Superfluidity in atomic Fermi gases: a set of open questions

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Questions

- ***A1. How strong are the cases for a condensation of Fermion pairs, or a transition to a new phase, in the JILA and MIT experiments ?***
- ***A2. What is the nature of this condensate : "molecular" or "fermion" rich ? Or else: What kind of pairs can be detected by the fast sweep experiments?***
- ***A3. Is the observed boundary of vanishing condensed molecular fraction a boundary between normal and superfluid, or a crossover from one type of superfluid (molecular rich) to another (fermion rich)? If it is the latter, how to reveal the true superfluid to normal phase boundary?***
- ***A4. How to further reveal the nature of the ground state, should it be either kind of these condensates?***
- ***A5. Are there fundamental differences between single channel and two channel models near resonance?***
- ***A6. What are the key predictions of these models? What are the major differences, especially near resonance?***
- ***A7. How much of these predictions have been measured or are consistent with current experiments ?***

Introduction

Fermion pairing, superfluidity (superconductivity) and BEC are intimately connected (key-) concepts relevant in many cond-mat problems

Examples

* ***Bosonic fluids:*** ***superfluidity corresponds to macroscopic occupation of lowest energy state, as for ^{4}He*** [Hohenberg&Martin 1965]. ***Experiments with bosonic alkalis provided unanmbigous demonstration!!*** [JILA & MIT 1995]

* ***Fermionic fluids:*** ***attractive interactions between particles lead to pairing with simultaneous condensation of the pairs to lowest energy state, as for superconducting metals (particles are electrons) where*** $T_c / T_F :\ 10^{-4}, k_F \xi ?\ 1$ [BCS 1957]

Strength and type of Interactions, and Dimensionality govern rich phase diagrams determining

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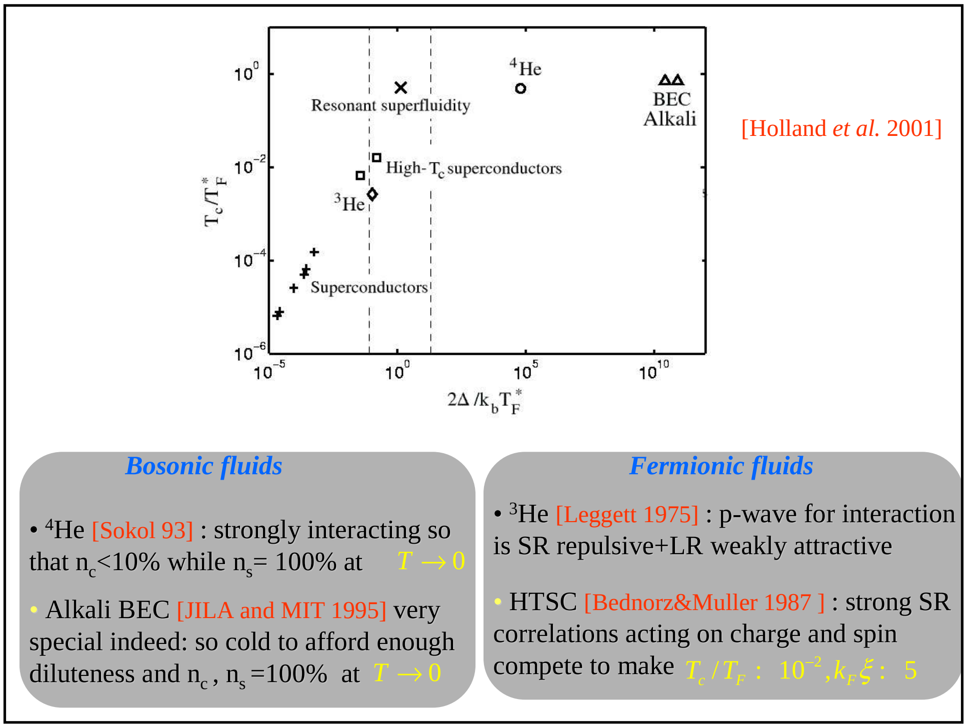
Nature and Symmetry of Normal and Super State, T_c **and** Δ



