

KITP-UCSantaBarbara Aug-Sep '08



Fluctuation-induced forces and wetting phenomena  
in colloid-polymer mixtures:  
Are renormalization group predictions measurable?

Y. Hennequin, D. G. A. L. Aarts, J. O. Indekeu, H. N. W. Lekkerkerker and D. Bonn, "*Fluctuation forces and wetting layers in colloid-polymer mixtures*", Phys. Rev. Lett. **100**, 178305 (2008).

1. Colloidal suspension
2. Hard spheres + tunable attractions
3. Scales
4. Functions and variables
5. Walls and wetting
6. Free interface: capillary wave roughness
7. Unbinding from a wall: RG predictions vs experiment
8. Line tension

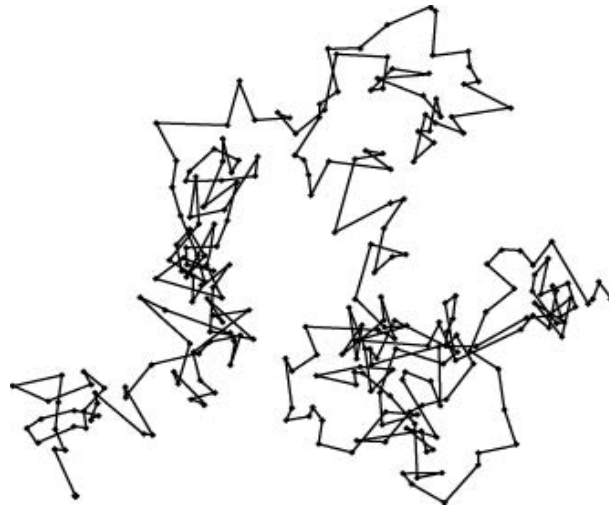
[1] H.N.W. Lekkerkerker, V.W.A. de Villeneuve, J.W.J. De Folter, M. Schmidt, Y. Hennequin, D. Bonn, J.O. Indekeu and D.G.A.L. Aarts, "*Life at Ultralow Interfacial Tension: wetting, waves and droplets in demixed colloid-polymer mixtures*", Eur. Phys. J. B (2008), to appear ([www.itf.fys.kuleuven.ac.be/~joi/](http://www.itf.fys.kuleuven.ac.be/~joi/))

[2] Y. Vandecan and J.O. Indekeu, "*Theoretical study of the three-phase contact line and its tension in adsorbed colloid-polymer mixtures*", J. Chem. Phys. **128**, 104902 (2008).

# 1. Colloidal suspension

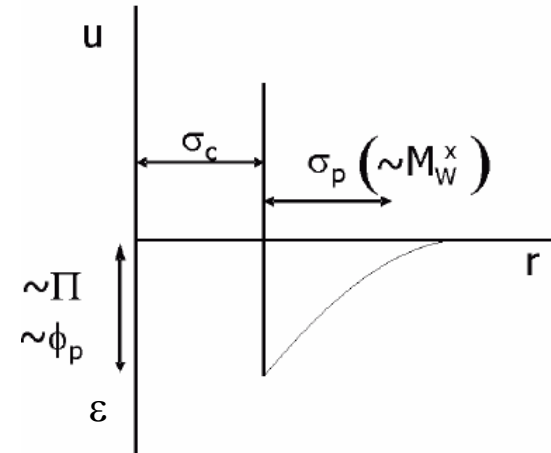
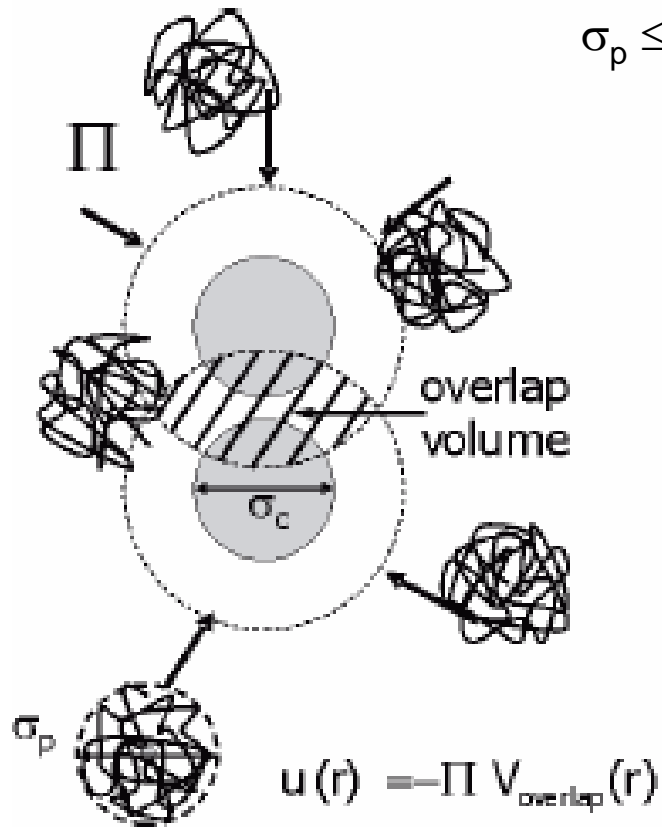
Brownian motion of a colloid:

