Snow Avalanche Dynamics — Change and Exchange

Fluid-Mediated Particle Transport
in Geophysical Flows @ KITP, UCSB
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Outline

1 The practical needs and the «Standard Model»

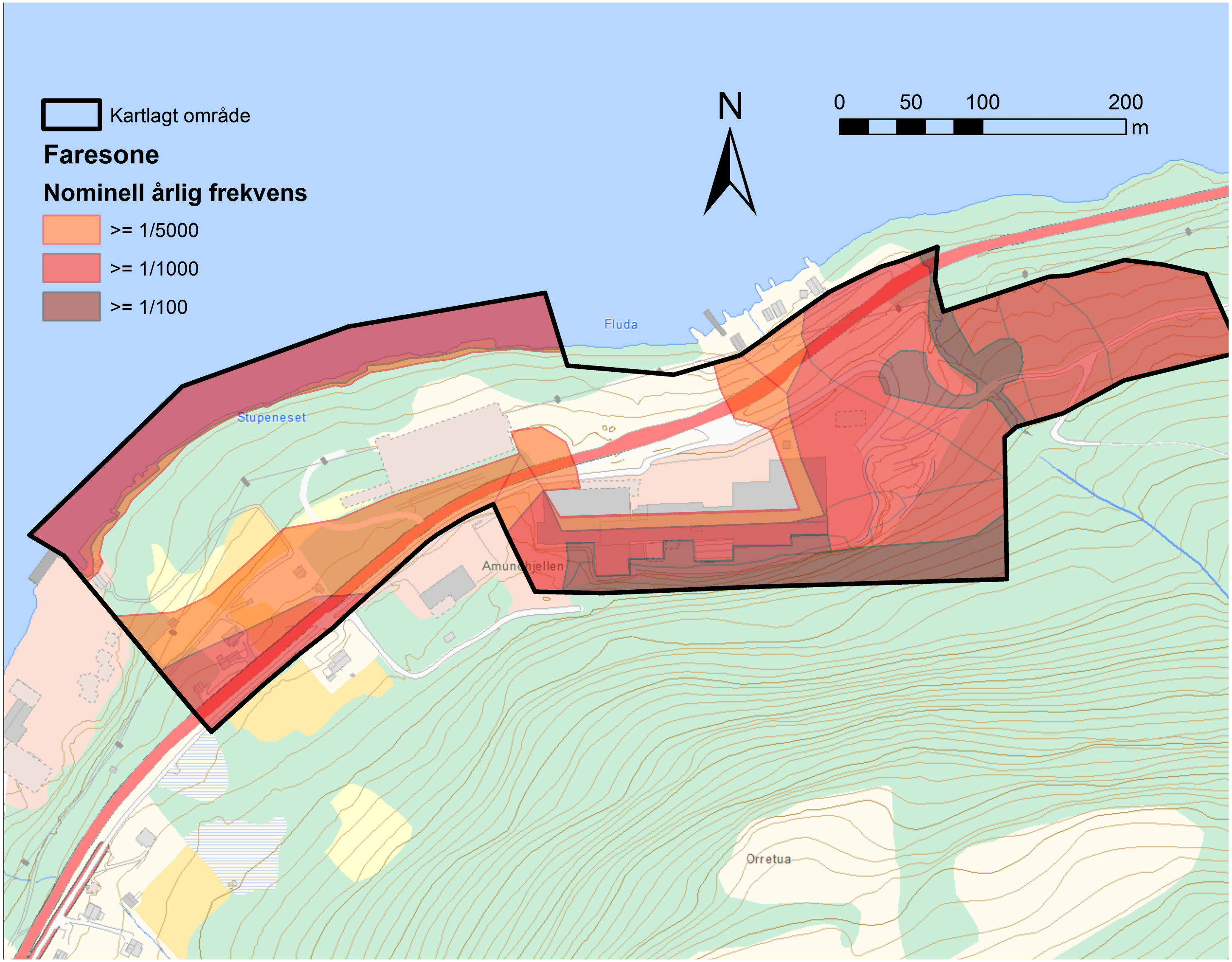
2 The challenges

3 A first look at flow-regime changes

4 Mass exchange between layers

5 Another look at flow-regime changes

6 The next frontiers



Example of a Norwegian GMF hazard map for return periods 100, 1000 and 5000 y

Deflection dam system protecting Flateyri, NW Iceland

Destroyed house in Davos, Switzerland, 1968. Photo SLF.

Brief summary of typical dynamical avalanche models (e.g., RAMMS):

- Mostly based on Voellmy friction ansatz (1955):

$$\begin{aligned} \partial_t h + \nabla\cdot(h\mathbf{u}) &= w_e \\ \partial_t(h\mathbf{u}) + \nabla\cdot(h\mathbf{u}\mathbf{u}) &= h\mathbf{g} - \frac{\mathbf{u}}{\|\mathbf{u}\|}\left(-\mu h g_3 + k\|\mathbf{u}\|^2\right) + \nabla\frac{g_3 h^2}{2} \end{aligned}$$

$\mathbf{u} = (u_1, u_2)$

$\mathbf{g} = (g_1, g_2)$

- $\mu = 0.1...0.4,\ \ k = 0.0025...0.025$ from calibration against observed run-out distances.
  Recommended parameters and initial conditions depend strongly on avalanche size and return period.

- Erosion model for w_e : none or empirical, e.g., $w_e = c\,\|\mathbf{u}\|$.

Some attempts to go beyond this:

- Norem–Irgens–Schieldrop (1987, 1989) model based on a complete rheological relation, but reduces to similar structure as Voellmy model in 1D depth-averaged (quasi-2D) formulation (except for longitudinal stresses).

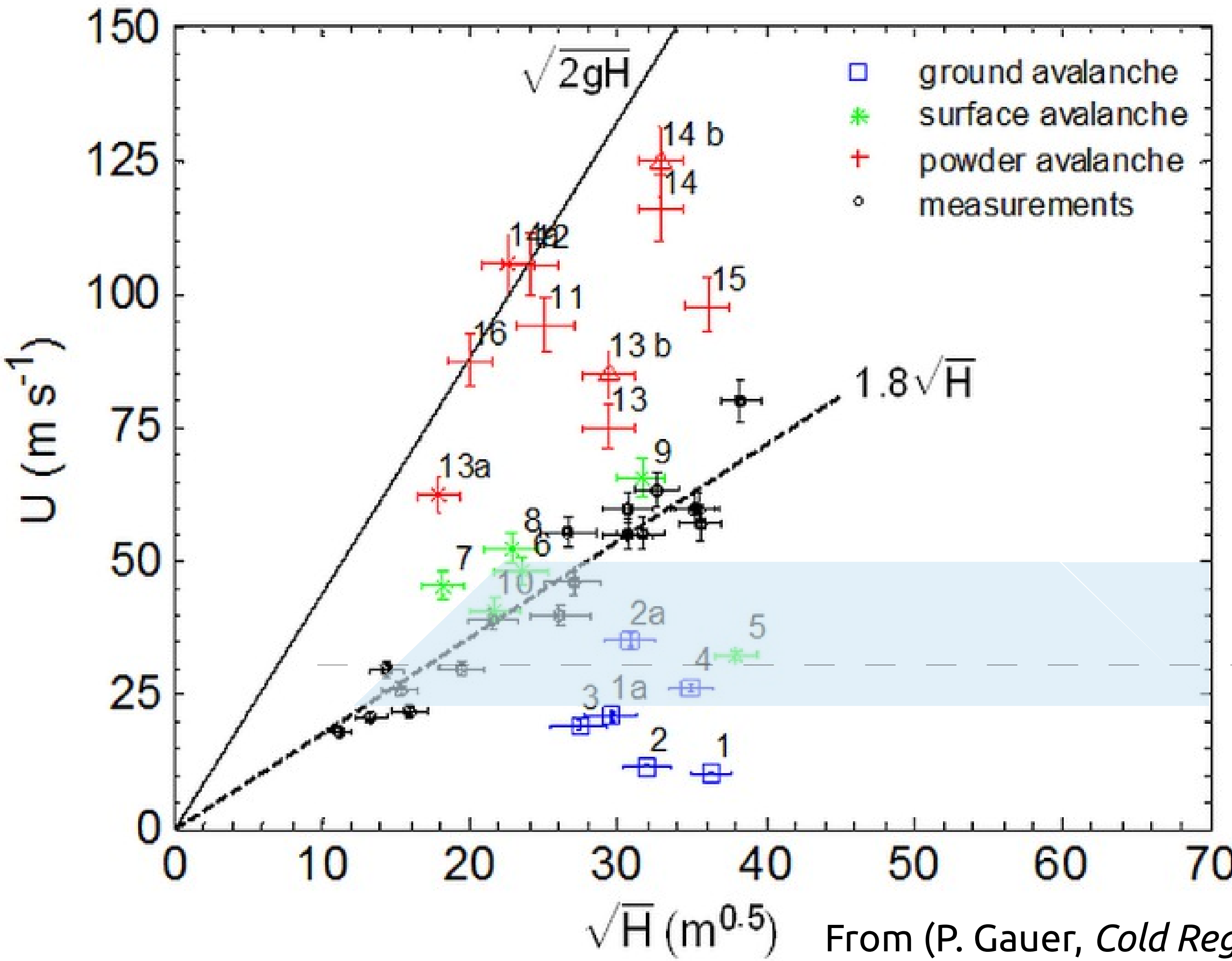
- Jop–Forterre–Pouliquen rheology (2006) boils down to friction law

$$\sigma_{\text{bed}} = \left(\mu_0 + \frac{\Delta\mu}{I_0/I+1}\right) g_3 h \quad \text{with} \quad I \equiv \frac{\dot{\gamma}\, d}{\sqrt{P/\rho_p}}$$

  and non-hydrostatic normal stresses.