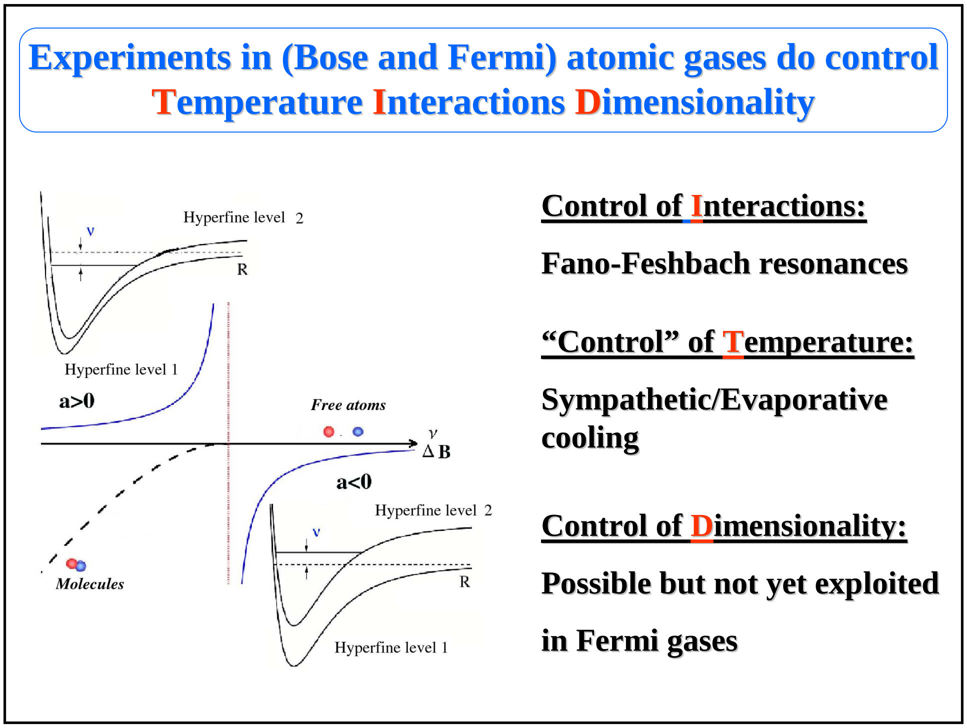
[Holland *et al.* 2001]

Bosonic fluids

- ^{4}He [Sokol 93] : strongly interacting so that n_c<10% while n_s= 100% at $T \to 0$
- Alkali BEC [JILA and MIT 1995] very special indeed: so cold to afford enough diluteness and n_c , n_s =100% at $T \to 0$

Fermionic fluids

- ^{3}He [Leggett 1975] : p-wave for interaction is SR repulsive+LR weakly attractive
- HTSC [Bednorz&Muller 1987] : strong SR correlations acting on charge and spin compete to make $T_c/T_F \sim 10^{-2}, k_F\xi \sim 5$



Experiments in (Bose and Fermi) atomic gases do control
Temperature Interactions Dimensionality

Control of Interactions:

Fano-Feshbach resonances

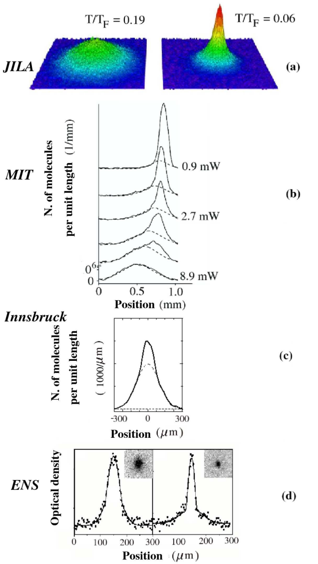
“Control” of Temperature:

Sympathetic/Evaporative cooling

Control of Dimensionality:

Possible but not yet exploited in Fermi gases

BECs of 2-fermion molecules



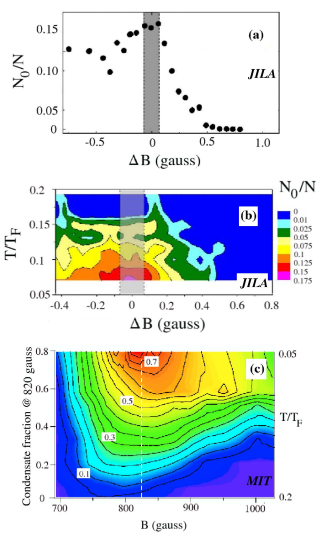
[Greiner *et al.* '03]

