P1

IS THERE A SPIN GLASS IN YOUR CELLULAR PHONE ?

YES, AND IT'S TRYING TO MINIMIZE ITS BETHE FREE-ENERGY.

[OR THERE WILL BE SOON!!]

A.M.
LPTENS, PARIS

N. SOURLAS
S. FRANZ
M. LEONE
F. RICCI-TERSENGHI

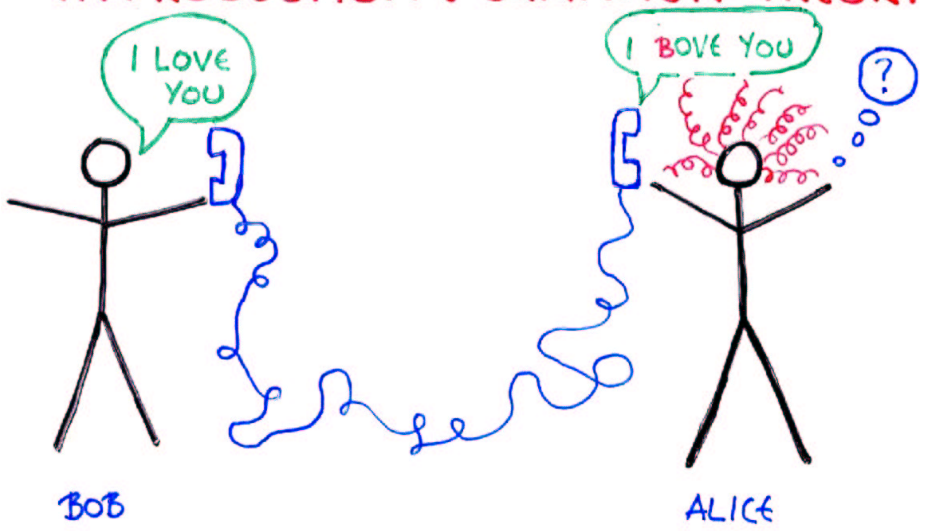
P2

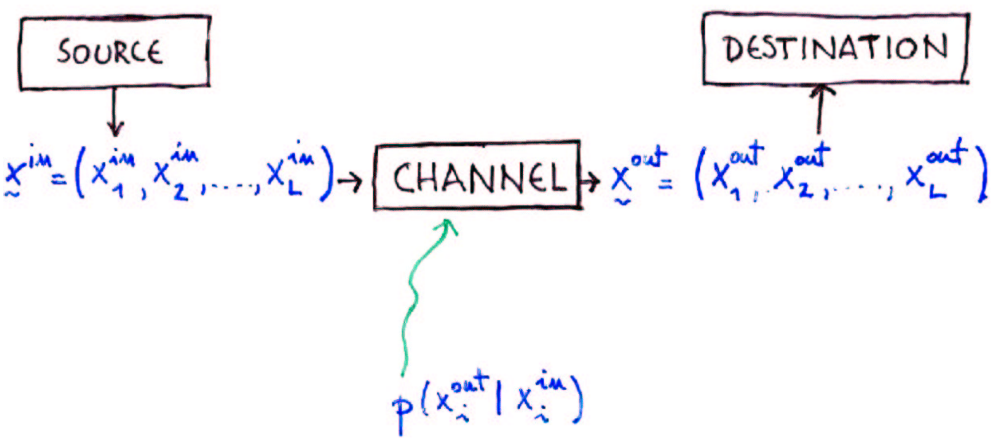
PLAN :

- INTRODUCTION : SHANNON THEORY
- INTRODUCTION : LOW DENSITY PARITY CHECK CODES
- INTRODUCTION : ITERATIVE DECODING
- THE PHASE DIAGRAM
  [OR: WHAT CODING THEORY CAN LEARN FROM PHYSICS]
- CONCLUSION
  [OR : WHAT PHYSICS CAN LEARN FROM CODING THEORY]

P3

INTRODUCTION : SHANNON THEORY

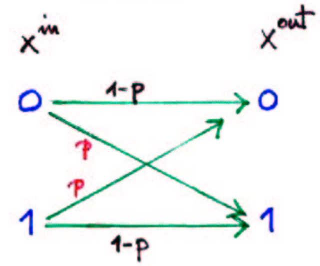




P4

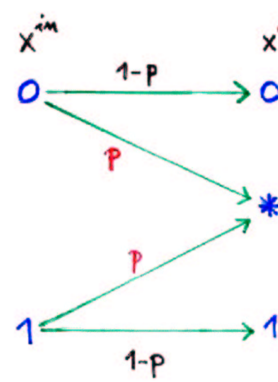
NOISY CHANNELS

- example 1 : BINARY SYMMETRIC CHANNEL (BSC)



$p \equiv$ "FLIP PROBABILITY"

- example 2 : BINARY ERASURE CHANNEL (BEC)



$p \equiv$ "ERASURE PROBABILITY"

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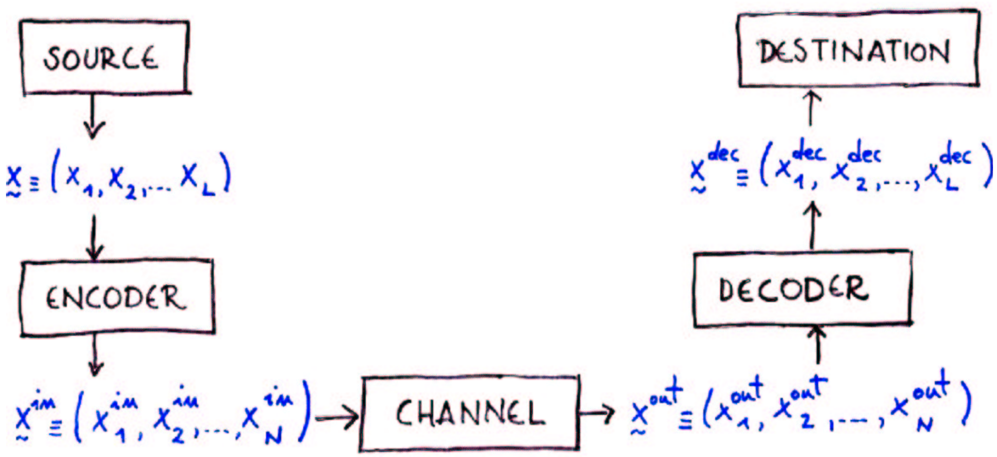
ADD REDUNDANCY!



