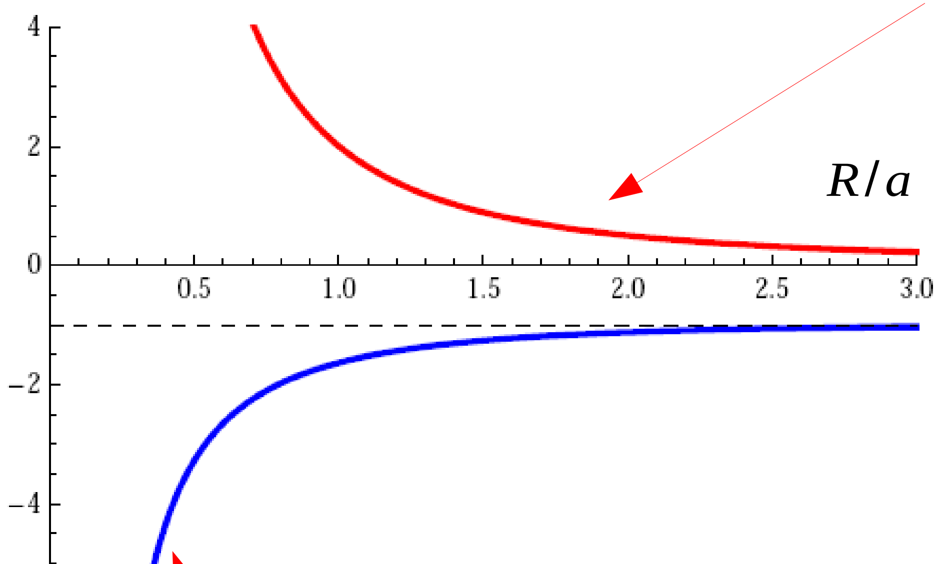
$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$U_{eff}(R) \approx -0.16\frac{\hbar^2}{mR^2}$$

Born-Oppenheimer approximation



$$\frac{U_{eff}(R)}{\epsilon_0}$$



$$U_{centrifugal}(R)=\frac{\hbar^2 l(l+1)}{MR^2}$$

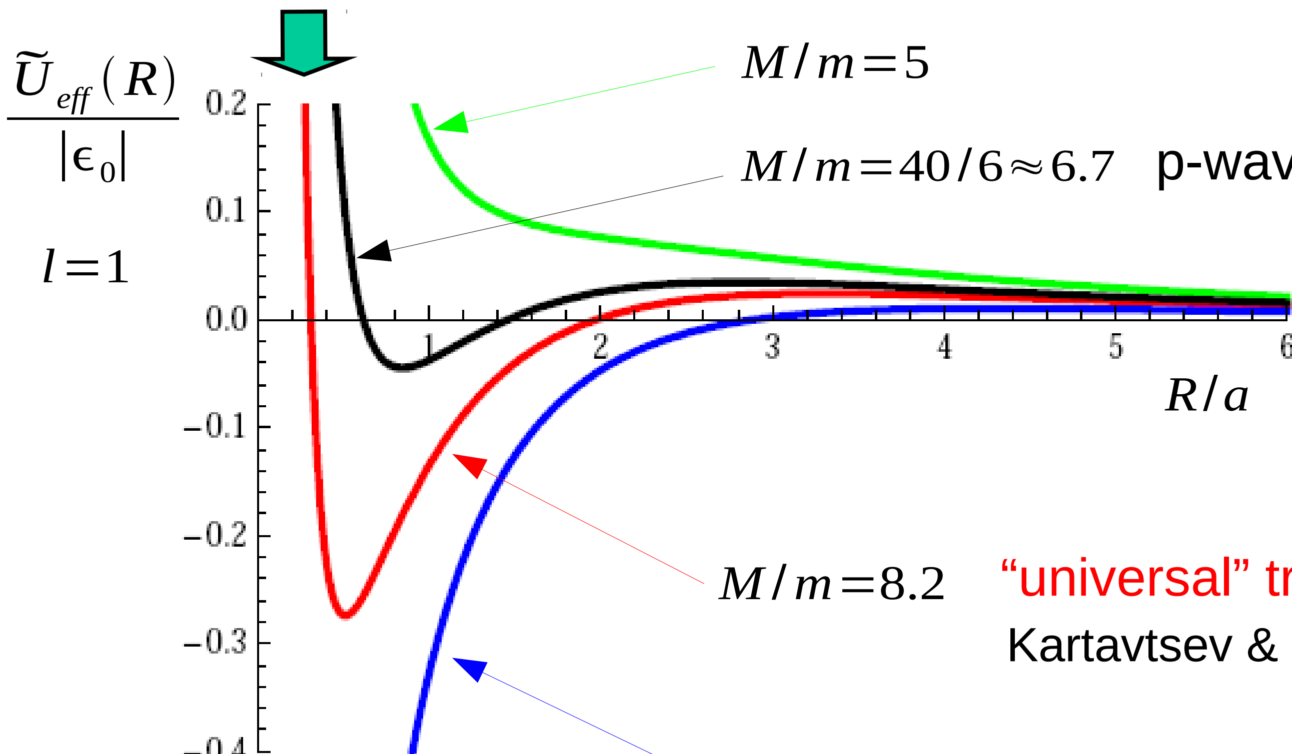
$$U_{eff}(R)\approx-\frac{\hbar^2}{2ma^2}=-|\epsilon_0|$$

