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# QED in a Pencil Trace

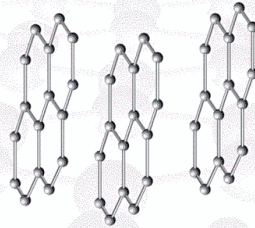
Andre Geim

in collaboration with

K. Novoselov, S. Morozov, F. Schedin, D. Jiang, T. Booth,  
M. Katsnelson, J. Meyer & I. Grigorieva

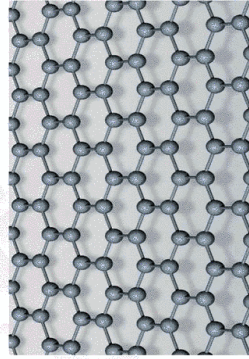
## GRAPHENE ALLOTROPES

3D



Graphite

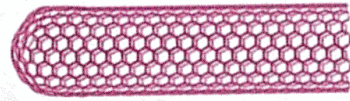
2D



graphene

**PRESUMED  
NOT TO EXIST  
IN THE FREE STATE**

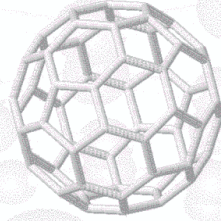
1D



Carbon  
Nanotube

multi-wall:  
1952 to Iijima 1991  
single-wall: 1993

0D

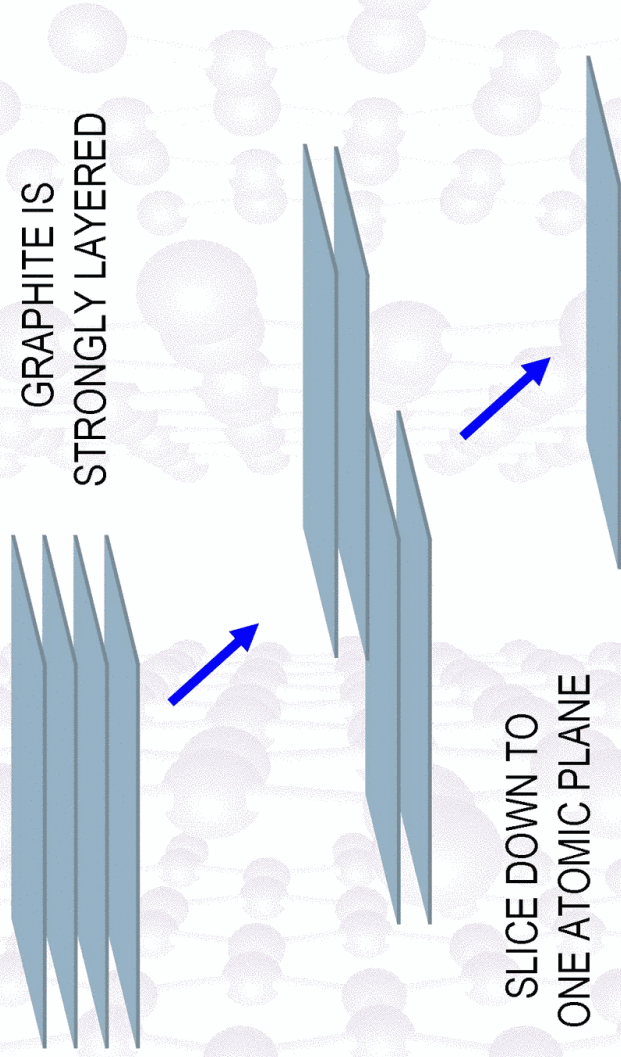


Buckyballs

Kroto et al 1985

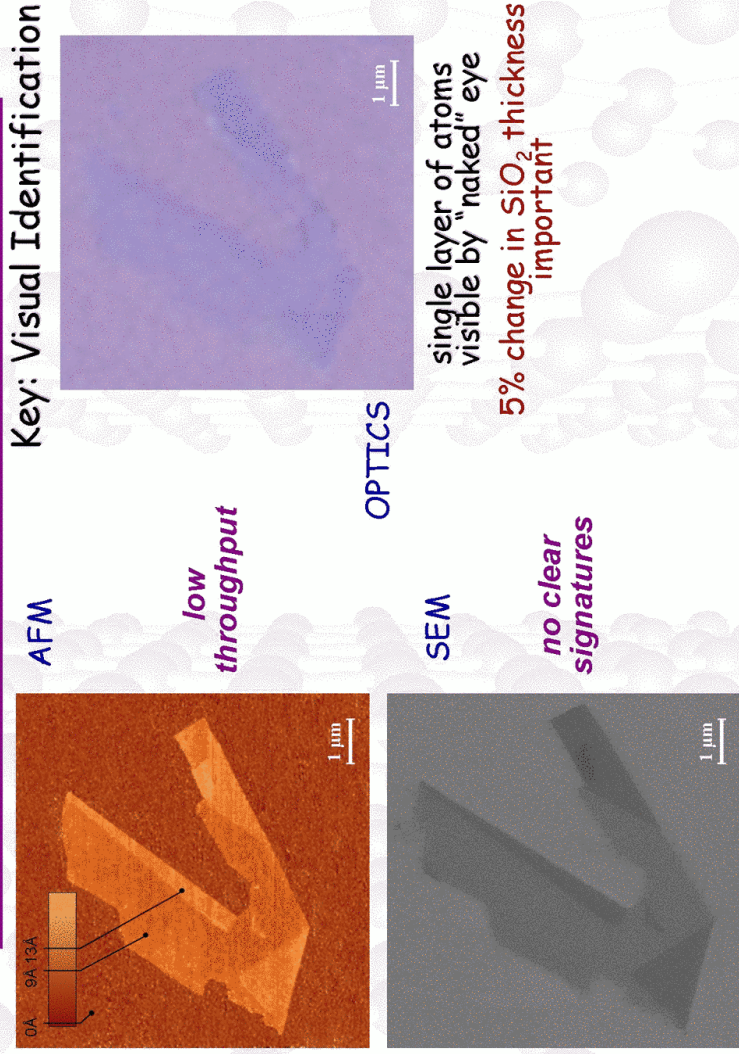


## Extracting a Single Plane



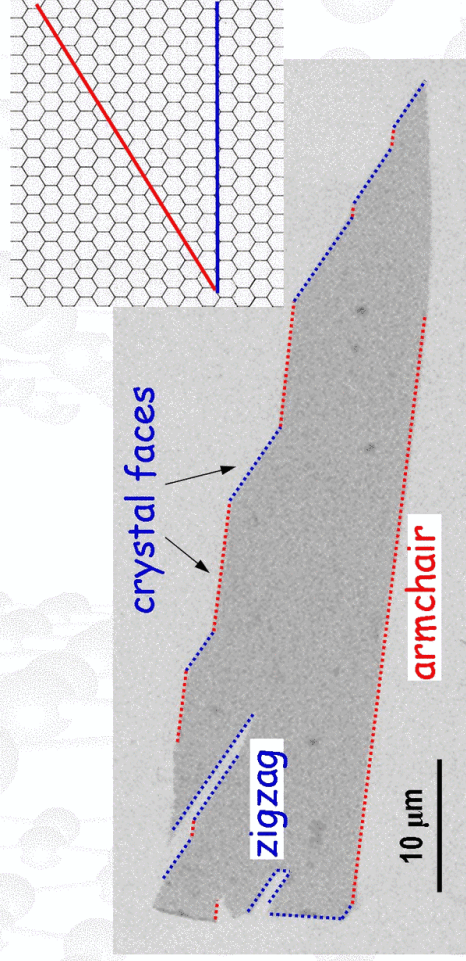
**individual atomic sheets: do they exist?**

## Free-Standing Graphene



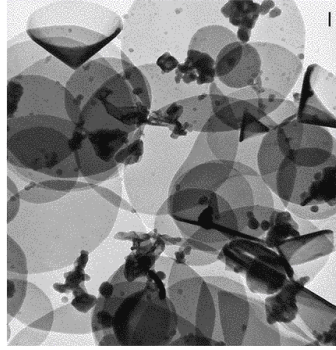


