Energy partition and segregation for an intruder in a vibrated granular system under gravity

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Introduction

Two observed peculiarities of vibrated granular systems:

- Breakdown of energy equipartion $\Leftrightarrow$ different species have different granular temperatures

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- Both effects will be investigated here for a dilute mixture in the tracer limit by means of kinetic theory. It will be shown that there is a close relationship between them

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- The system exhibits an inhomogeneous steady state with gradients only in the direction of the external field (Huntley's talk). The hydrodynamic profiles are not affected by the presence of the impurity

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- $\zeta^{(0)}$ and $\zeta_0^{(0)}$ are functional of the distribution functions. A good approximation for them is obtained by using Maxwellians*

- In this way, it is obtained

$$(1+\phi)^{1/2}\left(1-h\frac{1+\phi}{\phi}\right)=\frac{\beta}{h}$$

$$\phi=\frac{mT_0(z)}{m_0T(z)} \qquad h\equiv\frac{m(1+\alpha_0)}{2(m+m_0)} \qquad \beta\equiv\frac{1-\alpha^2}{4\sqrt{2}}\left(\frac{\sigma}{\overline{\sigma}}\right)^{d-1}$$

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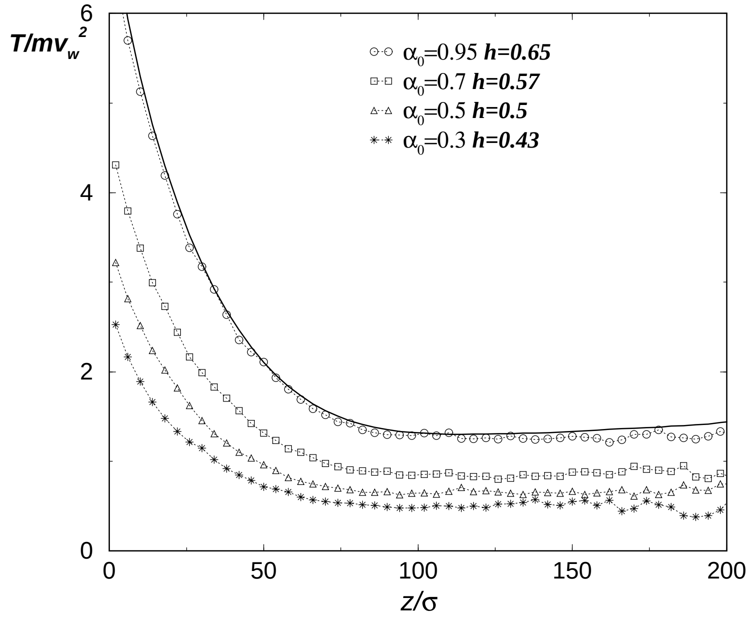
- Cubic equation for ϕ, with a unique real solution for all the physical values of h and ϕ

- $\Rightarrow$ ϕ does not depend on z (Feitosa and Menon, 2002), but it strongly depends on the inelasticity. Outside the tracer limit, also densities dependence

MD simulations I. Temperature profiles

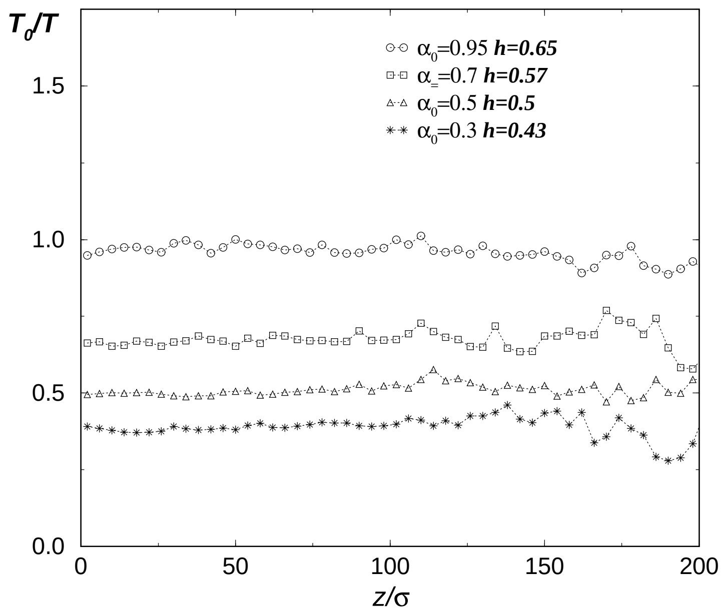
- $d = 2$, $N = 359$, $\alpha = 0.95$, $\sigma_0 = \sigma$, S (width) $= 50\sigma$, wall moves in a sawtooth way with $v_w = 5(\sigma g)^{1/2}$

