A Two-Phase Continuum Theory for Aeolian Transport

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Suspension and transport of sand by saltation, turbulent suspension, and inter-particle collisions in a turbulent shearing flow

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Scenario

A grain lifted from the bed by a strong turbulent eddy is accelerated by the mean turbulent shear flow and, upon colliding with the bed, rebounds and ejects other particles.

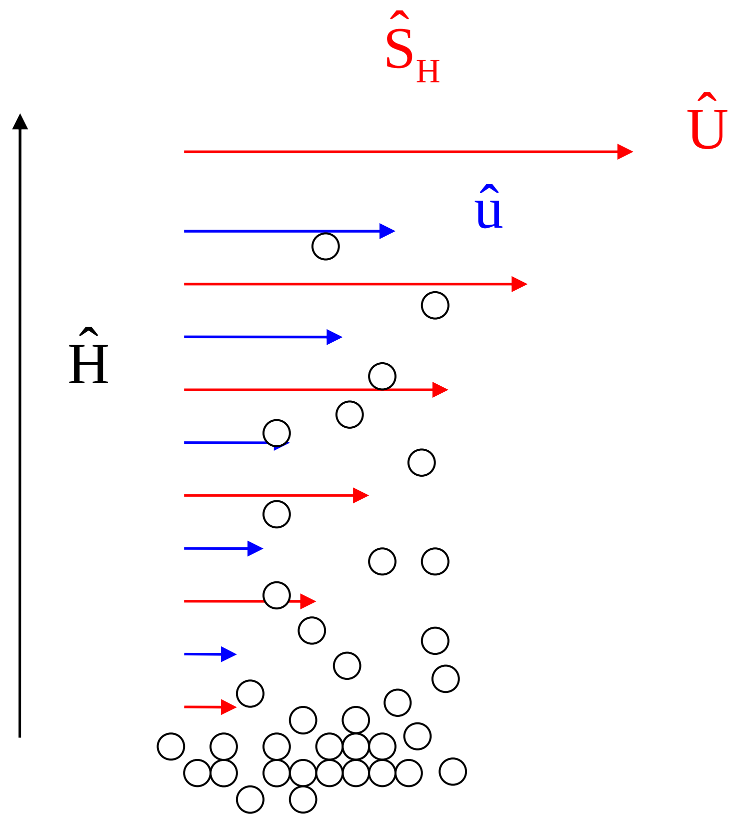
The drag of the particles on the wind eventually limits the number of particles that can participate.

At higher wind speeds, turbulent suspension, associated with velocity fluctuations of the wind that increase with distance above the bed, and collisional suspension, due to the increasing likelihood of collisions between particles near the bed, begin to play a role.

Goal

Describe splash, drag, turbulent suspension, and inter-particle collisions in a two-phase continuum theory for a steady, fully-developed turbulent shearing flow.