$$\boxed{R \equiv \frac{L}{N} < 1}$$

RATE

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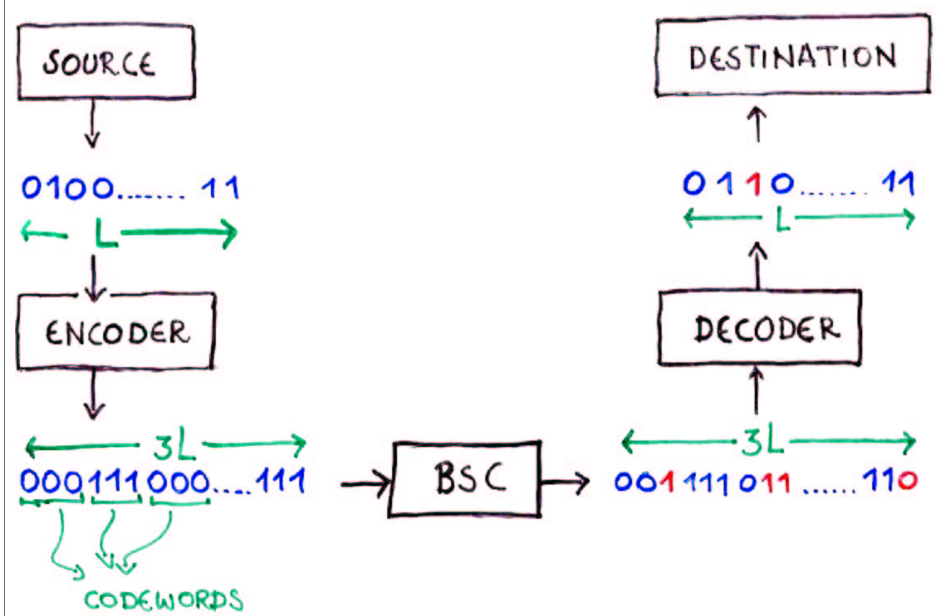
example : REPETITION CODE



$$R = \frac{1}{3}$$

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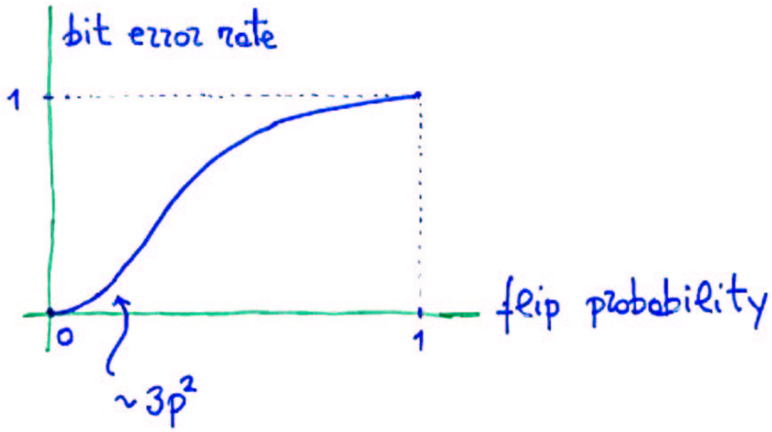
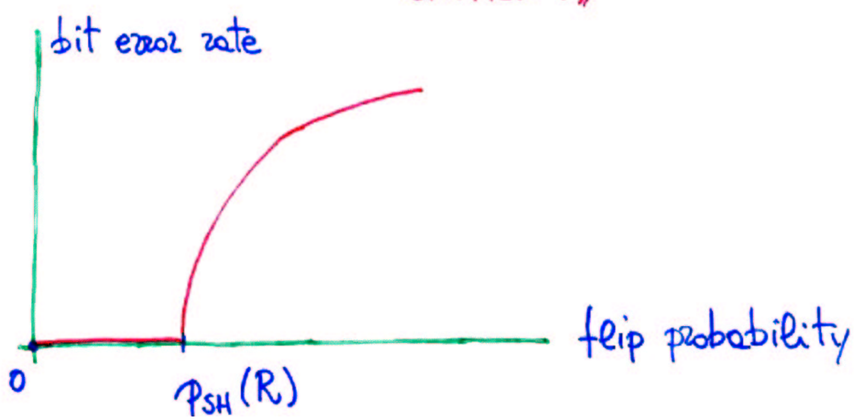
PERFORMANCES



SHANNON SAID [1948]: "YOU CAN COMMUNICATE ERROR-FREE BELOW CAPACITY"



P8

GOOD NEWS ⟶ "ALMOST ANY CODE ACHIEVE SHANNON BOUND"

BAD NEWS ⟶ "DECODING IS (IN GENERAL) AN NP-COMPLETE PROBLEM"

[BERKELAMP, MC ELIECE, VAN TILBORG]

INTRODUCTION: LOW DENSITY PARITY CHECK CODES

[GALLAGER, MAC KAY-NEAL, RICHARDSON-URBANKE, LUBY, SPIELMAN, ...]

