Tonks Girardeau Gas in an Optical Lattice

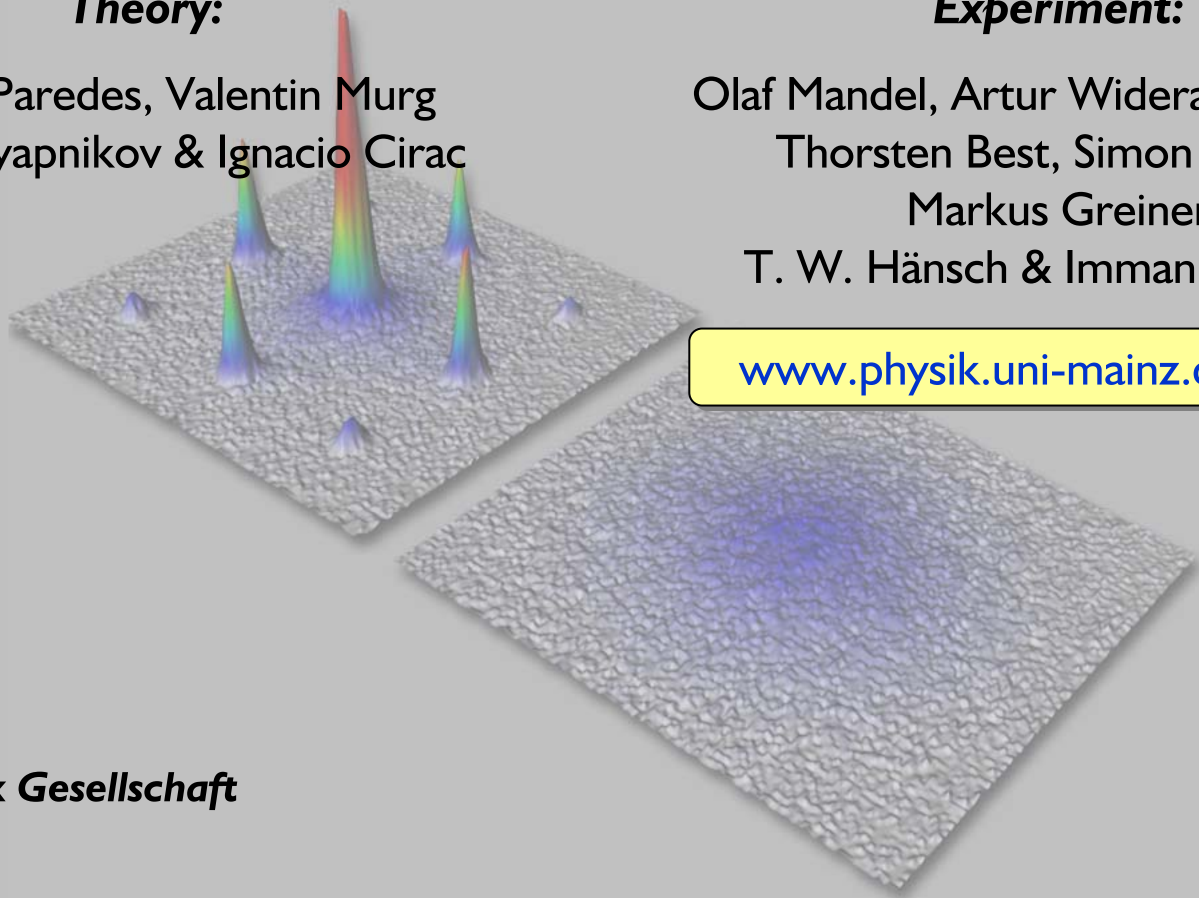
Theory:

Belén Paredes, Valentin Murg
Gora Shlyapnikov & Ignacio Cirac

Experiment:

Olaf Mandel, Artur Widera, Tim Rom,
Thorsten Best, Simon Fölling,
Markus Greiner
T. W. Hänsch & Immanuel Bloch

www.physik.uni-mainz.de/quantum



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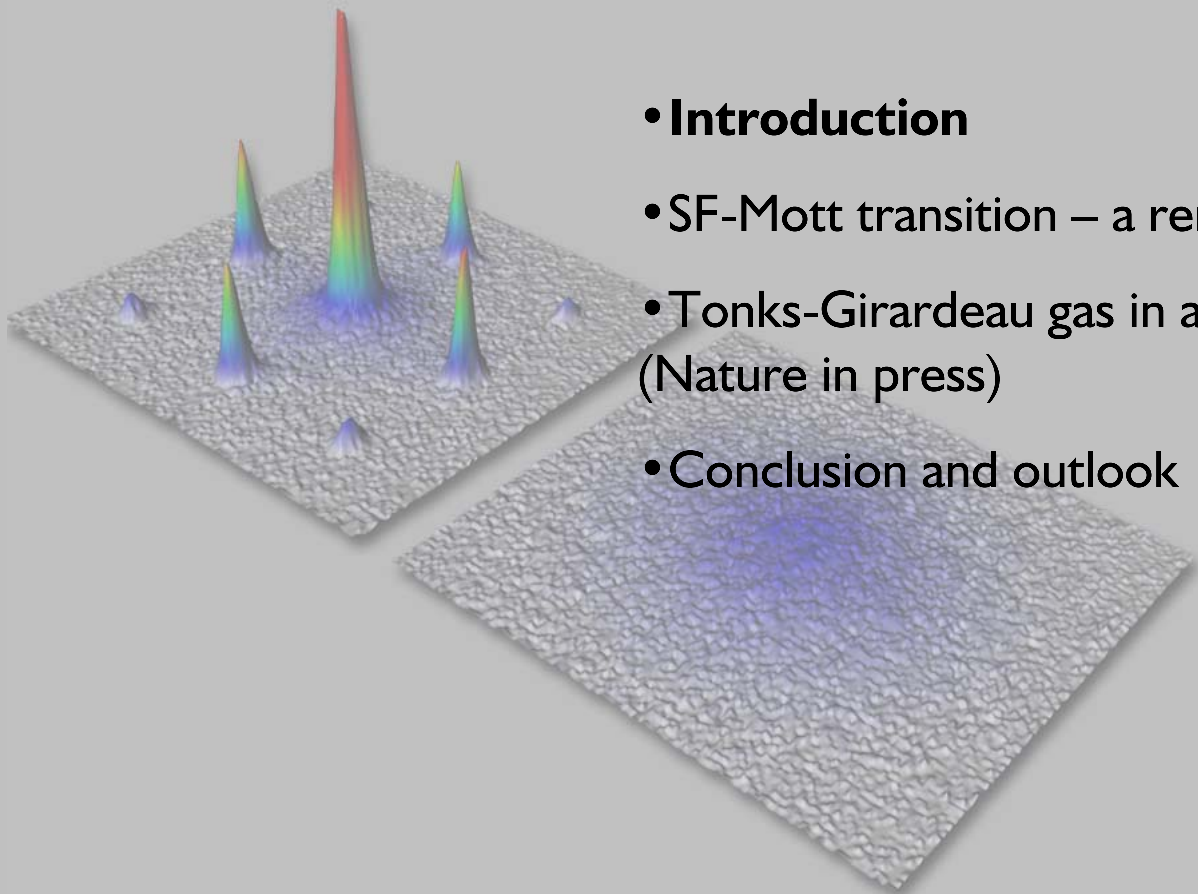
Johannes Gutenberg-Universität, Mainz, Germany

Outline



- **Introduction**
- SF-Mott transition – a reminder
- Tonks-Girardeau gas in an optical lattice (Nature in press)
- Conclusion and outlook

