Jamming: from grains to glasses & back again

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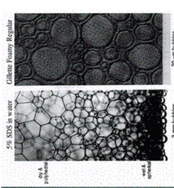
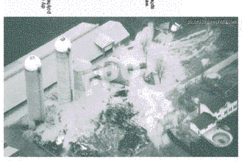
Acknowledgements:
Corey O'Hern, Yale
Matthieu Wyart, CEA
Thomas Witten, U. Chicago
NSF

- Definition of jamming transition
- Scaling near jamming threshold
- Critical nature of transition
- Frictional packings + fun stuff

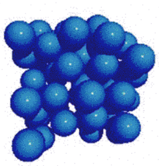
Jamming Transitions



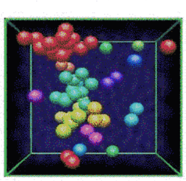
- **Dynamic Jamming Transition**
  - Foams, emulsions jam as **shear stress** drops
    - foods, cosmetics, oils



  - Granular materials jam as **driving force** drops
    - pharmaceuticals, ceramics, avalanches, slurries, grains



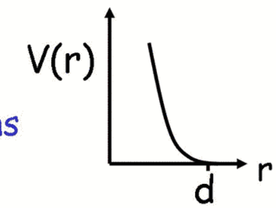
- **Colloidal Glass Transition**
  - Colloidal suspensions jam as **density** increases
    - complex fluids, paints, cytoplasm, novel materials



- **Glass Transition**
  - Supercooled liquids jam as **temperature** drops
    - metallic glasses, alloys, structured materials

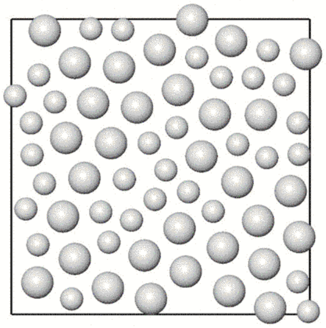
Jamming in a Model System: T=0 , μ=0

- Interact only on contact (soft)
- Finite-range and purely repulsive
- Mechanically stable configurations
  - 3D [$16 \le N \le 10000$]

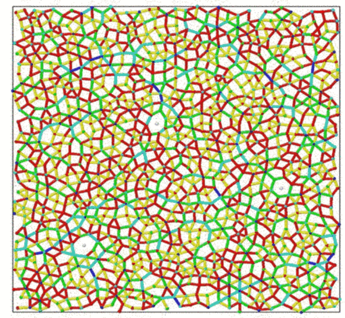


2D visualisation: vary volume fraction of particles

ϕ increasing

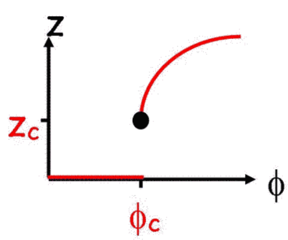


ϕ decreasing



Onset of jamming occurs at ϕ_c

Properties of Ideal Jamming Transition



Coordination Number

$z = 0$ for $\phi < \phi_c$

$(z-z_c) \sim (\phi-\phi_c)^{0.5}$ for $\phi > \phi_c$

$z_c \approx 6$ 3D

$\phi_c \approx 0.64$ 3D

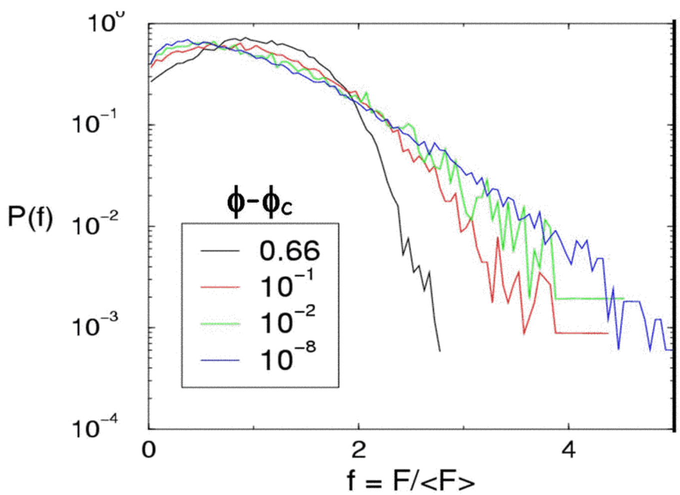
- Pressure: $p \sim (\phi-\phi_c)^{\beta}$
- Shear Modulus: $G \sim (\phi-\phi_c)^{\gamma}$

