Localization & anisotropic correlations in a mesoscopic model of amorphous plasticity

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Outline

- Motivation: plasticity of oxide glasses
- Local reorganization and internal stress
- Modelling plasticity of amorphous plasticity at mesoscopic scale

Credits

- Stéphane Roux, LMT, ENS Cachan
- Jean-Christophe Baret, master ESPCI, Paris 6
- Mehdi Talamali, Ph.D. thesis Paris 6
- Viljo Petaja, Post-doc CNRS

References

J.-C. Baret, D. Vandembroucq and S. Roux PRL **89**, 195106 (2002)

M. Talamali, V. Petaja, D. Vandembroucq and S. Roux PRE **78**, 016109 (2008)

M. Talamali, V. Petaja, S. Roux and D. Vandembroucq arXiv:1005.2463 (2010)

Motivation: plasticity of oxide glasses

Oxide glasses flow at microscopic scale

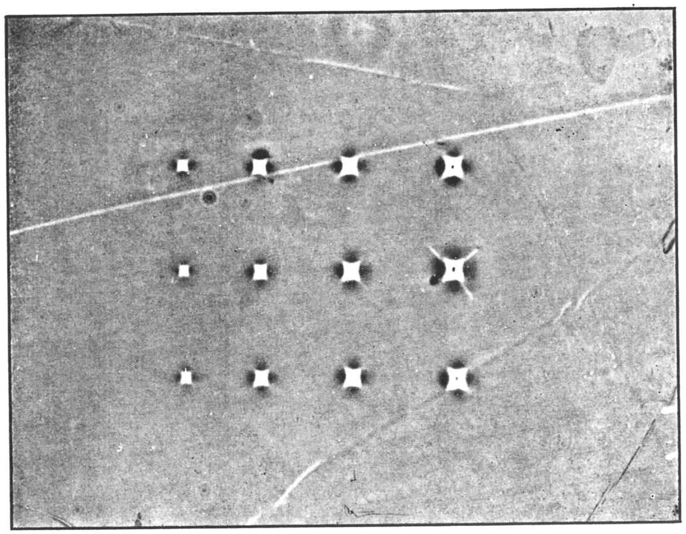


Fig. 5.
Diamond Impressions in Extra Dense Flint Glass. Loads 20, 15, 10 and 5 gm.
Phase-contrast Photograph. 560.

Taylor, Nature 1949

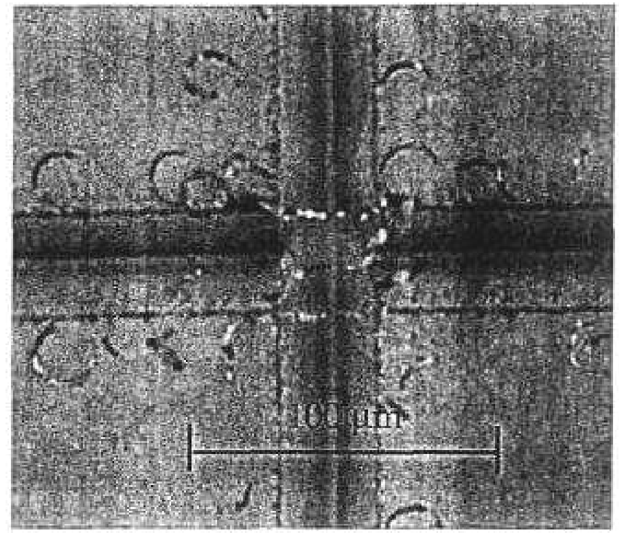


Figure 6 Micro-photograph of a new less-brittle glass scribed by a diamond tool.

Bruche, Schimmel 1956, Ito 2000

Plastic flow in glass

By D. M. Marsh

Tube Investments Research Laboratories, Hinxton Hall, Saffron Walden, Essex

(*Communicated by A. H. Cottrell, F.R.S.—Received* 2 *December* 1963)

[Plates 18 to 20]

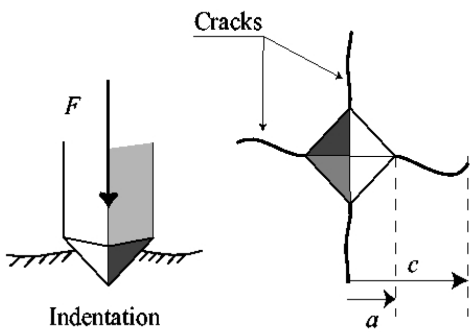
The classical brittle fracture mechanisms fail to account for the mechanical properties of glasses, but the widespread evidence of plastic flow in all glass fracture phenomena even at room temperature suggests that this flow might provide a consistent glass strength theory. This first paper discusses the evidence for flow, evaluates the flow stresses by re-interpretation of the indentation hardness figures, and shows that the flow stresses are much lower than the theoretical cohesive strengths.