Status of the Experiments in Mainz

After disentangling classical objects…

And rather not wanting to speak, hear or see anything about the move…

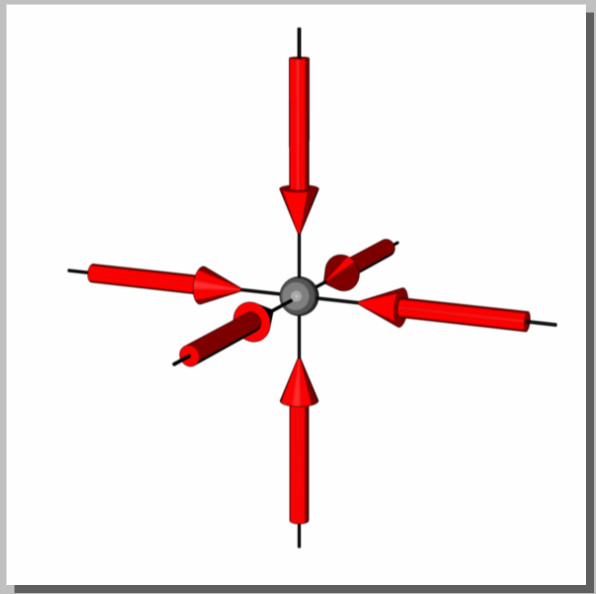
The Experiment Finally Moves to Mainz

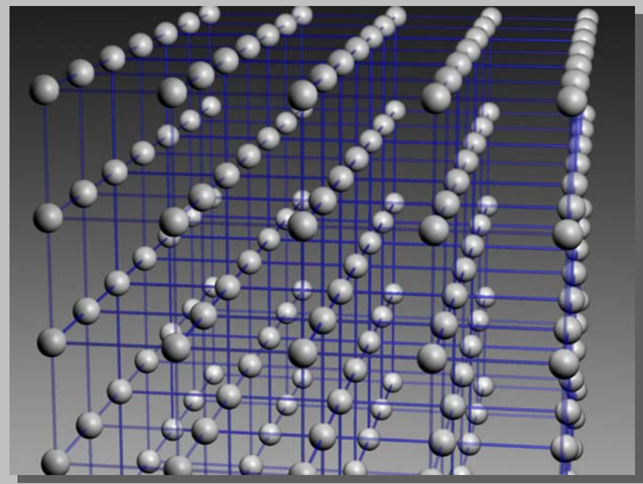
November 26, 2003

Current Status of Experiments

- BEC machine operational
- 3D Lattices almost completed and gearing up for new round of experiments

3D Lattice Potential





- ***Resulting potential consists of a simple cubic lattice***
- ***BEC coherently populates more than 100,000 lattice sites***

V_0 ***up to 40*** E_{recoil}

ω_r ***up to*** $2\pi \times 45$ ***kHz***

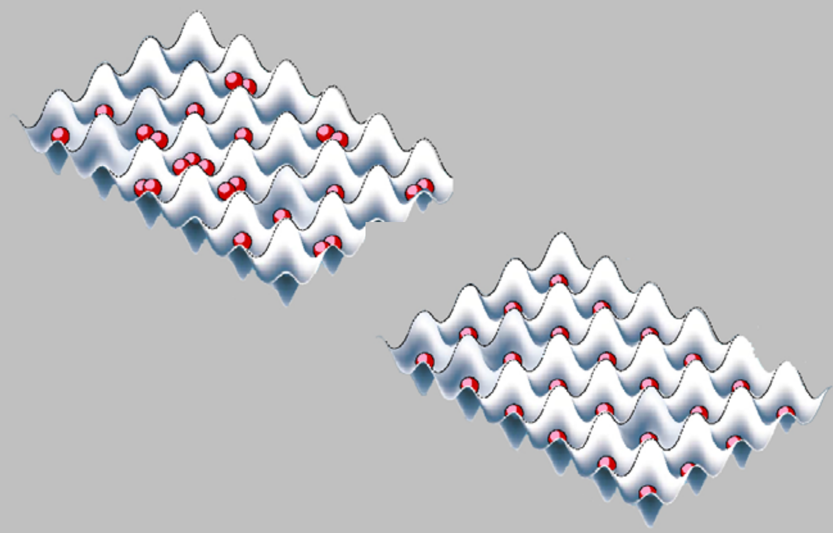
$n \approx$ ***1-5 atoms on average per site***

The SF-Mott Insulator Transition

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Characteristic parameter for the behaviour of the system

U/J



M. Greiner et al., Nature, 415, 39 (2002)

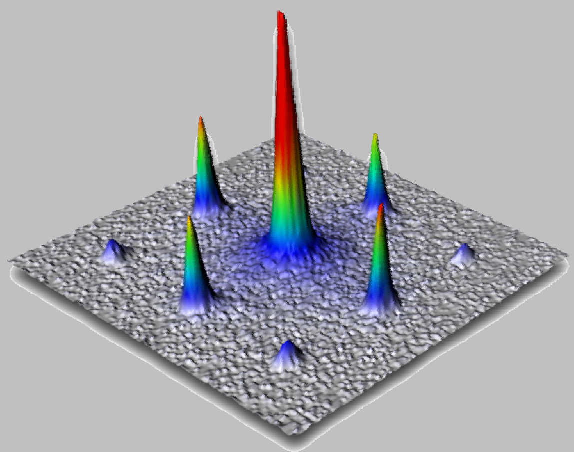
D. Jaksch et al. *PRL*, M. Fisher et al. *PRB*, R. Roth & K. Burnett, K. Braun-Munzinger, B. Svistunov et al., M. Lewenstein, L. Santos et al. M. Kasevich, Yale, W.D. Phillips, NIST, T. Esslinger ETHZ

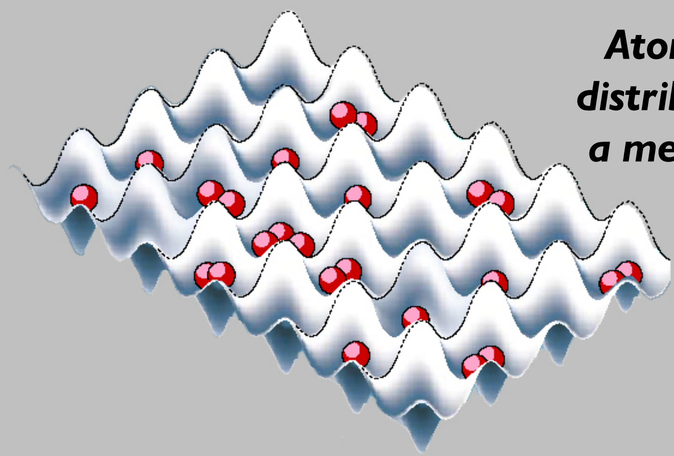
Superfluid Limit

$$H = -J\sum_{i,j}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are delocalized over the entire lattice !
Macroscopic wave function describes this state very well.

$$\left|\Psi_{SF}\right\rangle \propto \left(\sum_{i=1}^{M}\hat{a}_i^\dagger\right)^N \left|0\right\rangle$$





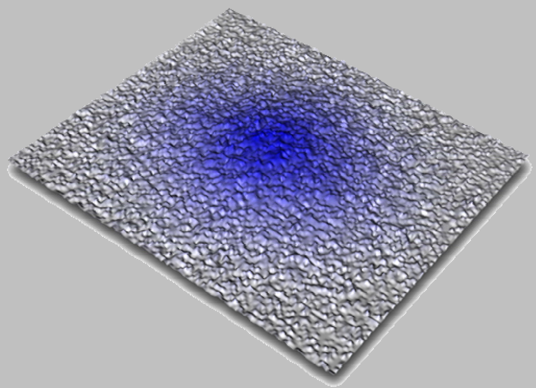
Atom number distribution after a measurement

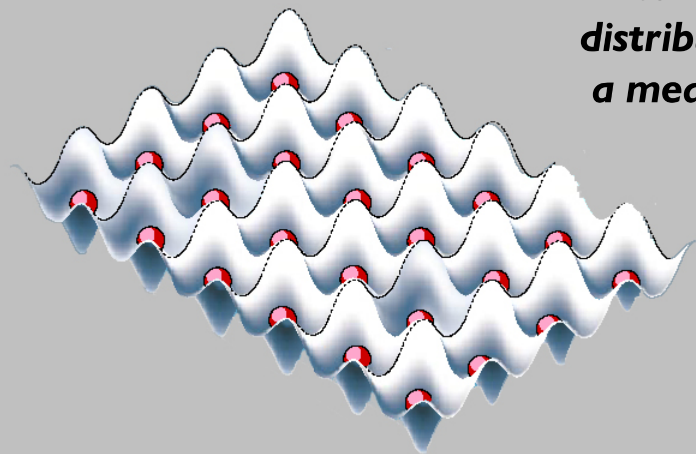
"Atomic Limit" of a Mott-Insulator

$$H = -J\sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

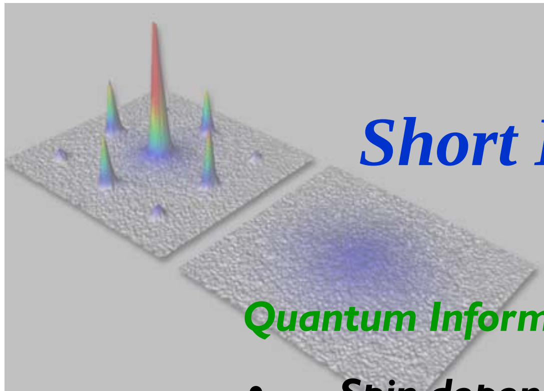
Strong repulsion between atoms leads to a kind of „fermionization"

