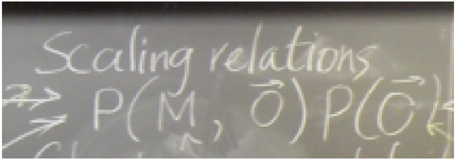
# Scaling relations: important, but hard to write down



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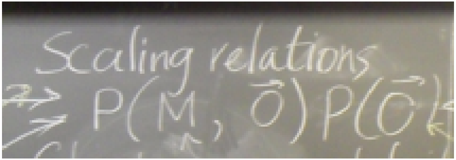


For cosmology, we need

$$P(\vec{O}|M, z)$$

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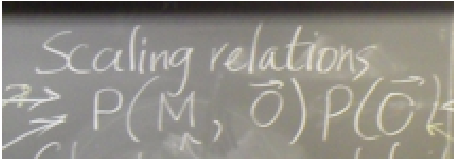
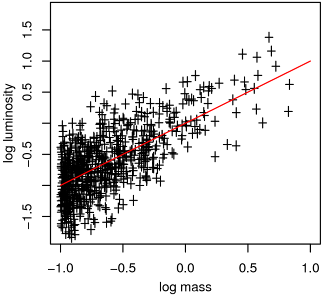



For cosmology, we need

$$\langle N_{\rm det}(M,z)\rangle = \int_{\vec{O}_{lim}}^{\infty} \langle N(M,z)\rangle \, {\color{red} P(\vec{O}|M,z)}$$

# Scaling relations

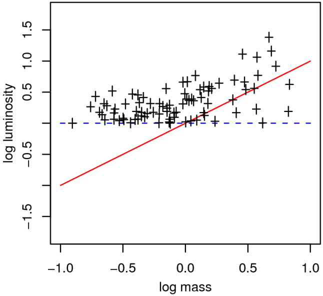



We need to model a population using incomplete information.

Difficulties:

- Selection bias
- Intrinsic covariance
- Measurement covariance

# Scaling relations

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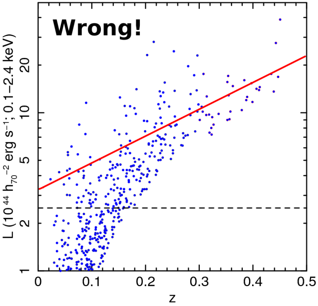


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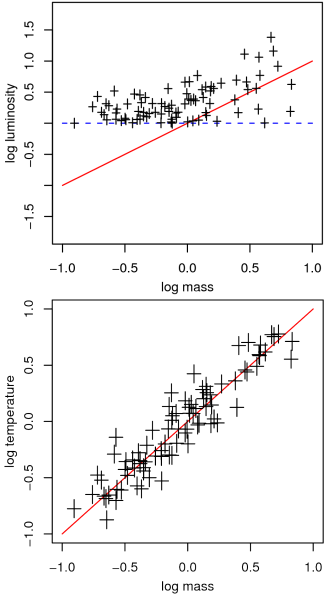


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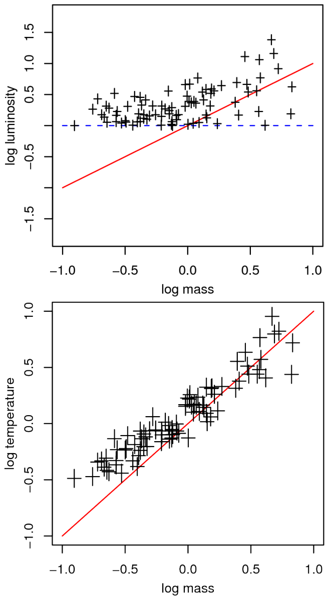
$\rho = 0.1$

# Scaling relations




We need to model a population using incomplete information.

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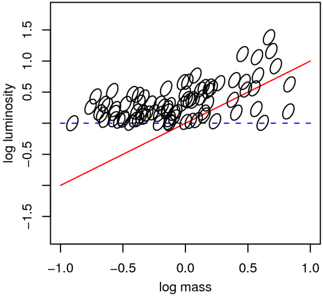
- Measurement covariance

$\rho = 0.9$

# Scaling relations




We need to model a population using incomplete information.

