Particle Segregation in Binary Mixtures of Disks

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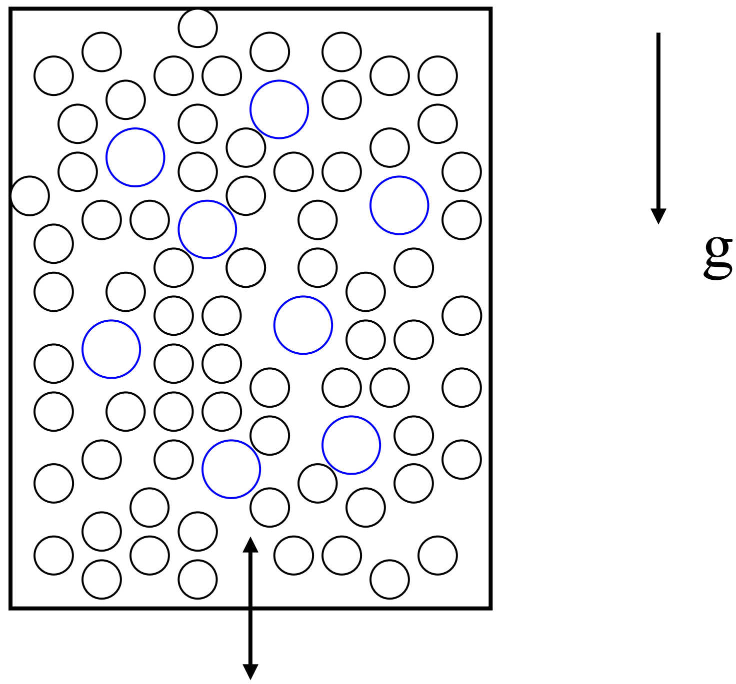
The influence of non-equipartition of energy on the segregation of disks with different sizes and/or masses in an agitated system under gravity

System

Masses
m_A, m_B

Radii
r_A, r_B

Number densities
n_A, n_B



Mass densities: $\rho_A \equiv m_A n_A, \ \rho_B \equiv m_A n_A$

$$n \equiv n_A + n_B \text{ and } \rho \equiv \rho_A + \rho_B$$

Diffusion velocities: $w_A, w_B; \ \rho_A w_A + \rho_B w_B = 0$

$$w_{BA} \equiv w_B - w_A$$

Temperatures: $T_A, T_B; \ nT \equiv n_A T_A + n_B T_B$

$$T_A = T + \theta_A, \ T_B = T + \theta_B$$

History

Simulation/Heuristic Theory

D.C. Hong, P.V. Quinn & S. Luding, Phys. Rev. Letts. 86, 3423 (2001).

Kinetic Theory: Equipartition

J.T. Jenkins & D. K. Yoon, Phys. Rev. Letts. 88, 194301 (2002).

Experiment

A.P.J. Breu, H.-M. Ensner, C.A. Kruelle & I. Rehberg, Phys. Rev. Letts. 90, 014302 (2003).

Kinetic Theory: Non-Equipartition

L. Trujillo & H.J. Herrmann, Gran. Matter 5, 85 (2003).

L. Trujillo, M. Alam & H.J. Herrmann, Europhys. Letts. 64, 190 (2003).

Segregation

Species momentum balance

Partial pressures: π_A, π_B; Collisional exchange ϕ

$$0 = -\pi'_A - \rho_A g + \phi$$

$$0 = -\pi'_B - \rho_B g - \phi$$

Sum

$$-p' - \rho g = 0$$

$$p \equiv \pi_A + \pi_B$$

Weighted difference

$$-\rho_B \pi'_A + \rho_A \pi'_B + \rho\phi = 0$$

Species energy balance

Collisional dissipations γ_A, γ_B; Energy source s

$$0 = -\gamma_A + \rho_A s$$

$$0 = -\gamma_B + \rho_B s$$

Sum

$$0 = -(\gamma_A + \gamma_B) + \rho s = 0$$

Weighted difference

$$-\frac{\gamma_A}{\rho_A} + \frac{\gamma_B}{\rho_B} = 0$$

Constitutive Relations: Momentum

$$\pi_i = n_i(T+\theta_i) + \sum_{k=A,B} \frac{\pi}{2} g_{ik} r_{ik}^2 n_i n_k \left(T + \frac{m_i\theta_k + m_k\theta_i}{m_{ik}} \right)$$