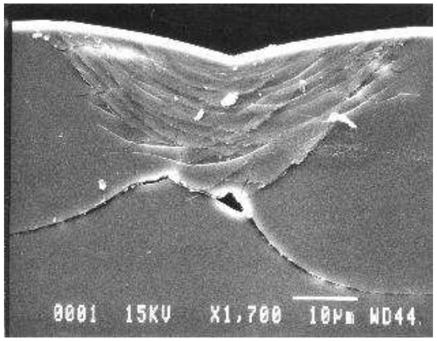
Plasticity of oxide glasses

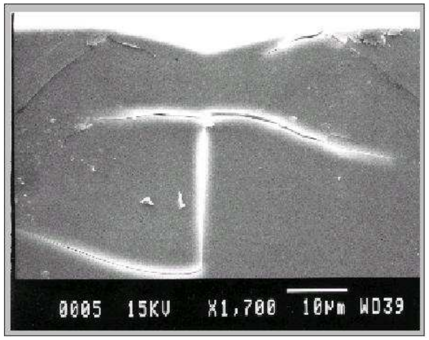
Indentation induces plastic imprints No crack under low enough loading. Plastic deformation confined at micro-scale ;

Permanent densification under indentation or hydrostatic test, amplitude depends on glass nature, up to 20 % in the case of silica glass ; strong coupling shear/pressure ;

No dislocation deformation results of series of local reorganizations.







A. Perriot et al. J. Am. Ceram Soc **89**, 596 (2006)

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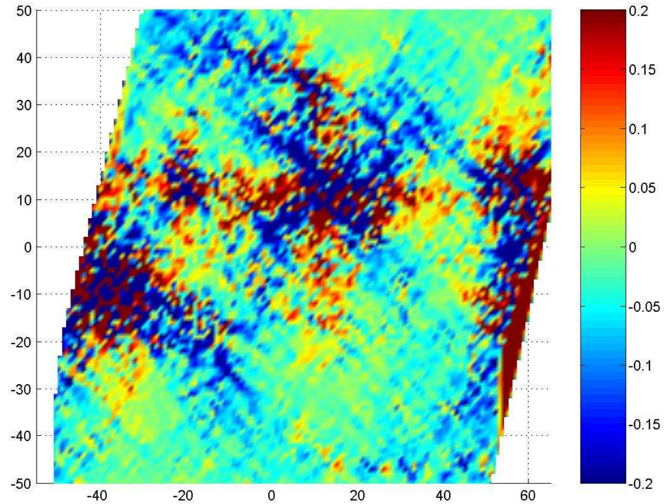
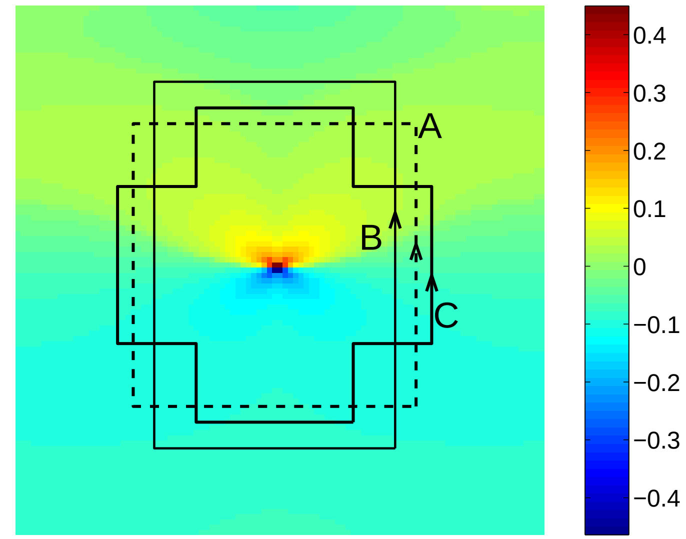
Local reorganization and internal stress

Local reorganizations induce anisotropic internal stress





MD simuations of LJ binary glass (Tanguy et al. PRB 06).

Numerical solution of elastic displacement field induced by a plastic inclusion.

The (far field) internal stress induced by a plastic reorganization obeys a quadrupolar symmetry:

$$\sigma_{xy}(r,\theta) = A\frac{\cos 4\theta}{r^2}$$

Potential formulation of plane elasticity

Stress and displacement can be written as complex fields

$$\mathbf{U} = U_x + iU_y \,, \qquad S_0 = \sigma_{xx} + \sigma_{yy} \,, \qquad \mathbf{S} = \sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} \,.$$

General solution of balance and compatibility equations can be obtained from 2 harmonic potentials φ and ψ (Kolossov & Muskhelichvili)

$$2\mu\mathbf{U} = \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \,,$$

$$S_0 = 2\left[\varphi'(z) + \overline{\varphi'(z)}\right] \,,$$

$$\mathbf{S} = 2\left[\bar{z}\varphi''(z) + \psi'(z)\right] \,,$$

where μ is the elastic shear modulus and $\kappa = (3-4\nu)$ for plane strain.