Onset of jamming occurs at ϕ_c

[O'Hern et al., PRL (2002), PRE (2003)]

Force Distribution

Normal Forces between particles in contact

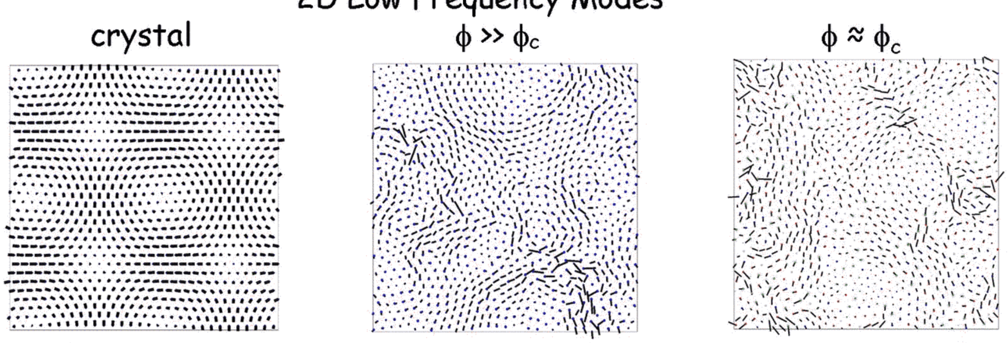


- Distribution becomes broader close to jamming transition
- Peak shifts to smaller f

Normal Mode Analysis

- Compute Dynamical Matrix: $\mathbf{D}\mathbf{u} = \omega^2\mathbf{u}$
  - eigenvalues – normal mode frequencies ω^2
  - eigenvectors – polarization vectors **p**
- Density of States
  - $D(\omega) \sim \omega^{D-1}$ Debye theory of crystals
  - In 3D: $D(\omega) \sim \omega^2 \Rightarrow C_p \sim T^3$
- **Long-wavelength propagative phonon modes**

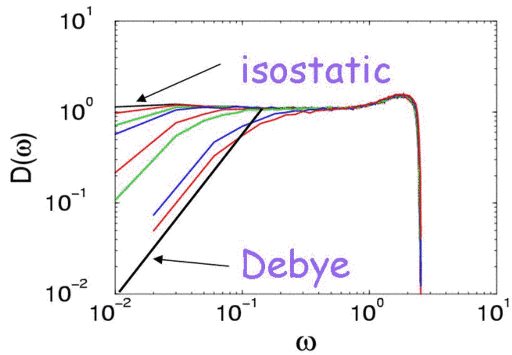
2D Low Frequency Modes



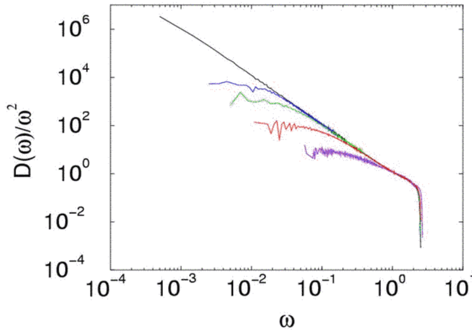
Lose Debye character close to jamming threshold

Density of States

Characteristic feature in $D(\omega)$ signals onset of jamming



Dramatic excess of low-freq modes signals onset of jamming transition

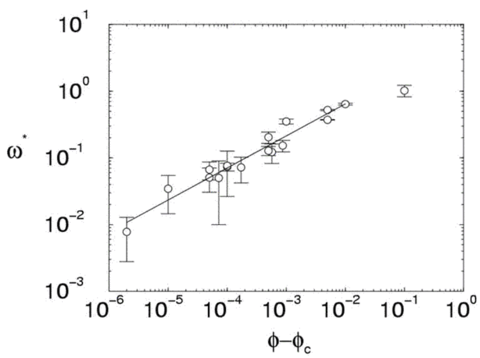


"Boson peak"

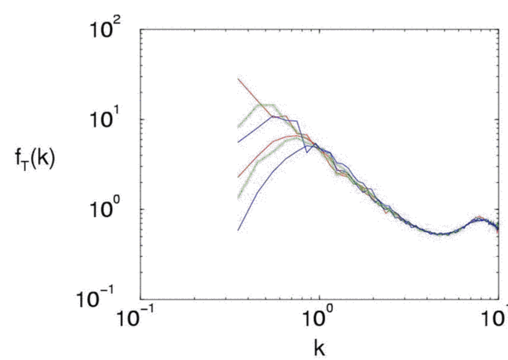
$D(\omega)$ ~ constant:
$C_p \sim T$ [low-T glasses]