$$U_{eff}(R)\approx-0.16\frac{\hbar^2}{mR^2}$$

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\widetilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$

$$\frac{\hbar^2}{MR^2}\left(l(l+1)-0.16\frac{M}{m}\right)$$

$$\widetilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$$



$M/m=5$

$M/m=40/6\approx 6.7$ p-wave scattering resonance !

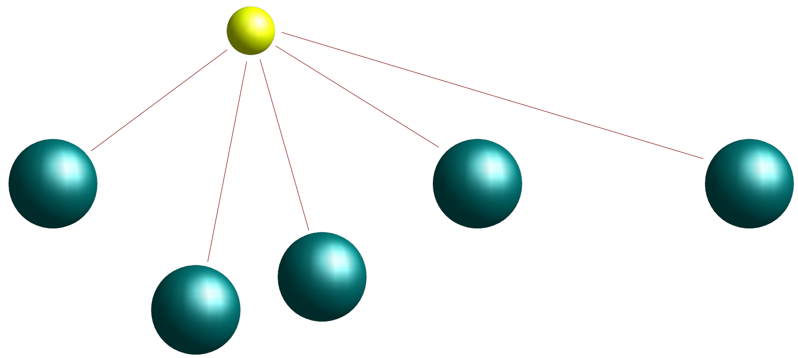
$M/m=8.2$ “universal” trimer state at threshold

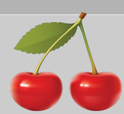
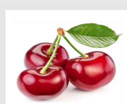
Kartavtsev & Malykh (2006)

$M/m>13.6$ many trimer (Efimov) states

(N+1)-body problem

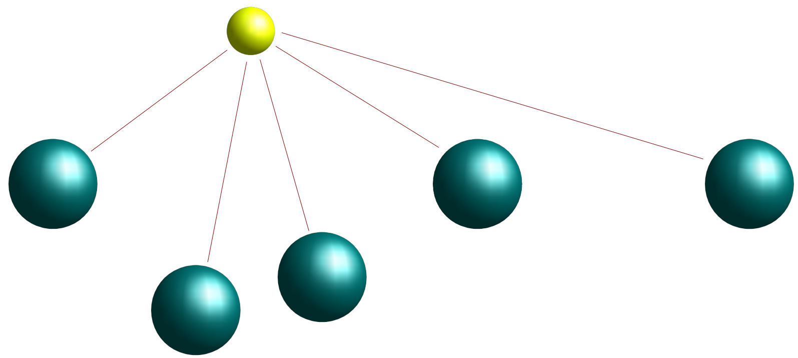
How many heavy fermions can be bound by a single light atom?

