“Zero Bias” Anomaly in the Absence of Equilibrium

Igor Lerner

THE UNIVERSITY
OF BIRMINGHAM

School of Physics & Astronomy

in collaboration with

Volodya Kravtsov
(ICTP)

OUTLINE

- Tunneling current in a non-equilibrium case
- What is ZBA?
- Interplay of disorder and interaction
- Tunnelling DoS or else?

Glassy States of Matter & Nonequilibrum Quantum Dynamics, KITP, 17 Apr 2003

Non-equilibrium distribution

Energy Distribution Function of Quasiparticles in Mesoscopic Wires

H. Pothier, S. Guéron, Norman O. Birge,* D. Esteve, and M. H. Devoret

PHYSICAL REVIEW LETTERS
VOLUME 79, NUMBER 18
3 NOVEMBER 1997

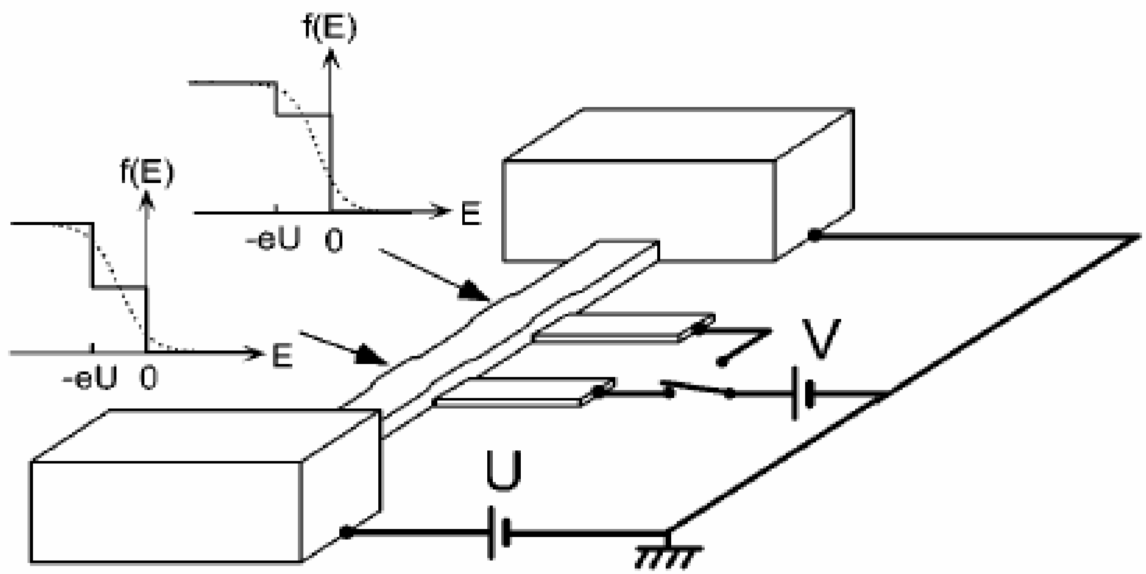


FIG. 1. Experimental layout: a metallic wire of length L is connected at its ends to reservoir electrodes, biased at potentials 0 and U. In the absence of interaction, the distribution function at a distance $X = xL$ from the grounded electrode has an intermediate step $f(E) = 1 - x$ for energies between $-eU$ and 0 (solid curves) (we assume $U > 0$).

Double Step Function

Steady-state kinetic equation:

$$\frac{1}{\tau_D}\frac{\partial^2 f(x,\varepsilon)}{\partial x^2} + \mathrm{St}(x,\varepsilon) = 0$$

In the absence of relaxation ($\mathrm{St}=0$), $f(\varepsilon)$ is a double-step:

$$f_0(x,\varepsilon) =$$

$$(1-x)\ f_0\left(\varepsilon + eU\right) + x\, f_0\left(\varepsilon\right)$$

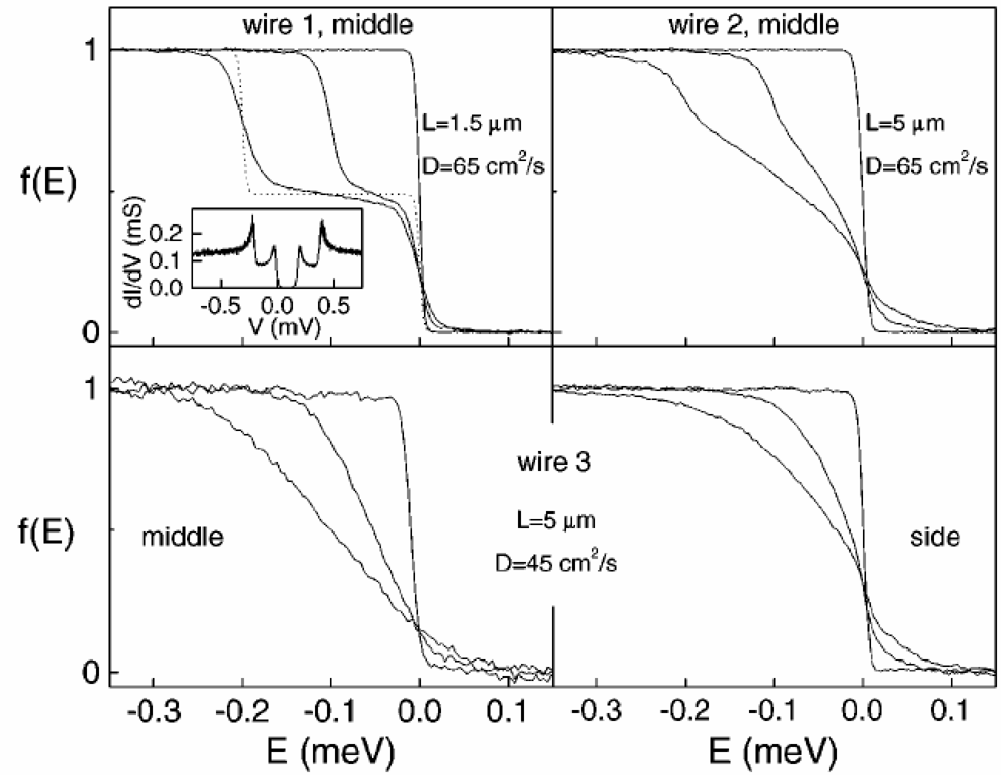


FIG. 2. Inset of the top left panel: Measured $dI/dV(V)$ of the tunnel junction to wire 1 for $U = 0.2$ mV. In the four panels, distribution functions, obtained from the deconvolution of such $dI/dV(V)$ curves, for $U = 0$, 0.1, and 0.2 mV in

