Hysteresis in Spin Glasses

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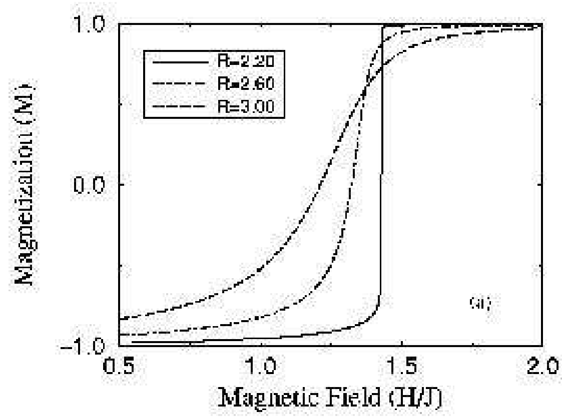
Motivation

- Recording media community and spin glass community can mutually benefit from each other's experience.
- Central object of interest for recording media: hysteresis
- In spite of nearly 30 years of research, the hysteresis of spin glass systems is not studied in the same detail as other aspects

Hysteresis of the RFIM



J. Sethna, K. Dahmen
PRL '93

$$H = J\sum_{<ij>} S_i S_j - \sum_i (h_i + H) S_i$$

T=0

Non-equilibrium

Tune disorder R

Phase transition at $R=R_c$ $H=H_c$

M(H) jumps for $R<R_c$

Avalanche distributions

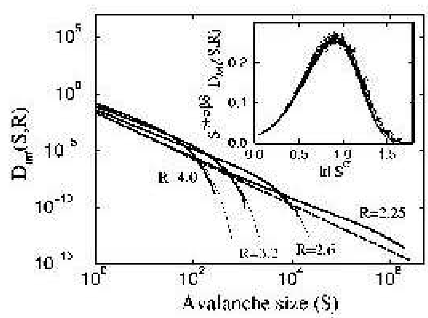


FIG. 3. Integrated avalanche size distribution curves in three dimensions for 320^3 spins and disorders $R=4.0, 3.2$, and 2.6. The last curve is at $R=2.25$, for a 1000^3 spin system. The 320^3 curves are averages over up to 16 initial random-field configurations. The inset shows the scaling collapse of the integrated avalanche size distribution curves in three dimensions, using $r=(R-R_c)/R$, $\tau+\sigma\beta\delta=2.03$, and $\sigma=0.24$, for sizes 160^3, 320^3, 800^3, and 1000^3, and disorders ranging from $R=2.25$ to $R=3.2$ ($R_c=2.16$). The two top curves in the collapse, at $R=3.2$, show no-

Calculate avalanche size distribution along hysteresis loop

Scaling collapse with $r=(R-R_c)/R_c$

Determine exponents

Critical region unusually large

(scaling 50% away from Rc)

No passing (Middleton)

-> Return point memory

Exponents and RG

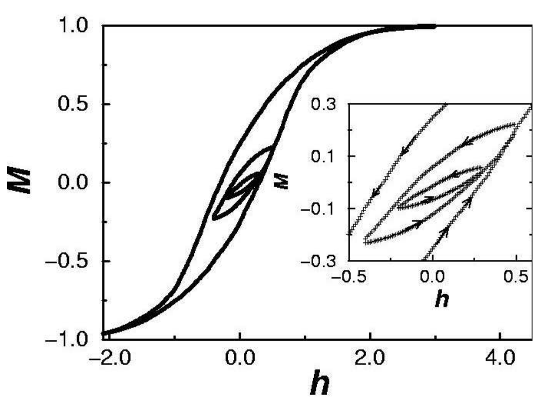
Measured exponents	3d	4d	5d	Mean field
$1/\nu$	0.71 ± 0.09	1.12 ± 0.11	1.47 ± 0.15	2
θ	0.015 ± 0.015	0.32 ± 0.06	1.03 ± 0.10	1
$(\tau+\sigma\beta\delta-3)/\sigma\nu$	-2.90 ± 0.16	-3.20 ± 0.24	-2.95 ± 0.13	-3
$1/\sigma$	4.2 ± 0.3	3.20 ± 0.25	2.35 ± 0.25	2
$\tau+\sigma\beta\delta$	2.03 ± 0.03	2.07 ± 0.03	2.15 ± 0.04	9/4
τ	1.60 ± 0.06	1.53 ± 0.08	1.48 ± 0.10	3/2
$d+\beta/\nu$	3.07 ± 0.30	4.15 ± 0.20	5.1 ± 0.4	7 (at $d_c=6$)
β/ν	0.025 ± 0.020	0.19 ± 0.05	0.37 ± 0.08	1
$\sigma\nu z$	0.57 ± 0.03	0.56 ± 0.03	0.545 ± 0.025	1/2

- Renormalization group study (Dahmen & Sethna) (6-ε expansion):
- This non-eq. RG can be mapped on RG of equilibrium RFIM:
  exponents are the same
- Transition in 2D??
  Numerically, Rc reported in the range of 0.9 – 1.3,
  but Rc=0 is suspected.

Hysteresis of the Sherrington-Kirkpatrick model

F. Pazmandi, G. Zarand, GTZ, Phys. Rev. Lett. **83**, 1034 (1999)

$$H=\sum_{ij} J_{ij} S_i S_j - H\sum_i S_i$$



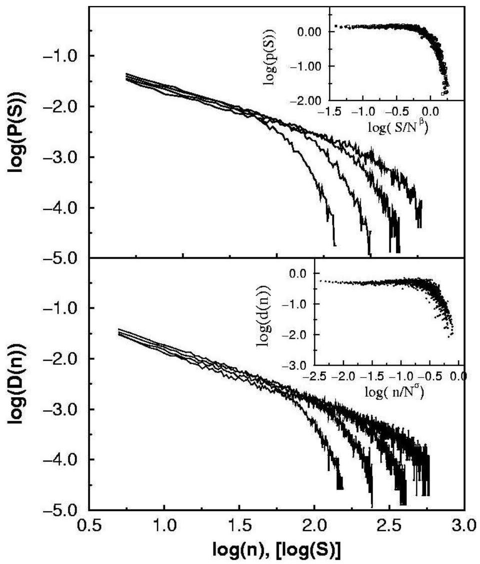
Remarkably, in 25 years there was minimal work on the hysteresis of the SK model (early work: Bertotti et al. ’91)

Procedure:

- T=0
- Start from saturation
- Decrease H with small step
- Calculate local field
- If spin points against local field, flip it
- Recalculate local fields
- Continue until all spins are aligned with local field

# of spins N ~ 4000
Realizations ~ 100-500

Avalanche distributions



No scaling with r=(R-Rc)/Rc

Only scaling variable:N!

