Curved graphene revisited

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Summary

• Fermi velocity is the only parameter in the continuum model.
• Energy dependence: disorder and interactions.
• Space dependence: ripples, strain.
• Reconciling the GR and TB approaches.
• Homework for the experimentalists.

Artistic view of Dirac cones from Manchester’s group
Summary of graphene features

- The electronic properties described by 2D massless spinors.

Not a tight-binding feature: $C_3$ symmetry + low energy.

- Spinor structure given by the two sublattices A and B

- They come in two flavors associated to the two Fermi points (related by time reversal symmetry)

- Real spin did not play much a role until the recent advent of the topological insulators

The Fermi velocity is the only free parameter in the continuum model. All observable quantities depend on it.
Fermi points are zeroes of the determinant. A perturbation will open a gap if the loop $k = Re^{i\theta} \rightarrow (h_x, h_y, h_z) = R(\cos \theta, \sin \theta, 0)$ is contractible in the space of Hamiltonians with non-vanishing determinant. If TI holds this space is homeomorphic to $R^2 - \{0\}$.

Fermi points robust to lattice deformations and interactions preserving TI.
Coulomb interactions: Graphene vs QED

\[ L = \int d^2 r \ dt \ \bar{\psi}(\mathbf{r},t) \gamma^\mu (\partial_\mu - ieA_\mu) \psi(\mathbf{r},t) \]

Non-relativistic QED (2+1)?

\[ j^\mu \sim (\bar{\psi} \gamma^0 \psi, \mathbf{v} \bar{\psi} \gamma_5 \psi) \]
The Fermi velocity

From cyclotron mass-
Suspended. Clean.
(Elias et al Nat. Phys.
2011)

From ARPES. Epitaxial
(Lanzara’s group PNAS 2011)

The Fermi velocity
It plays the same role as the effective mass in usual 2DEG

Energy dependent!

Coulomb interactions make it grow at lower energies.
Disorder does the opposite.
If you see it constant as decreasing energies it intrinsically grows.

Also space dependent?
Graphene as a bridge between high and low energy physics

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The running of the constants

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\[ \alpha_{QED} = \frac{e^2}{4\pi\hbar c} \quad \alpha_G = \frac{e^2}{4\pi \hbar v_F} \]

Renormalization is not just numerical correction!

\[ \alpha_{QED}^{-1} (1 GeV) \approx 137; \]
\[ \alpha_{QED}^{-1} (100 GeV) \approx 128 \]

\[ v_F (0.2 \text{ meV}) \approx 2.3 \times 10^6 \text{ m/s}; \]
\[ v_F (100 \text{ meV}) \approx 1.4 \times 10^6 \text{ m/s} \]

Renormalization is not just numerical correction!

Infrared stable. QED is probably “trivial” free fixed point. Graphene runs to \( \alpha_{QED} \).

\[ \Downarrow \quad E \to 0 \quad \Downarrow \]

Downwards Upwards

E-\to 0
Curved and strained graphene

Atomically resolved STM image of a monolayer of graphene on SiC(IIT).

Low, Guinea, Katsnelson 2011

Artistic

Graphene wrinkle, Sun et al, Nanotec. (09)
“Controlled ripple texturing of suspended graphene and ultrathin graphite membranes” Bao et al., Nat. Nanotech. 4 562 (2009)

“Impermeable atomic membranes from graphene sheets” Scott Bunch et al., Nanolett. 8, 2458 (2008)

