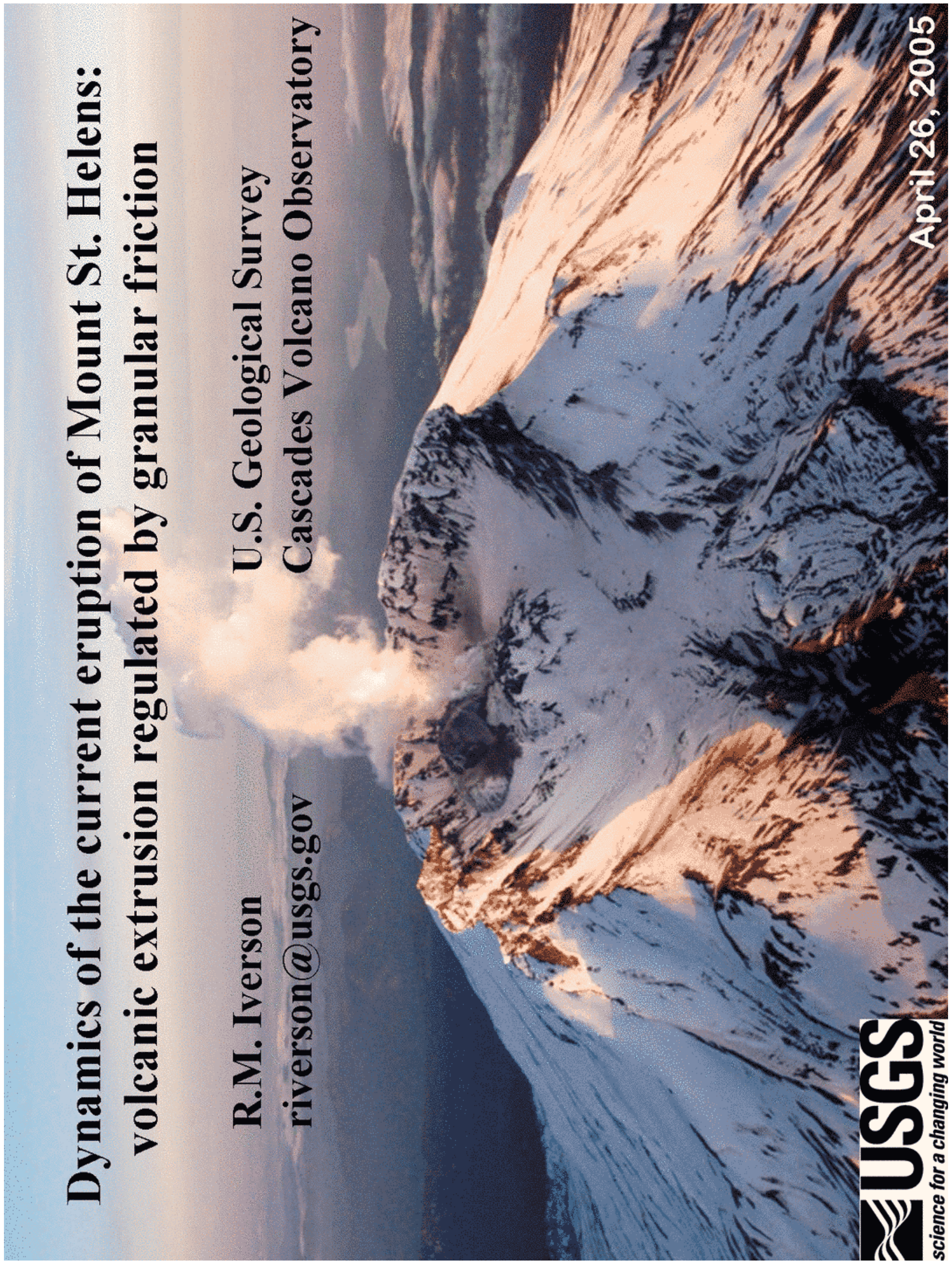
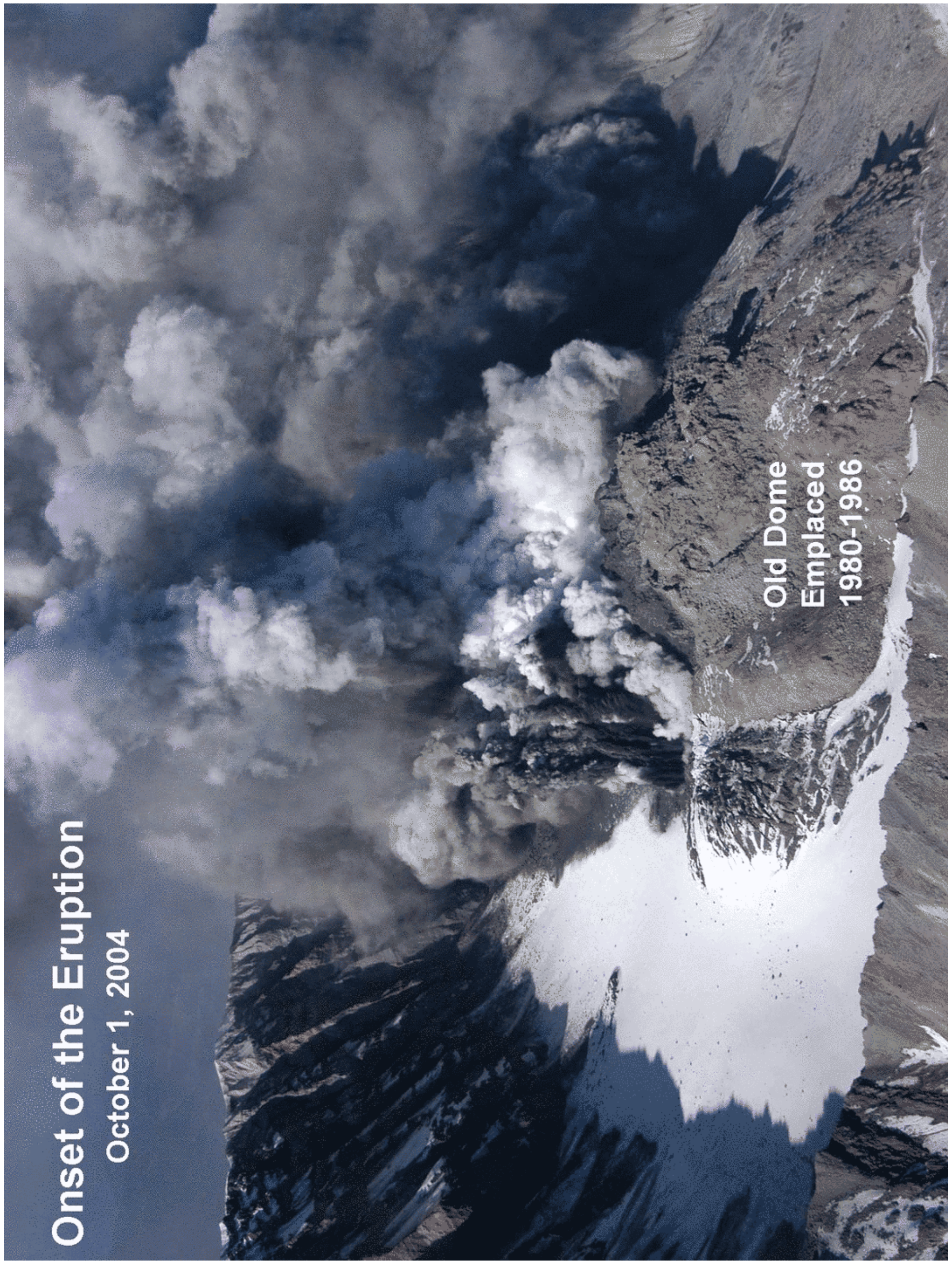
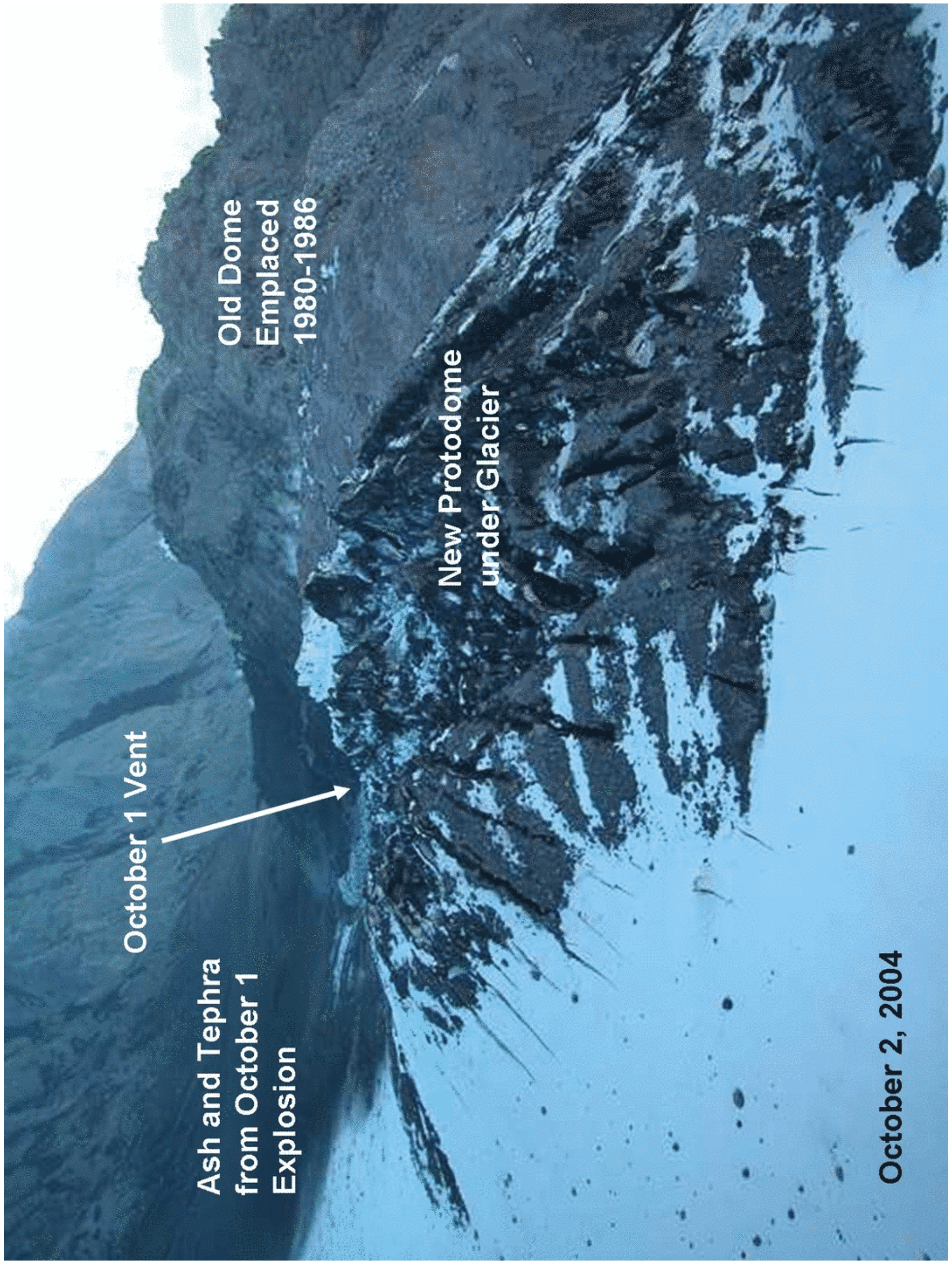
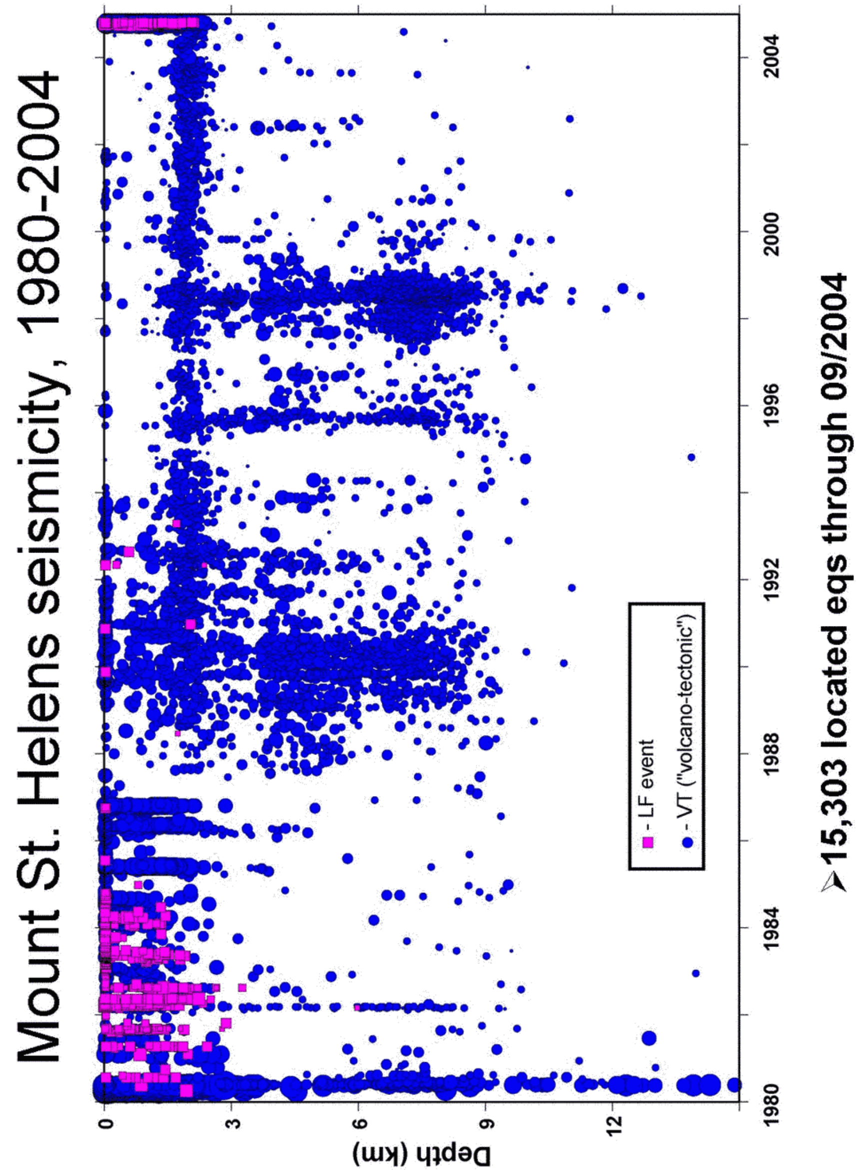
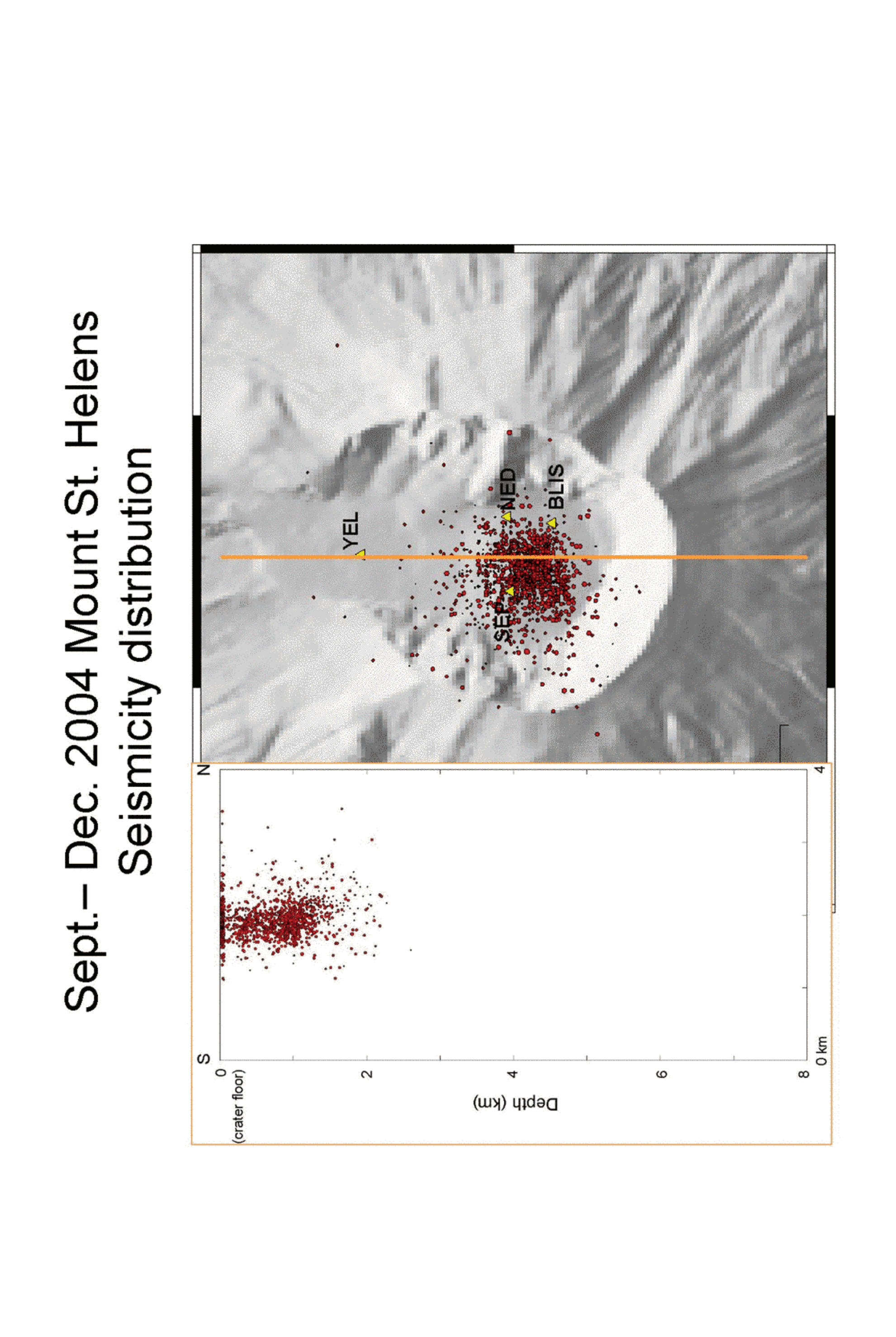
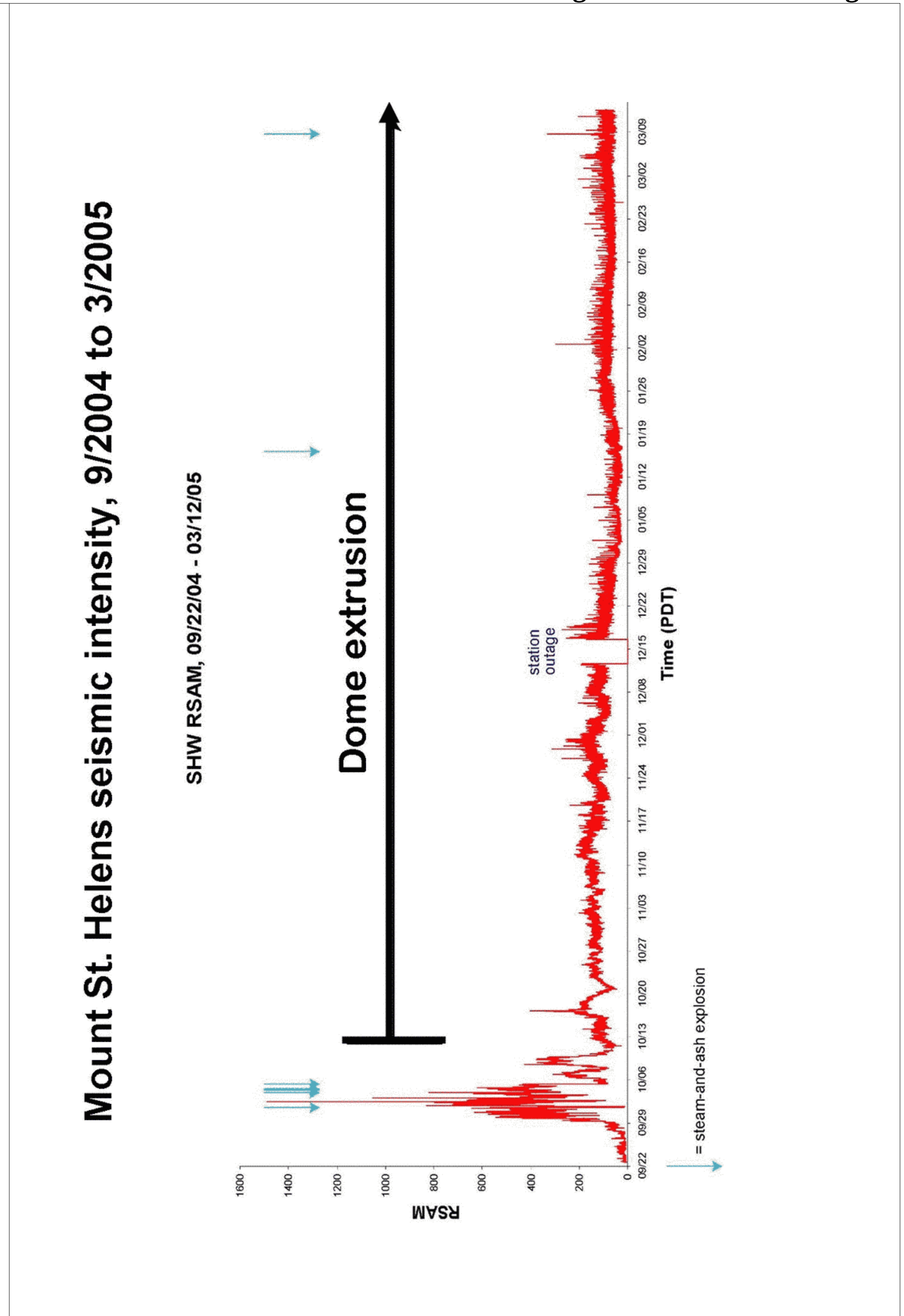