“dance driven by random collisions with atoms according to the kinetic theory of heat” (Thirion, Delsaux, Carbonelle, s.j.; 1874);  
proof of the existence of atoms (Perrin 1910, Einstein 1905)

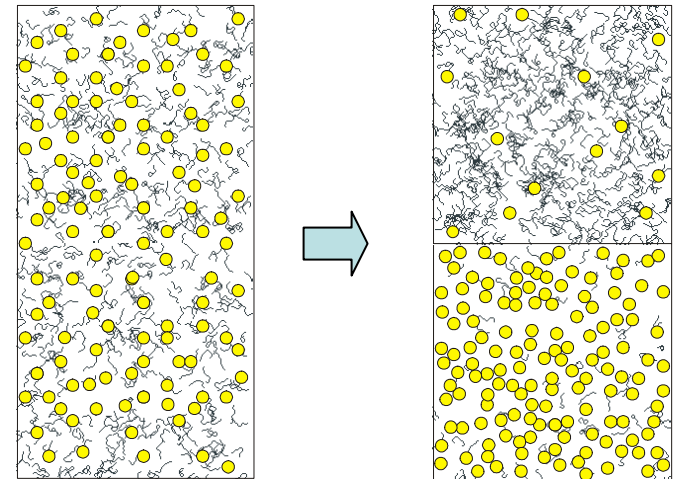


“Levitation by agitation”:  $Pe = (m_c - m_f)gR_c / kT < 1$

## 2. Hard spheres + tunable attractions



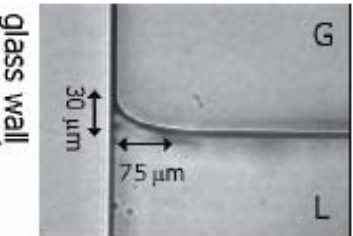
$\varepsilon / kT \approx 1$



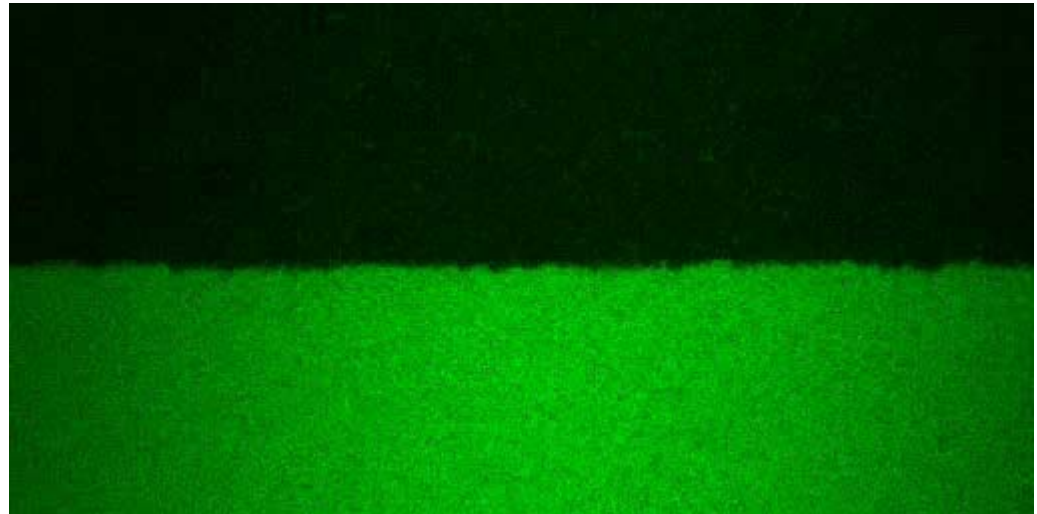
Depletion attraction (Asakura-Oosawa-Vrij)