[KABASHIMA, SAAD, KANTER, A.M, SOURLAS ..]

$$\underset{\sim}{z} = (z_1 .. z_L) \to \boxed{\text{ENCODER}} \to \underset{\sim}{x} = (x_1, x_2, \ldots x_N)$$

$$\boxed{\underset{\sim}{x} = \mathbb{G}\, \underset{\sim}{z} \quad (\text{mod } 2)}$$

$$\mathbb{G} = \begin{array}{c} \left[\begin{array}{l} 0110110\ldots0 \\ 1101001\ldots1 \\ 100111\ldots \\ 110\ldots \\ \vdots \\ \\ 0 \end{array}\right] \\ \xleftarrow{\quad L \quad} \xrightarrow{\quad} \end{array} \begin{array}{c} \uparrow \\ N \\ \downarrow \end{array}$$

CODEWORDS FORM A LINEAR SPACE :

$$\{ \underset{\sim}{x} = \mathbb{G}\underset{\sim}{z} : \underset{\sim}{z} \in \{0,1\}^L \} =$$

$$= \{ \underset{\sim}{x} \in \{0,1\}^N : \mathbb{H}\underset{\sim}{x} = 0 \}$$

$$\mathbb{H} = \begin{array}{c} \xleftarrow{\quad N \quad} \xrightarrow{\quad} \\ \left[\begin{array}{l} 000100000000110000000\,1 \\ 010000000100000100100 \\ 000010000001000011000 \\ 010000000000000001110 \\ \vdots \quad\quad\quad\quad \vdots \\ \\ 0 \ldots \end{array}\right] \end{array} \begin{array}{c} \uparrow \\ M = N - L \\ \downarrow \end{array}$$

PARITY CHECK MATRIX :

RANDOM AND SPARSE

- k ONES PER ROW
- ℓ ONES PER COLUMN

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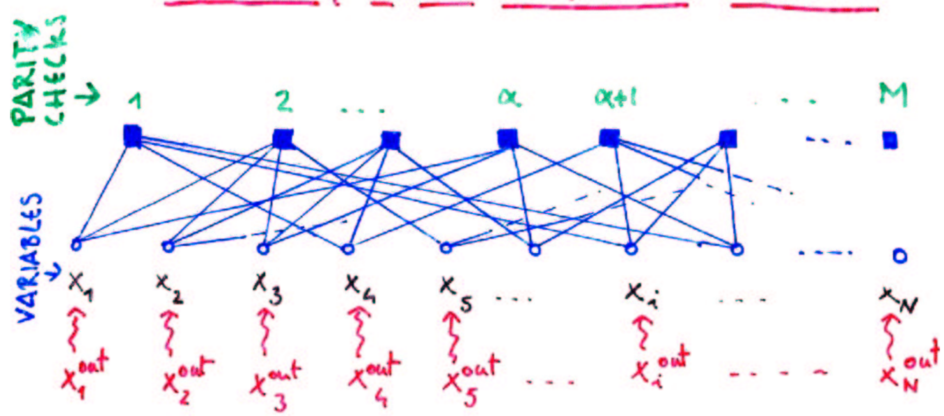
DECODING IS AN INFERENCE PROBLEM



$\boxed{\alpha=2} \Rightarrow x_1 \oplus x_2 \oplus x_3 \oplus x_5 = 0$

$$\boxed{P(\underset{\sim}{x} \mid \underset{\sim}{x}^{out}) = \frac{1}{Z} \prod_{\alpha=1}^{M} \psi(x_{\alpha_1}, \dots x_{\alpha_k}) \prod_{i=1}^{N} p(x_i^{out} \mid x_i)}$$

INTERACTIONS

RANDOM FIELD

PROBABILITY FOR $\underset{\sim}{x}$ TO BE THE CHANNEL INPUT

$$\psi(x_1 \dots x_k) = \begin{cases} 1 & \text{if } x_1 \oplus x_2 \oplus \dots \oplus x_k = 0 \\ 0 & \text{ow.} \end{cases}$$

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DECODING PROBLEM : COMPUTE

$$\boxed{P(x_i \mid \underset{\sim}{x}^{out}) = \sum_{\{x_j\}_{j \neq i}} P(\underset{\sim}{x} \mid \underset{\sim}{x}^{out})}$$

EXPONENTIAL NUMBER OF SUMMANDS!!

$$x_i^{dec} = \arg\max_{x_i} P(x_i \mid \underset{\sim}{x}^{out})$$

DECODER OUTPUT

[MAP DECODING]

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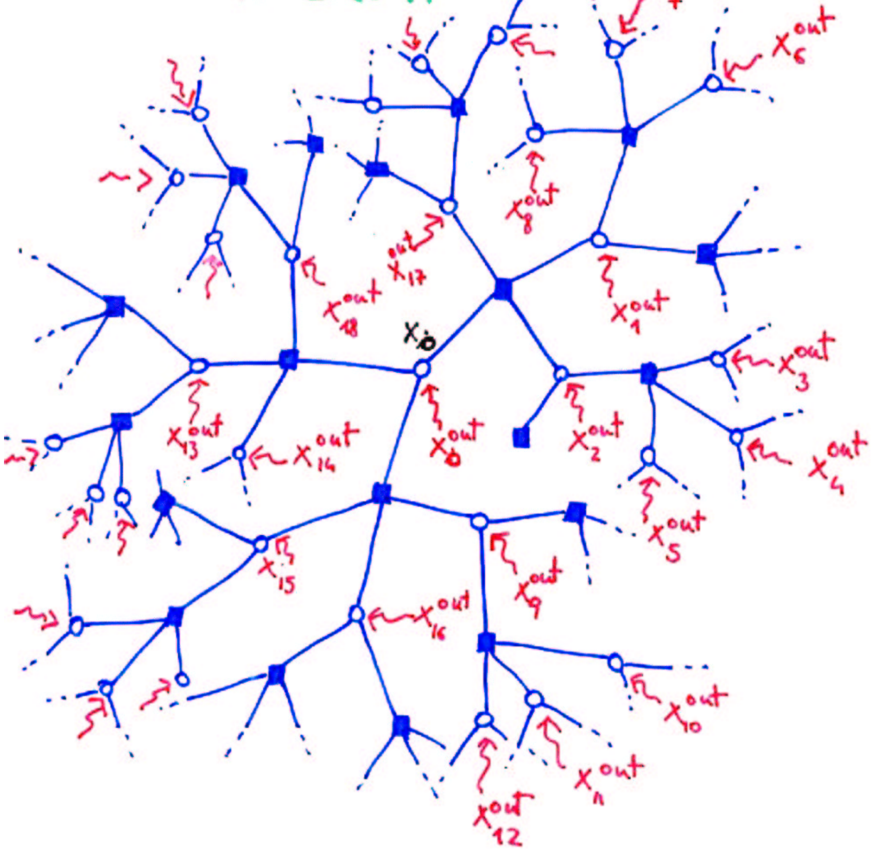
ITERATIVE DECODING

