Towards a theory of thermal fluctuations and rare events in glasses

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Outline

- Disorder-induced glasses
- Droplet phenomenology and random elastic models – glassy aspects
  - rare large thermal fluctuations
  - activated dynamics
- Functional Renormalization Group
- Inside the cusp: Thermal Boundary Layer
  - field theoretic structure of rare events + activation

Disorder-induced glasses

- When do you call a system a glass?
  - Dramatic slow-down of dynamics, history dependence
  - System with a large multiplicity of low energy metastable states induced by frustration
- In disordered systems, competition amongst random and/or non-random interactions provides obvious frustration
  - Structural glasses?
- Disordered systems in statistical mechanics
  - Weak random bonds: diluted FM, dirty SC in zero field
  - Random fields: diluted AF in field, pinned elastic media
  - Maximum randomness: spin glass, gauge glass

(+ Disordered free particle/random matrix problems)

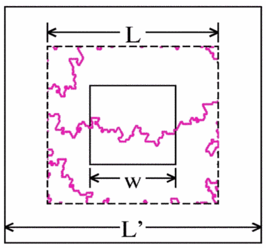
Physical Pictures of Glasses

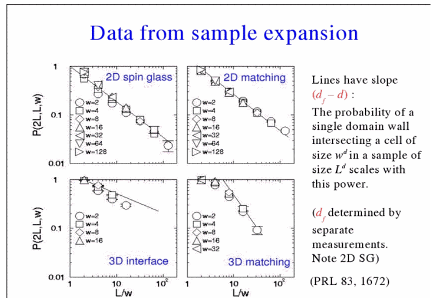
$$H = \sum_{ij} J_{ij} \sigma_i \sigma_j$$

- Old controversy: Replica Symmetry Breaking or Droplets?
  - “Strict” RSB: e.g. solution of SK model:
    - many non-symmetry related global “ground states”
  - Droplets:
    - unique ground state up to symmetry (prob ! 1)
    - designed for: local interactions with finite couplings
- Recent numerical simulations give support to droplet picture in some cases (Middleton, Young)

Testing many states

(From A.A. Middleton's KITP talk)





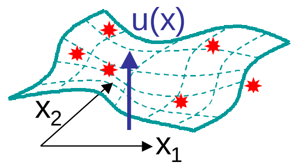
❖ Probability that interior subsystem "changes state" upon doubling system size vanishes rapidly
- In some 2d and 3d systems where numerically tractable

What to do theoretically?

- Need calculational tools for droplets
  - Theory is wholly phenomenological
    - Many unknown arbitrary parameters
    - No way to know which "guesses" are correct!
  - Challenge: droplet theory crucially involves *rare events*
- What about RSB?
  - There may be many ground states in some systems
    - If so, droplet ideas are probably still relevant to *excitations* above ground state
    - Probably some detailed modifications (fractal…)
  - Even if not, RSB is a computational tool
    - Exact at d=1 or N=1
    - Just means this limit is non-uniform and judiciously chosen quantities may be usefully extracted *if* we understand theoretical structure for d,N<1

http://online.itp.ucsb.edu/online/lnotes/balents/

Random Elastic Models



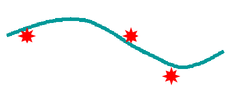
describes:
- interfaces
- vortex lines
- CDWs
- Random Field XY model…

$$H = \int d^d\mathbf{x} \left[\frac{1}{2}|\nabla u|^2 + V(u,\mathbf{x})\right]$$

$u(x) \in \mathbb{R}^N$
$x \in \mathbb{R}^d$

- Merits:
  - Describes many broad classes of materials
  - Demonstrably in glassy phase in most situations
  - Simple exposition of droplet physics
  - Amenable to both mean field/RSB (N=1) *and* functional renormalization group (d=4-ε) methods

Nature of the glassy phase



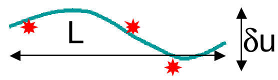
- Interface deforms to minimize V(u,x)
  - -Thermal fluctuations insufficient to “delocalize” interface (at low T and sometimes at all T)
- Larkin argument for ground state:
  - -Elasticity is strong in large d: sd^{d}x |ru|^2 » L^{d-2} u^2
    - ▪ assume small deformations

$$u_q = \frac{F_q(u=0)}{q^2} \Longrightarrow \overline{[u(x)-u(0)]^2} \sim \int \frac{d^d\mathbf{q}}{q^4}(1-\cos(\mathbf{q}\cdot\mathbf{x})) \sim |x|^{4-d}$$

- Interface deformations are large for d<4
  - -Expect non-trivial balance of disorder and elastic energy
  - -Many metastable solutions
  - -Expansion in ε=4-d?