Difficulties:

- Selection bias
- Intrinsic covariance
- Measurement covariance

# Scaling relations

On paper, the problem isn't too hard. The likelihood is:

$$\mathcal{L}(\hat{O}) = \sum_{N=N_{\rm det}}^{\infty} \left[ \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!} \right] \left[ \frac{N!}{N_{\rm det}! N_{\rm mis}!} \right] \times \prod_{i=1}^{N_{\rm det}} P_{\rm det}(\hat{O}_i) \prod_{j=1}^{N_{\rm mis}} P_{\rm mis}\left(\bar{I}\right)$$

# Scaling relations

$$P_{\rm det}(\hat{O}_i) = \int dM \int dz \int dO \; P(M,z)\; P(O|M,z)\; P(\hat{O}_i|O)\; P({\rm det}|\hat{O}_i, z)$$

1. Mass function
2. Scaling relations
3. Sampling distribution (measurement errors)
4. Selection function

# Scaling relations

Observations:

- This isn't as simple as minimizing $\chi^2$, but it's still feasible.
- Mass function dependence means that the scaling relation constraints depend on cosmology.

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- Mass function dependence means that the scaling relation constraints depend on cosmology.

Consequence:

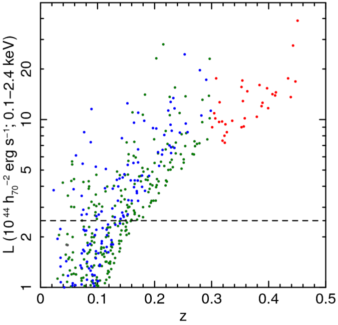
- We need to be careful when assessing goodness of fit (comparing scaling/scatter/evolution models) since those degrees of freedom can be degenerate with cosmology.
- Multiwavelength data and overlapping surveys with different selection will help with this.
- Simulations can inform the family of models that need to be tested (evolution, skewness, etc.)

# Scaling relations

$$P_{\mathrm{det}}(\hat{O}_i) = \int dM \int dz \int dO \; P(M,z)\; P(O|M,z)\; P(\hat{O}_i|O)\; P(\mathrm{det}|\hat{O}_i,z)$$

1. Mass function: known for a given cosmology
2. Scaling relations: ???
3. Sampling distribution (measurement errors): understood
4. Selection function: understood

# In practice: data




238 X-ray selected clusters from RASS (complete redshift follow-up)

Pointed X-ray follow-up of 94

- luminosity
- temperature
- gas mass

Gas mass used as a proxy for total mass.

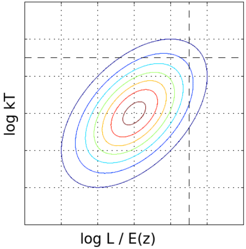
# In practice: model

Flat $\Lambda$CDM plus mean $L(M)$ and $T(M)$ as power laws:

$$\begin{aligned}
\log_{10}\left[\frac{L_{500}}{E(z)}\right] &= \beta_0^{\ell m} + \beta_1^{\ell m}\log_{10}\left[E(z)M_{500}\right] \\
\log_{10}\left[kT_{500}\right] &= \beta_0^{tm} + \beta_1^{tm}\log_{10}\left[E(z)M_{500}\right] \\
E(z) &= H(z)/H_0
\end{aligned}$$

Intrinsic scatter in $L, T|M$ as bivariate log-normal:

$$\begin{pmatrix} \sigma_{\ell m}^2 & \rho\sigma_{\ell m}\sigma_{tm} \\ \rho\sigma_{\ell m}\sigma_{tm} & \sigma_{tm}^2 \end{pmatrix}$$



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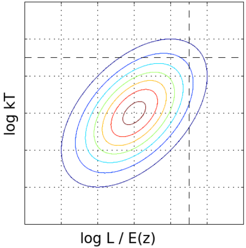

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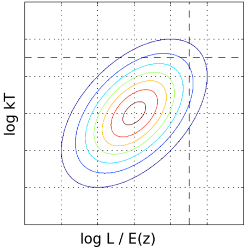

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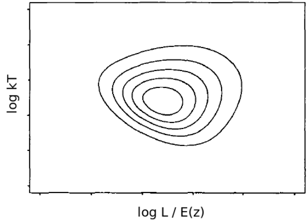
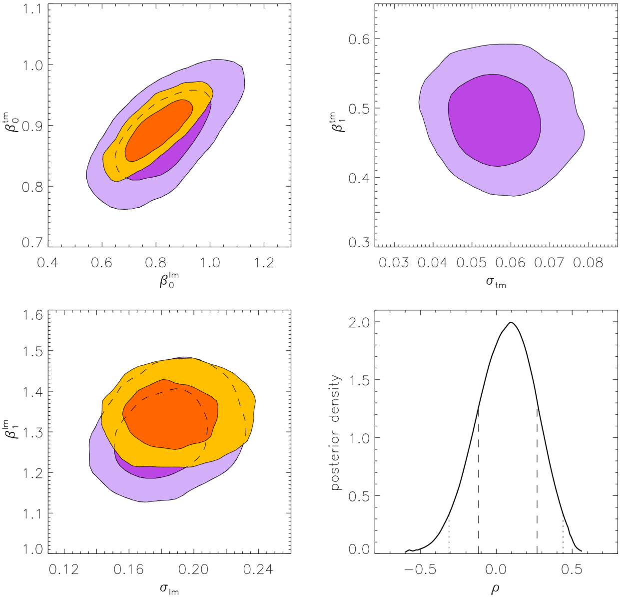
Intrinsic scatter in $L,T|M$ as bivariate skew log-normal:

$$\begin{pmatrix} \sigma_{\ell m}^2 & \rho\sigma_{\ell m}\sigma_{tm} \\ \rho\sigma_{\ell m}\sigma_{tm} & \sigma_{tm}^2 \end{pmatrix}, \quad \begin{pmatrix} \lambda_{\ell m} \\ \lambda_{tm} \end{pmatrix}$$



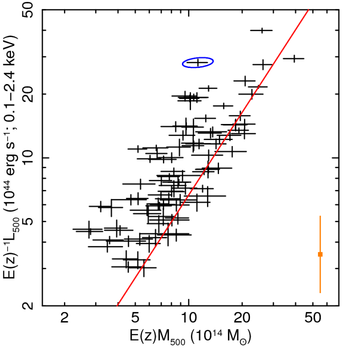
# In practice: results



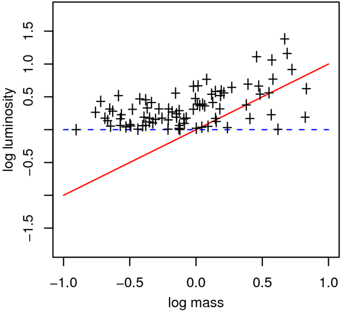

Purple: clusters only

Gold: with other data (WMAP etc.)

# In practice: results

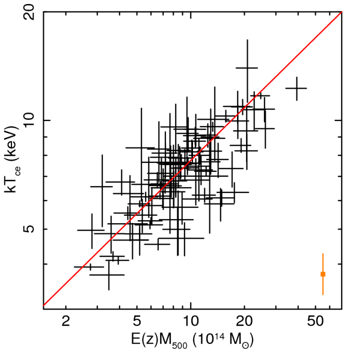


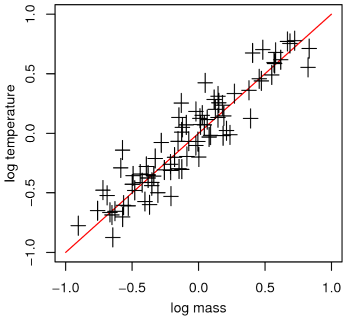

Real data
Complicated selection function




Fake data
Simplistic selection function

# In practice: results



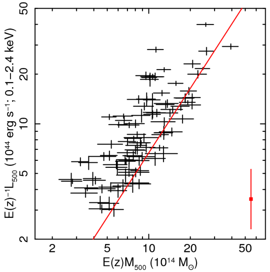

Real data

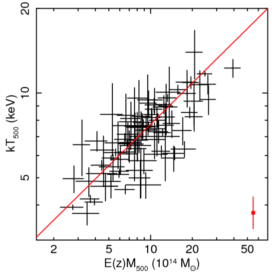
Selection on a different observable

Fake data

Mild correlation with selection observable

# In practice: goodness of fit



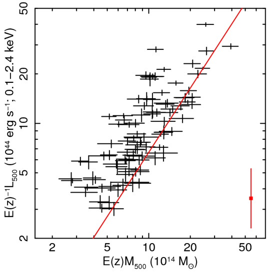

Constrain cosmology as much as possible (flat $\Lambda$CDM, use CMB et al.)

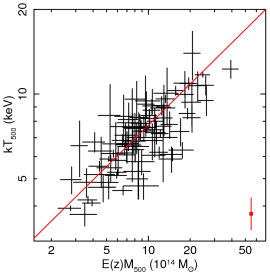
Test how well best fitting model predictions match the data

- Fold in cosmology, selection function, ...
- Check cluster abundance, survey distribution, measured masses, luminosities, etc.

Result: The $\Lambda$CDM+self-similar evolution model is acceptable (to these data).

