Strongly-Correlated Groundstates in Rotating Atomic Bose Gases

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Questions

- Can the correlated limit be achieved?

- What are the most promising schemes to realize the strongly correlated states?

- What are the signatures of the correlated states?

Parameters and Regimes

Energy scales:

Trapping energies, $\hbar\omega_{\perp,\parallel}$ $\left[a_{\perp,\parallel} \equiv \sqrt{\frac{\hbar}{m\omega_{\perp,\parallel}}}\right]$

Interaction energy scale, $g\bar{n}_{3\mathrm{D}}$ $\left[g \equiv \frac{4\pi\hbar^2 a_s}{m}\right]$

Weakly-interacting limit: [Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

$g\bar{n}_{3\mathrm{D}} \ll \hbar\omega_\perp \Rightarrow$ lowest Landau level limit.
$g\bar{n}_{3\mathrm{D}} \ll \hbar\omega_\parallel \Rightarrow$ quasi 2D limit.

$$\langle x, y|m\rangle \propto z^m e^{-|z|^2/2} \qquad [z \equiv (x+iy)/a_\perp]$$

Filling fraction: $\nu = \frac{N}{N_V}$ [NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

$\nu > \nu_c$: Vortex lattice groundstate – "mean-field" regime.
$\nu < \nu_c$: Incompressible liquid groundstates – "quantum Hall" regime.

$$\Psi_L(\{z_i\}) = \prod_{i<j}^{N}(z_i - z_j)^2 \, e^{-\sum_i |z_i|^2/2}$$

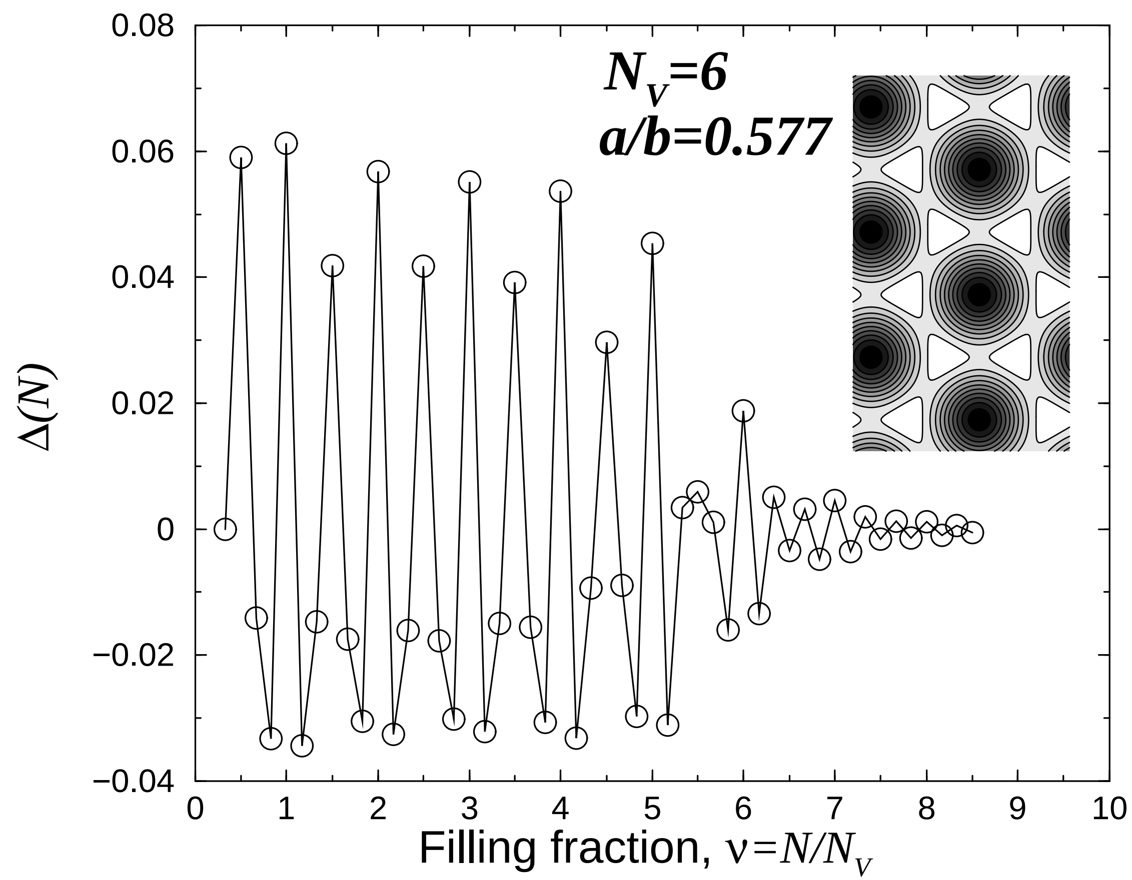
Theoretical Evidence for ν_c

(i) Lindemann criterion: The vortex lattice is stable to quantum fluctuations for $\nu > \nu_c \simeq 14, 8, 17$. [NRC, Wilkin & Gunn, PRL (2001); Sinova, Hanna & MacDonald, PRL (2002); Baym, cond-mat (2003)]