### 3. Scales

Interfacial tension  $\gamma = \varepsilon / \sigma^2$ ; 10nN/m for colloids, 10mN/m for molecular fluids

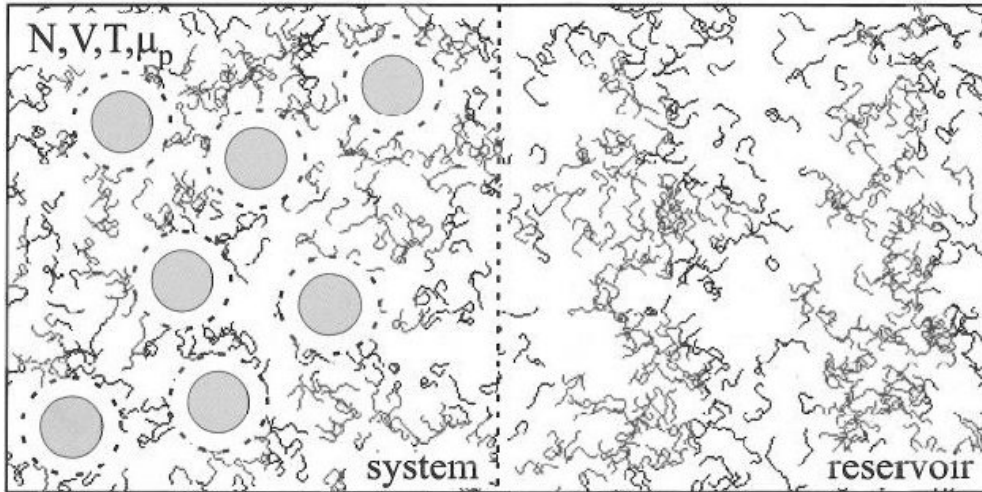


Thermal agitation of the interface:  $L_T = (kT / \gamma)^{1/2} \approx \sigma$  ;  
0.1  $\mu\text{m}$  for colloids, 0.1 nm for molecular fluids



Capillary length  $L_{\text{cap}} = (\gamma / g\Delta\rho)^{1/2}$  ;  $\mu\text{m}$  for colloids, mm for molecular fluids

## 4. Functions and variables



Semi-grand canonical potential

$$\Omega \approx F(T, V, N_c) - kT N_p(\mu_p),$$

ideal polymers ( $\Pi = n_p r kT$ )