"Anomaly": sharp features in $\mathrm{d}I/\mathrm{d}V$

The aim of the talk

- a survey of anomalies in differential conductance
- new features in disorder/interaction corrections in the absence of equilibrium

What is “anomaly” in

differential conductance

$$\frac{\mathrm{d}I}{\mathrm{d}V} = \frac{1}{R} \equiv G$$

Normal: Ohm’s law, $G = \mathrm{const}$

Anomalous: (almost) anything else

Expected “anomaly”: “high” voltage

“Unexpected” anomaly: low voltage (bias), V→0

At low T well-known for ages

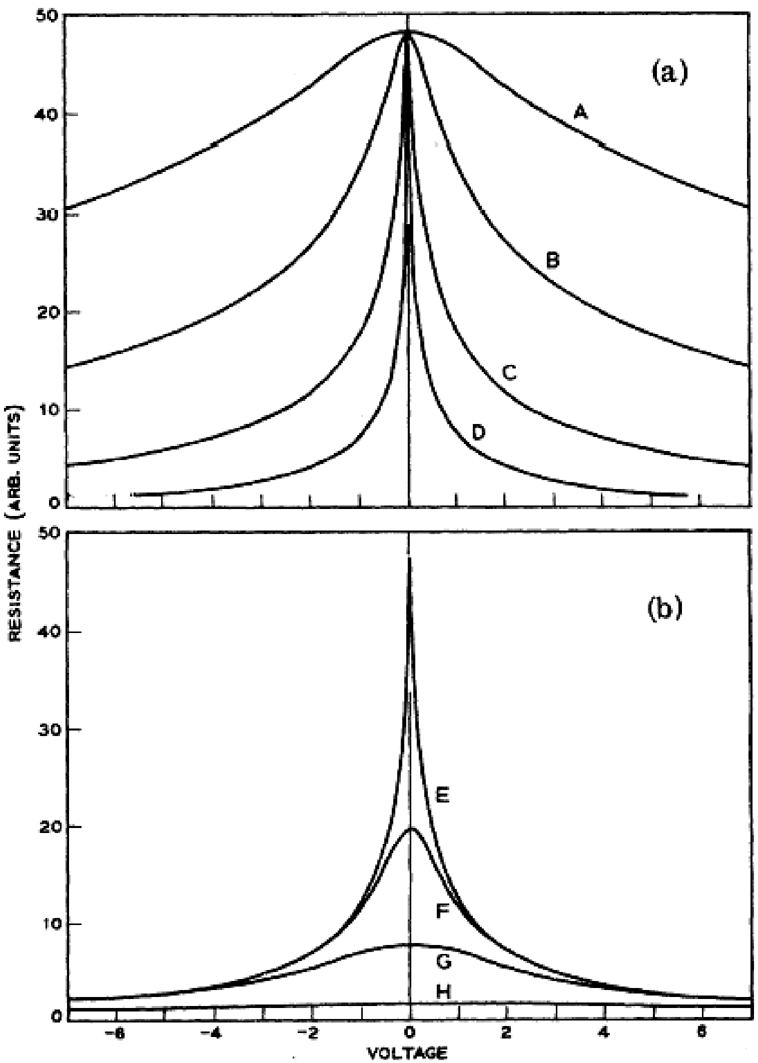
A. Sommerfeld and H. Bethe, <u>Handbuch der Physik</u>, edited by H. Geiger and K. Scheel (Verlag von Julius Springer, Berlin, 1933), Vol. 24, Pt. 2, p. 450.

Two types of ZBA

ZERO-BIAS ANOMALIES IN NORMAL METAL TUNNEL JUNCTIONS

J. M. Rowell and L. Y. L. Shen
Bell Telephone Laboratories, Murray Hill, New Jersey

PHYSICAL REVIEW LETTERS
VOLUME 17, NUMBER 1
4 JULY 1966



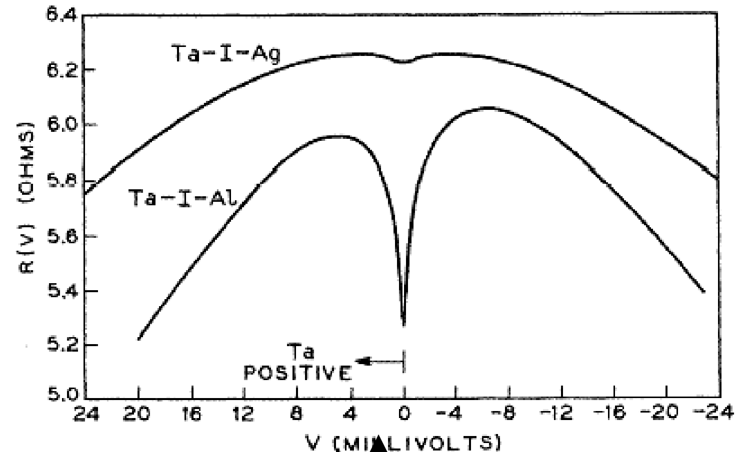


FIG. 3. Dynamic resistance versus voltage for Ta-*I*-Al and Ta-*I*-Ag junctions at 0.9°K. A field of 3 kG was used to drive the tantalum normal.

s-d exchange (Appelbaum,1966)

FIG. 1. (a) The dynamic resistance versus voltage for a Cr-*I*-Ag junction at 0.9°K. The voltage scales are $A = 0.2$ mV/division, $B = 1.0$ mV/division, $C = 5$ mV/division, $D = 20$ mV/division. (b) The dynamic resistance versus voltage for a Cr-*I*-Ag junction at various temperatures. $E = 0.9$, $F = 20.4$, $G = 77$, and $H = 290$°K. The voltage scale is 10 mV/division.