⇒ *Repulsion mimics Pauli principle, but connection still vague*





Atom number distribution after a measurement



Short Resum… Mott State

Quantum Inform… …ntum register

- ***Spin depend…***
  O. Mandel et a…
- ***Collaps and…***
  M. Greiner et…
- ***Controlled c… …rray – controllable Ising interaction***
  O. Mandel et a…

Atomic/Molecula…

- ***Entangleme… …of atomic scattering properties***
  A. Widera et a…
- ***State Selectiv…***
  T. Rom et al. (…

The Tonks-Girardeau Gas
– A Fermionized 1D Quantum Gas -

<u>Requirements (1):</u> ***1D bosonic quantum gas, tightly confined in two dimensions and only weakly confined along the axial direction***

$$\mu \ll \hbar\omega_\perp$$

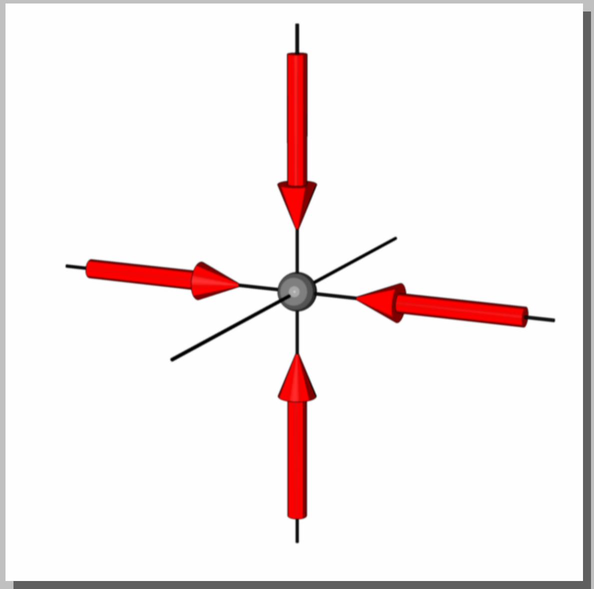
$$T < T_d \approx N\hbar\omega_{ax}$$

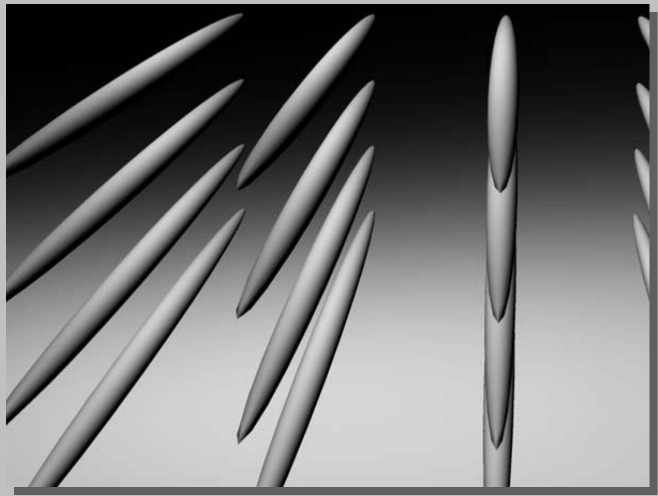
In Experiments here: aspect ratio typically100-200

<u>Experiments with 1D condensates:</u>
A. Goerlitz et al., PRL (2001), F. Schreck et al. PRL (2001), M. Greiner et al. PRL (2001)

more recently:
H. Moritz et al., PRL (2003), B. Laburthe Tolra et al., cond-mat (2003)

2D Lattice Potential





- *Resulting potential consists of an array of tightly confing potential tubes*
- *BEC is split into up to 10,000 1D quantum gases (radial motion confined to zero point oscillations)*

V_0 ***up to 30*** E_{recoil}

ω_r ***up to*** $2\pi \times 35$ ***kHz***

$n \approx$ ***up to 20 atoms per tube***

Tonks-Girardeau Gas

Requirements (2): ***Strong repulsive interactions between atoms***

$$\gamma \approx I / K \gg 1$$

Homogeneous case

Interaction energy $I \sim ng$

Kinetic energy $K \sim \frac{\hbar^2 n^2}{m}$

$$\Rightarrow \quad \gamma = \frac{mg}{\hbar^2 n}$$

D.S. Petrov et al. PRL (2000), M. Olshanii PRL (1998),
V. Dunjko & M. Olshanii PRL (2001), M.D. Girardeau & E.M. Wright, Laser Physics (2001)

General Theory of „Luttinger Liquids“ (see work of Haldane) can be applied to these quantum gases for arbitrary γ

Tonks-Girardeau Gas - Fermionization

In 1D for strongly interacting bosons, the many-body wave function can be mapped on to the one of non-interacting fermions.

(M.D. Girardeau, J. Math. Phys. 1960) *This lies at the heart of a TG gas!*

$$\Psi_B(x_1,\ldots,x_N) = \left|\Psi_F(x_1,\ldots,x_N)\right|$$

For example:

$$\Psi_B(x_1,\ldots,x_N) = \left|\det\left[\varphi_i(x_j)\right]\right| \qquad i,j = 1\ldots N$$

- ***Slater determinant ensures that two particles cannot be placed at the same position in space!***
- ***Absolute value ensures symmetrization***

Bosons behave like Fermions – Not Quite

Density distribution: $$\left|\Psi_B(x)\right|^2 = \left|\Psi_F(x)\right|^2$$

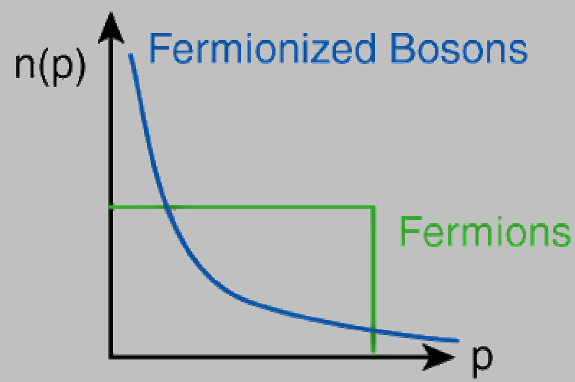
identical to the one of free fermions!
(absolute value of det does not matter)

Correlation function: $$g^{(1)}(x) = \left\langle \Psi_B^\dagger(0)\Psi_B(x)\right\rangle \neq \left\langle \Psi_F^\dagger(0)\Psi_F(x)\right\rangle$$

different to the one of free fermions!
(absolute value of det matters)

Momentum Distribution: $$n(p) \propto \int e^{-ipx} g^{(1)}(x)\, dx$$

different to the one of free fermions!
(FT of correlation function)



Momentum distribution is characteristic for a Tonks-Girardeau gas!

Status of Experiments

So far, experiments in 2D optical lattices have achieved $\gamma \approx 0.5\text{-}1$,

Still 1D mean-field regime (see H. Moritz et al. PRL (2003)), allthough correlations begin to be modified
(see B. Laburthe Tolra et al. cond-mat/0312003)

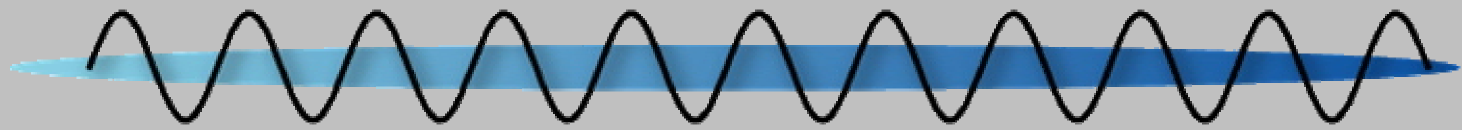
$$\gamma = \frac{mg}{\hbar^2 n}$$

Ways to increase γ:

1. ***Increase Interaction strength*** $g = 2\, a\, \hbar\omega_\perp$
2. ***Decrease density*** n
3. ***Increase of mass*** m

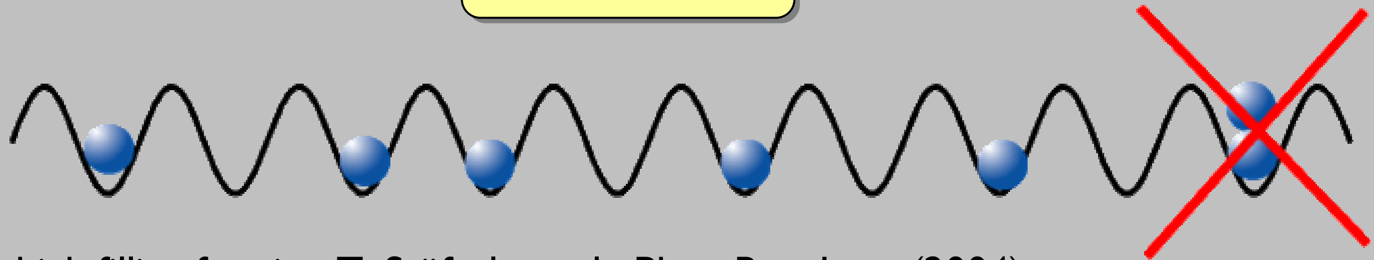
Increasing the Mass

Addition of lattice along the axial direction leads to an increase in the effective mass m*!