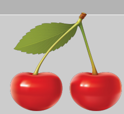
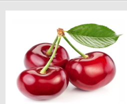


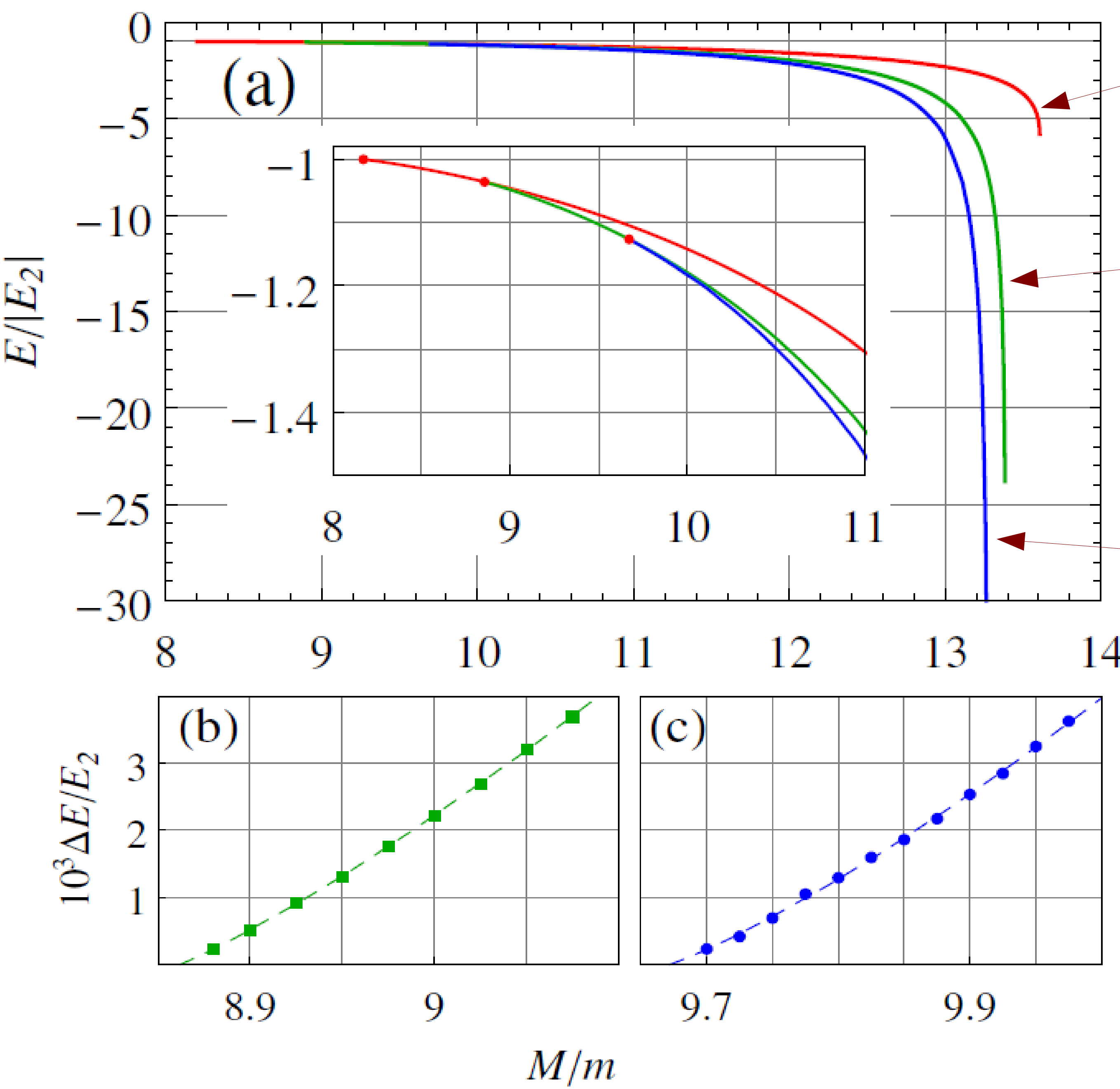
	Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer 	1^-	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer 	1^+	~9.5 Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer 	$?^?$	?	?
:	?	?	?
N+1-mer 	?	?	?