Outline

Challenge #1: Maximum velocity grows with drop height

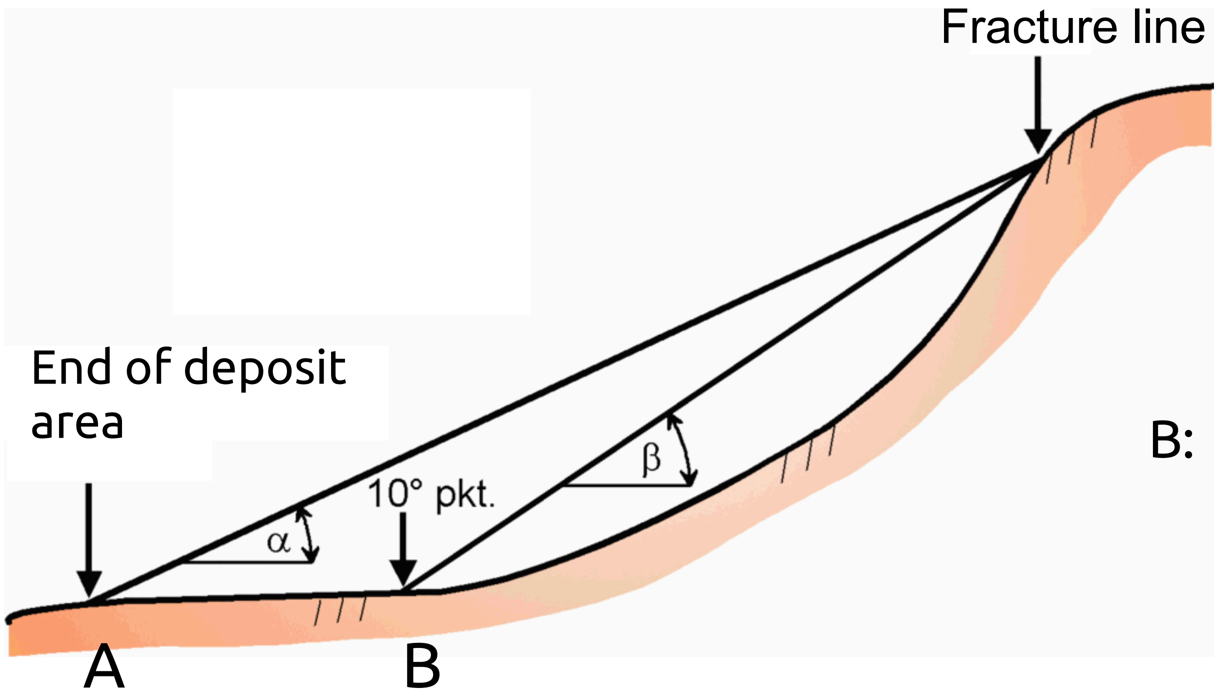


Typical limit velocity for Voellmy-type models with standard parameters

From (P. Gauer, *Cold Regions Sci. Technol.* , 2013)

Challenge #2: The statistical-topographical α-β model

(Lied and Bakkehøi, 1980)



B: Point where the slope angle falls below 10° (considered as beginning of run-out zone).

Norway: $\alpha = 0.96\,\beta - 1.4°$, SD = 2.3°, $R = 0.92$ (~ 200 avalanches)

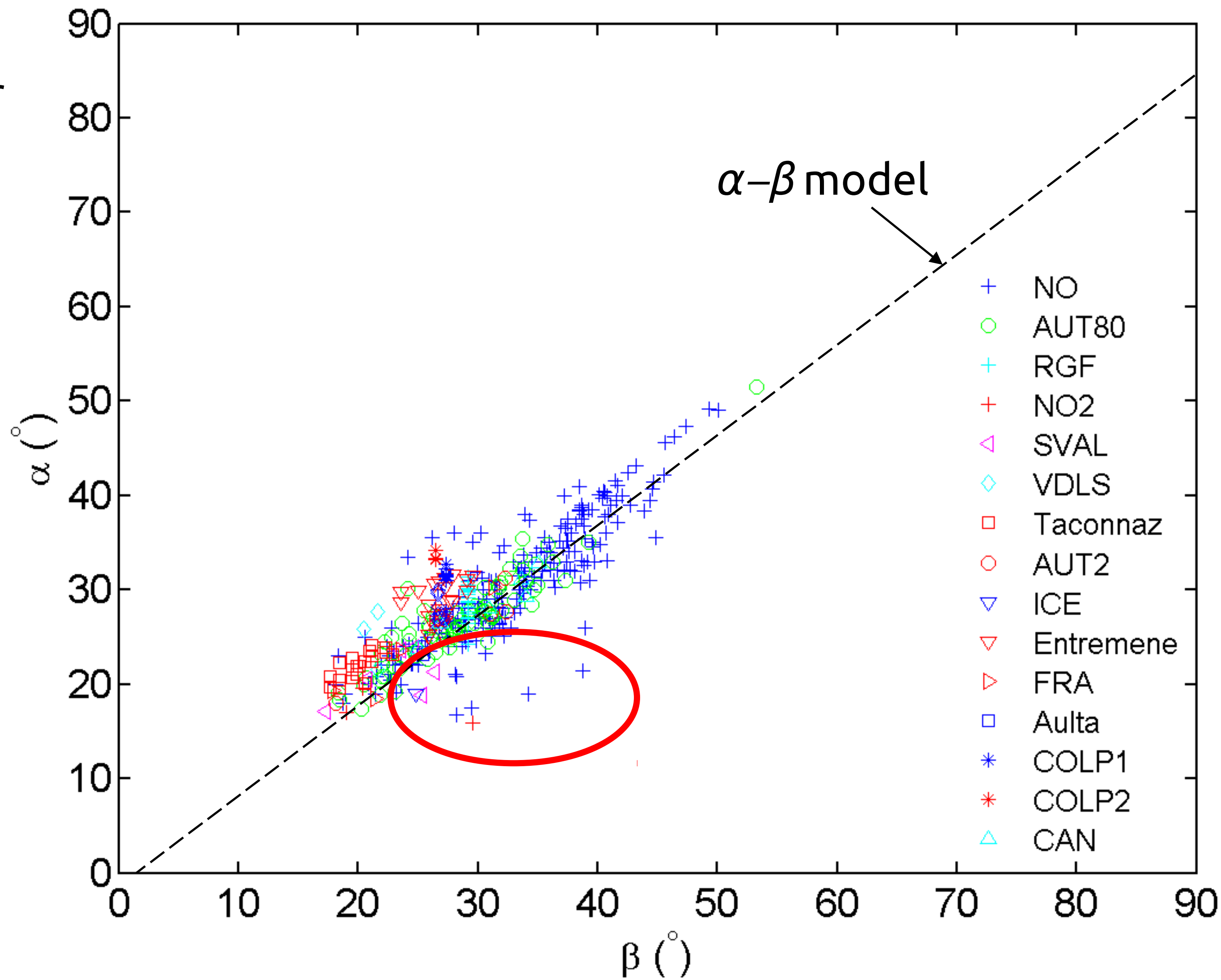
Austria: $\alpha = 0.95\,\beta - 0.8°$, SD = 1.5°, $R = 0.96$ (~ 70 avalanches)

(Databases contain supposedly «extreme» avalanches for each path.)

Challenge #3: Avalanches with very low values of run-out angle α

Some avalanches have *much* longer runout than expected from topographic-statistical model.

α = 14–16° in extreme cases!



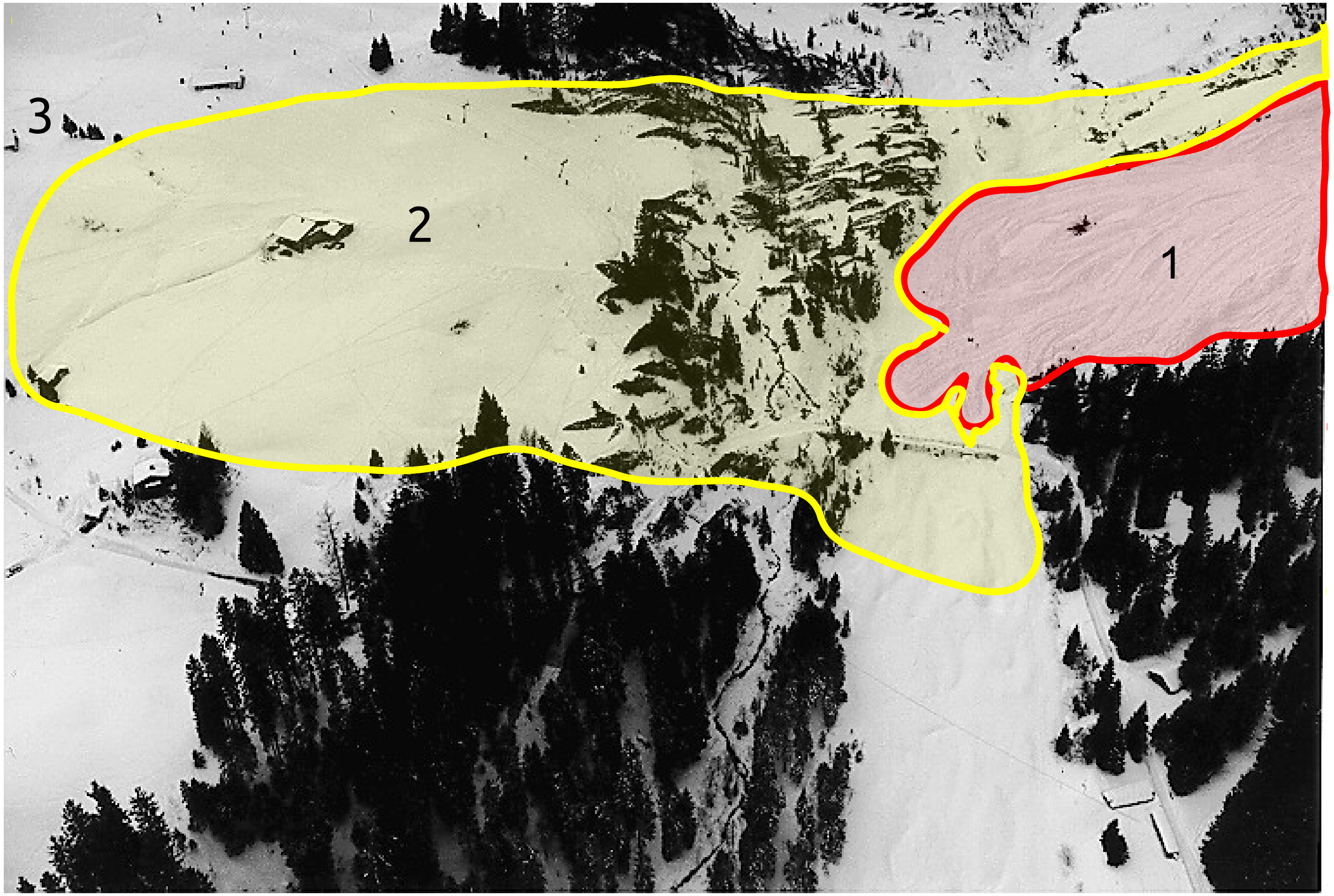
Plot courtesy
P. Gauer, NGI

Hint #1: Avalanches with 2 or 3 clearly differentiated deposit types

Deposit area of dense flow

Deposit of fluidized layer

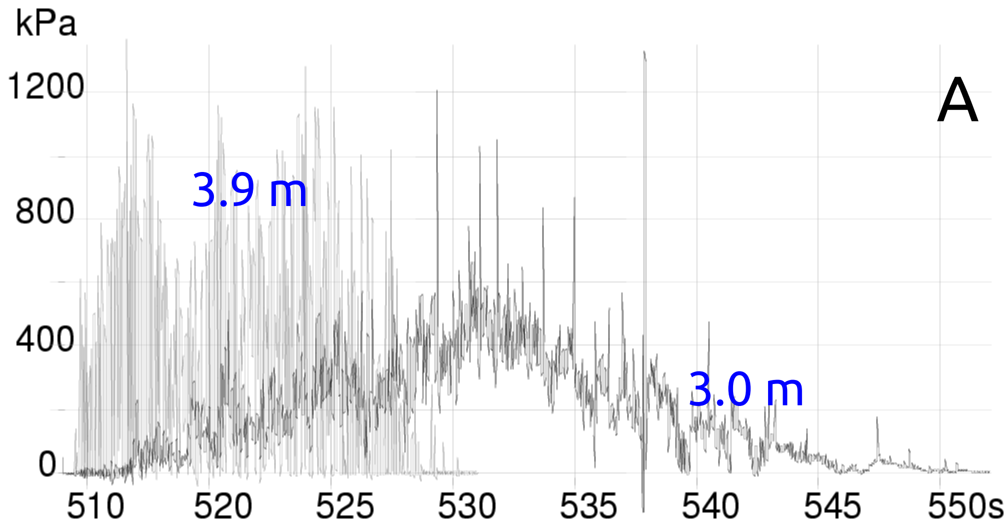
Powder-snow deposit extends ~ 500 m to left (uphill).