Continuum Theory



Friction velocity / Shields parameter

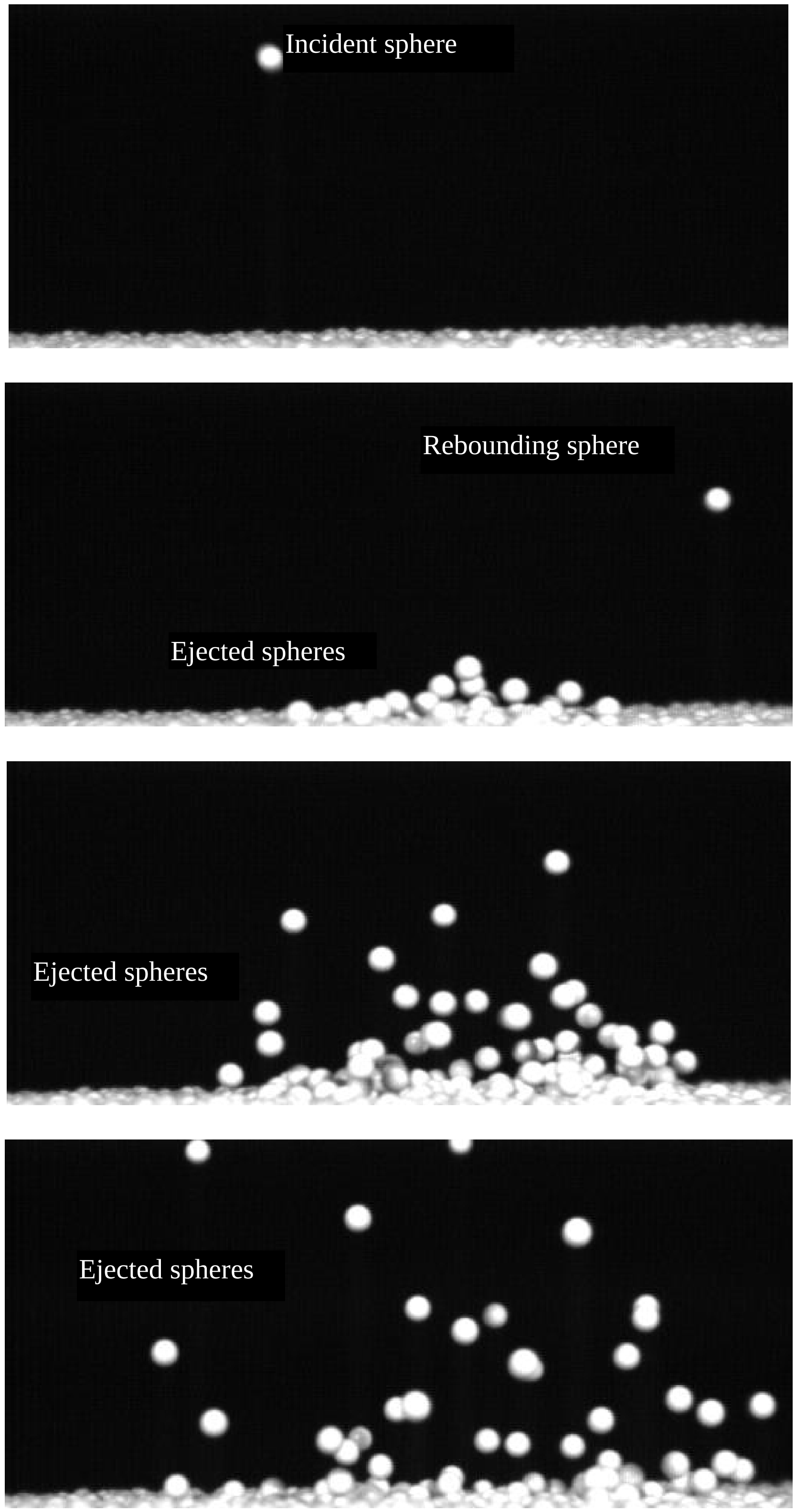
$$\hat{u}^* \equiv \left(\hat{S}_H / \hat{\rho}^f\right)^{1/2} \qquad S^* \equiv \hat{S}_H / \left(\hat{\rho}^s \hat{g} \hat{d}\right)$$

Drag force: $\nu \hat{D}(\hat{U} - \hat{u})$

$$\hat{D} \equiv \frac{3}{10}\frac{\hat{\rho}^f}{\hat{d}}\left[(\hat{U} - \hat{u})^2 + 3\hat{T}\right]^{1/2} + \frac{18\hat{\mu}^f}{\hat{d}^2}$$

Splash

Beladjine, et al. Phys. Rev. E 75, 061305 (2007)



Momentum of rebounding particles

$$\overline{\xi'} = e(\xi)\xi = (0.87 - 0.72\sin\theta)\xi$$

$$\overline{\xi'_y} = \varepsilon(\xi)\left|\xi_y\right| = \left(\frac{0.30}{\sin\theta} - 0.15\right)\left|\xi_y\right|$$

Number N of ejected particles

$$N(\xi) = \begin{cases} 13(1-e^2)\left(\dfrac{\xi}{40} - 1\right), & \text{if } \xi > 40 \\ 1, & \text{if } 1 \le \xi \le 40 \\ 0, & \text{if } \xi < 1 \end{cases}$$