However, in order to apply Fermionization, we need to work in a regime, where:

$$\nu \leq 1$$

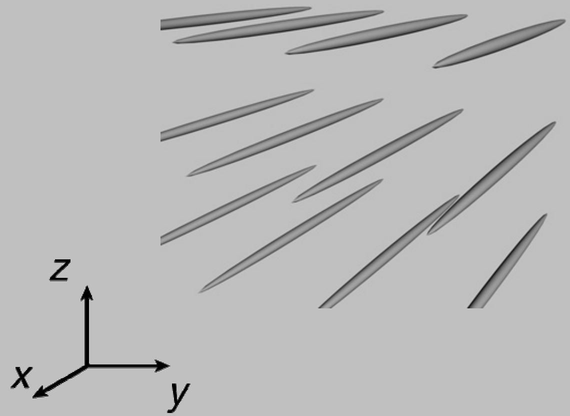


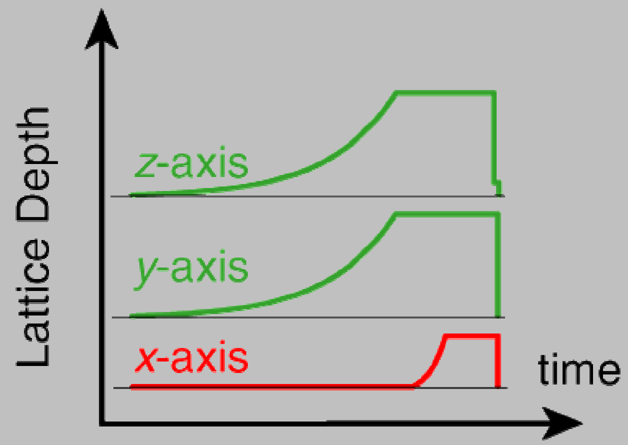
(exp. with high filling fraction T. Stöferle et al., Phys. Rev. Lett. (2004))

Tonks-Parameter in a lattice:

$$\gamma = U / J$$

Experimental Sequence to Prepare the 1D Quantum Gases





(1) Create array of 1D quantum gases

(2) Add lattice along axial direction

Experimental parameters:

V_0 (2D) approx 27 E_r

V_{ax} =0-19 E_r

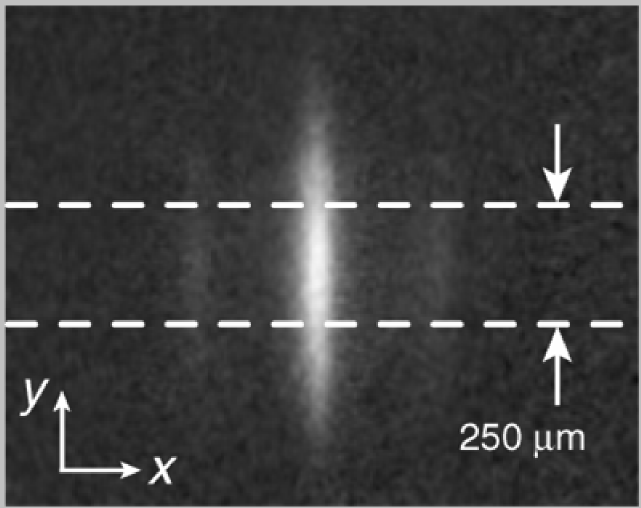
Lattice Wavelengths 825 nm

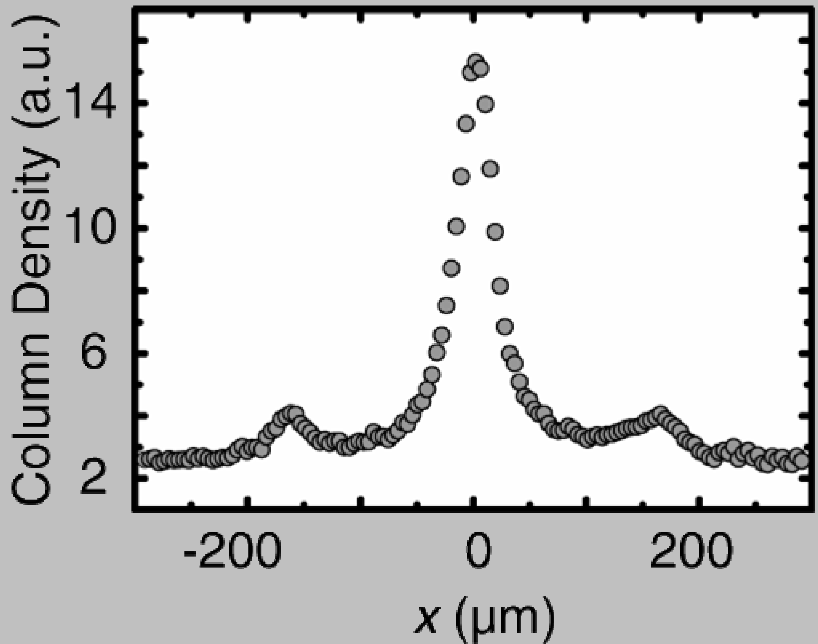
Atom number < 3-4×10^4

Harmonic confinements: $\omega_{ax} = 2\pi \times 60\,\mathrm{Hz}$, $\omega_{\perp} = 2\pi \times 35\,\mathrm{kHz}$

Typical Absorption Images After Time Of Flight

Observe fast expansion in radial direction





Due to the low atom number we average horizontal profiles within the white dashed lines.

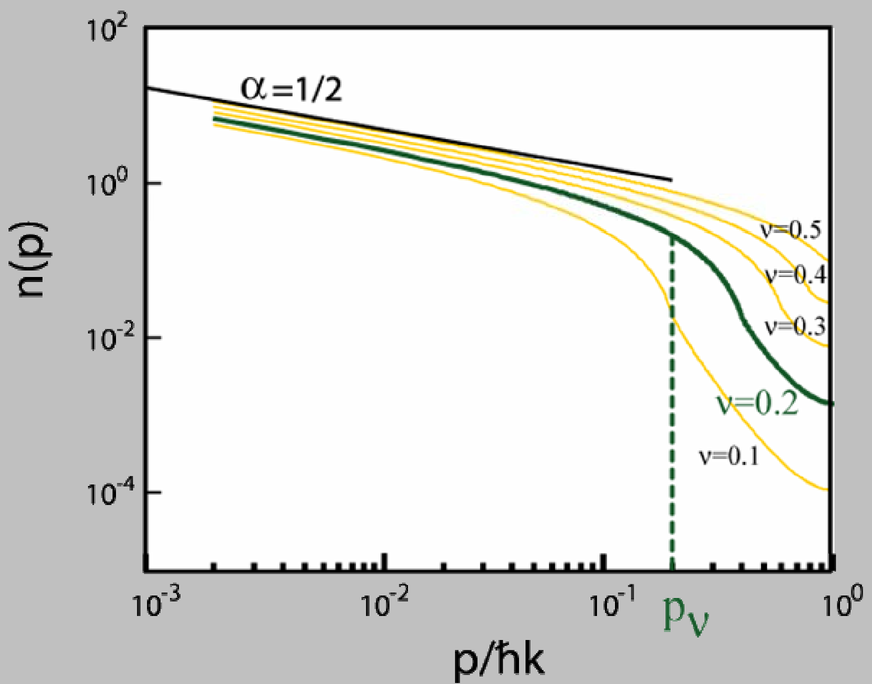
Challenge:
Fully explain momentum distributions!