S: magnetization change by avalanche

$$P(S) = (A/\ln N) S^{-\tau} p(S/N^{\beta})$$

$$\tau = 1 \pm 0.1 \quad \beta = 0.6 \pm 0.1$$

n: number of spins in avalanche

$$D(n) = (B/\ln N) n^{-\rho} d(n/N^{\sigma})$$

$$\rho = 1 \pm 0.1 \quad \sigma = 0.9 \pm 0.1$$

P(S) not equal D(n), because frustration forces many spins to flip against majority

Self-organized criticality

Only scaling variable: N – Self Organized Criticality (SOC)

Origin:

- Spin S_j flips, local field of spin S_i changes by $2J_{ij} \sim 1/N^{1/2}$
- Local field is spaced by dh ~ $1/N^{1/2}$
- External field has to be increased by $1/N^{1/2}$ to start new avalanche
- S is change of total magnetization, dm=S/N is change of magnetization/spin
- Plausibly, susceptibility χ = dm/dh is finite, confirmed numerically
- dm/dh ~ (S/N)/(1/$N^{1/2}$) ~ S/$N^{1/2}$ finite:
- S~$N^{1/2}$ $\rightarrow \beta = 0.5$ consistent with measured value $\beta = 0.6 \pm 0.1$
- In sum: AVALANCHES ARE MACROSCOPIC, LIKE IN SYSTEMS AT CRITICALITY
- Energy dissipation during an avalanche:

$\rightarrow \sigma = 1$ consistent with measured value $\sigma = 1 \pm 0.1$

Avalanches, stabilities

- Average avalanche size ~ $N^{1/2}$
- Typical avalanche size ~ $N^{-1/2}$
- Few large avalanches dominate the hysteresis/reversal: Barkhausen noise
- May characterize hierarchical structure of barriers

- Physical picture: local stability:

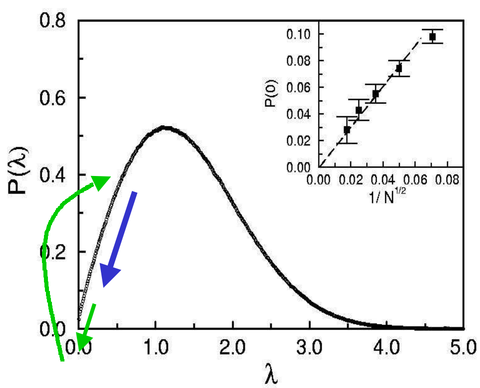
$$\lambda_i = S_i h_i = HS_i + \sum_j J_{ij} S_i S_j$$

The distribution of the local stabilities is the same along the hysteresis loop
We flip n_{flip} spins, and then calculate the number of spins which become unstable.
If $n_{unst.} \sim n_{flip}$, then the system is critical.
We showed that $n_{unst.} \sim n_{flip}$ requires:

$$P(\lambda) = C\,\lambda^{\alpha}$$
$$C = 1, \quad \alpha = 1$$

Numerically confirmed

Distribution of local stabilities



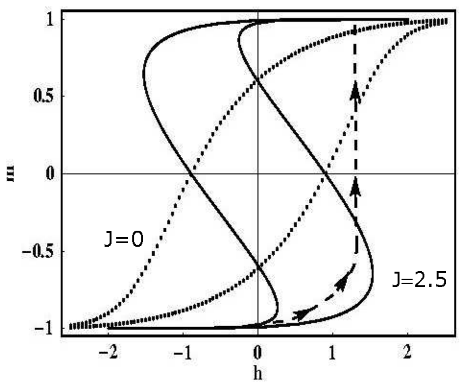
- Avalanche spins:
  Before the flip, small λ's were pulled negative by other just flipped avalanche spins. After the flip, λ is restored to a positive, non-small value.

- Non-avalanche spins
  The λ's of the non-flipping spins are changed by a random amount. But on a sloped distribution this produces a diffusion toward smaller λ values.

Analogy to sand-pile physics

Analogy to Coulomb gap

Analytic calculation



Number of single spin flip stable states:

$$<V(m,H)>=\left\langle Tr\left\{\prod_{i=1}^{N}\Theta(\lambda_i)\delta(mN-\sum_i S_i)\right\}\right\rangle$$

V~exp(NΩ),

Plotted: $\Omega=0$

Inside loop, V is exponentially large:
avalanches likely get trapped in
one of the single spin flip stable states.

Coercive field is off by a factor of 3:
Needed: replica symmetry breaking

Speculation: relaxation in self-organized critical systems

Guess based on self-organized criticality: **power law decay of M(t)?**

Our work: T=0, but e.g. θ=0.2 is insensitive to T. (Bray-Moore, Komori et al.)

Lot of work on out-of-equilibrium dynamics, aging:

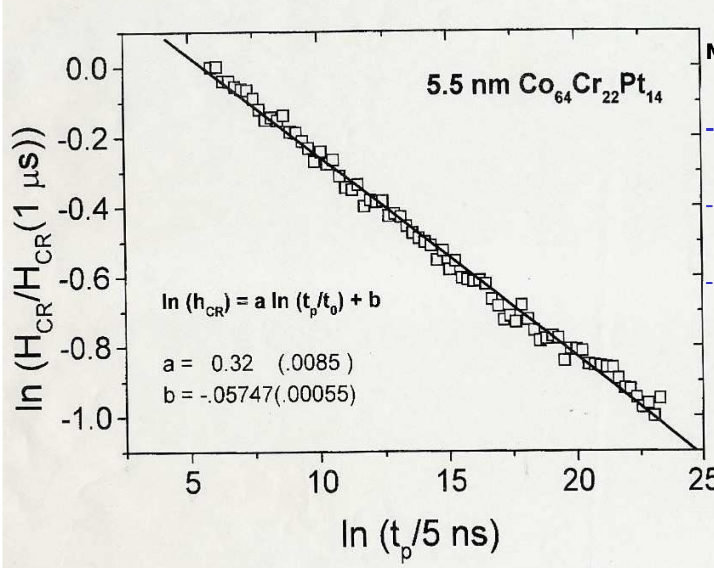
Replica theory-related work:
- Sompolinsky-Zippelius, Sompolinksy '81
- Franz-Mezard '93
- Cugliandolo-Kurchan '93

Numerical work:
- Binder '80s
- Kisker et al '93
- Rieger '96
- Marinari-Parisi '98
- Komori et al. '99-'03

Droplet work:
- Fisher-Huse '86
- Bray-Moore '84

Many of these works also report power law decay in finite D Ising glasses.

Relaxation in magnetic films with long range interactions

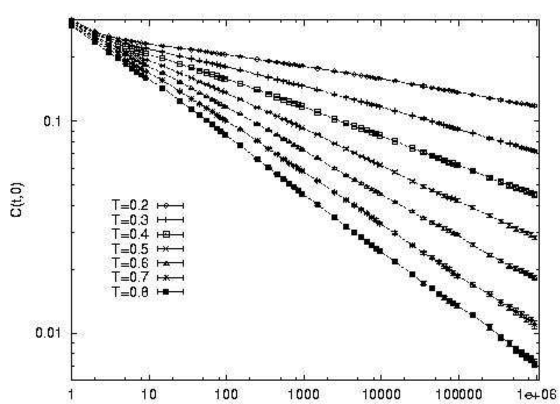


Moser, Weller '99

- strong anisotropy: Ising-like spins,
- long range interaction
- dipolar interaction changes sign: frustration

Power law over 6 decades!

Relaxation in the 3D Edwards-Anderson Spin Glass



Rieger '96

FIG. 1. The remanent magnetization of the 3D EA-model (1) for temperatures $T = 0.2$ to $T = 0.8$ (from top to bottom).

Power law

The EA Ising Spin-glass

Katzgraber, Pazmandi, Pike, Liu, Scalettar, Verosub, GTZ
Phys. Rev. Lett. 89, 257202 (2002)

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j + H \sum_i S_i \qquad S_i \in \{\pm 1\}$$

The sum ranges over the nearest neighbors on a hypercubic lattice in d dimensions of size $N = L^d$. The couplings J_{ij} are chosen according to a Gaussian distribution:

$$\mathcal{P}(J_{ij}) \sim e^{-J_{ij}^2/2J^2}$$

with $[J_{ij}] = 0$ and $[J_{ij}^2] = 1$. H is the applied field.