Dependence on disorder

Zero-Bias Anomaly in Irradiated Pb-GaAs Tunnel Junctions, and the Mott Transition*

Nabih A. Mora, Stuart Bermon, and J. J. Loferski

PHYSICAL REVIEW LETTERS

VOLUME 27, NUMBER 10

6 SEPTEMBER 1971

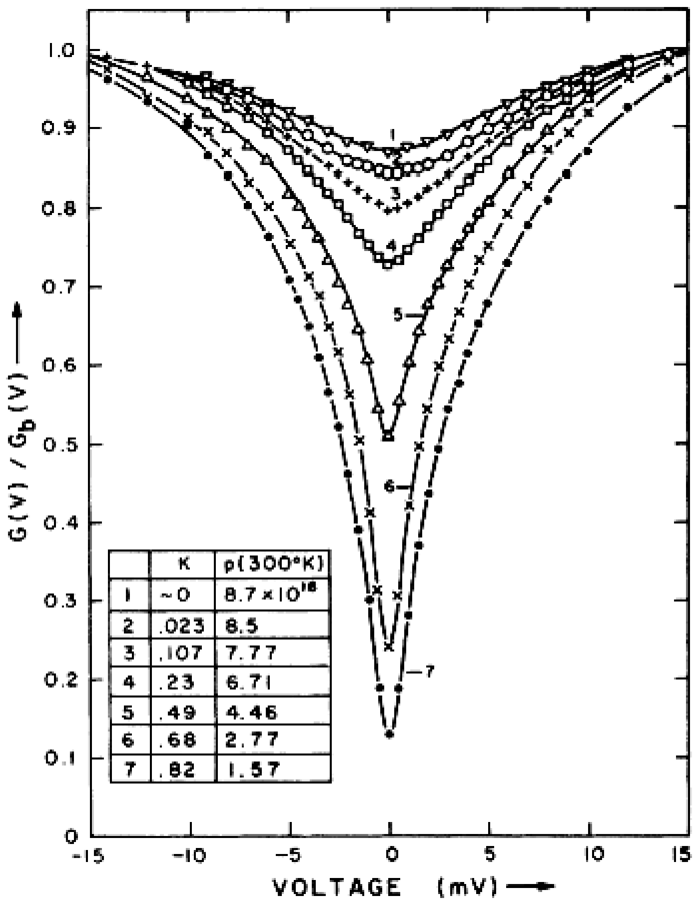


FIG. 2. Voltage dependence of the normalized conductance $G(V)/G_b(V)$ at 1.42°K for junction 8B-4 after successive neutron irradiations. The integrated neutron flux is (1) 0; (2) 0.6; (3) 2.8; (4) 5.9; (5) 12.1; (6) 18.3; (7) 24.5, all in units of 10^{16} n/cm^2.

conjecture – Mott transition (DoS goes to 0 in its vicinity).

Proved beyond doubt: disorder matters

Dependence on T and V

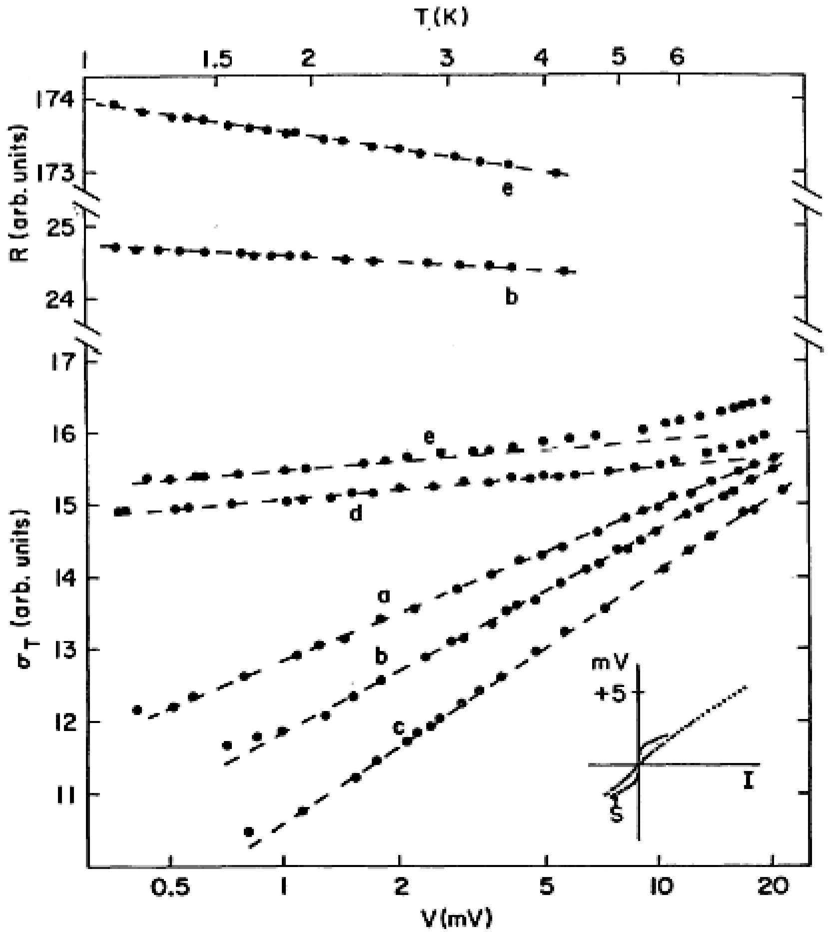


FIG. 1. Tunneling conductance vs voltage for five 2D samples (measured at 1.2 K) and typical resistance vs temperature for two of these samples (see Table I for sample identification). Inset depicts the low-bias I –V characteristics of sample b with the Pb electrode superconducting (curve s) and in the presence of 1 kOe magnetic field.