- $\frac{m_0}{m} = \frac{1}{2}$



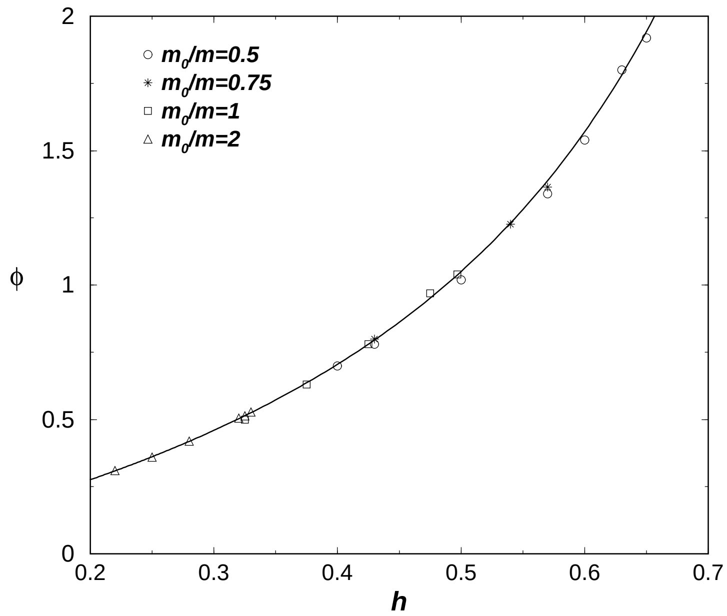
MD simulations II. Profiles of the temperature ratio

- Same values of the parameters as in the previous figure



Similar behavior for $m_0/m = 0.75, 1, 2$

MD simulations III. Comparison with the theoretical predictions



- ϕ can be larger than unity even if $m/m_0 < 1$, and viceversa (more)

Impurity density $n_0(z)$

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- To first order in the gradients, Chapman-Enskog gives

$$j_z = -m_0 D \partial_z x_0 - \frac{mn}{T(z)} D' \partial_z T - \frac{m}{T(z)} D_p \partial_z p(z)$$

with $x_0 \equiv n_0/n$, $p = nT$, D = diffusion coefficient, D' = thermal diffusion coefficient, D_p = pressure diffusion coefficient

$$D_p = \frac{n_0 T_0}{mn} \frac{\phi - 1}{\phi} \left(\nu - \frac{3\zeta^{(0)}}{2} + \frac{\zeta^{(0)2}}{2\nu} \right)^{-1} \qquad \phi = \frac{m T_0(z)}{m_0 T(z)}$$

ν= some collision frequency (*)

- The position (maximum density or center of mass) of the intruder relative to the fluid is determined by the sign of D_p. Similar to the elastic case (barometric formule), where $\phi = \frac{m}{m_0}$

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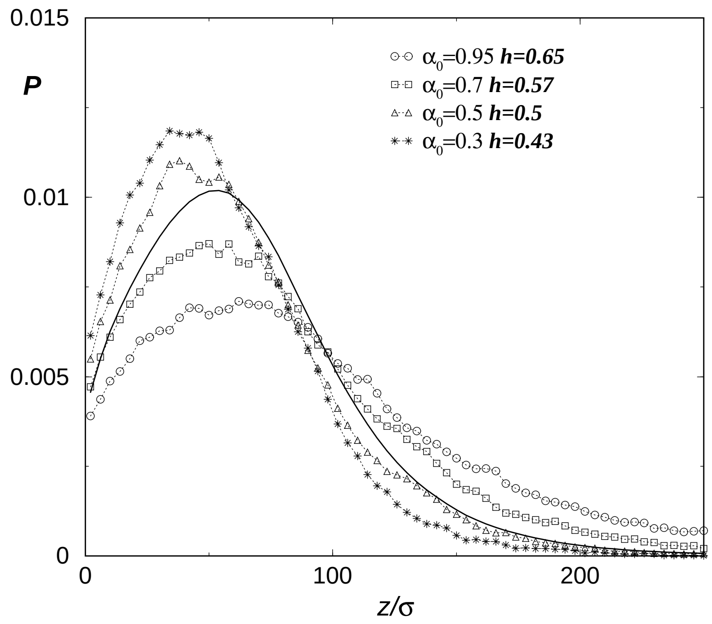
- -For $\phi > 1$, the impurity position is 'higher' than that of the gas

