Nematic Fractional Quantum Hall States

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One day in 1999 my phone rang…

``Hi. My name is Mike Freedman. I'm a mathematician at Microsoft Research interested in quantum computation. I've been reading one of your papers, and I hope that I can stop by your office when I'm in LA….''

Several weeks later in my office:

So I'm interested in the physics of modular tensor categories …

… oh, is that a photograph of Mont Blanc?

12 Years Later …

My cell phone rings:

``Hi Chetan. It's Mike. Is the velocity of an edge excitation in the 5/2 state around 10^5 m/s?''

Telecommunications technology has changed a lot.

Time evolution is not unitary.

Mike is now a condensed matter physicist.

Condensed Matter Theorist

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No Physics Allergies

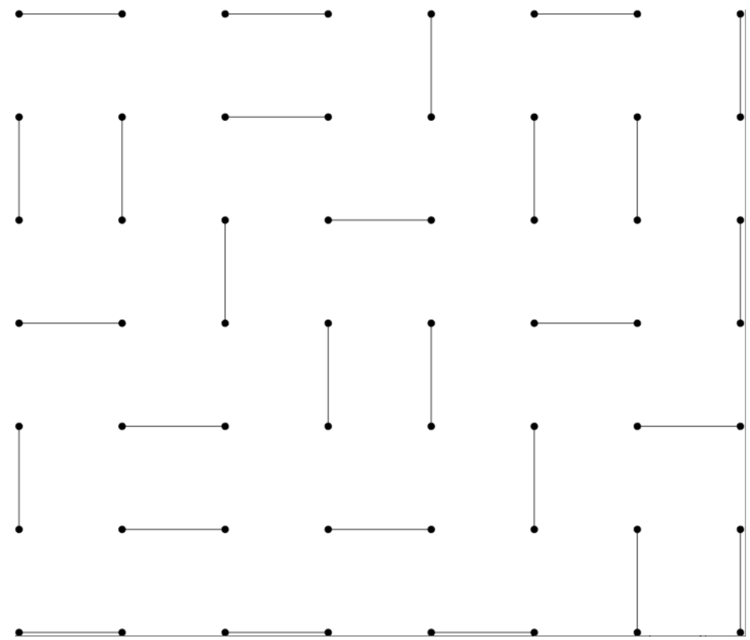
Quantum Lifschitz Transition

$$S = \int dt d^2x \Big[\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2} r (\nabla \phi)^2 - \frac{1}{2}\kappa^2 (\nabla^2 \phi)^2 - \frac{\lambda}{4} (\nabla \phi)^4 \Big]$$

$$r > 0 \Rightarrow \nabla \phi = 0$$

$$r < 0 \Rightarrow \nabla \phi = \sqrt{\frac{-r}{\lambda}}$$

<u>Quantum Dimer Model</u>:



$$H \;=\; -t\left(|\,=\,\rangle\langle\,\|\,| + h.c.\right) + v\left(|\,=\,\rangle\langle\,=\,| + |\,\|\,\rangle\langle\,\|\,|\right)$$

Rokhsar-Kivelson ’88; Henley ’97;
Moessner-Sondhi ’00; ….

Critical Point at r=0

$$S = \frac{1}{2}\int dt d^2x \left[(\partial_t \phi)^2 - \kappa^2(\nabla^2\phi)^2\right]$$

$$\Psi_0[\phi] = e^{-\frac{1}{2}\kappa\int d^2x(\nabla\phi)^2}$$

Related to Stat. Mech. height models, loop gases; see next talk.