THE INTERACTION GRAPH IS LOCALLY TREE-LIKE !!



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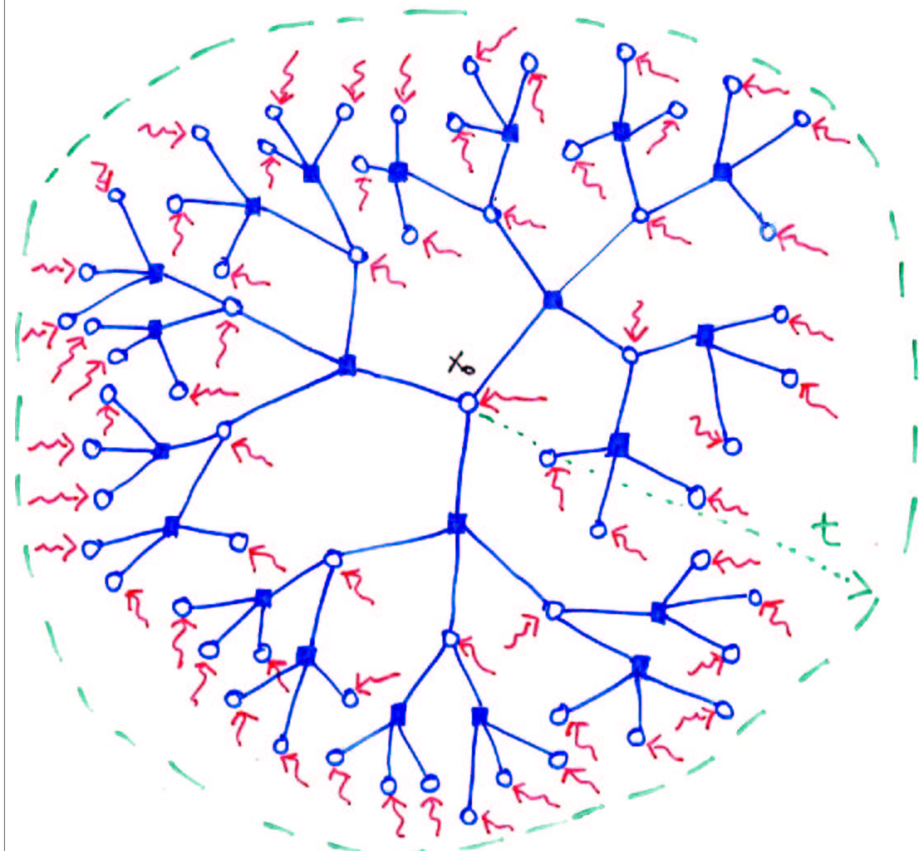
1) CUT THE GRAPH AT A DISTANCE t FROM THE ROOT....



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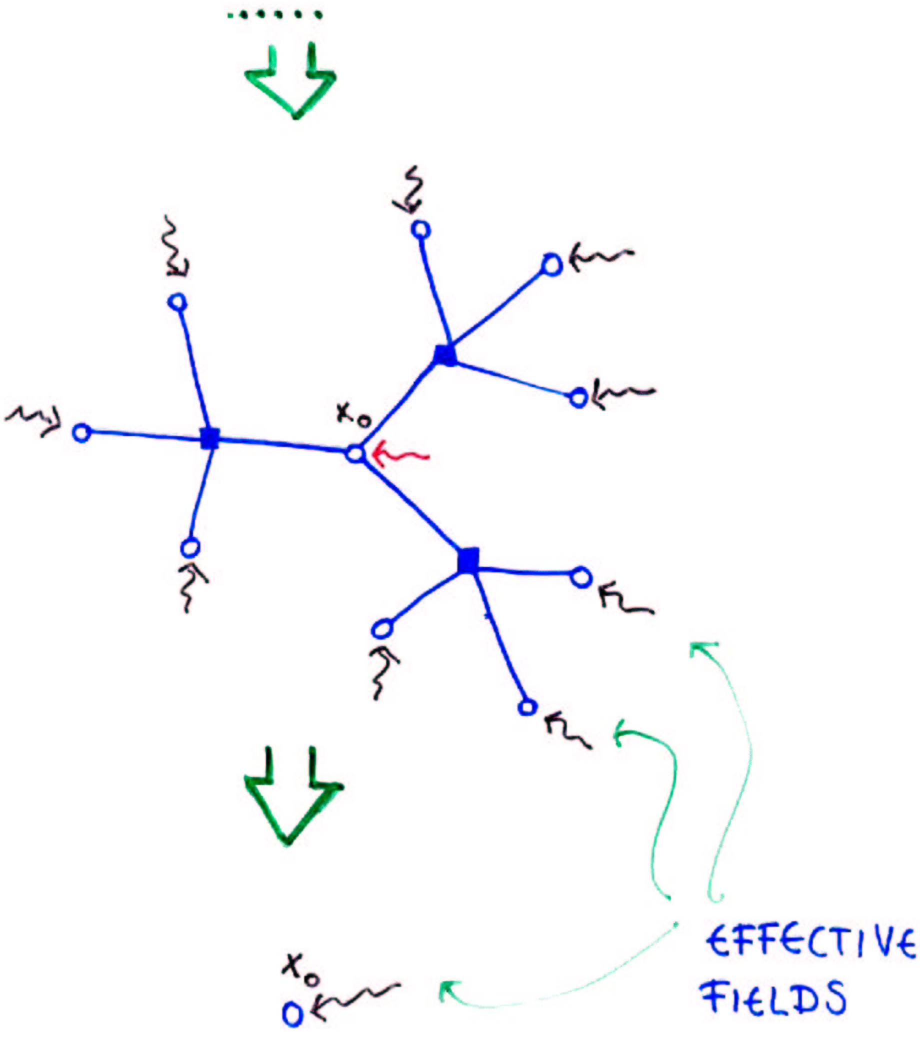
2)..AND SUM RECURSIVELY OVER THE LEAVES



