Numerical simulations of sediment transport and aeolian ripples

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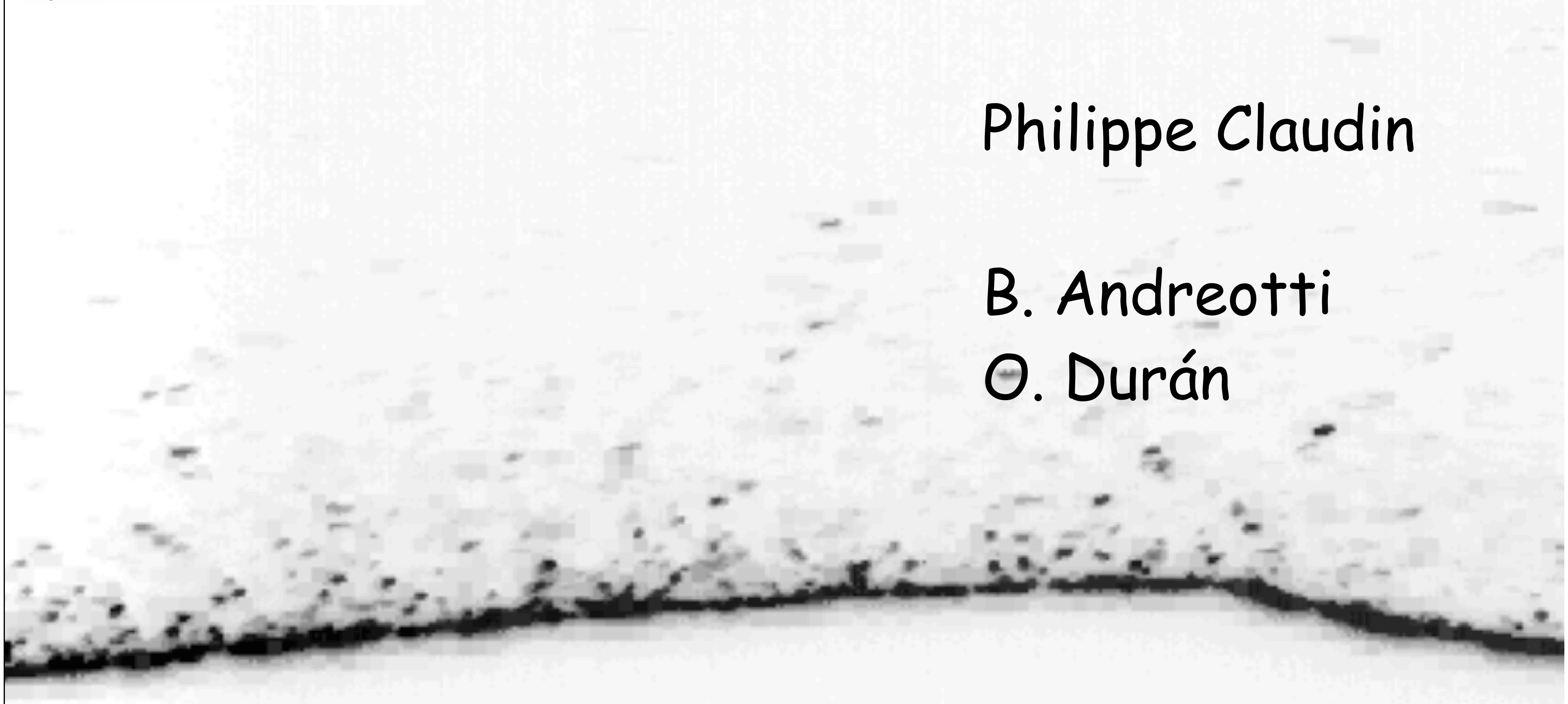

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Goal & outline

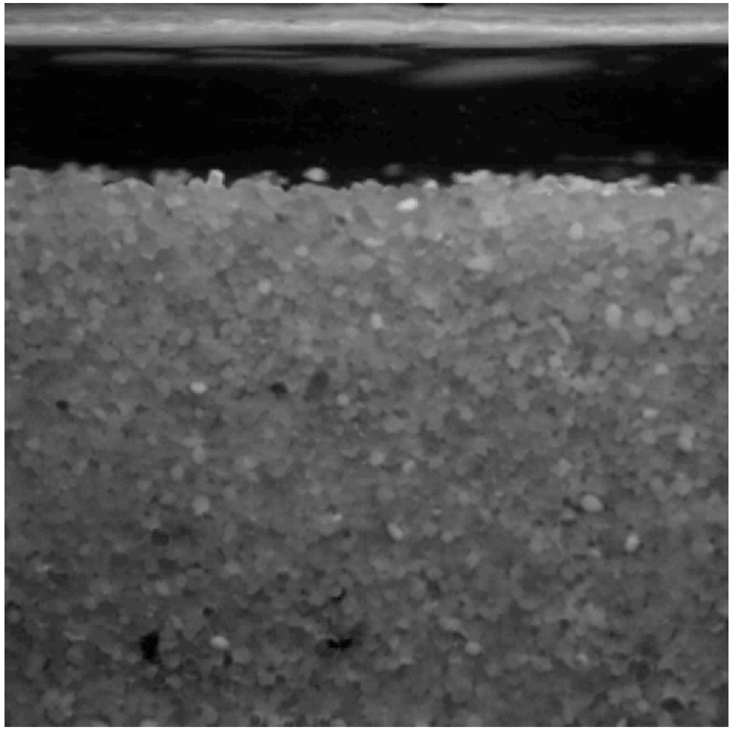
- Microscopic mechanisms of transport
- Empirical laws and modeling assumptions
- Model

  O. Durán, B. Andreotti and P. Claudin, Phys. Fluids **24**, 103306 (2012).
- Transport layer and sediment flux
- Scaling laws
- Aeolian ripples

  O. Durán, P. Claudin and B. Andreotti , submitted.

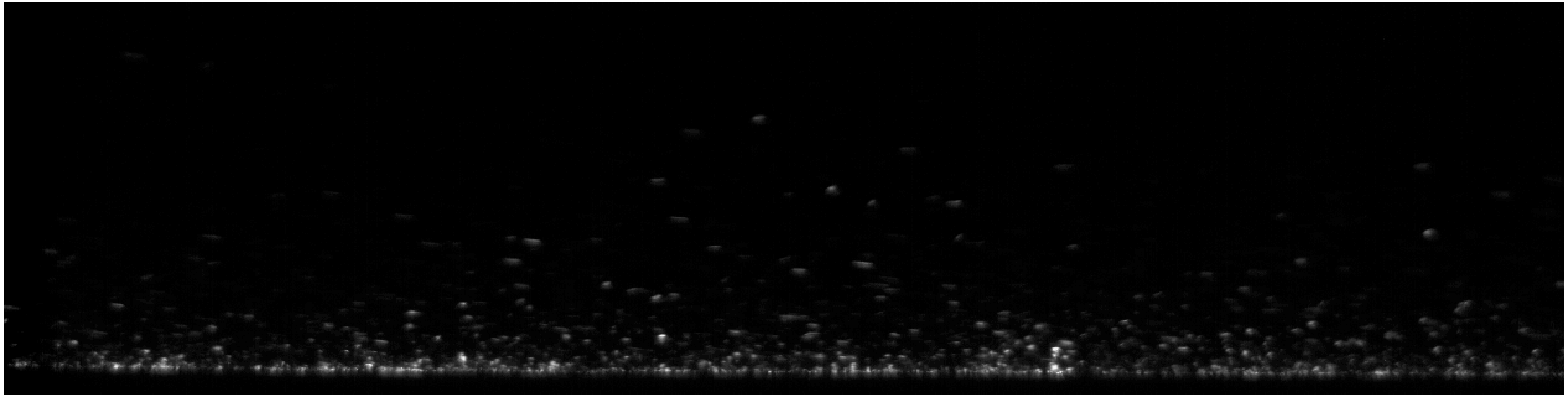
Sediment transport

Sub-aqueous bed load



From E. Lajeunesse (IPGP)

Aeolian saltation



Champ: 12 cm x 3 cm   Acquisition 1kHz

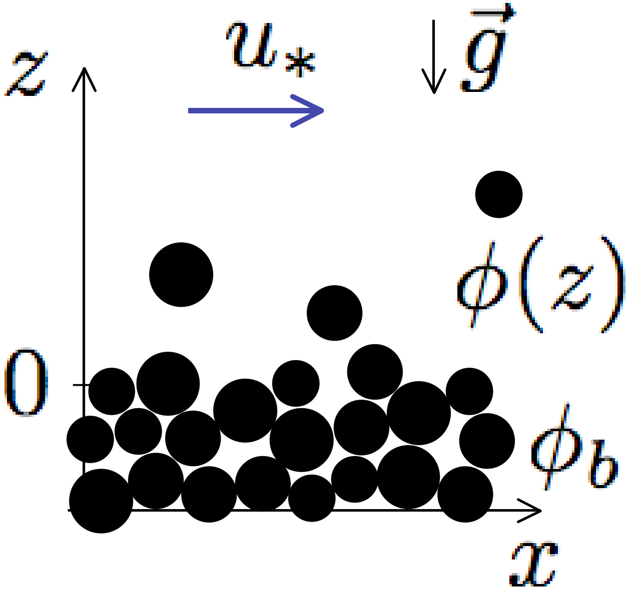
The numerical model

- Principle:

GRAINS **(DEM)** $$m\frac{d\vec{u}^p}{dt} = \ldots$$

FLUID **(RANS)** $$\rho_f(1-\phi)\frac{D\vec{u}^f}{Dt} = \ldots$$

COUPLING $$\vec{f}^p_{\text{fluid}} = \vec{f}^p_{\text{drag}} + \vec{f}^p_{\text{Arch}}$$



$1000d \times 1d \times 1000d$

15000 grains

- Control parameters:

$$\Theta = \frac{\rho_f u_*^2}{(\rho_p - \rho_f)gd}$$

Schields

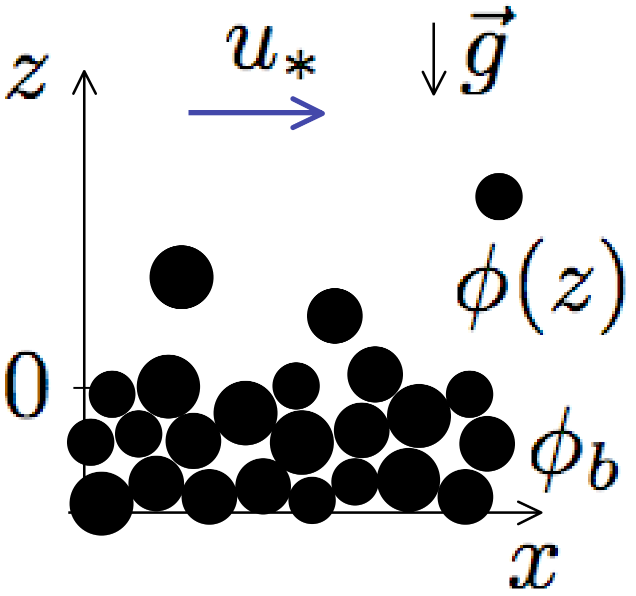
$$\rho_p/\rho_f$$

Density ratio

$$\mathcal{R} = \frac{d}{\nu}\sqrt{\left(\frac{\rho_p}{\rho_f} - 1\right)gd}$$

Grain Reynolds

The grains



$$m\frac{\mathrm{d}\vec{u}^p}{\mathrm{d}t} = m\,\vec{g} + \sum_q \vec{f}^{p,q} + \vec{f}^p_{\mathrm{fluid}}$$

$$I\frac{\mathrm{d}\vec{\omega}^p}{\mathrm{d}t} = \frac{d}{2}\sum_q \vec{n}^{p,q} \times \vec{f}^{p,q}$$

$$\vec{f}^p_{\mathrm{fluid}} = \vec{f}^p_{\mathrm{drag}} + \vec{f}^p_{\mathrm{Arch}}$$

$$\vec{f}^p_{\mathrm{drag}} = \frac{\pi}{8}\rho_f d^2 C_d(R_u)|\vec{u} - \vec{u}^p|(\vec{u} - \vec{u}^p)$$

$$C_d(R_u) = \left(\sqrt{C_d^\infty} + \sqrt{R_u^c/R_u}\right)^2$$

$$R_u = |\vec{u} - \vec{u}^p|d/\nu$$

$$\vec{f}^p_{\mathrm{Arch}} = \frac{\pi}{6}d^3\mathrm{div}\sigma^f$$

Hydrodynamics

RANS

$$\rho_f(1-\phi)\mathrm{D}_t u_i = -\partial_i p^f + \rho_f(1-\phi) g_i + \partial_j \tau^f_{ij} - F_i$$

$$\partial_z p^f = -\rho_f g$$

$$\partial_z \tau^f = F_x$$

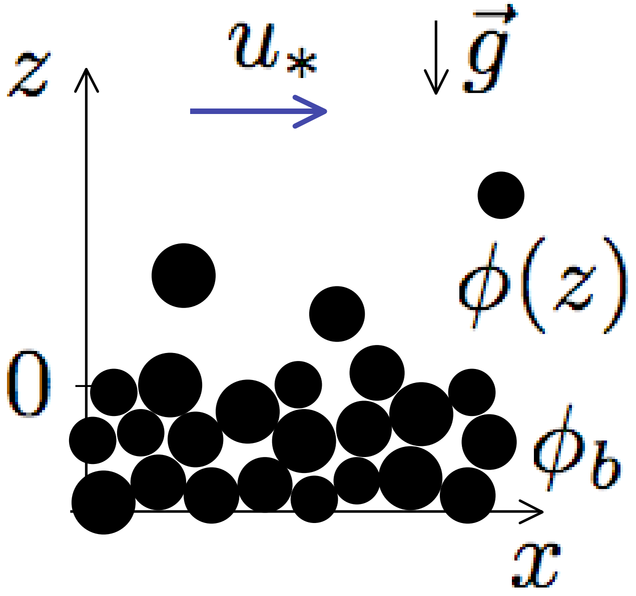
CLOSURE

$$\tau^f = \rho_f(\nu + \ell^2|\partial_z u|)\partial_z u$$

mixing length

COUPLING

$$\vec{F}(z) = \frac{1}{A\mathrm{d}z}\left\langle \sum_{p\in\{z;z+\mathrm{d}z\}} \vec{f}^{\,p}_{\mathrm{fluid}} \right\rangle$$