Ardonne et al. ’03; …. *Gauge/Gravity:* Kachru et al. ’08; …
Entang. Entropy: B. Hsu et al. ’08

Dual Description

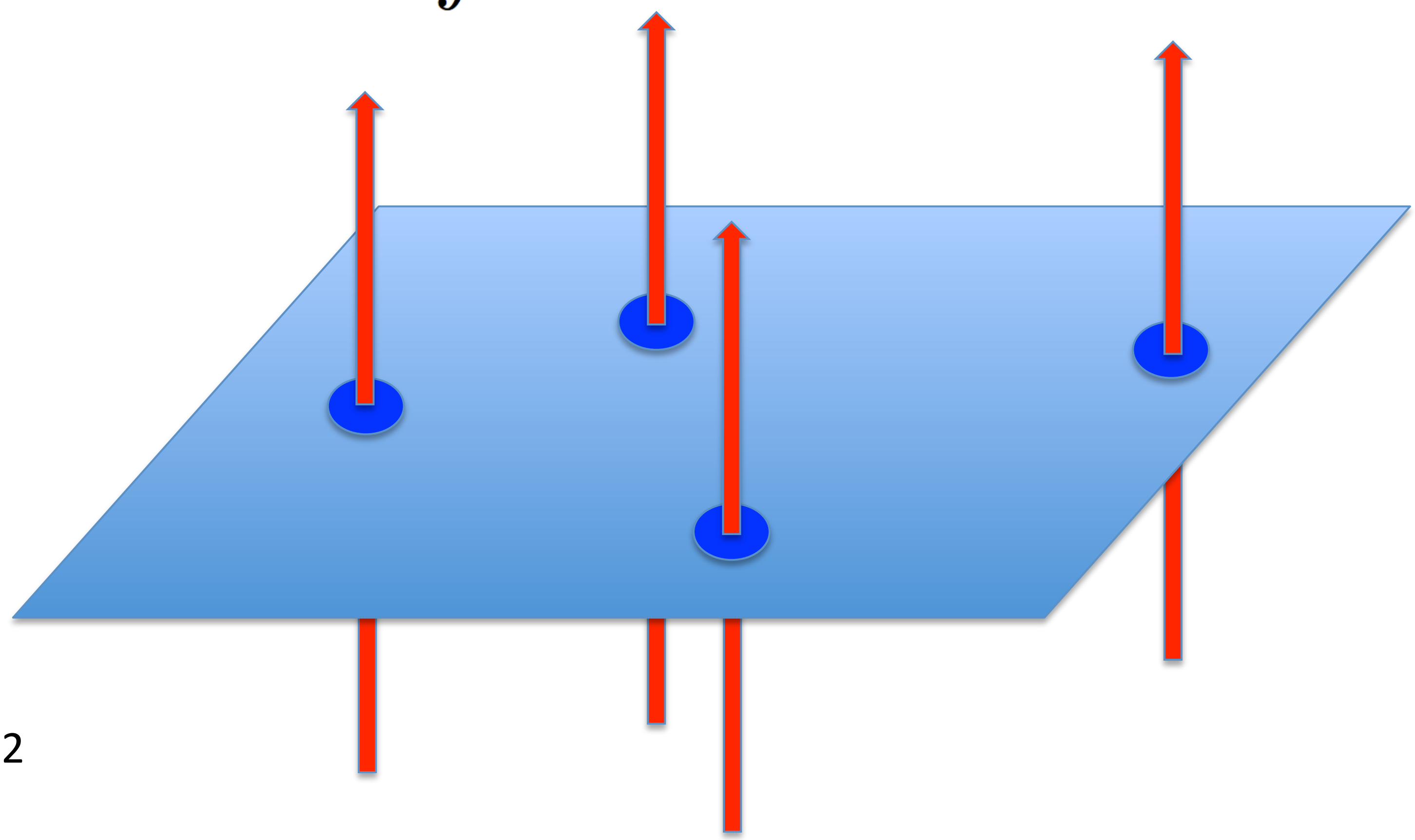
2+1-D Particle-Vortex Duality:

$$j_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda$$

$$S = \int dt\, d^2x \left[\frac{1}{g^2}\left(e_i \partial_t a_i + a_t \partial_i e_i - \frac{r}{2} e_i^2 - \frac{\kappa^2}{2}(\partial_i e_j)^2 - \frac{\lambda}{4}(e_i^2)^2 - \frac{1}{2} b^2 \right) + \frac{k}{4\pi}\epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right]$$

Chern-Simons Theory

$$S = \frac{k}{4\pi}\int dt\, d^2x\, \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$



Deser et al. '82
Witten '89
Zhang et al. '89, Read '89

$$S = \int dt\, d^2x \Big[-\frac{1}{2} f_{\mu\nu} f^{\mu\nu} + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \Big]$$

Fewer derivatives; more relevant.

$$\langle a_\mu(q) a_\nu(-q) \rangle = \frac{P^T_{\mu\nu}}{q^2 + (k/4\pi)^2} + \frac{k}{4\pi} \frac{\epsilon_{\mu\lambda\nu} q_\lambda}{q^2(q^2 + (k/4\pi)^2)}$$

Chern-Simons term acts as a mass term

Can ignore Maxwell in IR.