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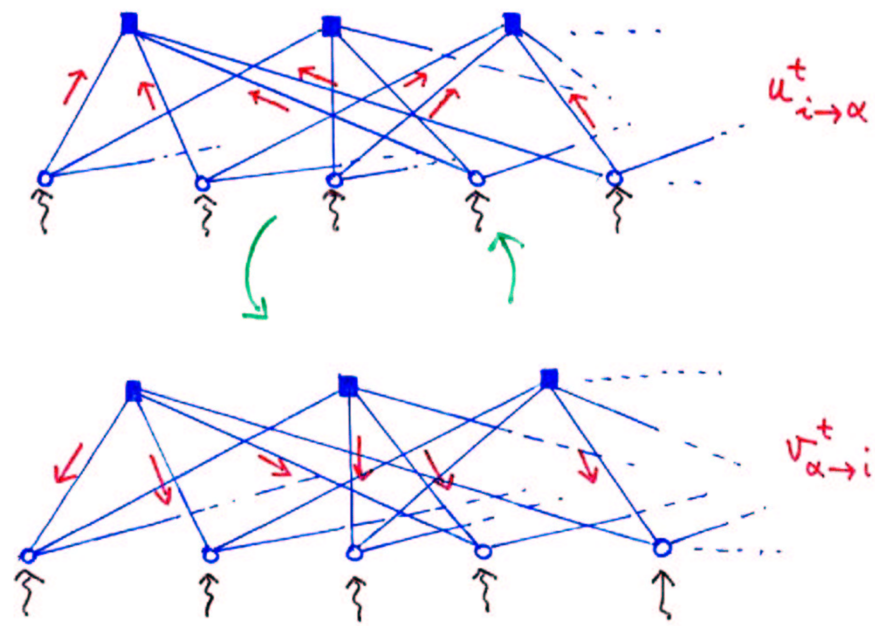
MESSAGE-PASSING SCHEDULING

(BELIEF PROPAGATION)



$\{u^t_{i\to\alpha}, v^t_{\alpha\to i}\} \in \mathbb{R}$

MESSAGES

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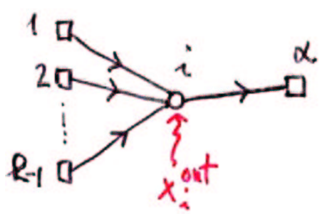
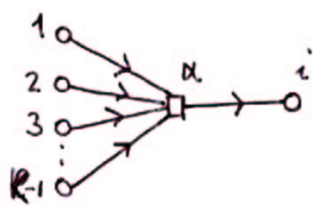
MESSAGE PASSING: UPDATES AT THE NODES

- VARIABLE NODES



$$\boxed{u^{t+1}_{i\to\alpha} = h(x_i^{out}) + \sum_{\beta=1}^{\ell-1} v^{t}_{\beta\to i}}$$

- CHECK NODES



$$\boxed{v^{t+1}_{\alpha\to i} = \mathrm{arctgh}\left[\prod_{j=1}^{k-1} \mathrm{tgh}\; u^{t}_{j\to\alpha}\right]}$$

CAVITY EQUATIONS

[≡ SADDLE POINT EQUATIONS
OF THE BETHE FREE ENERGY $\Phi(\underset{\sim}{u}, \underset{\sim}{v})$]

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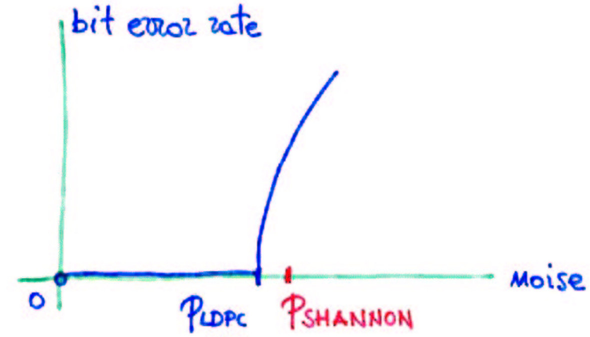
PERFORMANCES



AMAZINGLY CLOSE TO SHANNON BOUND !!!
[cf. CHUNG, FORNEY, RICHARDSON URBANKE]

AND THEORETICAL QUESTIONS

- DOES THE ALGORITHM COMPUTE THE CORRECT MARGINALS ?
- WOULD MAP-DECODING IMPROVE THE PERFORMANCES ?
- COULD A DIFFERENT LOW-COMPLEXITY

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A LITTLE BIT OF PHYSICS

$$P(\underset{\sim}{x} \mid \underset{\sim}{x}^{out}) = \frac{1}{Z} \prod_{\alpha} \phi(x_{\alpha_1} \ldots x_{\alpha_k}) \prod_i p(x_i^{out} \mid x_i)$$

$$\boxed{H(\sigma) = -\beta \sum_{\alpha} \sigma_{\alpha_1} \ldots \sigma_{\alpha_k} - \sum_{i=1}^{N} h_i \sigma_i}$$