Velocity distribution function

$$f(\xi) = \frac{n_0}{2\pi T_0}\exp\left[\frac{-(\xi_x - u_0)^2 - \xi_y^2}{2T_0}\right]$$

$$(\nu_0 = \pi n_0 / 6)$$

Mass flux

$$\dot{m} = \int_{\xi_y \le 0} (N-1)\xi_y f(\xi) d\xi$$

$$= \frac{13}{2\pi} \frac{n_0 T_0^2}{u_0 (40 - u_0)^2} \left[0.24 + 0.63 \left(\frac{\pi T_0}{20 u_0} \right)^{1/2} \right] e^{\frac{-(40-u_0)^2}{2T_0}}$$

$$- \frac{74\sqrt{2}}{\pi} \frac{n_0}{T_0} e^{\frac{-u_0^2}{2T_0}}$$

Momentum flux

$$\dot{\mathbf{M}} = -\frac{\pi}{6} \int_{\xi_y \le 0} (\bar{\xi}' - \xi)\xi_y f(\xi) d\xi$$

$$\dot{M}_x = \nu_0 T_0 \left(0.35 + 0.07 \frac{u_0}{T_0^{1/2}} - 0.33 \frac{T_0^{1/2}}{u_0} \right)$$

$$\dot{M}_y = \nu_0 T_0 \left[0.12 \left(\frac{u_0}{T_0^{1/2}} + \frac{T_0^{1/2}}{u_0} \right) - 0.08 \right] + \frac{\nu_0 T_0}{2}$$

With $\dot{M}_y = p_0 = \nu_0 T_0$, $\dfrac{u_0}{T_0^{1/2}} = 4.6$

and $\dot{M}_x = s_0 = 0.7 \nu_0 T_0$

Continuum Theory

Particle horizontal momentum

$$0 = \frac{ds}{dy} + \frac{\nu D}{\sigma}(U - u)$$

Particle vertical momentum:

$$0 = -\frac{dp}{dy} - \nu$$

Particle fluctuation energy

$$0 = -\frac{dq}{dy} + s\frac{du}{dy} - \gamma$$

Particle pressure

$$p = \nu T$$

Particle shear stress

?

Particle energy flux

?

Continuum Theory

Single particle trajectories without vertical drag

Upward: $\xi'_y \dfrac{d\xi'_x}{dy} = D(U - \xi'_x)$

Downward: $\xi'_y \dfrac{d\xi_x}{dy} = -D(U - \xi_x)$

Multiply by ν, sum, and average

$$\overline{\nu\xi'^2_y \frac{d}{dy}(\xi'_x + \xi_x)} = \nu\overline{D\xi'_y(\xi'_x - \xi_x)}$$

$$2u \equiv \overline{(\xi'_x + \xi_x)}$$

$$p \equiv \nu\overline{\xi'^2_y} = \nu T \quad 2s \equiv \nu\overline{\xi'_y(\xi'_x - \xi_x)}$$

$$p\frac{du}{dy} = \alpha D s$$

$$2q \equiv \nu\overline{\xi'_y\left[(\xi'_x - u)^2 + \xi'^2_y\right]} + \nu\overline{\xi_y\left[(\xi_x - u)^2 + \xi^2_y\right]}$$

$$\doteq \nu\left[\overline{\xi'_y(\xi'_x - \xi_x)}\,\overline{(\xi'_x + \xi_x)}/2 - \overline{\xi'_y(\xi'_x - \xi_x)}u\right]$$

$$q = 0$$

Boundary-Value Problem

Steady, fully-developed flow:

$\dot{m} = 0$, so $u_0 = 21$ and $T_0 = 21$.

$$\frac{d\nu}{dy} = -\frac{\nu}{T_0}$$

$$\frac{du}{dy} = 20D\frac{s}{p}$$

$$D = \frac{0.3}{\sigma}\left[(U-u)^2 + 3T_0\right]^{1/2} + \frac{18.3}{\sigma R}$$

$$\frac{ds}{dy} = -\nu D(U-u) \qquad \sigma = \frac{\hat{\rho}^s}{\hat{\rho}^f} \qquad R = \frac{\hat{d}(\hat{g}\hat{d})^{1/2}}{\hat{\mu}^f/\hat{\rho}^f}$$

$$\frac{dU}{dy} = \frac{(S^* - s)\sigma}{1/R + \left[(S^* - s)\sigma\right]^{1/2}\kappa y}$$