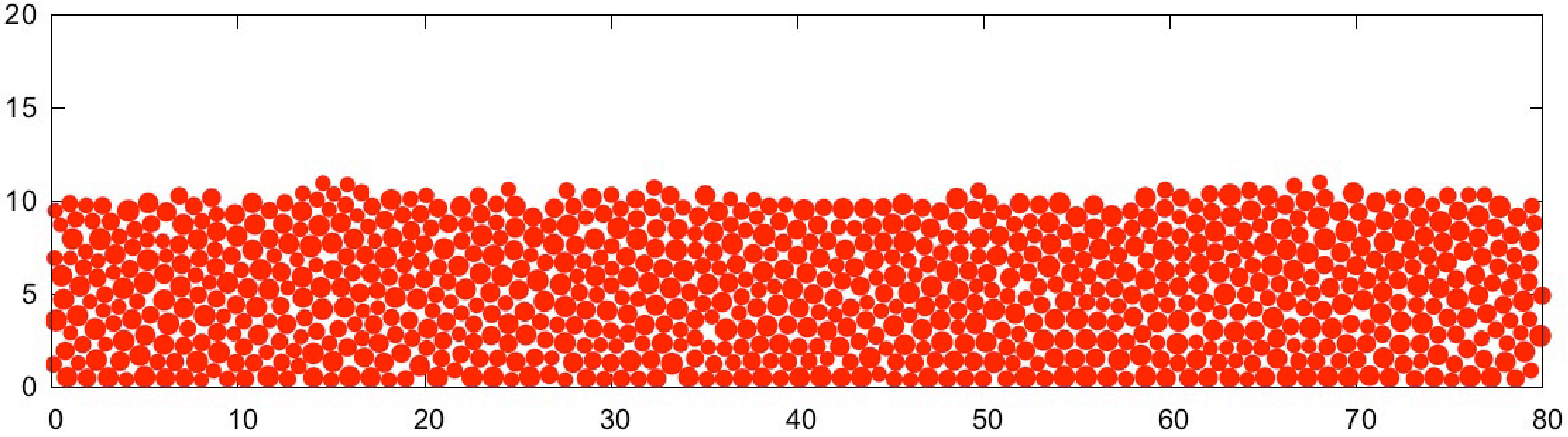
Movies

flow →

$$\sqrt{\Theta/\Theta_d} = 2$$

Water

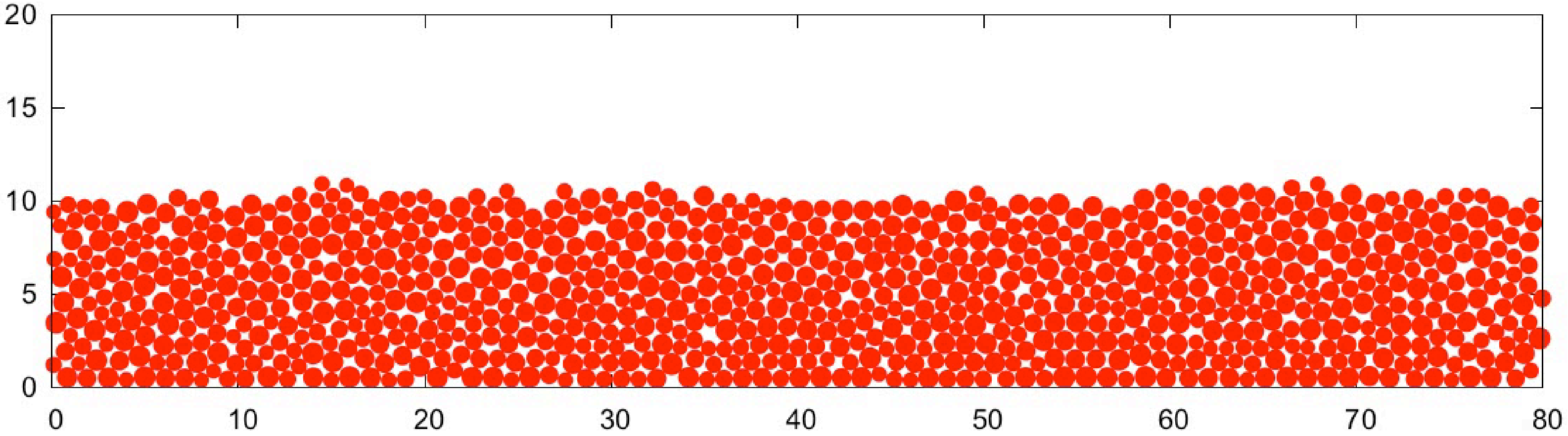
$$\frac{\rho_p}{\rho_f} = 2$$



$$\phi_b$$

Air

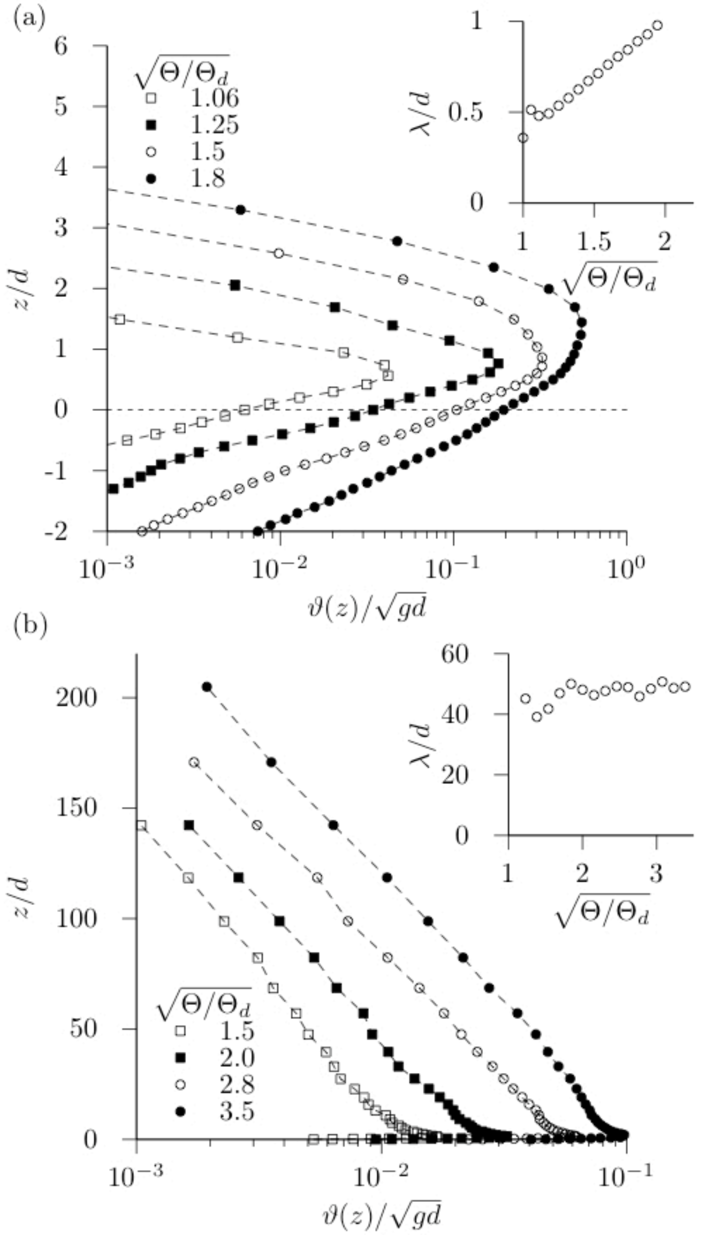
$$\frac{\rho_p}{\rho_f} = 2000$$

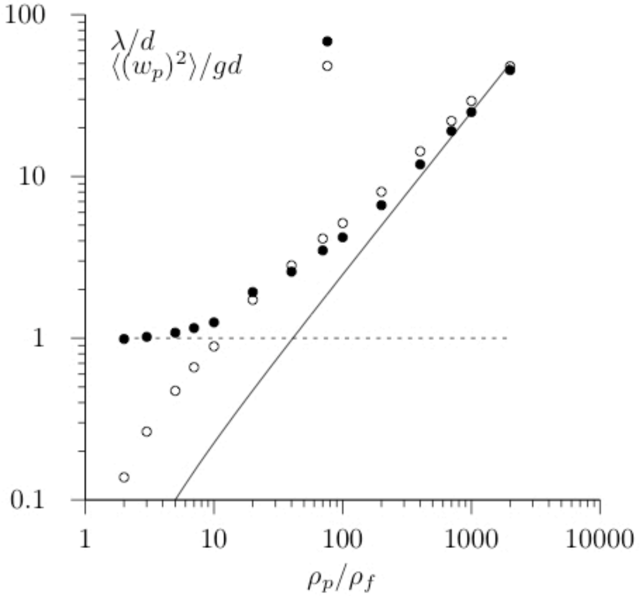


Transport layer

Water

Air

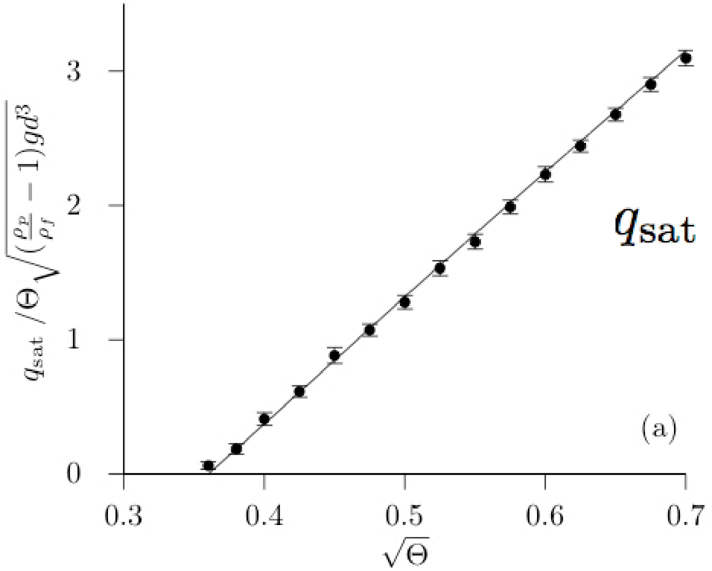




Saturated fluxes

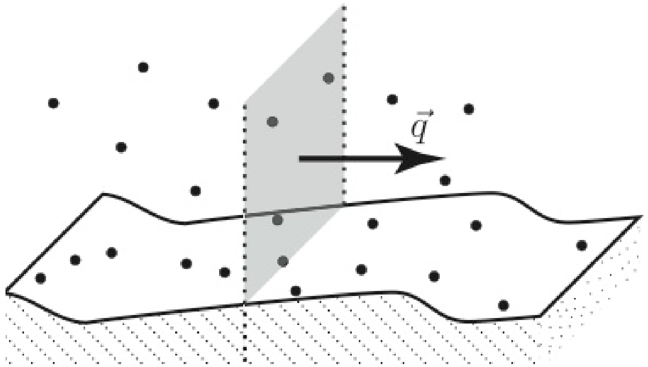
$$q_{\mathrm{sat}} = \frac{1}{A\phi_b}\frac{\pi}{6}d^3\sum_p u^p$$