Momentum Distribution of a (Lattice) 1D Gas

Important momentum scale (1/average interparticle spacing):

$$p_\nu = \hbar \times \frac{2\pi\nu}{\lambda}$$

ν : filling factor



(a) For $p \ll p_\nu$ the slope tends to 1/2

$$n(p) \propto \frac{1}{\sqrt{p}}$$

(b) For $p \gg p_\nu$ the momentum distribution is affected by short range correlations, which tend to **increase** the slope

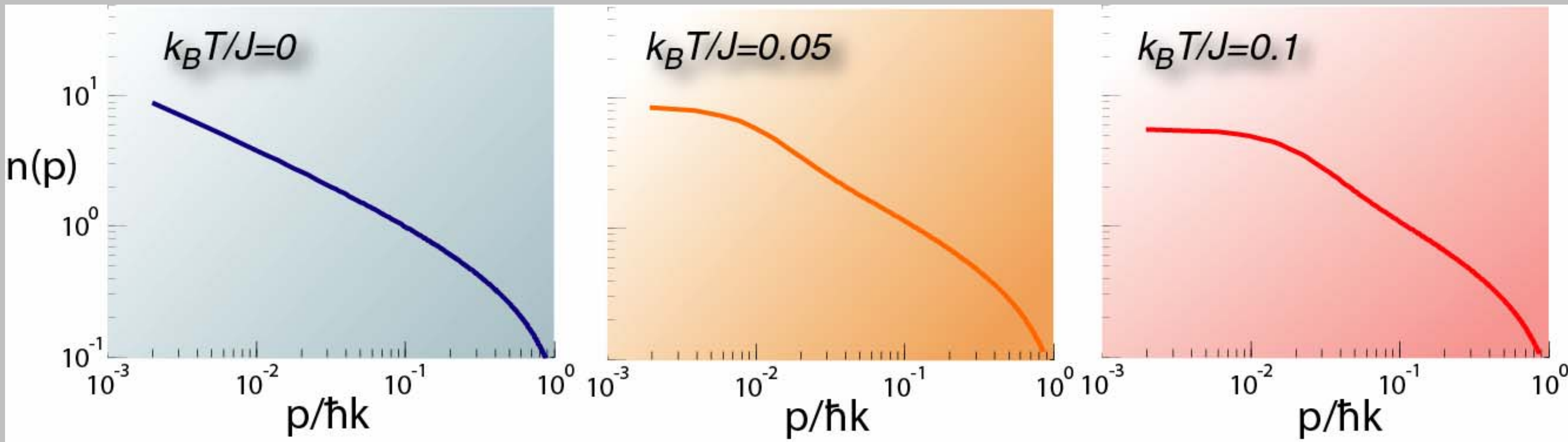
cp. M. Olshanii, PRL **91** (2003),
G.E. Astrakharchik & S. Giorgini
Phys. Rev. A (2003)

Finite Temperature Effects in a (lattice) 1D gas

Important momentum scale
(1/thermal phase coherence length):

$$p_T = \hbar \times \frac{\pi}{L_\phi}$$

$$L_\phi \approx \lambda J / k_B T \times \sin(\pi\nu)$$



$\nu = 1/2$, 1000 sites

Finite temperature affects low momenta
(long range coherence) and broadens peaks

Summary of Important Momentum Scales

$$p_\nu / \hbar = \frac{2\pi\nu}{\lambda} \approx n \approx k_F$$

Short range – long range correlations
(change slope)

$$p_T / \hbar = \frac{\pi}{L_\phi}$$

Thermal effects
(broaden momentum peaks)

$$p_L / \hbar = \frac{\pi}{L}$$

Finite size effects

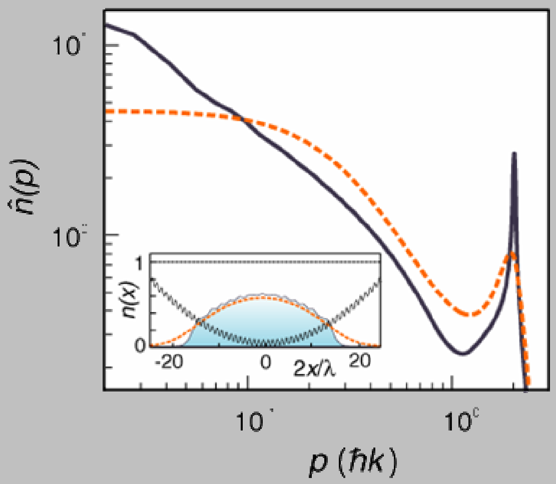
For our experimental parameters, we find:

$$p_L < p_T \sim p_\nu$$

Finite size effects are dominated by
finite temperature effects!

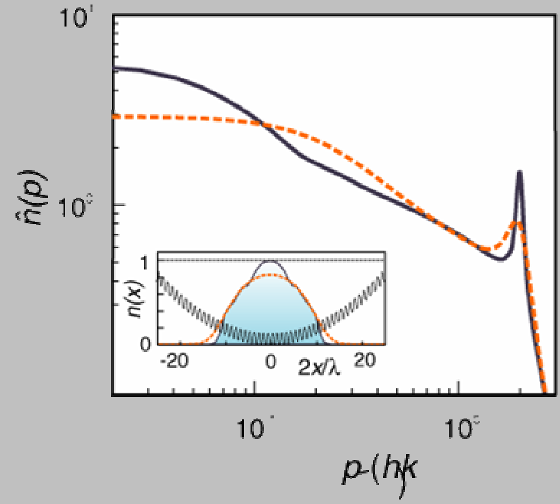
Momentum & Density Distribution for a Fermionized 1D (lattice) gas

