Wormholes and entangled states

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Two descriptions of black holes

- **Unitary:** From infinity, microstates

- **Local:** Infalling observer, general covariance, interior.

→ we should make them consistent!

It will probably require all of your ideas...
• Does gravity emerge as a result of an approximation?

• Why does this approximation sacrifice unitarity?
Emergence of spacetime

• New properties of matter due to collective behavior.
• Wilson ➔ Universality.
Geometry for the ground state of field theories

• Wilson $\rightarrow$ Usual RG picture.
• Condensed matter theorists found a convenient quantum mechanical, real time description of the states $\rightarrow$ Tensor networks.
\[ \Psi(s_1, \cdots, s_n) = Tr[T_{s_1} T_{s_2} \cdots T_{s_n}] \]
This representation works well for states with a mass gap. If we choose

\[ \log(D) \gg S_{\text{ent}} \]

Special wavefunctions. L sites.

\[ \dim(H) = 2^L \]
\[ \dim(\text{Space of Tensors}) = LD^2 \]

Of course, if we do arbitrary superpositions of tensor networks, we get a much bigger space:

\[ \dim(\text{Superposition of Tensors}) = 2^{LD^2} \]
Scale invariant wavefunctions

Each vertex is a five index tensor. Each line is an index contraction.

Indices $\rightarrow$ not ``real'' states.
This is similar to the geometry of AdS

Think of the tensors as representing the AdS vacuum wavefunction.

Tensor index contractions $\rightarrow$ entanglement
Entanglement & structure of space

Ryu-Takayanagi

Minimal surface
Conformal invariant system in a state with a mass gap.

eg: AdS space with an end of the world brane in the IR
Bulk effective field theory

Start with a wavefunction given by a tensor network.

This is the bulk vacuum.

Find new states as `small’’ deformations of the tensors. These are particles on top of the bulk vacuum.

\[ TT\delta TTT; \quad TT\delta TTT\delta TT.. \]

Local degrees of freedom \( \rightarrow \) indices of the tensor.
• If we consider superpositions of these networks we get a "semiclassical" fock space.
• It is an overcomplete space. The projection on to the correct space is obtained by evaluating the wavefunction from the network.
Time dependence

Start with a state with a gap and evolve it. Eg. Brane in Ads that falls into a black hole.
Time evolution produces a wavefunction that can be represented as a geometry which is simply longer.
brane

boundary

t > 0
Field theory picture. (focus on IR)

Network = history of the state.

→ geometry captures the history?
Start with basis of localized, unentangled states
Is a complete basis.
Each member evolves as:
Producing a more generic state
Over complete set of states in the interior. Many changes produce same boundary state.
Tensor networks for generic states?

Produce a generic state for the first system by considering a state entangled with a second system.

\[ |\Psi\rangle = \sum_{n} e^{-\beta E_n/2} |\bar{n}\rangle |n\rangle \]

We find a smooth geometry!

Israel, JM,...
Spatial direction along horizon
We could also have represented it in this way...
Captures better the entanglement pattern.
Seems more similar to the ``nice slices'’, which expand. The two horizons moving away...
Eternal AdS black hole

Entangled state in two non-interacting CFT’s.

\[ |\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle^{CPT}_L \times |E_n\rangle_R \]
ER = EPR

• Wormhole = EPR pair of two black holes in a particular entangled state.

• Large amounts of entanglement can give rise to a geometric connection.

• Geometry is a way to codify, or generate the entanglement between the two systems.
Some Lessons

• Do not confuse left exterior with interior.
• To describe this interior the microstates of one black hole is not enough. One CFT is not enough, we need the second.
• This is not $A = R_B$. We are not identifying the interior with the left exterior.
• The interior is constructed in a subtle way from both the left and right exterior its structure depends on the pattern of entanglement.
• The observer in the interior can receive signals from both, but cannot send arbitrary signals to either of the two exteriors.
• We cannot say that $A$ is some operator in the left Hilbert space. If there were so, an infalling Right-person could send a signal to a Left-person by changing $A$. 

\[
\begin{array}{c}
\text{Past Interior} \\
\text{Left Exterior} \\
\text{Future Interior} \\
\text{Right Exterior}
\end{array}
\]
Should the left and right horizons touch?