Water

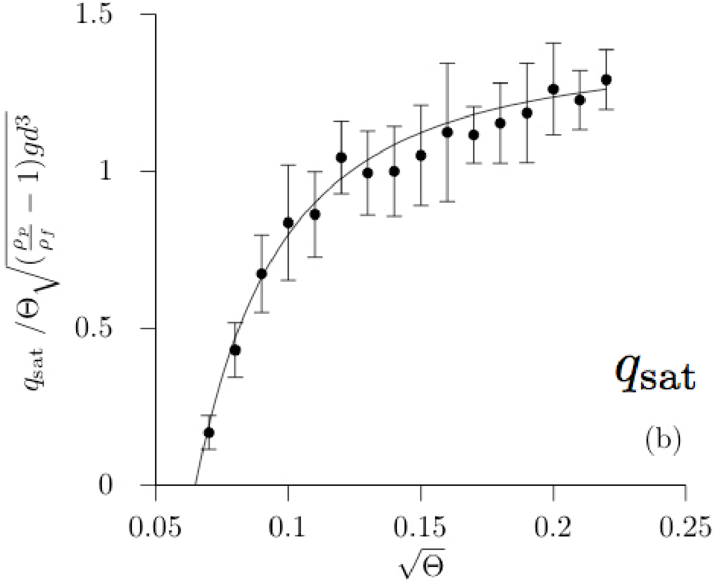


$$q_{\mathrm{sat}} = \frac{u_d d}{\phi_b \mu_d}(\Theta - \Theta_d)\left(\sqrt{\frac{\Theta}{\Theta_d}} - \sqrt{\frac{\mu_d}{\mu_s}}\right)$$

Meyer-Peter & Müller (1948)



Air

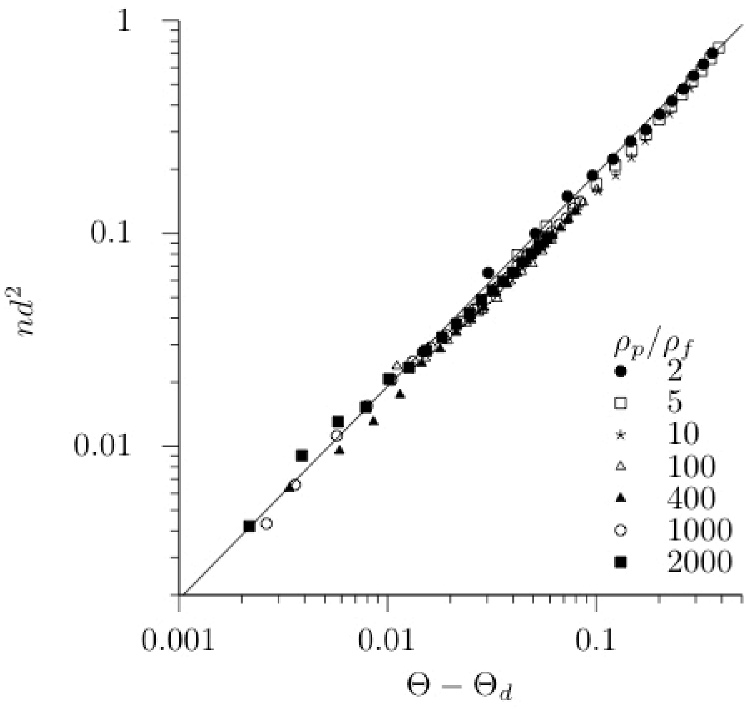


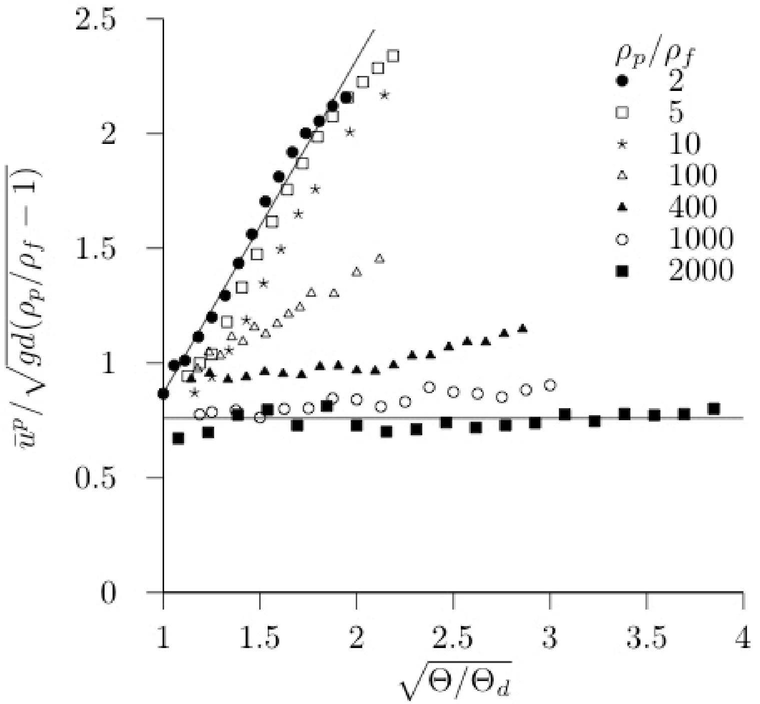
$$q_{\mathrm{sat}} \propto (1 - \rho_f/\rho_p)u_d d\,(\Theta - \Theta_d)$$

Rasmussen et al. (1991, 1996, 1999)

Flux decomposition

$$q_{\mathrm{sat}} = \frac{1}{\phi_b}\frac{\pi}{6}d^3 n\bar{u}^p$$





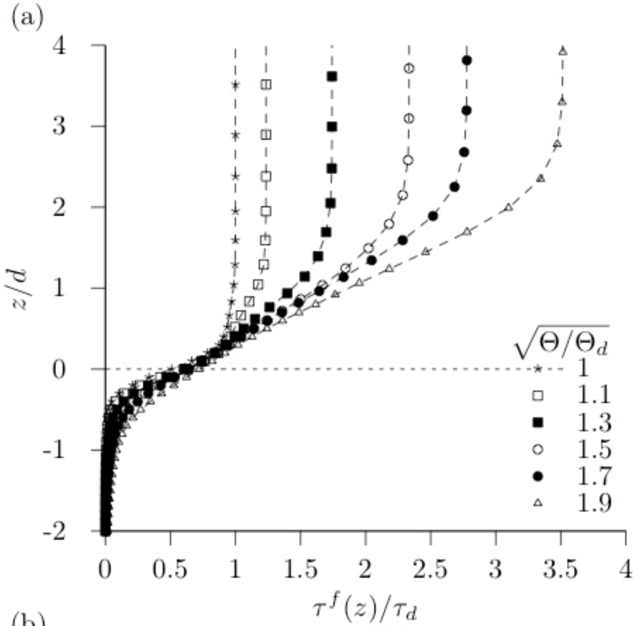
$$n = \frac{\left(\sum_p u_p\right)^2}{A\sum_p u_p^2}$$

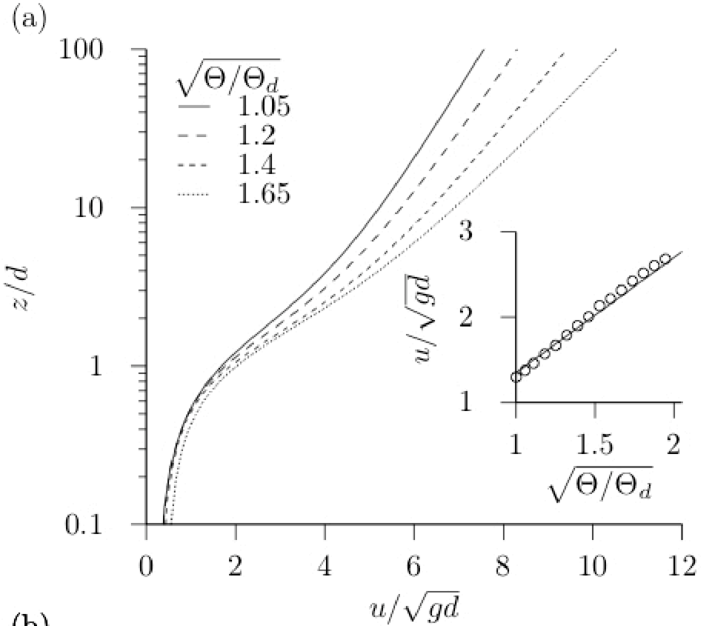
$$\bar{u}^p = \frac{\sum_p u_p^2}{\sum_p u_p}$$

Lajeunesse et al. (2010)

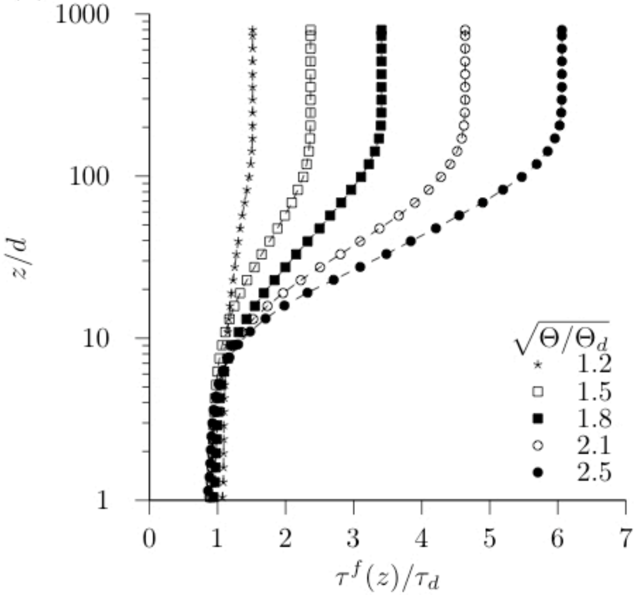
Grains' feedback on the flow

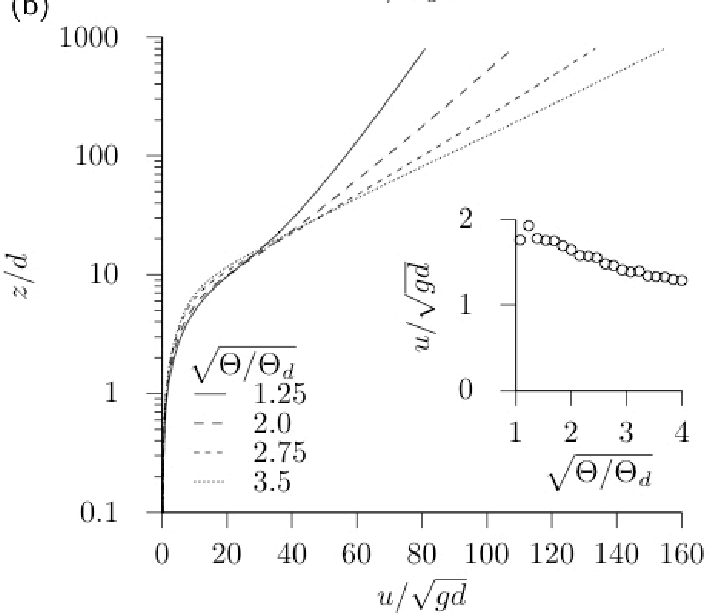
Water





Air





Interpretation

$$q_{\mathrm{sat}} = \frac{1}{\phi_b}\frac{\pi}{6}d^3 n \bar{u}^p$$

Water

- Bagnold's approach (1956)