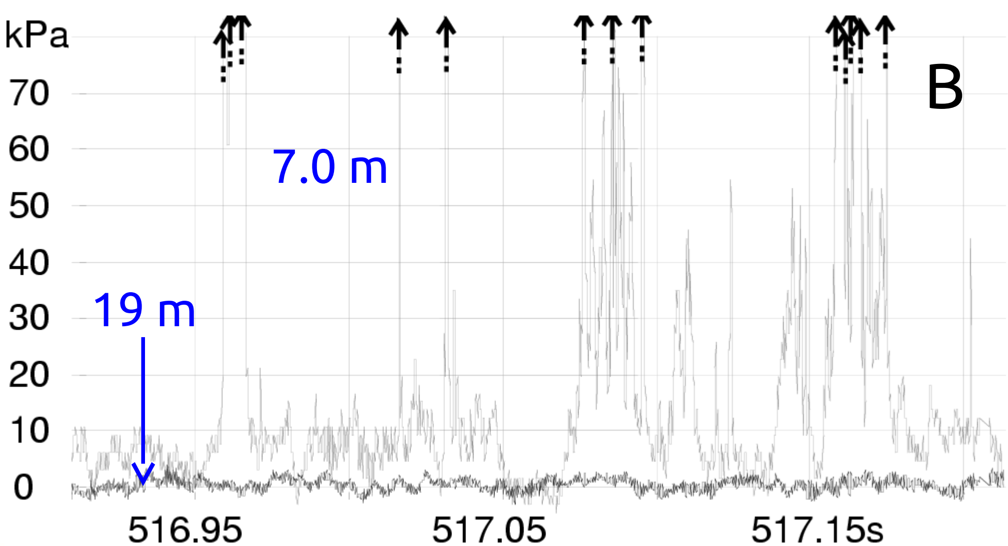
1995 avalanche at Albristhorn, Switzerland

Photo S. Keller

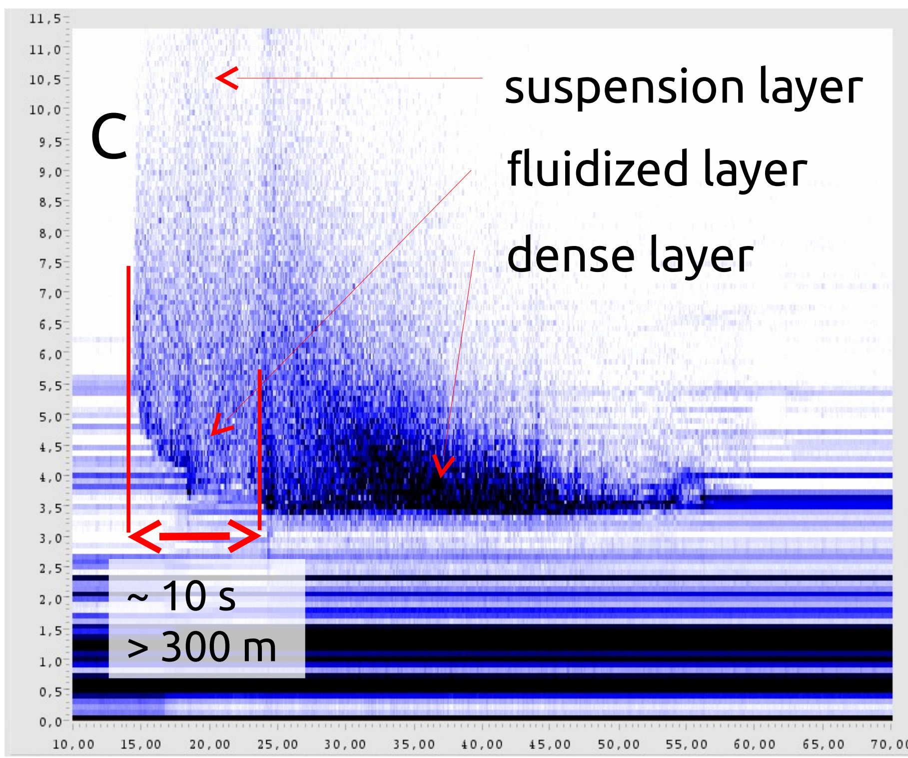
1999 measurements at Vallée de la Sionne
(from Schaer and Issler (2001). *Annals Glaciol.* **32**)



