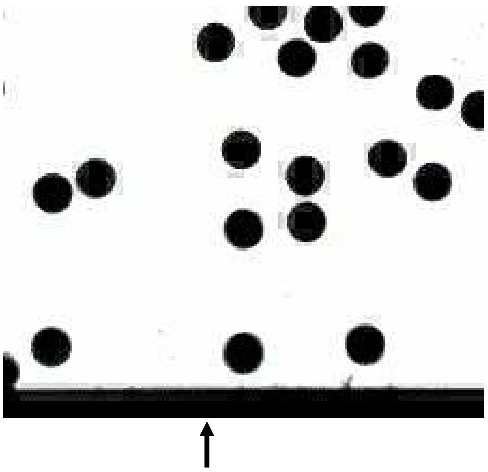
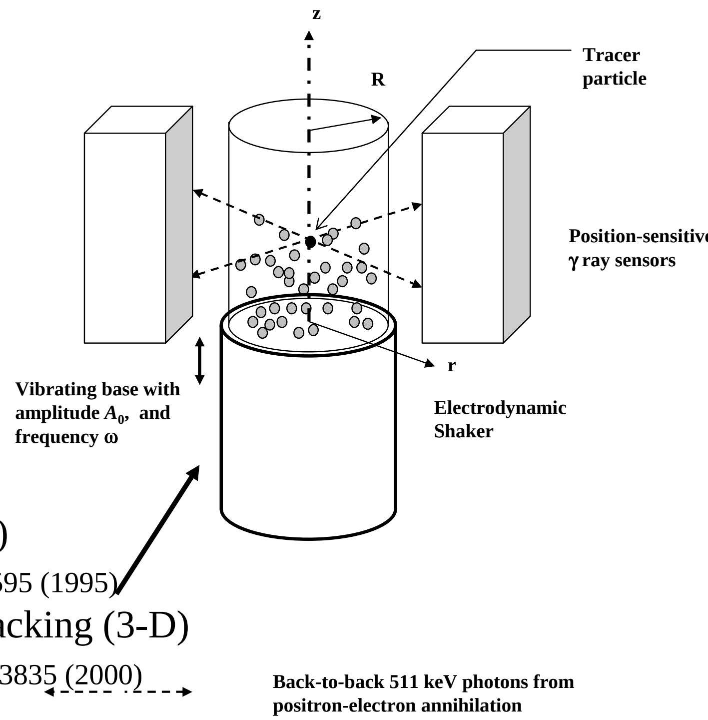
Knudsen regime in vibro-fluidized granular beds





High speed photography (2-D)

Warr et al., *Phys. Rev. E.* **52** 5583-5595 (1995)

Positron Emission Particle Tracking (3-D)

Wildman et al., *Phys Rev E* **62** 3826-3835 (2000)

1-D hydrodynamic model of vibro-fluidized bed

[to appear in *J. Fluid Mech.*]

Force balance (*z* is vertical):

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g$$

Equation of state:

$$P = \frac{6}{\pi d^3}\frac{\eta(1+\eta+\eta^2-\eta^3)}{(1-\eta)^3}T$$

(Carnahan and Starling 1967)

Energy flux:

$$J = -\kappa\frac{\mathrm{d}T}{\mathrm{d}z} - \mu\frac{\mathrm{d}\eta}{\mathrm{d}z}$$

Energy dissipation:

$$\frac{\mathrm{d}J}{\mathrm{d}z} = -\gamma_{gg} - \frac{2}{R}\gamma_{gw}$$

grain-grain   grain-wall

Hence set of 3 coupled first order ODEs

Transport coefficients

$$\kappa = \kappa^*(e)\kappa_0, \qquad \mu = \mu^*(e)\frac{T}{\eta}\kappa_0$$

Brey J J, Dufty J W, Kim C S, & Santos A
Phys. Rev. E **58**, 4638-4653 (1998)

$$\kappa_0 = \frac{75k_B}{64d^2}\left(\frac{k_B T}{\pi m}\right)^{1/2}$$

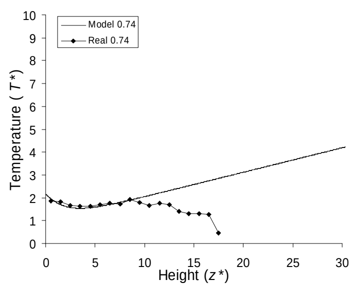
For large z we require $J \to 0$, and hence:

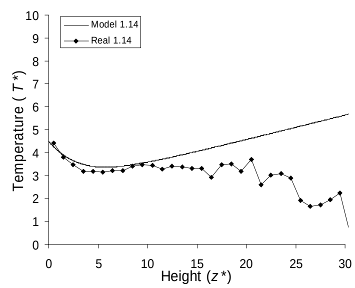
$$\lim_{z^* \to \infty}\left(\frac{\mathrm{d}T^*}{\mathrm{d}z^*}\right) = -\frac{\mu^*(1-\eta)^4}{\mu^*(1-\eta)\left(1+\eta+\eta^2-\eta^3\right) - \kappa^*\left(1+4\eta+4\eta^2-4\eta^3+\eta^4\right)}$$

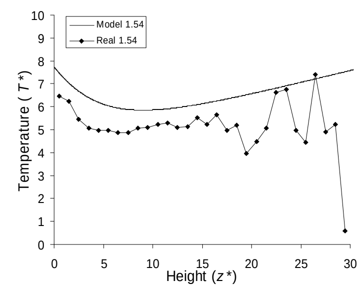
(See also: Brey J J & Ruiz-Montero M J, *Europhys. Lett.* **66**, 805-811 (2004))

Comparison of numerical solutions with experiment

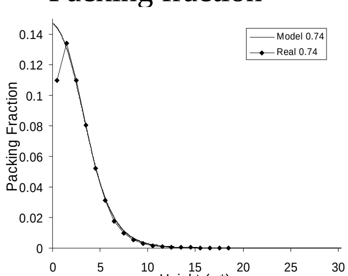
Temperature

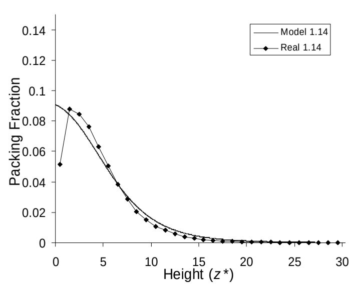


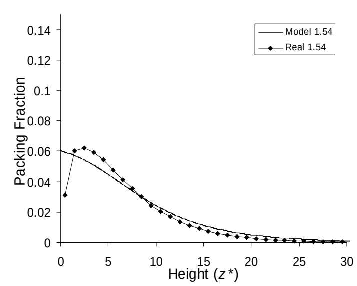




Packing fraction







Explanation for non-physical temperature profiles

- **Hydrodynamic description breaks down at high altitudes**
- Two possible criteria for onset of Knudsen regime:
- h_H – mean collision time large enough for grain to traverse the cell
- h_V – mean collision time large enough for gravity to reverse particle motion