Occam's Razor: *``entia non sunt multiplicanda praeter necessitatem''*

``entities must not be multiplied beyond necessity''

Lifschitz-Chern-Simons

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(e_i \partial_t a_i + a_t \partial_i e_i - \frac{\kappa^2}{2} (\partial_i e_j)^2 - \frac{1}{2} b^2 \right) + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right]$$

Can we ignore Lifschitz and focus on CS in the IR? No.

Scale-Invariant under:

$$t \to \lambda^2 t, \quad x \to \lambda x, \quad a_t \to \lambda^{-2} a_t, \quad a_i \to \lambda^{-1} a_i, \quad e_i \to \lambda^{-1} e_i$$

$$\langle e_i(-i\omega_n, -\mathbf{p})\, e_j(i\omega_n, \mathbf{p})\rangle = -\frac{g^2p^2}{\omega_n^2 + \tilde{\kappa}^2p^4}\, P_{ij}^T(p) - \frac{g^6\kappa^2\tilde{k}^2p^2\delta_{ij} + g^4\tilde{k}\omega_n\epsilon_{ij})}{\omega_n^2 + \tilde{\kappa}^2p^4}$$

$$\langle e_i(-i\omega_n, -\mathbf{p})\, a_j(i\omega_n, \mathbf{p})\rangle = -\frac{g^2\omega_n}{\omega_n^2 + \tilde{\kappa}^2p^4}\, P_{ij}^T(p) - \frac{g^4\kappa^2\tilde{k}p_i\epsilon_{jk}p_k}{\omega_n^2 + \tilde{\kappa}^2p^4}$$

$$\langle e_i(-i\omega_n, -\mathbf{p})\, a_t(i\omega_n, \mathbf{p})\rangle = \frac{-ig^2\Big((\omega_n^2 + \kappa^2p^4)p_i - \kappa^2g^2\tilde{k}\omega_n\epsilon_{ij}p_j\Big)}{p^2(\omega_n^2 + \tilde{\kappa}p^4)}$$

$$\langle a_i(-i\omega_n, -\mathbf{p})\, a_j(i\omega_n, \mathbf{p})\rangle = -\frac{g^2\kappa^2p^2}{\omega_n^2 + \tilde{\kappa}^2p^4}\, P_{ij}^T(p) - \frac{g^2}{p^2}\left(\frac{p_ip_j}{p^2}\right)$$

$$\langle a_t(-i\omega_n, -\mathbf{p})\, a_i(i\omega_n, \mathbf{p})\rangle = \frac{ig^2\omega_np_i}{p^4} - \frac{i\kappa^4g^4\tilde{k}\epsilon_{ij}p_jp^6)}{p^4(\omega_n^2 + \tilde{\kappa}^2p^4)}$$

$$\langle a_t(-i\omega_n, -\mathbf{p})\, a_t(i\omega_n, \mathbf{p})\rangle = \frac{g^2(\omega_n^4 + (\kappa^2 + \tilde{\kappa}^2)\omega_n^2p^4 + \kappa^4p^8)}{p^4(\omega_n^2 + \tilde{\kappa}^2p^4)}$$

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(e_i \partial_t a_i + a_t \partial_i e_i - \frac{r}{2} e_i^2 - \frac{\kappa^2}{2} (\partial_i e_j)^2 - \frac{\lambda}{4} (e_i^2)^2 - \frac{1}{2} b^2 \right) + \frac{k}{4\pi} \epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right]$$