Typical/Ground State Properties

- Many properties of *individual states* are non-trivial due to the frustrated Hamiltonian
  - Roughness:



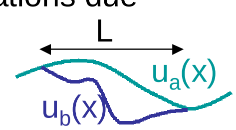
δu » L^{ζ}

  - Sample to sample energy fluctuations: δE » L^{θ}
  - Exponent relation: $\theta=d-2+2\zeta$

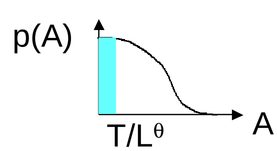
- These properties are non-trivial but do not directly reflect glassiness
  - Good agreement exists for such quantities between RSB, FRG, and numerical methods

Equilibrium Glassy Aspects

- Why equilibrium?
  - *Local* equilibrium occurs in finite time
  - Still anomalously strong thermal fluctuations due to many low energy excitations



- Local metastable excitations = "droplets"
  - Excitation energy $E_{ab}=E[u_b]-E[u_a]$ » AL^{θ}



- Typically, E_{ab} » AL^{θ} À T

- In rare samples/regions, $E_{ab} < T$ ⇒ thermally active fluctuations between two "wells"

Anomalous Fluctuations

- Long-time (equilibrium) averages (Boltzmann):

$$\langle (u - \langle u \rangle)^2 \rangle = \begin{cases} O(e^{-L^\theta/T}) & \text{prob } 1 - T/L^\theta \\ L^{2\zeta} & \text{prob } T/L^\theta \end{cases}$$

rare but not rare enough to be unimportant

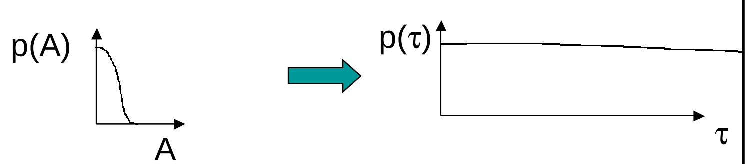
- Disorder averages

$$\overline{\langle (u - \langle u \rangle)^2 \rangle^n} \sim \frac{T}{L^\theta} L^{2n\zeta} \qquad \text{e.g.} \sim \frac{T}{L^\theta} \quad \text{for } \zeta = 0$$

- Replicas
  - -Two well quantities $\overline{\langle (u_a - u_b)^2 \rangle} \sim \frac{T}{L^\theta} L^{2\zeta}$
  - -Three well quantities $\overline{\langle (u_a - u_b)^2 (u_a - u_c)^2 \rangle} \sim \left(\frac{T}{L^\theta}\right)^2 L^{4\zeta}$
  - …

Activated Dynamics

- Near-equilibrium dynamics: $\eta \partial_t u = \nabla^2 u + F(u,x) + \zeta(x,t)$
- Relaxation controlled by *barriers*
  - -Activated dynamics $\tau \sim \tau_0 e^{E_B/T} \sim \tau_0 e^{AL^\psi/T}$ ($\psi = \theta$?)
  - -Broad distributions



- How to determine (and define) $p(\tau)$? Is $\tau_{typical}$ meaningful?

- Response functions $R_k(t) \sim \underbrace{\frac{1}{\tau_k} e^{-t/\tau_k}}_{\text{naive}} \overset{?}{\rightarrow} \frac{1}{t} F(Tk^\theta \ln t)$
- Aging » coarsening?

Challenge

- How can a field theory handle both typical and rare events?
  - – Typical properties are “critical” (u» L^{ζ} etc)
    - Such power-laws suggest continuum field theory
  - – This “critical” theory is completely defined by T=0 ground state properties!
  - – Anomalous thermal fluctuations and broad distributions must also be present!

Functional RG

KITP Talks:
Kay Wiese
Pierre Le Doussal

Method: D.S. Fisher, 1986

- Recall Larkin: $(\delta u)^2$ » L^{4-d} ⇒ expect “small” for d=4-ε
- Perturbative in V(u,x) i.e. u » L^{ζ}, ζ = O(ε)
- Scaling of all polynomials in u is O(ε)!

$$\overline{V(u,x)V(u',x')} = \delta(x-x')R(u-u')$$

-replicas
$$H_{rep} = \int_x \frac{1}{2}\sum_a (\nabla u_a)^2 - \frac{1}{2T}\sum_{ab} R(u_a - u_b)$$

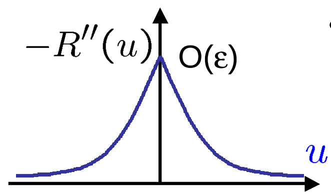
- Functional RG: x! e^{l} x, u! $e^{\zeta l}$ u

$$\partial_l R(u) = (\epsilon - 4\zeta)R + \zeta u \partial_u R + T_l R'' + \frac{1}{2}R''^2 - R''R''(0)$$