PHYSICAL REVIEW LETTERS

VOLUME 49, NUMBER 11

13 SEPTEMBER 1982

Density-of-States Anomalies in a Disordered Conductor:

Yoseph Imry[a]

IBM Research Center, Yorktown Heights, New York 10598

Zvi Ovadyahu[b]

Brookhaven National Laboratory, Upton, New York 11973

Excellent log dependence – in an agreement with (then) new theory:

PHYSICAL REVIEW LETTERS

VOLUME 44, NUMBER 19

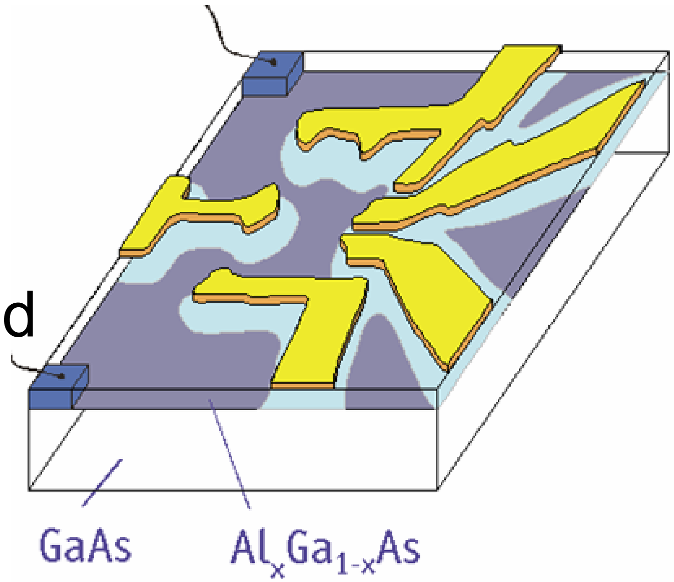
12 MAY 1980

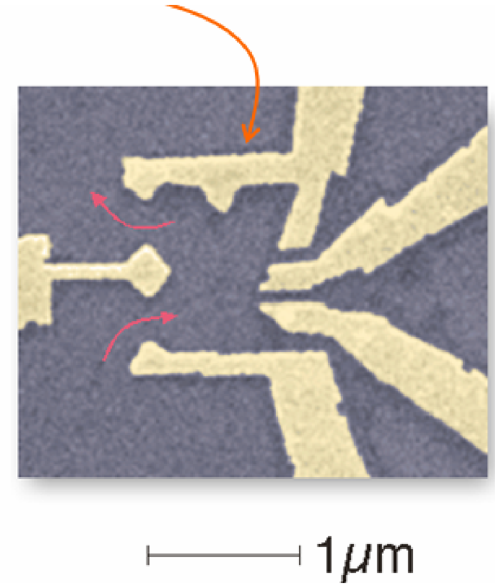
Interaction Effects in Disordered Fermi Systems

B. L. Altshuler A. G. Aronov P. A. Lee

"Extreme ZBA:" Coulomb blockade

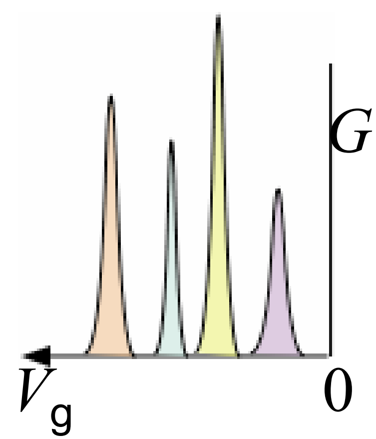
Quantum dot with tunneling leads (tunneling is controlled by gate voltage V_g

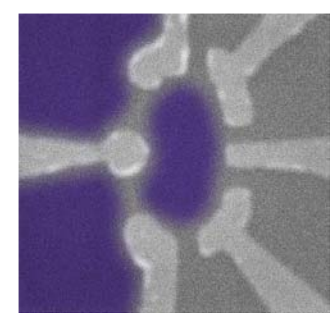




Conductance vanishes at $V \to 0$ and $V_g \to 0$:

Charging energy e^2/C blockades electron from entering the dot

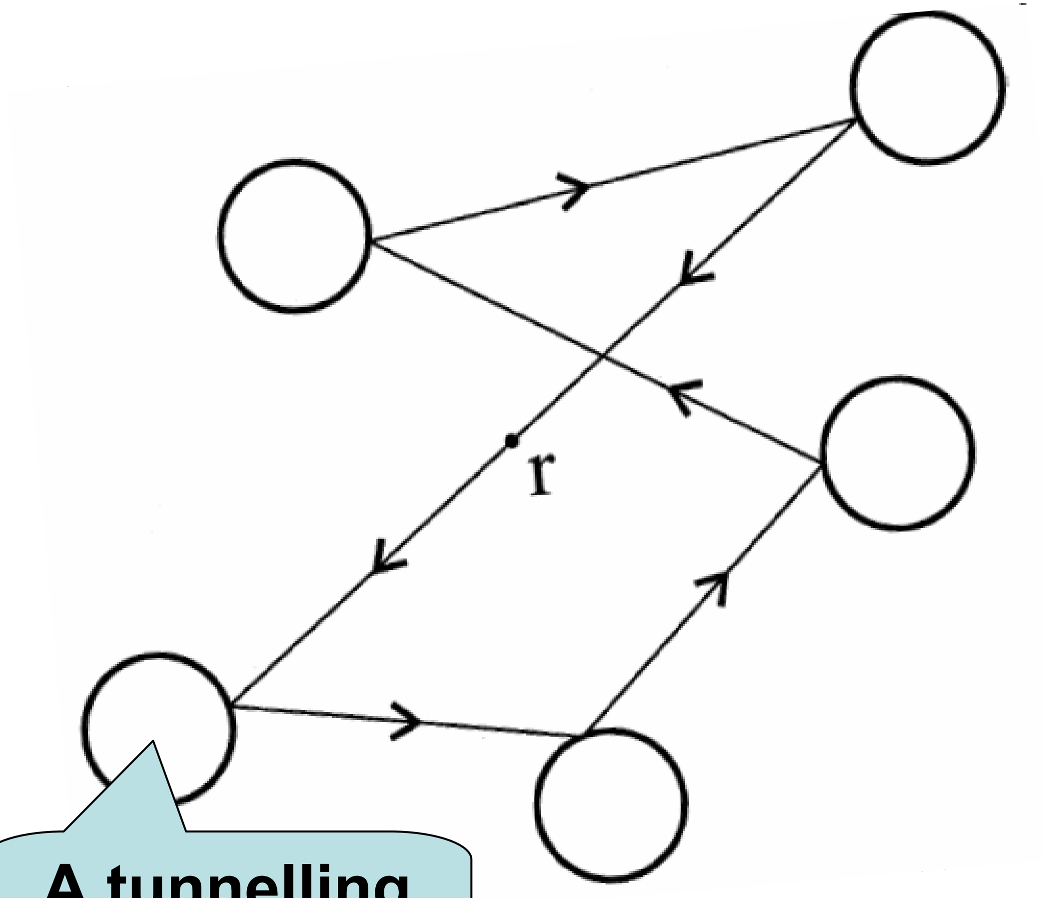




WEAK TUNNELING: $\Gamma < \Delta$

Disorder+Interaction ⇒ “miniblockade”

Scatterting from impurities creates self-returning trajectories

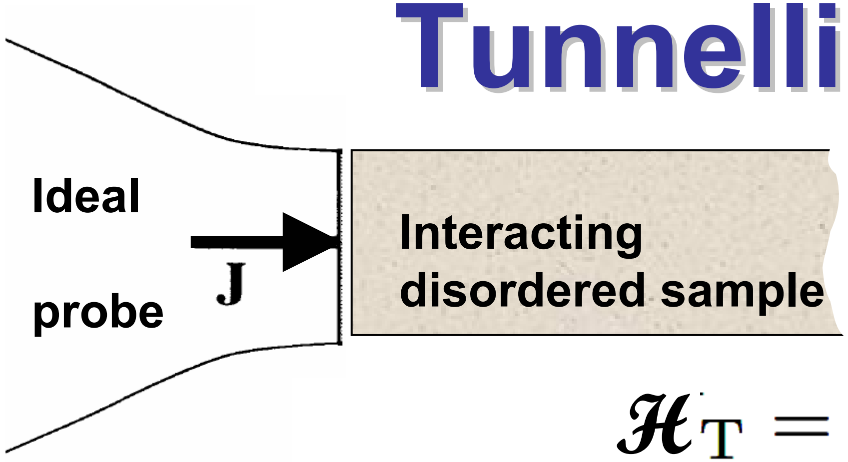


A tunnelling point contact

The lower dimension d, the higher return probability

This **disorder-driven** effect makes tunnelling more difficult for an additional electron **interacting** with the rest

Tunnelling Current



$$\mathcal{H} = \mathcal{H}_{\mathrm{P}} + \mathcal{H}_{\mathrm{S}} + \mathcal{H}_{\mathrm{T}}$$

$$\mathcal{H}_{\mathrm{T}} = \int \gamma(\boldsymbol{r})\, \psi^{\dagger}_{\mathrm{P}}(\boldsymbol{r}) \psi_{\mathrm{S}}(\boldsymbol{r})\, \mathrm{d}^d r + \mathrm{h.c.}$$

integration over the contact volume

$$\mathcal{I}(t) = -e\dot{N}_{\mathrm{P}} = -\frac{ie}{\hbar} \langle [H_{\mathrm{T}}, N_{\mathrm{P}}] \rangle = -\frac{ie\gamma}{\hbar} \int \mathrm{d}^d r \left[\left\langle \psi^{\dagger}_{\mathrm{P}}(\boldsymbol{r}, t) \psi_{\mathrm{S}}(\boldsymbol{r}, t) \right\rangle - \mathrm{h.c.} \right]$$

$$\equiv \frac{e\gamma}{\hbar} \int \mathrm{d}^d r \left[G^{<}_{\mathrm{PS}}(\boldsymbol{r}, \boldsymbol{r}; t, t) - G^{<}_{\mathrm{SP}}(\boldsymbol{r}, \boldsymbol{r}; t, t) \right]$$

$$\hat{G} \equiv \begin{pmatrix} G^{++} & G^{<} \\ G^{>} & G^{--} \end{pmatrix}$$

– Kelsdysh' Green's function, with

$$G^{++}(1, 1') \equiv -i \left\langle \hat{T} \psi(1) \psi^{\dagger}(1') \right\rangle$$

$$G^{++} + G^{--} = G^{<} + G^{>}$$

Weak Tunnelling Limit

$$\mathcal{I} = \frac{i\pi e\widetilde{\gamma}^2}{\hbar}\int_{-\infty}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\Big\{\nu_{\mathrm{P}}(\varepsilon - eV)\Big[G_{\mathrm{S}}^{\mathrm{K}}(\boldsymbol{r},\boldsymbol{r};\varepsilon) - h_0(\varepsilon - eV)\Big(G_{\mathrm{S}}^{\mathrm{R}} - G_{\mathrm{S}}^{\mathrm{A}}\Big)(\boldsymbol{r},\boldsymbol{r};\varepsilon)\Big]\Big\}$$

here $h \equiv 1-2f$; the probe is assumed ideal and local

$G^{\mathrm{R}} - G^{\mathrm{A}} = 2\pi i \mathrm{A}(r,\varepsilon)$ →LDoS; no information on distribution

all non-equilibrium effects are in $G^{\mathrm{K}}{}_{\mathrm{S}}$

Keldysh ansatz: $G^{\mathrm{K}}{}_{\mathrm{S}} = H_{\mathrm{S}}(G^{\mathrm{R}} - G^{\mathrm{A}})_{\mathrm{S}}$