$$r_{AB} \equiv r_A + r_B \text{ and } m_{AB} \equiv m_A + m_B$$

$$g_{ik} = \frac{1}{1-\nu} + \frac{9}{8}\frac{r_i r_k}{r_{ik}} \frac{\pi\left(n_A r_A + n_B r_B\right)}{(1-\nu)^2}$$

$$\nu_A = n_A \pi r_A^2 \quad \nu_B = n_B \pi r_B^2 \quad \nu = \nu_A + \nu_B$$

$$\phi = \frac{\pi}{2} g_{AB} r_{AB}^2 n_A n_B T \left[\frac{m_B - m_A}{m_{AB}} (\ln T)' + \left(\ln \frac{n_A}{n_B} \right)' + \frac{4}{r_{AB}} \left(\frac{2}{\pi} \frac{m_A m_B}{m_{AB} T} \right)^{1/2} w_{BA} \right]$$

Constitutive Relation: Energy

$$\gamma_i = \sum_{k=A,B} 4 g_{ik} r_{ik} n_i n_k T \left[\left(\frac{2\pi m_i m_k T}{m_{ik}} \right)^{1/2} \left(\frac{\theta_i - \theta_k}{m_{ik} T} \right) \right.$$

$$\left. + \frac{1}{2} \frac{m_k}{m_{ik}} (1 - e_{ik}) \left(\frac{2\pi m_{ik} T}{m_i m_k} \right)^{1/2} \left(1 + \frac{3}{2} \frac{m_i \theta_k + m_k \theta_i}{m_{ik} T} \right) \right]$$

Energy

$$\theta_B - \theta_A = \pi \frac{n m_{AB}}{\rho} D_{AB} (1-e) \left(\frac{T}{\pi}\right)^{1/2} \left[2 m_n n_A r_A \frac{g_{AA}}{m_A^{1/2}} + 2^{1/2} \left(m_B^2 n_B - m_A^2 n_A \right) r_{AB} \frac{g_{AB}}{(m_A m_B m_{AB})^{1/2}} - 2 m_A n_B r_B \frac{g_{BB}}{m_B^{1/2}} \right]$$

$$D_{AB} \equiv \frac{1}{2 n r_{AB} g_{AB}} \left(\frac{m_{AB} T}{2 \pi m_A m_B} \right)^{1/2}$$

Momentum

(nearly homogeneous)

$$w_{BA} \frac{\pi}{2} g_{AB} r_{AB}^2 n_B T \frac{4}{r_{AB}} \left(\frac{2}{\pi} \frac{m_A m_B}{m_{AB} T} \right)^{1/2} = \frac{n_B}{n_A R + n_B} \left(m_A - R m_B \right) g$$

$$R \equiv \frac{\pi_A}{\pi_B} \frac{n_B}{n_A}$$

B dilute in dense A

$$(n_B / n_A \ll 1), \quad \eta \equiv r_A / r_B$$

$$g_{AA} \doteq \frac{1}{1-\nu} + \frac{9}{16}\frac{\nu}{(1-\nu)^2}$$

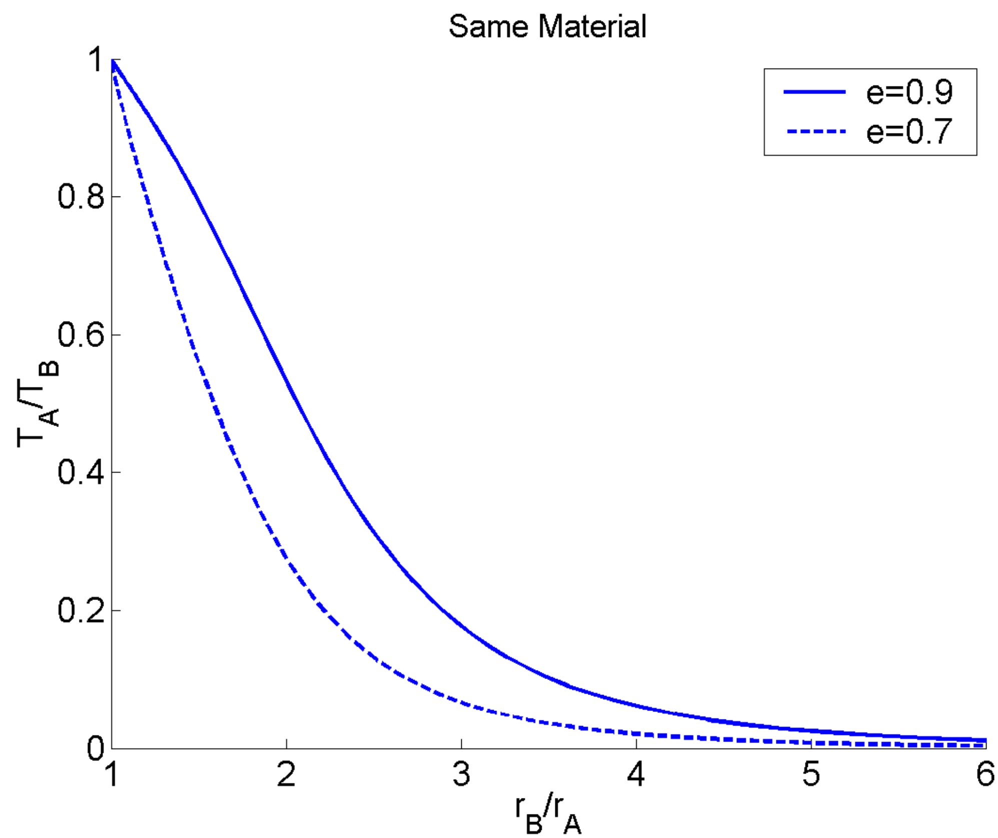
$$g_{AB} \doteq \frac{1}{1-\nu} + \frac{9}{8}\frac{1}{1+\eta}\frac{\nu}{(1-\nu)^2}$$