Phases on Either Side

$\underline{r > 0}$:

$$\sigma_{xx}(\omega) = \frac{i}{2\pi k} \cdot \frac{\omega/g^4\tilde{k}r}{1 - (\omega/g^4\tilde{k}r)^2}$$

$$\sigma_{xy}(\omega) = \frac{1}{2\pi k} \cdot \frac{1}{1 - (\omega/g^4\tilde{k}r)^2}$$

$\underline{r < 0}$:

$$\sigma_{xx}(\omega) = 0$$

$$\sigma_{yy}(\omega) = \frac{g^4 r}{8\pi^2}\delta(\omega) + \frac{i g^4 r}{8\pi^2 \omega}$$

$$\sigma_{xy}(\omega) = 0$$

But, if we assume D_4, rather than O(2) symmetry:

$\underline{r < 0}$:

$$\sigma_{xx}(\omega) = \alpha \frac{i}{2\pi k} \cdot \frac{\omega / g^4 \tilde{k} r \alpha}{1 - (\omega / g^4 \tilde{k} r \alpha)^2}$$

$$\sigma_{yy}(\omega) = \frac{1}{\alpha} \cdot \frac{i}{2\pi k} \cdot \frac{\omega / g^4 \tilde{k} r \alpha}{1 - (\omega / g^4 \tilde{k} r \alpha)^2}$$

$$\sigma_{xy}(\omega) = \frac{1}{2\pi k} \cdot \frac{1}{1 - (\omega / g^4 \tilde{k} r \alpha)^2}$$

Fractional quantum Hall state, but anisotropic at finite-frequency.

At the Critical Point

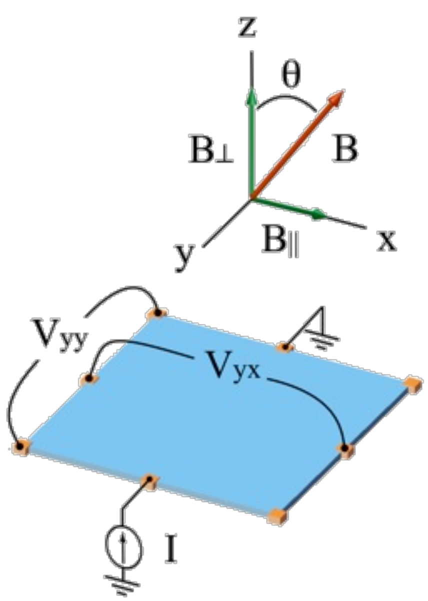
$$\mathrm{Re}\,\sigma_{xx}(\omega) = \mathrm{Im}\,\sigma_{xx}(\omega) = 0$$

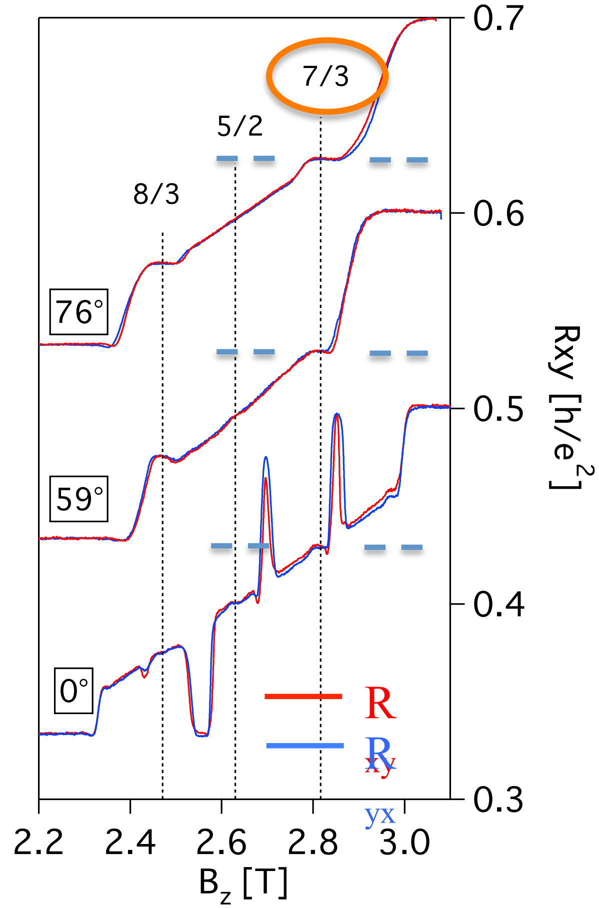
$$\mathrm{Re}\,\sigma_{xy}(\omega) = \mathrm{Im}\,\sigma_{xy}(\omega) = 0$$

Gap collapses but the system becomes *more insulating*.