Load cell measurements

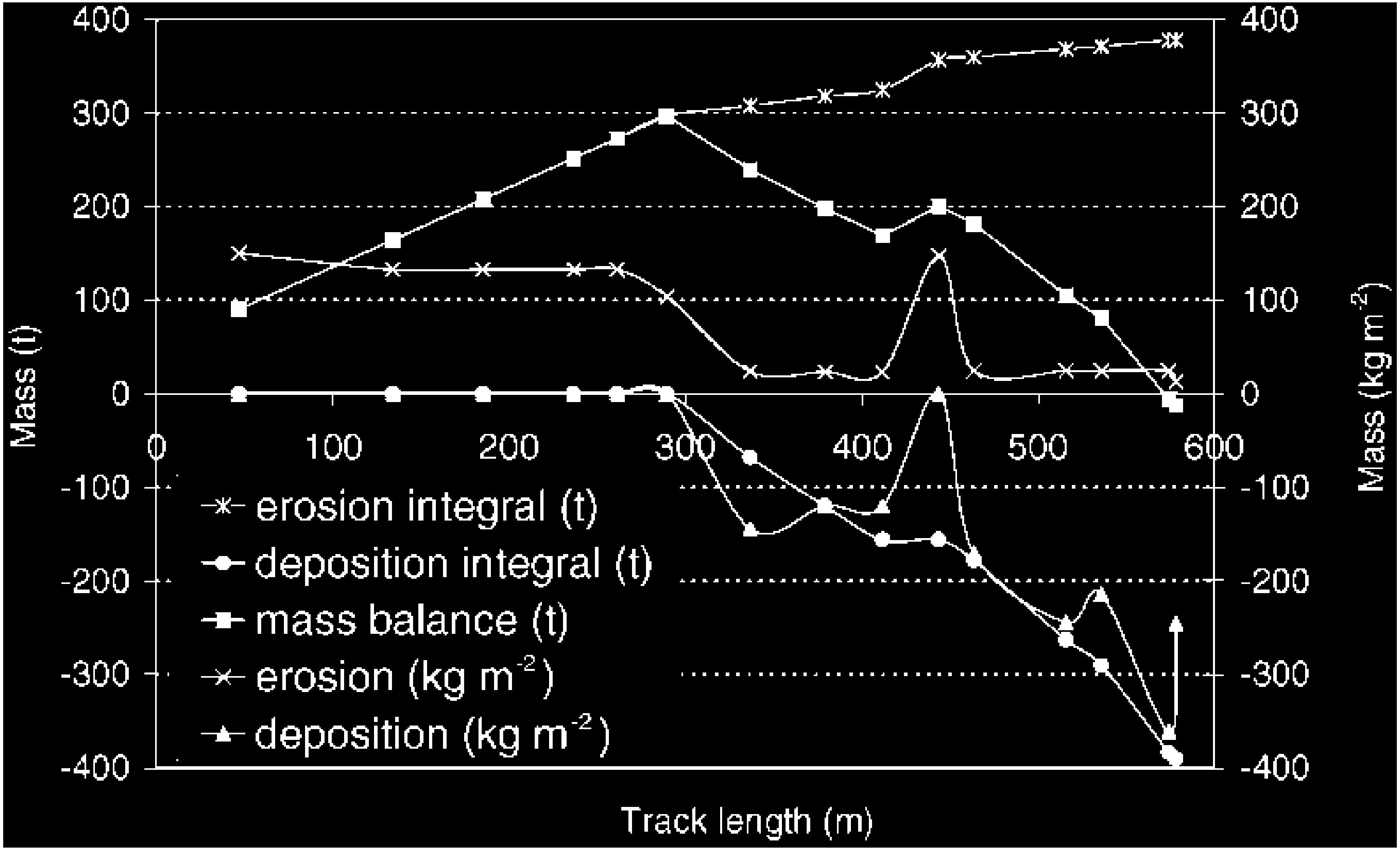


FMCW radar profile



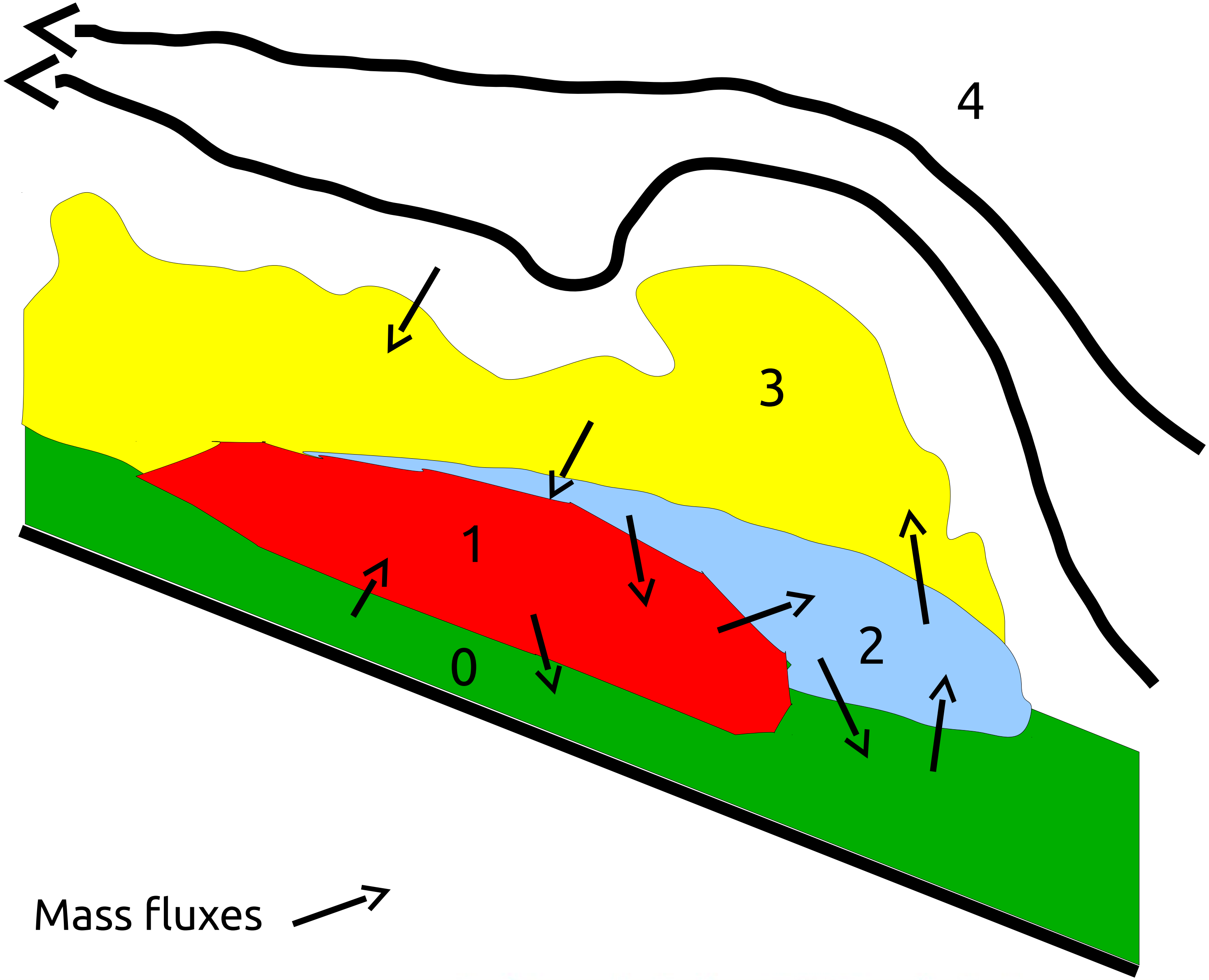
Hint #2: Erosion and deposition

Spatial mass balance in a snow avalanche (Monte Pizzac test site, Italy, 1998)



From Sovilla et al., *Annals Glaciol.* **32** (2001), 230–236.

Present view of avalanche structure



4 ambient air
3 suspension layer
2 fluidized layer
1 dense flow
0 snow cover

Mass fluxes →

Order-of-magnitude estimates:

Flow type	*Density*	*Concentration*	*Mean free path*	*Granular flow regime*
	(kg/m³)	(—)	(Particle diam.)	
Dense	100–500	0.1–0.6	0–1	Frictional/ collisional
Fluidized	10–100	0.01–0.1	1–4	Collisional/ grain-inertial
Suspension	1–10	$< 10^{-2}$	> 4	Macro-viscous (turbulent)

Physical properties and transport processes differ substantially between flow types!

Outline

B2FR – a block model switching between two flow regimes

- Norem–Irgens–Schieldrop (NIS) rheology incorporates dry friction and dispersive stresses.
- At a critical shear rate, effective stress and dry friction vanish.
- B2FR extends NIS model to variable density.
- Rheological parameters depend on density in B2FR.
- Potential for richer dynamics if fluidization modifies the rheological parameters!

(Norem et al.,1989, *Annals Glaciol.* **13**, 218–225.
Issler and Gauer, 2008. *Annals Glaciol.* **49**, 193–198.)

Amended formulation of NIS rheology:

For simplicity, consider plane shear flow in *x-z* plane here.

$$\sigma_{xx} = -p_e - p_u - \rho(\nu_0 + \nu_2 - \nu_1)\dot{\gamma}^2$$
$$\sigma_{yy} = -p_e - p_u - \rho\nu_0\dot{\gamma}^2$$
$$\sigma_{zz} = -p_e - p_u - \rho(\nu_0 + \nu_2)\dot{\gamma}^2$$
$$\tau_{xz} = \mu p_e + \rho\nu_s\dot{\gamma}^2$$

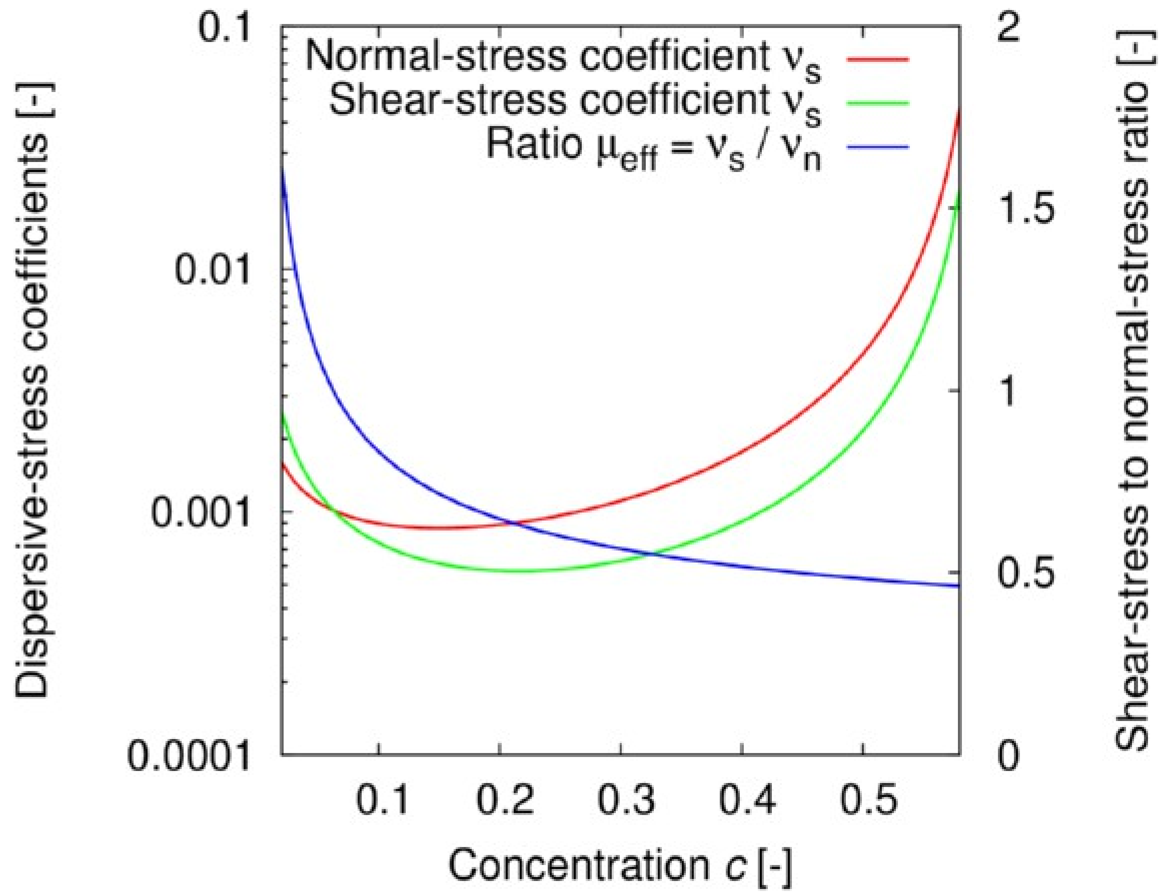
p_e effective pressure transmitted through skeleton
p_u pore pressure
μ dry friction coefficient [–]
$\nu_{0,1,2}$ dispersive stress coefficient [m²]
ν_s shear viscosity [m²]

Extension to variable density:

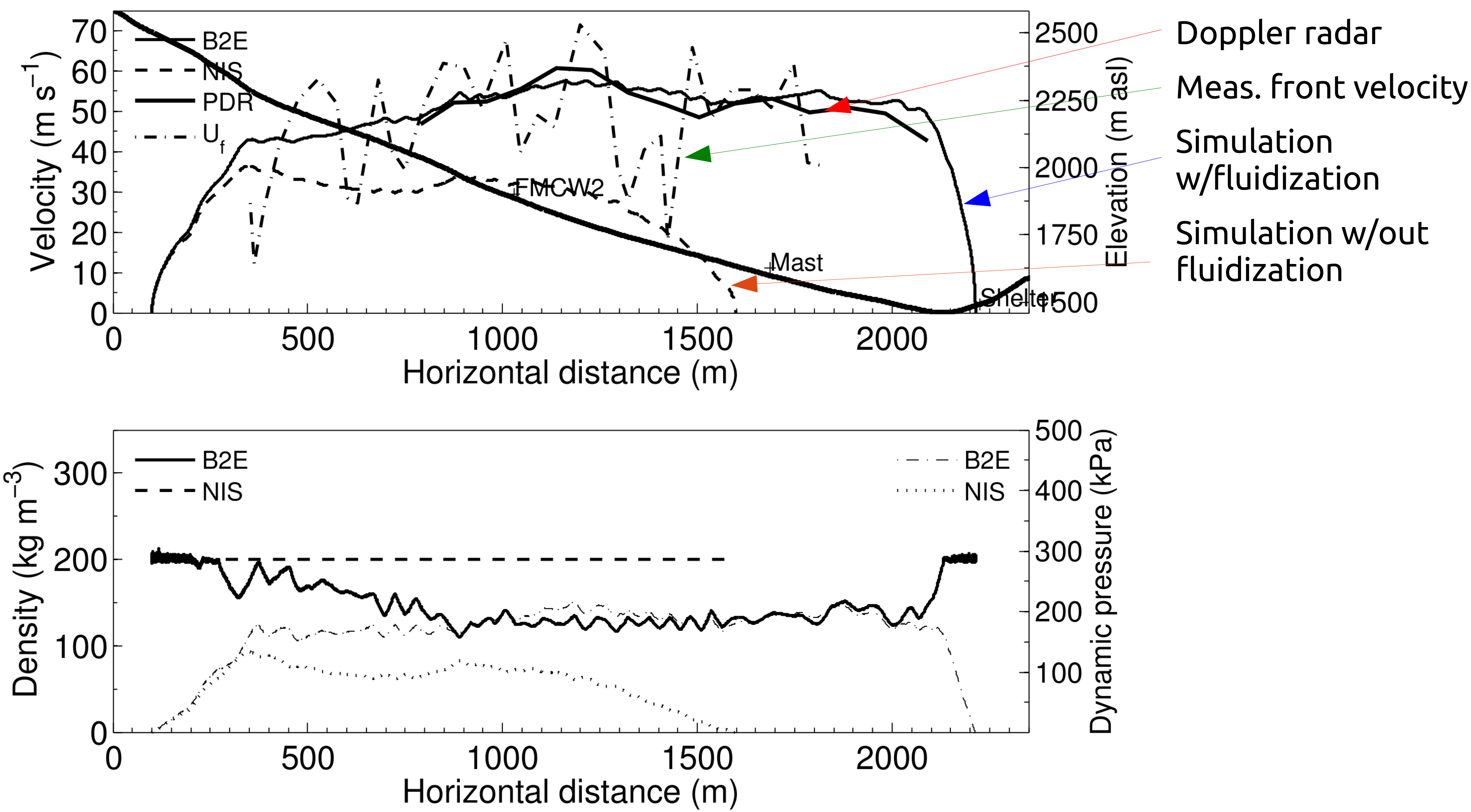
Theoretical calculations (Pasquarell et al., 1988) and numerical simulations (Campbell and Gong, 1986) of 2D stress tensor as a function of particle concentration may be approximated by

$$\begin{aligned} \nu_n(c) &= Q\,c^{-q}(c_{\max}-c)^{-r} \\ \nu_s(c) &= R\cdot\left(1+S\,c^{-s}\right)\nu_n(c) \end{aligned}$$

with $q, s \approx 0.5$,
$r \approx 1.5$,
$Q \approx 10^{-4}$,
$R \approx 0.2$,
$S \approx 1$,
$c_{\max} \approx 0.6$.