Standard Contribution to d$\mathcal{J}$/dV

Differentiate $\mathcal{J}$ to find $\mathrm{d}\mathcal{J}/\mathrm{d}V = (\mathrm{d}\mathcal{J}/\mathrm{d}V)_1 + (\mathrm{d}\mathcal{J}/\mathrm{d}V)_2$:

$$\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_1 = \frac{1}{2R_T}\int \mathrm{d}\varepsilon \, \frac{\nu_{\mathrm{S}}(\varepsilon)}{\overline{\nu}_{\mathrm{S}}} \frac{\partial}{\partial \varepsilon} h_0(\varepsilon - eV) \approx \frac{\nu_{\mathrm{S}}(eV)}{\overline{\nu}_{\mathrm{S}} R_T}$$

here $1/R_T = \left(2\pi e^2/\hbar\right) \, \widetilde{\gamma}^2 \overline{\nu}_{\mathrm{P}} \, \overline{\nu}_{\mathrm{S}}$

This standard contribution is proportional to the tunnelling DoS of the sample

Additional Contribution to d$\mathcal{I}$/dV

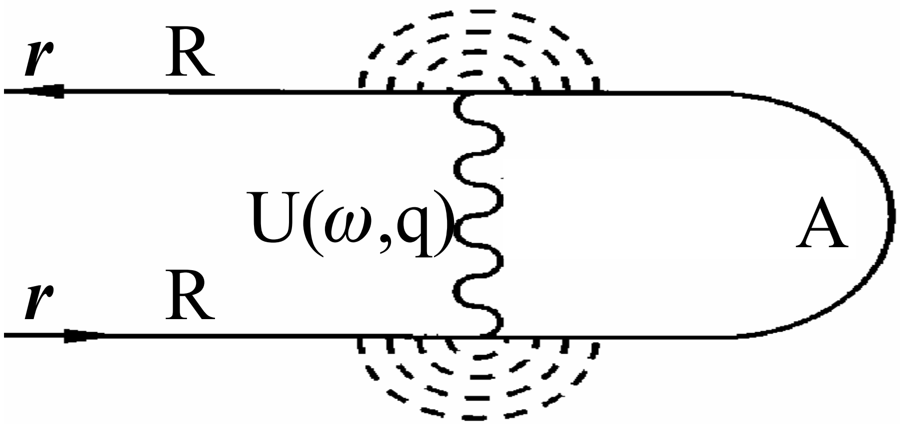
$$\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_2 = \frac{1}{2R_T\overline{\nu}_{\mathrm{P}}}\int \mathrm{d}\varepsilon\,\frac{\partial \nu_{\mathrm{P}}(\varepsilon)}{\partial \varepsilon}\left[h_0(\varepsilon) - H_{\mathbf{S}}(\widetilde{\varepsilon})\right] A_S(\widetilde{\varepsilon}, \boldsymbol{r})$$

here $\widetilde{\varepsilon} \equiv \varepsilon + eV$

Two conditions for its existence:

- the probe DoS is non-flat near ε_{F}
- H_S differs essentially from h_0

Tunnelling DoS contribution



In the diffusive limit,
$\lambda_F, \kappa^{-1} \ll \boldsymbol{\ell} \ll L$,
1st order correction in the inter-n is enough

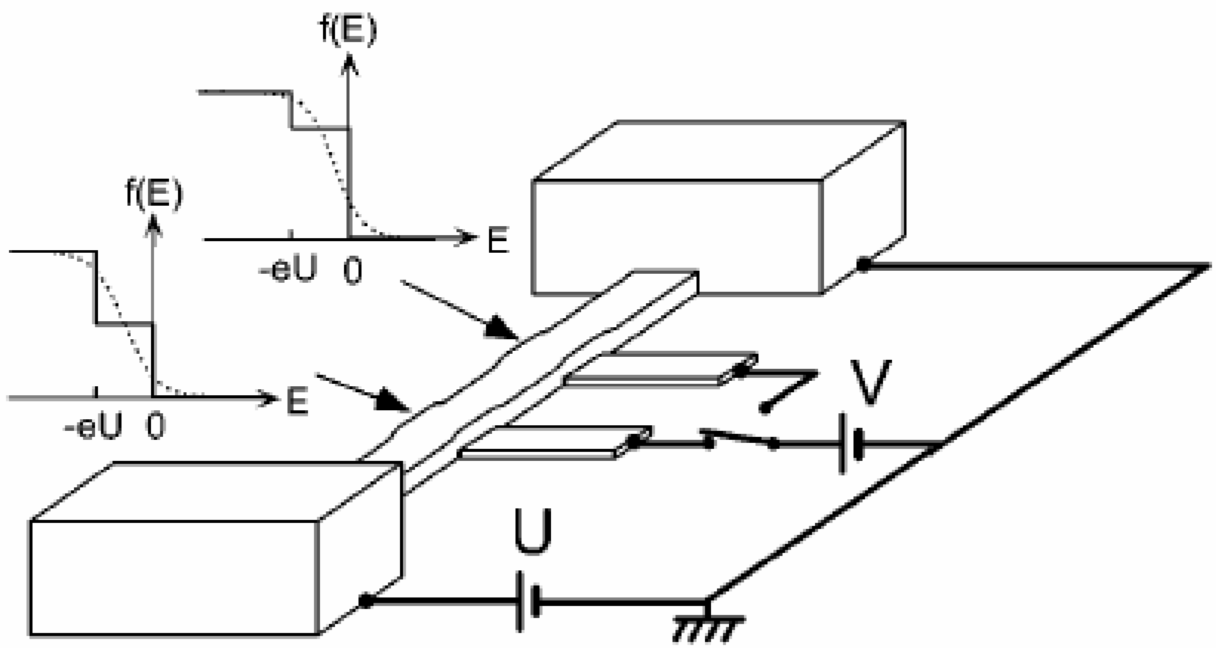
$$U^R(\omega, \boldsymbol{q}) = \frac{\varkappa}{2\nu} \frac{Dq^2 - i\omega}{q(Dq\varkappa - i\omega)} \mapsto \frac{1}{2\nu}\left[1 - \frac{i\omega}{Dq^2}\right]$$

is universal (no dependence on int.strength at $\omega \ll D\kappa q$)

$$\frac{\delta\nu(\bar{\epsilon})}{\nu_0} = -\frac{\hbar}{16\pi E_F \tau}\left\{ \ln\left(\frac{\bar{\epsilon} a_B^4}{\hbar D^2 \tau}\right) \ln(\bar{\epsilon}\tau/\hbar) \right\}$$