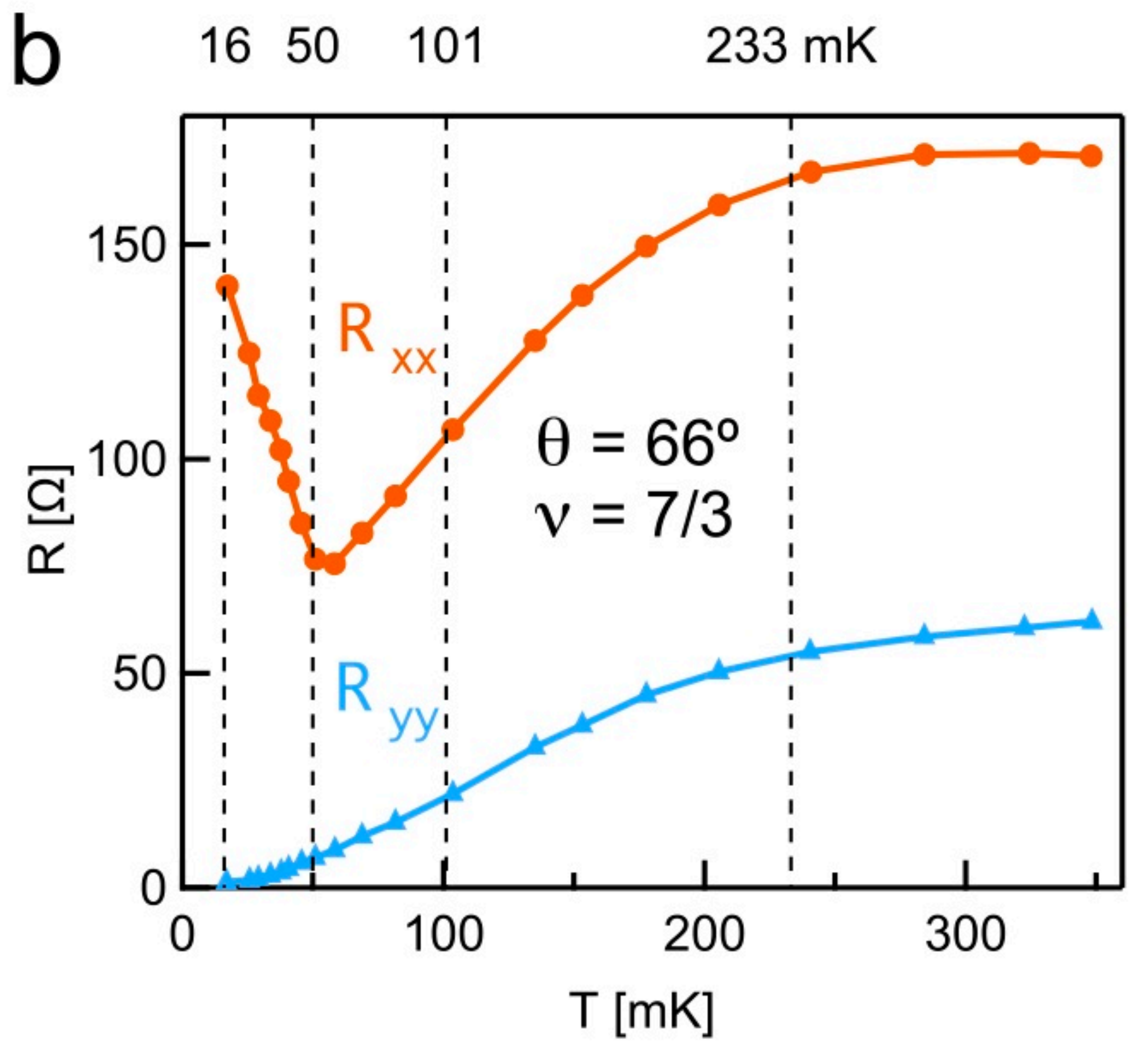
A funny thing happened on the way to hep-th …