$V_{ax}=5\ E_r$



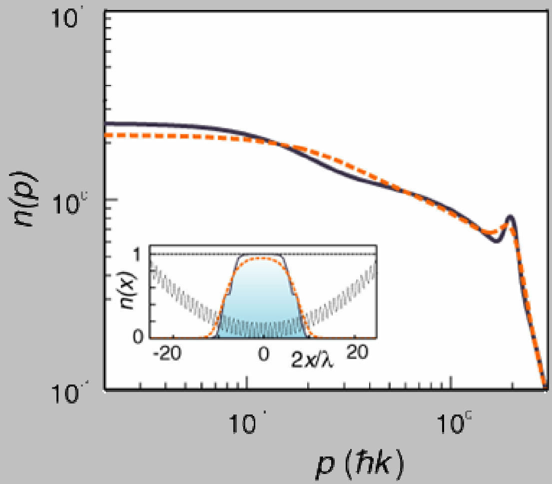
$\alpha=0.8,\ T=0$

$V_{ax}=9.5\ E_r$



$\alpha=0.5,\ T=0$

$V_{ax}=12\ E_r$

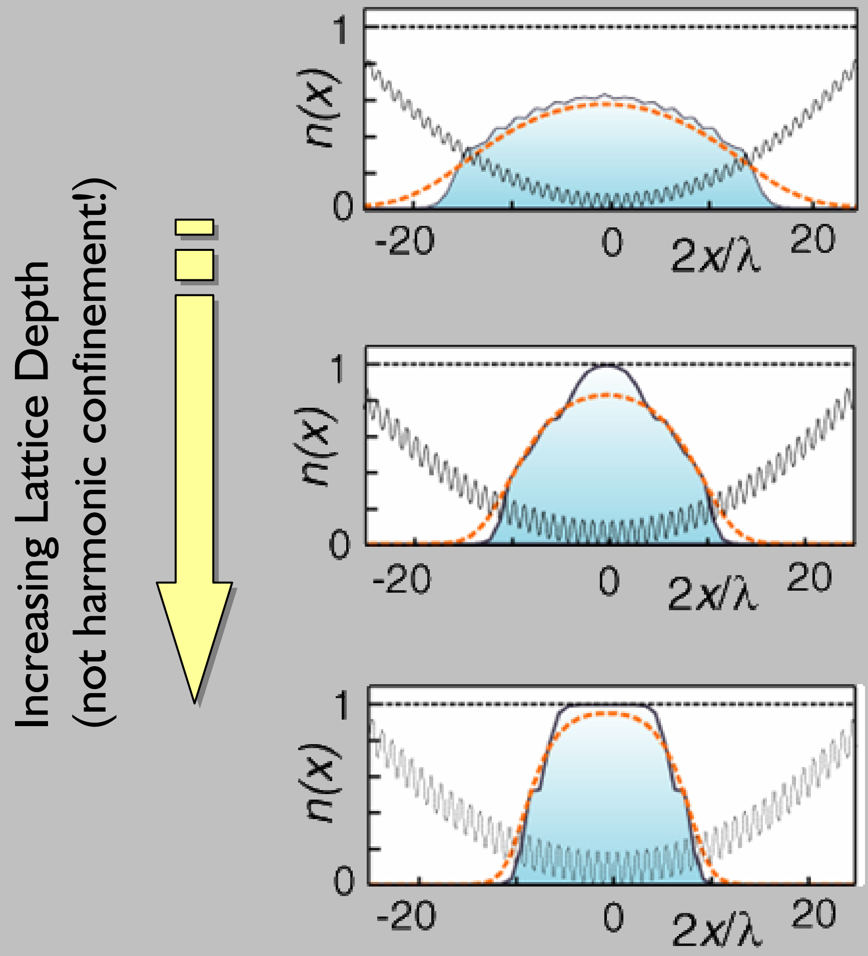


$\alpha=0.2,\ T=0$

Red curve
$k_BT/J=0.75$

Single tube result: N=15

Increase of Lattice Depth Changes Filling Factor in the Inhomogeneous System

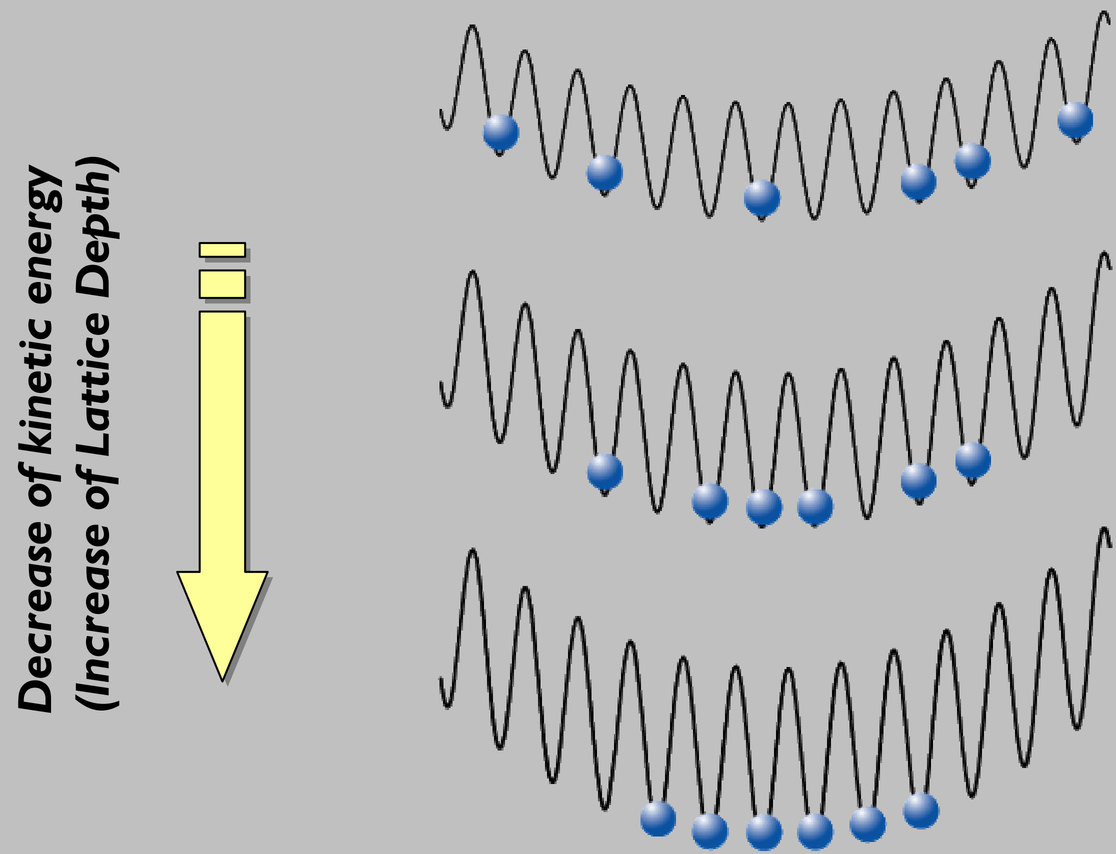


Even for $U \to \infty$ the system spreads out due the kinetic energy *J* !

⇨ If *J* decreases (deeper lattice), the system shrinks until a Mott state with n=1 is formed in the center!

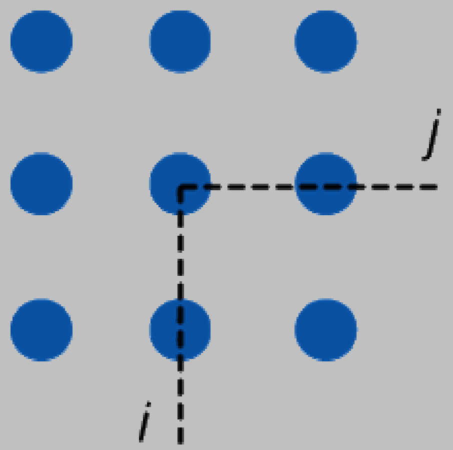
Fermionization describes all filling factor regimes up to $n \leq 1$, provided $\gamma >> 1$!

Simple Picture for Change in Filling Factor



$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^{\dagger}\hat{a}_j + \sum_i \frac{1}{2}m\omega^2\left(\frac{\lambda}{2}\right)^2 i^2\hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Averaging over the Different 1D Gases



Atom number in potential tube i,j

$$N_{i,j} = N_{0,0}\left(1 - \frac{5}{2\pi}\frac{N}{N_{0,0}}\left(i^2 + j^2\right)\right)^{3/2}$$

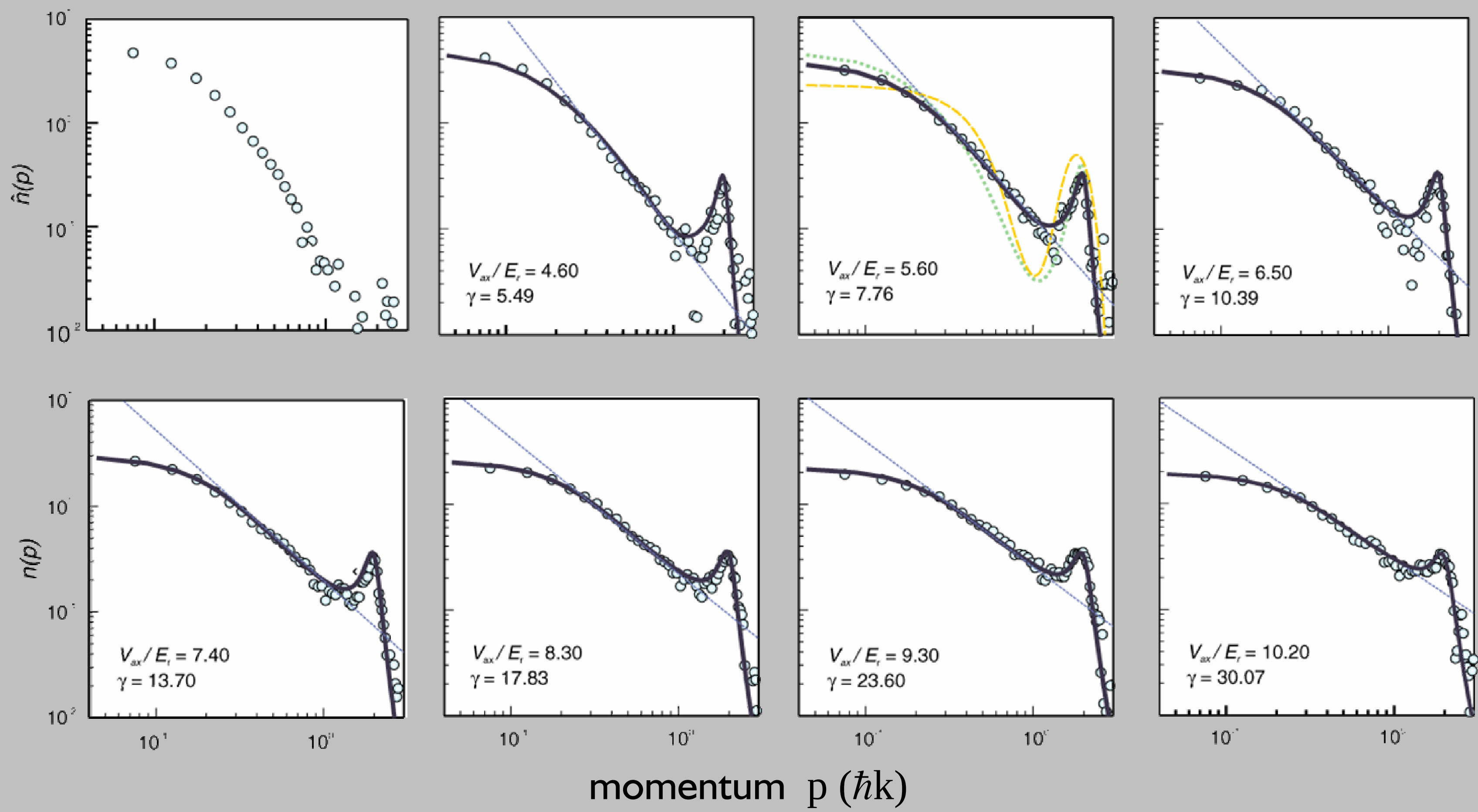
Probability for finding tube with M atoms

$$P(M) = \frac{2}{3}\frac{1}{N_{0,0}^{2/3}M^{1/3}}, \quad M \le N_{0,0}$$

$N_{0,0}$ atom number in central tube !