Dynamics of the current eruption of Mount St. Helens:
volcanic extrusion regulated by granular friction
R.M. Iverson
riverson@usgs.gov
U.S. Geological Survey
Cascades Volcano Observatory
April 26, 2005
USGS
science for a changing world

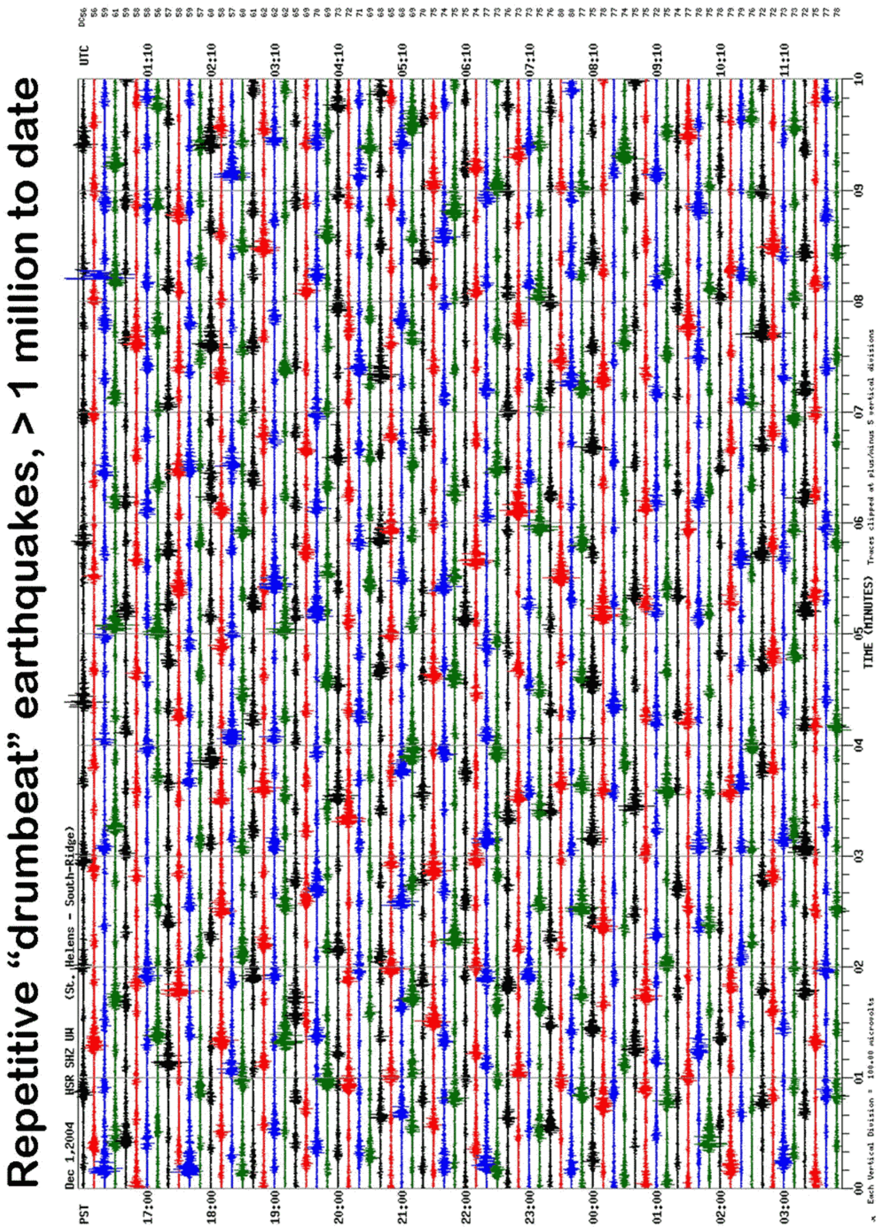

Onset of the Eruption
October 1, 2004
Old Dome
Emplaced
1980-1986

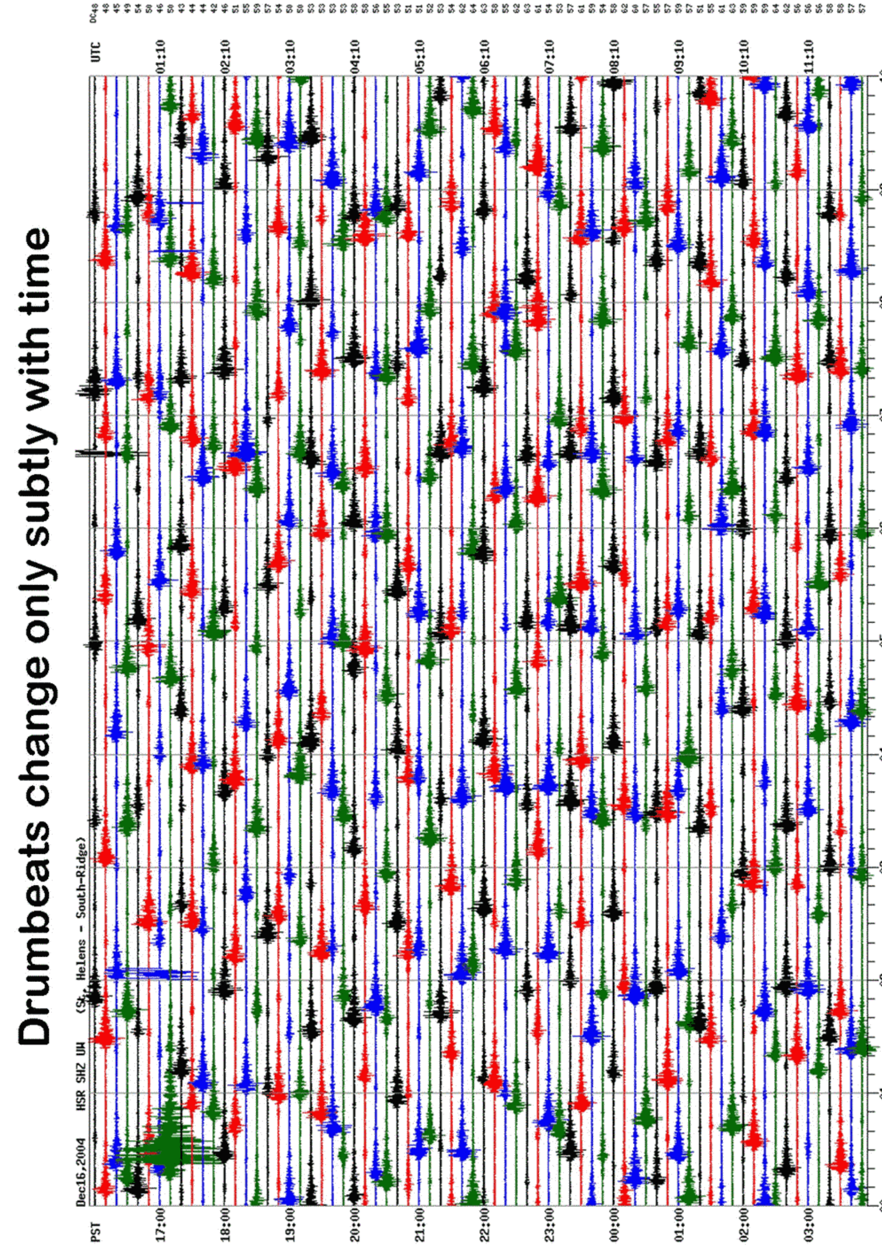

Old Dome
Emplaced
1980-1986
New Protodome
under Glacier
October 1 Vent
Ash and Tephra
from October 1
Explosion
October 2, 2004

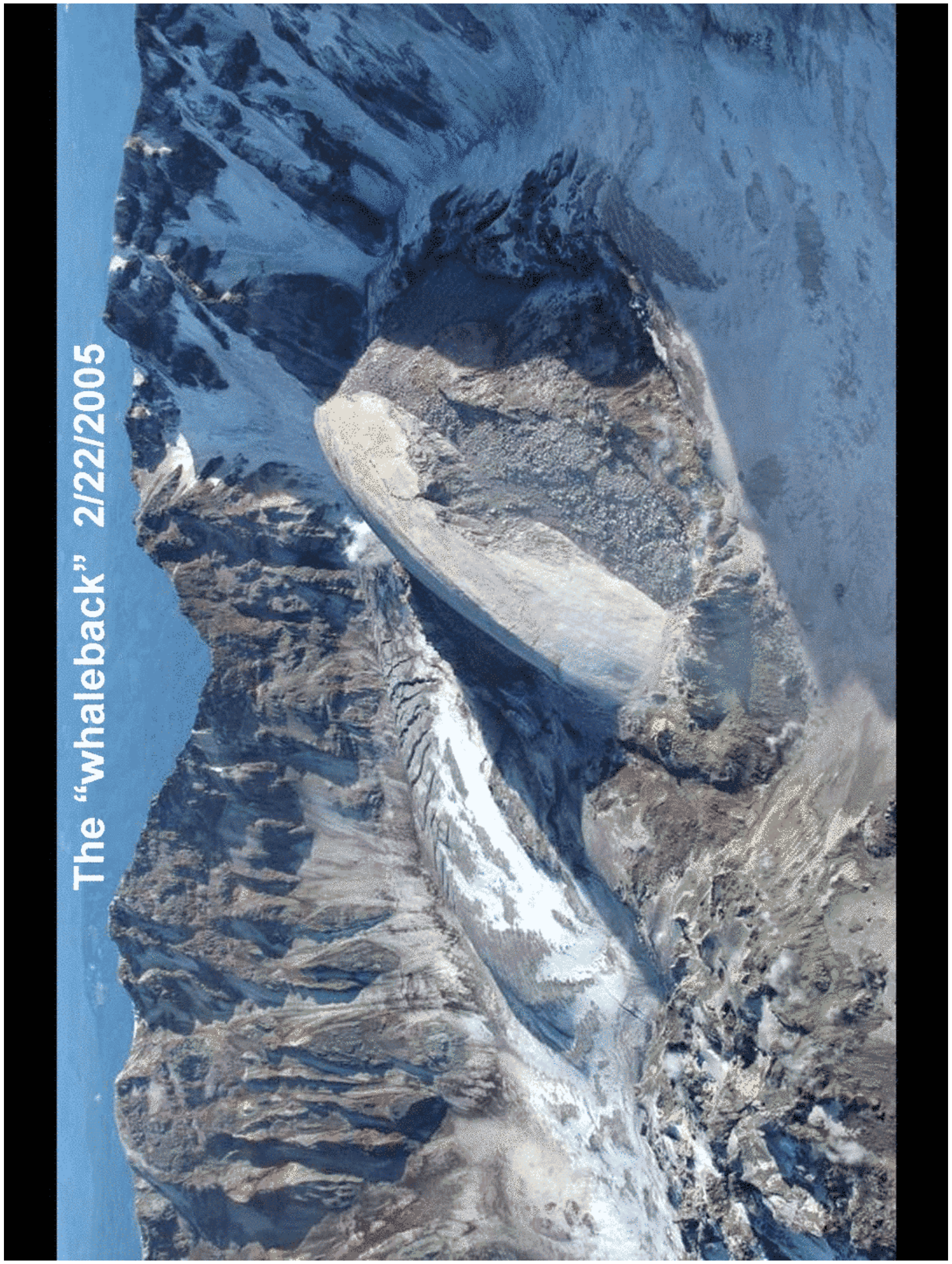

Mount St. Helens seismicity, 1980-2004
Depth (km)
0
3
6
9
12
1980
1984
1988
1992
1996
2000
2004
- LF event
- VT ("volcano-tectonic")
15,303 located eqs through 09/2004

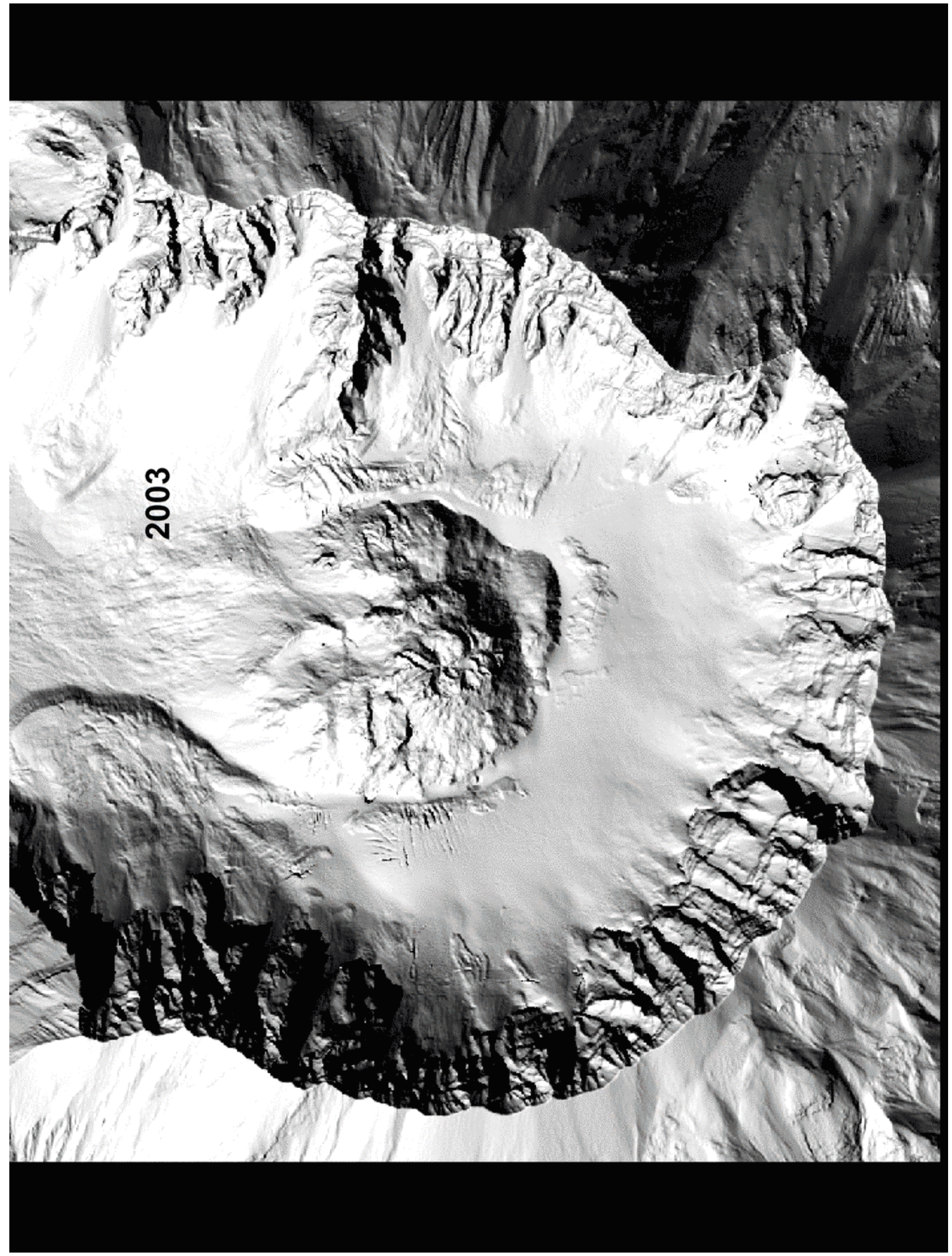

Sept.– Dec. 2004 Mount St. Helens
Seismicity distribution
YEL
NED
BLIS
SEP
N
S
Depth (km)
(crater floor)
0
2
4
6
8
0 km
4