Then $\theta_A / \theta_B = -n_B / n_A \ll 1$ and with $\mu \equiv \rho_B^s / \rho_A^s$

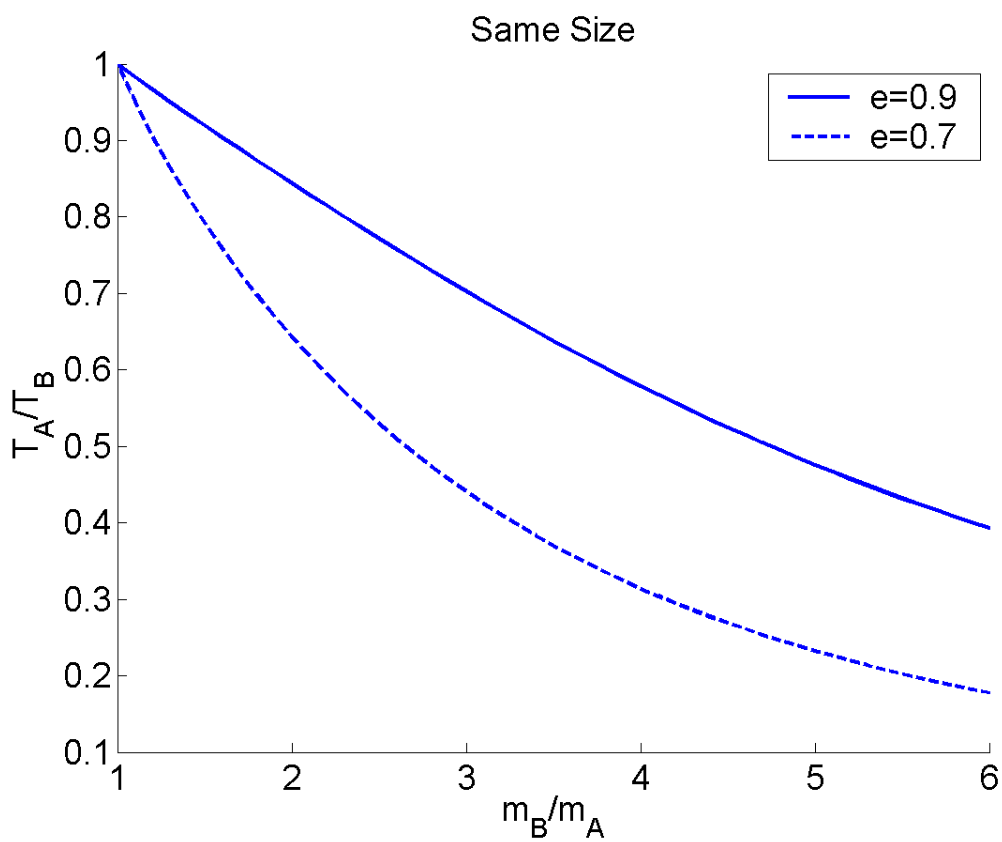
$$\frac{\theta_B}{T} \doteq \frac{1-e}{2}\left(\frac{\mu+\eta^2}{\eta^2}\right)^{3/2}\left[\frac{2^{1/2}\mu^{1/2}}{1+\eta}\frac{g_{AA}}{g_{AB}} - \frac{\eta^3}{\mu\left(\mu+\eta^2\right)^{1/2}}\right]$$