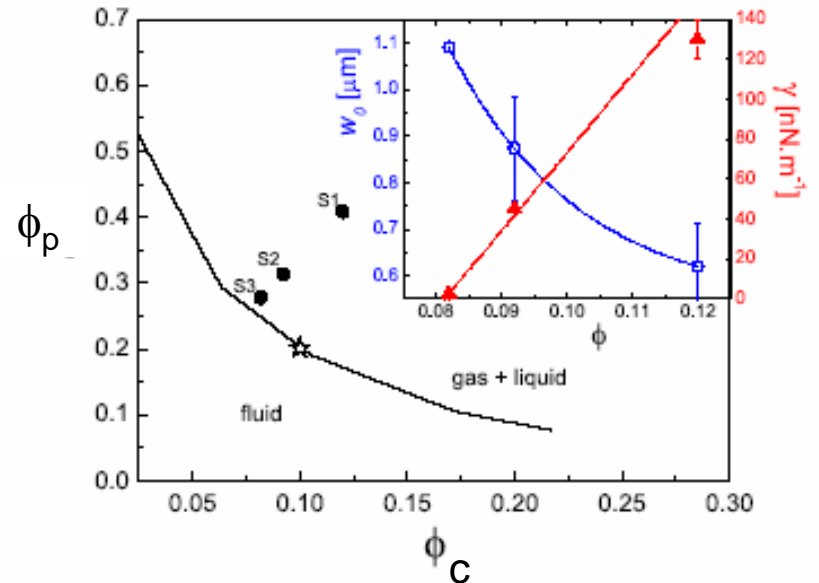
$$N_p = n_p r \alpha V$$

$$\text{Free volume } \alpha V \approx V - N_c (4/3)\pi(R_c + \Delta)^3$$

$$\alpha(\phi_c, \Delta/R_c) < 1 - \phi_c; \quad \phi_p = \alpha \phi_p^r$$

$R_g$

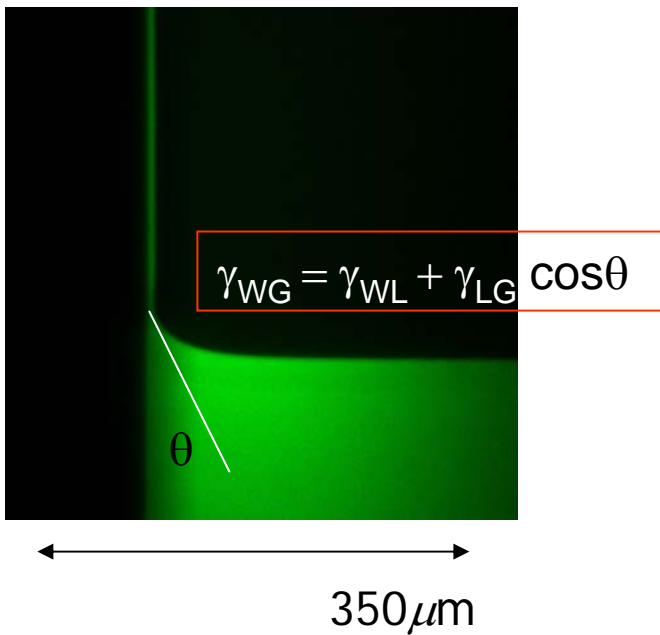
Excluded-volume-interacting polymers...