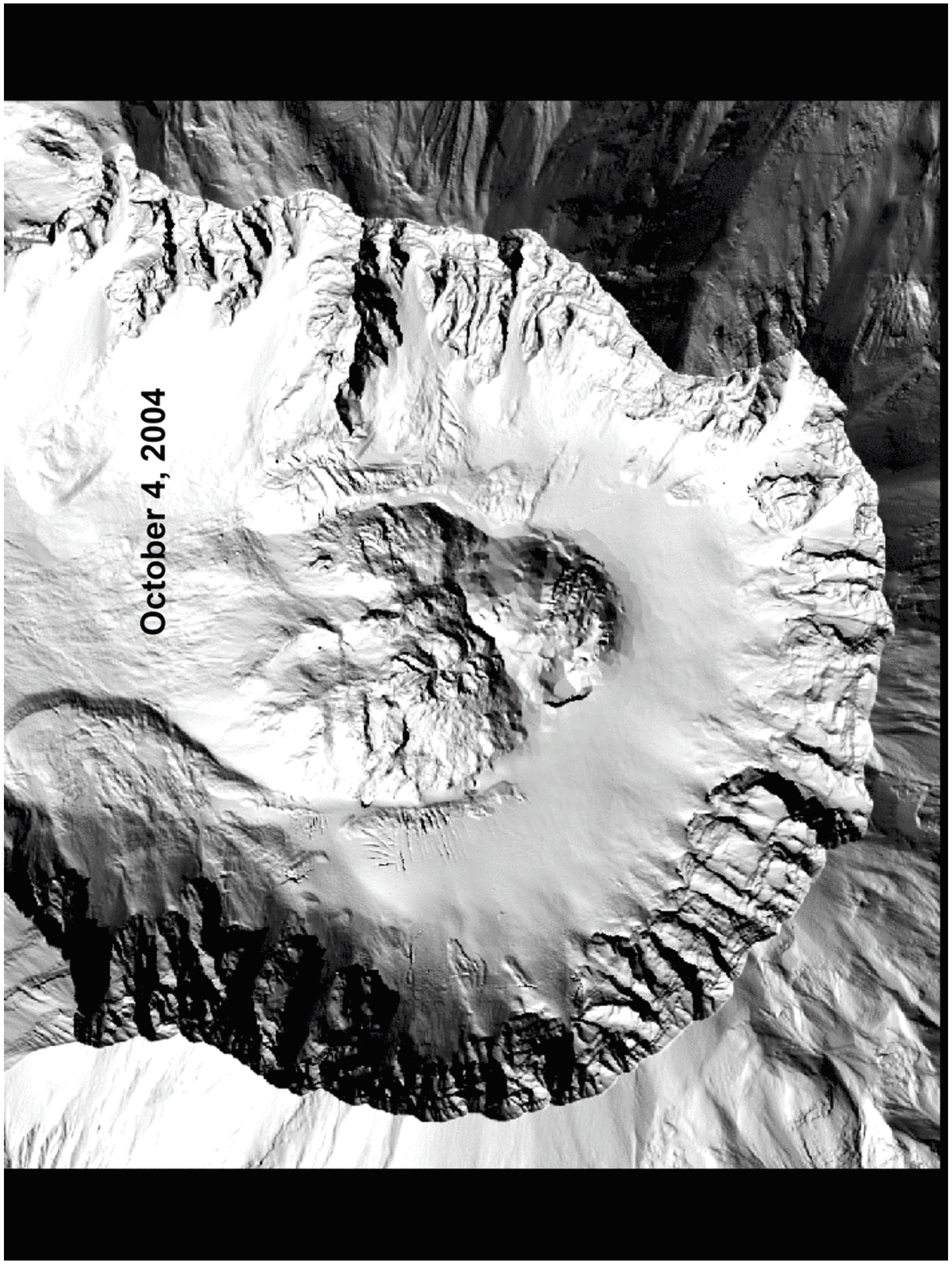

Mount St. Helens seismic intensity, 9/2004 to 3/2005
SHW RSAM, 09/22/04 - 03/12/05
Dome extrusion
station outage
RSAM
0
200
400
600
800
1000
1200
1400
1600
09/22
09/29
10/06
10/13
10/20
10/27
11/03
11/10
11/17
11/24
12/01
12/08
12/15
12/22
12/29
01/05
01/12
01/19
01/26
02/02
02/09
02/16
02/23
03/02
03/09
Time (PDT)
= steam-and-ash explosion

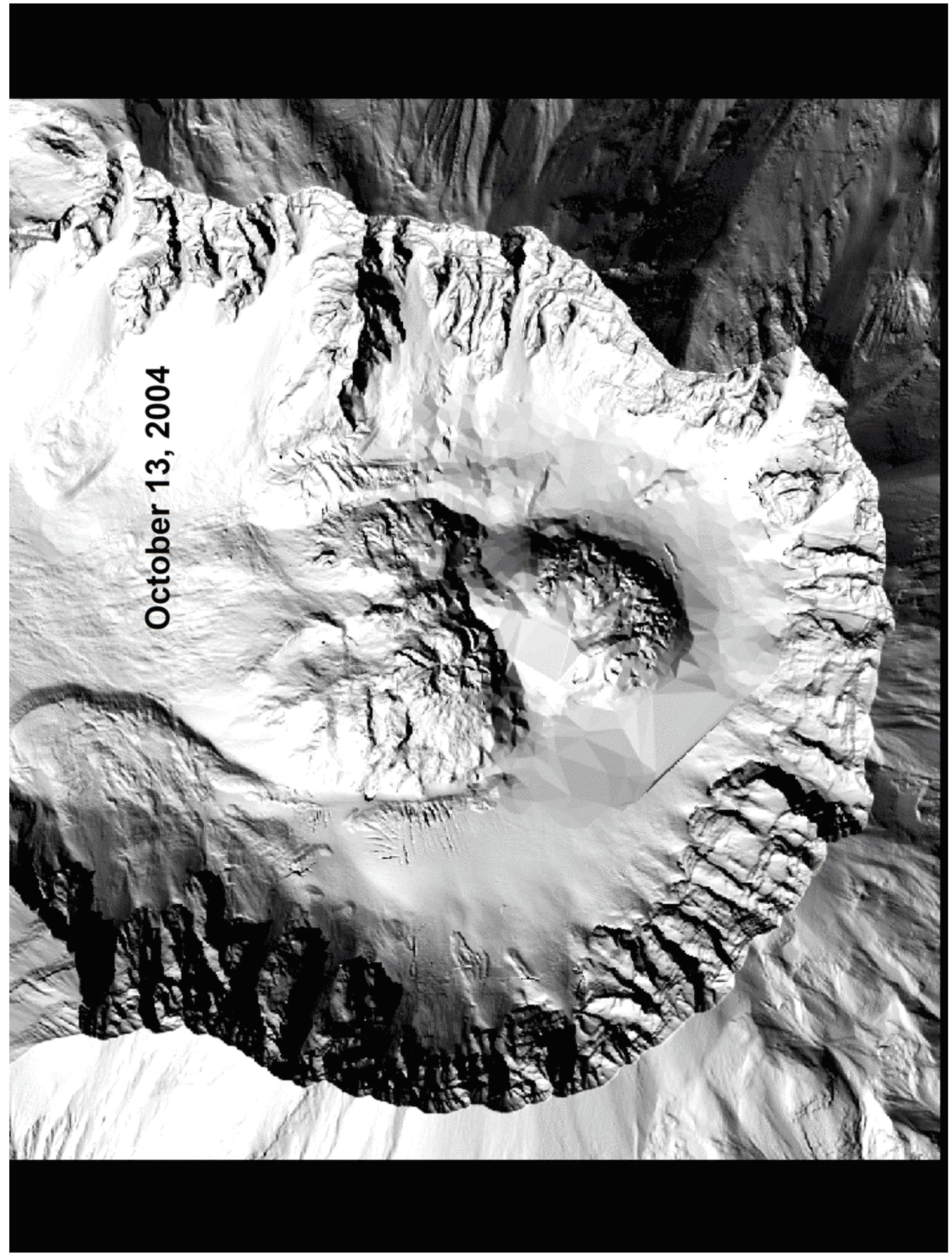

Repetitive “drumbeat” earthquakes, > 1 million to date
Dec 1,2004 HSR SHZ UW (St. Helens - South-Ridge)
TIME (MINUTES)

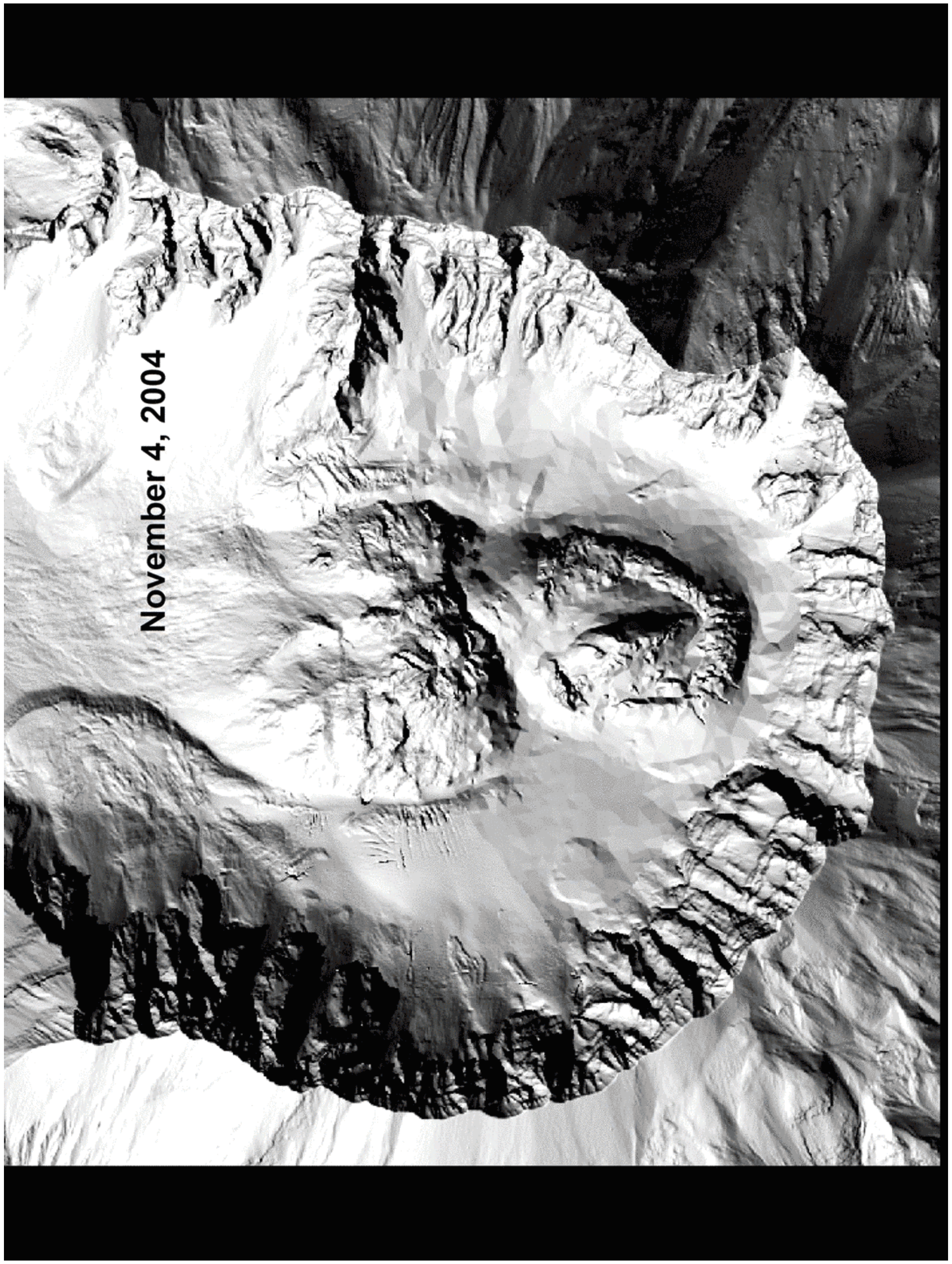

Drumbeats change only subtly with time
Dec16,2004 HSR SHZ UW (St. Helens - South-Ridge)
TIME (MINUTES)