$\sigma_i = (-1)^{x_i}$

FERROMAGNETIC MULTI-BODY INTERACTIONS ($\beta \to \infty$)

"SELF-INDUCED" FRUSTRATION

RANDOM MAGNETIC FIELD

LOW NOISE $\leftrightarrow$ $h_i > 0$

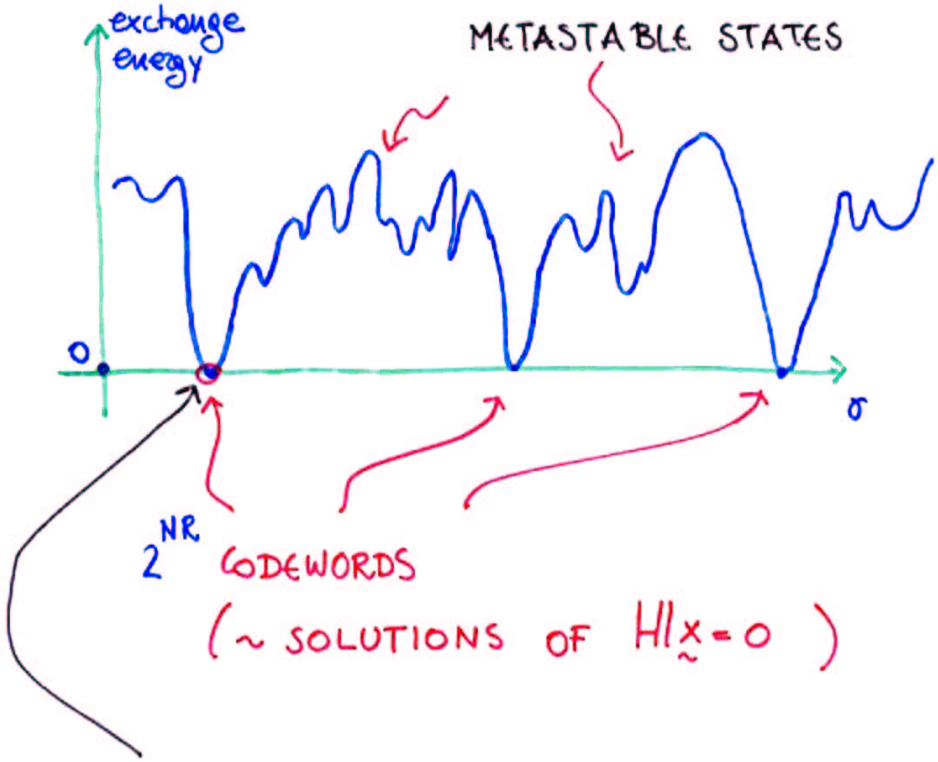
HIGH NOISE $\leftrightarrow$ $h_i \gtrless 0$

MORE BIASED | LESS BIASED

P20

THE HANDWAVING PICTURE:

WITHOUT EXTERNAL FIELD



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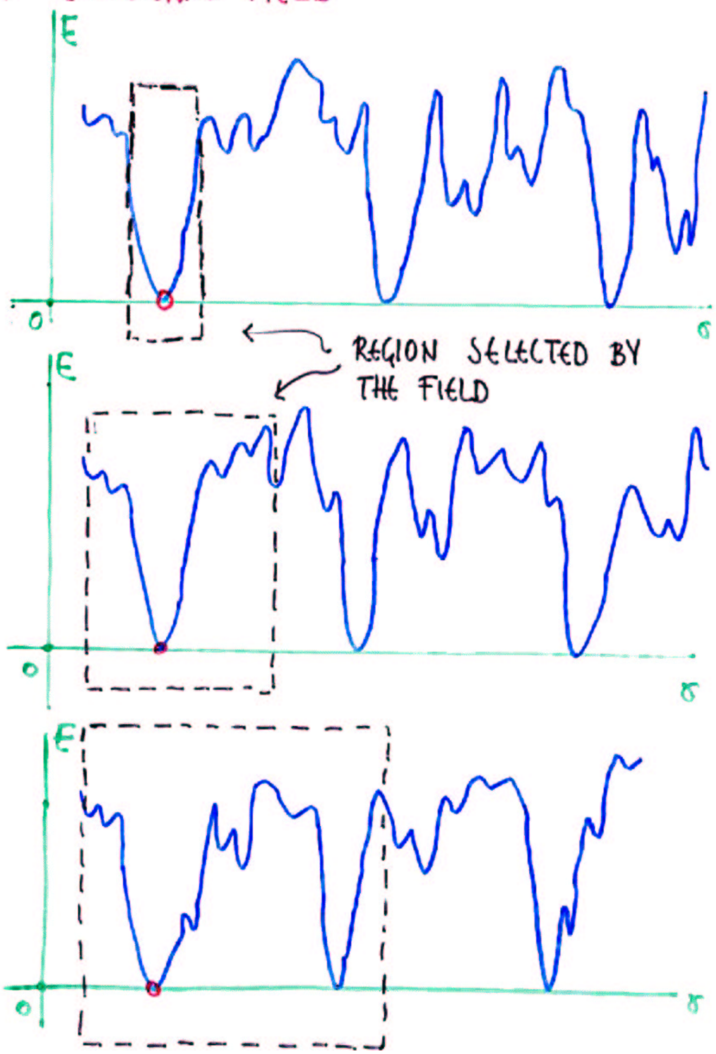
THE HANDWAVING PICTURE : WITH EXTERNAL FIELD