$$\tau^p = n f_d \qquad \tau^p = \rho_f u_*^2 - \tau_d$$

$$f_d \simeq C_d \rho_f (u - \bar{u}^p) d^2 \simeq \mu(\rho_p - \rho_f) g d^3$$

$$n \simeq (\Theta - \Theta_d)/d^2$$

$$\bar{u}^p \simeq u_d \left(\sqrt{\Theta/\Theta_d} - 1\right)$$

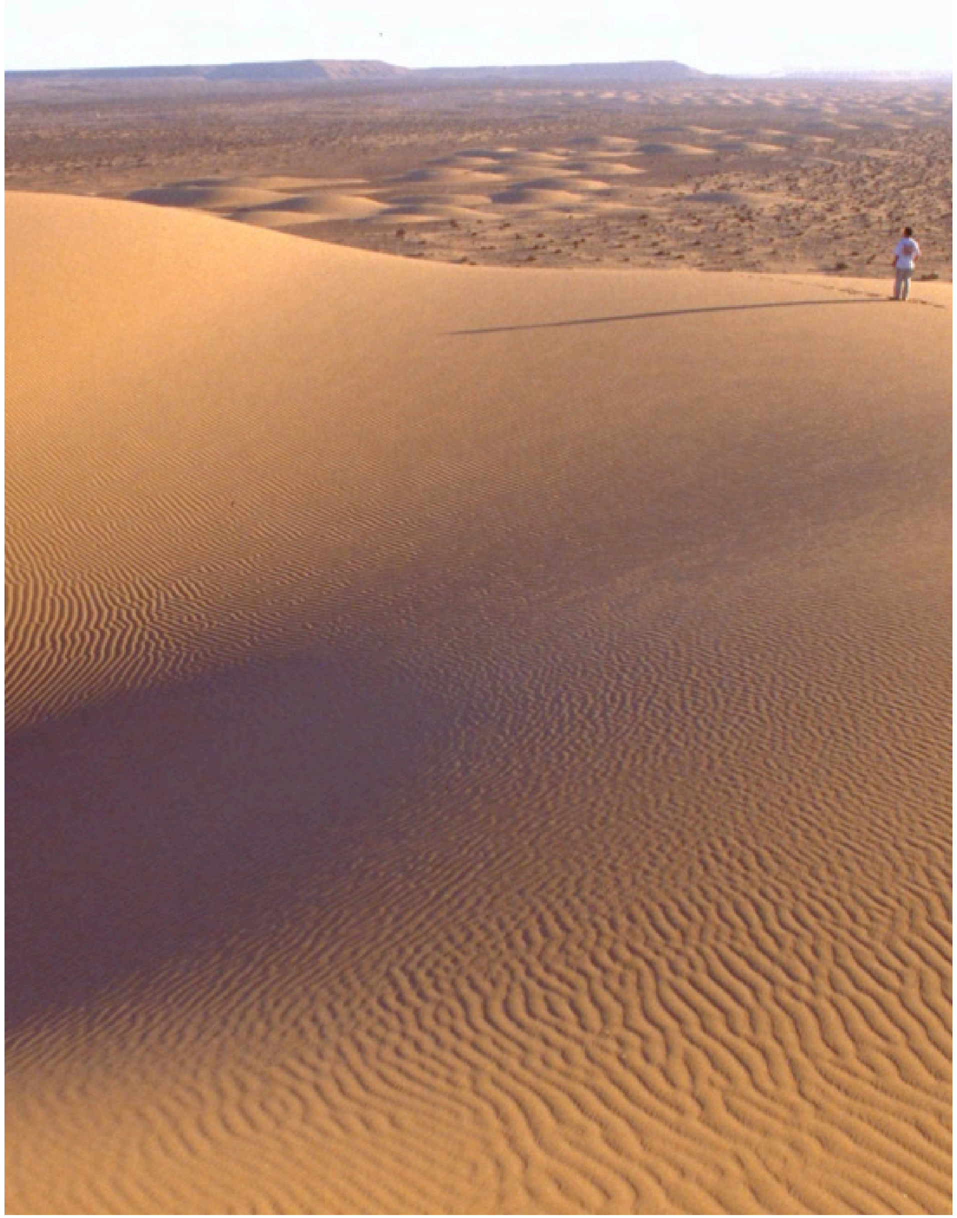
Air

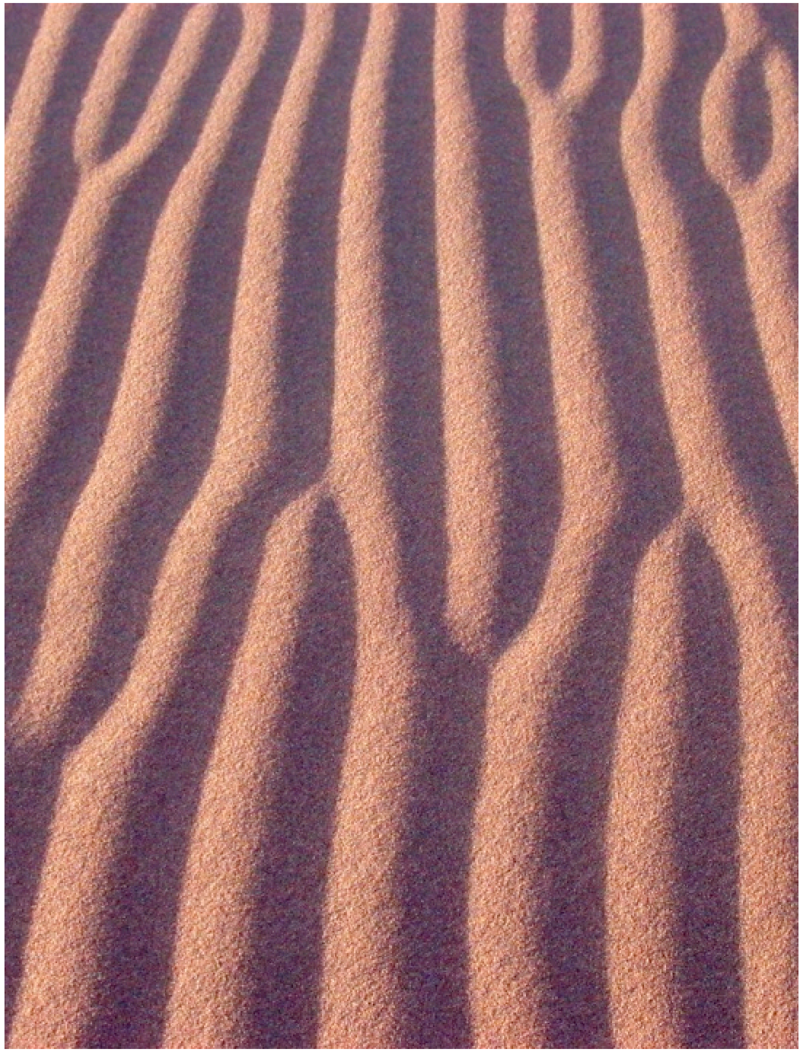
- Approach of Owen (1964) and Ungar & Haff (1987)

$$\bar{u}^p \simeq u_d$$

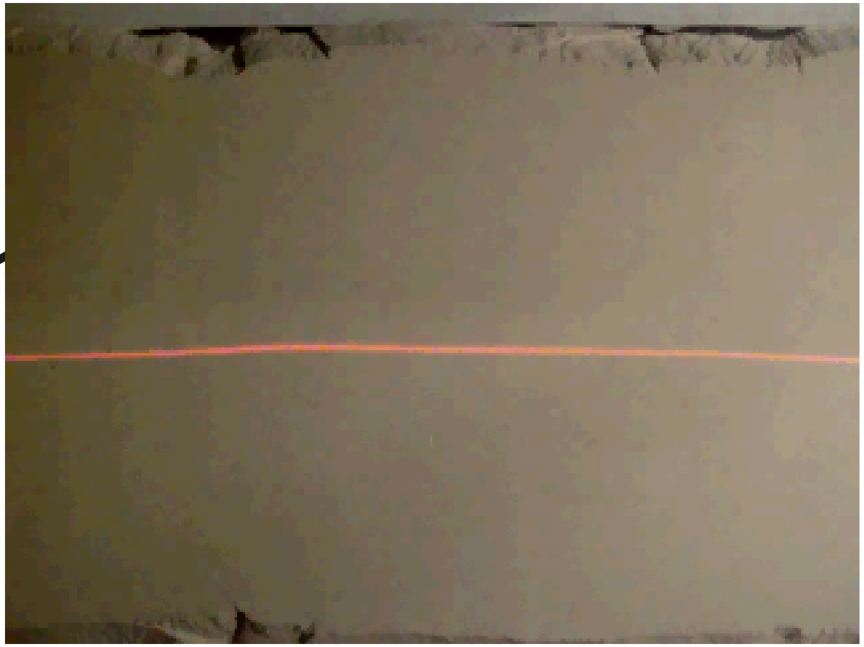
replacement capacity = 1

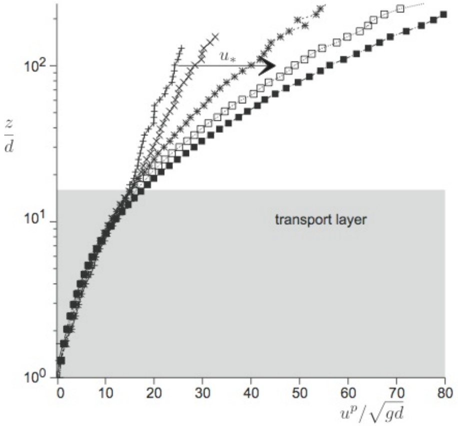
Aeolian ripples





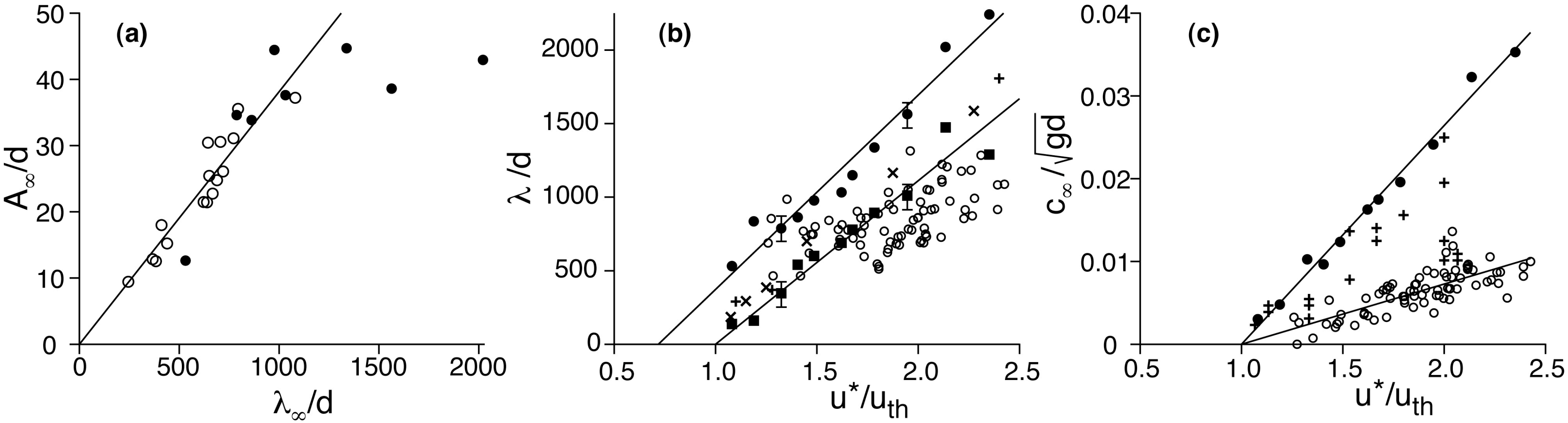
Ripple size and speed





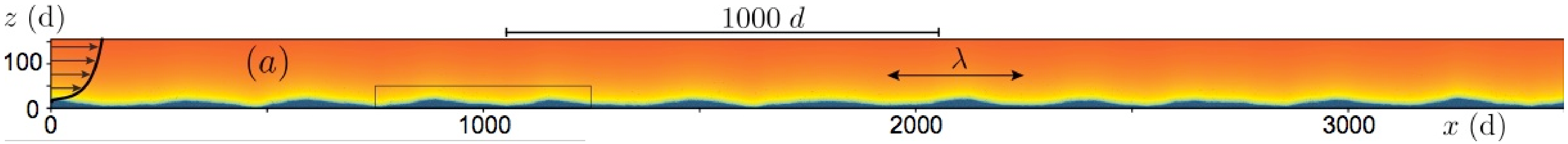
Anderson 1987
Csahok et al. 2000
Yizhaq 2004

Screening mechanism



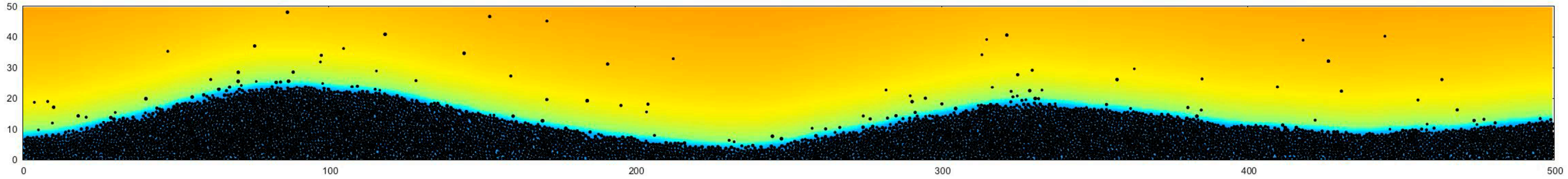
Andreotti et al. 2006

Numerical ripples



$\rho_p/\rho_f = 500$ $\mathcal{R} = 22$ $\Theta/\Theta_{\text{th}} = 1\text{–}25$

