Quench physics with few-body atomic systems in an optical lattice

Eite Tiesinga

NIST

Joint Quantum Institute

Joint Center for Quantum Information and Computer Science

Background

- Prepare the superfluid ground-state of ultra-cold atoms in a 2D or 3D optical lattice.
- On average a few-atoms per unit cell (<n>=2.5.)
- "Suddenly", turn off tunneling between unit cells. Then let atoms evolve and interact.
- In each site we end up with a few-body state of the kind

$$|0\rangle + e^{-iE_1 t}|1\rangle + e^{-iE_2 t}|2\rangle + e^{-iE_3 t}|3\rangle + \cdots$$

think coherent state missing amplitudes

- Interferometer can measure differences of (multiple) energies E_n. (Momentum distribution.)

Bosonic examples

- Paul, Johnson and Tiesinga, Phys. Rev. A **93**, 043616 (2016).
- Paul and Tiesinga, Phys. Rev. A **92**, 023602 (2015).

Fermionic example

- Nuske, Mathey and Tiesinga, Phys. Rev. A **94**, 023607 (2016).

Bosonic example

- Atoms in a cubic 3D lattice with weak interactions
- Quench prepares in each site the state

$$|0\rangle + e^{-iE_1 t}|1\rangle + e^{-iE_2 t}|2\rangle + e^{-iE_3 t}|3\rangle + \cdots$$

- In 2009 we studied the elastic three-body contribution!

See NJP **11**, 093022 (2009)

$$E_1$$

single-particle energy

$$E_2 = 2E_1 + U_2$$

$$E_3 = 3E_1 + 3U_2 + U_3$$

Three-body contribution!

Observed by S. Will, … I. Bloch, Nature **465**, 197 (2010)

Three-body strength

- Imagine "weak" delta-function

$$\mathbf{g}\,\delta(\mathbf{r}_1 - \mathbf{r}_2)(\partial/\partial r)r$$

don't mind the units

interaction between atoms. Then

$$U_2 \propto g \ll \Delta E_1$$

energy scale of trap

$$U_3 = c_0 \frac{(U_2)^2}{\Delta E_1}$$

a constant (known for a harmonic trap)

- Can we create situation where $U_2 << U_3$?
- As $U_3 \sim (U_2)^2$, the answer seems to be no

Zero-point energy

- We realize that collisions do not occur at zero collision energy and

$$\left(g + V\,\nabla^2_{\text{rel.}} + \cdots\right)\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

collision-energy dependent effective-range correction

- Now we find

energy scale of trap

$$U_2 \propto (g + V\Delta E_1)(1 + \cdots) \to 0$$

when $V \cdot \Delta E_1 = -g$

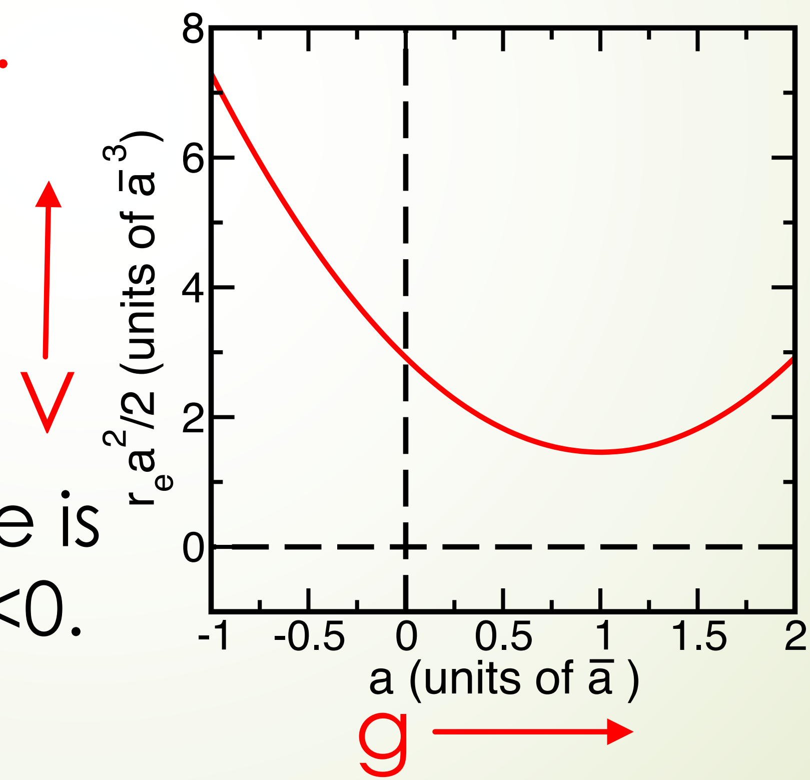
and still $U_3 = d_0 \dfrac{g^2}{\Delta E_1}$

Based on Phys. Rev. A **93**, 043616 (2016)

How realistic is this?