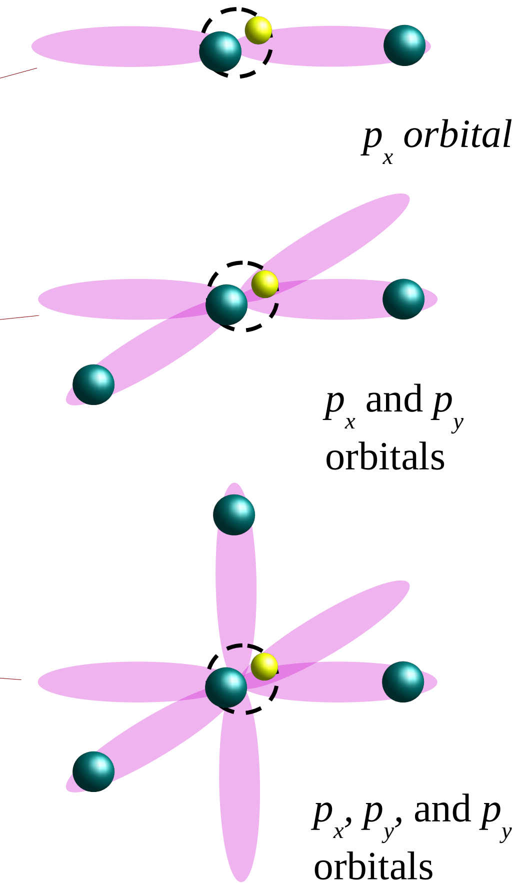
(N+1)-body problem

How many heavy fermions can be bound by a single light atom?



	Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer 	1^-	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer 	1^+	~9.5 → 8.862(1) Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer 	0^-	9.672(6)	13.279(2)
:	?	?	?
N+1-mer 	?	?	?