• Only for special states.

Shenker Stanford
Not all entangled states have a smooth geometry

\[ |\Psi\rangle = \sum_n e^{-\beta E_n/2} |n\rangle |n\rangle |n\rangle |n\rangle \]

GHZ–like state:

\[ I = S(A \cup B \cup C) - S(A \cup B) - S(A \cup C) - S(B \cup C) + S(A) + S(B) + S(C) \leq 0 \]

States connected by a smooth geometry obey the following inequality for the triple mutual information

\[ S(A) = S(A \cup B) = S(A \cup B \cup C') \]

Here we have:

\[ I > 0 \]

(Hayden, Maloney)

Gharibyan, Penna
Changing the entangled state

• Time evolution $\rightarrow$ Different slicings $\rightarrow$ phases

$$|\Psi\rangle = \sum_{n} e^{-2iE_{n}t} e^{-\beta E_{n}/2} |E_{n}\rangle^{CPT}_{L} \times |E_{n}\rangle^{R}_{R}$$

Each time: Whole yellow region, slices related by the Wheeler de Wit equation.

Heemskerk, Marolf, Polchinski, Sully
There is a standard projector operator \( P_0 = |\Psi_0\rangle \langle \Psi_0| \) that tests whether the system is the entangled state \( |\Psi_0\rangle \) or a different one that tests whether it is in the other state \( |\Psi_1\rangle \).

We claim that we should think of the bridges associated to these two states as being different. In fact, we can see this clearly in the case of the eternal black hole. In this case, we can consider the following family of Schrödinger picture states:

\[
|\Psi_t\rangle \sim |n\rangle - \beta E_n/2 e^{-2iEt} |n, n\rangle.
\]

Two states with different values of \( t \) are related by forward time evolution on the two sides. However, consider them as possible alternative states at the same instant of time and view \( t \) in \( \beta \) as a parameter labeling alternative states at a common instant of time. All these states have "maximal" entanglement and the same density matrix on each side. There is a projection operator \( P_t \) into each of these states. However, there is no projection operator onto the whole family, since considering linear combinations such as

\[
|\Psi_t\rangle = d e^{2iE_0t} |\Psi_0\rangle,
\]

projects us into a particular state \( |n_0, n_0\rangle \) which is the one having the energy \( E_0 \). This state is not maximally entangled.

Note that region A is common to more than one state.
such a way that we create a state with finite Minkowski energy. We can ask what this operation corresponds to in the AdS example. For simple unitary operators such states are expected to add additional particles on top of the Hartle-Hawking vacuum as discussed in section 5.

Let us study the gravitational back reaction in one very particular case. Imagine that what we do is to add the phase $\theta \omega = -\omega t$ so that we consider the particular state $U_{\theta}|0\rangle_M = \exp \left\{ \int d\omega e^{-\beta \omega/2} e^{-i\omega t} b_{L,\omega}^\dagger b_{R,\omega}^\dagger \right\} |0\rangle_R$

This is a state with infinite Minkowski energy. However, this state $|0\rangle_M$ can also be viewed as the expression for the Minkowski vacuum but quantized along a different spatial slice, a slice with a kink as in figure (a).