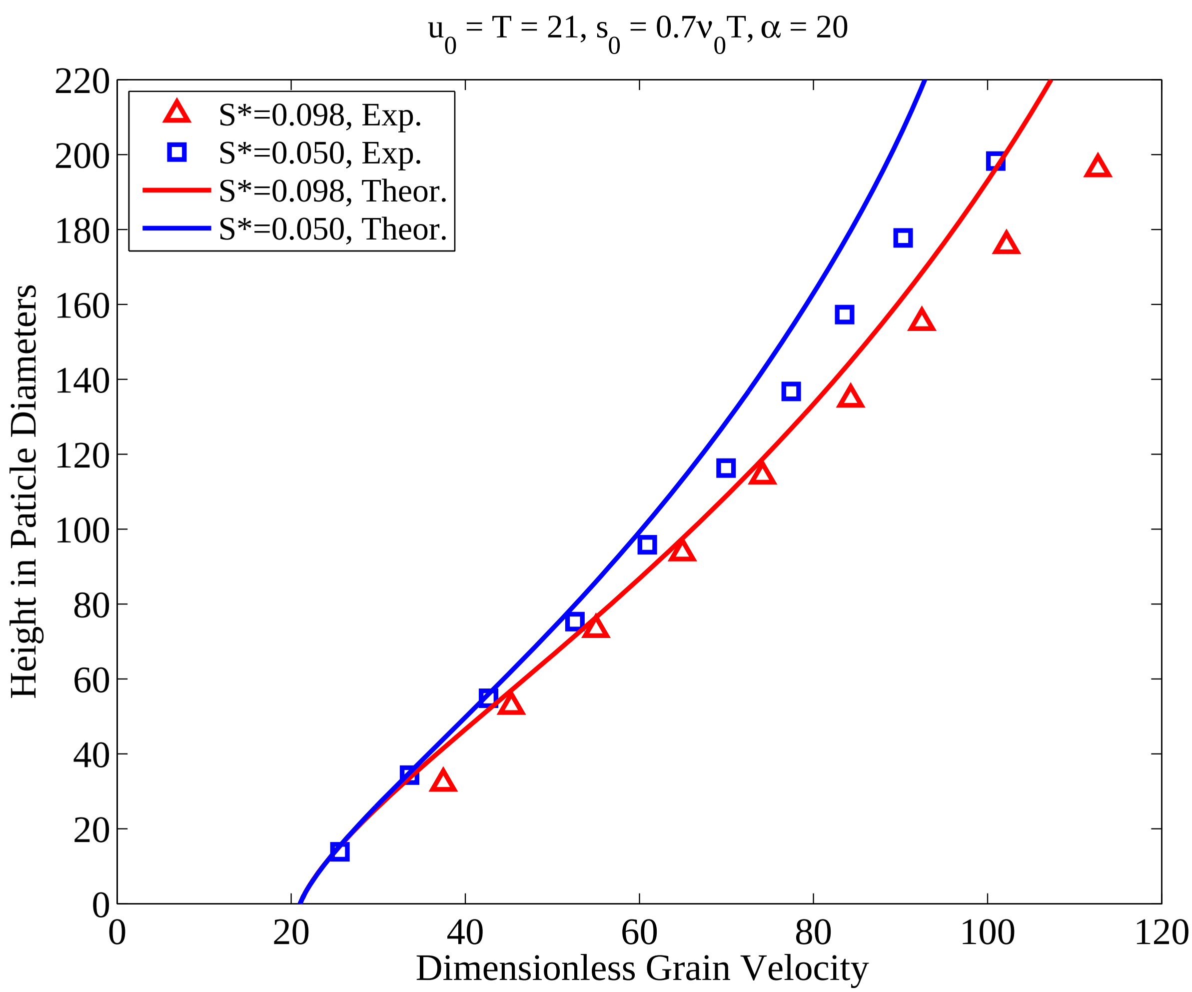
$$y = 0:\ T_0 = 21,\ u = 21,\ s = 0.7\nu_0 T_0,\ U = 0$$