1 loop corrections

T_l = T $e^{-\theta l}$ » T/L^{θ}: temperature is “irrelevant”

T=0 Fixed Point



• Features
  - Unique solution for each universality class (B.C.s)
  - Non-analytic at u=0
    - avoids "dimensional reduction"

• Non-analytic effective action!
  - Usually integrating out finite wavevector modes *cannot* induce any singularity
  - At T=0, integration) minimization of H

• Cusp signals metastability despite V^2 » ε

$$\underline{R''''(0) = \overline{[V''(u)]^2} = \infty}$$

otherwise H'' >0

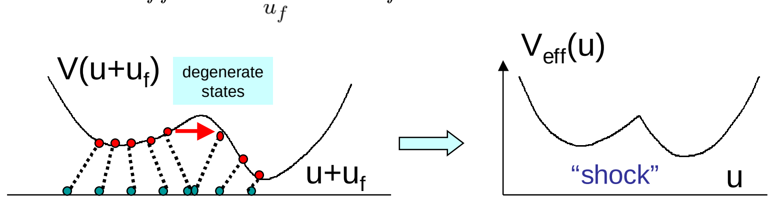
Meaning of the Cusp?

w/ J.-P. Bouchaud, M. Mezard

• "Toy RG"

$$H = \frac{q^2}{2}u^2 + \frac{\Lambda^2}{2}|u_f|^2 + V(u+u_f)$$

$$H_{eff}[u] = \underset{u_f}{\text{Min}}\, H[u,u_f]$$



• Linear cusp in –R''(u) reproduced in many limits
  - Large N
  - d=0 "toy model"
  - Relations to Burger's turbulence (d=1)

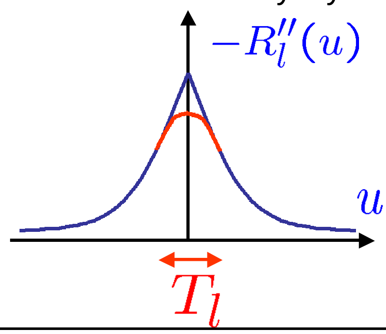
Thermal Boundary Layer (naïve)

LB 93
LB,MC Marchetti,+L Radzihovsky 98
Chauve, Giamarchi, Le Doussal 98

- T>0) no singularities from finite coarse-graining
- Naïve one-loop analysis:

$$(\epsilon - 4\zeta)R + \zeta u \partial_u R + T_l R'' + \frac{1}{2}R''^2 - R'' R''(0) = 0$$

- Thermal boundary layer for T_l ¿ ε^2 and ε u/T_l » O(1)



$$R_l(u) = R''(0)\frac{u^2}{2} + \frac{T_l^3}{\epsilon^2} r(\epsilon u / T_l)$$

⇩

$$-R_l''''(0) \sim \frac{\epsilon^2}{T_l} \sim \frac{L^\theta}{T}$$

Worry:
Large?
Non-perturbative?

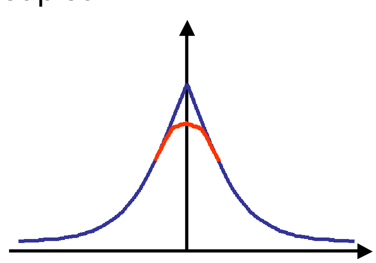
The TBL is Non-Perturbative

- Usual small parameter ε is not sufficient in the TBL!
- Replicated Hamiltonian:

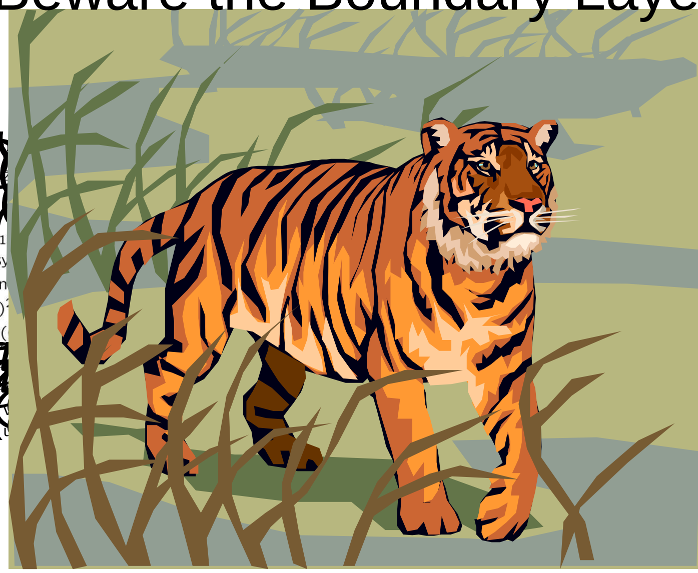
$$H_{rep} = \int_x \frac{1}{2}\sum_a (\nabla u_a)^2 - \frac{1}{2T}\sum_{ab} R(u_a - u_b) - \frac{1}{6T^2}\sum_{abc} S(u_a, u_b, u_c) - \cdots$$

- All (multilocal) cumulants are coupled and non-negligible in the TBL!