Departure from isostatic state occurs at characteristic frequency ω^*

Characteristic Frequency

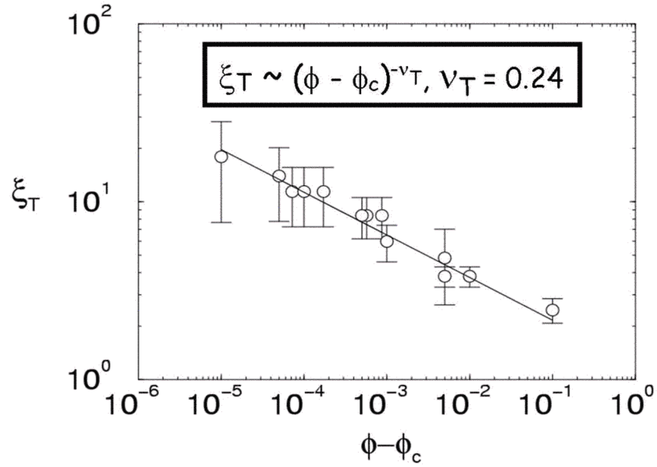


$\omega^* \sim (\phi - \phi_c)^{\Omega}$, $\Omega = 0.48$



- Fourier transform of polarisation vectors at ω^*
- Peak in $f_T(k,\omega^*)$ at k^*
- Define length scale $\xi_T = 2\pi/k^*$

Characteristic Lengthscale



- Consistent with speed of sound
  - $k^* = \omega^*/c_T(\phi)$
  - $\omega^* \sim (\phi - \phi_c)^{\Omega}$
  - $c_T(\phi) \sim \sqrt{G}$, $G \sim (\phi - \phi_c)^{\gamma}$

$$\nu_T = \Omega - \gamma/2$$

(independent of potential)

[Silbert et al. 2005]

Frictional Packings

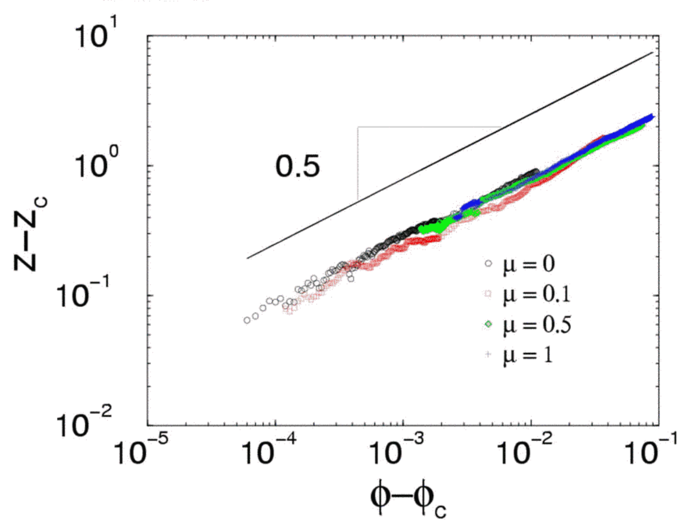
Scaling of Coordination number

[Makse]