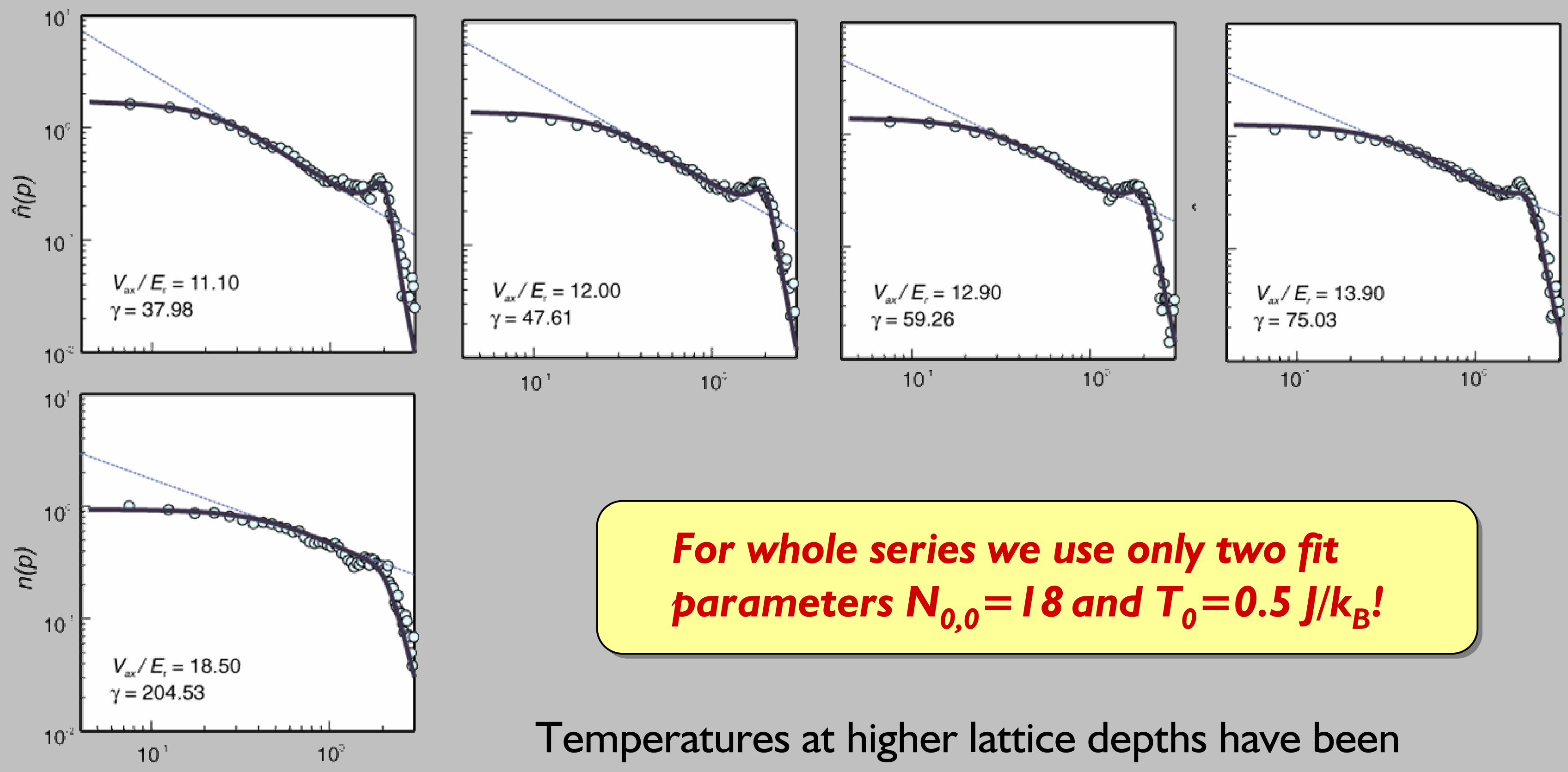
We use this probability distribution to average over the momentum distributions of different tubes.

Comparison Experiment-Theory



Already for low axial lattice depths we observe a pronounced power-law decay with slopes <2

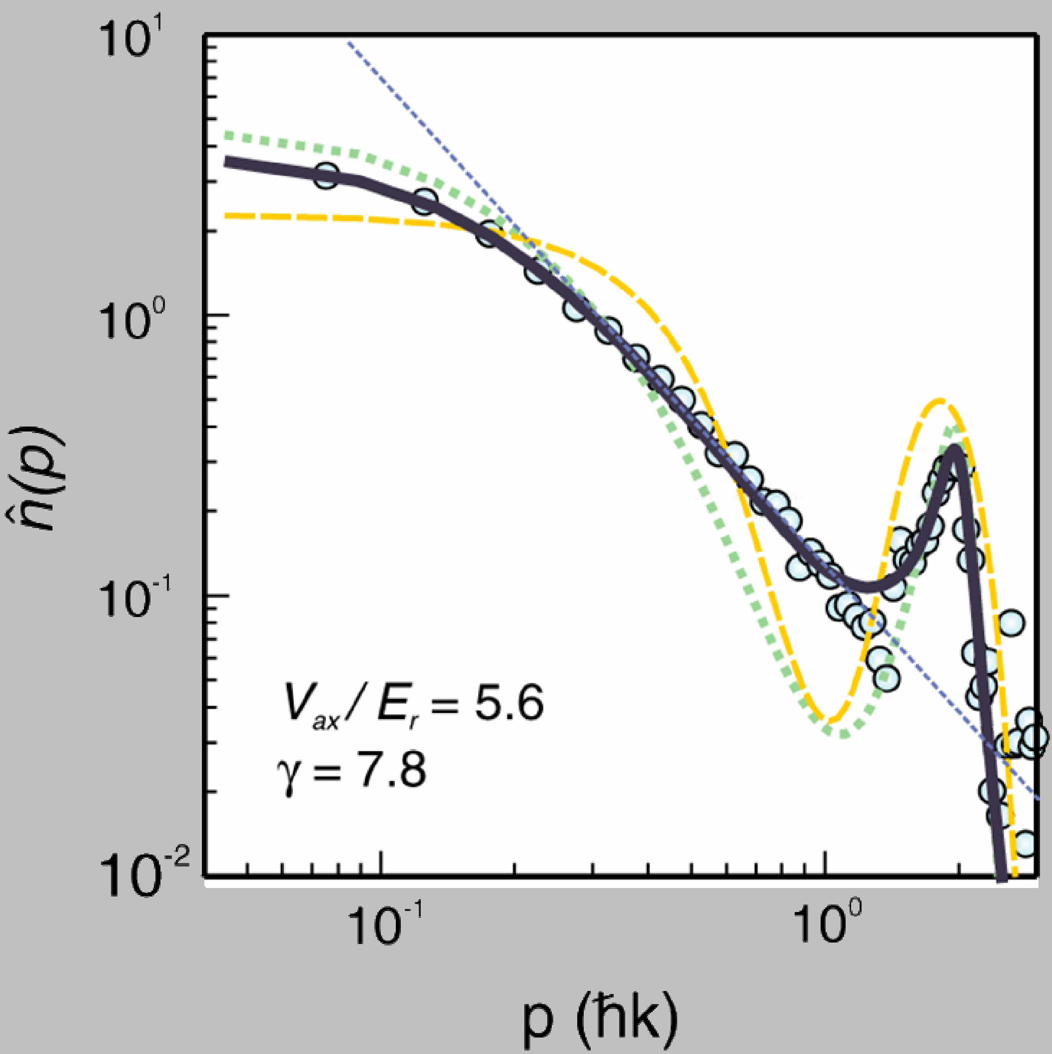
Comparison Experiment-Theory (2)