- Spin-less atoms like those in second column, ytterbium, or even alkali-metal atoms well away from resonances.
- van-der-Waals physics gives relationship between g and V.
  - parabola !
  - V is positive!
- The only valid case is ^{88}Sr with “small” g<0.

$a = -2a_0$



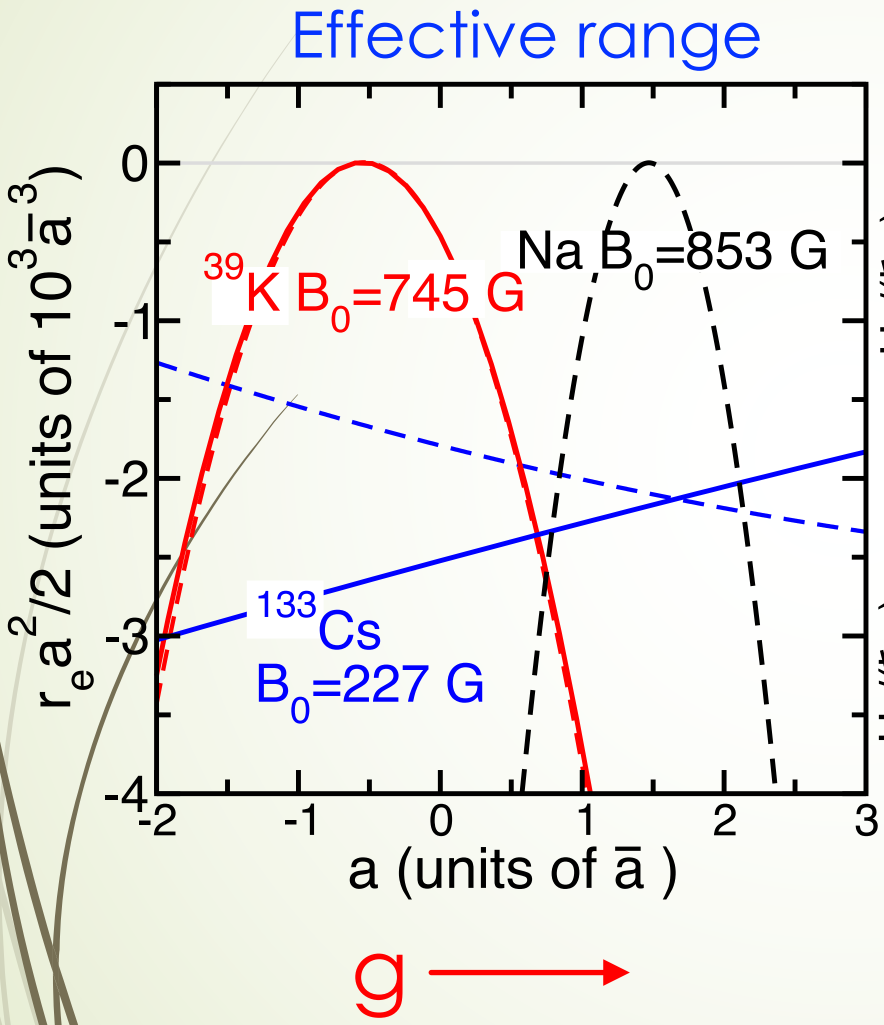
Feshbach resonances

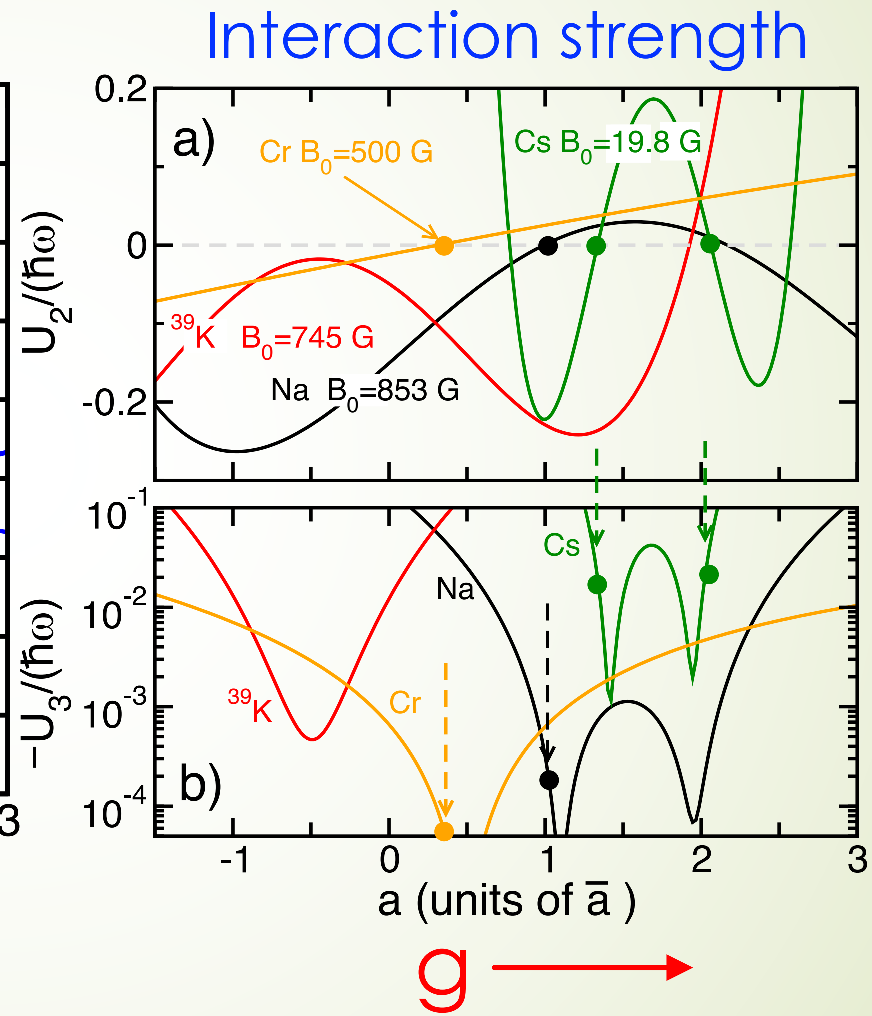
- near resonances the relationship between V and g is more favorable.
- Still a (inverted) parabola, but also V<0 and "large".
- We can now satisfy the relationship for "reasonable" g>0.

Scattering lengths on order of van der Waals length

- Within the theory we have restrictions on how large V can be.
- Nevertheless, we suggest to try a few actual resonances. Both broad and narrow.

Feshbach resonances





Caveat

- Three-body recombination for reasonable g

$$K_3 = 25\frac{\hbar^2 \bar{a}^4}{m} \quad \text{when} \quad |a| \sim \bar{a}$$

- For ^{88}Sr the experiment will be hard.
- For resonances loss puts upper limits on the depth of the lattice

$$\frac{|a|}{\bar{a}} \gg \frac{\bar{a}}{\ell}$$

harmonic oscillator length

Fermionic atoms in a lattice

- Spin up/down fermions hop around between wells (with energy J).
- Occupy only lowest vibr. level in each well.
- Only up and down atoms interact (with strength U_2.)

See Phys. Rev. A **94**, 023607 (2016).

Superfluid are Cooper pairs

- For attractive $U_2 \sim J$, a BCS ground state of *paired* up/down atoms with momentum *k* and *-k*.
- Formally,