[Zwierlein *et al.* '03]

[Jochim *et al.* '03]

[Bourdel *et al.* '03]

Tuning across resonance



[Regal *et al.* '04]

[Zwierlein *et al.* cm/2004]

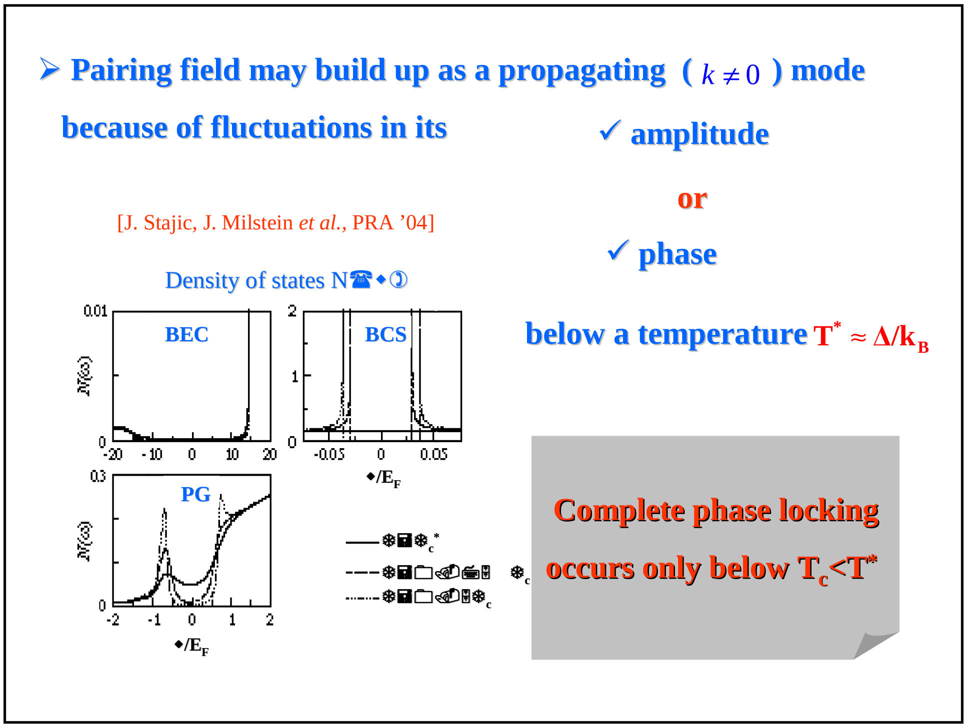
Theoretical Issues on the Crossover from Bosonic to Fermionic Superfluidity

The key-point:

- **The formation of Cooper pairs and their condensation to the lowest energy state do not necessarily occur at the same time (BCS in metallic SC is exception!)**

- **Tuning of attractive or resonant interactions may create "pairs" that populate higher energy states (on the energy scale of the interactions) and leave states $\approx \Delta$ above E_F depleted**

→ **(pseudo)gap formation**



- **Pairing field may build up as a propagating ($k \neq 0$) mode**

because of fluctuations in its

✓ **amplitude**

or

✓ **phase**

below a temperature $T^* \approx \Delta/k_B$

[J. Stajic, J. Milstein *et al.*, PRA ’04]

Density of states N☎◆☽

Complete phase locking occurs only below $T_c<T^*$

Crossover

between two extreme limits depending on “pair” size ☒

Super / Normal

BCS / ”fermionic” *n* ☒▤ >>1

BEC / ”bosonic” *n* ☒▤ <<1

Theoretical Issues…

1. Observability

- **Normal *vs.* Superfluid State**

  Signatures of S-state: "transverse probes" *(2-fluid vs. Maxwell equations)*

  Signatures of N-state: (pseudo)gap *(several papers in cm in the last month)*

  Collective modes: depending on collisional regime

  - **Bose-Einstein Condensation *vs.* Superfluidity**

    Interactions make n_c differ from ❒$_s$

    Phase-coherence probes

- **Dynamics *vs.* Thermodynamics (equilibrium)**

  Are dynamical effects affecting the formation of the pairs?
  (*Dynamical theories needed)*