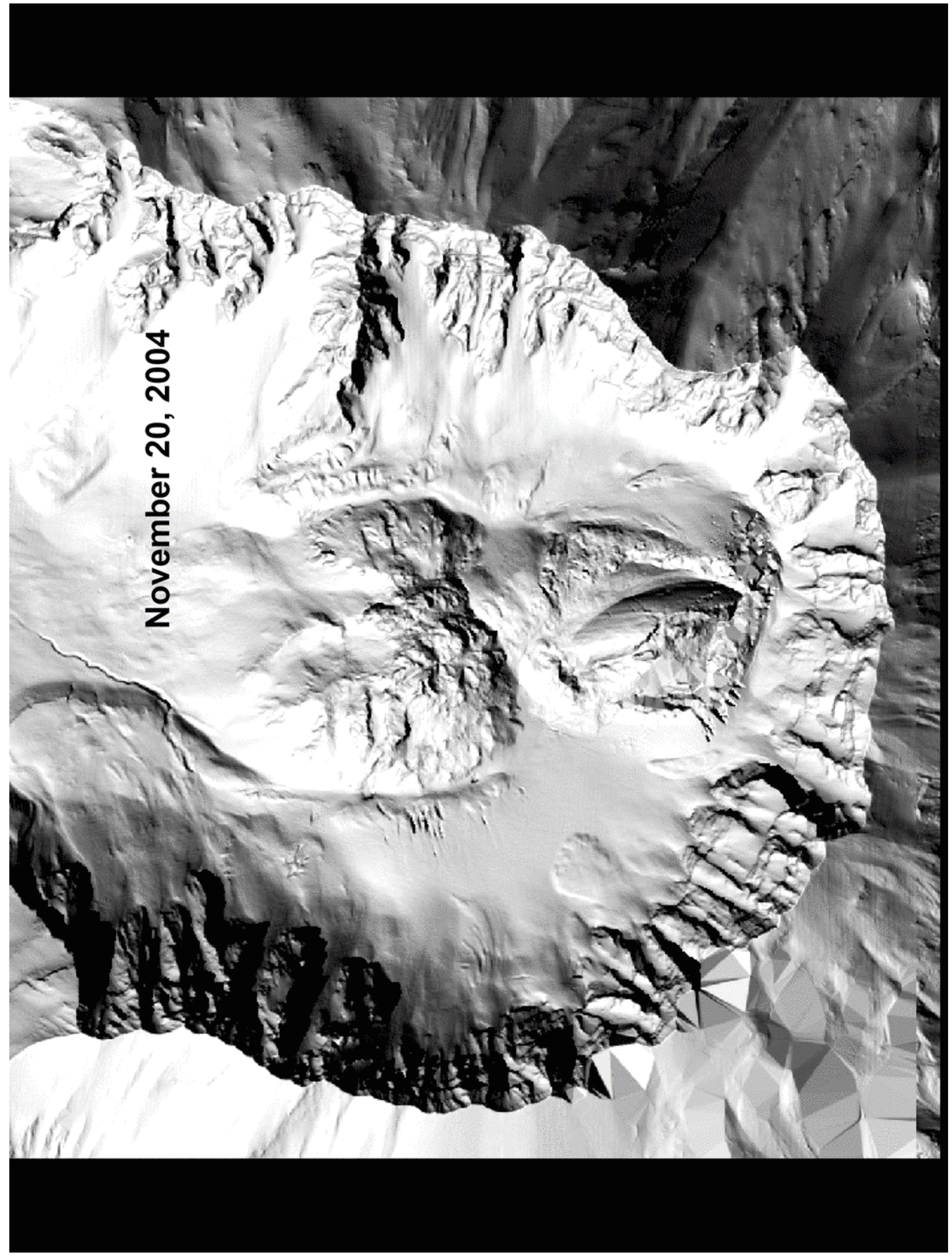

The “whaleback” 2/22/2005

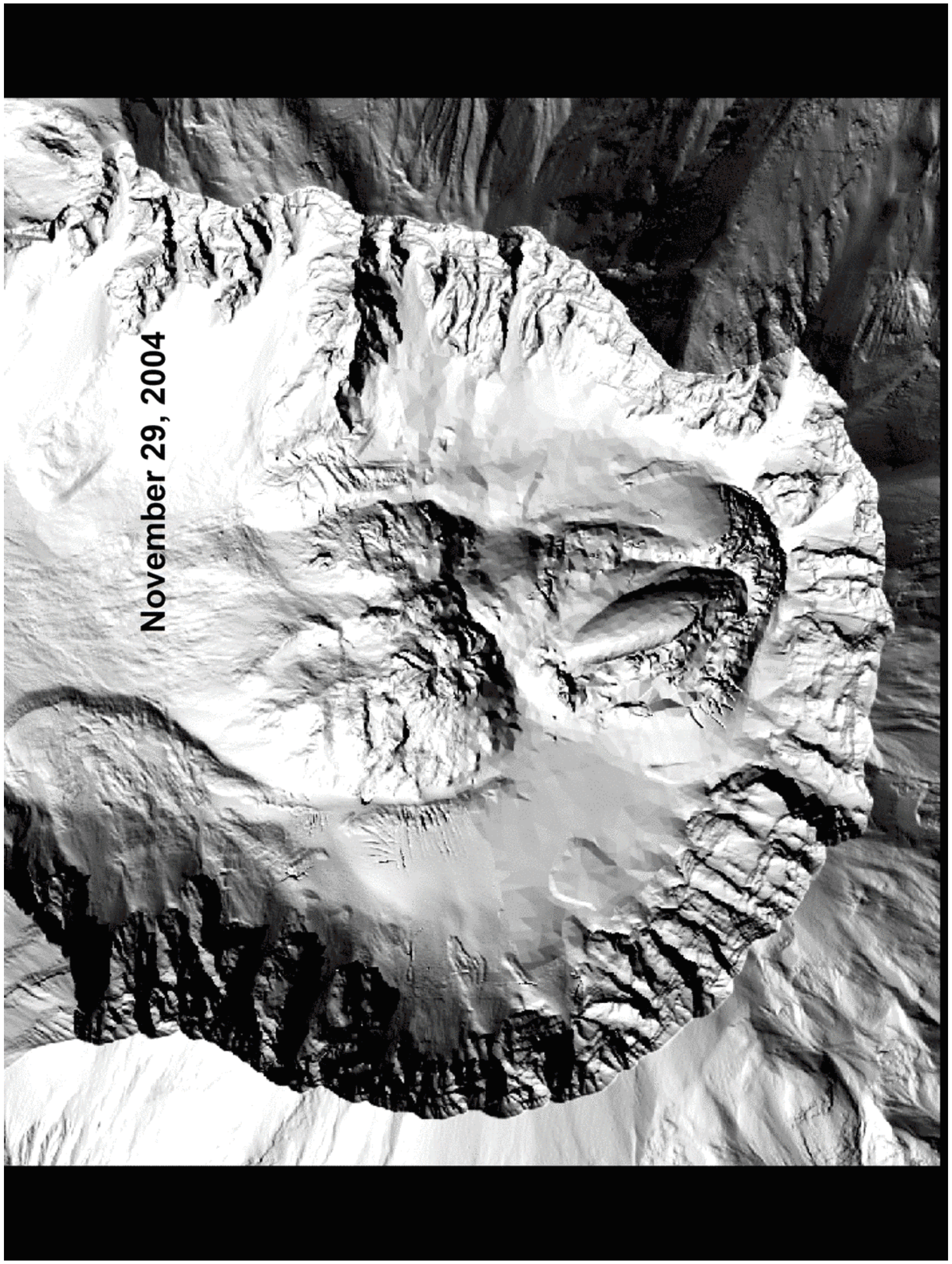

2003

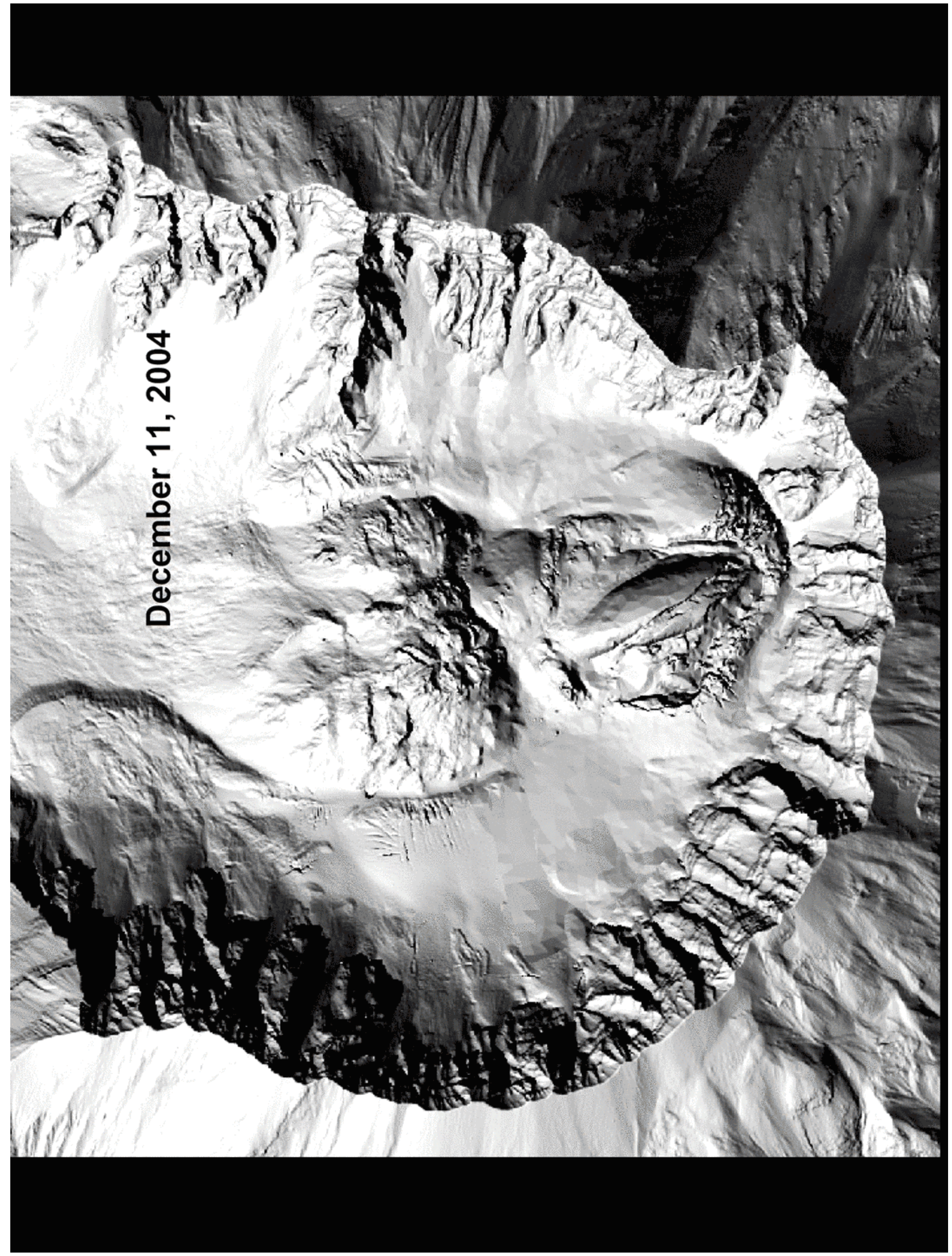

October 4, 2004


October 13, 2004

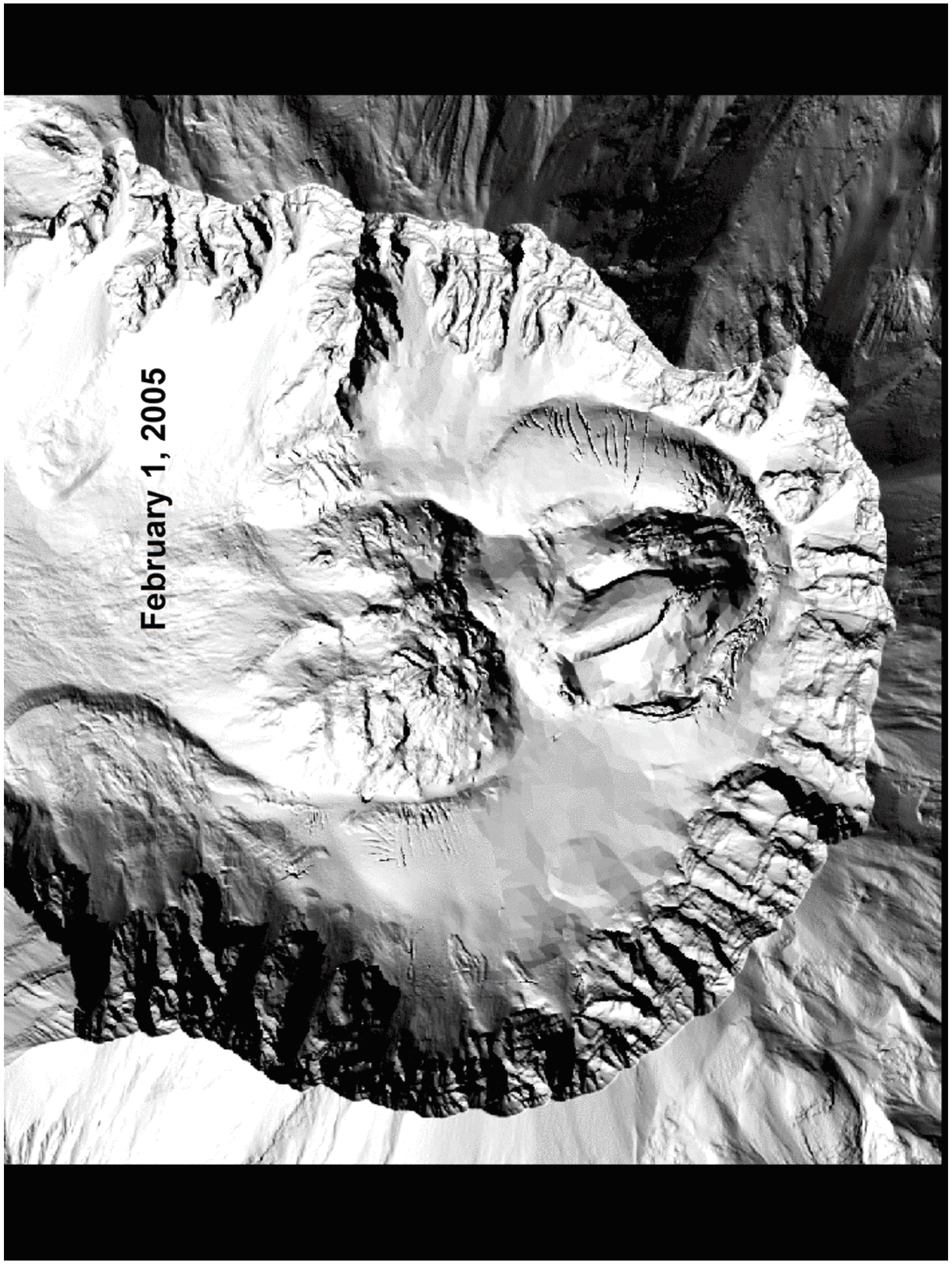

November 4, 2004

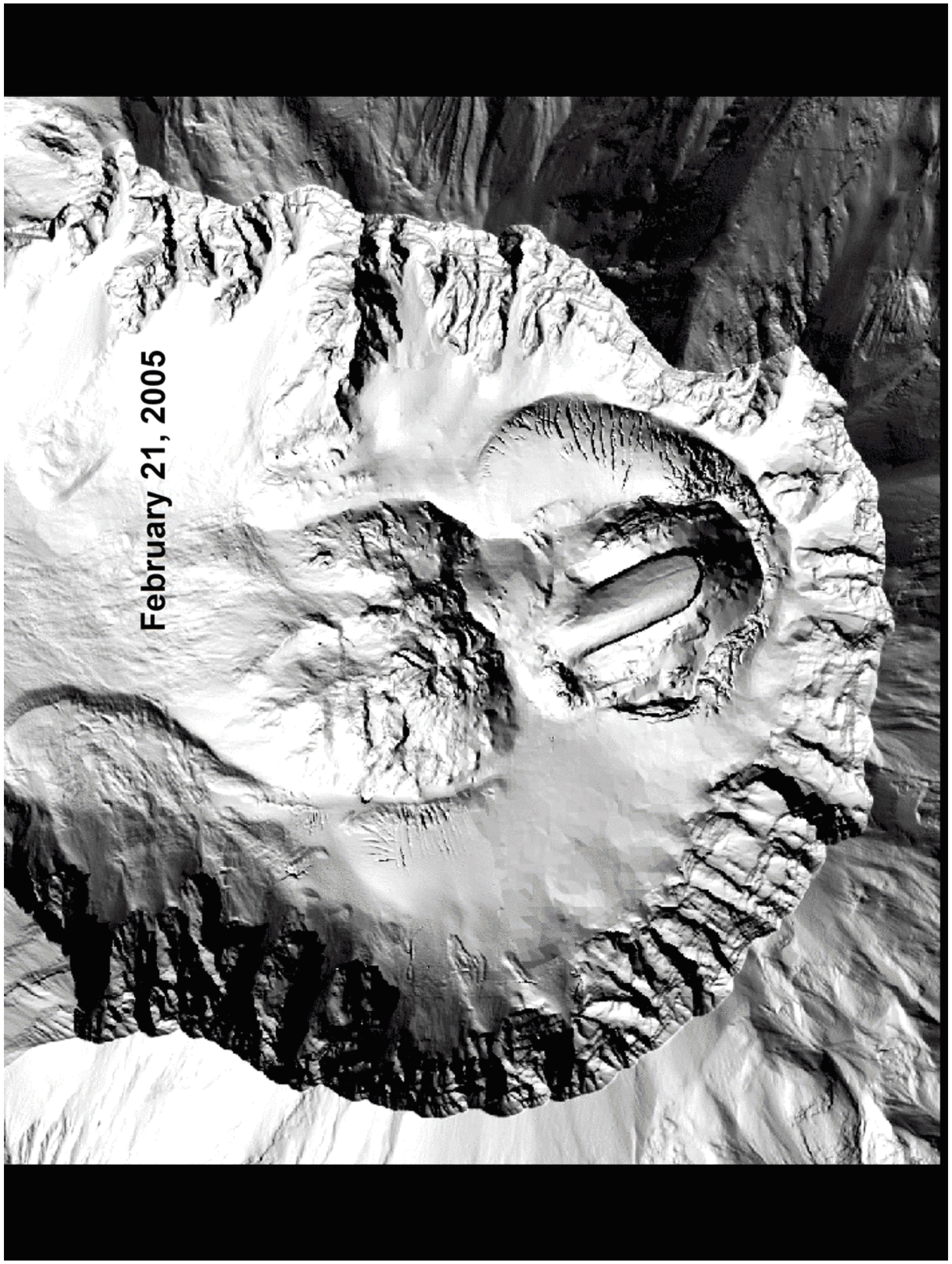

November 20, 2004

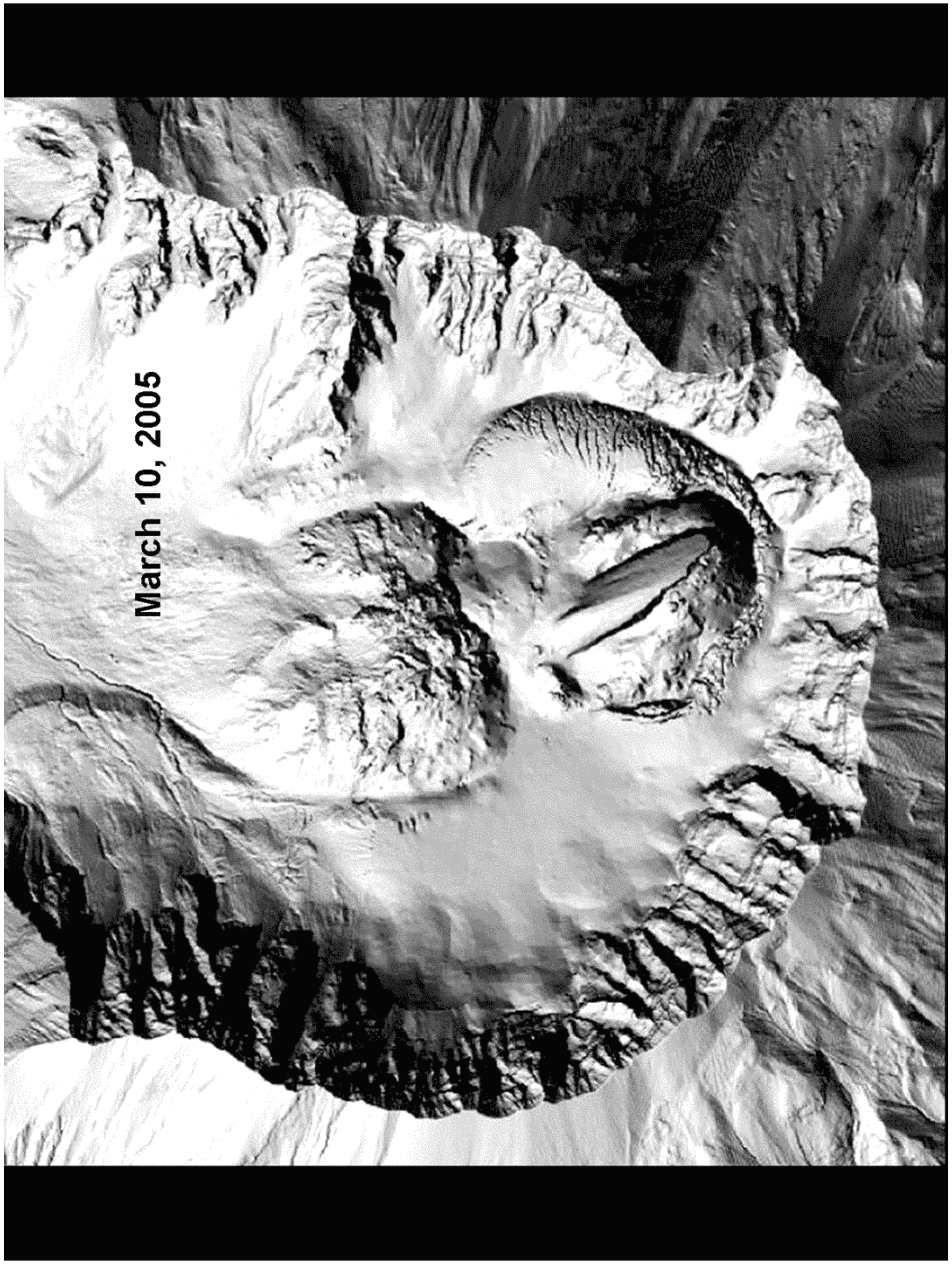

November 29, 2004

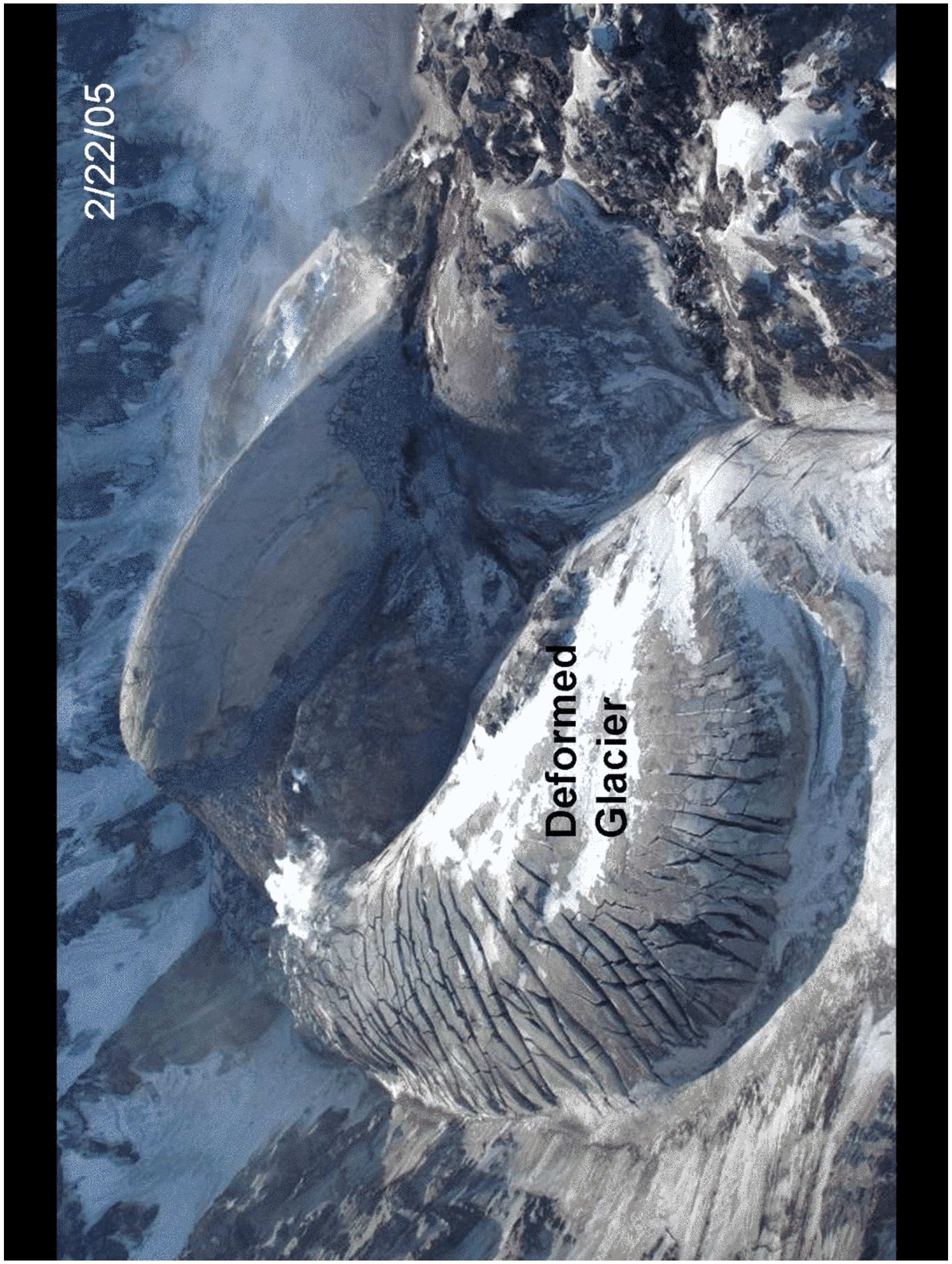

December 11, 2004

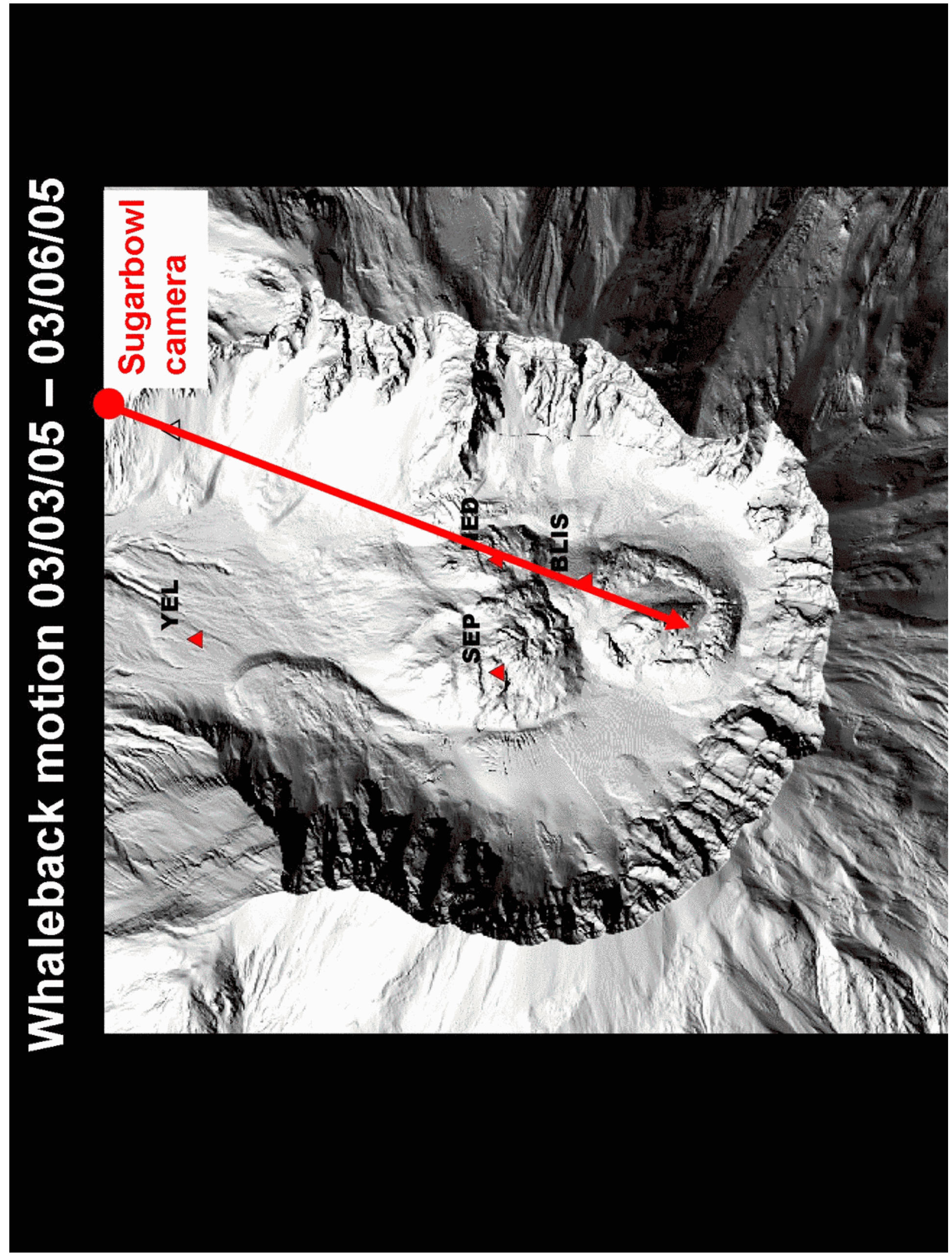

January 3, 2005

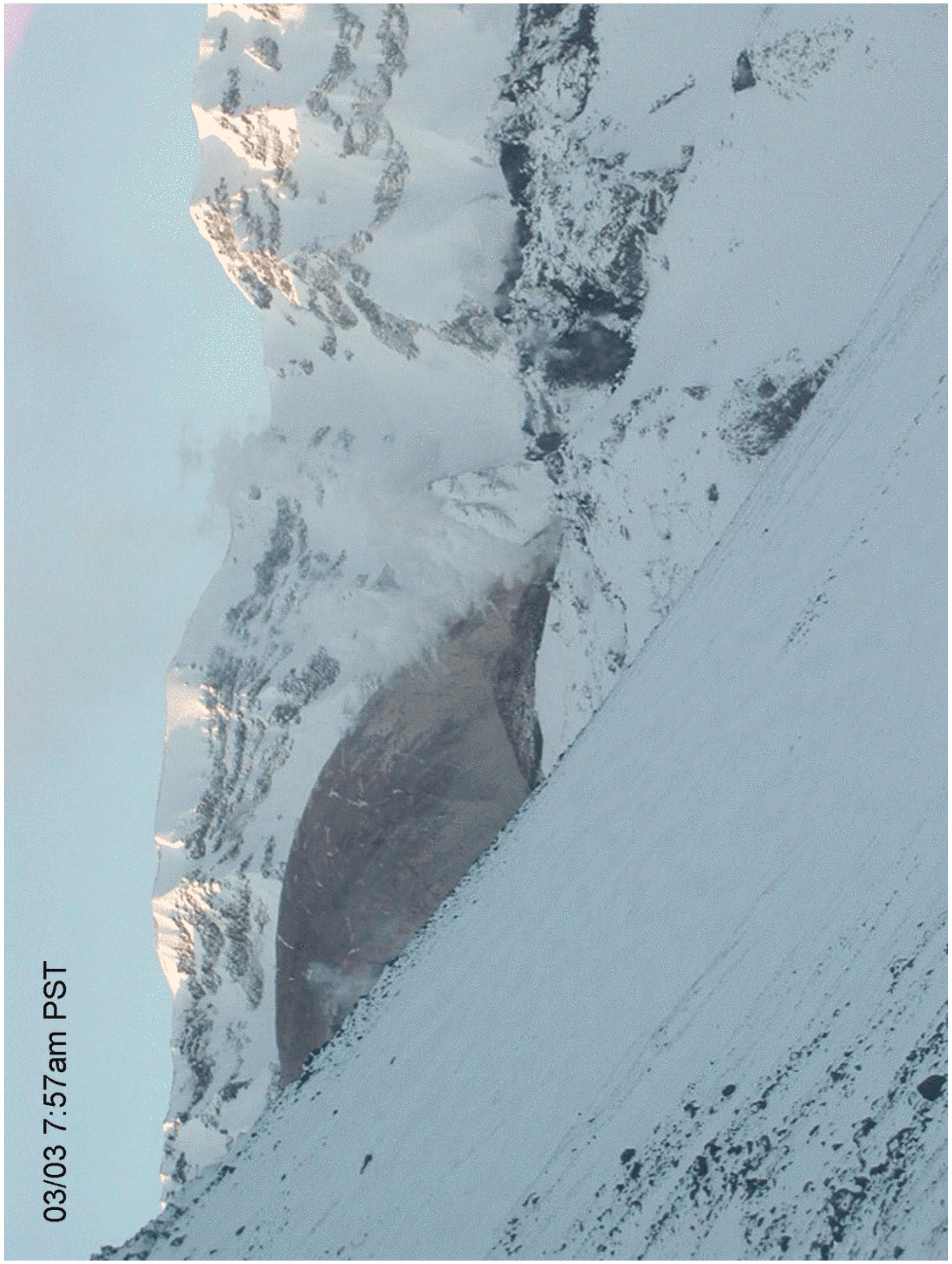

February 1, 2005


February 21, 2005


March 10, 2005

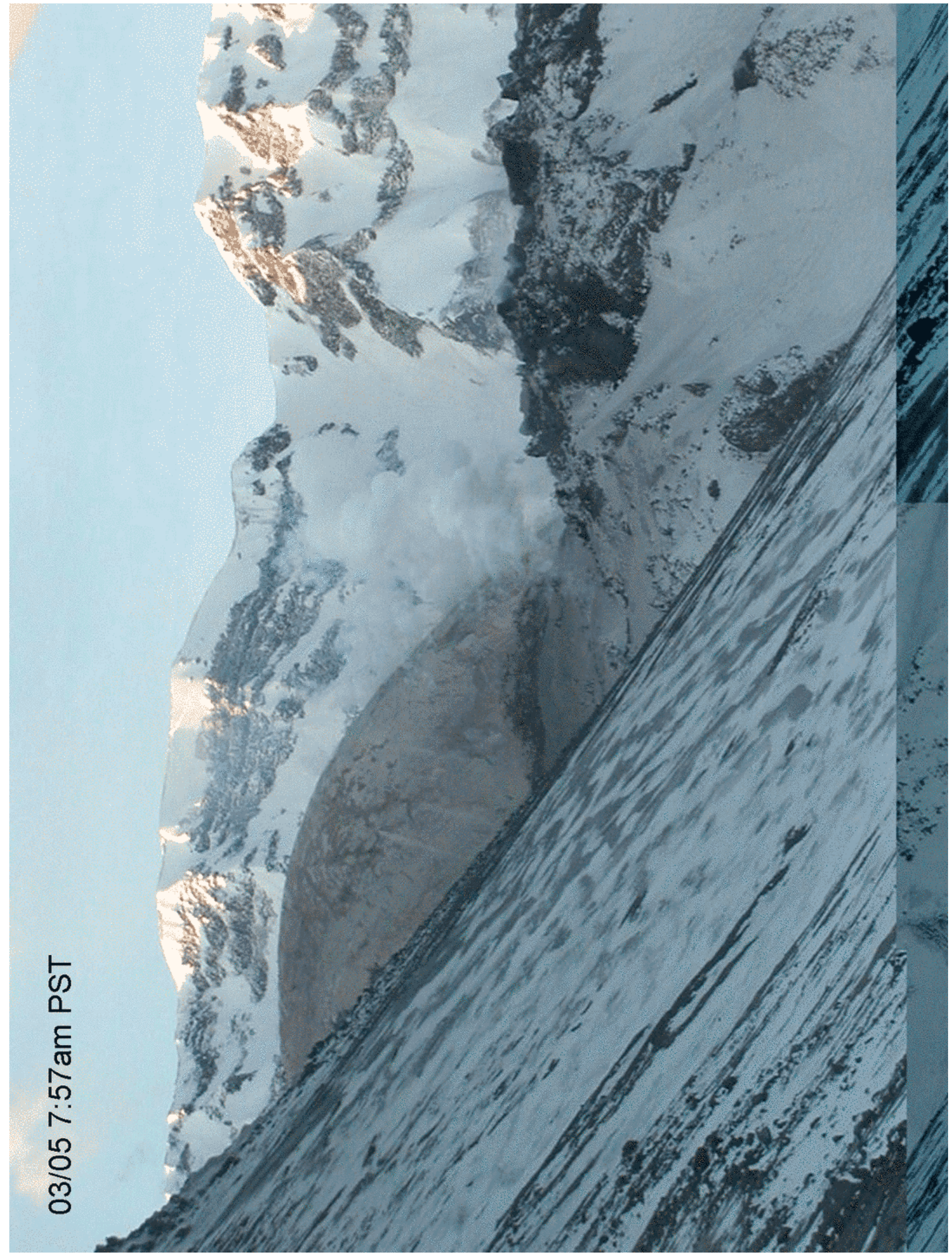

2/22/05
Deformed
Glacier

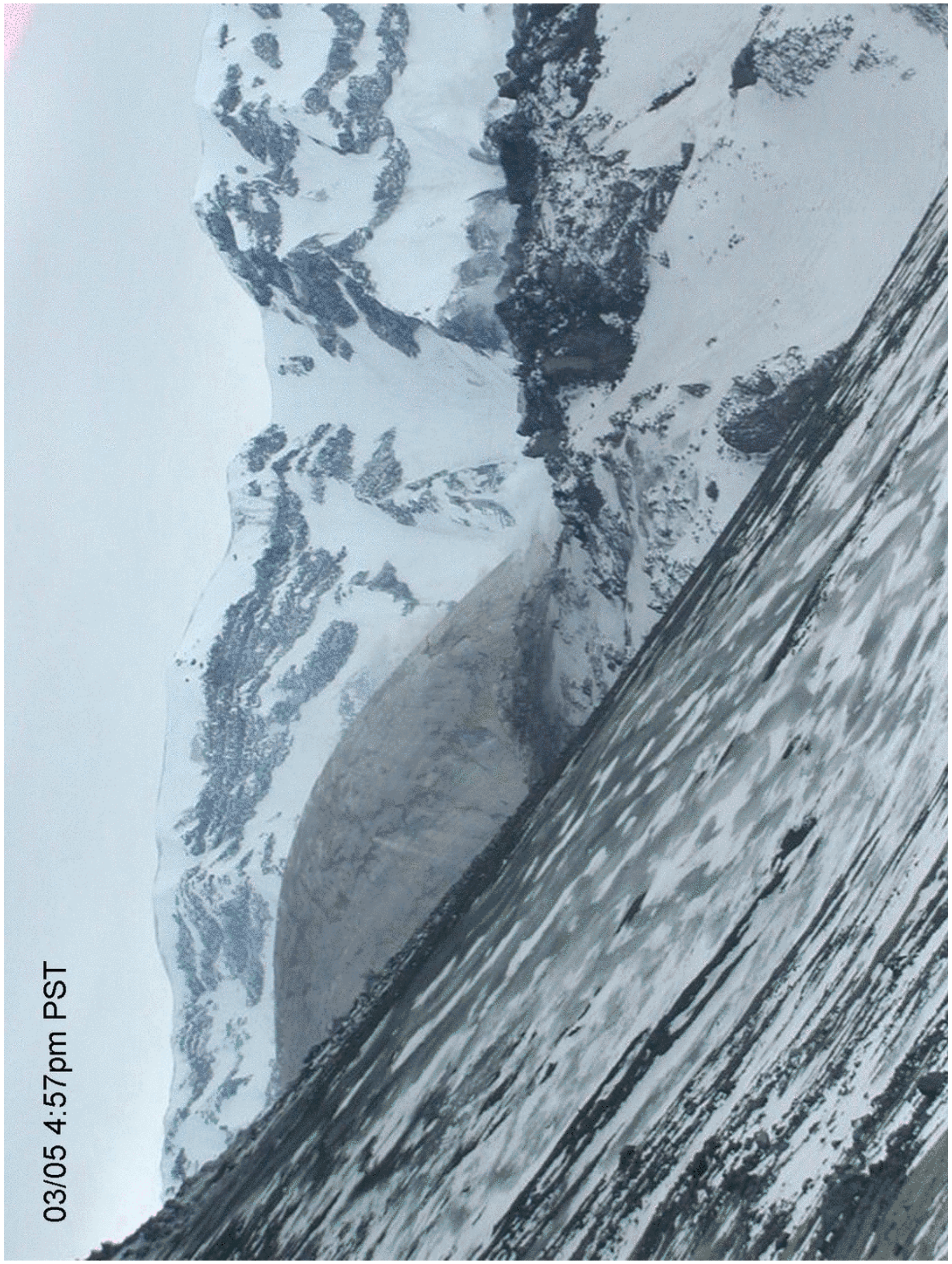

Whaleback motion 03/03/05 – 03/06/05
Sugarbowl
camera
YEL
SEP
BLIS

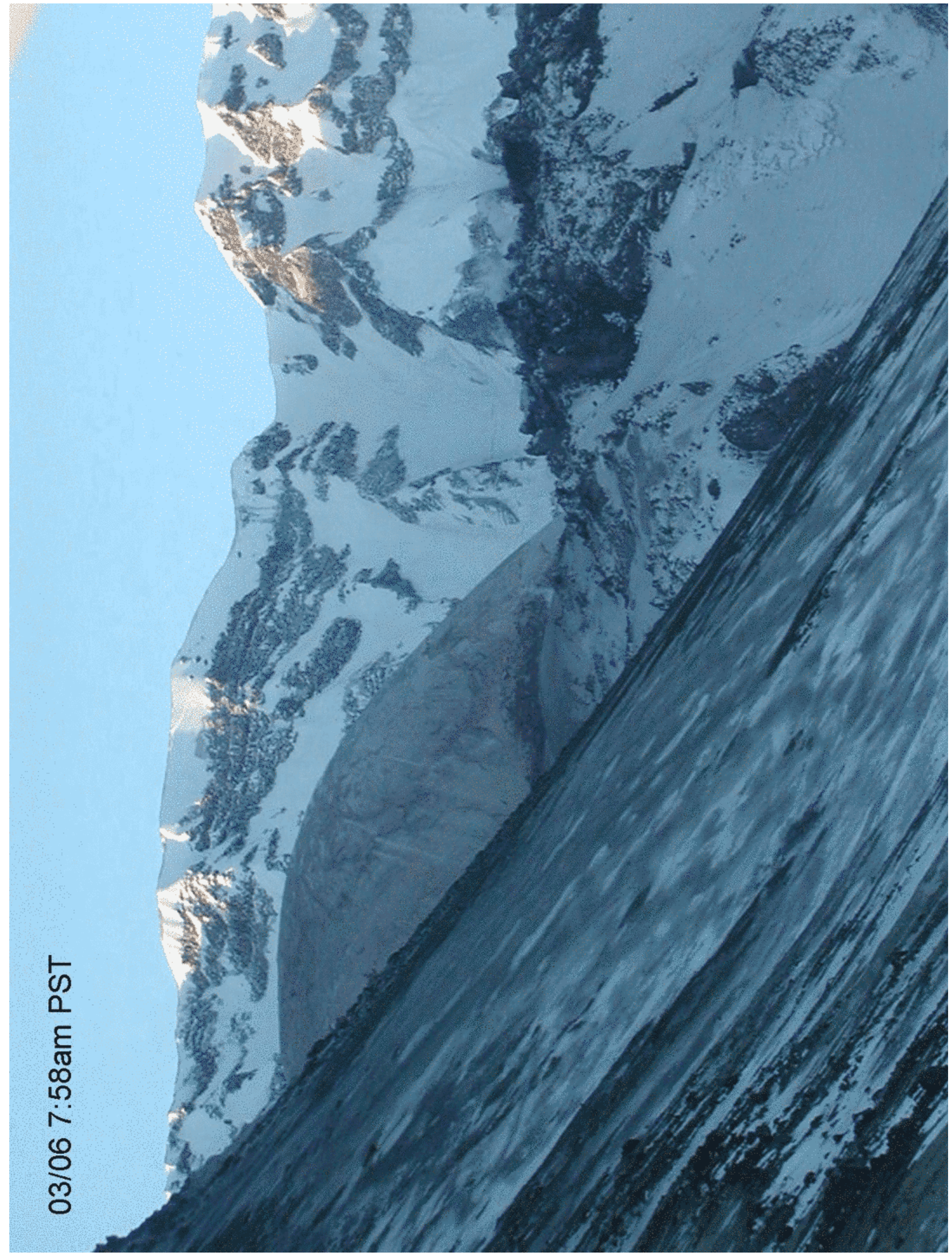

03/03 7:57am PST


03/03 4:58pm PST

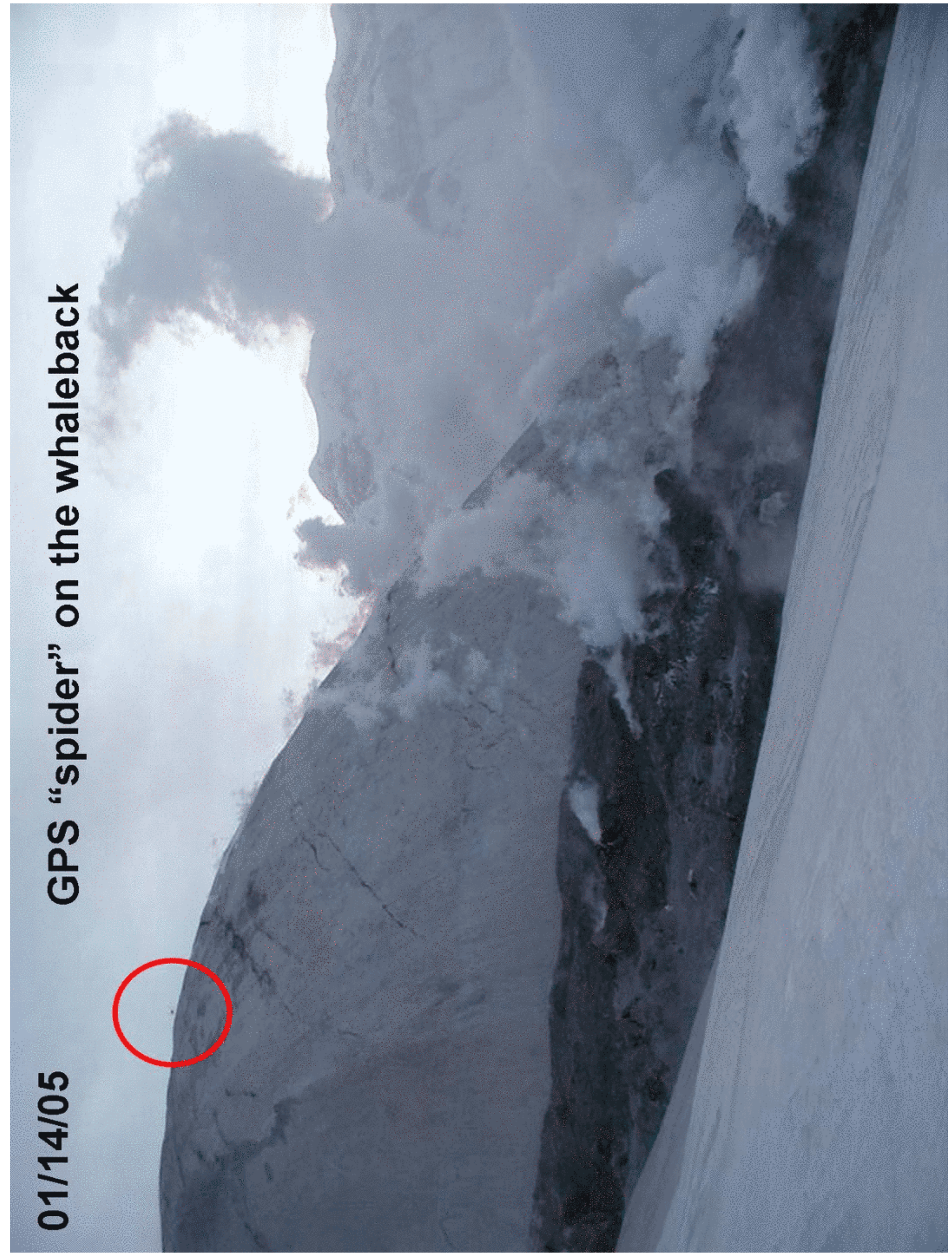

03/04 7:57am PST


03/05 7:57am PST

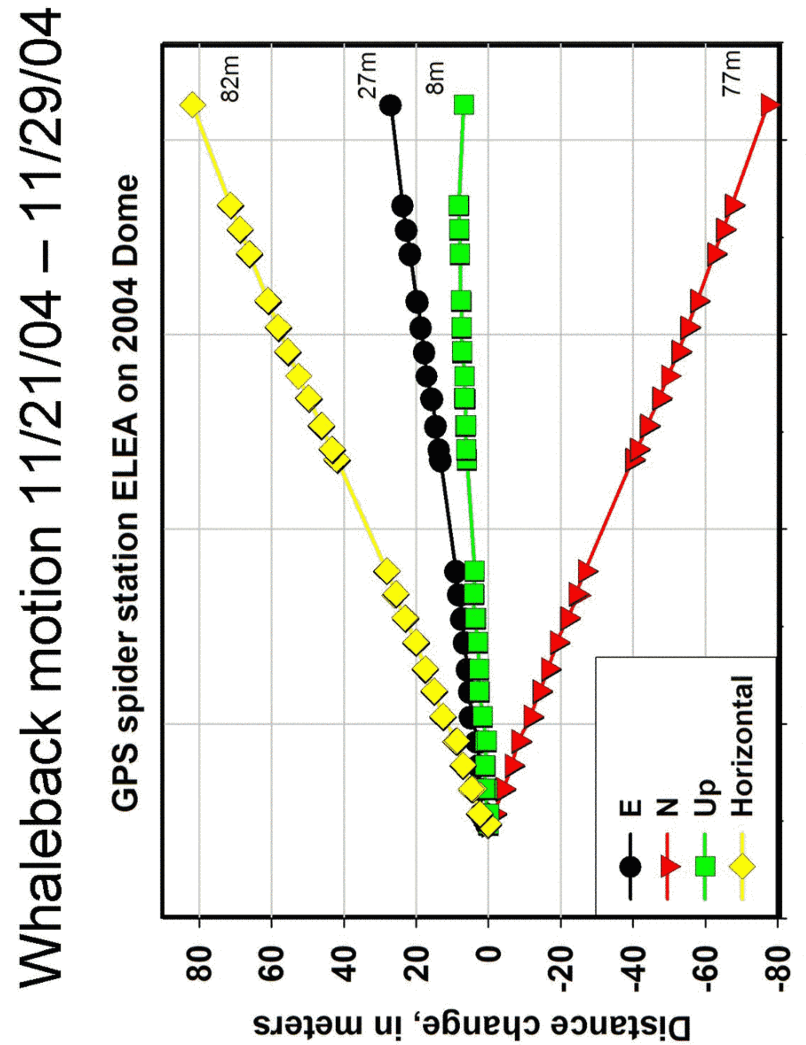

03/05 4:57pm PST


03/06 7:58am PST

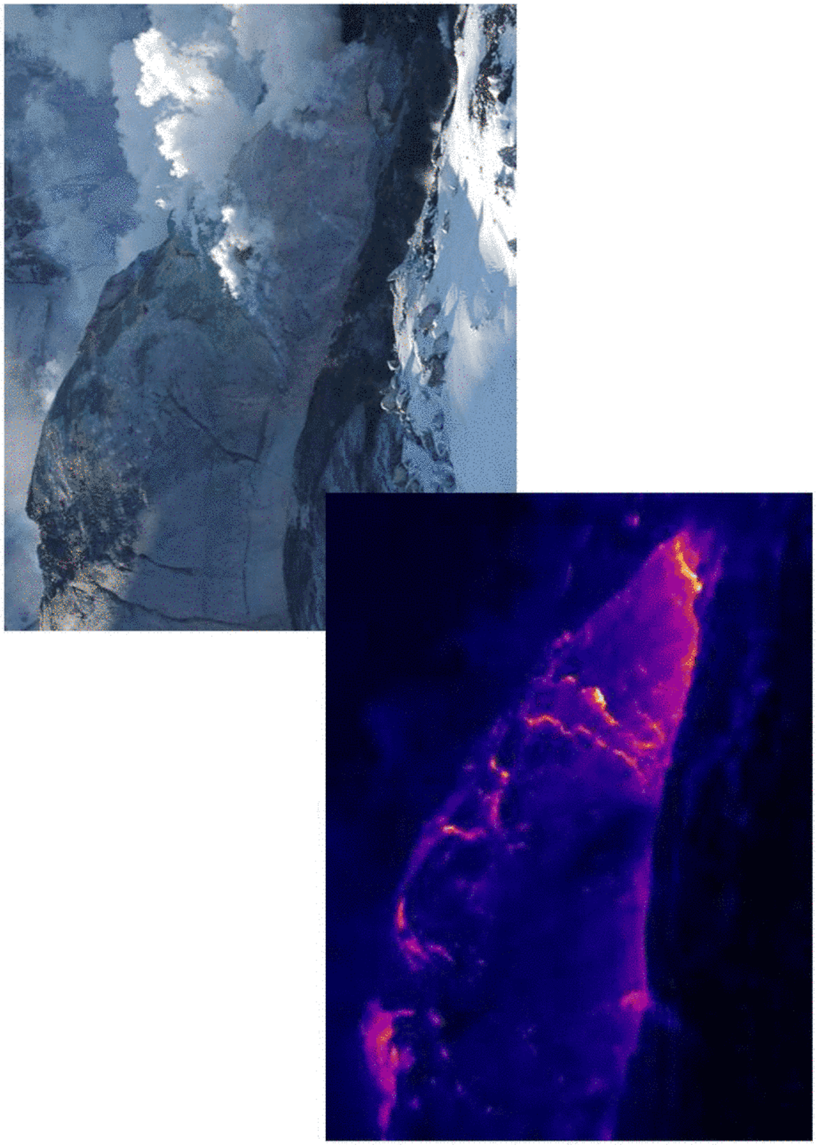

03/06 5:57pm PST

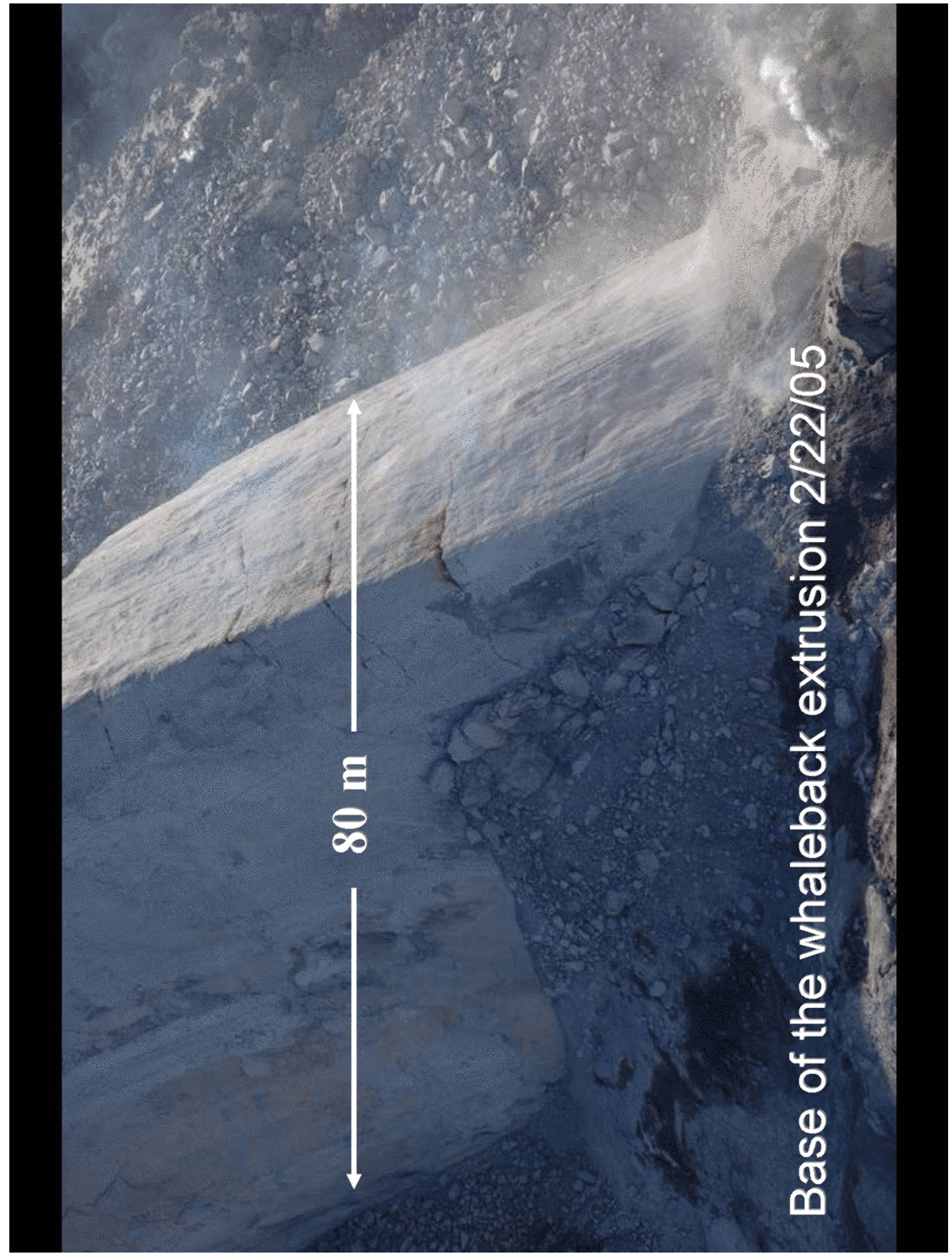

GPS “spider” on the whaleback
01/14/05

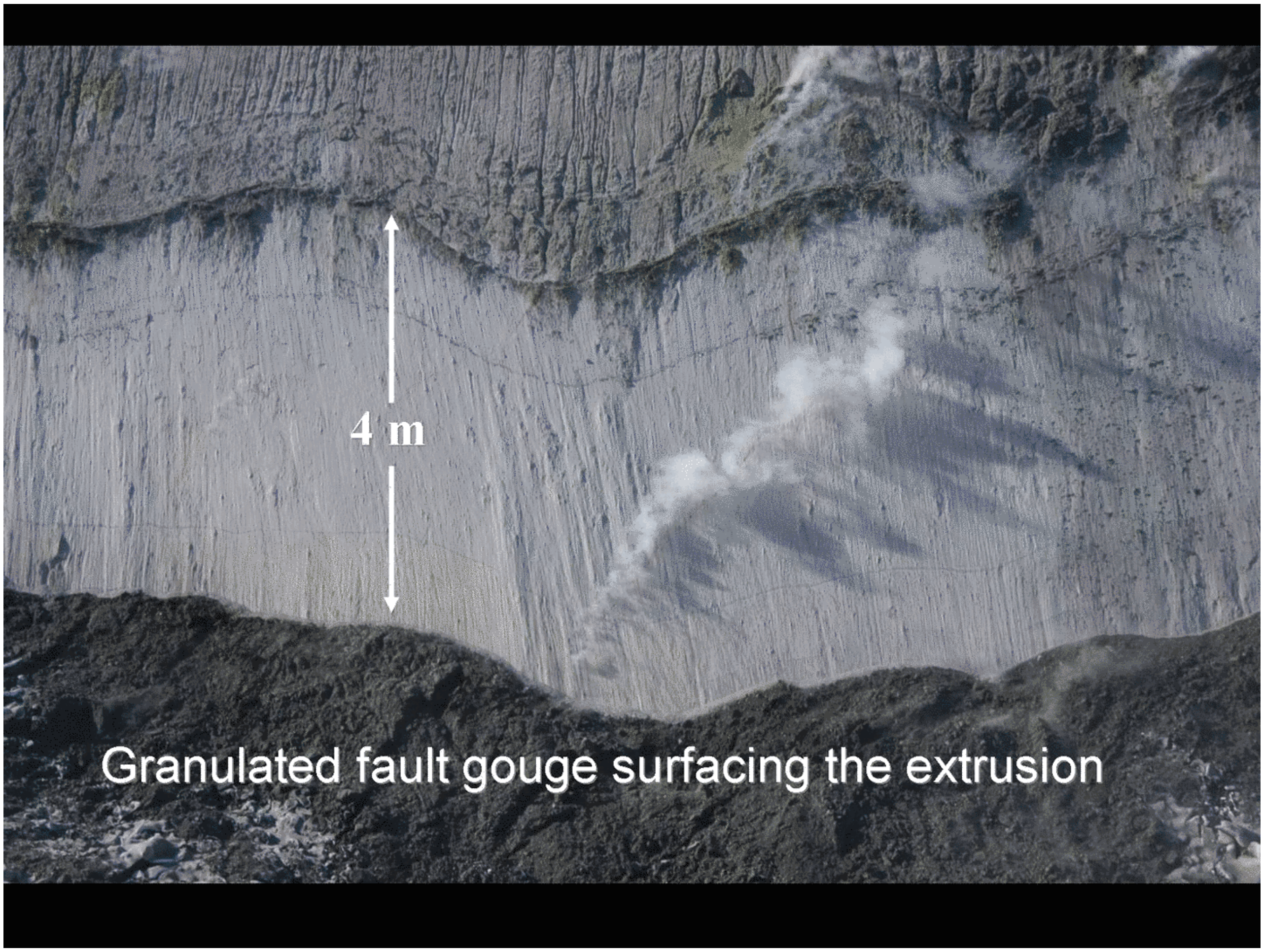

02/08/05


Whaleback motion 11/21/04 – 11/29/04
GPS spider station ELEA on 2004 Dome
Distance change, in meters
80
60
40
20
0
-20
-40
-60
-80
11/20/04
11/22/04
11/24/04
11/26/04
11/28/04
82m
27m
8m
77m
E
N
Up
Horizontal

FLIR (Forward Looking Infrared) imagery



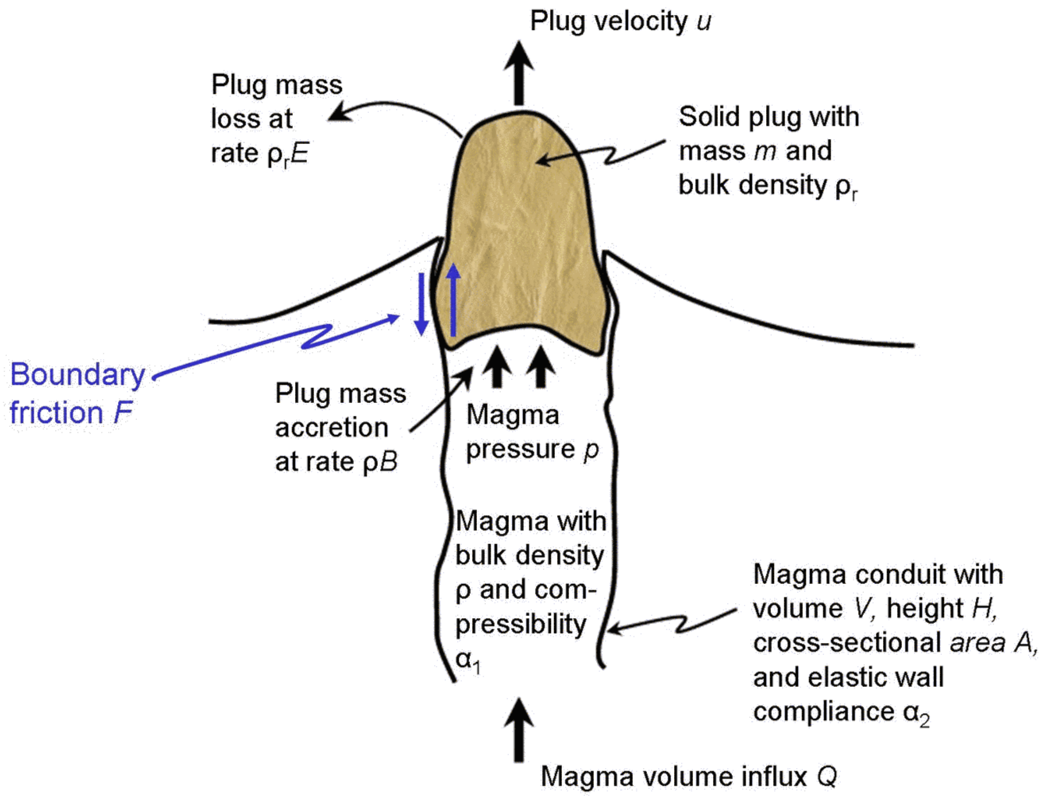




FACTS: Quasi-steady, solid-state extrusion at long-term rates ranging from 1 - 3 m^3/s is accompanied by repetitive, quasi-periodic small earthquakes. The extruded plug is surfaced with striated, granulated fault gouge.