Plastic inclusion and universal elastic singularities

Outside inclusion, the harmonic potentials can be expanded:

$$\varphi(z) = \alpha^{out} z + \sum_{n=1}^{\infty} \frac{\varphi_n}{z^n}, \qquad \psi(z) = \beta^{out} z + \sum_{n=1}^{\infty} \frac{\psi_n}{z^n}$$

Dominant singular terms φ_1/z (and ψ_1/z) associated to the elastic stress induced by an equivalent circular inclusion of area $\mathcal{A}$ experiencing a plastic deviatoric γ_p (or volumetric strain δ_p)

$$\varphi_1 = \frac{2i\mu\mathcal{A}\gamma_p}{\pi(\kappa+1)}, \quad \psi_1 = -\frac{2\mu\mathcal{A}\delta_p}{\pi(\kappa+1)}.$$

All details of the local rearrangement are carried in the prefactor $\mathcal{A}\gamma_p$. In particular, for a pure shear plastic event:

$$\sigma_{xy} = -\frac{2\mu}{\kappa+1}\frac{\mathcal{A}\gamma_p}{\pi r^2}\cos(4\theta)$$

M. Talamali, V. Petäjä, DV, S. Roux PRE 08

Modelling plasticity of amorphous plasticity at mesoscopic scale

A scalar depinning-like model of amorphous plasticity

A scalar plastic criterion $\sigma_{xy} > \sigma_Y$

Disorder Local plastic criterion : $\sigma_{xy}(i,j) > \overline{\sigma_Y} + \delta\sigma_Y(i,j)$

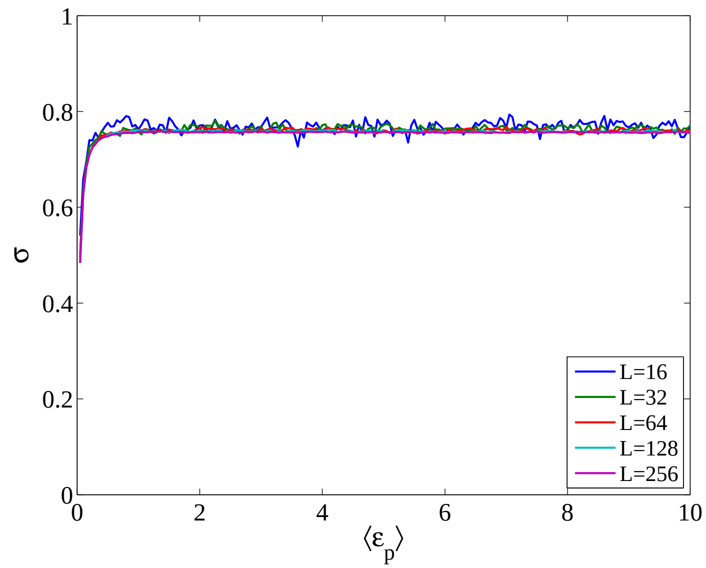
Local reorganization local slip increment $\delta\gamma$ and update of local plastic threshold

Anisotropic elastic response A local slip induces a stress redistribution σ^{el} all over the system, $\sigma^{el} \propto \mu \cos 4\theta / r^2$

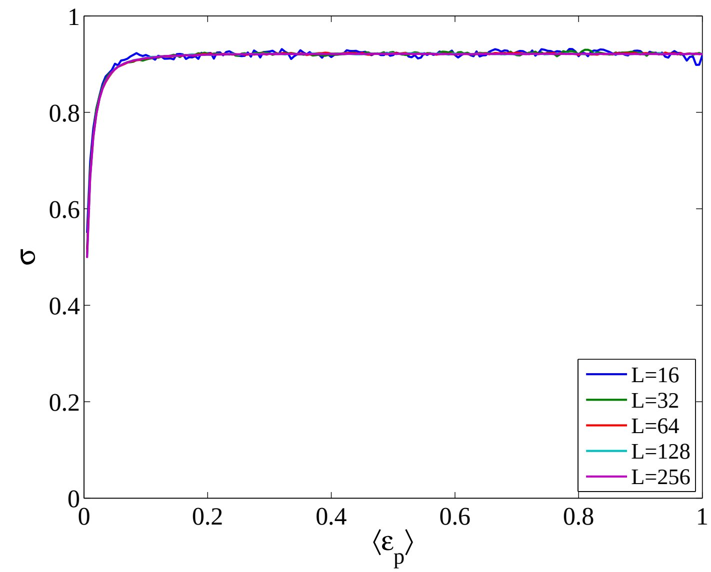
Extremal dynamics

Parameters of the model: $\mu = 1$, $\delta\sigma_Y \in rand[0,1]$, $\delta\gamma \in rand[0,d]$.