pentamer = closed *p*-shell

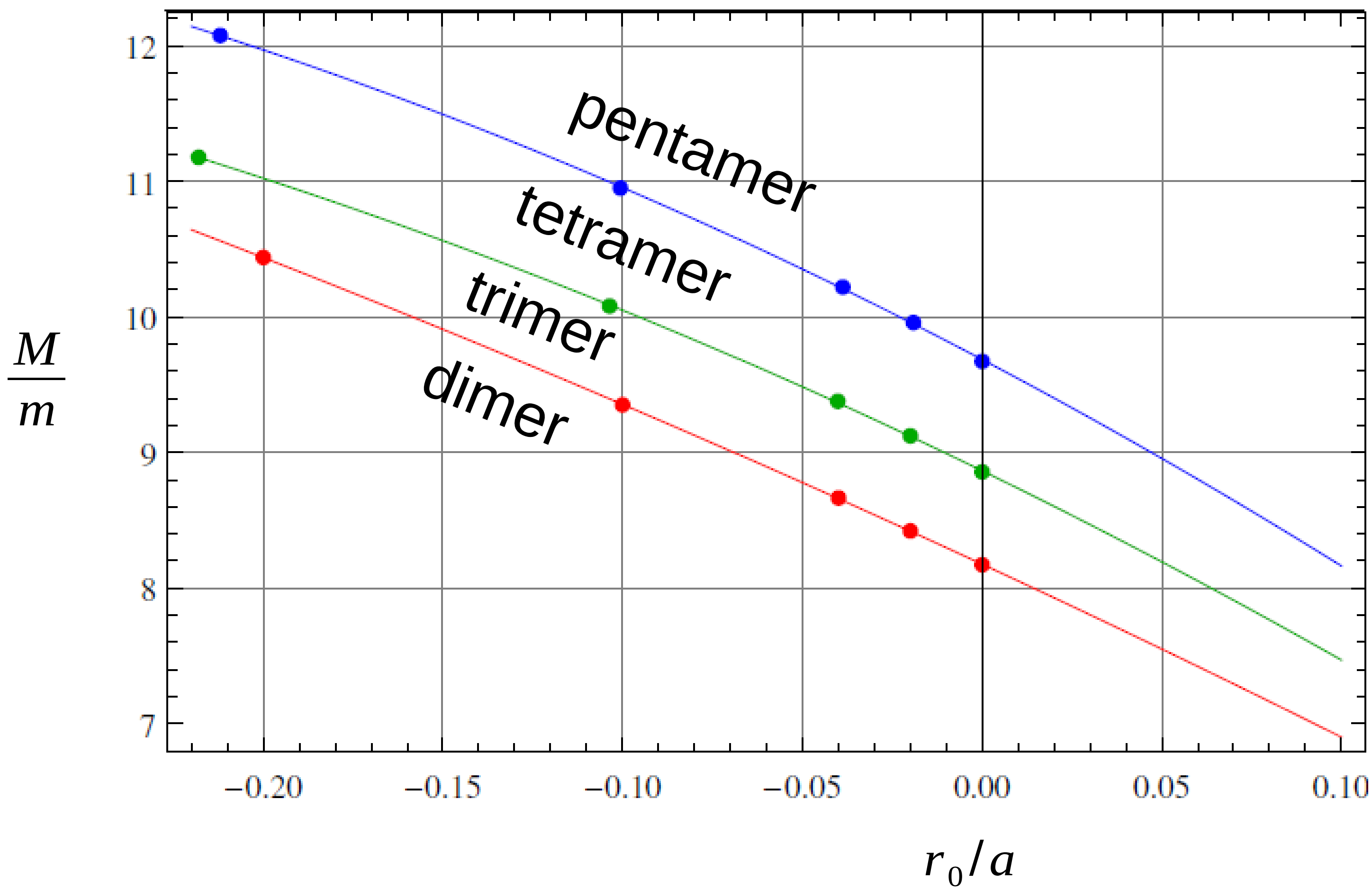
CONJECTURE:

No hexamer!

(requires justification)