- “Introducing Nonuniform Strain to Graphene Using Dielectric Nanopillars” (cond-mat/1106.1507 Tomori et al.)
- “Graphene bubbles with controllable curvature” (Cond-mat/1108.1701, Manchester group)
- “Topological properties of artificial graphene assembled by atom manipulation”, where they produced atomically engineered strains. (Manoharan group, APS 2011)
- And the list goes on...
Effective gauge fields (from tight binding)

\[ H = \begin{pmatrix} 0 & t_1 e^{i \mathbf{k} \cdot \mathbf{a}_1} + t_2 e^{i \mathbf{k} \cdot \mathbf{a}_2} + t_3 e^{i \mathbf{k} \cdot \mathbf{a}_3} \\ t_1 e^{-i \mathbf{k} \cdot \mathbf{a}_1} + t_2 e^{-i \mathbf{k} \cdot \mathbf{a}_2} + t_3 e^{-i \mathbf{k} \cdot \mathbf{a}_3} & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{3t a}{2} (k_x + i k_y) + \Delta t \\ \frac{3t a}{2} (k_x + i k_y) + \Delta t & 0 \end{pmatrix} \]

A modulation of the hoppings leads to a term which modifies the momentum: an effective gauge field.

The induced “magnetic” fields have opposite sign at the two corners of the Brillouin Zone.

Courtesy of F. Guinea
Model for curved graphene: QFT in curved space
Dirac in curved space

We can include curvature effects by coupling the Dirac equation to a curved space

\[ \gamma^a e^\mu_a \left( \partial_\mu - \Omega^\mu (x) \right) \psi = E \psi \]

Need a metric and a “tetrad”.

\[ e^a_\mu e^b_\nu \eta_{ab} = g_{\mu \nu} \]

Generate r-dependent Dirac matrices and an effective “gauge” field.

\[ \Omega^\mu = \frac{1}{4} \gamma^a \gamma^b e^\nu_{a;\mu} e^b_{\nu} \]
Effects of the curvature

1. The curved gamma matrices:

\[ \gamma^\mu(r) = \gamma^a e^\mu_a(r) \]

\[ H = iv_F \left( \sigma^1(r) \partial_1 + \sigma^2(r) \partial_2 \right) = \left( v_1(r) \sigma^1 \partial_1 + v_2(r) \sigma^2 \partial_2 \right) \]

Can be seen as a position-dependent Fermi velocity

2. The spin connection:

- It can be seen as an effective gauge field. It is constructed with derivatives of the metric and depends on the \( \gamma \) matrix representation (Dirac point).

- It has different signs at the two Fermi points (time reversal symmetry preserved).
Physical reality of the fictitious gauge fields
Observing fictitious gauge fields

Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles


30 JULY 2010 VOL 329 SCIENCE www.sciencemag.org

\[ A_x = \frac{\beta}{a} (u_{xx} - u_{yy}) \]
\[ A_y = \frac{2 \beta}{a} u_{xy} \]
\[ \beta = \frac{\partial \log (t)}{\partial \log (a)} \approx 2 \]

Need strong deformations giving rise to strong-uniform fictitious fields
Aharonov–Bohm interferences from local deformations in graphene

Fernando de Juan¹, Alberto Cortijo², María A. H. Vozmediano³* and Andrés Cano⁴

\[ u_r = u_0 \exp \left( -\frac{r^2}{2\sigma^2} \right) r^2 \sin 3\theta \]

\[ u_\theta = u_0 \exp \left( -\frac{r^2}{2\sigma^2} \right) r^2 \cos 3\theta \]

\[ z(r) = A \exp \left( -\frac{r^2}{b^2} \right) \]

Magnetic field Interference pattern

5nm 2nm
Profiles of fictitious magnetic fields

\[ z(r) = A \exp \left( -\frac{r^2}{b^2} \right) \]

\[ u_r = u_0 \exp \left( -\frac{r^2}{2\sigma^2} \right) r^2 \sin 3\theta \]

\[ u_\theta = u_0 \exp \left( -\frac{r^2}{2\sigma^2} \right) r^2 \cos 3\theta \]

Covariant

\[ B_z = -\frac{1}{r} \partial_r (rA_\theta) = \frac{1}{4r} \frac{\alpha z'}{(1 + \alpha z)^{3/2}} \]

Axial symmetry

TB-strain

\[ A_x = \frac{\beta}{a} (u_{xx} - u_{yy}) \]

\[ A_y = \frac{2\beta}{a} u_{xy} \]