Stress-strain relation



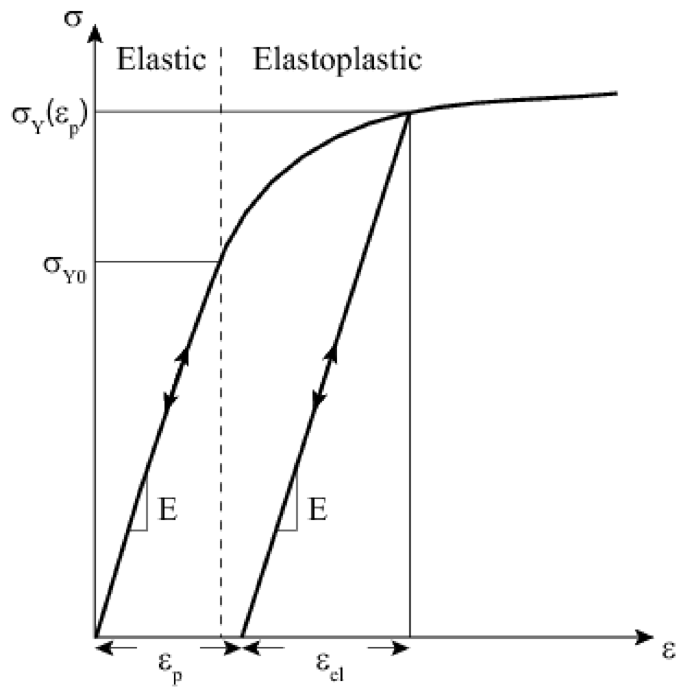
$d = 0.1$

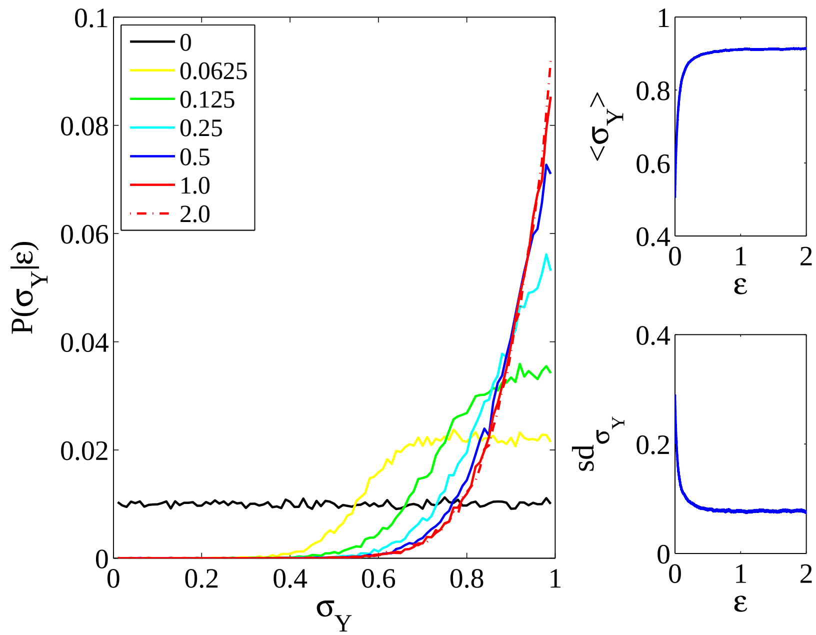


$d = 0.01$

After a transient regime (hardening) we obtain a plateau stress dependent on the local slip amplitude d which can also be seen as the amplitude of a mechanical noise (stress redistribution).