HYPOTHESIS: Extrusion is driven by magma pressure pushing against the base of an accreting solid plug that slips frictionally and incrementally against the conduit wall rock. Repetitive stick-slip cycles generate quasi-periodic small earthquakes.

SPASM Model (Seismogenic Plug of Ascending, Solidifying Magma)



Model Components

<u>Physical Conservation Laws</u>
- Linear momentum of solid plug with evolving mass
- Mass of solid plug (evolves due to basal accretion and surface erosion)
- Linear momentum of compressible conduit fluid (laminar flow in tube)
- Mass of conduit fluid (evolves due to change in density and conduit volume)

<u>Constitutive "Laws"</u>
- Elastic compliance of conduit walls
- Compressibility of conduit fluid
- Friction rule for solid plug interacting with conduit walls

<u>Simplifying Assumptions</u>
- One-dimensional dynamics
- System is driven by constant influx of magma at depth

Reduced system of governing equations

Evolution of Plug velocity

$$\frac{du}{dt} = -g + \frac{1}{m_0 + \kappa t}\left[pA - \kappa u - F(u/u_{ref})\right]$$

possible strong nonlinearity in friction rule

Evolution of Magma pressure on plug base

$$\frac{dp}{dt} = \frac{1}{\alpha_1 + \alpha_2}\frac{Q - Au}{V}$$

weak nonlinearity due to quotient u/V

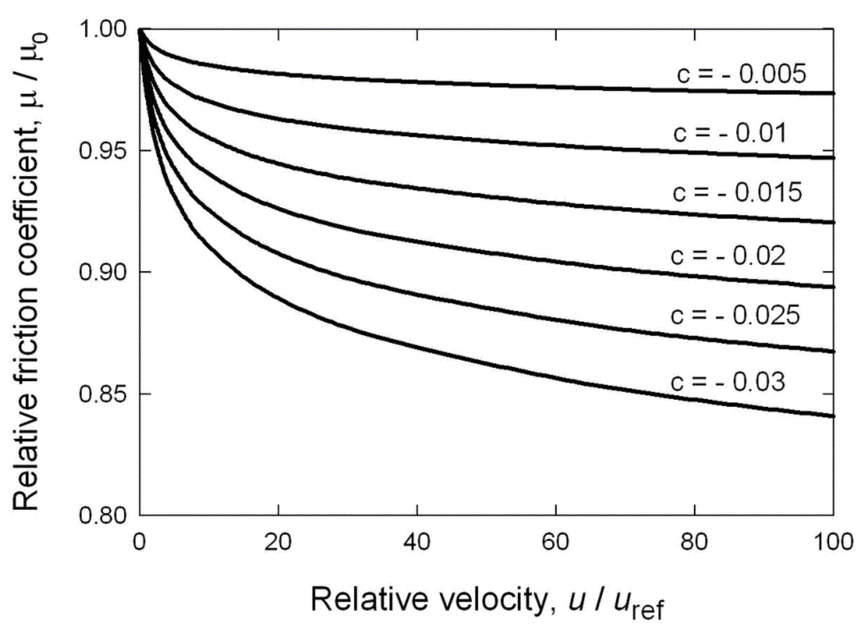
Evolution of Volume of magma-filled conduit

$$\frac{dV}{dt} = \frac{\alpha_1}{\alpha_1 + \alpha_2}Au + \frac{\alpha_2}{\alpha_1 + \alpha_2}Q - B$$

Nonlinear friction rule for sliding of solid plug