LOW-NOISE : $p < p_d$

INTERMEDIATE NOISE $p_d < p < p_c$

HIGH NOISE $p_c < p$



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THE CALCULATION

$H(\sigma)$

$\downarrow$

REPLICAS

$\downarrow$

1RSB [FUNCTIONAL ORDER PARAMETER]

$\downarrow$

$\phi(m)$ [REPLICATED FREE ENERGY]

$\downarrow$ cf Monasson

$$\Sigma_1(\epsilon) = \frac{1}{N} \log \mathcal{N}_{MS}(\epsilon)$$

COMPLEXITY

NUMBER OF METASTABLE STATES

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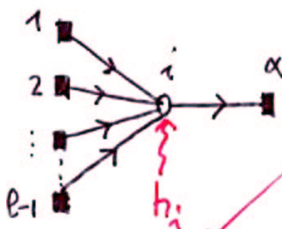
THE CALCULATION [AN ALTERNATIVE POINT OF VIEW]

LET US COUNT THE NUMBER OF FIXED POINTS OF THE ALGORITHM



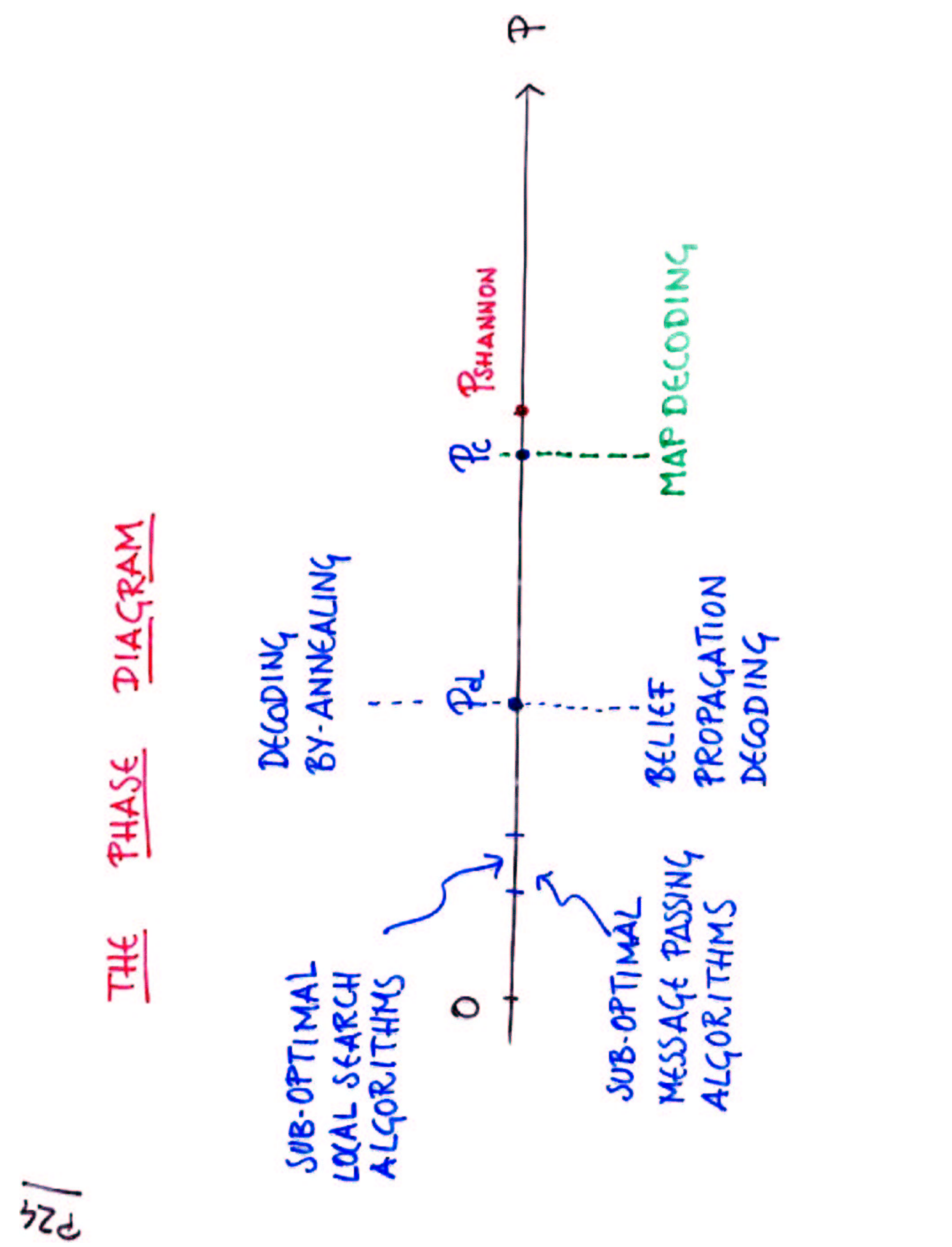
NUMBER OF FIXED POINTS S.T. $u_{i\to\alpha} = u$ AND WITH FREE ENERGY F