Fitting parameters: z_c & ϕ_c

μ	z_c	ϕ_c
0	6.0	0.6390
0.1	5.2	0.6121
0.5	4.5	0.5716
1	3.9	0.5610

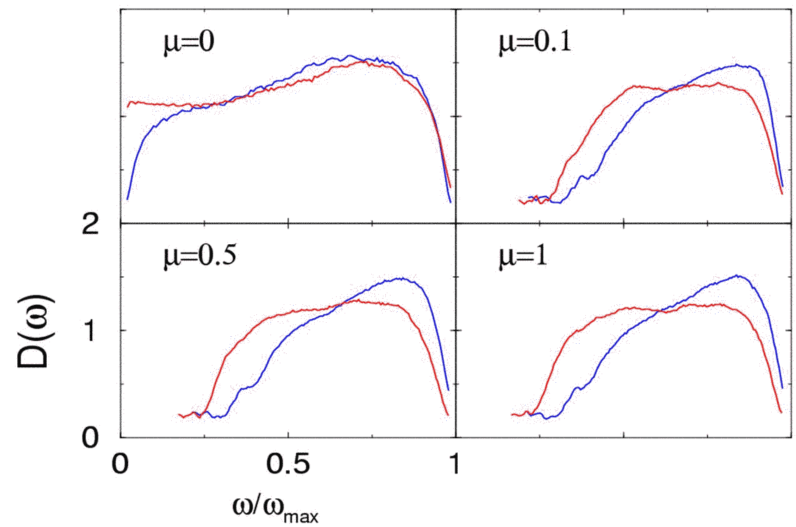
$$(z-z_c) \sim (\phi-\phi_c)^{0.5}$$



$\mu > 0$: z_c & ϕ_c history dependent

Frictional Packings

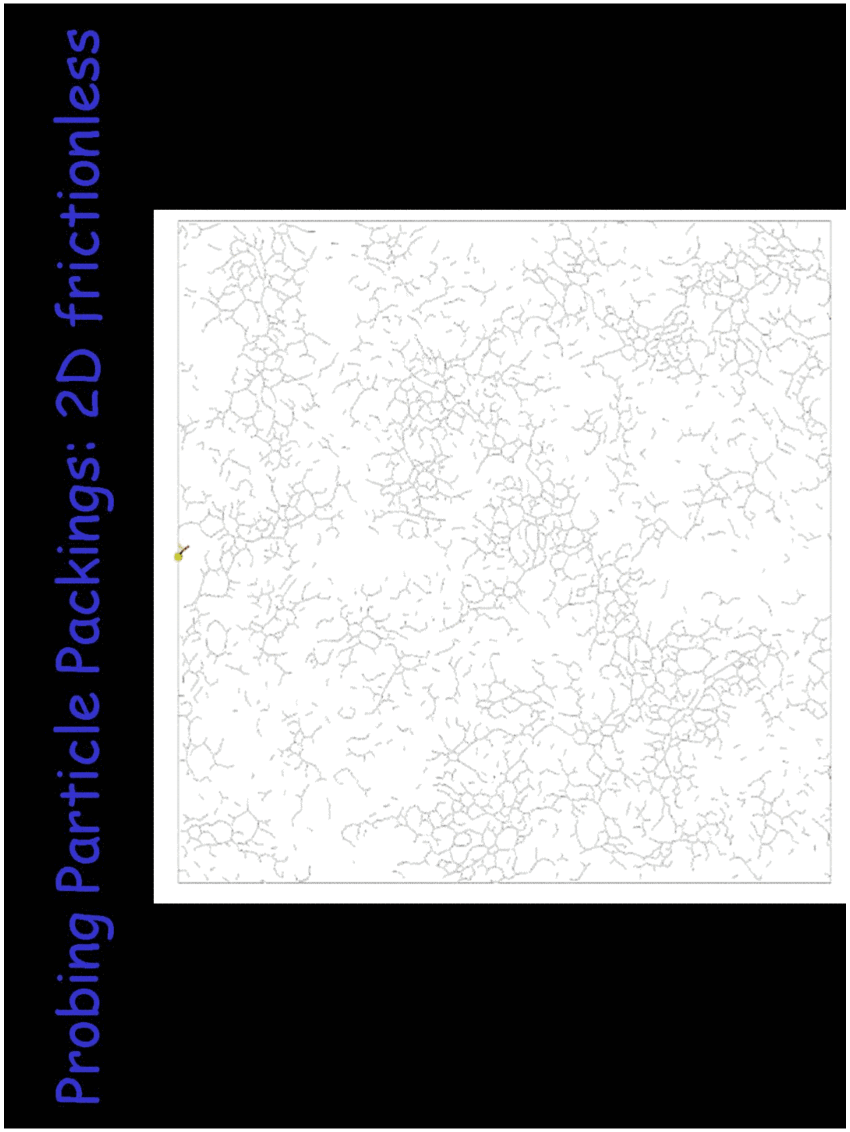
Density of States

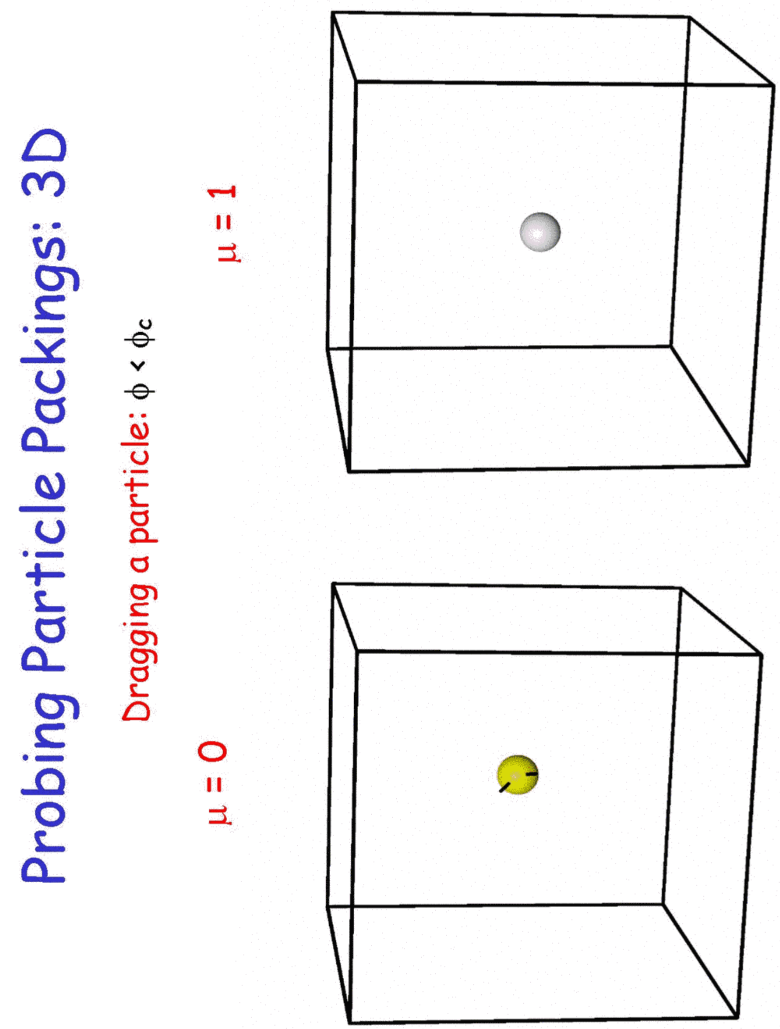


Frictional packings exhibit similar trend as frictionless systems

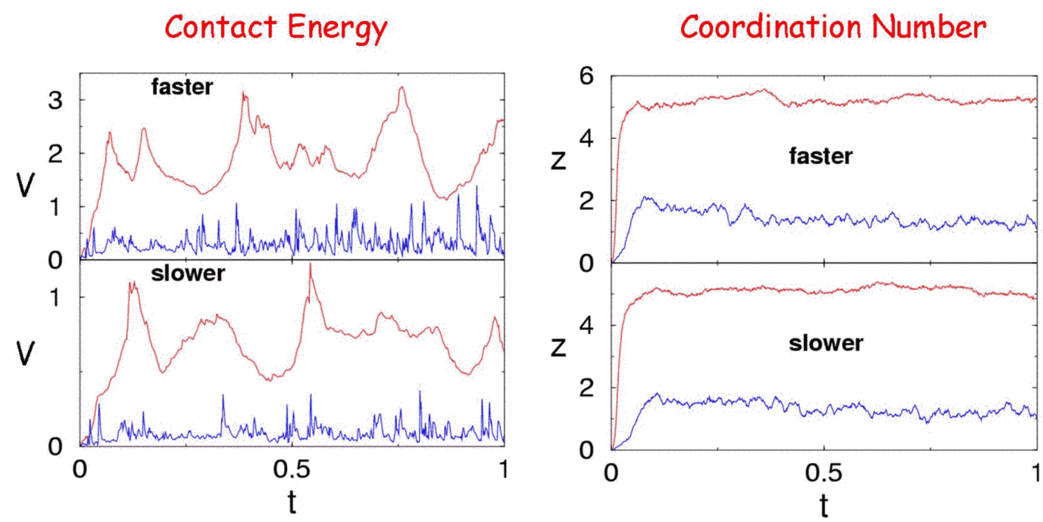
Implications for structural stability, modes of failure... summary

- Motivation for recent theoretical analyses:
  - Wyart et al. :- density of states, scaling and length scale, soft modes
  - Schwarz et al. :- k-coordinated clusters, scaling and length scale, mixed first/second order nature
- Critical nature of jamming: control properties near threshold
- Relation to granular materials
  - force perturbations and force chains [Behringer, Clement]
  - granular-elastic crossover [Goldenberg, Wittmer]
  - probing particle packings [LANL, Schiffer, Weeks]
- How does friction fit in?


Probing Particle Packings: 2D frictionless


Probing Particle Packings: 3D
Dragging a particle: $\phi < \phi_c$
$\mu = 1$
$\mu = 0$

Dragging Analysis



Harder to drag through frictionless packings because ϕ higher