Altshuler, Aronov, Lee, 1980 **Rudin, Aleiner, Glazman, 1997**

Non-equilibrium distribution

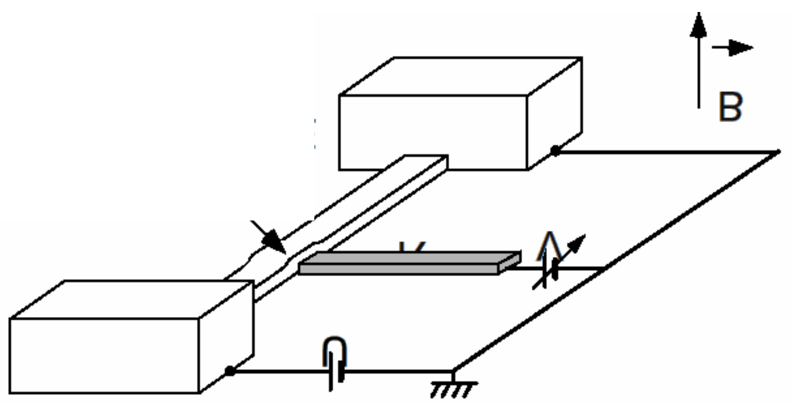


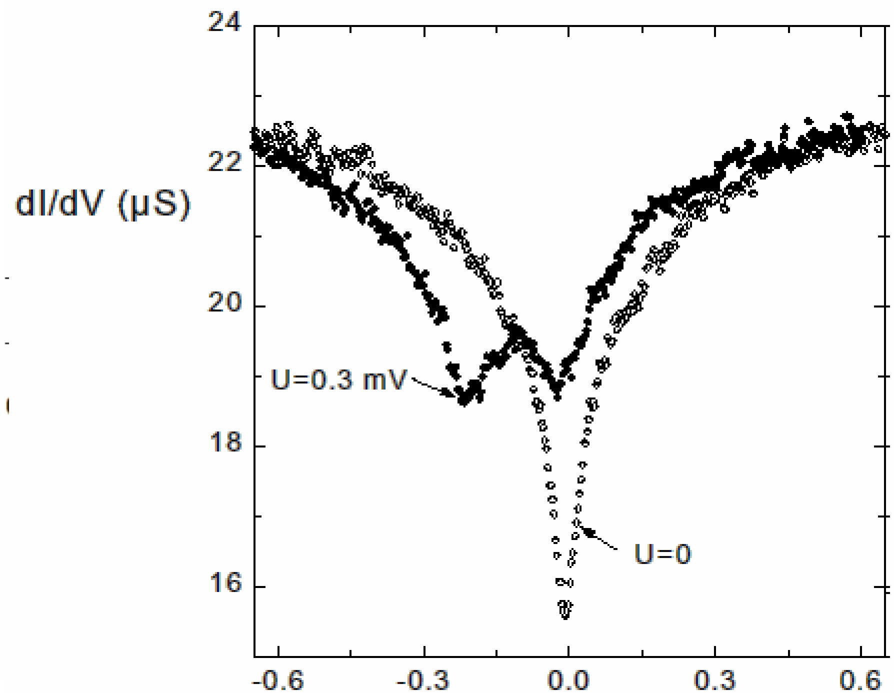
Probes are superconducting: $\nu_{\mathrm{P}}(\varepsilon) = \overline{\nu}/\sqrt{\varepsilon^2 - \Delta^2}$

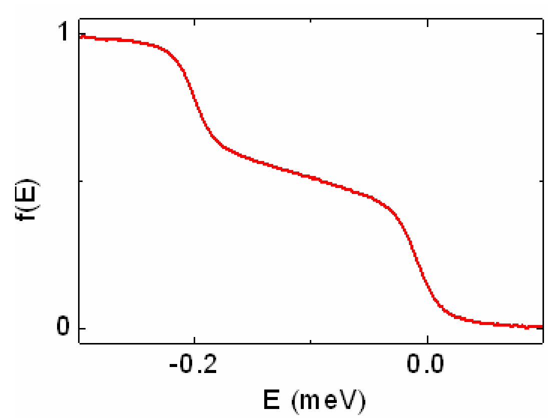
⇒The standard ZBA is suppressed; only the additional contribution to $d\mathcal{J}/dV$ is measured