Comparison with measured avalanche at Vallée de la Sionne, 1999-02-10:
Dry friction $\mu = 0.50$, aerodynamic lift coeff. $C_L = 1.7$



Assessment of first attempt at modeling fluidization

- Block-model approach is too simplistic, but front behavior is reproduced.
- Fluidization does not progress to the assumed densities of the fluidized layer (30–100 kg/m^3), but stops at 100–150 kg/m^3.
- Very diverse avalanches are reproduced well with small (and explainable) variations of μ (±10%) and aerodyn. lift coeff. C_L (±20%).
- However, entrainment plays an important role!
- General trend $U_{\text{max}} \propto \sqrt{g H_{\text{drop}}}$ is reproduced within observable range.
- Empirical correlation between α and β angle is **poorly** reproduced.

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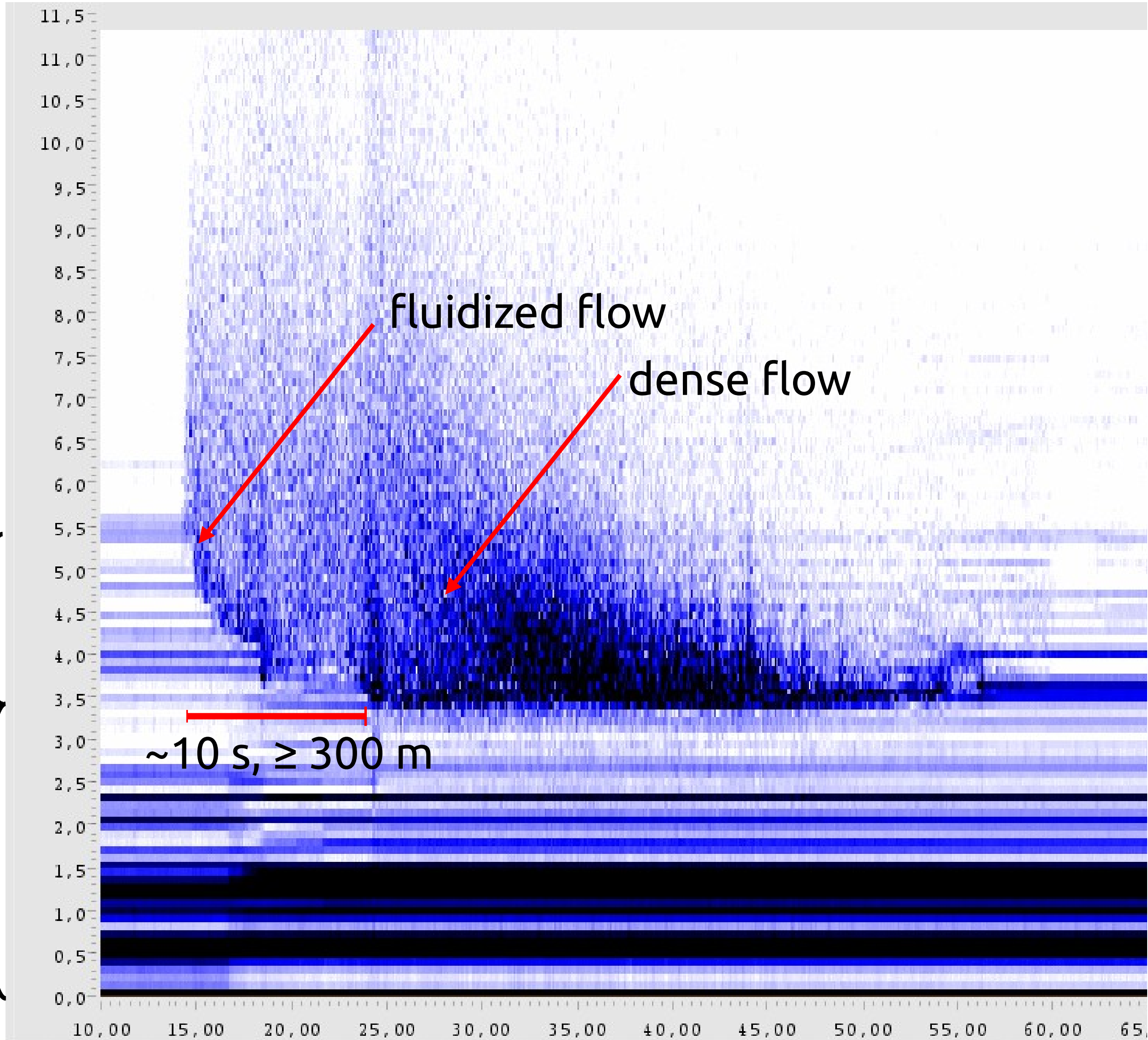
FMCW radar plot of snow avalanche at Vallée de la Sionne

Observed entrainment rate:
10–200 kg m^{-2} s^{-1}, diminishing with time and erosion depth.

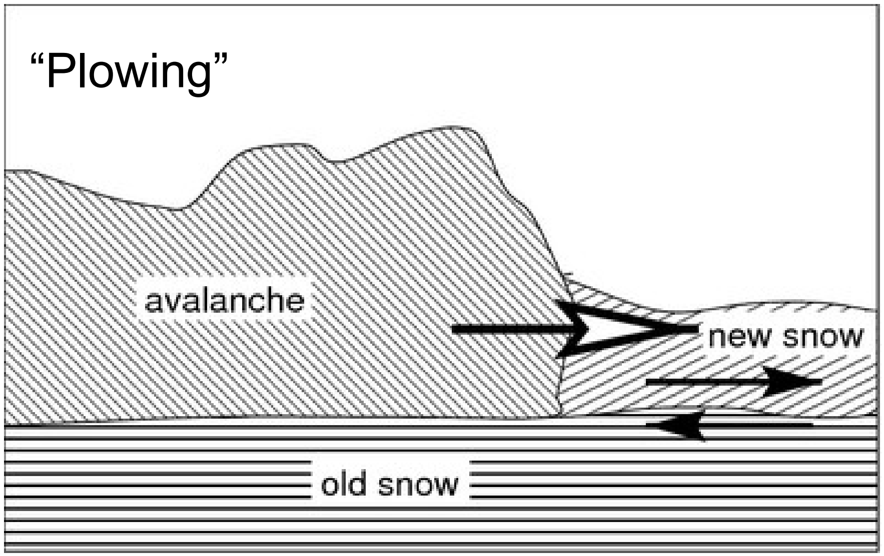
2 m of fresh snow eroded

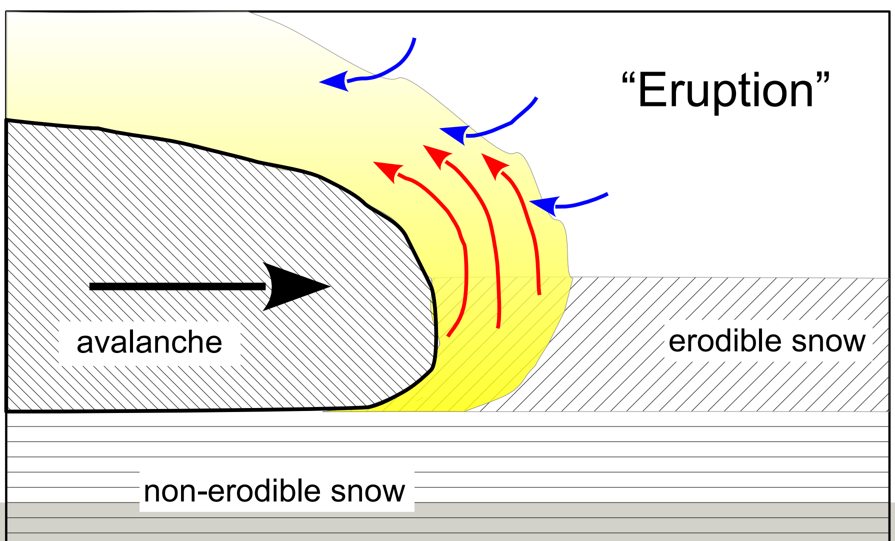
Hard old snow not eroded



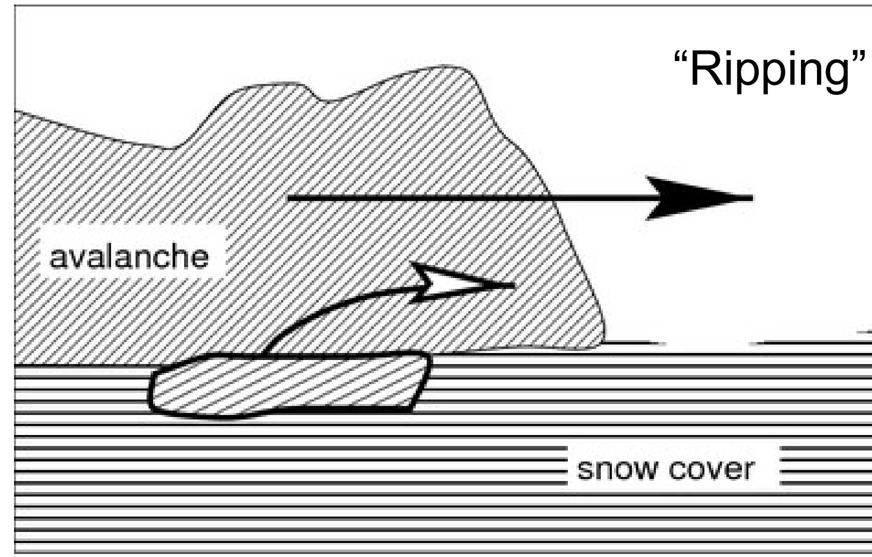
Conjectured erosion mechanisms (Gauer & Issler, 2004)