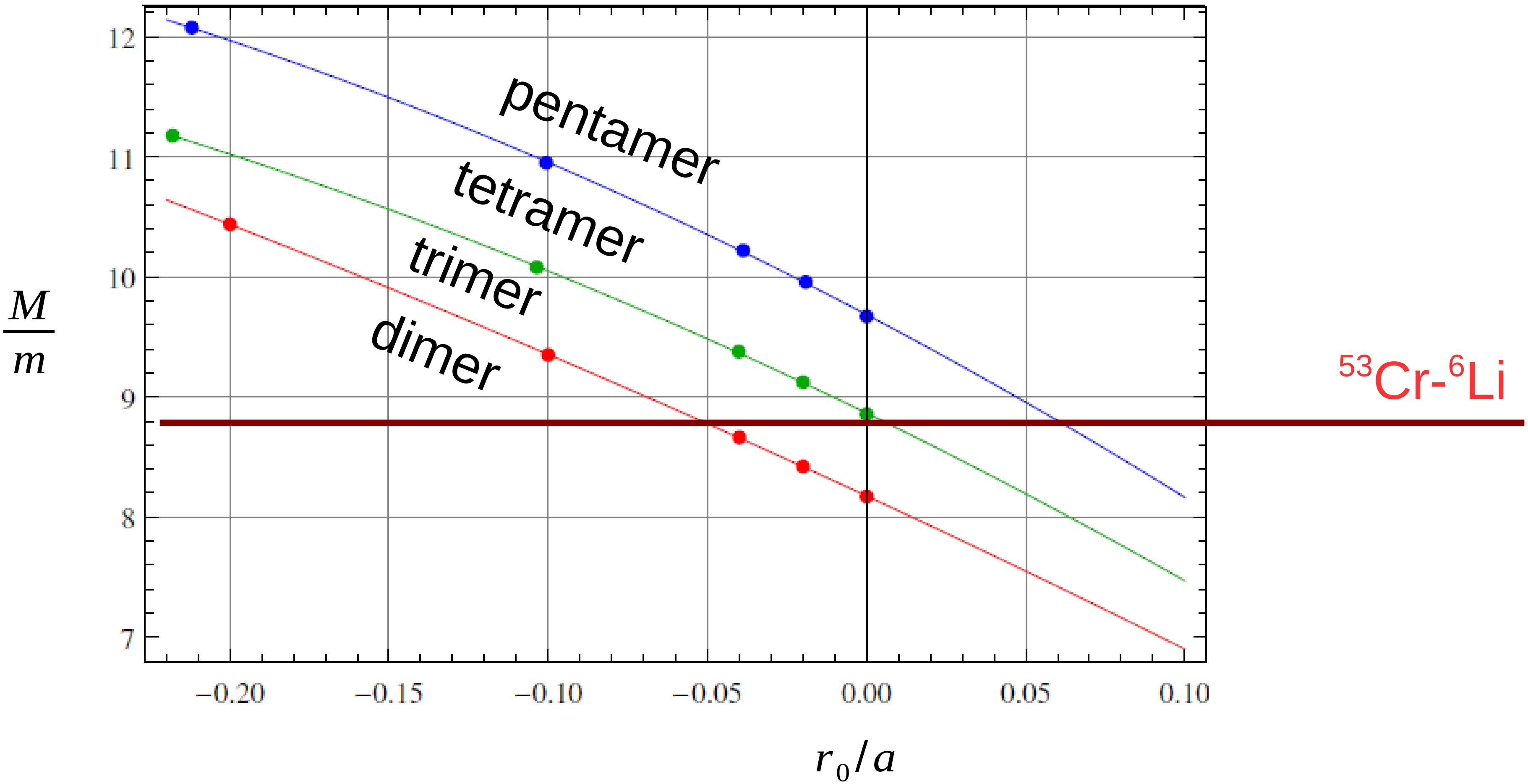
Effective-range effects

$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2}k^2$$



Effective-range effects

$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2} k^2$$



Physics at *a*=infinity (& zero range)

Small-hyperradius behavior of the (*N*+1)-body wave function:

$$\left[-\frac{\partial^2}{\partial R^2} - \frac{3N-1}{R^2}\frac{\partial}{\partial R} + \frac{s^2-(3N/2-1)^2}{R^2}\right]\Psi(R) = 0$$