$3400d \times 1d \times 1000d$
45000 grains



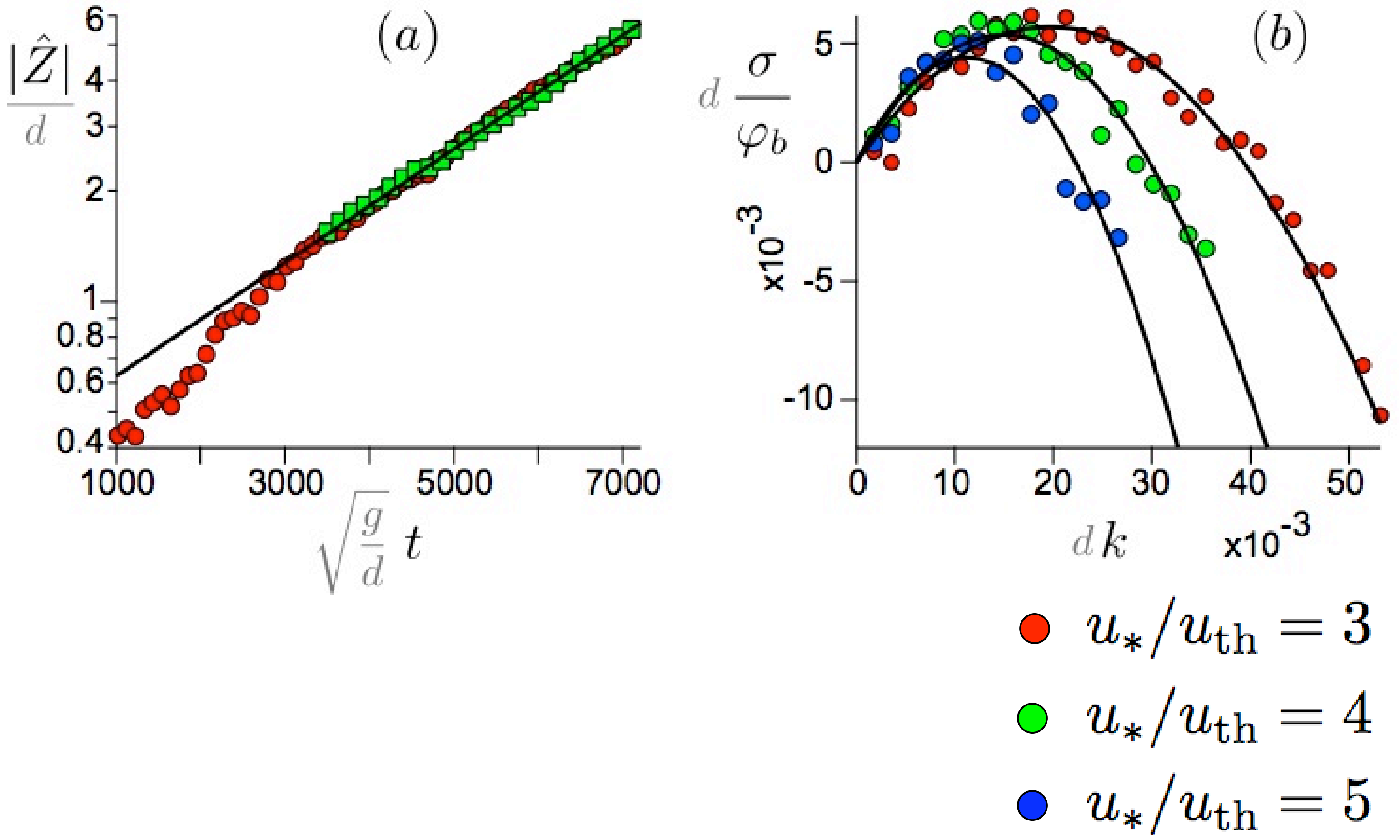
Bed elevation profile $$Z(x) = \frac{1}{\phi_b}\int_0^\infty \left[\phi(x,z) - \frac{1}{A}\int_0^A \phi(x,z)\,\mathrm{d}x\right]\mathrm{d}z$$

Dispersion relation

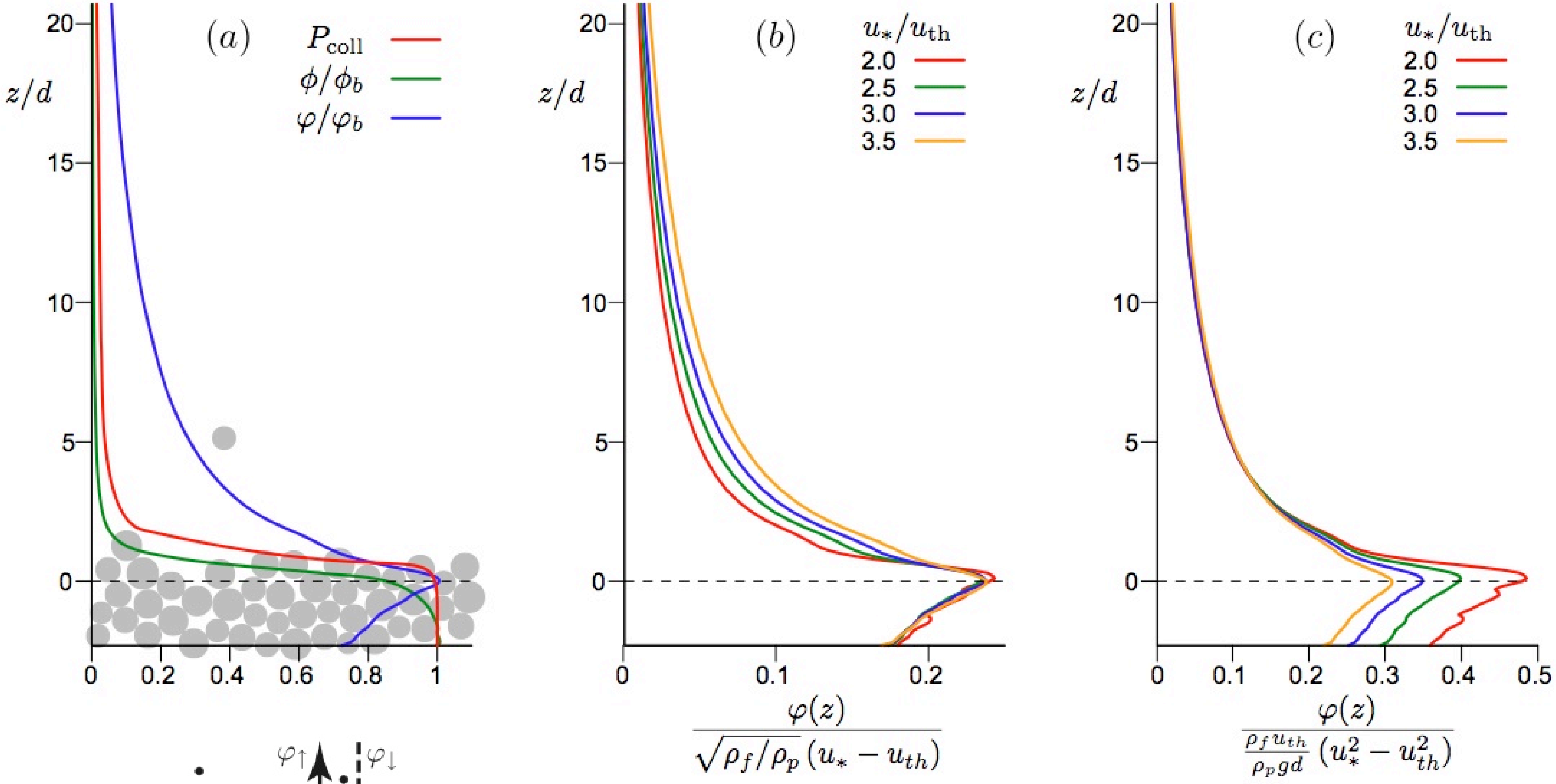
From a flat bed

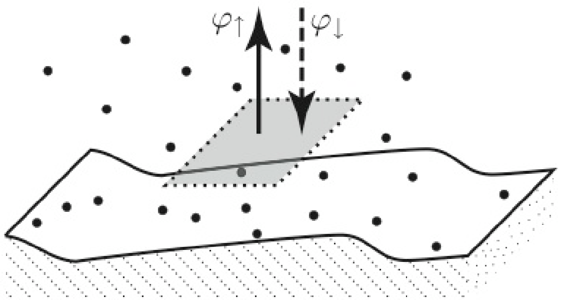
From a modulated bed e^{ikx}

$$\rightarrow |\hat{Z}|(t) = |\hat{Z}|(0)\, e^{\sigma t}$$



Flat bed: upper and basal layers

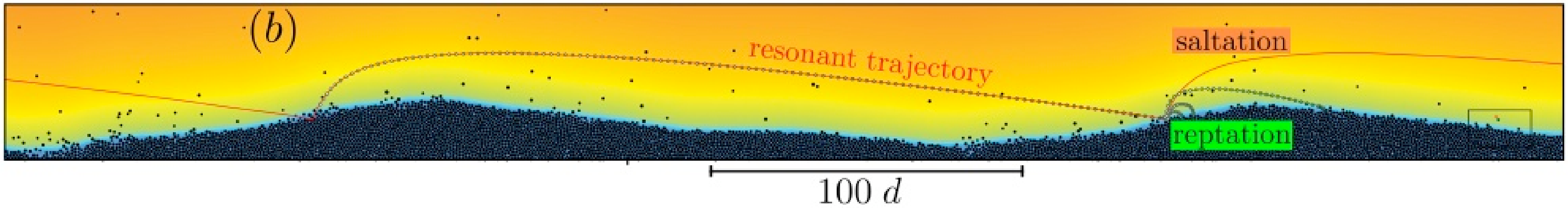




$$\varphi = \frac{1}{\phi_b A \mathrm{d}z}\frac{\pi}{6}d^3 \sum_{p\in\{z,z+\mathrm{d}z\}} u_\uparrow^p$$

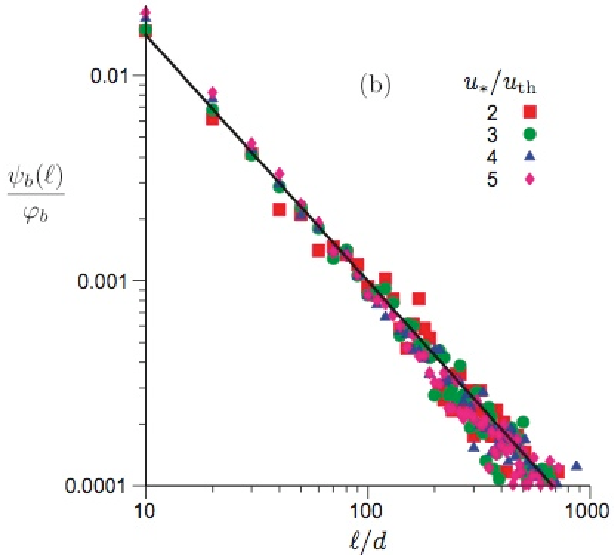
collisional (gaseous) layer $\tau^p \sim \rho_p u_b^2$ $\varphi_b \sim u_b$