Thus the state $|0\rangle_M$ is very singular if we view it as quantized along the slice in figure (a) but non-singular along the slice in figure (b). Now if we take this second point of view, we are not making any statement about "generic" unitary operators. Singularity in flat space QFT.

$$|\Psi\rangle = \sum_n e^{-iE_n t} e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

Non-singular in gravity.
Other states

Adding particles to the Hartle-Hawking state. Precise translation between states in the CFT and states in the bulk.
Comments

• Entangled states can be connected by a smooth geometry.
• Each entangled state corresponds to a whole region of the bulk, with slices related by the WdW equation.
• Different entangled states correspond to different geometries, or the same geometry plus extra particles.
• We did not make a statement about the generic entangled state.
• We can view the left side as “processed” radiation.
• What we do to the radiation matters for what an infalling observer sees.
• The AMPS paradox is real (if we ignore computational constraints).
• Some states are not smooth.
• What happens if we do nothing?. What is the particular entangled state produced by the “natural” evolution of an evaporating black hole?

Harlow Hayden
Whether or not they were initially scrambled after a time of order $M \log M$, they will become scrambled and therefore highly entangled in all combinations. It seems reasonable to expect the nucleus of figure 12 will evolve into the interior of the black hole. In other words, after the scrambling time $\tau$ but long before the Page times, the interior of the black hole is the Einstein-Rosen bridge system that connects the massively entangled near-whorizon system of a black hole.

3.6 Hawking Radiation

The Hawking radiation of a black hole is a scrambled cloud of radiation entangled with the black hole. The obvious configuration of the Einstein-Rosen bridge would resemble the standard two-black-hole case except that Alice’s black hole would be replaced by the Hawking radiation. We can draw a very impressionistic cartoon of the black hole connected to the radiation by a Einstein-Rosen bridge with many exits, see figure 13.

Another representation is shown in figure 14. This figure shows only the geometrical Einstein-Rosen bridge part of spacetime. On the far left, the interior of a young, one-sided black hole is shown. The black circle represents the horizon which should be identified with the horizon as seen from the exterior side. In the beginning, there is no Hawking radiation. As we move to the right, Hawking quanta are emitted and since they are entangled with the black hole, they have to be connected to the bridge. The red dots represent the places where the Hawking quanta connect to the main body of the bridge. The earlier quanta are to the right of the later quanta. The green circles represent slices through the bridge that divide the system into two parts. To the right of the circle, the quanta were emitted.
A possible diagram for the bridge connecting an old evaporating black hole to the radiations. The radiation little wormhole mouths would join along the thick black lines. When we look at an old black hole, we are looking at the upper corner of the Penrose diagram of the original black holes. When we do a time translation or boost to focus on the late time region, we squeeze the trajectories of the early radiation along the past horizons. These diagrams do not take into account the complete evaporation of the black holes.

In the language of qubits, the simplicity of an operator represents the number of computational qubits that are involved in its definition. In the black hole radiation, the concept of a computational qubit is replaced by the local modes of the radiation fields. If we ignore states with more than one quantum in a mode, then the localized modes can be replaced by computational qubits. The simple operators in this context are made of a single radiation mode. They are easy to measure or to encode in another system.

By contrast, the operators $R$ and $B$ in $\mathcal{W}$ are extremely complex. These are the operators that Harlow and Hayden identify as computationally difficult to access. They are nonlocally distributed over at least half the total number of radiation modes. If the initial entropy of the black hole is $S$, then complex operators involve of order $S$ radiation modes.

In our ADStCFT-based model, we will work in the Schrödinger picture. The simple units which are easily accessed are the local single-trace operators in the boundary CFTs. The most complex operators are very nonlocal expressions in the gauge theory. They may involve larger-scale Wilson loops and even more complicated objects. Experience has shown that the deeper one probes into the interior of AdS, the more complex the probes have to be. An example is the precursor operators in $\mathcal{W}$. 

Smooth horizon

or

Firewall
Conclusions

• We gave an EPR interpretation to the ER bridge.
• The topology of space can be modified by massive amounts of entanglement.
• A black hole entangled with radiation could produce a similar geometric bridge. Its interior could depend on what we do with the radiation.
• We discussed some qualitative similarities between the tensor network description of quantum states and the spacetime description.