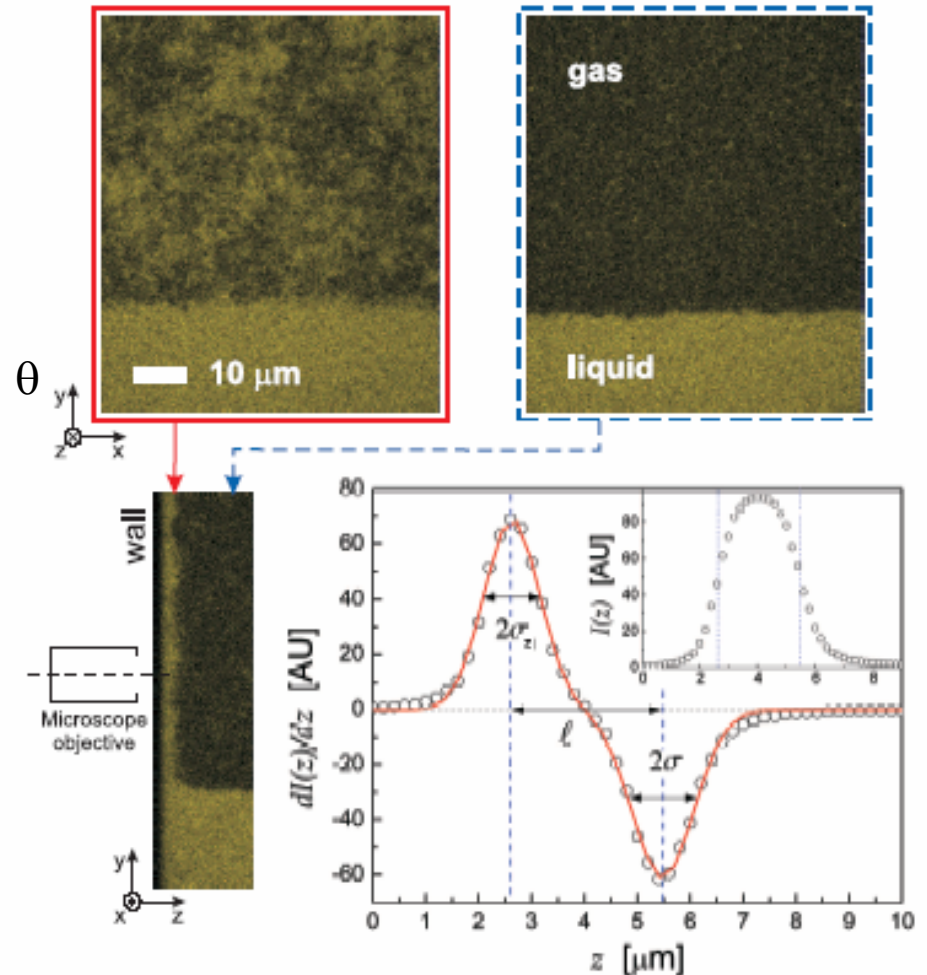
## 5. Walls and wetting

Cahn-Landau theory

$$\chi[\rho] = \int_0^\infty dz \left( f(\rho) - \mu_c \rho(z) + p_c + m(\rho) \left( \frac{d\rho}{dz} \right)^2 \right) - h_1 \rho_1 - \frac{1}{2} g \rho_1^2, \quad \rho = \phi_c$$

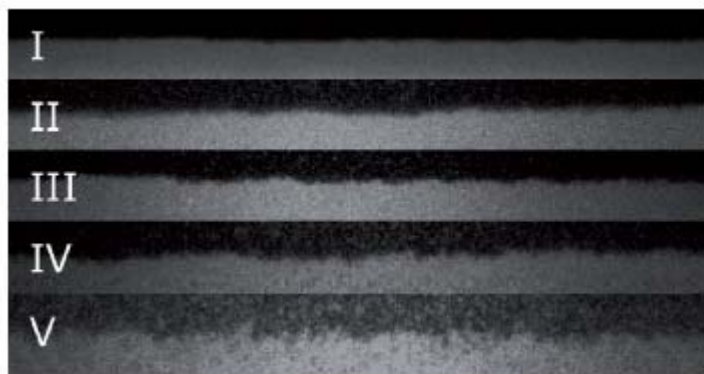


Laser scanning confocal microscopy



First-order wetting transition is predicted; complete wetting is observed

## 6. Free interface: capillary wave roughness



←  $L_x$  →

↑  $L_T = (kT/\gamma)^{1/2}$  “fluctuate with moderation”

→  $L_{cap} = (\gamma/g\Delta\rho)^{1/2}$  “stay horizontal”

$$\langle h(x,y)^2 \rangle \propto L_T^2 \log \left[ \frac{((2\pi/R_c)^2 + (1/L_{cap})^2)}{((2\pi/L_x)^2 + (1/L_{cap})^2)} \right]$$

Without gravity:  $\langle h(x,y)^2 \rangle \propto L_T^2 \log [L_x / R_c] \rightarrow \infty$

Thermal wandering in space dimension  $d$ :  $\langle h(x,y)^2 \rangle \propto L_x^{2\zeta}$ ;  $\zeta = (3-d)/2$

Also favourable time scales for direct visualization:  $\tau = (\eta/\gamma)L_{cap}$  ;  
seconds for colloids,  $10\mu\text{s}$  for molecular fluids



## 7. Unbinding from a wall: RG predictions vs experiment

$$\langle h(x,y)^2 \rangle = (1/2\pi) L_T^2 \log(\xi_{||} / R_c)$$

wetting parameter  $\omega = kT/4\pi\gamma\xi^2 \approx 0.8$ , Ising model

RG theory for wetting with short-range forces in  $d=3$

D.S. Fisher and D.A. Huse, Phys. Rev. B **32**, 247 (1985);