## Two Dimensional Crystallites

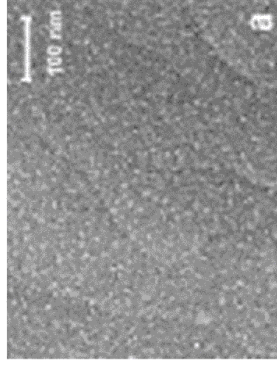


not just flakes  
but graphene crystallites

## No Free Standing Monolayers Known

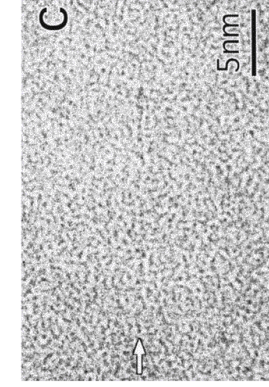


**CHEMICAL DECOMPOSITION**  
up to  $\varnothing$  2μm; > 80 layers; Krishnan 1998



**EPITAXIAL GROWTH**

McConville 1986 (on Ni); Land 1992 (on Pt)  
Affoune 2001 (on HOPG); Nagashima 1993 (on TiC)  
**quality & continuity (?)**



**CHEMICAL EXFOLIATION**

Dresselhauses (2002 review)  
**restacked and scrolled SOOT**  
*individual folds*: Horiuchi 2004

**SiC**: Bommel 1975; Forbeaux 1998; Charrier 2002;  
*Berger 2004-2006; Rotenberg 2006*



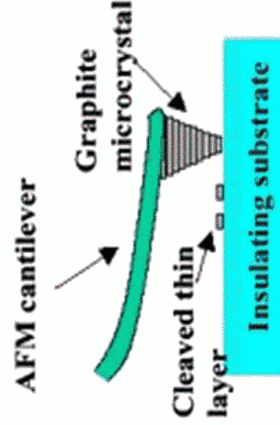
## mechanical cleavage in retrospect

Ohashi (*Tanso* 1997, 2000)  
from 1000 down to 50 layers

**our work (*Science* 2004): single layer**

Philip Kim's & Paul McEuen's groups  
(*PRL* 2005 & *Nanoletters* 2005) down to 35 layers