TB-Magnetic field
### Relating TB-elasticity and geometric

**Geometric**

- Space dependent Fermi velocity
- Effective gauge fields with the symmetry of the deformation
- Dimensions: \( A : \partial g : \partial u \)
- Need intrinsic curvature
- Material independent

**TB-elasticity**

- \( g^{ij} = \eta^{ij} + u^{ij} \)
- Effective gauge fields with different symmetry
- In-plane strain OK
- Material dependent

\[ \beta = v_F \frac{\partial t}{\partial a} \approx 2 \text{ eV} \]

**An important issue:** is \( v(x) \) a physical effect? (observable)
Strain + symmetry

Build an effective $H$ at low energy with $C_3$ symmetry

What can we build with $(\sigma^i, q^i, u^{ij})$?

$H_0 = v_F \mathbf{\sigma} \cdot \mathbf{q}$

$C_3$ invariant tensor:

$$f^{ijk} = \frac{1}{a^3} \sum_{n=1}^{3} a_n^i a_n^j a_n^k$$

Terms compatible with $C_3$ symmetry:

- Even # indices: contract with the flat metric
  - $\sigma_i \partial_j$, the flat Hamiltonian
  - $\sigma_i (\partial_j u_{kl})$ the geometric gauge field
  - $u_{kl} \sigma_i \partial_j$ the space dependent Fermi velocity

- Odd # indices: contract with $f$ or $\varepsilon_{ijl}$
  - $\sigma_i u_{jk}$ the trigonal gauge field
  - $\sigma_i \partial_k \partial_j$ the trigonal warping term
Deformed graphene: TB beyond linear approximation

\[ H = -\sum_{n=1}^{3} (t + \delta t_n) \begin{pmatrix} 0 & e^{-i(K+q) \cdot d_n} \\ e^{i(K+q) \cdot d_n} & 0 \end{pmatrix} \]

Expand in \(q\):
Dirac fermions

\[ H_{\text{trigonal}} = f^{ijk} \sigma^i q^j q^k \]

Expand in both:

\[ A^i = \frac{2}{3a} e^{ij} f^{jkl} u^{kl} \]

Expand in \(\delta t_n\):
Gauge fields

\[ H_{\text{vF}} = \sigma^i q^j u^{kl} f^{ijkl} \]

Related works

J. L. Manes 2007 *Phys. Rev. B* **76** 045430
Winkler R and Zulicke U 2010 *Phys. Rev. B* **82** 245313
T. L. Linnik arXiv1111.3924

See also Pereira on Thursday

F. de Juan, M. Sturla, MAHV, work in progress.
A working example

\[ z(r) = A \exp \left( - \frac{r^2}{b^2} \right) \]

\[ v_r(r) = \frac{1}{\sqrt{1 + z'(r)^2}} \]

\( 1 + z'(r)^2 \equiv \alpha f(r), \)

\( \alpha = \left( \frac{A}{b} \right)^2 \approx 10^{-2} \), \( f(r) = \frac{4}{b^2} r^2 \exp\left(-2 \frac{r^2}{b^2}\right) \)

\( \alpha \) measures the departure from flat space \((h/L)^2\). The effects are of order \( \alpha \).

Notice:
A distribution of ripples will give rise to a landscape of \( v_F \).
Relating morphology and electronics

Evidence for Strain-Induced Local Conductance Modulations in Single-Layer Graphene on SiO2

Correlation of the tunneling spectrum with strain tensor ->
Conclusion

Pay attention to the Fermi velocity!

(It is not the constant that it looks like)
Some tasks for the experimentalists

(May 2010)

• Establish the existence (or not) of a gap in the clean, neutral system.

• Measure the variations (or not) of the Fermi velocity with energy at low energies.

• Measure the variations (or not) of the Fermi velocity with the curvature in rippled samples.

(January 2012)

• Establish the correlation (or not) of morphology and electronics (see Lau).

• Revise the experiments where the value of $v_F$ is a crucial quantity. Notice that it varies with the energy and with location on the sample.