and

$$\frac{T_B}{T_A} \doteq \frac{T+\theta_B}{T} = 1 + \frac{\theta_B}{T}$$



Same Material
e=0.9
e=0.7
T_A/T_B
r_B/r_A



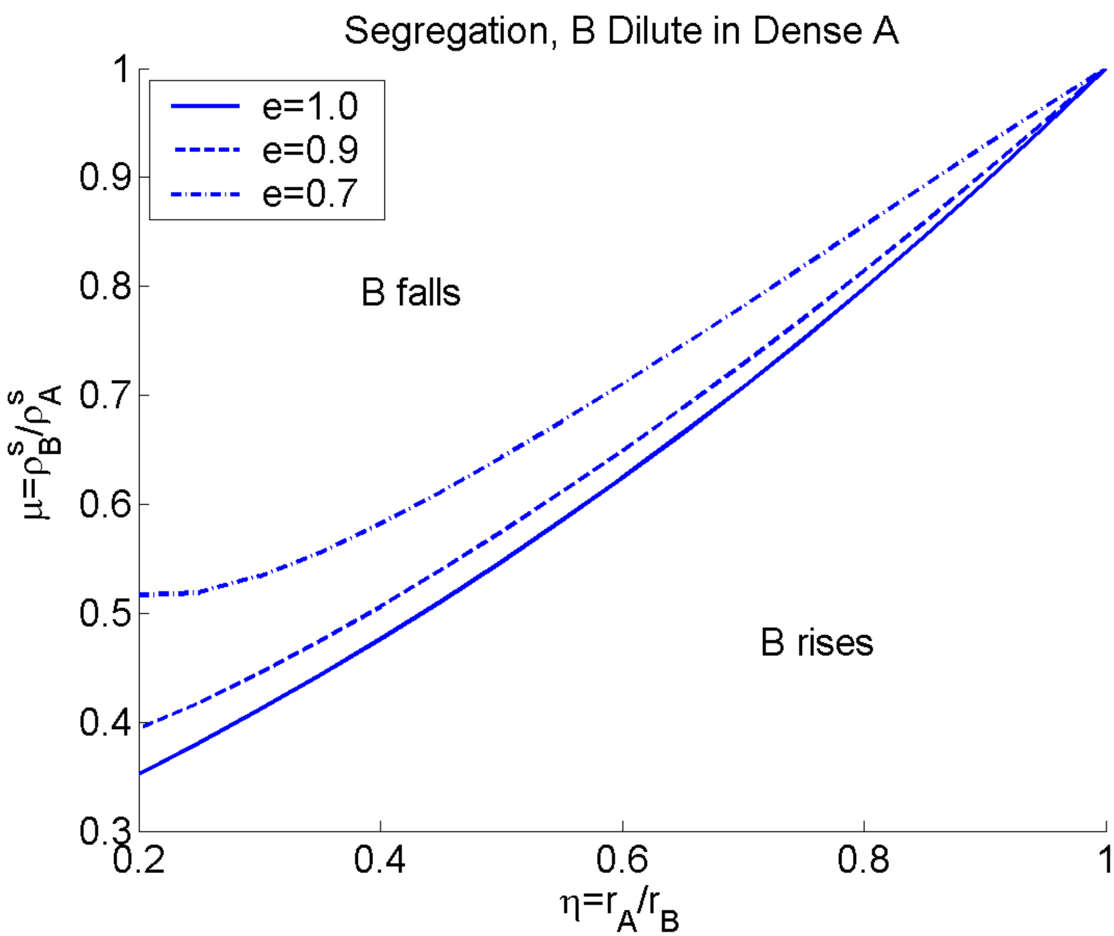
Same Size
e=0.9
e=0.7
T_A/T_B
m_B/m_A

B dilute in dense A

$$R \doteq \frac{g_{AA}}{g_{BA}} \frac{4r_A^2}{r_{AB}^2} \frac{1}{1 + \dfrac{\theta_B / T}{1 + m_B / m_A}}$$

$$w_{BA} > 0 \text{ (B rises) when } R < m_A / m_B$$

Plot $R - m_A / m_B = 0$ for $e = 0.7,\ 0.9,\ 1.0$


Segregation, B Dilute in Dense A
e=1.0
e=0.9
e=0.7
B falls
B rises
μ=ρ^s_B/ρ^s_A
η=r_A/r_B
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6
0.8
1