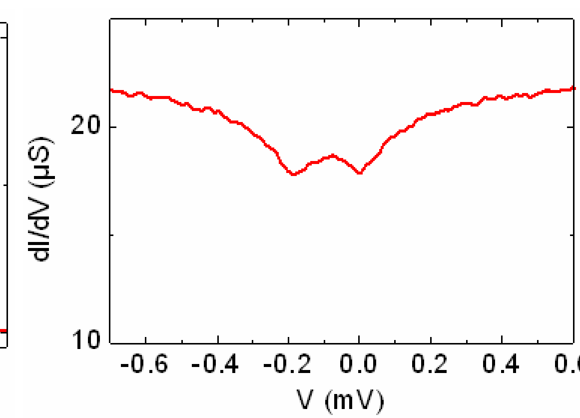
Measuring f with “zero-bias” probe

A. Anthore, F. Pierre*, H. Pothier, D. Esteve, and M. H. Devoret



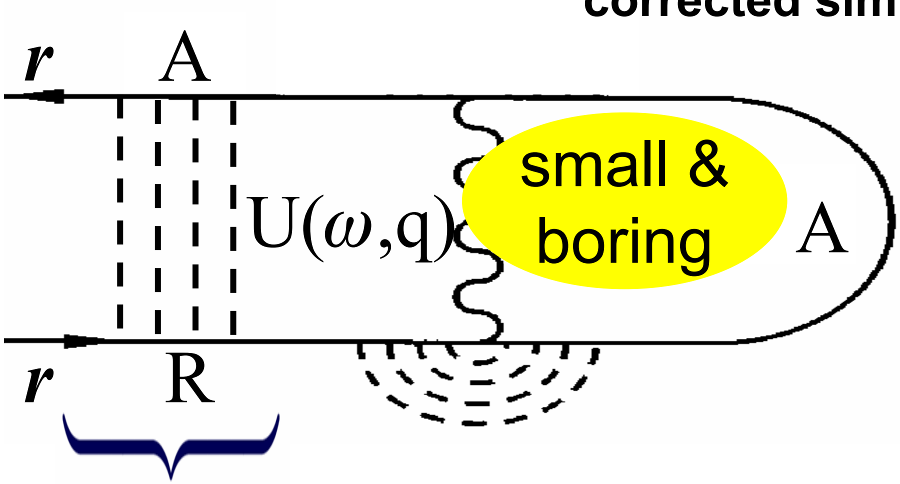






Kinetic contribution to d$\mathcal{J}$/dV

$$\int \mathrm{d}\varepsilon \, \frac{\partial \nu_{\mathrm{P}}(\varepsilon)}{\partial \varepsilon} \left[h_0(\varepsilon) - \underbrace{H_{\mathrm{S}}(\tilde{\varepsilon})\right] A_S(\tilde{\varepsilon}, \boldsymbol{r})}_{\textbf{corrected simultaneously}} \quad \mapsto G^{\mathrm{K}}_{\mathrm{S}}(\tilde{\varepsilon}, \boldsymbol{r})$$



$=\tau_{\mathbf{esc}}$

An extra diffuson is at $\boldsymbol{q}=\omega=0$: divergency!

Can only be cut off at $1/\tau_{\mathrm{esc}}$: particle conservation allows only dN/dt in the denominator

$$\frac{1}{Dq^2 - i\omega} \longrightarrow \frac{1}{Dq^2 - i\omega + 1/\tau_{\mathrm{esc}}}$$

General results:

$$\frac{\mathrm{d}\mathcal{I}^{\mathrm{add}}}{\mathrm{d}V} = \frac{\tau_{esc}}{\overline{\nu} R_T} \int \mathrm{d}\varepsilon \, \frac{\partial \nu_{\mathrm{P}}(\varepsilon)}{\partial \varepsilon} \, \mathrm{St}(\varepsilon + eV)$$

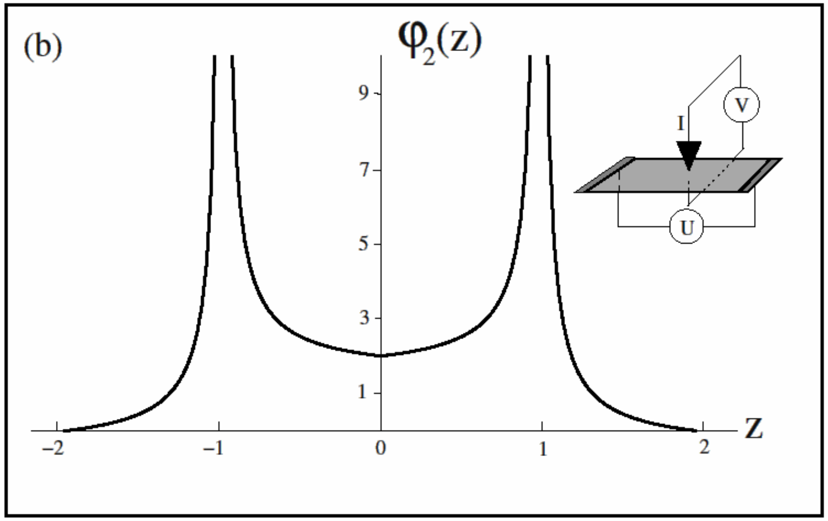
$$\mathrm{St}(\varepsilon) = -\frac{1}{2\pi\nu_d(\epsilon)} \int \frac{\mathrm{d}\epsilon'}{2\pi} \int \frac{\mathrm{d}\omega}{2\pi} K(\epsilon, \epsilon', \omega) \times \Big\{ \left[h(t, \varepsilon' - \omega) - h(t, \varepsilon')\right] \left[1 - h(t, \varepsilon - \omega) h(\varepsilon)\right] - \left[h(t, \varepsilon - \omega) - h(t, \varepsilon)\right] \left[1 - h(t, \varepsilon' - \omega) h(t, \varepsilon')\right] \Big\}$$

At equilibrium ($h = h_0$), $\mathrm{St} \equiv 0$.

Results for a double-step f and a Breit-Wigner DoS of the probe ($\partial\nu/\partial\varepsilon \propto \delta(\varepsilon)$):

$$-\left(\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}V}\right)_2 = \frac{\tau_{\mathrm{esc}}}{\overline{\nu} R_{\mathrm{T}}} \times \begin{cases} \left(g_1\sqrt{2\tau_{\mathrm{D}}U}\right)^{-1} \varphi_1(\varepsilon/U), & \text{quasi-1d} \\ (1/2g_2)\ \varphi_2(\varepsilon/U), & \text{2d} \end{cases}$$

The singularity is smeared out by temperature $T \ll U$



Summary

- For non-equilibrium f, tunnelling current at small V is governed by the standard tunnelling DoS singularity and by a singular kinetic term proportional to St
- Kinetic contribution may be dominant, and is controllable by changing the escape time
- There exist quantum disorder/interaction corrections to the collision integral (St)