For the rest of this talk we choose $d = 2$ and $N = 50^2$ spins.

Algorithm

Change the external field H in small steps. After each field step the local fields

$$h_i = \sum_j J_{ij} S_j + H$$

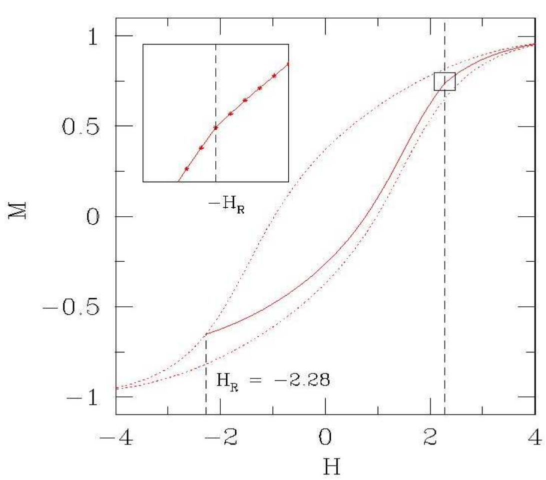
of each spin S_i are calculated. A spin is unstable if $h_i \cdot S_i < 0$. We use the following zero temperature Monte Carlo dynamics:

i) flip a randomly chosen unstable spin;

ii) update the local fields at neighboring sites;

iii) go back to (i) until all spins are stable.

Reversal Field Memory in the Edwards-Anderson Spin Glass

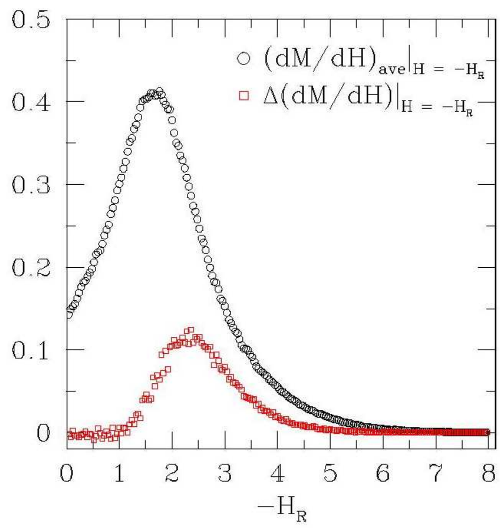


- **Saturate the system**
- **Lower field to reversal field H_R**
- **Increase field back up to H**
- **Kink in M(H) at $H=H_R$**

Memory is present even
- after averaging
-in finite systems

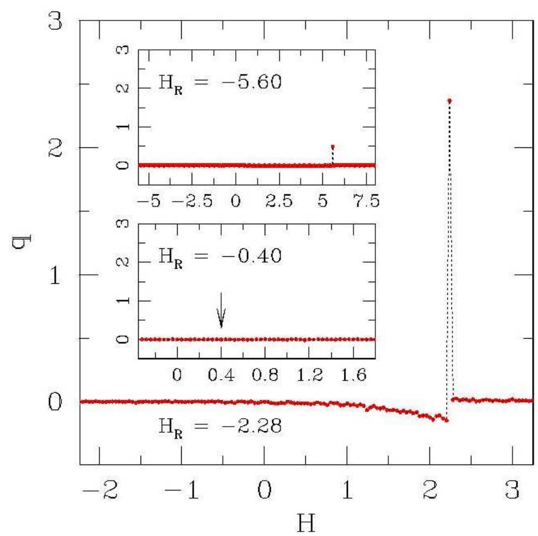
-2D EASG: 10,000 spins
1,000 realizations

Size of reversal field effect



- **If sweep is reversed in the middle region of the hysteresis loop: jump in dM/dH can be 25-30% of average of dM/dH.**
- **If sweep is reversed in tail regions of the hysteresis loop: effect small**

Possible origin: Symmetric Clusters



Overlap of spins, flipped down at H_R and spins flipped up at H:

$$q(H) = \frac{1}{4}\sum_i \left[S_i(H_R+\delta) - S_i(H_R)\right] \times \left[S_i(H+\delta) - S_i(H)\right]$$

Clearly, clusters of spins, which flipped down at H_R, are flipping up at H.

Physical Picture

1) **Local spin reversal symmetry**:
If the magnetic field is reversed and
all neighboring spins are reversed, then
the effective local field is completely reversed.

The Edwards-Anderson Spin Glass has this symmetry,
the RFIM does not.

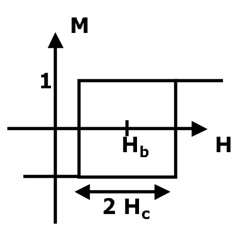
2) **Symmetric clusters:**
A few central spins are coupled to each other strongly,
coupled to their neighbors weakly.
("soft on the outside, crunchy on the inside")

These central spins will flip after all of their neighbors flipped.
By virtue of local spin reversal symmetry, they will flip up
exactly at the negative of the field, where they flipped down.

3) **The number of symmetric clusters can be estimated**:
the number of symmetric clusters is macroscopic, i.e. they are
a possible candidate for explaining the reversal field memory.

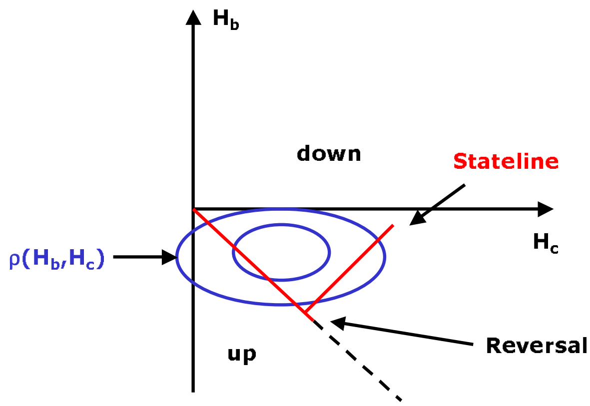
Preisach Hysteron Picture

- Hysteretic behavior modeled as a collection of hysterons (Preisach 1935)
- Hysterons are two state systems (~ Ising spins), but the up-switching fields H_u and the down-switching fields H_d are different.
- Characterize with $H_b=(H_u+H_d)/2$ Bias field (local effective field)
  $H_c=(H_u-H_d)/2$ Coercivity



- $\rho(H_b,H_c)$ is distribution of hysterons with H_b bias field and H_c coercivity
- Assume phenomenological $\rho(H_b,H_c)$ functions, fit the experimental data