Observables:

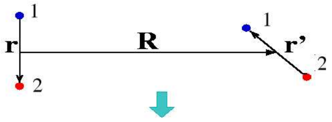
- ✓ **Scalar: Energies (gs, interaction,…)**
  Susceptibilities (compressibility,…)
- ✓ **Vectorial: N-body density matrices**
  (normal and anomalous)
- ✓ **……………………………………..**

One and Two body density matrix

$$h^{(1)}(\vec{x}_1,\vec{x}'_1) = \frac{1}{n}\langle\phi^\dagger(\vec{x}_1)\phi\ (\vec{x}'_1)\rangle = \rho(\vec{x}_1-\vec{x}'_1)$$

→ Momentum distribution

$$h^{(2)}(\vec{x}'_1,\vec{x}'_2;\vec{x}_2,\vec{x}_1) = \langle\phi^\dagger(\vec{x}'_1)\phi^\dagger(\vec{x}'_2)\phi(\vec{x}_2)\phi(\vec{x}_1)\rangle$$



Asymptotic behaviour $R >> r, r'$

- Normal State $\rightarrow h^{(1)}(\vec{x}'_1,\vec{x}_1)h^{(1)}(\vec{x}'_2,\vec{x}_2)$
  oscillating to zero
- Off Diagonal Long-Range Order
  $\rightarrow \Delta^*(r)\Delta(r')$ BCS limit
  $\rightarrow \alpha n\varphi^*(r)\varphi(r')$ BEC limit

2. Models

Single-channel (only c-fermion) with single parameter

$$H = \sum_{k\sigma} \varepsilon_k c^+_{k\sigma} c_{k\sigma} + \sum_{qkk'} V_{kk'} c^+_{q/2+k\uparrow} c^+_{q/2-k\downarrow} c_{q/2-k'\downarrow} c_{q/2+k'\uparrow}$$

$V_{kk'} \leftrightarrow a_F$ at given ν

Two-channel (b-boson/a-fermion) with three parameters

$$H_{res} = \sum_{k\sigma} \varepsilon_k a^+_{k\sigma} a_{k\sigma} + \sum_q \left(\frac{\varepsilon_q}{2} + \nu\right) b^+_q b_q + \sum_{qk} g_k \left(b^+_q a_{q/2-k\downarrow} a_{q/2+k\uparrow} + hc\right) + \sum_{qkk'} U_{kk'} a^+_{q/2+k\uparrow} a^+_{q/2-k\downarrow} a_{q/2-k'\downarrow} a_{q/2+k'\uparrow}$$

$U_{kk'} \leftrightarrow a_{F,bg}$: background a_F

ν: detuning from resonance

g : coupling close-open channel

Single-channel with single parameter *a*

Two-channel (boson-fermion) with three parameters

Eagles (1969), Leggett (1980), Nozieres&Schmitt-Rink (1985)
Electron gas, BCS ground-state with large attractive interactions

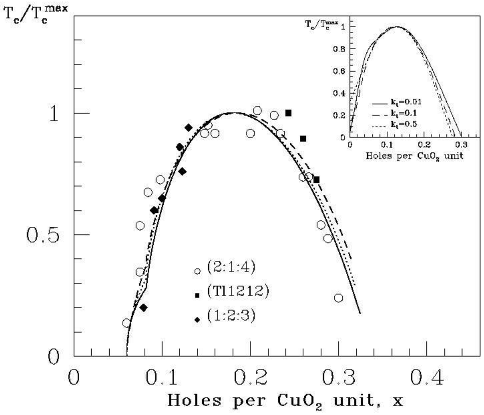
Randeria *et al.*(1992), Chen *et al.*(1999), Pieri&Strinati (2000)
Electron gas, higher-order expansions

Perali *et al.*(2003)
Atomic Fermi gases with Fano-Feshbach resonances, higher-order expansions

Ranninger&Robaszkiewicz (1985), Friedberg&T.D.Lee (1989), Chiofalo *et al.* (1995)
Electron gas, BCS-like groundstate