Xia and Eisenstein '11





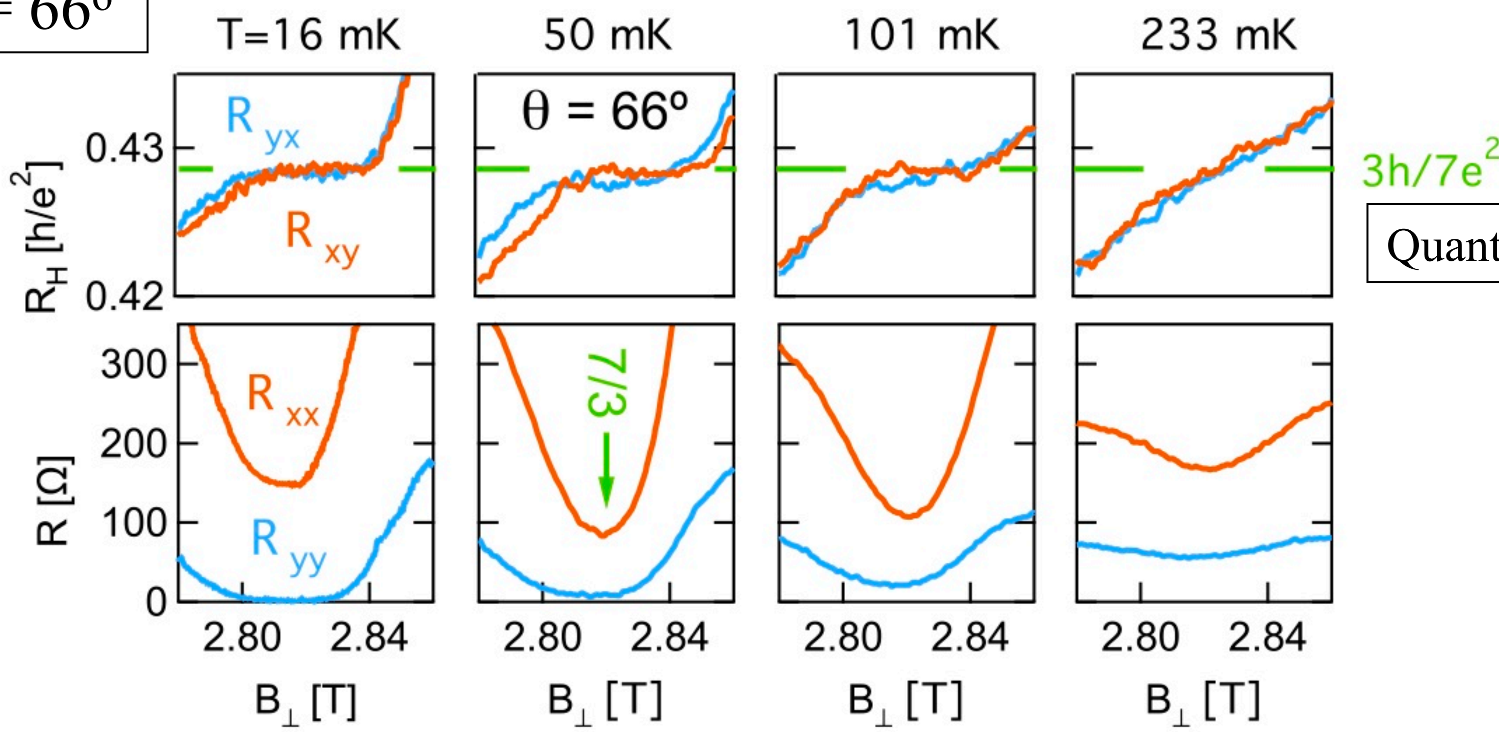
7/3: FQHE with broken rotational symmetry