The Preisach plane

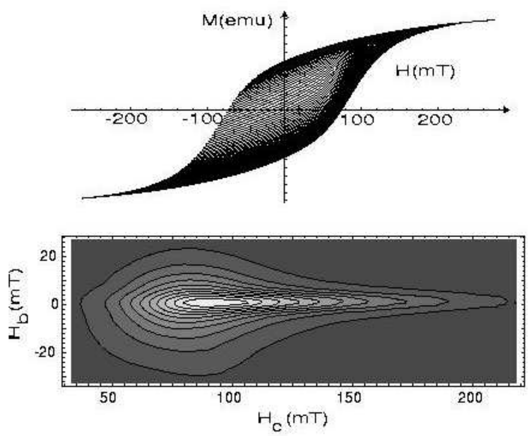


$$M = \iint dH_b\, dH_c\, \rho(H_b, H_c)\,[above\ SL - below\ SL]$$

Preisach – FORC technique

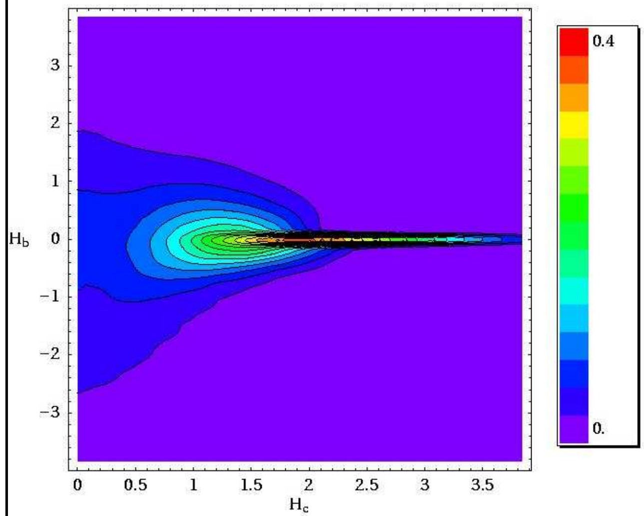
- Experimentally or numerically determine First Order Reversal Curves
- Measure $M(H_b, H_c)$, determine $\rho(H_b, H_c)$:

$$\rho(H, H_R) = -\frac{1}{2}\frac{\partial^2 M(H, H_R)}{\partial H \partial H_R}$$



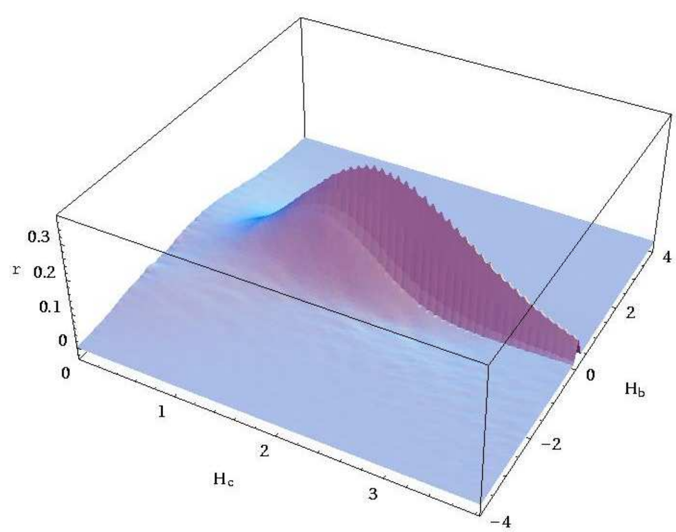
"Magnetic fingerprinting"

FORC of the Edwards-Anderson Spin Glass

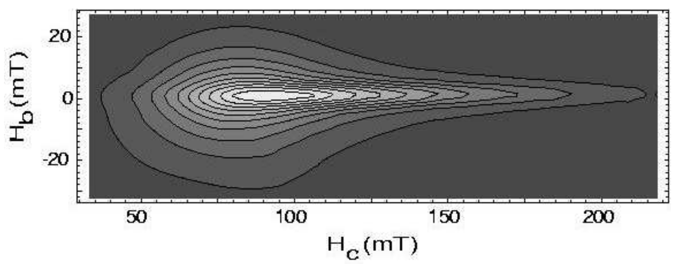


- The kink of the reversal memory effect gives rise to a sharp ridge in FORC: very sensitive technique
- The ridge is at Hb=0, manifestly capturing that the kink's hysterons are symmetric

3D FORC of the 3D Edwards-Anderson Spin Glass



Experimental FORC of FeO Kodak recording media



Our group developed:

- **Fast data taking technique (Alternating Gradient Magnetometer)**
- **Fast algorithm to determine FORC**

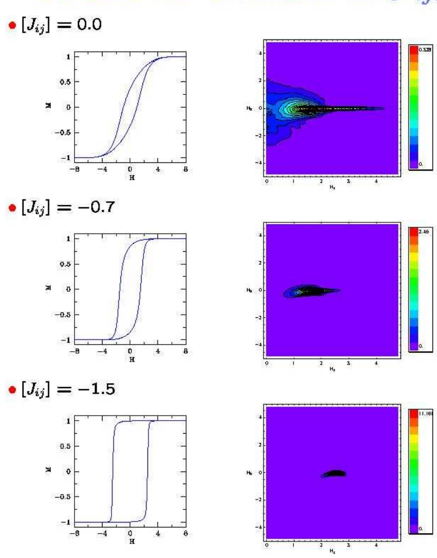
- **Previous interpretation of H_b=0 ridges: weakly-interacting systems.**

FORC: ridges can arise in strongly interacting systems.

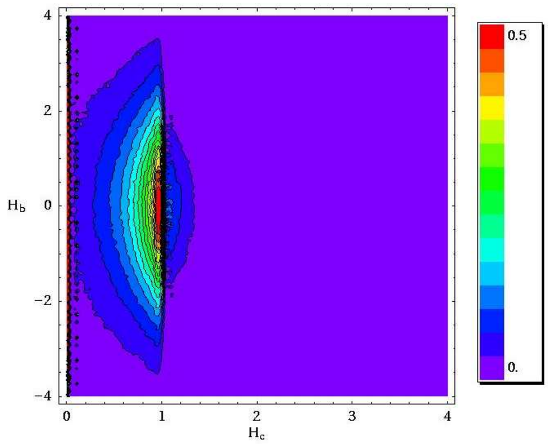
- **Reversal field memory was not noticed on ridge samples.**

FORC: reversal field memory captured profoundly.

Effects of Changes in $[J_{ij}]$

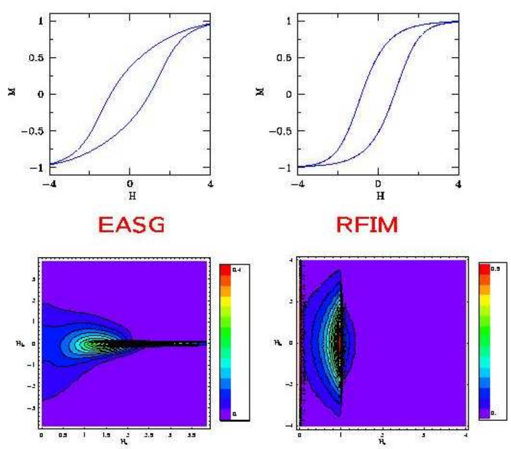


FORC of the Random Field Ising Model



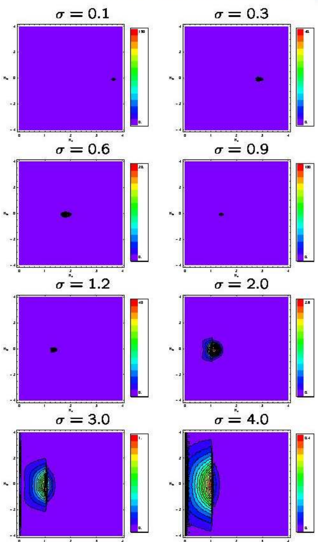
- No horizontal ridge in FORC
- No reversal field memory in hysteresis loop
- RFIM does not have local spin reversal symmetry
- Vertical ridge: single spin flips
- Wake: avalanches

Sensitivity of FORC technique



Similar hysteresis loops yield profoundly different FORCs

RFIM σ-scan (SFH)



For small disorder a single domain wall reverses the system (Drossel & Dahmen 98).