Statistical hardening

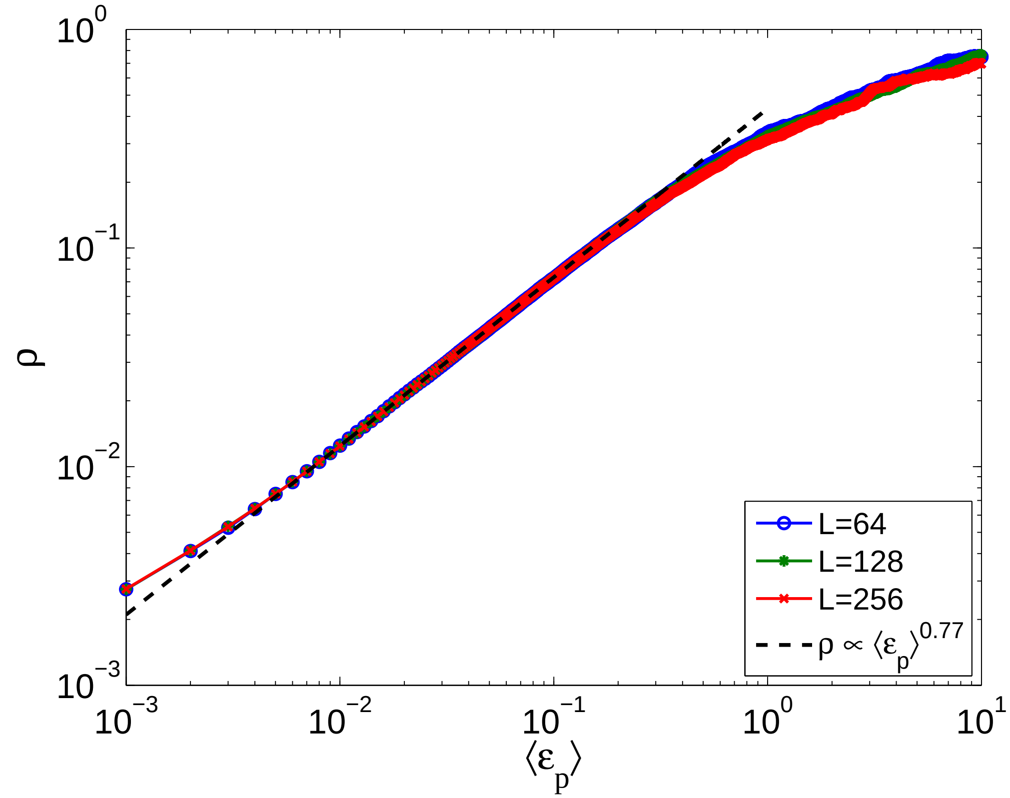




Hardening behavior: the plastic yield stress gets higher whith deformation.

Interpretation: "Darwinian" or Bak-sneppen-like dynamics; progressive exhaustion of the weakest sites during the transient stage induces a systematic bias in the distribution of local plastic thresholds.

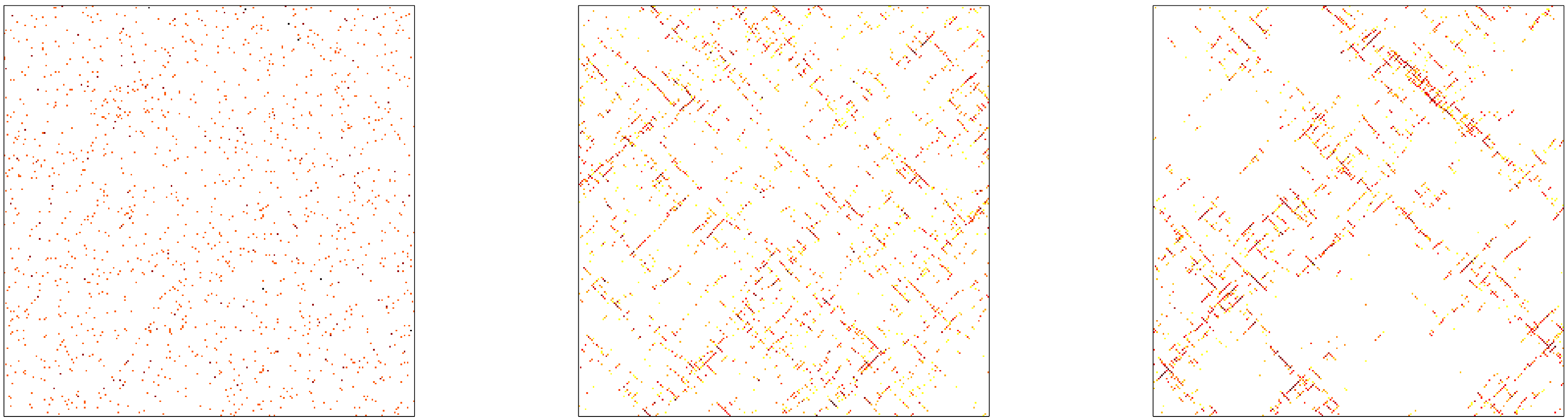
Fluctuation of plastic strain



In case of localization along a band of extension L, the plastic strain standard deviation should grow as $\rho(\epsilon_p) \propto L^{1/2}\varepsilon_p$ while in absence of localization, $\rho(\epsilon_p)$ should saturate.