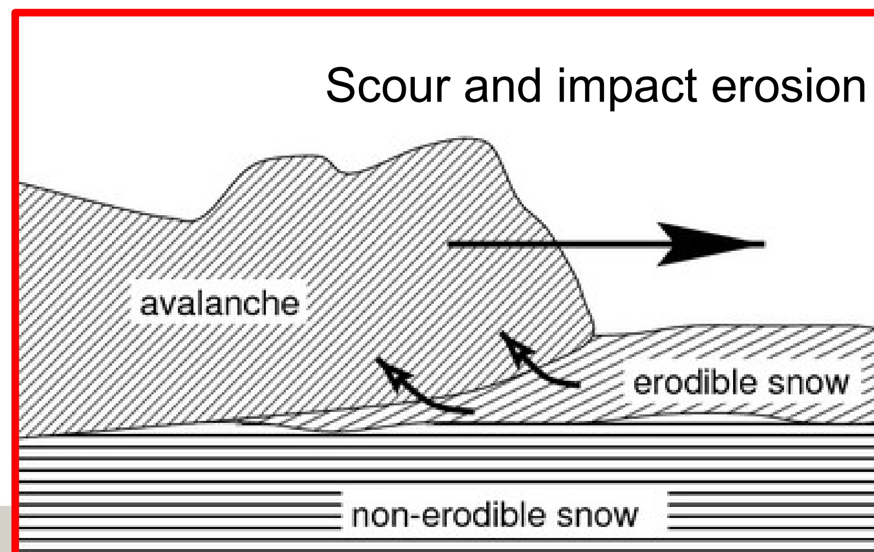
Frontal mechanisms





Mechanisms acting along bottom





Assumptions:

- Concentrate on erosion by scour along flow bottom, neglect frontal entrainment.

- **Bed material is perfectly brittle – breaks at stress τ_c.** Neglect energy required to break the snow cover.

- Interior of snow cover remains stable when surface is eroded:

$$\hat{\tau}_c(z) = C(z) + \mu_{sc}\,\hat{\sigma}_n(z)$$

with suitable choice of cohesion $C(z)$ and μ_{sc}.

- Stress tensor depends on overburden and shear rate $\partial_z u_x \equiv \dot{\gamma}$:

$$\sigma_{ij}(x, z, t) = \sigma_{ij}(z, h, \dot{\gamma}),$$

where $h = h(x,t)$ and $\dot{\gamma} = \dot{\gamma}(x, z, t)$.

Equation for the erosion rate

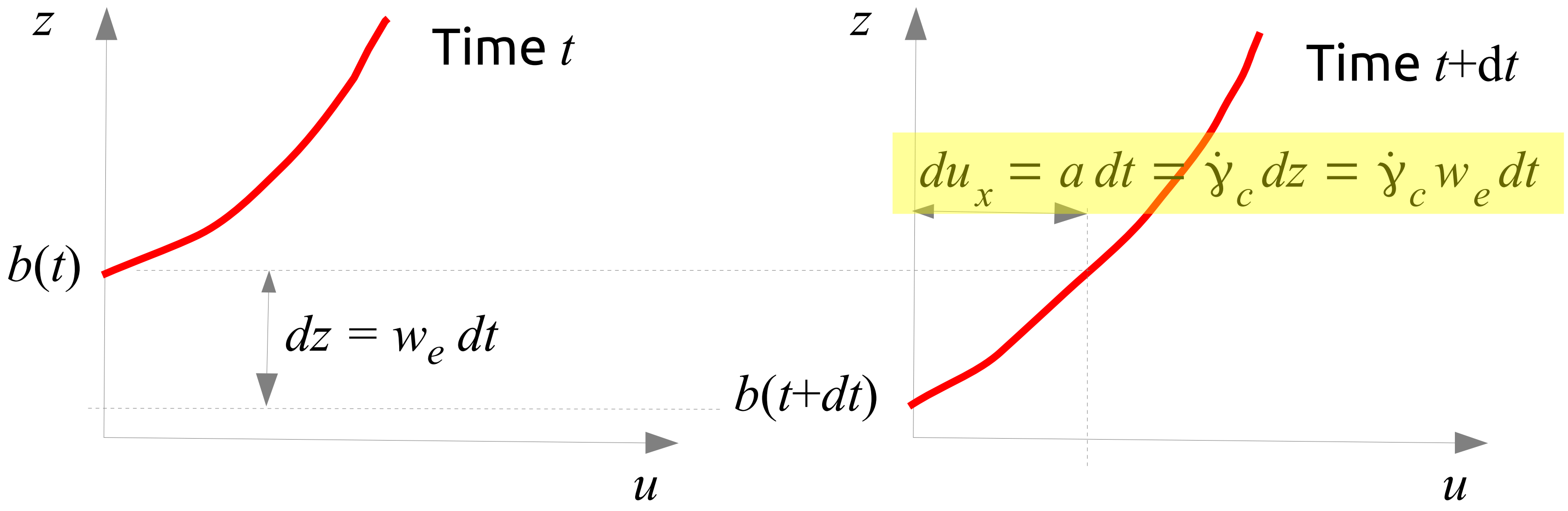
- Equation of motion in the flow (2D for simplicity):

$$\frac{Du_x}{Dt} = g \sin\theta + \partial_x \hat{\sigma}_{xx} + \partial_z \hat{\sigma}_{xz}$$

- Boundary conditions: $u_x(b,t) = 0, \quad \sigma_{zz}(h,t) = 0, \quad \boldsymbol{\sigma_{xz}(b,t) = \tau_c}.$

  No erosion: $\sigma_{xz}(b,t) < \tau_c.$
  Erosion: $\sigma_{xz}(b,t) = \tau_c.$
  If shear stress exceeds τ_c, erosion rate increases instantaneously and shear stress drops because of acceleration of eroded particles. Self-regulating mechanism similar to Owen's hypothesis for blown sand or snow.)

- Kinematic relation between erosion rate, velocity profile and particle acceleration at the bed-flow interface:



- Velocity at time $t + dt$ of particles eroded at time t:

$$u\left(b(t), t+dt\right) = 0 + \left(g \sin\theta + \partial_z \hat{\sigma}_{xz}\right) dt \; .$$

- Shear rate at erosion front is locked to critical shear rate:

$$\dot{\gamma}(b,t) = \frac{u(b,t+dt) - 0}{dz} = \frac{\left(g \sin\theta + \partial_z \hat{\sigma}_{xz}(b,t)\right) dt}{w_e(t)\, dt} \overset{!}{=} \dot{\gamma}_c$$

Equation for the erosion rate

- Momentum balance equation in the flow (2D for simplicity):

$$\frac{du_x}{dt} = g \sin\theta + \partial_x \hat{\sigma}_{xx} + \partial_z \hat{\sigma}_{xz}$$

- Boundary conditions: $u_x(b,t) = 0$, $\sigma_{zz}(h,t) = 0$, $\boldsymbol{\sigma_{xz}(b,t) = \tau_c}$.

- Kinematic relationship between erosion rate, velocity profile and particle acceleration at the bed-flow interface:

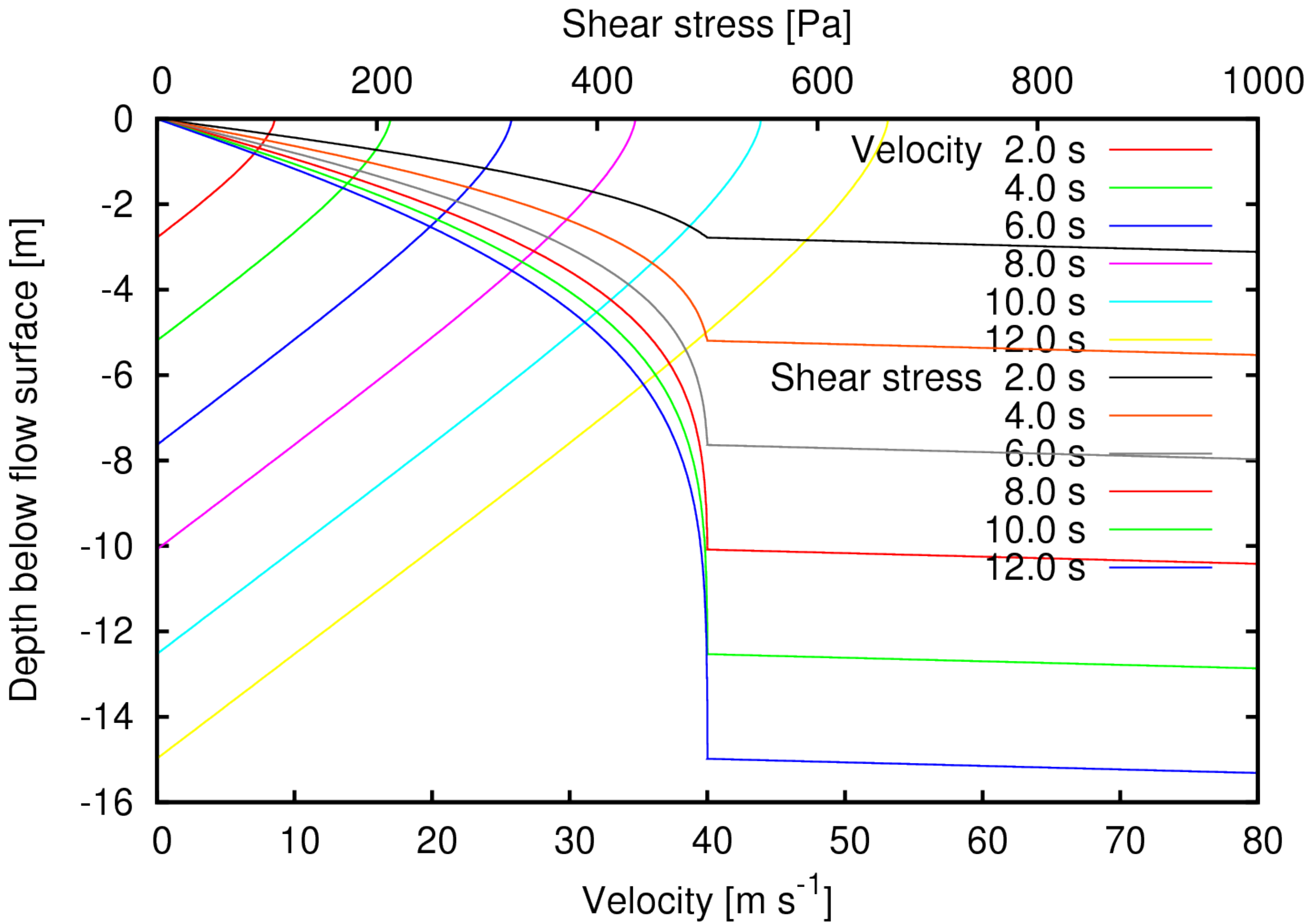
$$a_x\, dt = \partial_z u_x\, dz \quad \Rightarrow \quad a_x = \dot{\gamma}\, w_e$$

- Shear stress at interface equals shear strength → critical shear rate $\dot{\gamma}_c$.