Increase disorder: Roughening of domain wall. Transition from weak disorder to self-affine/self-similar?

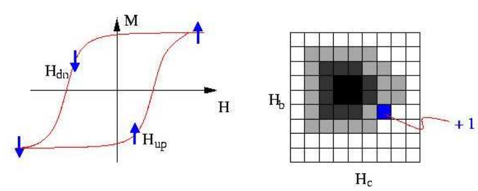
Onset of transition to multi-domain behavior ($\sigma_{\text{crit}} \approx 1$ – Vives). Amplitude shrinks.

Multiple domains nucleating (wake).

What happens with frustrated systems?

Switching-field Histograms

Re-parametrize the major hysteresis loop to test if we can predict the corresponding FORC diagram:



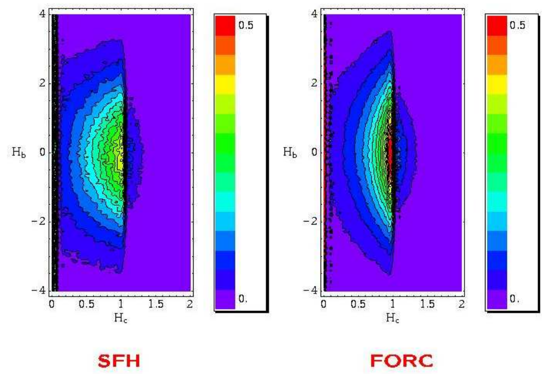
Calculation of a switching-field histogram:

- Saturate the system.
- Change the external field H in small steps.
- After each field step store the field H_{dn} at which each spin flipped down until negative saturation is reached.
- Do the same procedure while increasing the field and store H_{up}.
- Make a histogram of all spins with

$$H_c = (H_{\text{up}} - H_{\text{dn}})/2$$

$$H_b = (H_{\text{dn}} + H_{\text{up}})/2$$

SFH of RFIM



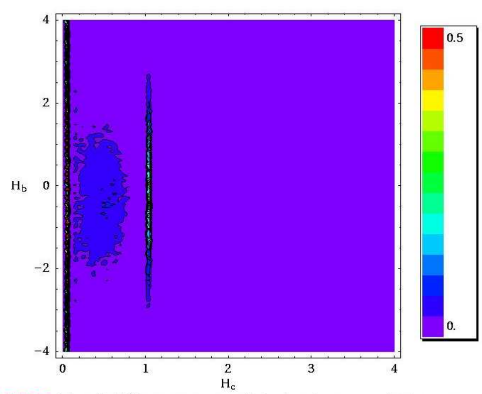
Very good agreement:

- Overall features very similar.
- Small differences ($\sim 10\%$) probably due to statistics.

$\longrightarrow$ For systems with no frustration, SFHs predict FORC diagrams accurately.

We can also compute the difference of both. . .

$\Delta =$ (SFH - FORC)



HERE: Small differences, possibly due to numerical error ($\sim 10\%$).

In general, the differences between SFHs and FORC diagrams quantify:

- The robustness of the hysteron picture.
- Corrections due to, e.g., hysteron correlations.

WHY BOTHER? SFH's are much faster to calculate. We can scan the parameter space $\sim 10^2$ times faster!

FORC technique

- Rich in details
- Provides insight into the physics:
  - notice effects (reversal field memory)
  - provides hints of physics (symmetry of FORC)
  - separates avalanches from single spin events
- Provides a new way to characterize a disordered magnet:
  - distribution of bias fields and coercivities
- Very sensitive:
  - similar hysteresis loops have very different FORCs
- Useful for developing recording media:
  - by comparing simulated FORCs of realistic models with experimental FORCs, the parameters of the media and their distributions can be determined

Hysteretic Optimization

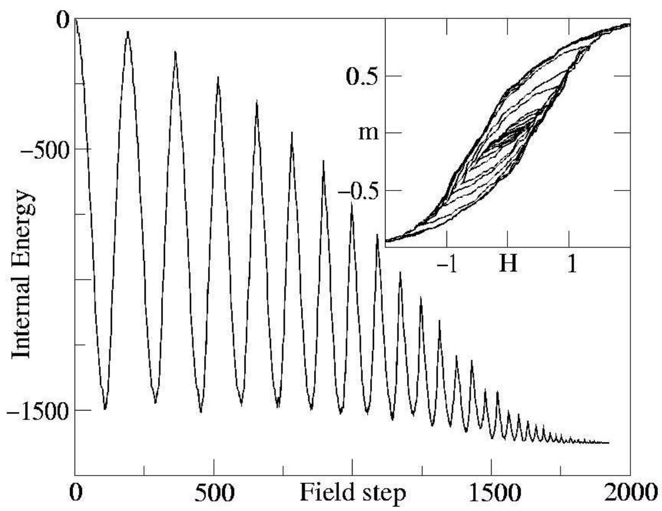
G. Zarand, F. Pazmandi, K. Pal, GTZ, Phys. Rev. Lett. 89, 150201 (2002)

- How to find low energy states of disordered systems efficiently?
  - Simulated Annealing (Kirkpatrick, Gelatt, Vecchi)
  - Many more techniques developed since, but simulated annealing (SA) remains a standard method

- Simulated Annealing: cycling with temperature
- Why not cycle with magnetic field?
- Experimentally practiced for millennia:

  AC, or hysteretic demagnetization

Basic hysteretic optimization procedure



Improvements

1) **Random sign for the magnetic field** (by gauge symmetry it is equivalent)

$$H = \sum_{i,j}^{N} J_{ij} S_i S_j - H \sum_{i}^{N} \xi_i S_i$$

2) **Small fluctuations in return point ratio** $\gamma_n = \left| H_{n+1}^{ret} / H_n^{ret} \right|$

$$\gamma_n = \gamma \pm \delta_n \qquad \gamma = 0.9, \quad \delta_n^{\max} = 0.1$$

3) Correlations between ξ_i and S_i "freeze" up convergence :

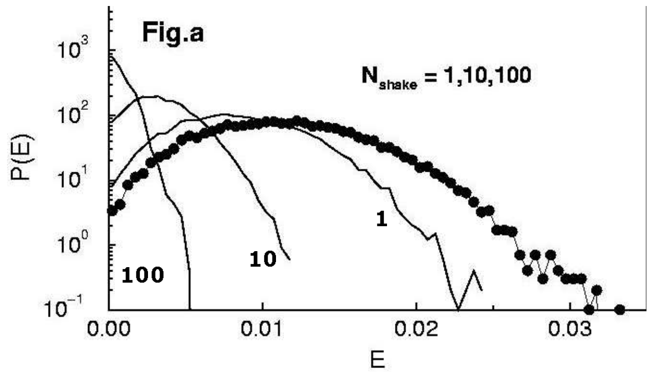
Improvement: **Shake up**:

- Once the hysteretic demagnetization is finished
- Change the ξ_i configuration
- Increase H to an intermediate value
- Repeat hysteretic demagnetization