Here we have $\rho(\epsilon_p) \propto \varepsilon_p^{0.77}$ in the hardening regime then a diffusive-like regime $\rho(\epsilon_p) \propto \sqrt{\varepsilon_p}$

Development of localization

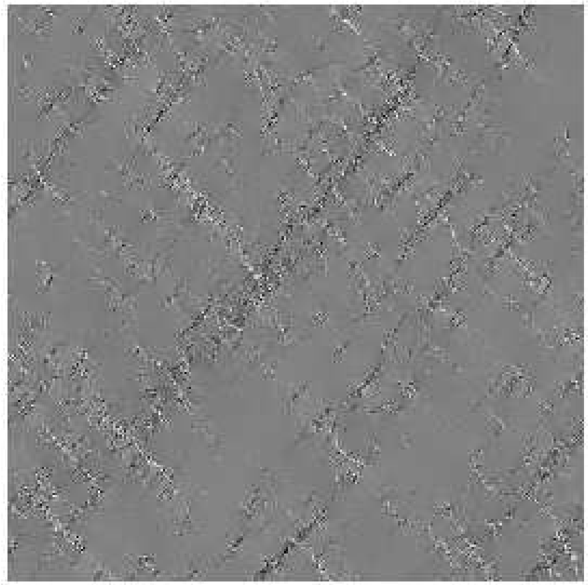


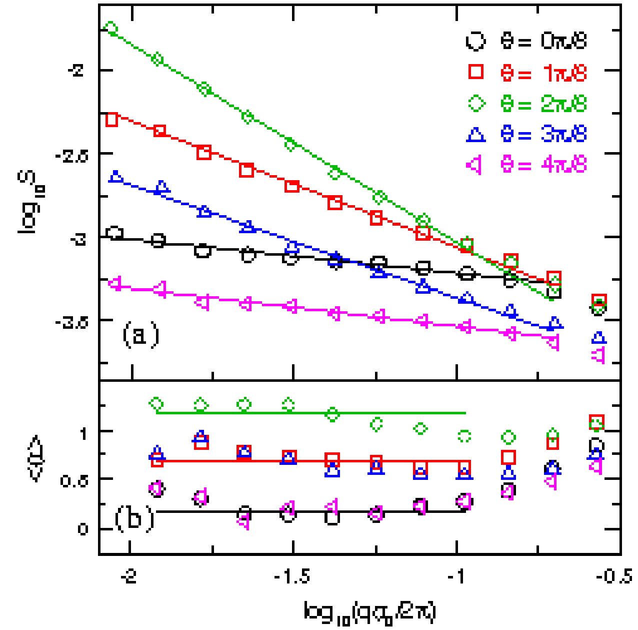
During the transient/hardening stage, plastic deformation gets progressively more and more correlated with the same quadrupolar symmetry as the elastic interaction.

Localization *vs* diffusion

In the stationary regime, plastic deformation remains localized but, localization patterns are not persistent, rather they diffuse throughout the system.

Comparison with atomistic simulations - Maloney/Robbins



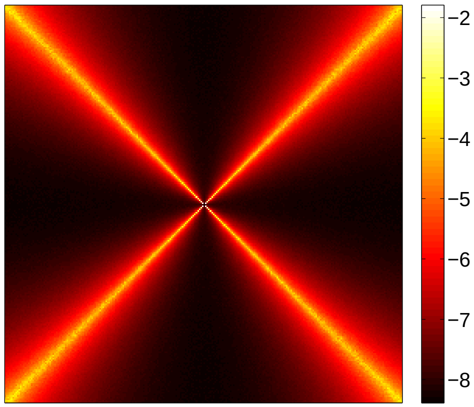


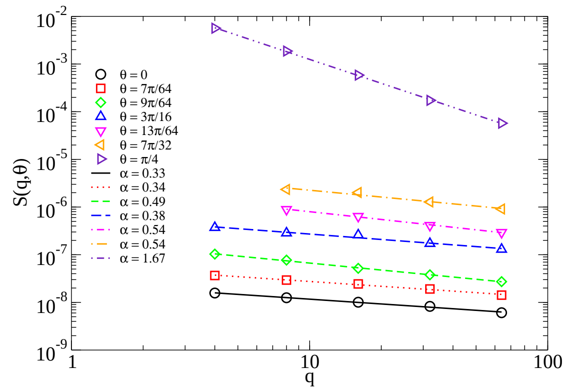
Localization of the plastic deformation; avalanches