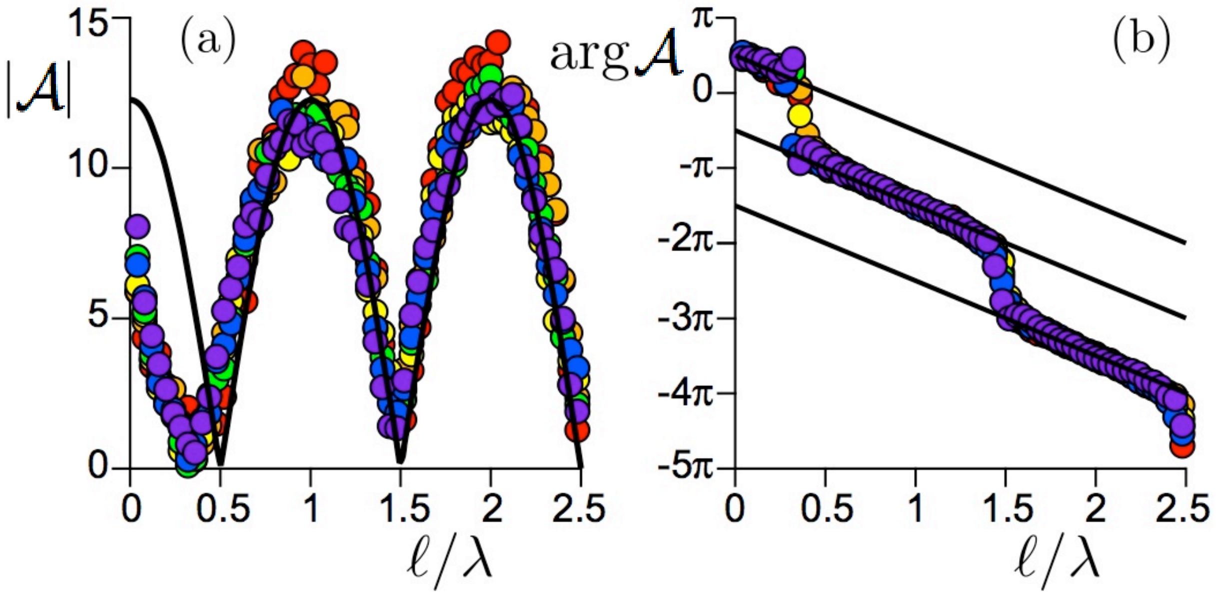
Resonant trajectories



Hop length distribution

$$\psi(\ell)$$





$$\hat{\psi}(\ell) = \mathcal{A}\,\psi_b(\ell) ik\hat{Z}$$

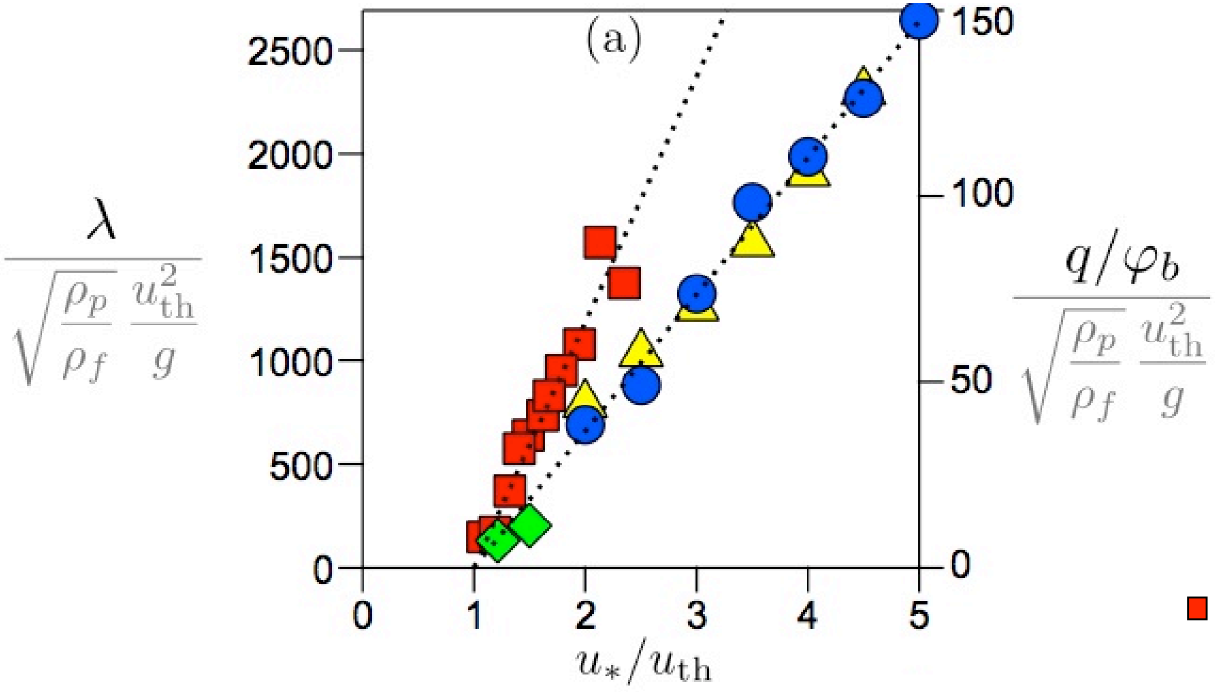
Ripple instability (1)

Gravity-induced diffusion
of transport (saltons)

Growth rate

$$\sigma = a\varphi_b k - b q_s k^2$$

Small-trajectory grains (reptons)
ejected by resonant grains

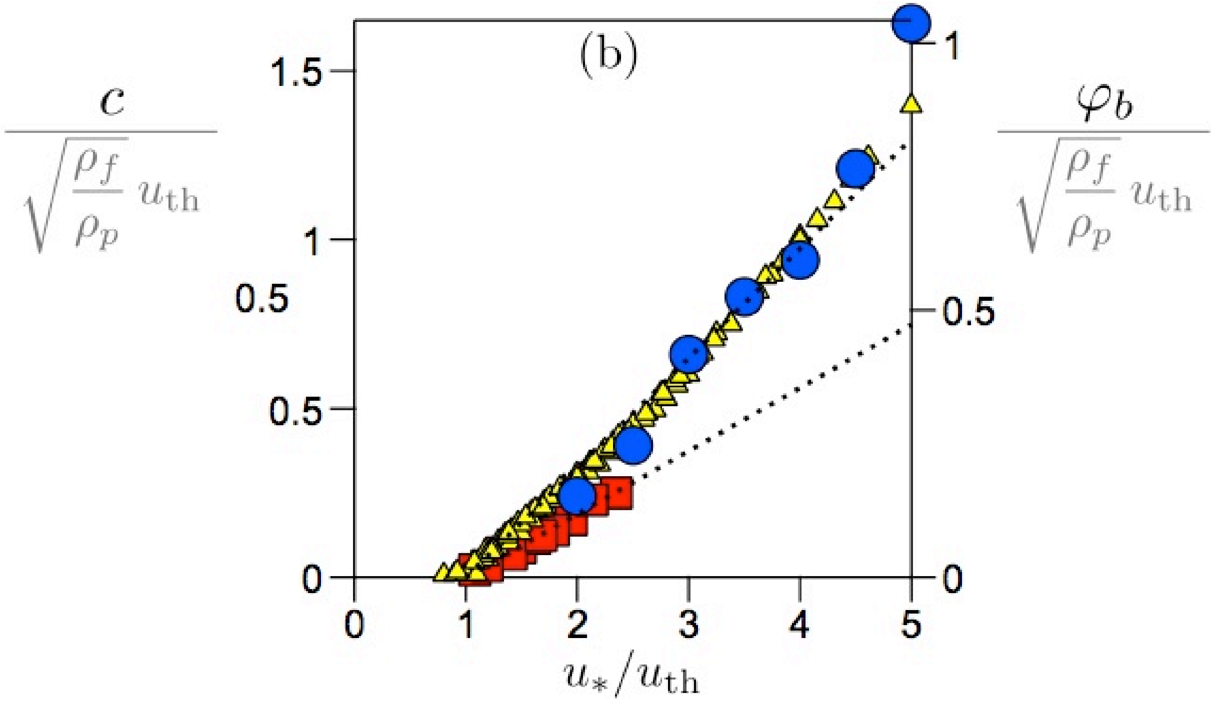


$$\lambda \sim q_s/\varphi_b$$

🟥 Experimental data from
Andreotti et al. (2006)

Ripple instability (2)

Propagation speed $$c \sim \varphi_b$$



Summary & perspectives

- Mechanisms and scaling laws for sediment transport
- Mechanisms and scaling laws for aeolian ripples
- Non-homogeneous situations
  transients → saturation length L_{sat}
- Transition to from bed load to suspended load

Supplementary slides

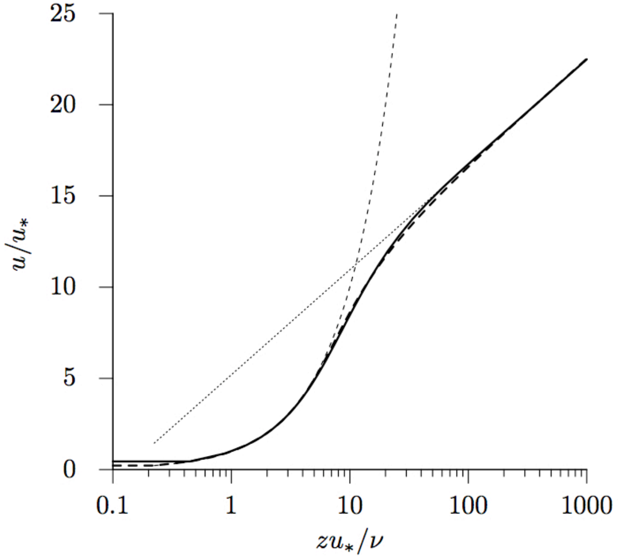
Mixing length

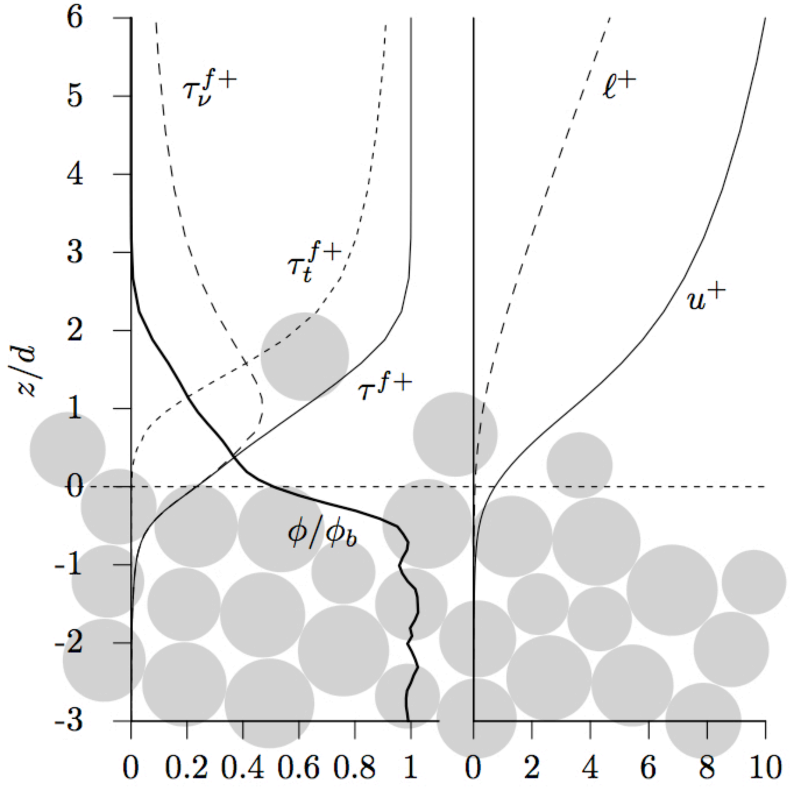
$$\tau^f = \rho_f(\nu + \ell^2|\partial_z u|)\partial_z u$$

$$\ell = \kappa z\left[1 - \exp\left(-\frac{1}{R_{\mathrm{vD}}}\frac{z u_*}{\nu}\right)\right]$$

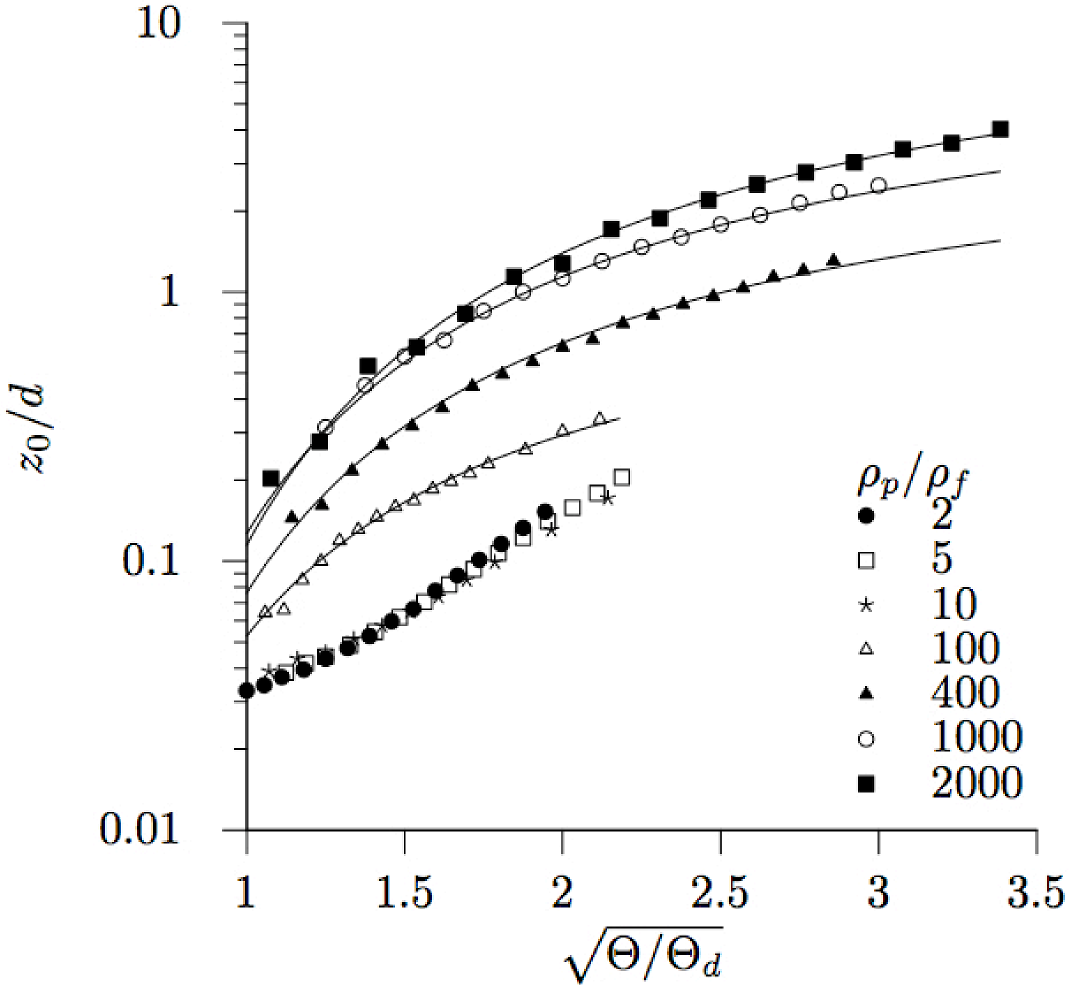
van Driest (1956)

$$\partial_z \ell = \kappa\left[1 - \exp\left(-\sqrt{\frac{1}{R_c}\left(\frac{u\ell}{\nu}\right)}\right)\right]$$