With these improvements: Hysteretic Optimization

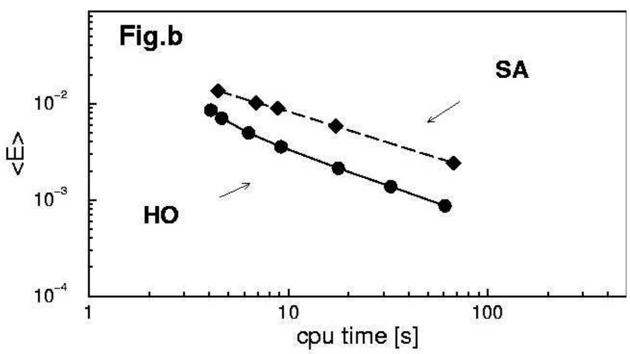
Hysteretic optimization of the SK model

P(E): Probability of finding a state within E of the exact ground state E_{GS}



N~1,000

Comparison of performance of SA and HO



<E> = average energy / spin

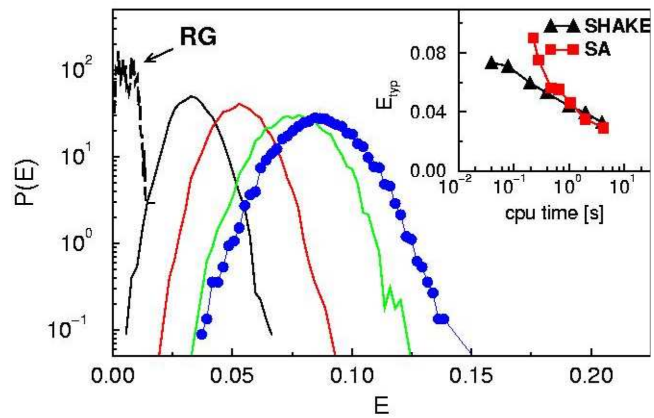
Hysteretic Optimization can be 3-10 faster than Simulated Annealing

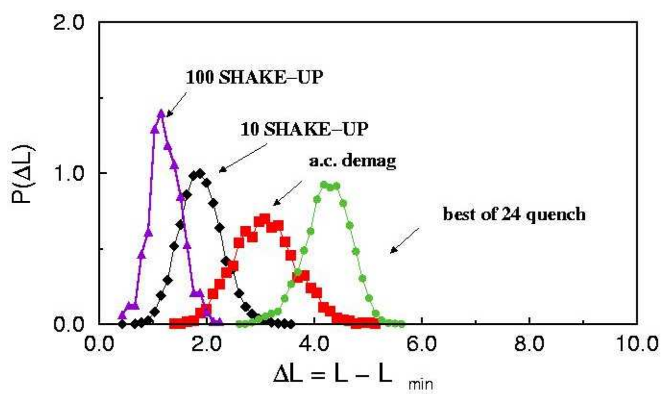
Hysteretic Optimization can be used in combination

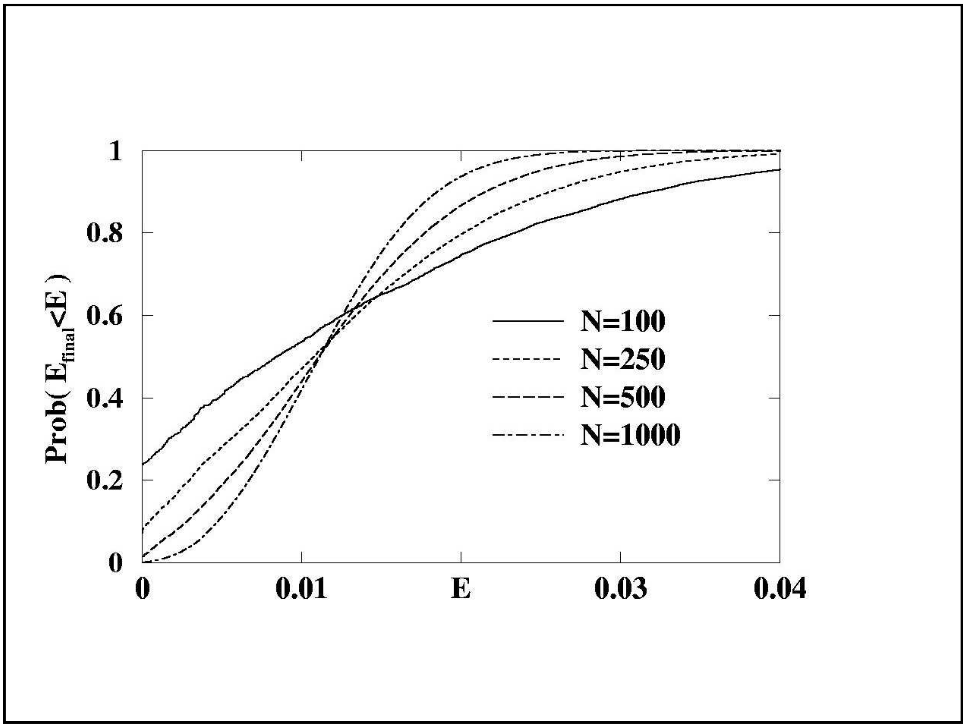
- Hysteretic Optimization can be used in combination with other techniques:
  - genetic algorithms
  - cluster renormalization group (Kavashima, Houdayer, Martin)
- During the first down sweep, we identify, which clusters of spins flipped at the same field
- In subsequent cycles, propose the reversal of the identified clusters
- The speed of HO+RG is comparable to the best genetic and parallel tempering methods

SUMMARY

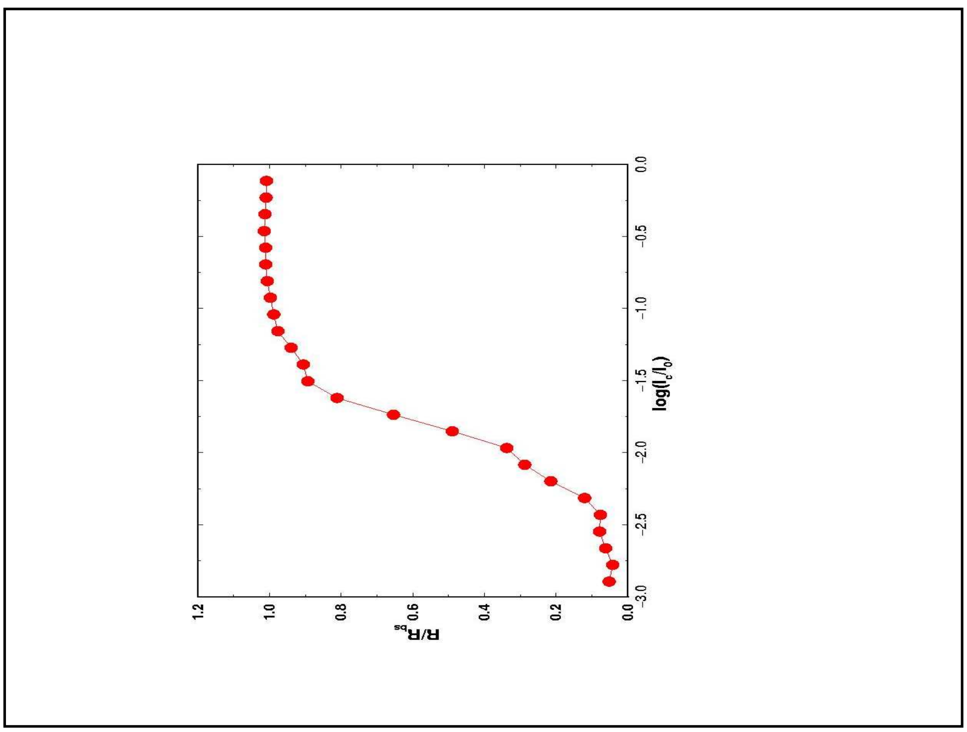
1) The hysteresis of the Sherrington-Kirkpatrick model exhibits Self Organized Criticality
2) The hysteresis of the Edwards-Anderson spin glass exhibits Reversal Field Memory
3) The hysteresis of disordered magnetic systems can be characterized with the very sensitive FORC diagnostics.
4) The low energy states of disordered systems can be reached efficiently with the powerful Hysteretic Optimization.
5) The disordered vortex matter may freeze like the structural glasses: Vortex Molasses scenario