Holland *et al.*, Timmermans *et al.* (2001), Chiofalo *et al.* (2002), Ohashi&Griffin (2002), Milstein *et al.* (2002), Stajic&Milstein *et al.* (2004)
Atomic Fermi gases with Fano-Feshbach resonances, BCS-like with effective interaction mediated by pairs (the “phonons”)

Critical temperature in HTSC cuprates *vs.* carrier density



[Chiofalo *et al.* 1995]

❑ Share: key-concept of non-simultaneous pair formation and condensation

❑ Fermi gases: formally equivalent when the resonance state has a sufficiently short lifetime [Holland et al., cm/0404234] **with following pairing-function correspondence**

$$< c_{-k\downarrow} c_{k\uparrow} > \Rightarrow < a_{-k\downarrow} a_{k\uparrow} > - \sum_q \frac{g_k}{2\varepsilon_k - E} \left(b_q a^+_{q+k\uparrow} a_{k\uparrow} - b_{-q} a^+_{-k\downarrow} a_{-q-k\downarrow} \right)$$

$$E = \nu - \sum_k \frac{g_k^2}{2\varepsilon_k - E}$$

❑ Resonance Hamiltonian separates energy scales. Thus advantageous when $|a_F| \mapsto \infty$ (no easy way of incorporating energy dependence in single-channel model)

3. BCS & BEC limits

Theories have to reproduce correct BCS and BEC limits

(at variance than HTSC in Fermi gases there's room for quantitative understanding)

✓ **BCS: easy as most calculations start from BCS ground state**

✓ **BEC:** Petrov et al cm/0309010 **point out that the boson-boson scattering length is** $\mathbf{a_B}$; $\mathbf{0.6a_F}$ **from solution 4-body Schroedinger equation** ($\mathbf{a}_F$? r_0 potential range)

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4. Universality&Unitarity limit

Theories have to cope with the unitarity limit $|a_F| \to \infty$

☐ **At resonance thermodynamic properties are expected to be independent of** a_F **as the relevant length scale is the interparticle distance** $\approx \mathbf{n}^{-1/3}$ [e.g. Heiselberg 2001, Ho and Mueller cm/0306187]

☐ **Experiments consistent with the "universal" parameter (**$a_F<0$**)**

$$\beta \equiv \frac{E_{int}}{E_F} \approx -0.25 \text{ over the range } 0.1 < \frac{T}{T_F} < 1$$

But Innsbruck measures –0.68 *(temperature effect? Width of resonance as compared to Fermi energy?)*

☐ **Theories range from** $\beta = -0.56$ [Carlson et al. 2003, QMC T=0] **to** $\beta = -0.67$ [Baker 1999] **to** $\beta = -0.3$ [Bruun 2004], …………

☐ **Strictly, universality has theoretically been demonstrated in the Boltzmann limit. But is universality always the case?**

On the BCS side, analysis of Resonance Hamiltonian suggests that universality holds only for broad resonances

$g^2 ? \frac{4\pi k_F}{m^2}$ [Bruun&Pethick 2004]

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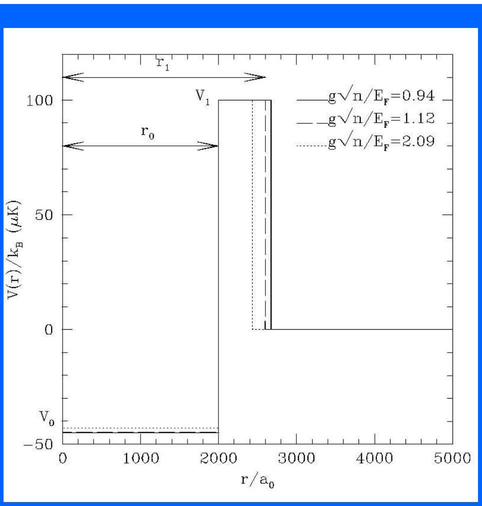
BCS ground state. Does universality hold?