up

down

vacuum

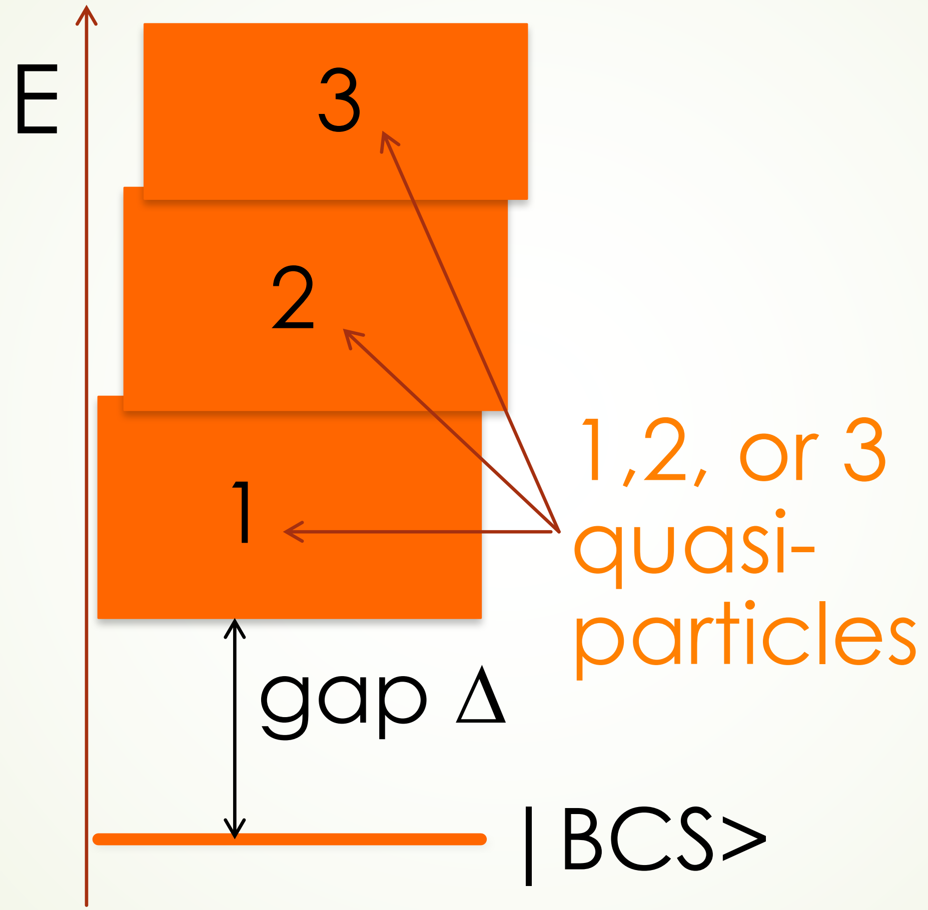
$$|BCS\rangle = e^{\sum_k u_k a_k^\dagger b_{-k}^\dagger}|0\rangle$$

(missing band index)

- More importantly in each well/site

$$\text{“}\ |0\rangle + |\uparrow\rangle + |\downarrow\rangle + |\uparrow\downarrow\rangle\ \text{“}$$

Spectrum near half filling



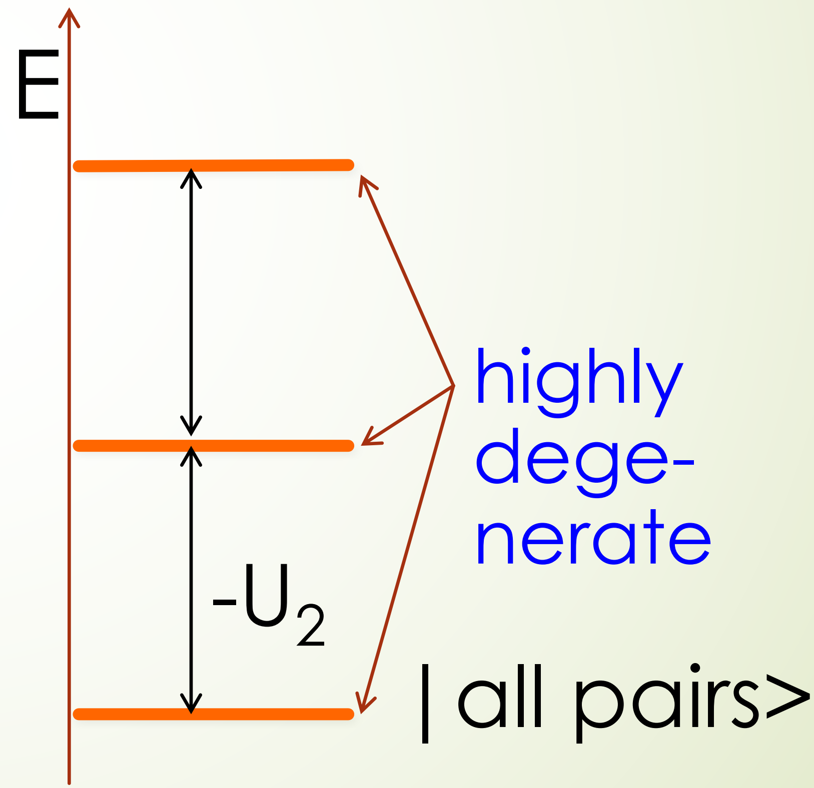
From Zhao & Paramekanti, PRL **97**, 230404 (2006)

Now Quench to

$$\frac{1}{2}U_2\, b_i^\dagger a_i^\dagger\, b_i a_i$$

a,b -- spin up/down
i -- unit cell

- Spectrum is exactly solvable
- Near half filling, ground state has *every other site* occupied with 2 atoms, 1 spin up and 1 spin down.
- Excited states have broken pairs.