(ii) Exact Diagonalisations on a Torus [NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

Vortex lattice at $\nu \gtrsim 6$

Incompressible liquids at $\nu \lesssim 6$



Parameter Values

Experimental Status: [V. Schweikhard *et al.* [JILA], PRL **92**, 040404 (2004)]

$\mu/(2\hbar\omega_\perp) < 1$ $\Rightarrow$ lowest Landau level

$\nu \gtrsim 500$ $\Rightarrow$ vortex lattice groundstate

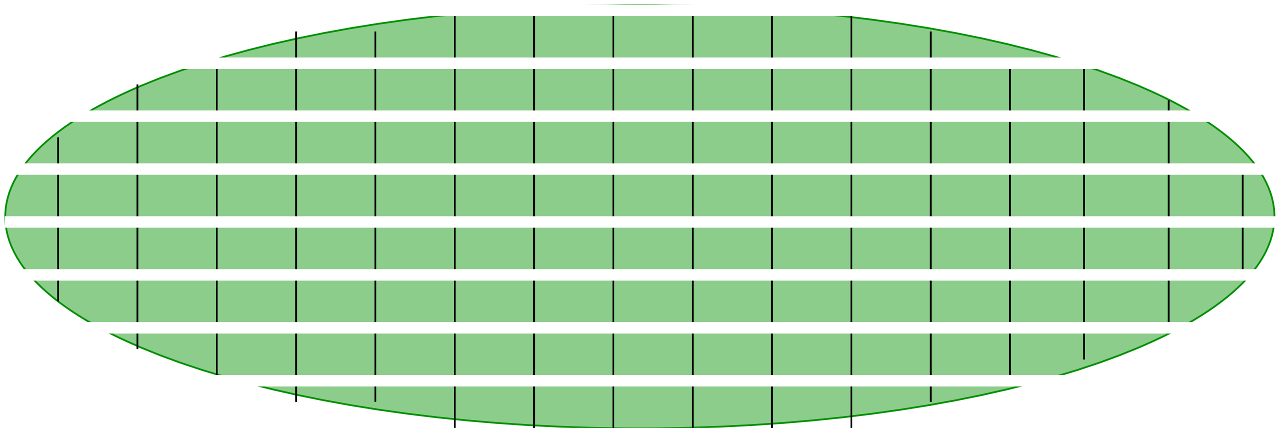
Theoretical Estimates:

Incompressibility gaps, $0.15\frac{4\sqrt{2\pi}\hbar^2 a_s}{m a_\parallel a_\perp^2} \simeq 1.5\,\frac{a_s}{a_\parallel}\hbar\omega_\perp$

[NRC, Wilkin & Gunn, PRL 2001; Regnault & Jolicoeur, PRL 2003]

$$\left.\begin{array}{l} \hbar\omega_\perp \simeq h \times 100\text{Hz} = 5\text{nK} \\ \\ \frac{a_s}{a_\parallel} \simeq 5\text{nm}/3\mu\text{m} = 2\times 10^{-3} \end{array}\right\} \Rightarrow \text{Incompressibility gaps} \simeq 0.015\text{nK!}$$

1D Optical lattice [E. Cornell]



Filling fraction of each layer reduced: $\nu^{\text{layer}} = \nu \frac{1}{N_L}$.

Strong confinement of the optical lattice: $\omega_{\parallel}^{\text{layer}} \simeq 2\pi \times 40\text{kHz} \Rightarrow a_{\parallel}^{\text{layer}} \simeq \frac{1}{60} a_{\parallel}$.

Incompressibility gaps $\simeq 0.2\ \hbar\omega_{\perp}$.

$\Rightarrow$for $\hbar\omega_{\perp} \simeq h \times 200\text{Hz} = 10\text{nK}$, energy gaps are of order 2nK.

What are the signatures of the correlated states?

– fractional statistics; [B. Paredes, P. Fedichev, J. I. Cirac & P. Zoller, PRL 2001]

– vanishing condensate fraction; [J. Sinova, C. Hanna & A. MacDonald, PRL 2003]

– collective excitations – gapless edge modes [M. Cazalilla, PRA 2003]
bulk modes are gapped

– many-body wavefunctions in LLL survive expansion
so all density correlation functions are available; [N. Read & NRC, PRA 2003]

– incompressibility is reflected in the density distribution.
[F. van Lankvelt, J. Reijnders, K. Schoutens, APS March Meeting (2004)]

Density Distribution

[NRC, unpublished]

Local approximation:

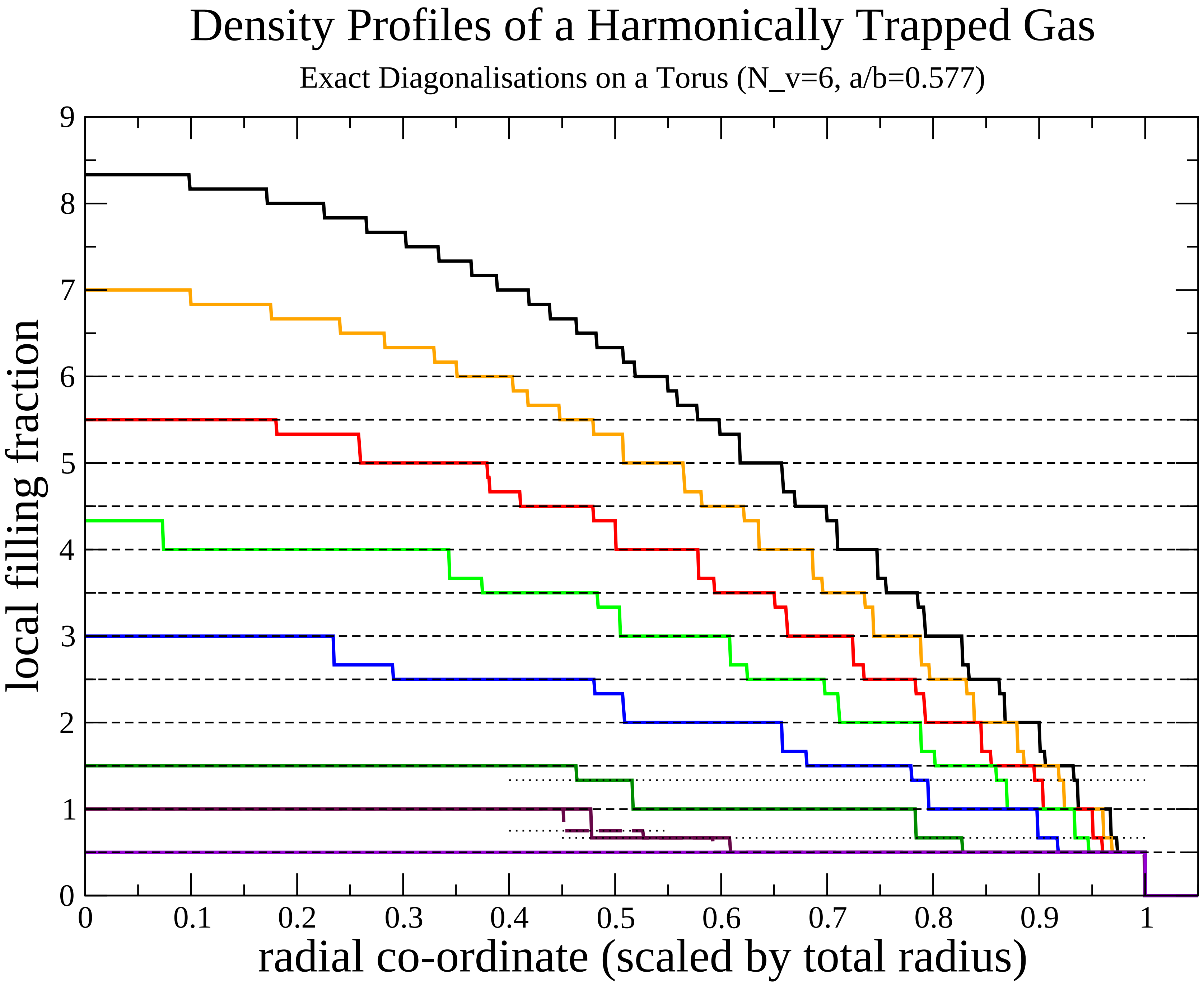
$$\nu(\vec{r}) \equiv \frac{n_{2\mathrm{D}}(\vec{r})}{n_V}$$

$$E_{\mathrm{int}} \simeq \frac{g}{(2\pi)^{3/2} a_{\parallel} a_{\perp}^2} \int \epsilon[\nu(\vec{r})] \frac{1}{\pi a_{\perp}^2} \, d^2\vec{r}$$

Vortex Lattice: $\epsilon(\nu) = b\nu^2 \quad \Rightarrow \quad \nu(r) = \nu_{\max}\left[1 - \frac{r^2}{R^2}\right]$

[Watanabe, Baym & Pethick, cond-mat/0403470]

Strongly-correlated regime: take $\epsilon(\nu)$ from exact-diagonalisation results.



Density Profiles of a Harmonically Trapped Gas
Exact Diagonalisations on a Torus (N_v=6, a/b=0.577)
local filling fraction
radial co-ordinate (scaled by total radius)
0
1
2
3
4
5
6
7
8
9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Summary

Need $\nu = \frac{N}{N_V} \lesssim 10$ for strongly-correlated groundstates.

Incompressibility gaps $\simeq 1.5\frac{a_s}{a_\parallel}\hbar\omega_\perp$ must be sufficiently large.

Incompressibility has dramatic consequences on the density distribution of a confined gas.