☞

$$w_e(t) = \frac{g \sin\theta + \partial_z \hat{\sigma}_{xz}(\dot{\gamma}_c, h)}{\dot{\gamma}_c}.$$

Numerical solution for a Bagnoldian fluid: Evolution of velocity and shear stress profiles



Approximate solution for the NIS model

➔ Aim for *depth-averaged* model with erosion rate as source term.

- Use Norem–Irgens–Schieldrop (NIS) model for rheology:

$$\hat{\sigma}_n = -\hat{p}_e - \nu_0 \dot{\gamma}^2, \qquad \hat{\tau} = \mu \hat{p}_e + \nu_s \dot{\gamma}^2$$

effective pressure
(through skeleton)

collisional contribution
(dispersive stresses)

- Without erosion: Equilibrium profile functions are Bagnoldian,

$$u_x(z) = U \cdot \left[1 - \left(1 - \frac{z}{h} \right)^{3/2} \right].$$

- Critical shear rate for erosion:

$$\mu g h \cos\theta + (\nu_s - \mu \nu_0) \dot{\gamma}_c^{\,2} = \hat{\tau}_c$$

With erosion, approximate the velocity profile by (see next slide):

$$u_x(z) \approx U \cdot \left[1 - \left(1 - \frac{z}{h} \right)^{\alpha(x,t)} \right]$$

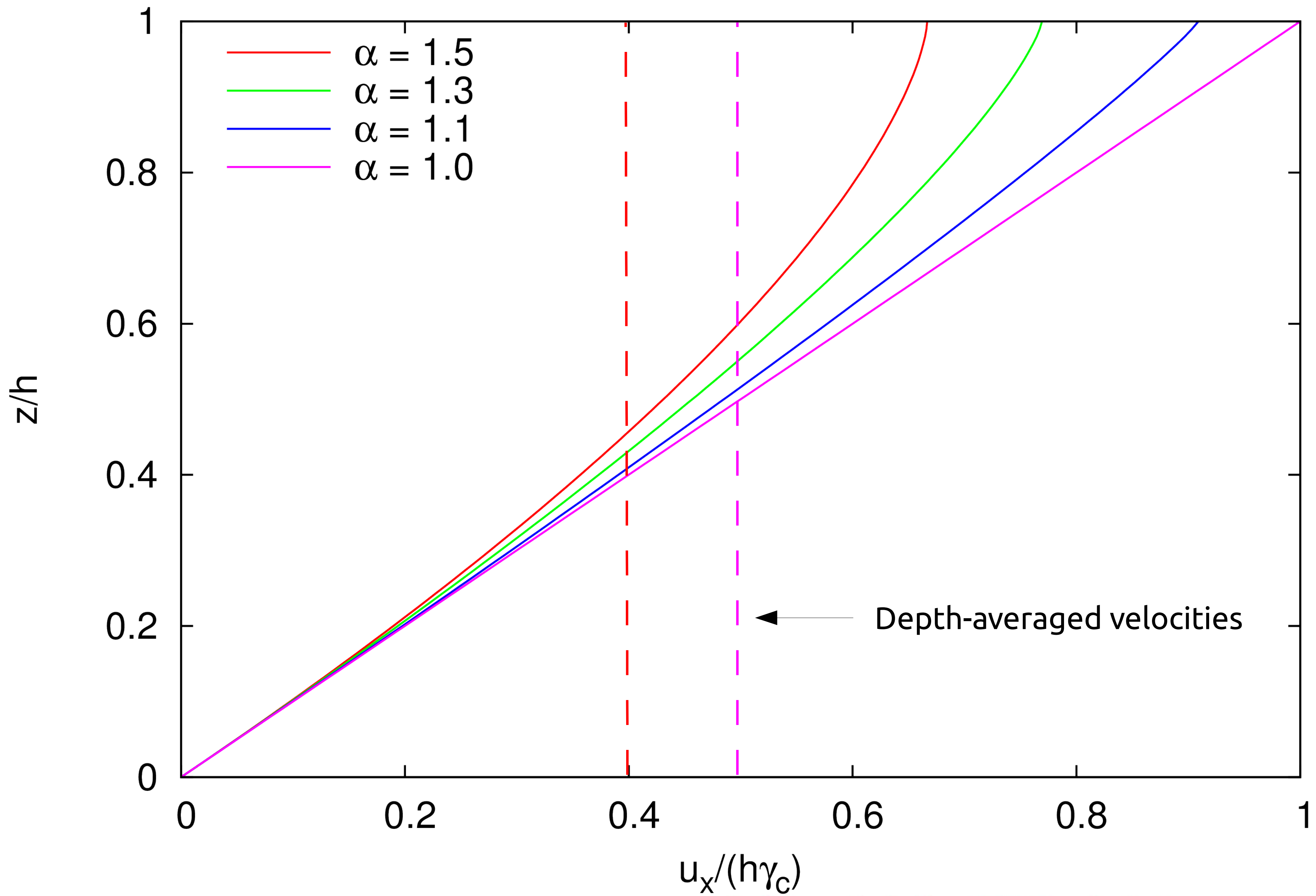
Choose $1 \leq \alpha \leq 3/2$ *variable* to match velocity $\bar{u}_x$ and shear stress at interface:

$$u_x(0) = 0, \quad \partial_z u_x(h) = 0, \quad \partial_z u_x(0) = \dot{\gamma}_c, \quad \bar{u} = \frac{\alpha}{\alpha + 1} U$$

Fulfilled by construction (first two conditions)

2 equations, 2 unknowns (last two conditions)

All relevant quantities can now be computed.



α = 1.5
α = 1.3
α = 1.1
α = 1.0
Depth-averaged velocities
z/h
0
0.2
0.4
0.6
0.8
1
u_x/(hγ_c)

From velocity profile compute $\partial_z \hat{\sigma}_{xz}(b,t)$ to obtain erosion formula:

☞ $$w_e = \Theta(\sigma_{xz}(0) - \tau_c) \left[\frac{h g \sin\theta - \hat{\tau}_c}{h \dot{\gamma}_c} + (\nu_s - \mu \nu_0) \left(5 \frac{\dot{\gamma}_c}{h} - 2 \frac{\dot{\gamma}_c^2}{\bar{u}_x} \right) \right]$$

- ✔ No free parameters in the erosion model!
- ✔ Erosion threshold: Shear stress must exceed bed strength.
- ✔ For given flow depth, erosion rate grows with velocity, but is limited.

- ➢ Model applies only if Coulomb friction is less than bed strength.
- ➢ For typical snow avalanche conditions:

$$w_e \lesssim 0.2\,\mathrm{m/s}.$$

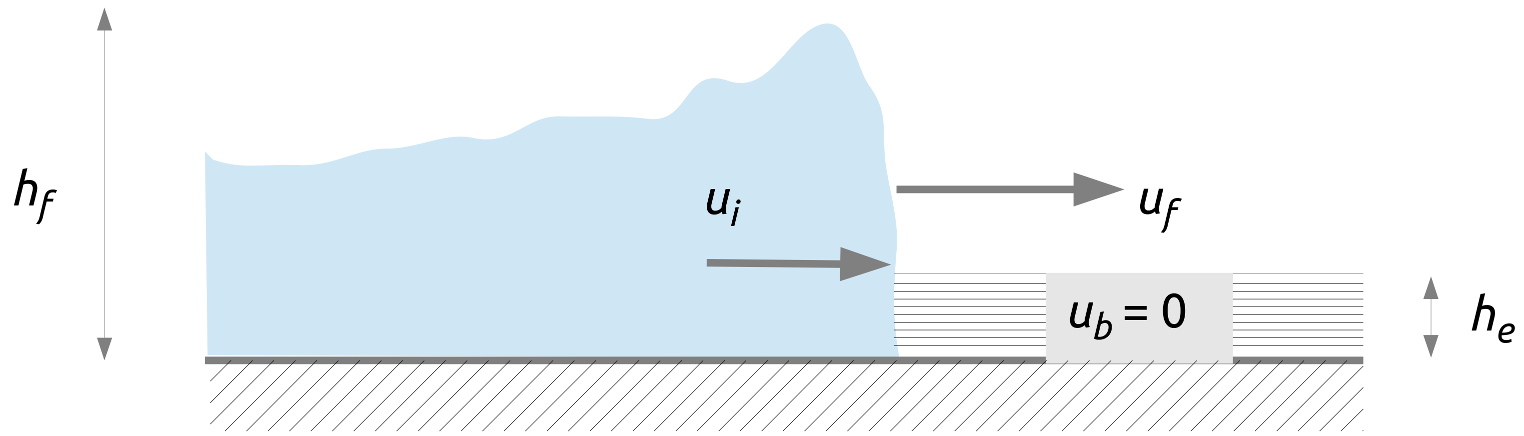
What do these limitations mean?

- As overburden increases, critical shear rate goes to zero, entrainment rate diverges.
- Maximum allowed velocity diminishes, avalanche becomes "supercritical".
- Snow cover fails catastrophically to a finite depth (e.g., entire new-snow layer at once). *Eroded* layer cannot be *entrained* at once.
- Velocity profile has inflection point.

➔ **Different erosion regime!**
Needs a completely different analysis.

➔ Description in terms of a shock condition might be appropriate (Eglit and coworkers, ca. 1965).

Early entrainment model by Eglit and coworkers implements plowing as jump boundary condition at avalanche front:



Mass balance:

$$h_b \rho_b u_f = h_f \rho_f (u_f - u_i)$$

Momentum balance:

$$h_b \rho_b \cdot 0 \cdot u_f + k_b \rho_b g\, h_b^2/2 + h_b \tau_c = h_f \rho_f u_i (u_f - u_i) + k_f \rho_f g\, h_f^2/2$$

Fracture strength of bed, τ_c, determines frontal dynamics.
Front moves more rapidly than flowing material at front.

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Note added in proof

The (preliminary and sketchy) work described in this section was triggered by a remark by David Mohrig (Univ. of Texas, Austin), who came to my office on Oct. 7 after my first talk at the workshop. Referring to the enigma of avalanches with exceptionally long run-out (see Challenge #3 in Sec. 2 of this presentation), he said:

«Dieter, I think this must have something to do with the substrate!»