$$\Psi(R) \propto R^{-3N/2+1\pm s}$$

$$s^2 > 0 \quad (s > 0)$$

$$\Psi(R) \propto R^{-3N/2+1+s}$$

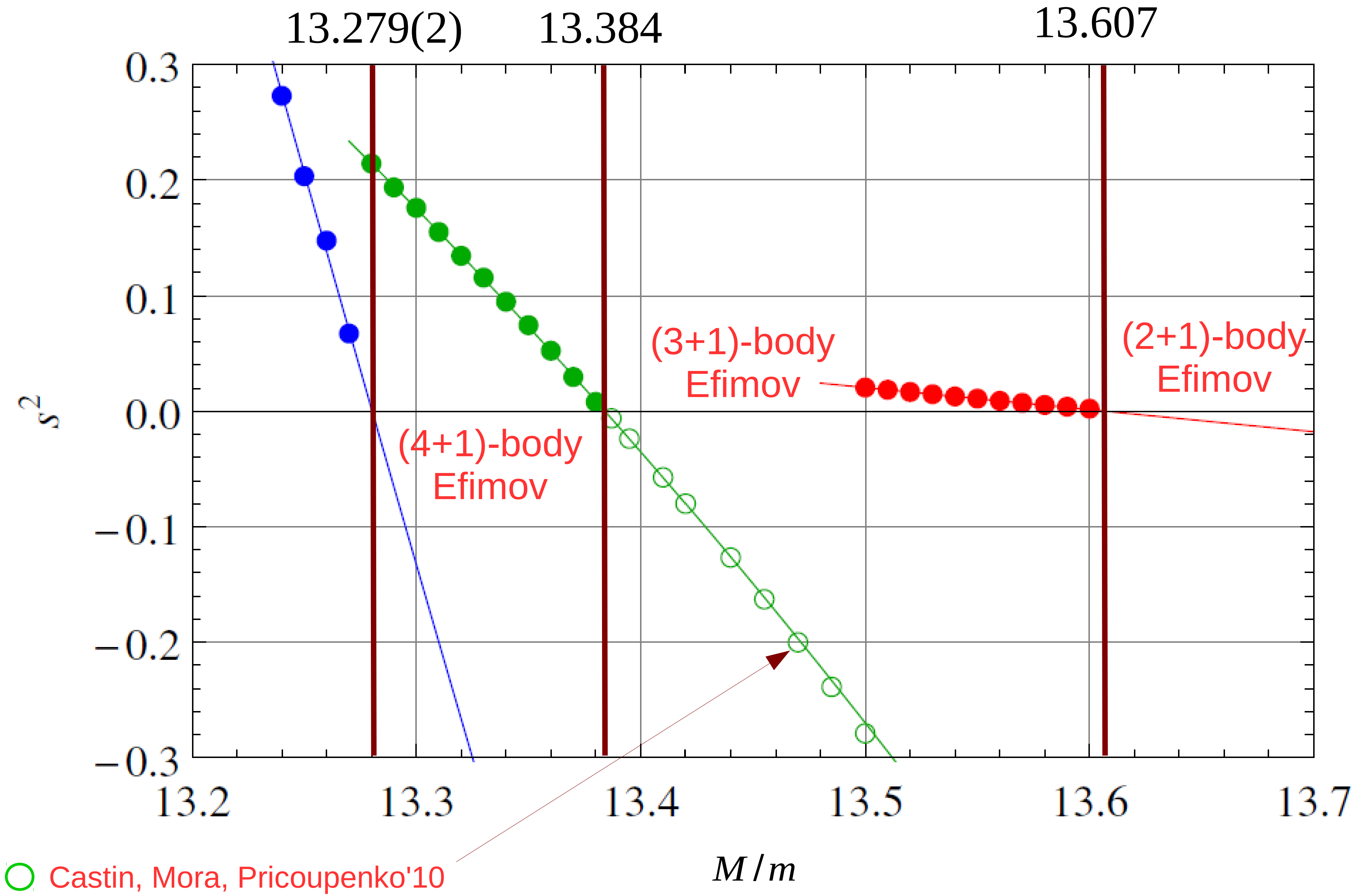
"Universal" regime in the sense that one needs no three-body parameter

Non-Efimovian regime

$$s^2 < 0 \quad (s = i s_0)$$

$$\Psi(R) \propto R^{-3N/2+1} \sin(s_0 \ln R/R_0)$$

"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wave function. Infinite number of trimer states. Efimov effect

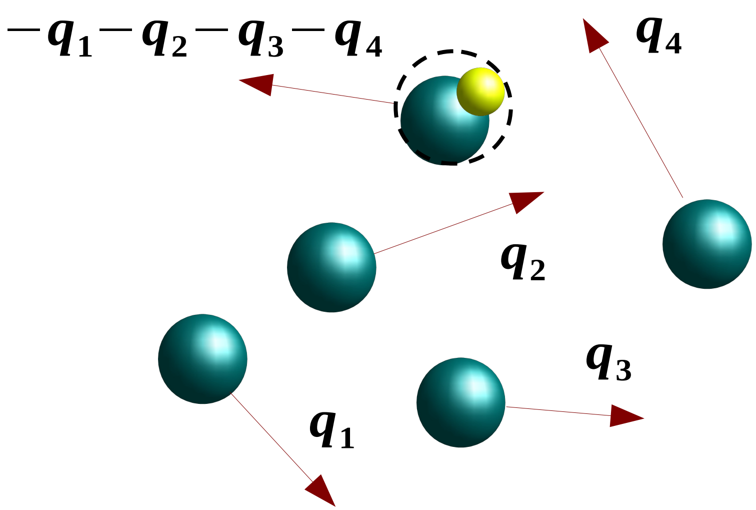

13.279(2)
13.384
13.607
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
s^2
(3+1)-body Efimov
(2+1)-body Efimov
(4+1)-body Efimov
13.2
13.3
13.4
13.5
13.6
13.7
M / m
Castin, Mora, Pricoupenko'10