Hydrodynamical roughness



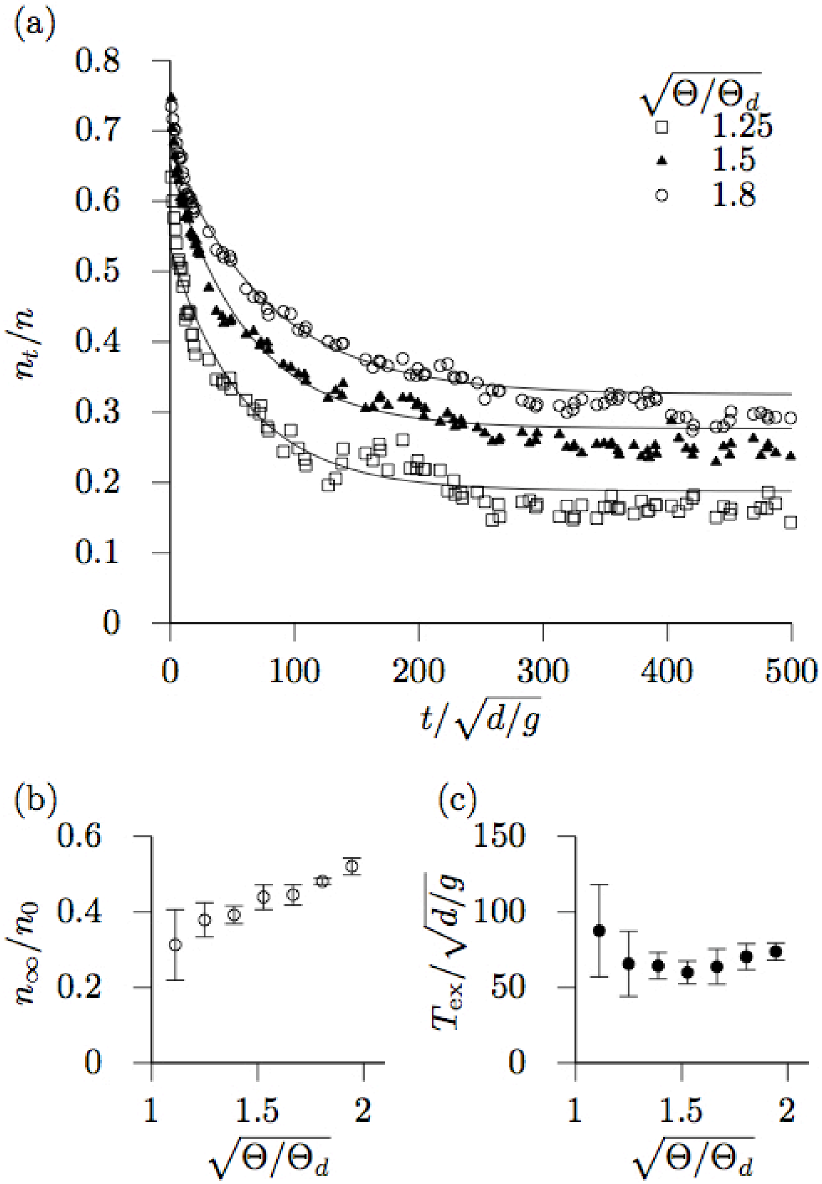
$$u = \frac{u_*}{\kappa} \ln \frac{z}{z_0}$$

Bagnold's focal point

$$z_0 = H_f \exp(-\kappa U_f / u_*)$$

Exchange time

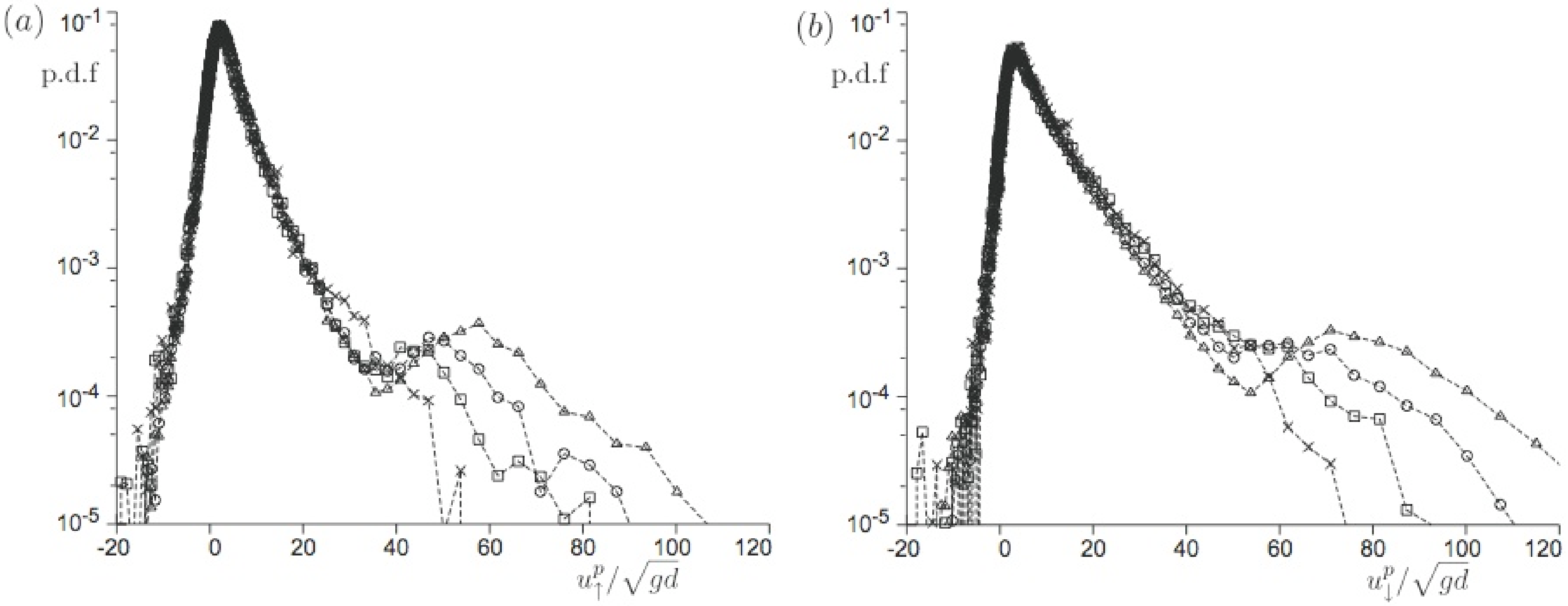
$$n_t = \frac{\left(\sum_{p\in\varepsilon} u_p\right)^2}{A\sum_{p\in\varepsilon} u_p^2}$$



$$n_t(t) = (n_0 - n_\infty)\exp(-t/T_{\mathrm{ex}}) + n_\infty$$

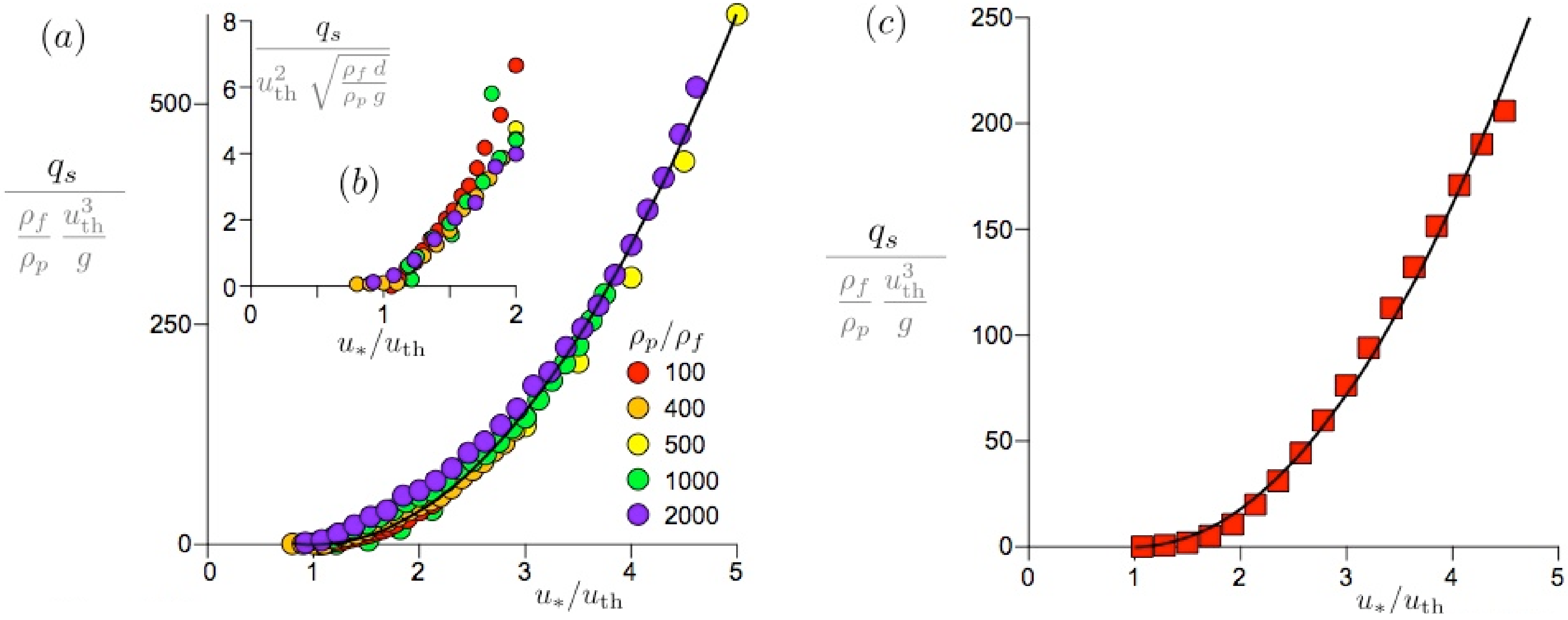
Particle velocity distribution

Aeolian transport



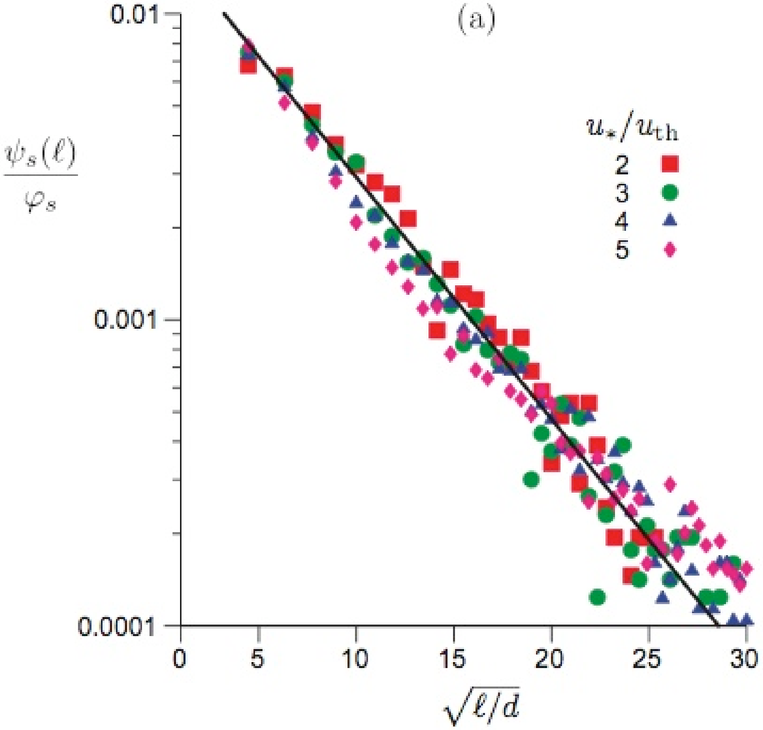
$$u/u_{\mathrm{th}} = 2.94\,(\times),\ 4.41\,(\square),\ 5.88\,(\bigcirc),\ 7.35\,(\triangle)$$

Sediment flux scaling



- 🟥 Experimental data from Iversen & Rasmussen (1999)