Slide courtesy of J. Eisenstein

7/3: FQHE with broken rotational symmetry

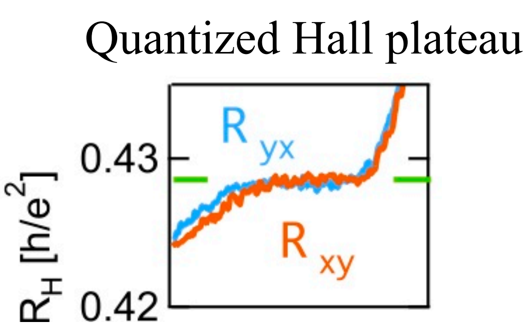
$\nu = 7/3$,
$\theta = 66^o$

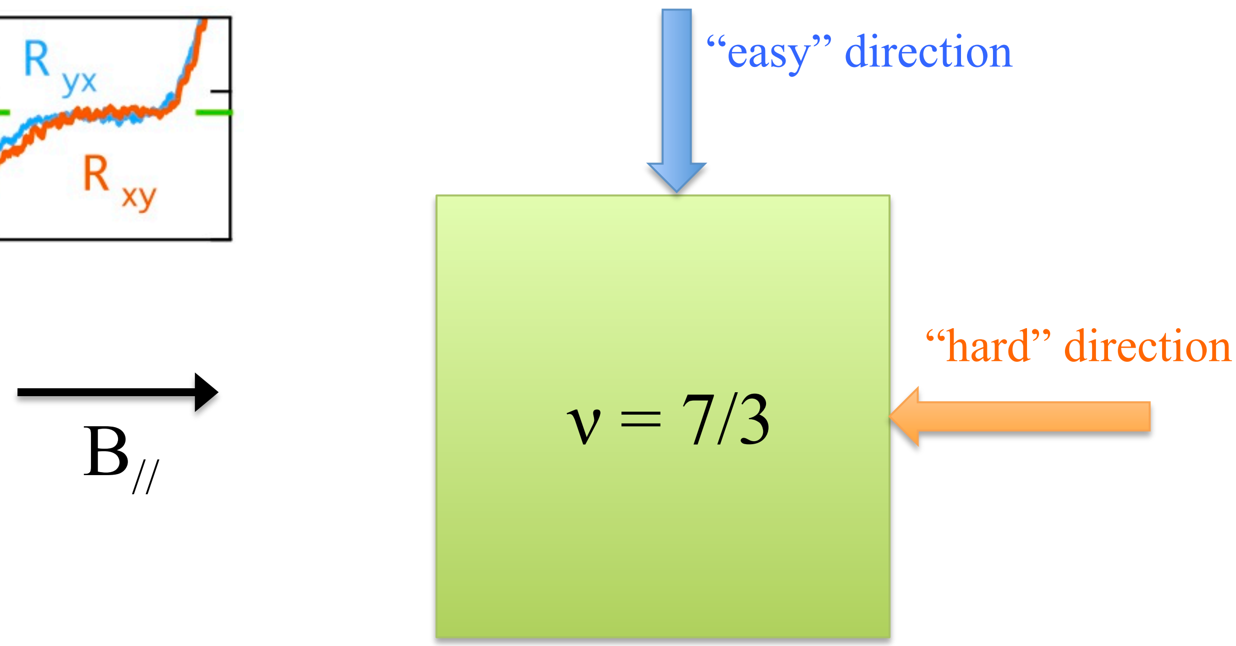


Quantized R_H

Slide courtesy of J. Eisenstein

Reminiscent of $r<0$ phase of LCS Theory:





But the anisotropy is showing up in T>0 DC transport, rather than T=0 AC transport.