  -Might be anticipated from shocks:
  - rare but large force jumps imply highly non-Gaussian disorder



Beware the Boundary Layer



TBLAnsatz

- Boundary layer scaling can be consistently found:

$$S^{(p)}(u_{a_1},..u_{a_p}) = \underbrace{f_p\, u_{a_1}..u_{a_p}}_{\text{“Larkin” Random forces}} + \frac{T_l^{p+1}}{\epsilon^2}\underbrace{s^{(p)}\left(\frac{u_{a_1}\epsilon}{T_l},..,\frac{u_{a_p}\epsilon}{T_l}\right)}_{\text{Thermal activation physics}}$$

- What good is it?

(c.f. ERG)

- Predicts droplet “two-well” scaling

e.g. $\overline{\langle (u-\langle u\rangle)^2\rangle^n} \sim \frac{T}{L^\theta} L^{2n\zeta}$

- Can verify “three-well” scaling and many other properties in d=0 toy model $\overline{\langle (u_a-u_b)^2(u_a-u_c)^2\rangle} \sim \left(\frac{T}{L^\theta}\right)^2 L^{4\zeta}$

-Believe TBLA encodes droplet scaling distributions, correlations…*violations of droplet picture?*

Dynamics

- TBLA describes activated dynamics:
- E.O.M.) Dynamical Field Theory (MSR)

$$S = \int_{xt} \left[\hat{u}_{xt}(\eta\partial_t - \nabla_x^2)u_{xt} - T\hat{u}_{xt}^2\right] - \frac{1}{2}\int_{xtt'} \hat{u}_{xt}\hat{u}_{xt'}\Delta(u_{xt}-u_{xt'})$$

- Force-force correlator $\Delta(u) = -R''(u)$

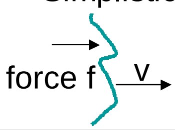
- Naïve renormalization:

$$\frac{\partial_l \eta_l}{\eta_l} = \Delta_l''(0) \sim \frac{\epsilon^2}{T_l}$$

$$\tau_L \sim \eta_l \sim e^{L^\theta/T}$$

(ψ=θ)

- Simplistic approach keeping this term gives “creep” law



$$v \sim e^{-1/Tf^\mu}$$

$$\mu = \frac{\psi}{d+\zeta-\psi} = \frac{\theta}{2-\zeta}$$

Chauve *et al*

Broadly Distributed τ’s

- Extremely rapid growth of η_l looks dangerous
  - Fortunately,statics is protected by “frequency conservation” diagrammatic property
- Higher moments of relaxation times are generated and grow rapidly

$$S_{friction} \to \sum_p \frac{(-1)^{p+1}}{p!}\eta_p \int_x \left[\int_t \hat{u}_{xt}\partial_t u_{xt}\right]^p$$

$$\underbrace{\eta(x)}\partial_t u = \nabla^2 u + F(u,x) + \zeta(x,t)$$

random: moments grow $\eta_p = \overline{\eta^p} \sim \tau_L^{\alpha(p)}$

α(p) ¼ $2p^2$-p approx. result

- static and additional dynamic TBLA hierarchies govern relaxation spectrum and response functions

Where We Stand

- "Matching": need to connect the TBLA to the ε-expansion outside the BL
  - -Determine if T=0 FRG is well behaved
  - -Give 1st principles derivation of "anomalous" T=0 terms

c.f. Kay $$\partial_l R(u) = \cdots + \frac{1}{2} R''(u) \left([R'''(u)]^2 - \underbrace{[R'''(0^+)]^2}_{\text{can } \textit{derive} \text{ this in d=0}} \right)$$

- Want to go beyond TBL*Ansatz* to TBL*Analysis!*
  - -universal amplitudes
  - -higher orders in ε
  - -dynamic response functions
  - -aging dynamics
  - -deep understanding of rare events and glasses

Conclusions

- We are beginning to understand the structure of an analytic theory of rare events and thermal fluctuations in disordered glasses outside MFT
- Such understanding has very broad applications (also to quantum glasses)

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