The well-barrier model for the Fano-Feshbach resonance



☐ **Conditions**

$nr_0^3 = 1$	diluted
$na^3 > 1$	unitarity limited

☐ **Regimes (tunable by this model)**

$\frac{g\sqrt{n}}{E_F} > 1$	"broad" resonance
$\frac{g\sqrt{n}}{E_F} < 1$	"narrow" resonance

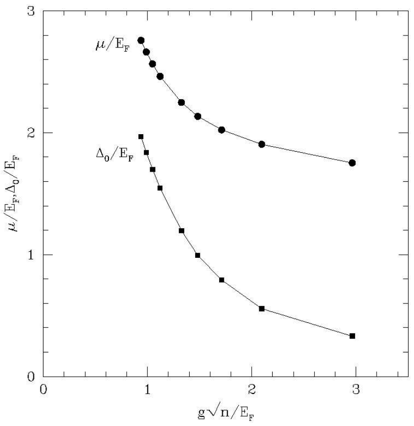
☐ **Parameters**

$r_0 = 2000a_0$

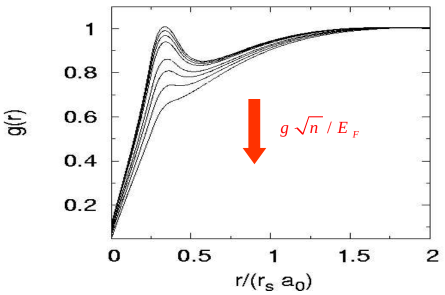
$a = +5000a_0$

$n = 1.054\times10^{14}\ cm^{-3}, \quad nr_0^3 = 0.125$

➢ Emergence of BCS-superfluidity with decreasing $g\sqrt{n}/E_F$



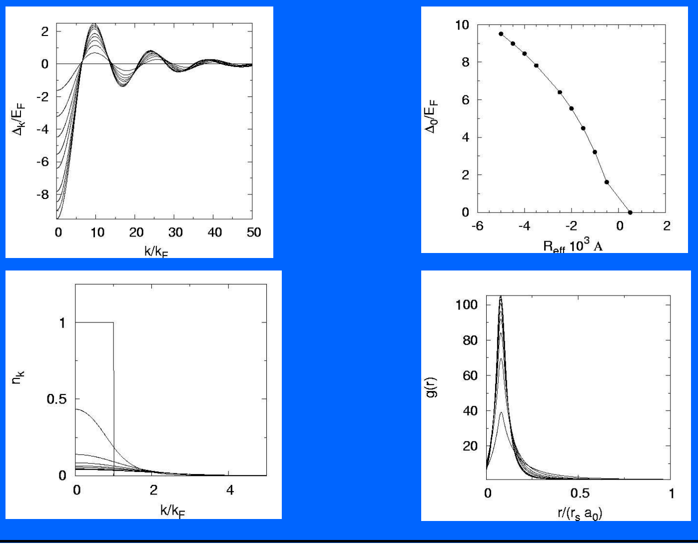
[S. De Palo et al., cm/0404xxx]



Would the results be confirmed after accounting for interactions?

❖ **We are currently resorting to Quantum Monte Carlo simulations of the Fermi gas with Fano-Feshbach resonance using well-barrier potential**

➢ Decreasing nr_0^3 (r_0=500 a_0)



Questions

- ***A1. How strong are the cases for a condensation of Fermion pairs, or a transition to a new phase? Qualitatively strong (until dynamical theory comes and/or QMC simulations)***
- ***A2. What is the nature of this condensate : "molecular" or "fermion" rich ? Depends on the detuning from resonance and on the species. What kind of pairs can be detected by the fast sweep experiments? Pairs with sufficient overlap with a BEC of "molecules" , namely with*** $n\xi \approx 1$
- ***A3. Is the observed boundary of vanishing condensed molecular fraction a boundary between normal and superfluid, or a crossover from molecular to fermion-rich superfluid? If it is the latter, how to reveal the true superfluid to normal phase boundary? Experiments wanted (see below)***
- ***A4. How to further reveal the nature of the ground state, should it be either kind of these condensates? Distinguish super- from normal fluid (transverse probes), Pseudogap probes, Bogolubov-Anderson mode, phase coherence***

- ***A5. Are there fundamental differences between single channel and two channel models near resonance? Formally no, provided resonant state has short lifetime. In practice yes, if approximations do not satisfy special limits***
- ***A6. What are the key predictions of these models? What are the major differences, especially near resonance? Two-channel in general expected to give better (easier?) account of thermodynamical quantities near resonance, at correspondent order of approximation (see also above)***
- ***A7. How much of these predictions have been measured or are consistent with current experiments ? ??????***