R. Lipowsky and M.E. Fisher, Phys. Rev. B **36**, 2126 (1987).

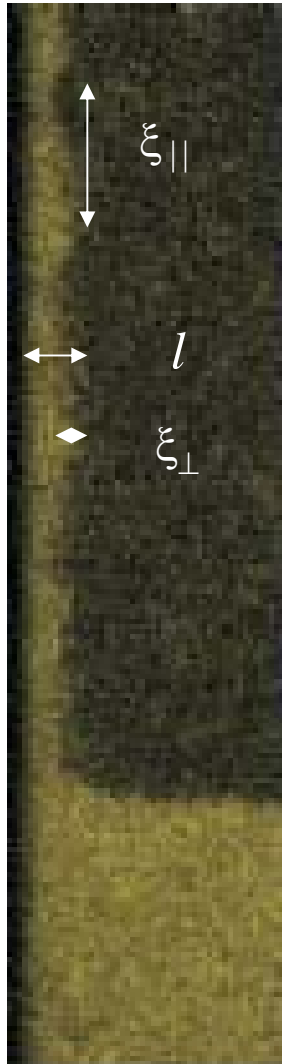
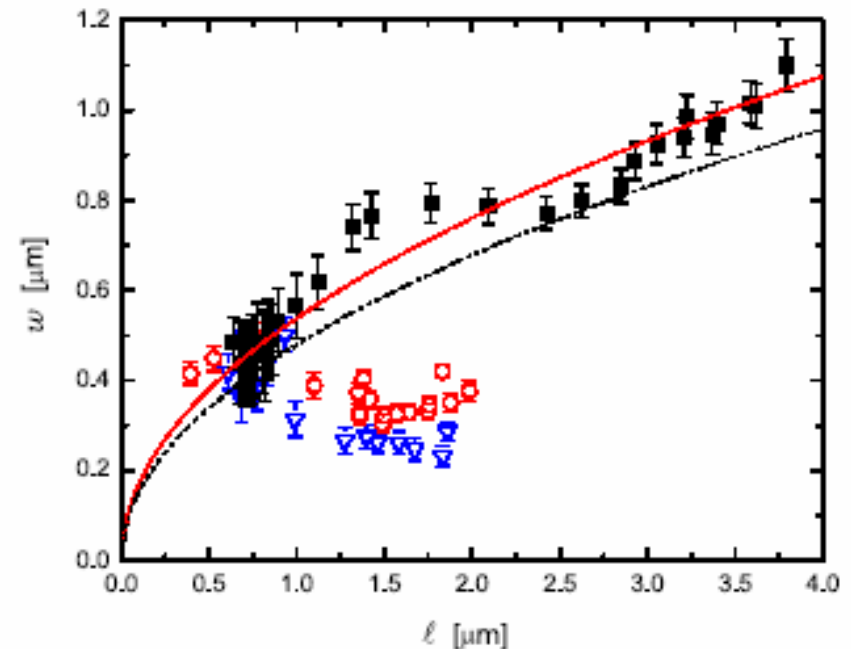
T. Kerle, J. Klein, and K. Binder, PRL **77**, 1318 (1996); Eur. Phys. J. B **7**, 401 (1999).

$$l/\xi = F(\omega) \log(\xi_{||} / \xi)$$

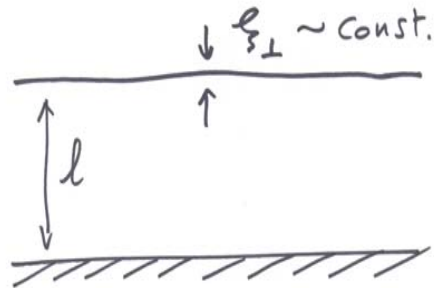
$$\xi_{\perp}^2 = f(\omega) L_T l$$

Expt.  $\xi_{\perp}^2 = 0.29 \mu\text{m} l$

Theory  $\xi_{\perp}^2 = 0.23 \mu\text{m} l$



Mean-field regime:  $\xi_{\perp}$  remains constant\* while  $l$  and  $\xi_{\parallel}$  diverge



e.g., systems with van der Waals forces in  $d=3$

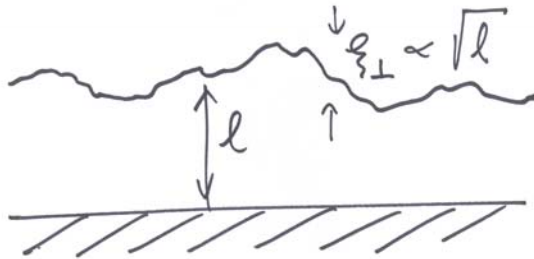
\* or diverges logarithmically

All fluctuation regimes:  $\xi_{\perp} \propto l$



$d < d_u$ , regardless of short- or long-range forces, critical or complete wetting, and including membranes