# In practice: extended models

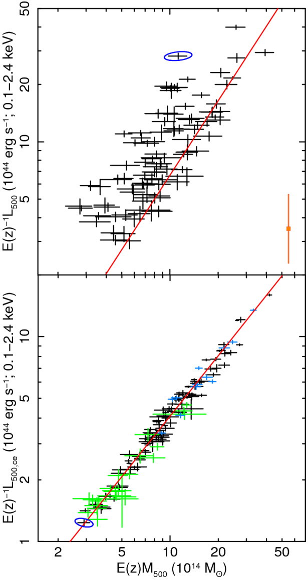



Comparing deviance information criteria (i.e. doing model comparison), there is no preference for

- departures from self-similar evolution
- evolution in the intrinsic scatter
- asymmetry in the intrinsic scatter

# Center-excised scaling relations




The $L$–$M$ relation has

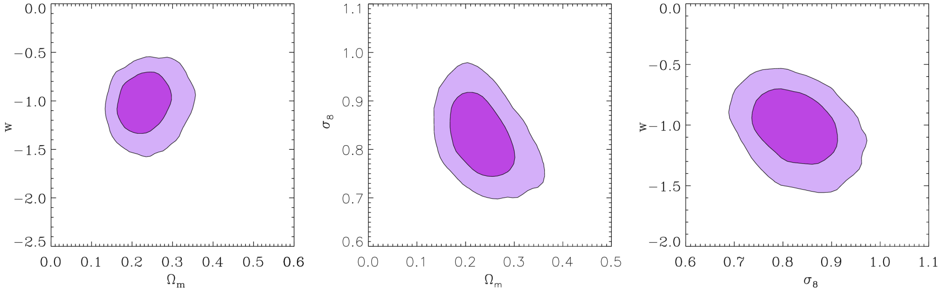
- large scatter ($\sim 40\%$)
- slope $> 4/3$ (virial theorem)

Exclude the central $0.15 r_{500}$ from $L$ ...
(e.g. Zhang '07, Maughan '07)

The $L_{\rm ce}$–$M$ relation has

- small scatter ($< 10\%$)
- slope $1.30 \pm 0.05$
- self-similar evolution with redshift

# Basic cosmology results (flat, constant $w$ models)




238 clusters, $z < 0.5$ (XLF)
Including systematics

$$\begin{aligned} \Omega_m &= 0.23 \pm 0.04 \\ \sigma_8 &= 0.82 \pm 0.05 \\ w &= -1.01 \pm 0.20 \end{aligned}$$

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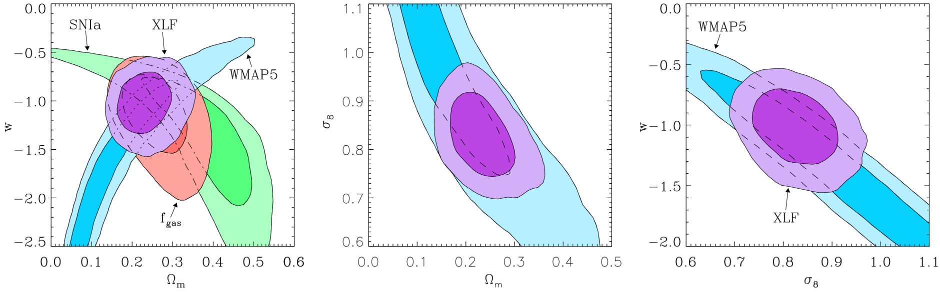


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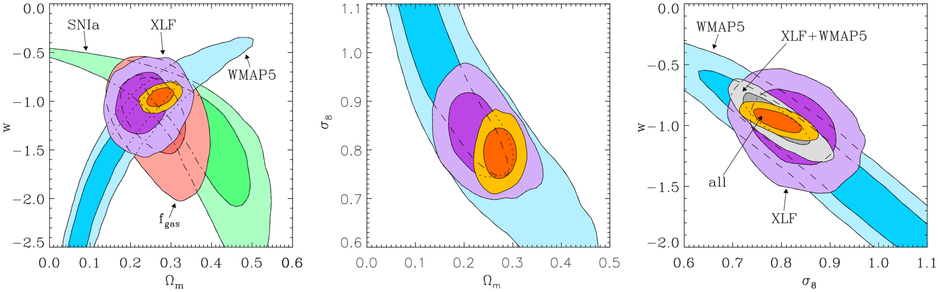


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XLF+WMAP5+SNIa+$f_{\rm gas}$+BAO

$$\Omega_{\rm m} = 0.272 \pm 0.016$$
$$\sigma_8 = 0.79 \pm 0.03$$
$$w = -0.96 \pm 0.06$$