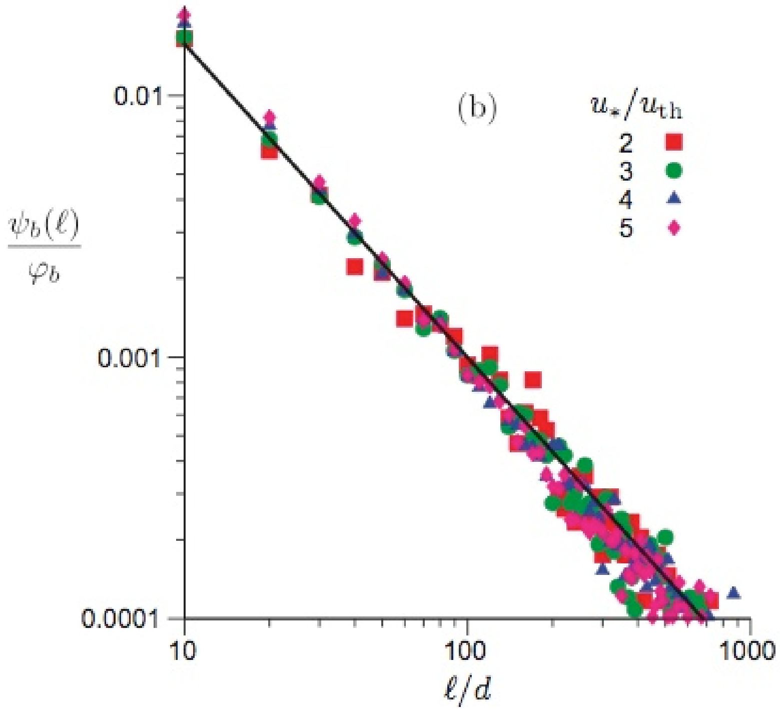
Hop length distribution



Upper transport layer

$$\psi_s(\ell) = \varphi_s \ell_s^{-1} \exp(-\sqrt{\ell/\ell_s})$$

$$\ell_s \simeq 30d$$

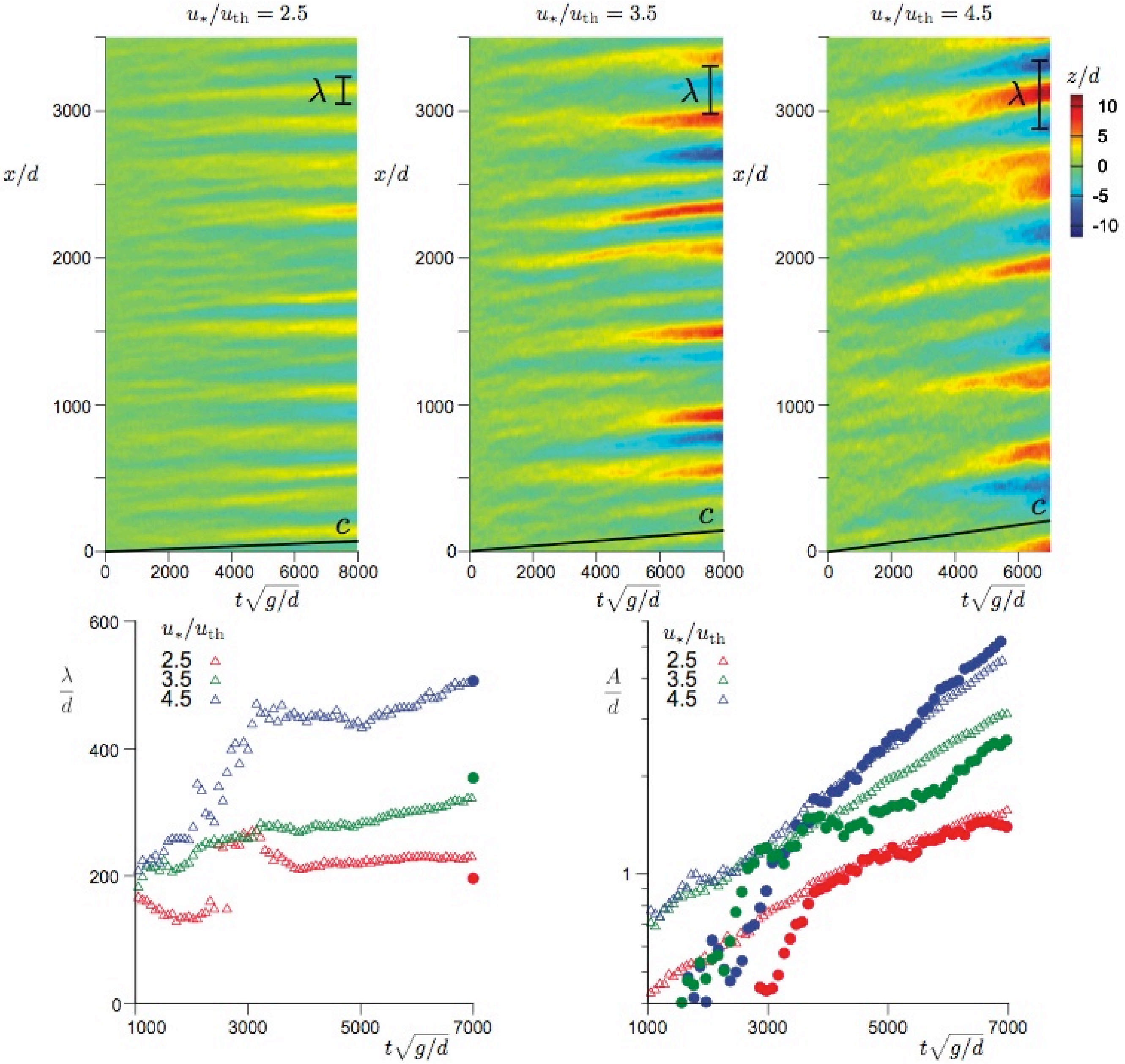


Basal collisional layer

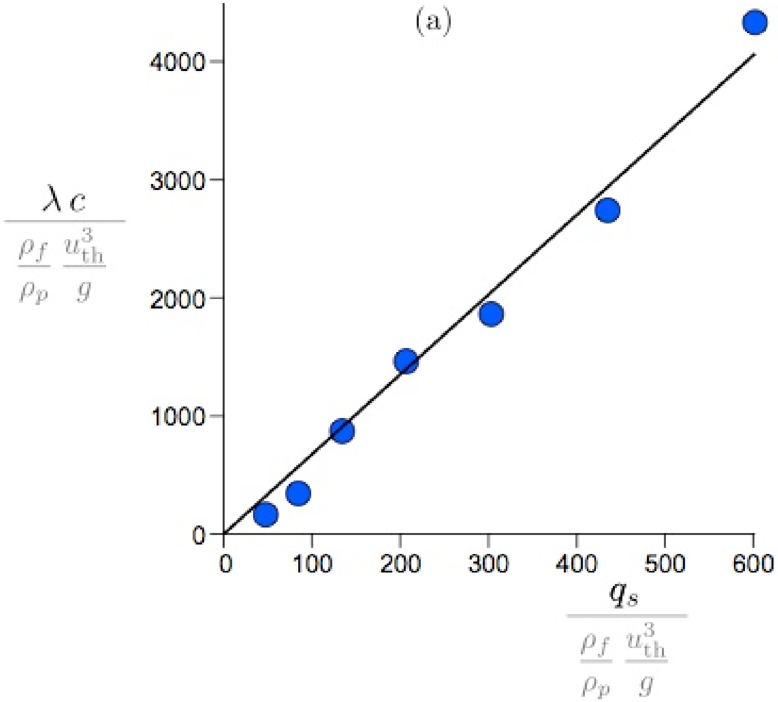
$$\psi_b(\ell) = \frac{\varphi_b}{d}\left(\frac{d}{\ell}\right)^{\alpha}$$

$$\alpha \simeq 1.2$$

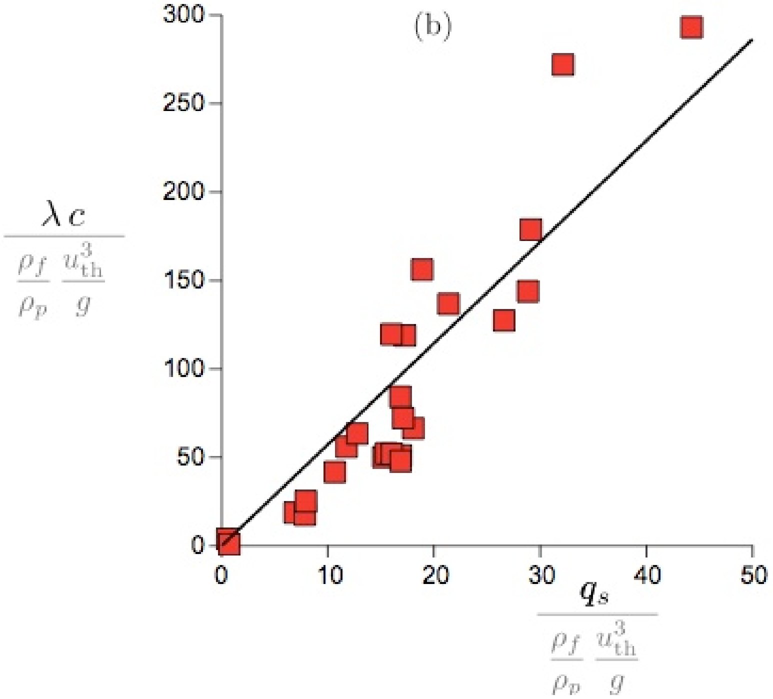
Ripple time evolution



A proxy for the sediment flux



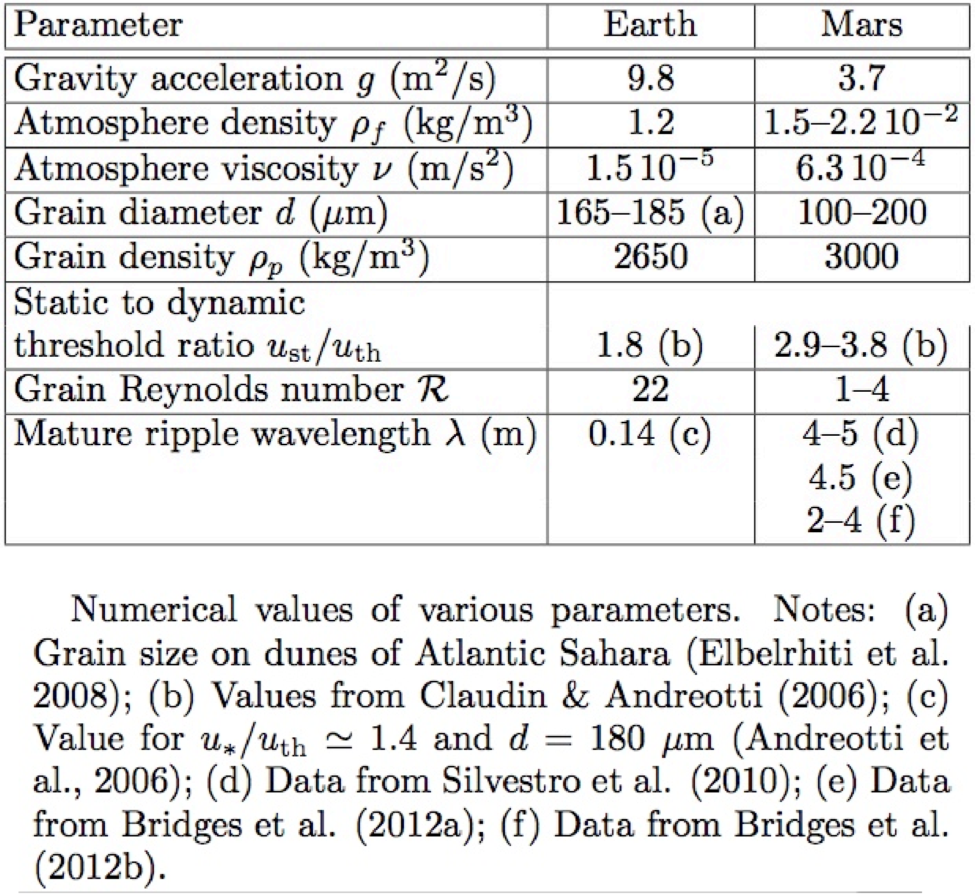
Wind tunnel measurements



Field measurements

data from Andreotti et al. 2006

Ripples on Mars



Parameter	Earth	Mars
Gravity acceleration g (m^2/s)	9.8	3.7
Atmosphere density ρ_f ($\mathrm{kg/m}^3$)	1.2	1.5–$2.2\,10^{-2}$
Atmosphere viscosity ν ($\mathrm{m/s}^2$)	$1.5\,10^{-5}$	$6.3\,10^{-4}$
Grain diameter d (μm)	165–185 (a)	100–200
Grain density ρ_p ($\mathrm{kg/m}^3$)	2650	3000
Static to dynamic threshold ratio $u_{\mathrm{st}}/u_{\mathrm{th}}$	1.8 (b)	2.9–3.8 (b)
Grain Reynolds number $\mathcal{R}$	22	1–4
Mature ripple wavelength λ (m)	0.14 (c)	4–5 (d) 4.5 (e) 2–4 (f)

Numerical values of various parameters. Notes: (a) Grain size on dunes of Atlantic Sahara (Elbelrhiti et al. 2008); (b) Values from Claudin & Andreotti (2006); (c) Value for $u_*/u_{\mathrm{th}} \simeq 1.4$ and $d = 180\ \mu$m (Andreotti et al., 2006); (d) Data from Silvestro et al. (2010); (e) Data from Bridges et al. (2012a); (f) Data from Bridges et al. (2012b).

Nili Patera, photo NASA

$$\lambda \sim q_s/\varphi_b$$