  -For $\phi < 1$, the impurity position is 'lower' than that of the gas

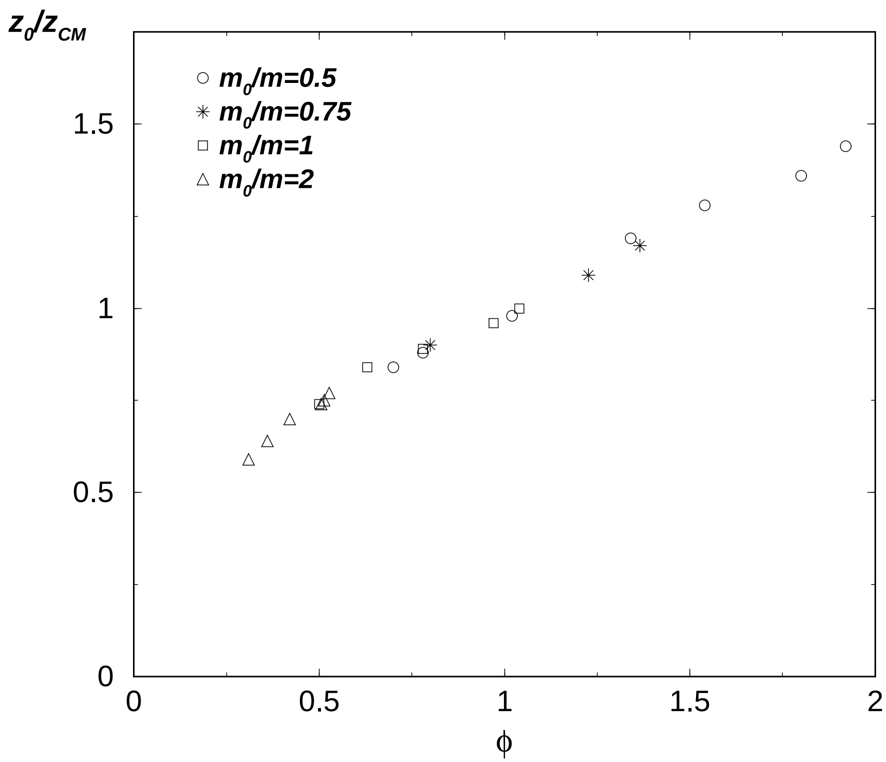
MD simulations IV. Normalized density profiles

$m_0/m = 1/2$, $\alpha = 0.95$



Qualitative agreement with theory (ϕ increases as α_0 increases)

MD simulations V. Ratio of the center of mass positions



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Some additional comments

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- Moreover, at high density, other effects also leading to segregation may become relevant, as for instance those discussed in: D.C. Hong, P.V. Quinn, and S, Luding, Phys. Rev. Lett. **86**, 3423 (2001); J.T. Jenkins and D.K. Yoon, Phys. Rev. Lett. **88**, 194301 (2002)

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- On the other hand, there is no reason to expect that the mechanism discussed here be no important at high densities

The local HCS cooling rates in the Maxwellian approximation

$$\zeta^{(0)*} \equiv \frac{\zeta^{(0)}(z)}{n(z)\sigma^{d-1}v_g(z)} = \frac{\sqrt{2}\pi^{(d-1)/2}}{\Gamma(d/2)d}(1-\alpha^2)$$

$$\zeta_0^{(0)*} \equiv \frac{\zeta_0^{(0)}(z)}{n(z)\sigma^{d-1}v_g(z)} = \nu_0^*(1+\phi)^{1/2}\left(1-h\frac{1+\phi}{\phi}\right)$$

where $n(z)$ is the number density of the gas,

$$v_g(z) = \left[\frac{2T(z)}{m}\right]^{1/2} \qquad h = \frac{m(1+\alpha_0)}{2(m+m_0)} \qquad \nu_0^* = \frac{8h\pi^{(d-1)/2}}{\Gamma(d/2)d}\left(\frac{\overline{\sigma}}{\sigma}\right)^{d-1}$$

$$\overline{\sigma} = \frac{\sigma+\sigma_0}{2} \qquad \phi = \frac{mT_0(z)}{m_0T(z)}.$$