RG
SHAKE
SA
P(E)
E
E_{typ}
cpu time [s]

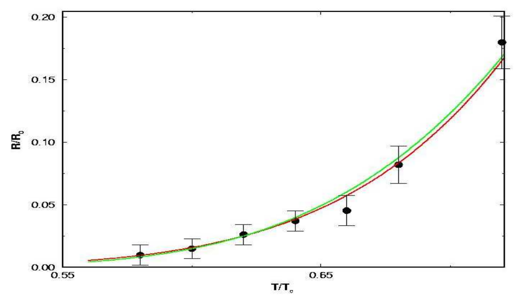

100 SHAKE–UP
10 SHAKE–UP
a.c. demag
best of 24 quench
P(ΔL)
ΔL = L – L min

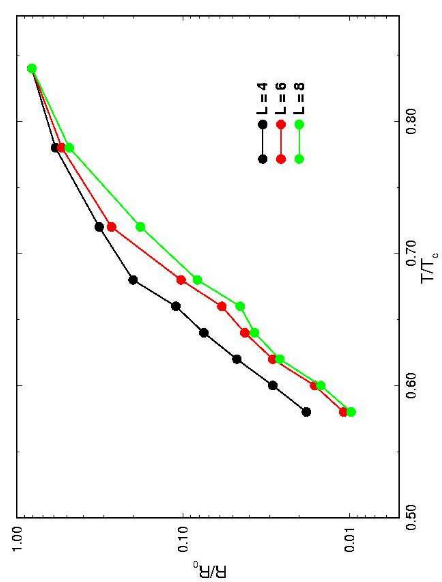


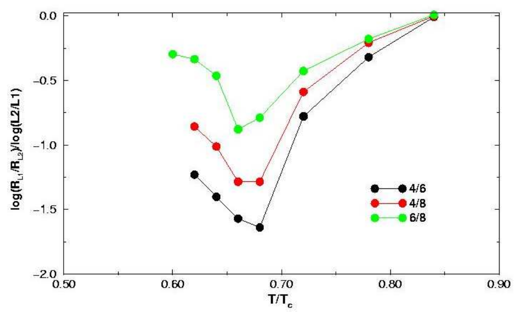
1
0.8
0.6
0.4
0.2
0
Prob(E_final<E)
N=100
N=250
N=500
N=1000
0
0.01
E
0.03
0.04

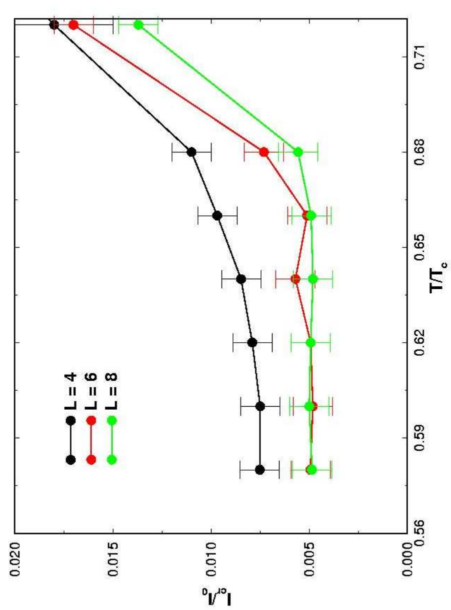


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
1.2
1.0
0.8
0.6
0.4
0.2
0.0

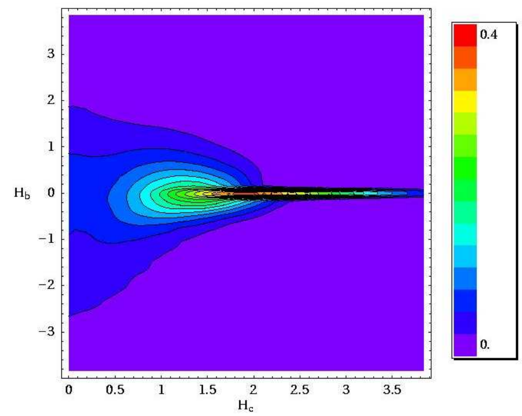

0.20
0.15
0.10
0.05
0.00
0.55
0.65
R/R_0
T/T_c


L = 4
L = 6
L = 8
1.00
0.10
0.01
R/R_0
0.50
0.60
0.70
0.80
T/T_c


log(R_L1/R_L2)/log(L2/L1)
T/T_c
4/6
4/8
6/8
0.0
−0.5
−1.0
−1.5
−2.0
0.50
0.60
0.70
0.80
0.90


L = 4
L = 6
L = 8
T/T_c
I_cr/I_0
0.020
0.015
0.010
0.005
0.000
0.56
0.59
0.62
0.65
0.68
0.71

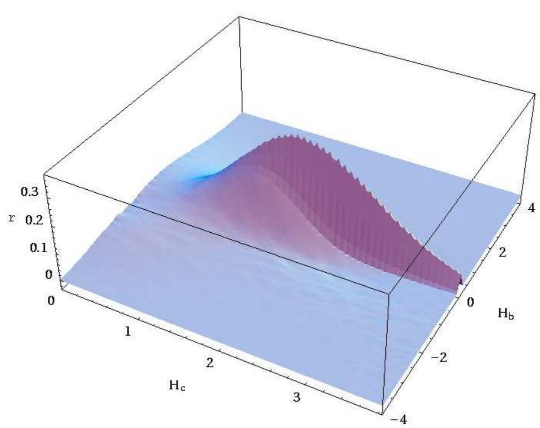
FORC Diagram of EASG



The kink in the reversal curves previously shown is clearly visible as a sharp ridge in the FORC diagram.

As the ridge is parallel to the H_c-axis, we have symmetric particles ($H_b = 0$) contributing to the kinks (symmetry in $\mathcal{H}$).

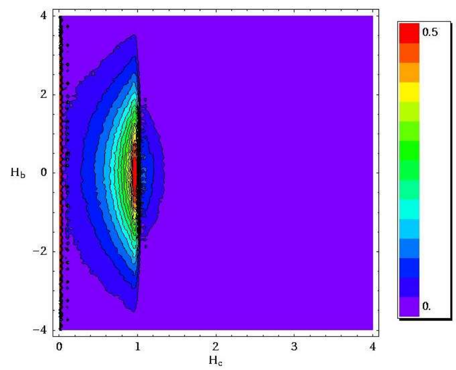
3D FORC Diagram EASG



The FORC diagram resembles experimental data from single-domain weakly interacting particles which are also expected to act as symmetric hysterons.

How robust is this effect with respect to changes in $[J_{ij}]$?