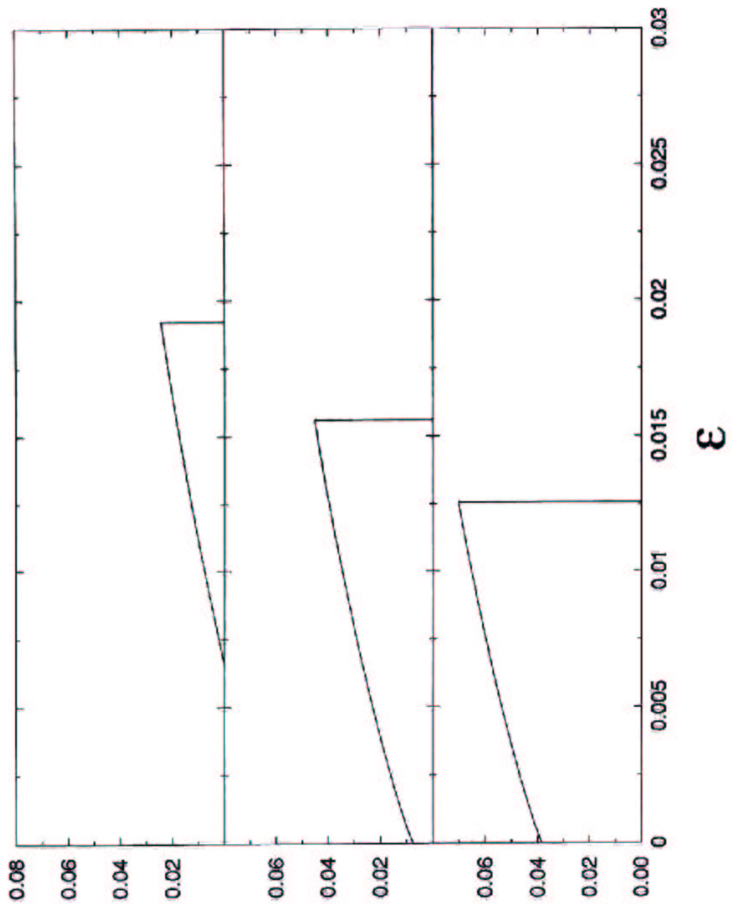
NUMBER OF ....

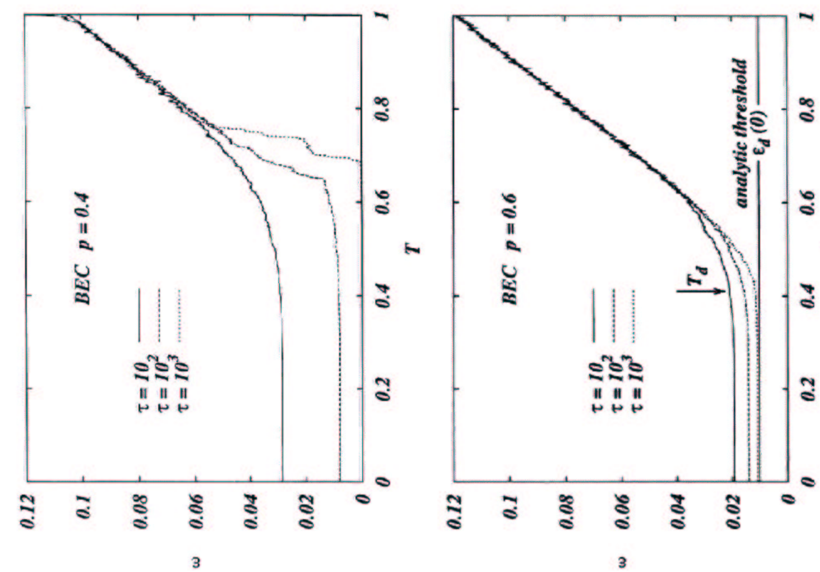
$$\mathcal{N}_{i\to\alpha}(u;F) = \int \prod_{\beta=1}^{l-1} d\mathcal{N}_{\beta\to i}(v_\beta;F_\beta) \cdot \delta\Big(u - h_i - \sum_\beta v_\beta\Big) \cdot \delta\Big(F - \sum_\beta F_\beta - \Delta F(\{v\})\Big)$$

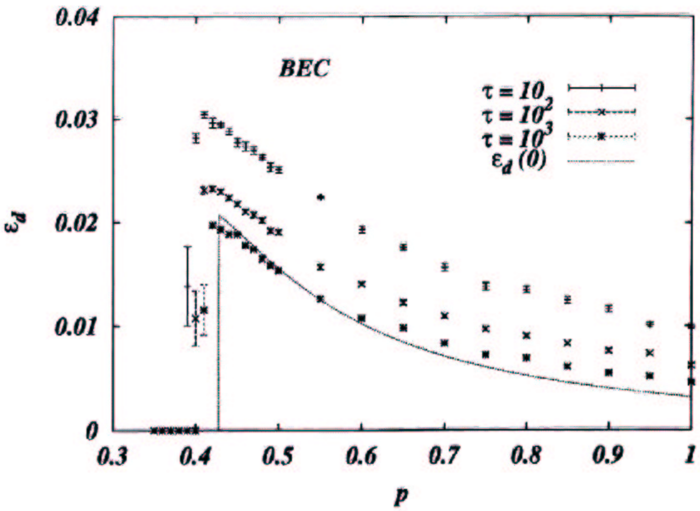
FIXED POINT CONDITION

[PLUS A SIMILAR EQUATION FOR THE CHECK NODES UPDATING]




ε
0.08
0.06
0.04
0.02
0.00
0
0.005
0.01
0.015
0.02
0.025
0.03


BEC p = 0.4
BEC p = 0.6
τ = 10
τ = 10^2
τ = 10^3
T_d
analytic threshold
$\varepsilon_d(0)$
T
ε
0.12
0.1
0.08
0.06
0.04
0.02
0
0.2
0.4
0.6
0.8
1



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DECODING-BY-ANNEALING

$$H(\sigma) = -\beta \sum_{\alpha} \sigma_{\alpha_1} \cdots \sigma_{\alpha_k} - \sum_{i=1}^{N} h_i \sigma_i$$

1. SET $\beta = 0$
2. RUN A MONTE CARLO ALGORITHM FOR n STEPS
3. IF $\beta < \beta_{MAX}$, SET $\beta \leftarrow \beta + \Delta\beta$ AND GOTO 2.
4. SET $\sigma_i^{dec} = \sigma_i$

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CONCLUSION

[OR: WHAT PHYSICS CAN LEARN FROM THIS STORY]

- SINGLE-SAMPLE POINT OF VIEW

- THINK TO PHYSICAL APPROXIMATIONS 'ALGORITHMICALLY'