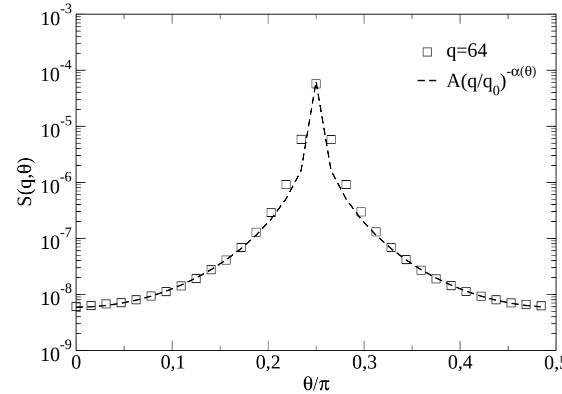
Anisotropic scaling of the strain correlations

Maloney, Robbins, JPCM 08, PRL 09

Anisotropic strain correlation





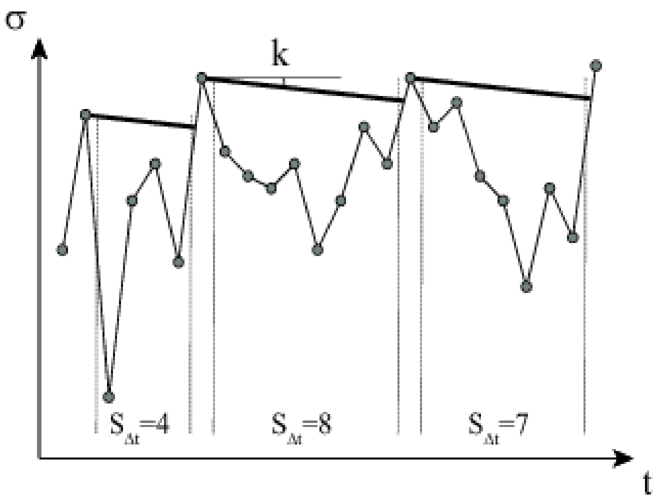


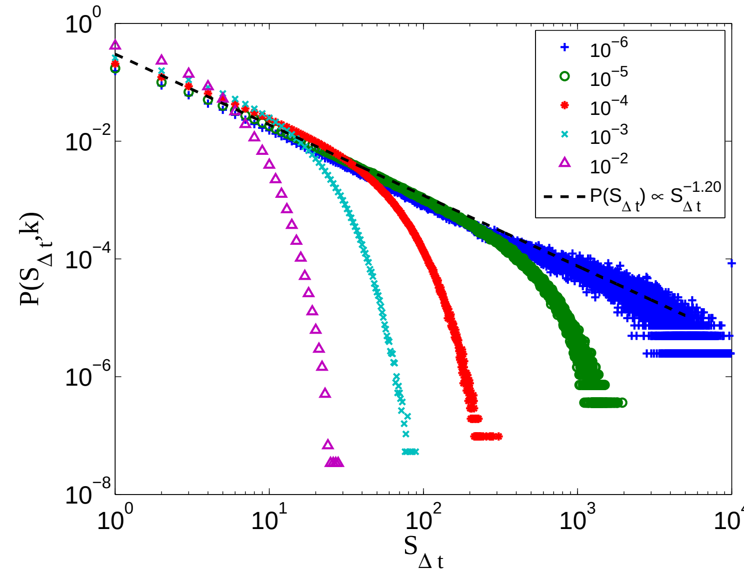
Power spectrum of plastic deformation exhibits **scaling** and **quadru-polar symmetry**

$$P(q,\theta) \approx A \left(\frac{q}{q_0}\right)^{-\alpha(\theta)} \quad ; \quad \alpha(\theta) = a - b|\cos 2\theta|^x$$

$$a \approx 1.7 \,, \quad b \approx 1.4 \,, \quad x \approx 1.4$$

Avalanches





Extremal dynamics gives access to all accessible configurations. Possiblity of reconstructing the dynamics corresponding to an arbitrary loading *e.g.* finite stiffess k.

Scaling invariance of avalanches with a stiffness dependent cut-off:

$$p(S) \propto S^{-1.2} \exp[S/S^*(k)] \,, \quad \langle S \rangle \propto L^{0.8}$$