$$F(u/u_{ref}) = \lambda\, m_0\, g\, \mu = \lambda\, m_0\, g\, \mu_0\, [1 + c \sinh^{-1}(u/u_{ref})]$$

For u/u_{ref} <1, behavior approximates linear; for u/u_{ref} >1, behavior approximates logarithmic



Part 1. Analytical Results

Exact steady-state solution
(for case with $Q = Au$, $\kappa = 0$)

$$u_0 = \frac{Q}{A} \qquad p_0 = \frac{m_0 g + F_0}{A} \qquad V = V_0$$

In this case plug velocity, plug mass, plug friction, conduit volume, and conduit pressure remain constant, and forces remain balanced.

Linearize about this steady state to examine behavior of small perturbations in plug velocity $u'(t)$. Assume rate-dependence of friction is linear, i.e. $dF/du = \lambda m_0 g \mu_0 c / u_{ref} = C$

Find velocity perturbations obey damped, forced oscillator equation

$$\left(1 + \frac{\kappa t}{m_0}\right)\frac{d^2u'}{dt^2} + \frac{1}{m_0}\left(2\kappa + \frac{CF_0}{u_0}\right)\frac{du'}{dt} + \frac{1}{m_0(\alpha_1 + \alpha_2)}\frac{A^2}{V_0}u' = -\frac{g\kappa}{m_0}$$

Acceleration rate — Damping rate — Restoring rate — Forcing rate

For simplest case with zero κ (no plug mass change) and zero damping ($C = 0$, no rate-dept. friction) this equation has a simple periodic solution

$$u' = u_0 \cos(t/t_0) \qquad \text{where} \qquad t_0 = \frac{[m_0 V_0(\alpha_1 + \alpha_2)]^{1/2}}{A}$$

Period of free oscillations is $T \approx \dfrac{2\pi\,[m_0 V_0(\alpha_1 + \alpha_2)]^{1/2}}{A}$

For a cylindrical plug and conduit this period can be expressed as

$$T \approx 2\pi\,[(\alpha_1 + \alpha_2)\,\rho_r\,H_{plug}\,H_{conduit}]^{1/2}$$

Insert some appropriate numbers for Mount St. Helens:
bulk compressibility $\alpha_1 + \alpha_2 = 10^{-8}$ Pa^{-1}