Method

(*N*+1)-body Skorniakov – Ter-Martirosian equation (STM) [Pricoupenko'11]:

$$\frac{1}{4\pi}\left(\frac{1}{a}+\frac{r_0\kappa^2}{2}-\kappa\right)F(\mathbf{q}_1,\ldots,\mathbf{q}_{N-1})=\int\frac{d^3q_N}{(2\pi)^3}\frac{\sum_{i=1}^{N-1}F(\mathbf{q}_1,\ldots,\mathbf{q}_{i-1},\mathbf{q}_N,\mathbf{q}_{i+1},\ldots,\mathbf{q}_{N-1})}{-\frac{2\mu E}{\hbar^2}+\frac{\mu}{M}\sum_{i=1}^{N}q_i^2+\frac{\mu}{m}\left(\sum_{i=1}^{N}\mathbf{q}_i\right)^2}$$

where $\kappa=\sqrt{-\frac{2\mu E}{\hbar^2}+\frac{\mu}{M}\sum_{i=1}^{N-1}q_i^2+\frac{\mu}{M+m}\left(\sum_{i=1}^{N-1}\mathbf{q}_i\right)^2}$



N=2: $F(\boldsymbol{q}_1)=\hat{\boldsymbol{q}}_1\cdot\hat{z}\,f(q_1)$

N=3 [Castin, Mora, Pricoupenko'10]:

$$F(\boldsymbol{q}_1,\boldsymbol{q}_2)=\hat{z}\cdot\hat{\boldsymbol{q}}_1\times\hat{\boldsymbol{q}}_2\,f(q_1,q_2,\boldsymbol{q}_1\cdot\boldsymbol{q}_2)$$

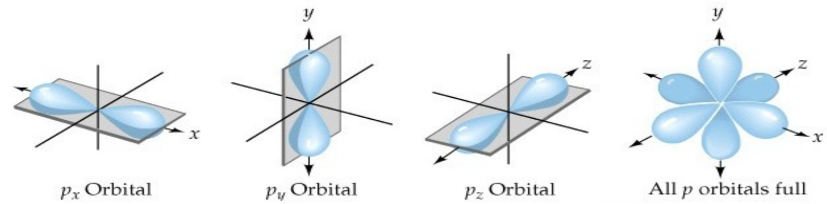
N=4:

$$F(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)=\hat{\boldsymbol{q}}_1\cdot\hat{\boldsymbol{q}}_2\times\hat{\boldsymbol{q}}_3\,f(q_1,q_2,q_3,\boldsymbol{q}_1\cdot\boldsymbol{q}_2,\boldsymbol{q}_1\cdot\boldsymbol{q}_3,\boldsymbol{q}_2\cdot\boldsymbol{q}_3)$$

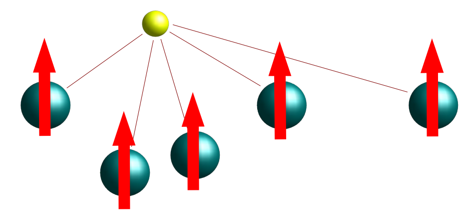
f – symmetric → ground state → *f*>0 → dens. distr. function → organize a diffusion process for which STM is the detailed balance equation

Summary

- pentamer = closed p-shell
  - no-go theorem for hexamer and six-body Efimov effect?



- Cr-Li (M/m=8.80) promising mixture
  - many-body physics with (*N*+1)-mers
  - few-body: include Cr-Cr dipole interaction?



- STM+DMC method
  - efficient in ``difficult'' corners of (4+1) problem
  - more particles? *N*+*M*?