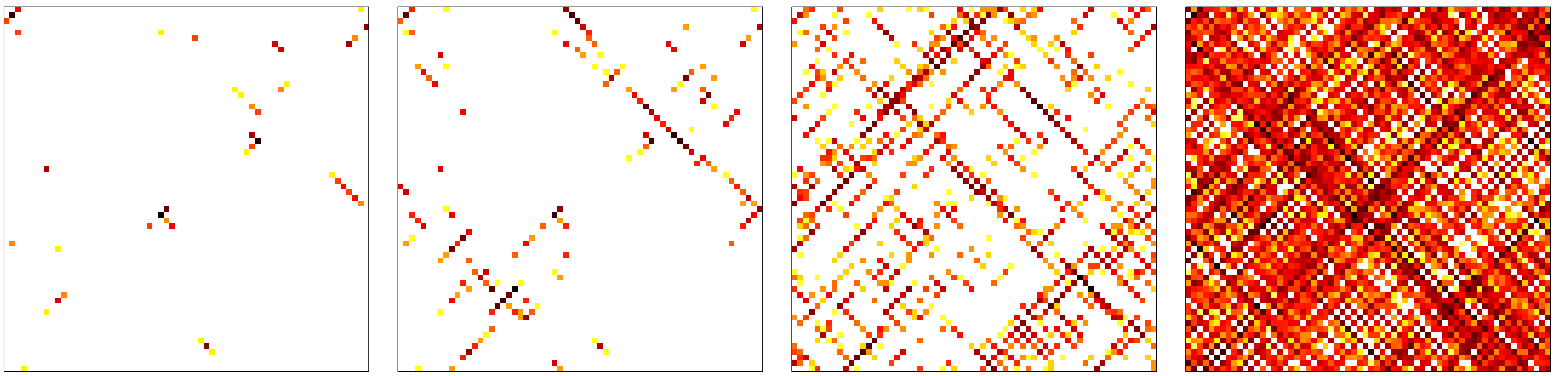
Summary

Depinning-like scalar model

Competition disorder – quadrupolar elastic interaction

Bak-Sneppen like statistical hardening

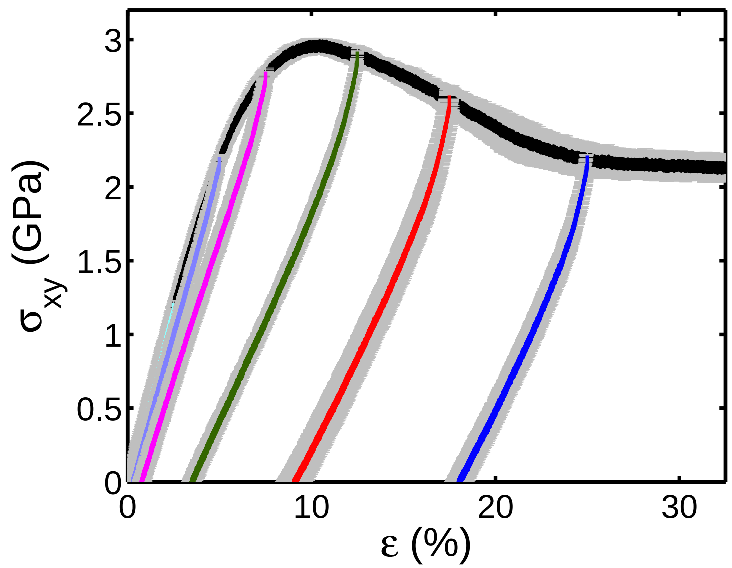
Localisation – Diffusion of deformation



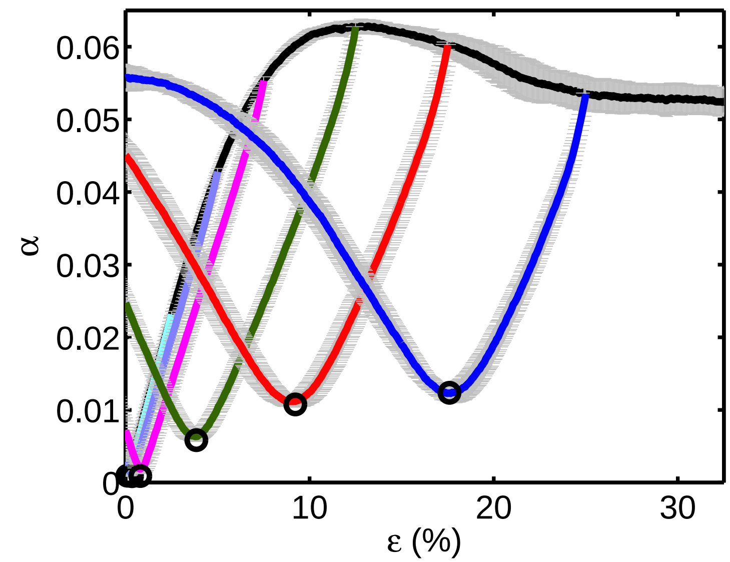
Avalanche and anisotropic strain correlation

quantitative results reminiscents of atomistic simulations (Maloney, Robbins 09)

Plasticity induced structural anisotropy in a model silica glass



MD simulation of amorphous silica under shear flow



Evolution of a fabric tensor like anisotropy index

The structure keeps a memory of the plastic deformation.

C.L. Rountree, DV, M. Talamali, E. Bouchaud, S. Roux, PRL **102** 195501 (2009)