$$y = 220:\ s = 0$$

Parameter: ν_0

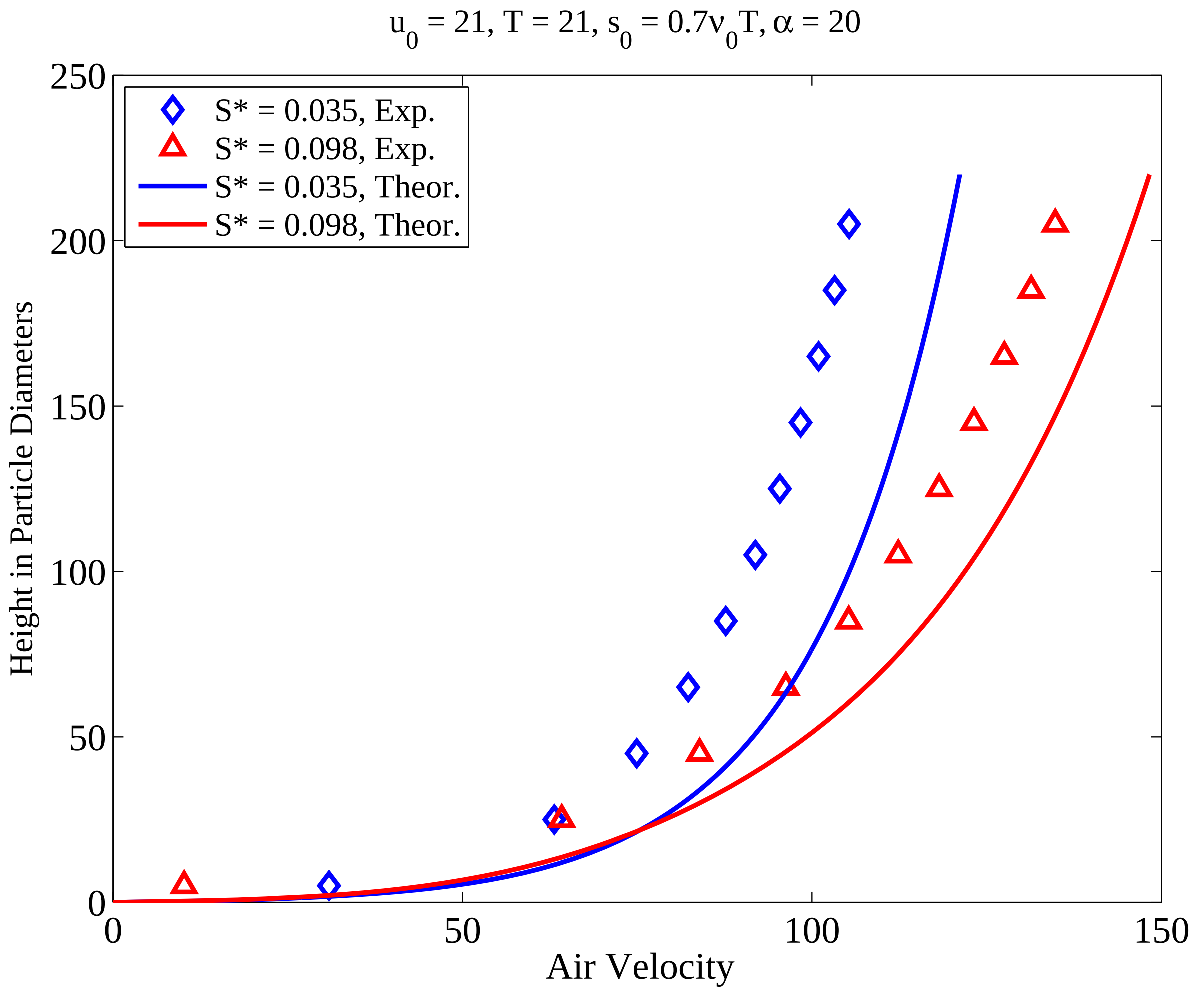
Comparison with Experiments

Grain Velocity



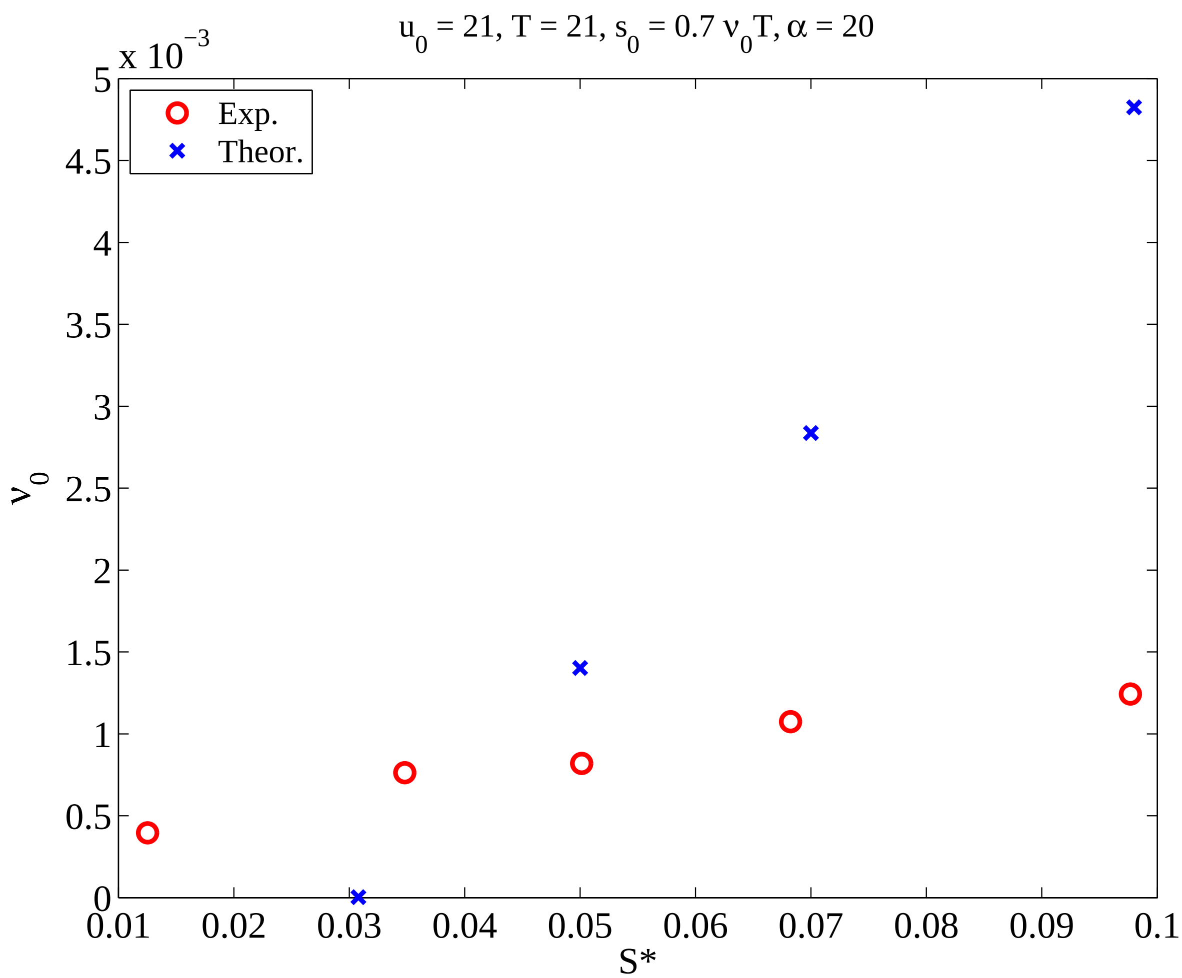
Comparison with Experiments

Air Velocity



Comparison with Experiments

Bed Volume Fraction



Also $T = 21$ versus $\ell_\nu = 40$

Include suspension by turbulent velocity fluctuations

Particle vertical momentum

$$0 = -\frac{d\hat{p}}{d\hat{y}} - \hat{\rho}^{s}\nu'\hat{g} + \hat{D}\overline{\nu'\Delta\hat{V}}$$

Turbulent suspension force

$$\overline{\nu'\Delta\hat{V}} = -\frac{\hat{\mu}^{T}}{\hat{\rho}^{f}}\frac{d\bar{\nu}}{d\hat{y}}$$

Turbulent viscosity

$$\hat{\mu}^{T} = \hat{\rho}^{f}\frac{0.09}{0.165}\kappa\hat{y}\hat{k}^{1/2}$$

Velocity fluctuations

$$\hat{k}^{1/2} = \frac{(0.09)^{1/2}}{0.165}\left[\frac{(\hat{S}_{H} - \hat{s})}{\hat{\rho}^{f}}\right]^{1/2}$$