Marginal regime:  $\xi_{\perp} \propto \sqrt{l}$



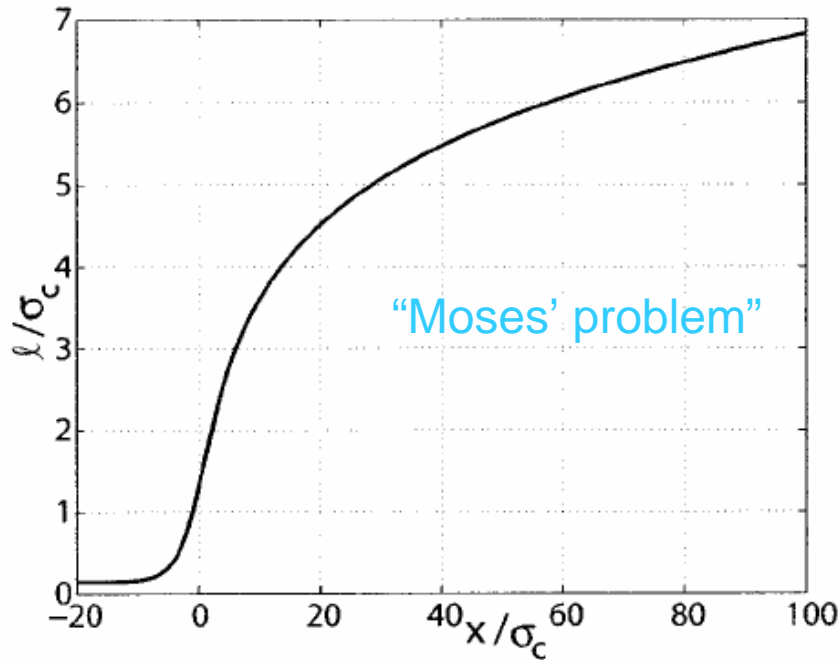
short-range forces and  $d = d_u = 3$ ,  
regardless of critical or complete wetting

## 8. Line tension

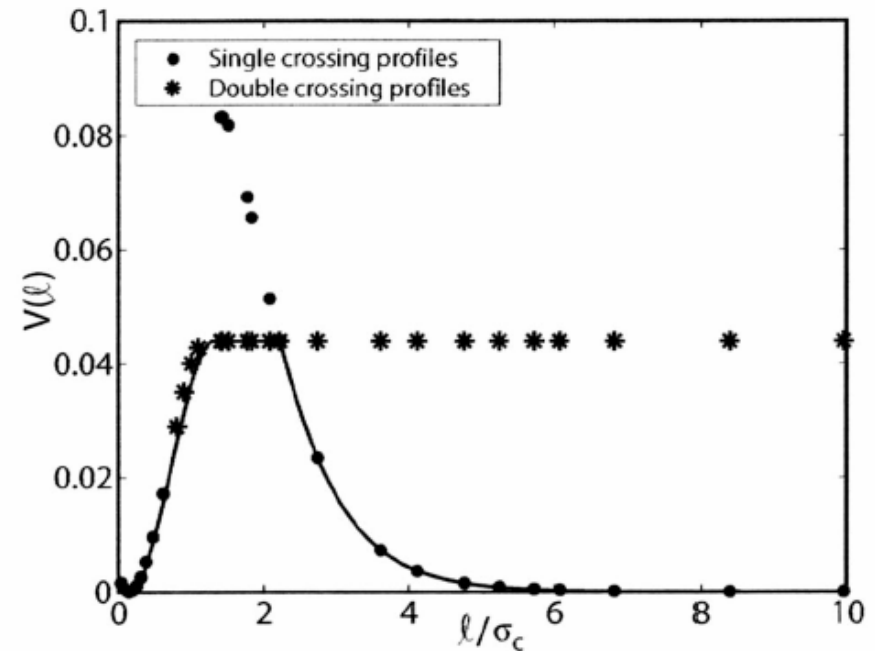
$$\tau[\ell] = \int_{-\infty}^{\infty} dx \left[ \gamma_{\text{LG}} \left( \sqrt{1 + \left( \frac{d\ell}{dx} \right)^2} - 1 \right) + V(\ell(x)) + c(x) \right],$$

$$\tau = (2\gamma_{\text{LG}})^{1/2} \xi \int_{\ell_1}^{\infty} d\tilde{\ell} [V(\tilde{\ell})^{1/2} - E^{1/2}],$$

Typically  $\tau \approx \gamma \xi$ ;  $10^{-13}$  N for colloids,  $10^{-11}$  N for molecular fluids



Interface displacement at first-order wetting



Interface potential