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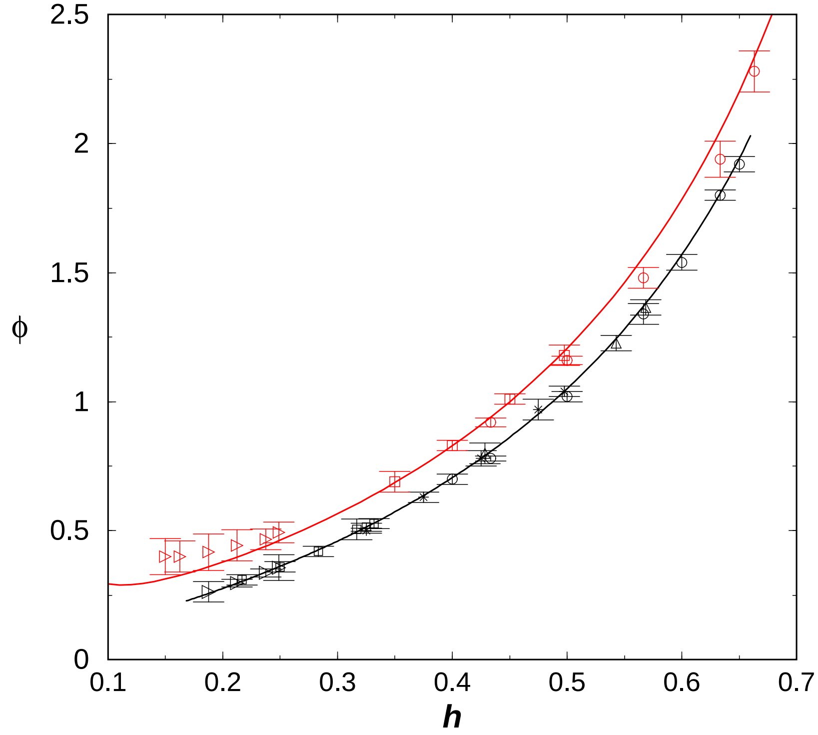
MD simulations IIIa. Comparison with theory



- black symbols $\alpha = 0.95$

- red symbols $\alpha = 0.8$

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Diffusion coefficients

$$D = \frac{nT_0}{m_0}\left(\nu - \frac{\zeta^{(0)}}{2}\right)^{-1} \qquad D' = -\frac{\zeta^{(0)}}{2\nu}D_p$$

$$D_p = \frac{n_0 T_0}{mn}\frac{\phi - 1}{\phi}\left(\nu - \frac{3\zeta^{(0)}}{2} + \frac{\zeta^{(0)2}}{2\nu}\right)^{-1}$$

Here,

$$\nu = \nu_e \frac{1+\alpha}{2}\left(1-\Delta\right)^{1/2}\left(1+\phi\right)^{1/2}$$

$$\Delta = \frac{m}{m+m_0} \qquad \nu_e = \frac{4\sqrt{2}\pi^{(d-1)/2}}{\Gamma(d/2)d}\overline{\sigma}^{d-1}\Delta^{1/2}n\left(\frac{T}{m_0}\right)^{1/2}$$

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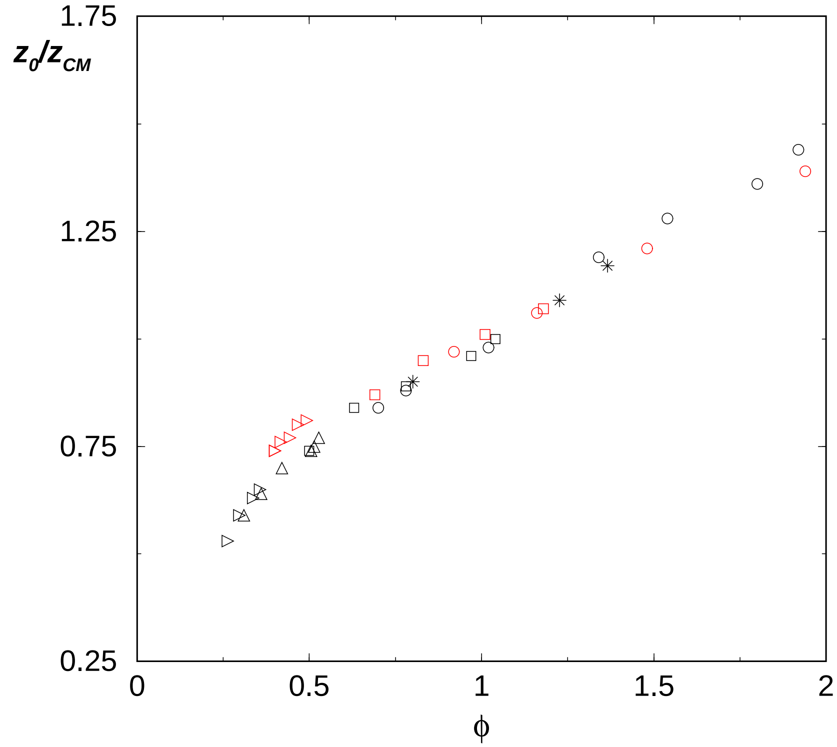
MD simulations Va. Ratio of the center of mass positions



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