Boundary-value Problem

$$\frac{d\nu}{dy} = -\frac{\nu}{T_0 + D\mu^T}$$

$$\mu^T \equiv \frac{0.09}{0.165}\frac{1}{\sigma}k^{1/2}\,0.41y$$

$$k^{1/2} \equiv \frac{(0.09)^{1/2}}{0.165}\left[(S^* - s)\sigma\right]^{1/2}$$

$$\frac{du}{dy} = 20D\frac{s}{p}$$

$$\frac{ds}{dy} = -\nu D(U - u)$$

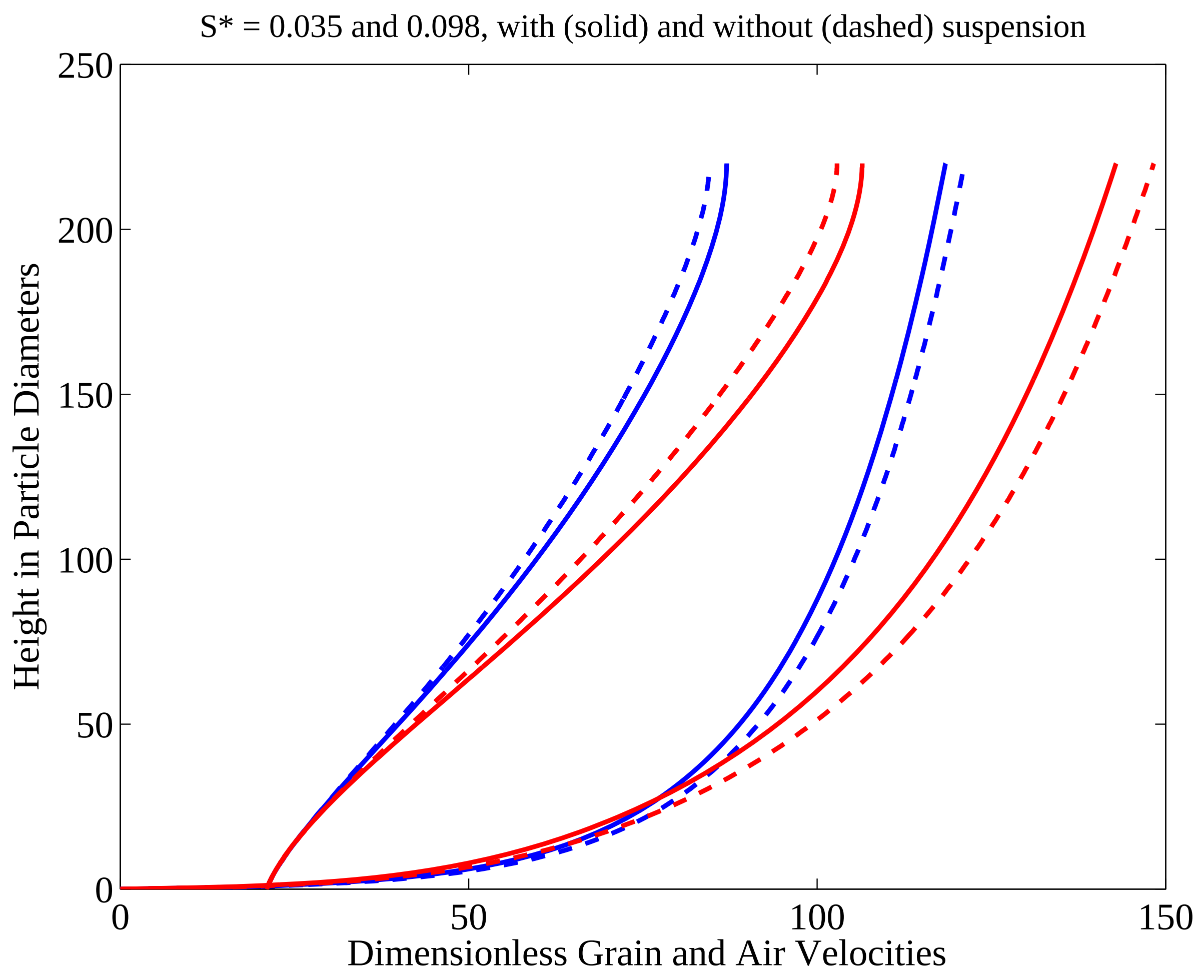
$$\frac{dU}{dy} = \frac{(S^* - s)\sigma}{1/R + \mu^T}$$