plug-rock density $\rho_r = 2400$ kg/m^3
plug height $H_{plug} = 1$ km
conduit height $H_{conduit} = 8$ km

These numbers yield free oscillation period $T = 87$ s, similar to common earthquake recurrence interval during 2004-2005 eruption of Mount St. Helens

Stability of quasi-steady extrusion subject to small oscillations

Case I : plug mass is constant ($\kappa = 0$)

Stability parameter

$$D = (T/4\pi)\, g\, \lambda\, \mu_0 \frac{c}{u_{ref}}$$

Oscillations decay if $D > 0$; grow if $D < 0$

Instability can be manifested as either growing oscillations or runaway extrusion, depending on the value of D.

Stability of quasi-steady extrusion subject to small oscillations

Case II : plug mass evolves ($\kappa \neq 0$)

Stability parameter

$$C/\kappa = \frac{1}{\kappa} m_0 \, g \, \lambda \, \mu_0 \, \frac{c}{u_{ref}}$$

$\kappa > 0$, $C > 0$	unconditionally stable
$\kappa < 0$, $C < 0$	unconditionally unstable
★ $\kappa > 0$, $C < 0$	stable oscillations if $-C/\kappa < 3/2$
$\kappa < 0$, $C > 0$	stable oscillations if $-C/\kappa > 3/2$

This is the only case in which persistent stick-slip cycles occur !

Part 2. Computational Results for Nonlinear System

$$\frac{du}{dt} = -g + \frac{1}{m_0 + \kappa t}\left[pA - \kappa u - F(u/u_{ref})\right]$$

$$\frac{dp}{dt} = \frac{1}{\alpha_1 + \alpha_2}\,\frac{Q - Au}{V}$$

$$\frac{dV}{dt} = \frac{\alpha_1}{\alpha_1 + \alpha_2} Au + \frac{\alpha_2}{\alpha_1 + \alpha_2} Q - B$$

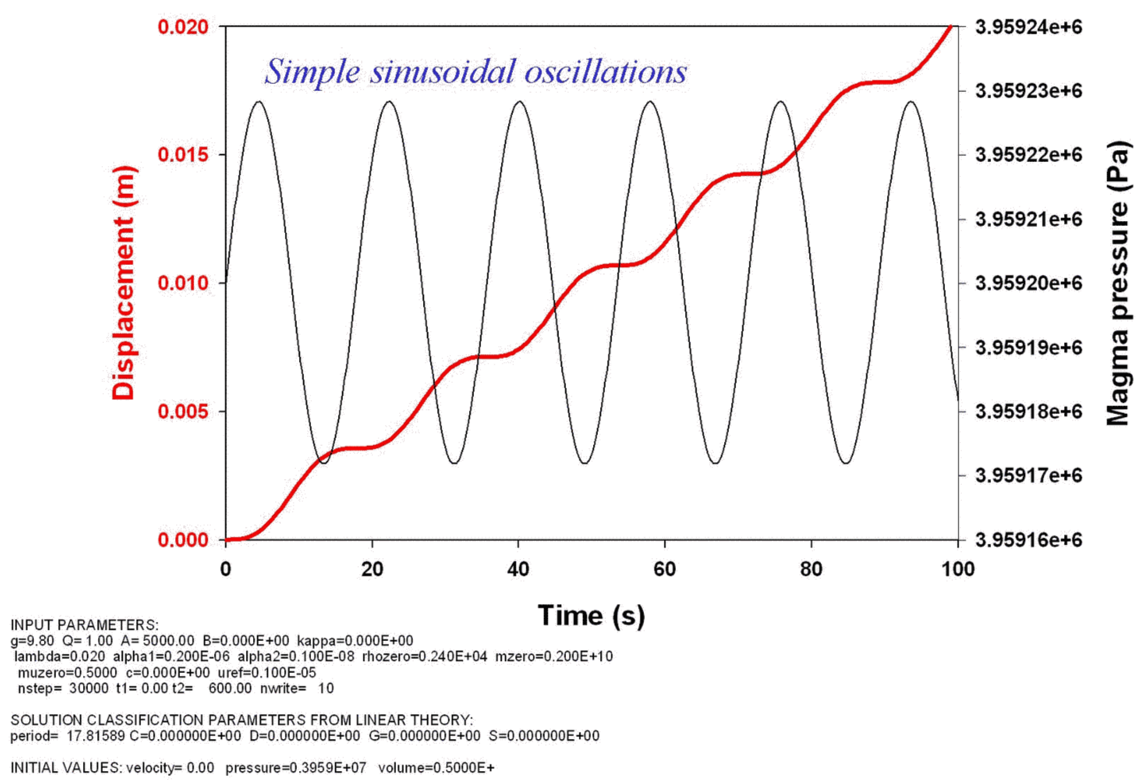
$$F(u/u_{ref}) = \lambda \, m_0 \, g \, \mu = \lambda \, m_0 \, g \, \mu_0 \left[1 + c \sinh^{-1}(u/u_{ref})\right]$$