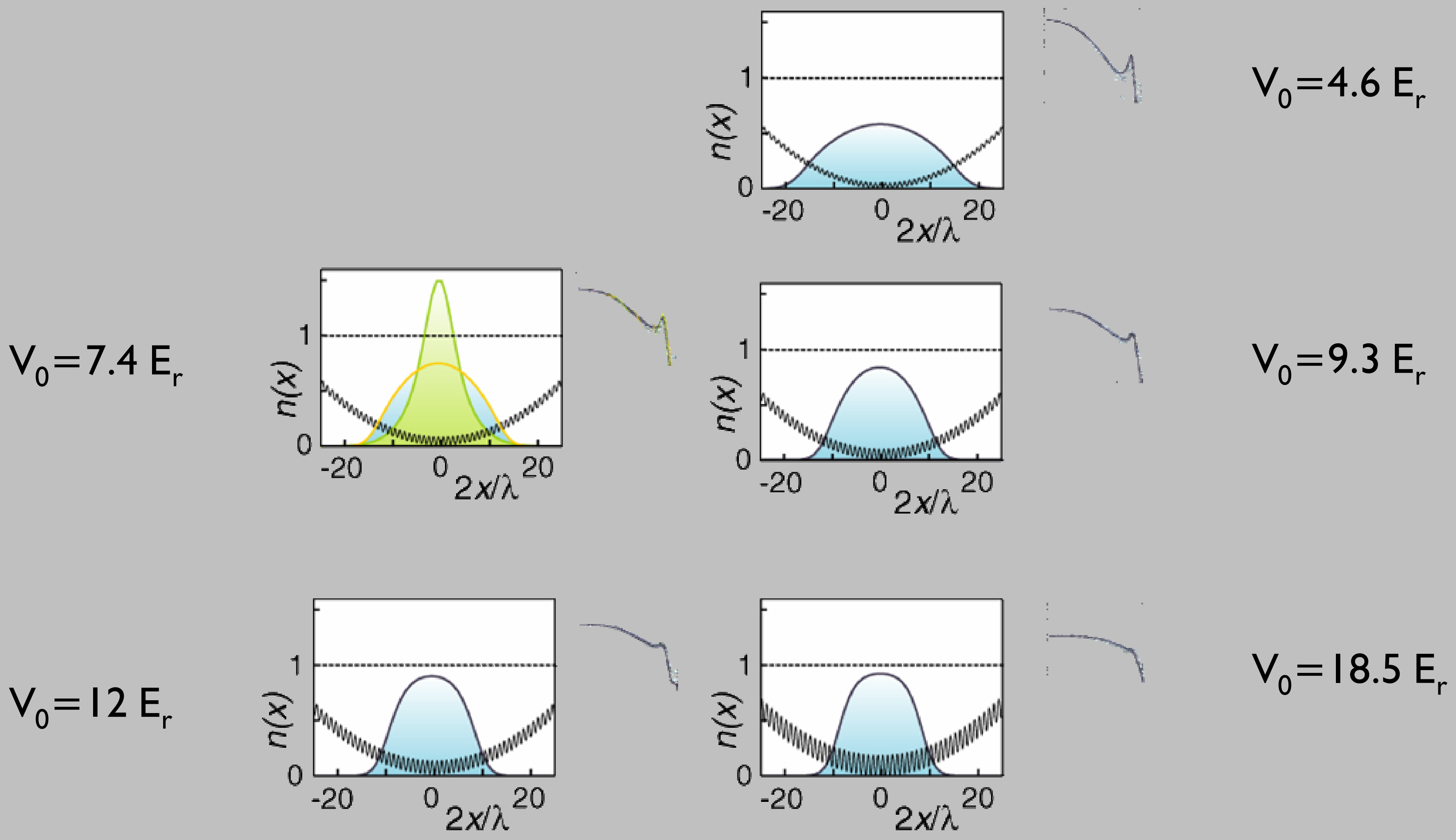
For whole series we use only two fit parameters $N_{0,0}$=18 and T_0=0.5 J/k_B!

Temperatures at higher lattice depths have been calculated from T_0 assuming an adiabatic evolution (conservation of entropy) of the system!

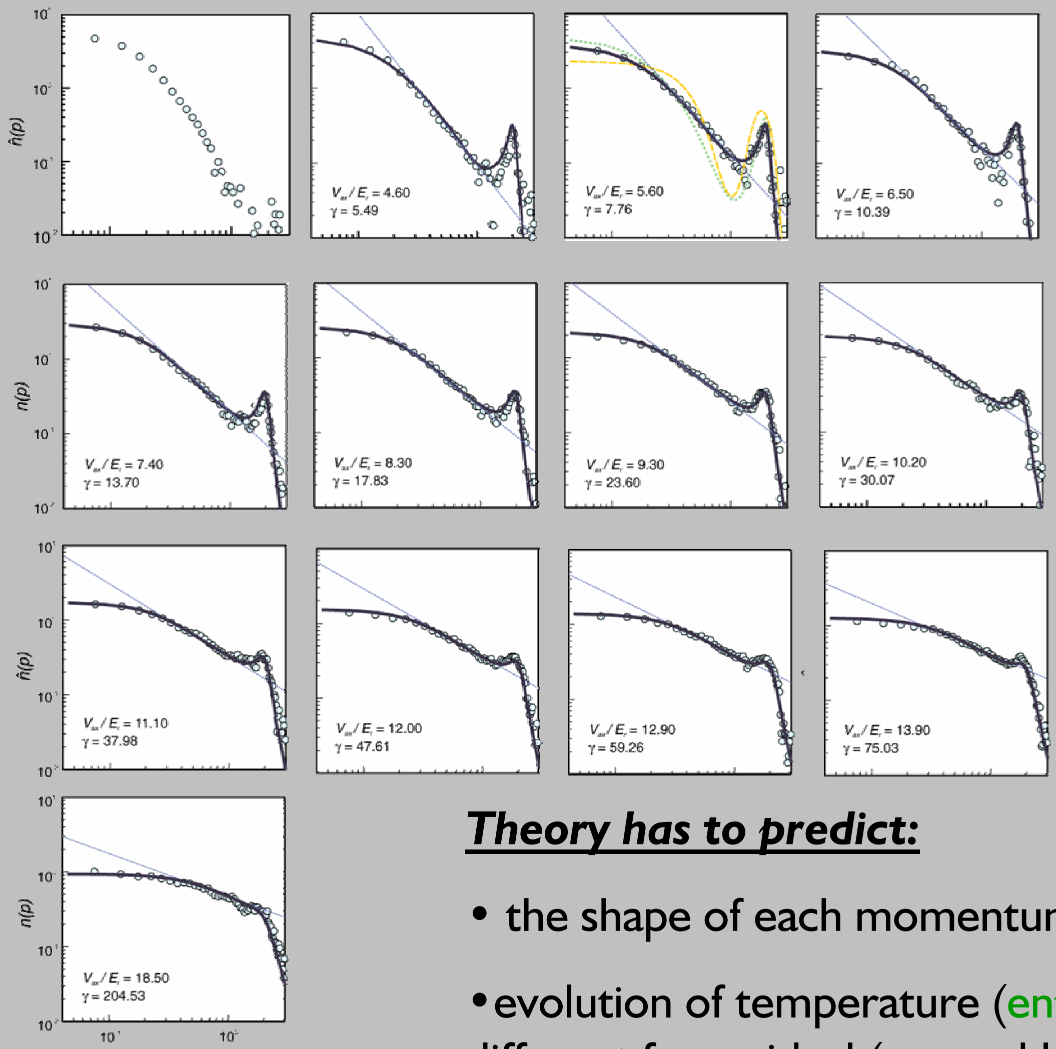
Example of Momentum Profile



Change in Densities



Comparison Theory-Experiment (All Series)



Only two fit parameters for whole series!

$$k_B T_0 / J \approx 0.5$$

$$N_{0,0} = 18$$

Theory has to predict:

- the shape of each momentum profile
- evolution of temperature (entropy) which is very different for an ideal (or weakly interacting) Bose and Fermi gas

Conclusion & Outlook
- Fermionization -

1D Quantum gases

- We have been able to enter the Tonks-Girardeau regime in a 2D array of one-dimensional quantum gases
- Increase in effective mass good way to increase interactions
- For Fermionization to be applicable it is however important to work at low filling factors
- We observe excellent agreement with the theory based on a fermionization approach

⇒ First quantitative comparison of momentum distribution with theory

- Good agreement has allowed us to determine temperature of the quantum gases