This made me instantaneously realize what years of exposure to my geotechnical colleagues at NGI talking about the importance of excess pore pressure strangely had failed to convey, namely that the flow of an avalanche over a compressible, highly porous snow cover must have profound effects on the dynamics of at least the front of the avalanche.

Tusen takk to David for the right words at the right time!

Erosion model indicates snow layer may collapse catastrophically.
What does this imply?

New snow is extremely contractive:

- Undisturbed: $\rho_{sc} = O(100\ \text{kg/m}^3)$, $c_{ice} = O(10\%)$
- In avalanche: $\rho_{sc} = O(200\text{–}300\ \text{kg/m}^3)$, $c_{ice} = O(20\text{–}30\%)$

Collapse requires

- Squeezing out air $\Delta V_a = (\tfrac{1}{2}\ldots\tfrac{2}{3})\, V_{sc}$
- Or compressing pore air to $\Delta p_u = 100\ldots200\ \text{kPa}$

Compare to typical weight of avalanche: $p_{ob} = 1\ldots5\ \text{kPa}$

⇒ Pore air cannot be compressed substantially, but must flow out gradually.

⇒ Analogy with fluidized-bed reactor!

Fluidization condition for reactor bed/ snow avalanche:

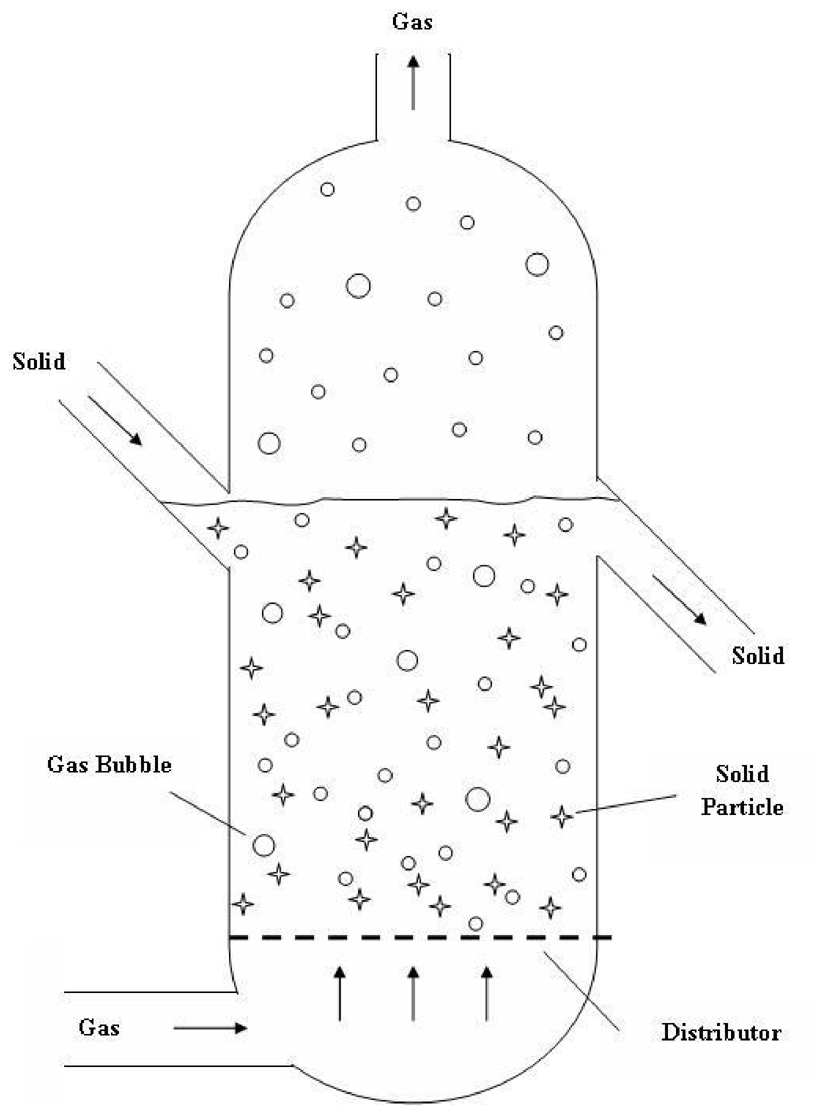
$$\Delta p_u \geq (\rho_p - \rho_f) c_p g h$$

Δp_u	Excess pressure at base of flow
ρ_p, ρ_f	Density of particles, fluid
c_p	Particle volume concentration
h	Flow height

Fluidized-bed reactor: Pressure gradient imposed externally.

Avalanche: Pressure gradient created by overburden (= avalanche) compresses air from the collapsed layer.

Air flows out as quickly as pressure gradient permits.



From http://commons.wikimedia.org

Treat avalanche as porous, but highly permeable medium:

- Darcy's law:

$$w_s = K \frac{\rho_p c_p g}{\mu_f}$$

w_s	Seepage speed	[m/s]
μ_a	Air (dynamic) viscosity	[kg/(m s)]
K	Permeability	[m²]

- *K* unknown – should be larger than for gravel, assume $O(10^{-6}\ \text{m}^2)$.
- **But:** Reynolds number in the pore space is too high for Darcy's law:

$$\Re = \frac{U\, D}{\nu_a} = O\left(10^3\right)$$

- Apparent permeability seems to tend to asymptotic value $O(10^{-3}\ K)$ at high rates. With that obtain crude estimate

$w_s = O(0.1 \ldots 1\ \text{m/s})$ and $T_s = h/w_s = O(0.1 \ldots 10\ \text{s})$

Speculation: (Possible) implications of snow-cover collapse

- Related to eruption mechanism (Carroll et al., *JGR* **F118**, 2013), but operates along extended area at avalanche front.

  Length of fluidized head: $L_f \leq u_f T_s = O(5 \cdot 10^{1\pm1}\ \mathrm{m})$

- Head has essentially unlimited supply of excess pore pressure, but body and tail have not

- Compression of snow cover under avalanche weight w/out erosion has similar effect, but probably achieves partial fluidization only.

- Effect of Coulomb friction disappears completely in the fluidized regime.
  Dispersive pressure and shear stress also diminish greatly, leading to high speeds.

- We may have to rethink snow avalanche dynamics completely – run-out distance may depend primarily on snow-cover properties rather than on rheology!

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Längenboden near Davos, Switzerland. Photo Thomas Wiesinger, SLF

Slab break-up and transformation of the flowing material

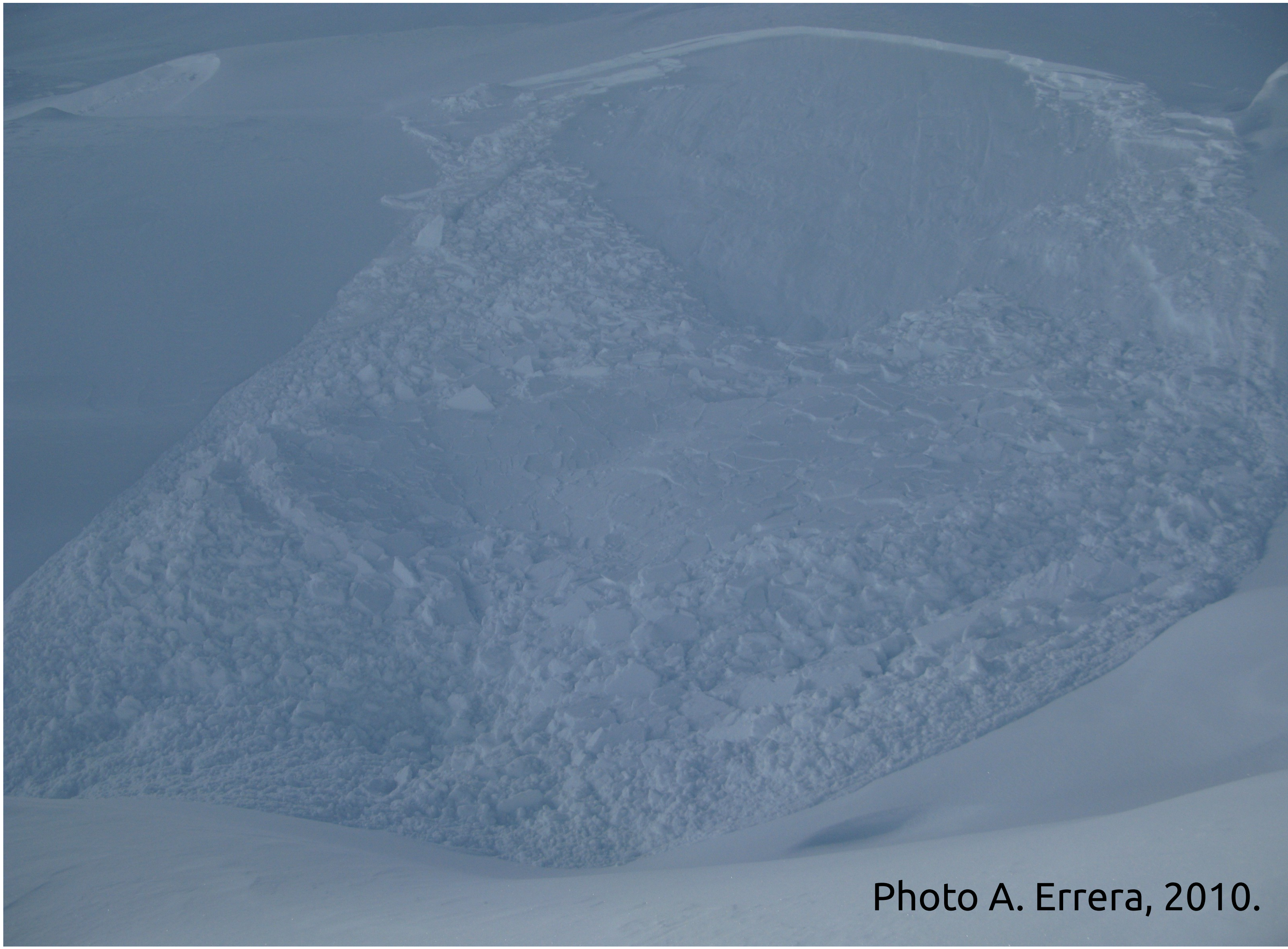

Photo A. Errera, 2010.

Progressive break-up of the flow material may be decisive in certain situations:

- General wisdom: Break-up of original slab usually is fast and need not be modeled explicitly.
- In small avalanches, break-up sometimes does not progress sufficiently and sliding stops on steep slopes.
- However, even in big avalanches, small particles may still be ground down to grain size.
- In granular materials in the collisional flow regime:

$$\sigma_{xz} \propto \dot{\gamma}^2 d^2$$

i.e., at constant shear stress, shear rate and velocity increase strongly with progressive comminution!