Transport due to Charged QPs

$$\mathcal{L}_{LCS} = \frac{1}{g^2}\int d^2x dt\Big(e_i\partial_t n_i + n_t\partial_i e_i - \frac{r}{2}e_i^2 - \frac{\kappa^2}{2}(\partial_i e_j)^2 - \frac{1}{2}(\epsilon_{ij}\partial_i n_j)^2 + \frac{g^2}{4\pi\nu}\epsilon_{\mu\nu\lambda}n_\mu\partial_\nu n_\lambda - \frac{\lambda}{4}(e_i^2)^2 + \frac{\alpha}{4}(e_x^4 + e_y^4) + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu n_\lambda\Big)$$

$$\mathcal{L}_{\mathrm{matter}} = \int d^2x dt\Big(\Phi^*\left(i\partial_t + n_t - \Delta + (i\partial_i + n_i)^2\right)\Phi + ue_x^2\,\Phi^*(i\partial_x + n_x)^2\Phi + ue_y^2\,\Phi^*(i\partial_y + n_y)^2\Phi\Big). \quad (6)$$

Quantized Hall resistance with anisotropic longitudinal resistances:

$$\rho_{xy} = -\rho_{yx} = k$$

$$\rho_{xx} = 2\pi\sigma^{\mathrm{qp}}_{yy} \qquad \rho_{yy} = 2\pi\sigma^{\mathrm{qp}}_{xx}$$

$$\sigma^{\mathrm{qp}}_{xx,yy} = \tfrac{\pi}{4}\left(1 + u\langle e_x\rangle^2\right)^{\pm 1/2} T\tau\, e^{-\Delta/T}$$

Scattering rate put in by hand.

More Microscopic Picture

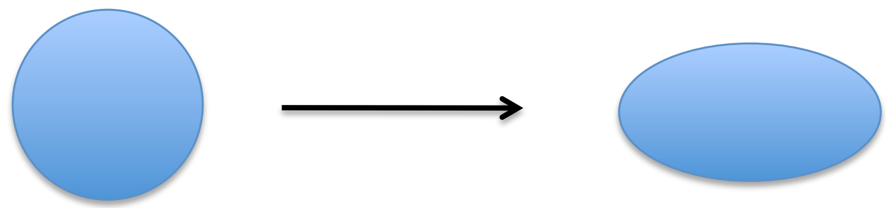
$$\mathcal{L}_{LG} = \int d^2x dt \Big(\phi^\dagger i(\partial_t - i(A_t + a_t))\phi$$
$$- \frac{1}{2m_e}|(\partial_i - i(A_i + a_i))\phi|^2 + \frac{\nu}{4\pi}\epsilon_{\alpha\beta\gamma}a_\alpha\partial_\beta a_\gamma$$
$$- \frac{1}{2}\int d^2y(\phi^\dagger\phi(x) - \bar{\rho})V(x-y)(\phi^\dagger\phi - \bar{\rho})\Big).$$

$$\delta\mathcal{L} = -c\int d^2x dt |(\partial_i - i(A_i + a_i))^2\phi|^2$$

If the effective parameter *m* can change sign, recover LCS theory.

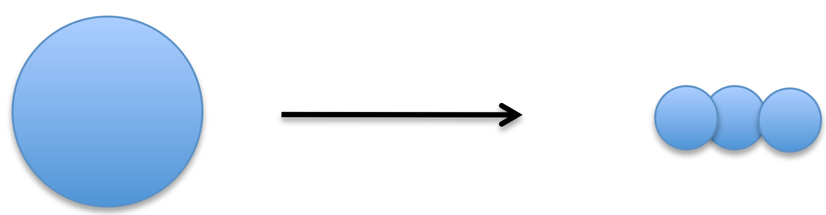
Ground State Wavefunction

$$\Psi([z_i]) = \prod_{i<j}(z_i - z_j)^3 \left(1 + \frac{\delta\bar{r}}{2|\bar{r}|3}\frac{(z_i-z_j)^2+(\bar{z}_i-\bar{z}_j)^2}{|z_i-z_j|^2}\right) e^{-\frac{1}{4\ell_0^2}\sum_i |z_i|^2}$$



Compare to Musaelian and Joynt '96:

$$\Psi([z_i]) = \prod_{i<j}(z_i - z_j)(z_i - z_j + \lambda)(z_i - z_j - \lambda)\, e^{-\frac{1}{4\ell_0^2}\sum_i |z_i|^2}.$$



Summary

- Interesting field theories (even simple ones) can inform our interpretation of experiments.

- The FQHE is still producing surprises.

- The combination of the two is a fertile playground for (some?) string theorists and ~~mathematicians~~ condensed matter physicists