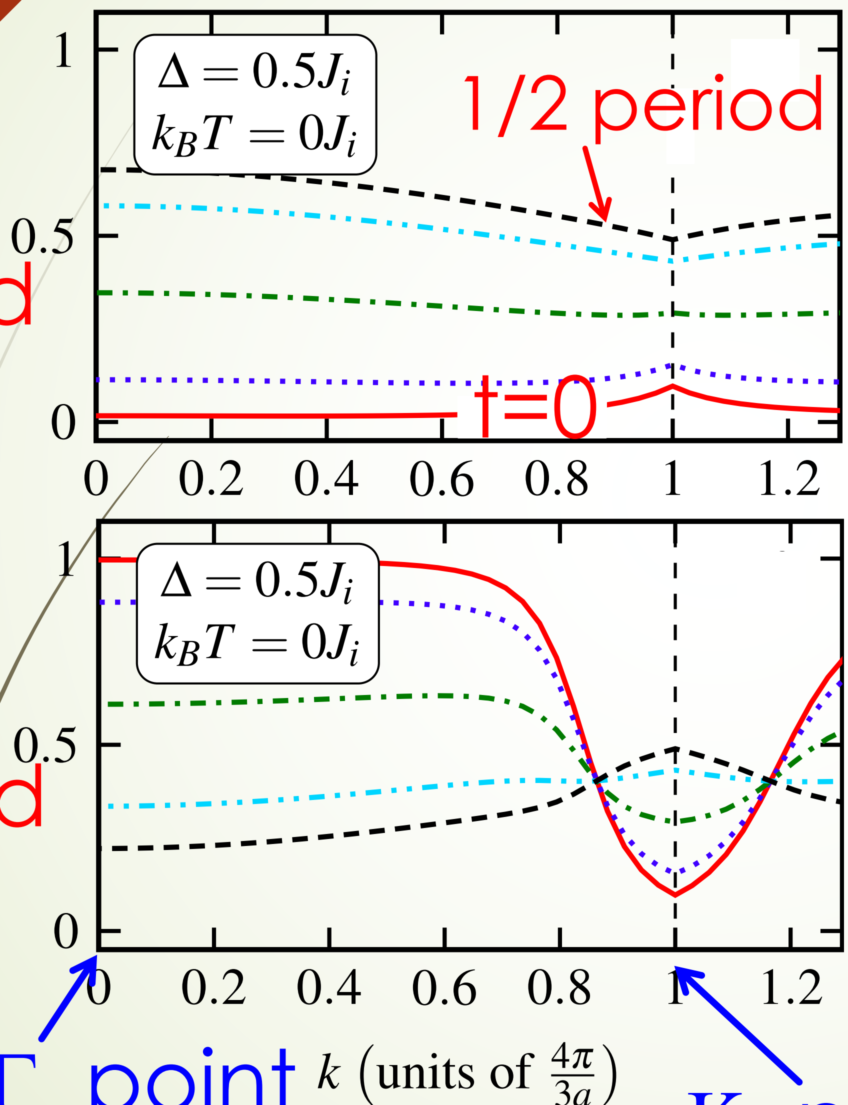


PRA **94**, 023607 (2016)

Momentum distribution in graphene

2nd band

1st band

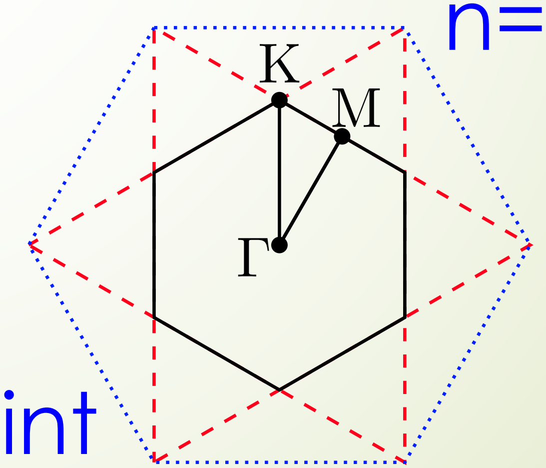


Γ point

K point

- U_2 periodic increase of population in 2nd band.
- but also true for gap Δ=0.

n=0.45



Conclusions

- Two examples of few-body physics with optical lattices using atom-number superposition states.

- References
  - Phys. Rev. A **94**, 023607 (2016)
  - Phys. Rev. A **93**, 043616 (2016)
  - Phys. Rev. A **92**, 023602 (2015)