First, fix values of all parameters except c at reasonable values:

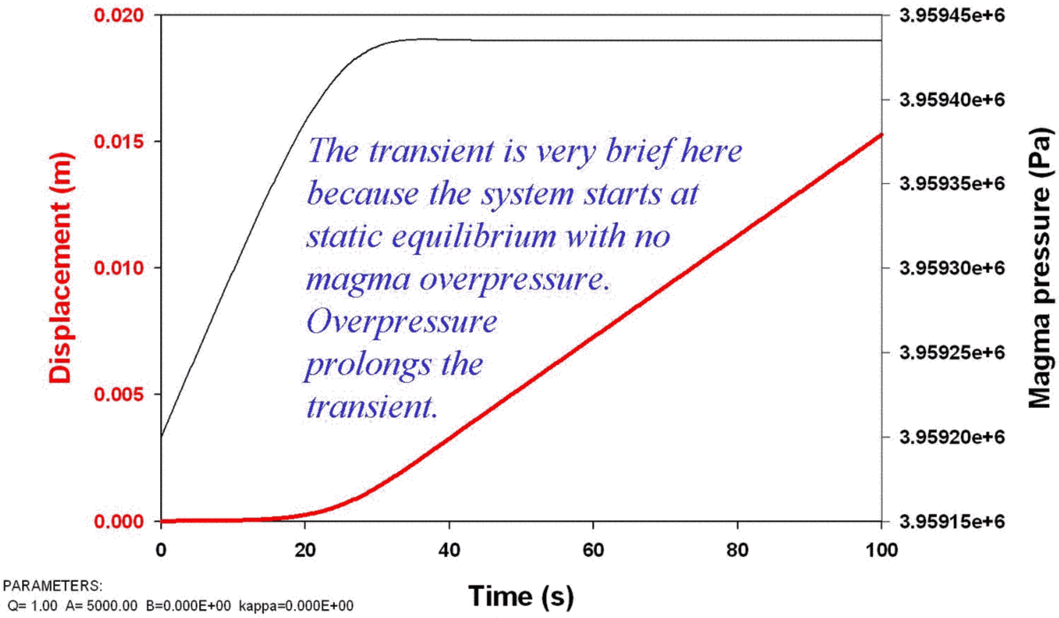
Magma influx rate Q = 1 m^3/s
Plug base area A = 5000 m^2 (80 m diameter circle)
Plug mass m = 2 x 10^9 kg
Plug mass growth rate $\kappa = 0$
Magma compressibility α_1 = 2 x 10^{-7} Pa
Conduit compliance α_2 = 1 x 10^{-9} Pa
Static friction coefficient μ_0 = 0.5
Effective stress coefficient λ = 0.02 (friction can support 2% of plug weight)
Reference sliding rate u_{ref} = 1 x 10^{-6} m/s (implies strong nonlinearity)

Then, use Runge-Kutta method to generate numerical results for different values of c

Baseline solution for c = 0 reproduces analytical results for linearized system, substantiating the inference that the nonlinearity involving u/V is weak.



In rate-strengthening case with c = 0.001, first a transient occurs in which forces equilibrate. Then the system reaches stable, dynamic equilibrium. This occurs for both linear (i.e., "viscous") and nonlinear rate strengthening.

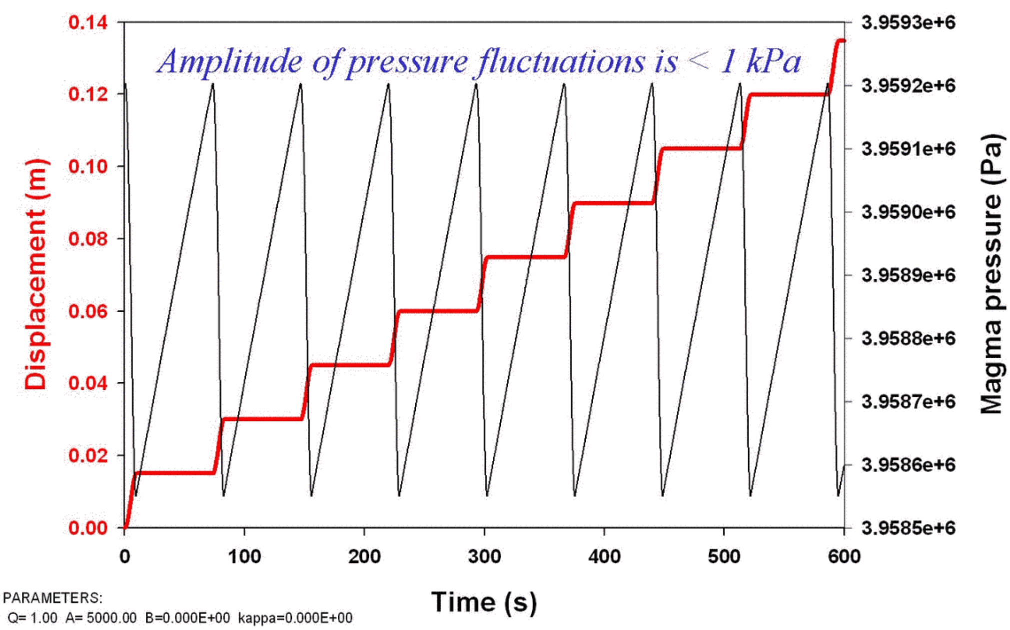


INPUT PARAMETERS:
g=9.80 Q= 1.00 A= 5000.00 B=0.000E+00 kappa=0.000E+00
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=0.100E-02 uref=0.100E-05
nstep= 30000 t1= 0.00 t2= 600.00 nwrite= 10

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=0.196000E+12 D=0.138939E+03 G=0.000000E+00 S=0.000000E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+06

In rate-weakening case with c = - 0.001, the system exhibits stick-slip oscillations with periods longer than than those predicted by the linear theory

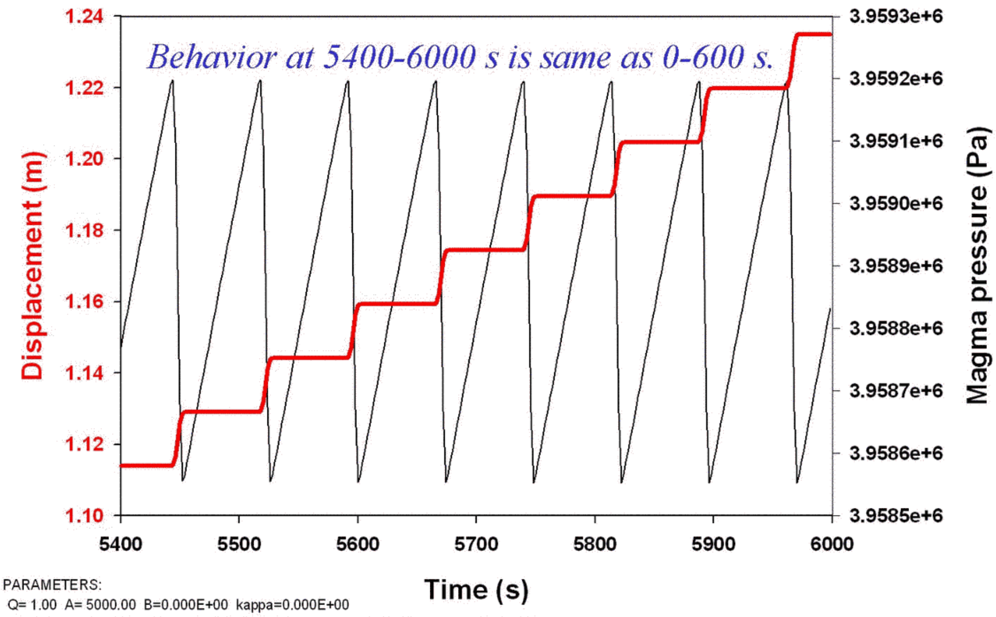


INPUT PARAMETERS:
g=9.80 Q= 1.00 A= 5000.00 B=0.000E+00 kappa=0.000E+00
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=-.100E-02 uref=0.100E-05
nstep= 30000 t1= 0.00 t2= 600.00 nwrite= 10

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=-.196000E+12 D=-.138939E+03 G=0.000000E+00 S=0.000000E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+06

The amplitude of the stick-slip oscillations does not grow with time, contrary to predictions of the linear theory. The nonlinearity *stabilizes* the oscillations by limiting positive feedback between slip velocity and frictional resistance. This stabilization may help explain the repetitive earthquake pattern at MSH

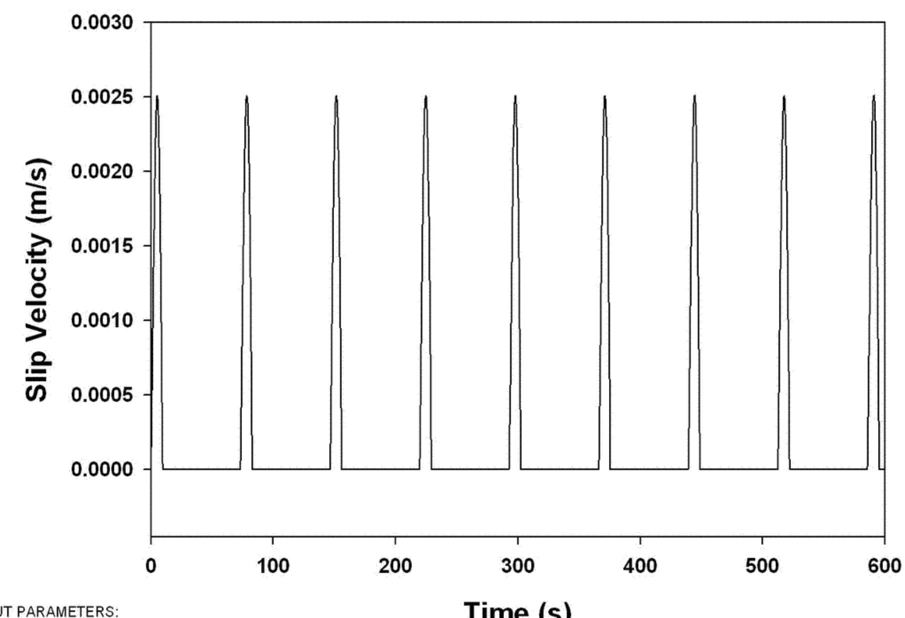


INPUT PARAMETERS:
g=9.80 Q= 1.00 A= 5000.00 B=0.000E+00 kappa=0.000E+00
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=-.100E-02 uref=0.100E-05
nstep= 300000 t1= 0.00 t2= 6000.00 nwrite= 100

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=-.196000E+12 D=-.138939E+03 G=0.000000E+00 S=0.000000E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+06

Velocity "spikes" accompany the slip events depicted in previous slides. Work done during each of these spikes is about 3 x10^8 J, similar to the energy release in a M 2.2 earthquake. Of course, some of the work may be done aseismically, resulting in smaller earthquakes.



INPUT PARAMETERS:
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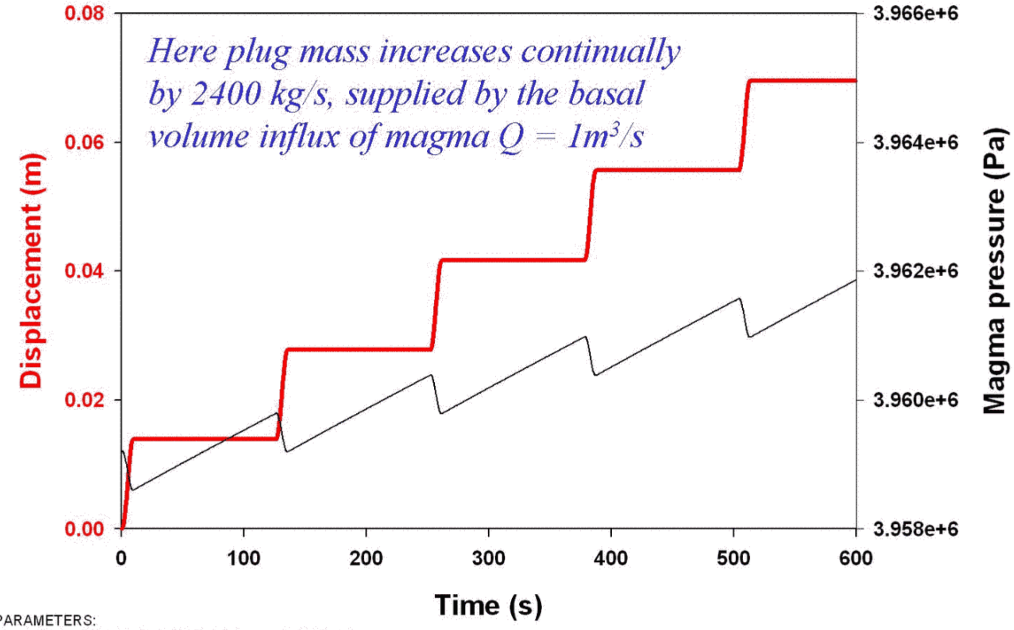
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=-.100E-02 uref=0.100E-05
nstep= 30000 t1= 0.00 t2= 600.00 nwrite= 10

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=-.196000E+12 D=-.138939E+03 G=0.000000E+00 S=0.000000E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+

Now, include mass accretion at base of plug, with no compensating erosion of surface. The added mass lengthens the oscillation period, reduces the extrusion rate, and causes the magma pressure to gradually increase. The converse occurs if there is net loss of plug mass.

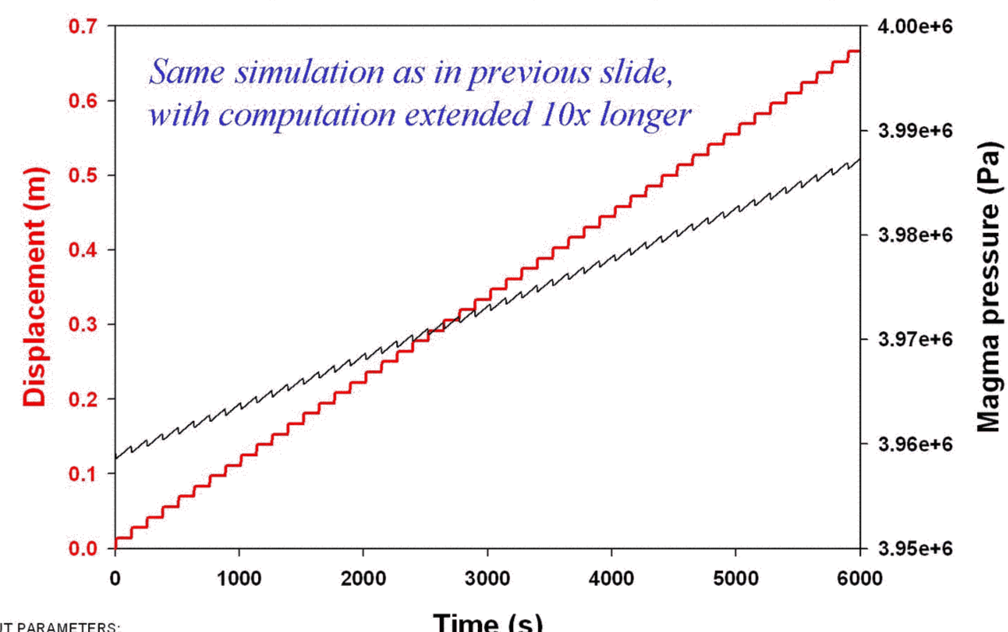


INPUT PARAMETERS:
g=9.80 Q= 1.00 A= 5000.00 B=0.100E+01 kappa=0.240E+04
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=-.100E-02 uref=0.100E-05
nstep= 30000 t1= 0.00 t2= 600.00 nwrite= 10

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=-.196000E+12 D=-.138939E+03 G=0.340259E-05 S=-.472752E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+

… and the beat goes on forever unless magma influx changes or one or more parameters evolve to change the system response. Note, however, that the magma pressure is gradually increasing…



INPUT PARAMETERS:
g=9.80 Q= 1.00 A= 5000.00 B=0.100E+01 kappa=0.240E+04
lambda=0.020 alpha1=0.200E-06 alpha2=0.100E-08 rhozero=0.240E+04 mzero=0.200E+10
muzero=0.5000 c=-.100E-02 uref=0.100E-05
nstep= 300000 t1= 0.00 t2= 6000.00 nwrite= 100

SOLUTION CLASSIFICATION PARAMETERS FROM LINEAR THEORY:
period= 17.81589 C=-.196000E+12 D=-.138939E+03 G=0.340259E-05 S=-.472752E+00

INITIAL VALUES: velocity= 0.00 pressure=0.3959E+07 volume=0.5000E+06

Normalized damped, forced oscillator equation

$$(1+Gt^*)\frac{d^2u^*}{dt^{*2}}+(2G+IR)\frac{du^*}{dt^*}+u^*=-GI$$

Dimensionless parameters also control numerical solutions

$I = \dfrac{t_0 g}{Q/A}$ O (1000) or larger applies to MSH

$G = \dfrac{t_0}{m_0/\kappa}$ O (1) or smaller applies to MSH

$R = \lambda \mu_0 c \dfrac{Q/A}{u_{ref}}$ O (1) or smaller *probably* applies to MSH

Conclusions

1. Nonlinear rate-weakening boundary friction can explain stick-slip oscillations during extrusion of magma plug.

2. Use of realistic (i.e. best-guess) parameter values produces stick-slip oscillations with roughly the correct period and amplitude to correspond to repetitive “drumbeat” earthquakes at MSH.

3. Nonlinearity of the friction rule stabilizes stick-slip oscillations and allows them to occur “forever” if plug mass remains constant or if it evolves.

4. If plug mass changes due to net erosion or accretion, the oscillation period and amplitude and the surface extrusion rate change in response.

5. Long-term, oscillatory behavior of the system is remarkably stable unless magma influx changes, friction evolves, or magma pressure increases sufficiently to cause a hydrofracturing “blowout.”