$y = 0:\ \ T_0 = 21,\ \ u = 21,\ \ s = 0.7\nu_0 T_0,\ U = 0$

$y = 220:\ \ s = 0$ Parameter: ν_0

Influence of Suspension

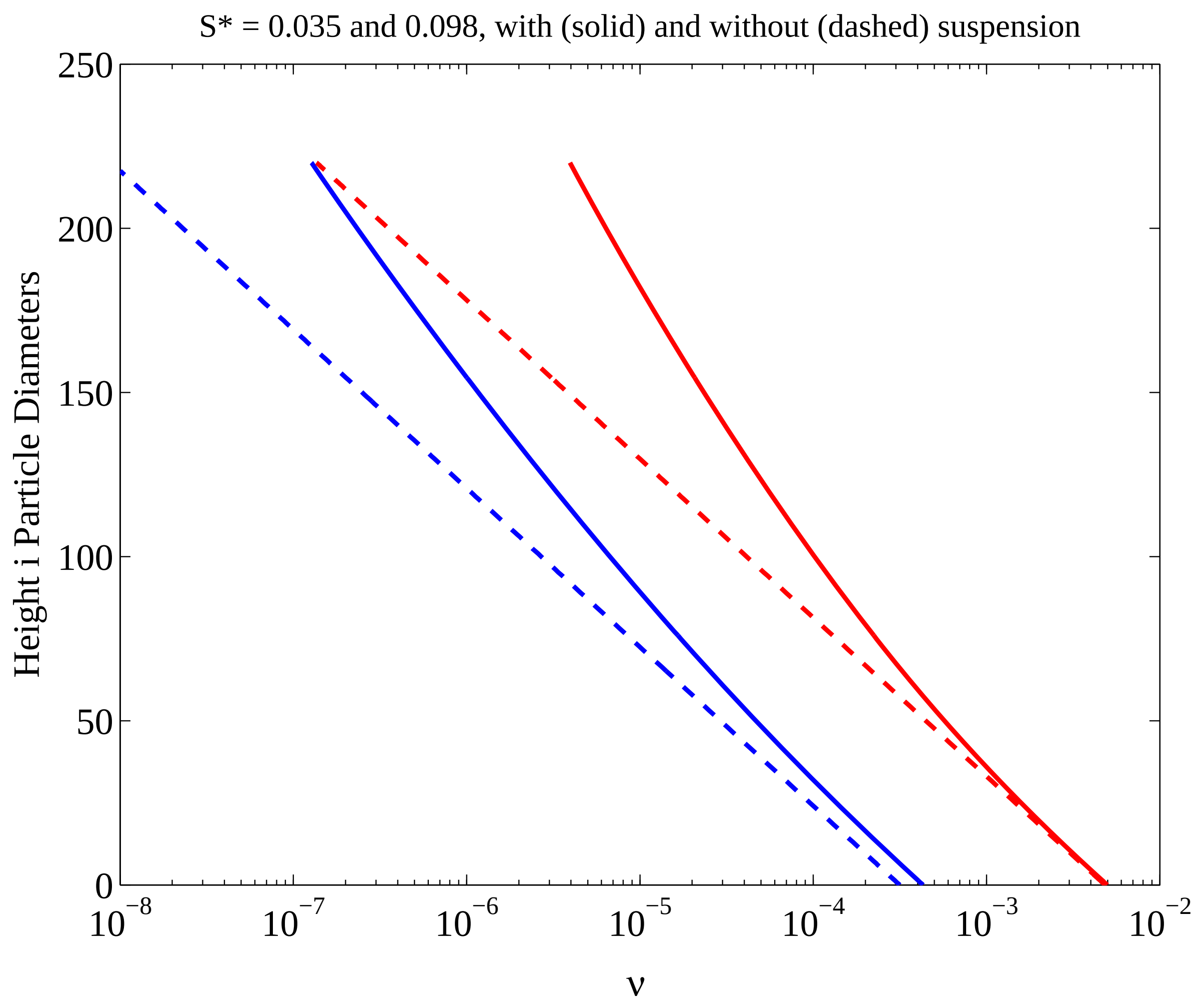
Grain and Air Velocities



Suspension moves both the grain and air velocities in the right directions.

Influence of Suspension

Volume Fraction



Suspension increases the dimensionless decay length from 21 to about 30.

Conclusions

Splash, saltation, and suspension by turbulent fluctuations have been incorporated into a two-phase continuum model.

There is reasonable agreement of the resulting predictions with existing experiments, and suggestions for further measurements.

The formulation for steady, uniform flows can be extended to describe unsteady and non-uniform situations.

The modeling for the Aeolian system may have analogs in other fluid-particle systems

Pasini & Jenkins, Phil. Trans. Roy. Soc. 363, 1625, 2005.
Jenkins, Cantat & Valance, Phys. Rev. E 82, 020301, 2010