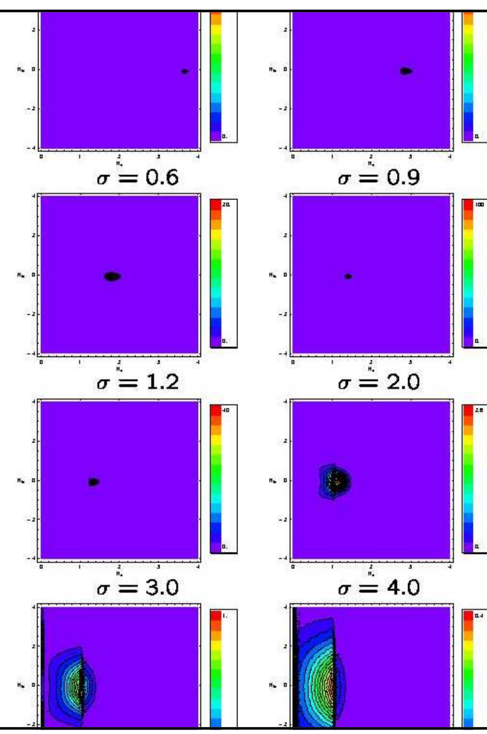
FORC Diagram of RFIM

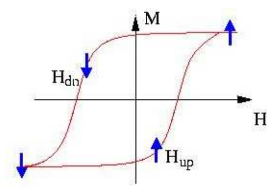


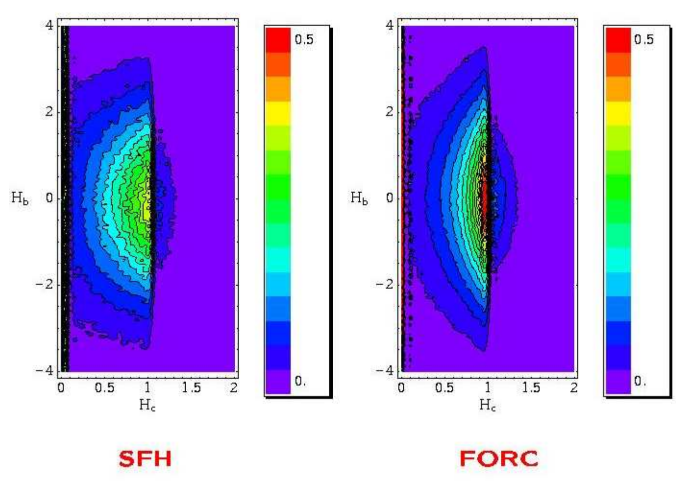
There is no horizontal ridge, i.e. no reversal-field memory effect (data for $\sigma = 4.0$). Note:

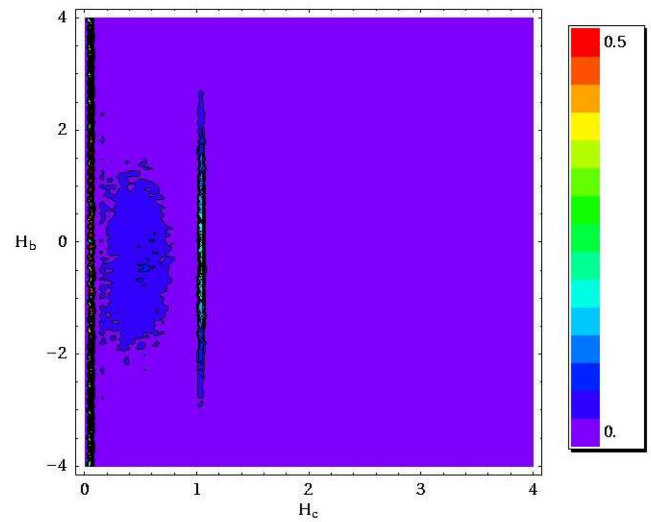
- Ridge corresponds to single-spin flip events ($H_c = 1$).
- Wake corresponds to avalanche propagation.

Can we predict the FORC diagram (first order reversal curves) from the major hysteresis loop (zeroth order)?




M
H
H_{dn}
H_{up}


H_b
H_c
0.5
0.
SFH
FORC



Do we see the effects of reversal-field memory in Hamiltonians which do not have spin-reversal symmetry:

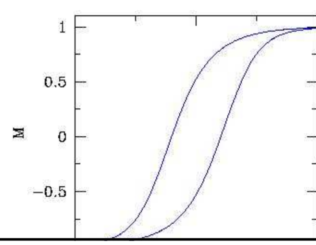
$$\mathcal{H}(-S_i, -H) = \mathcal{H}(S_i, H)$$

Example: The Random-Field Ising Model

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i S_i H_i - \sum_i S_i H \qquad S_i \in \{\pm 1\}$$

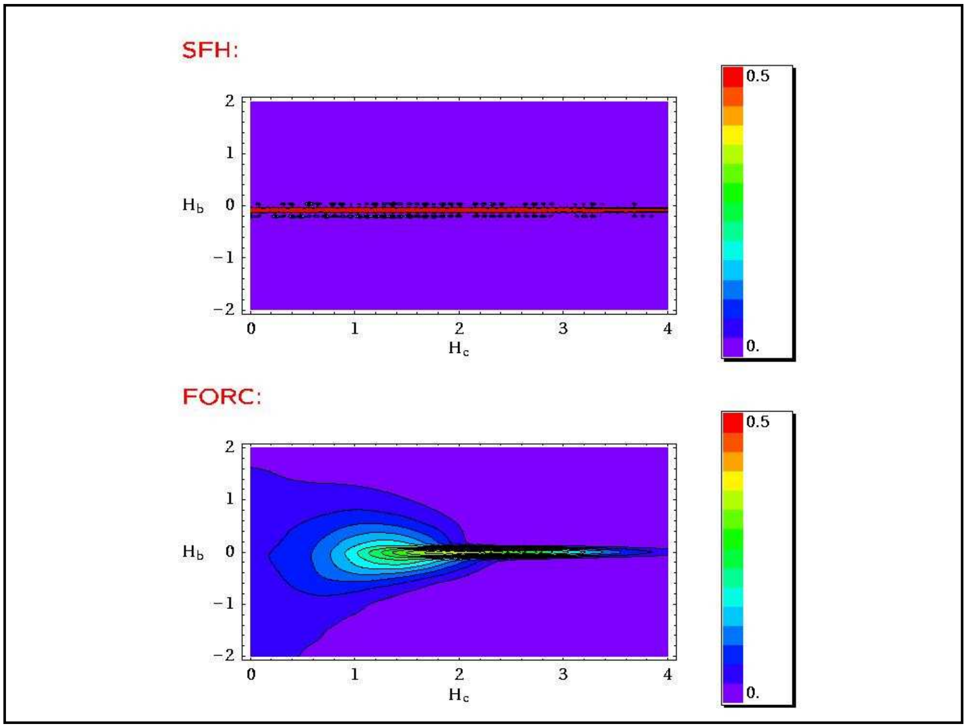
- The sum ranges over the nearest neighbors on a hypercubic lattice.
- The random fields H_i are site-dependent and chosen according to a Gaussian distribution with zero mean and standard deviation σ.
- Direct inspection shows that the Hamiltonian does NOT have the aforementioned symmetry.

Major hysteresis loop:



While the mayor loops look similar, the FORC diagram looks completely different...

(Data for $\sigma = 4.0$).


SFH:
FORC:
H_b
H_c
0.5
0.