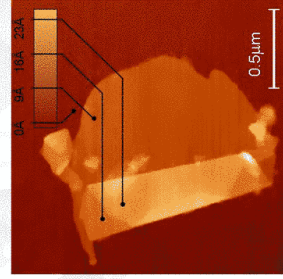
### Nanopencil



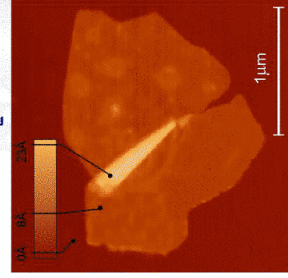
for >10 layers,  
electronic structure  
of bulk graphite  
(*Partoens 2006*)

## Other 2D Atomic Crystals

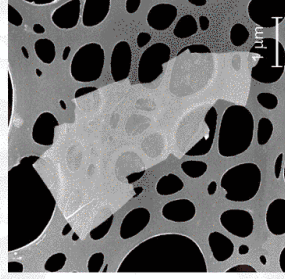
2D boron nitride in AFM



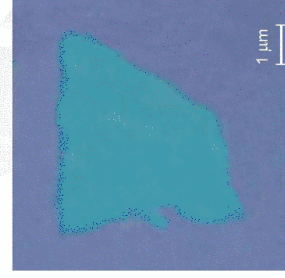
2D NbSe<sub>2</sub> in AFM



also,  
2,3,4... layers



2D Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>x</sub> in SEM



2D MoS<sub>2</sub> in optics

*PNAS* 102,  
10451 (2005)



## 2D ATOMIC CRYSTALS

**a new kind of materials**

wide choice of materials properties

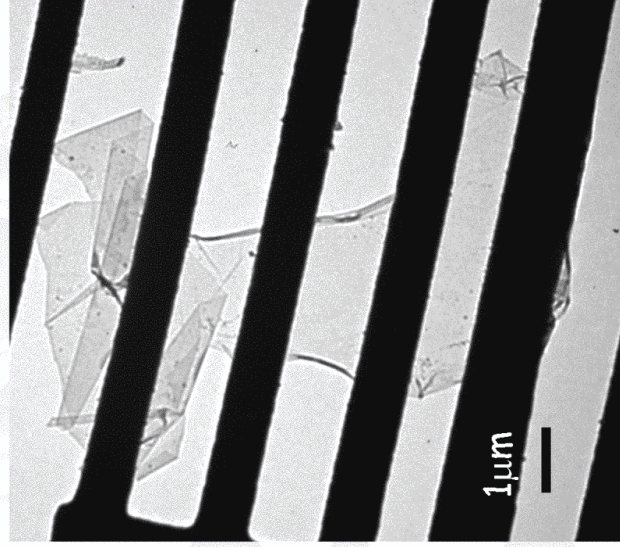
(electronic, mechanical, chemical, etc.)

## Why 2D Crystals Exist ?

- *stabilizing influence of substrate*
- *extracted from 3D, remain (meta)stable*
  - *something else*

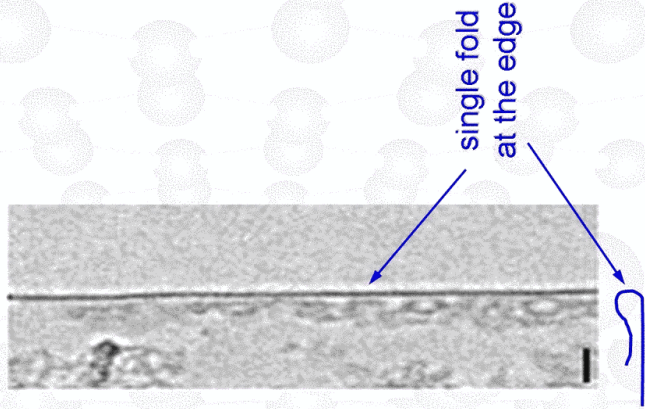


## "Free-Hanging" Graphene



one-atom-thick  
single-crystal fabric

TEM confirms single layer



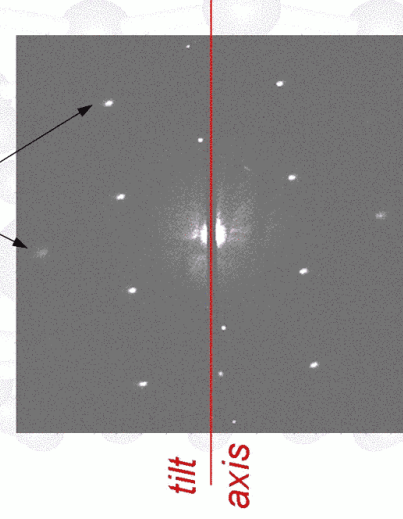
## Structure Of Suspended Graphene

local electron diffraction  
(beam  $\varnothing$  250 nm)



normal incidence

diffraction peaks  
away from tilt axis  
become blurred

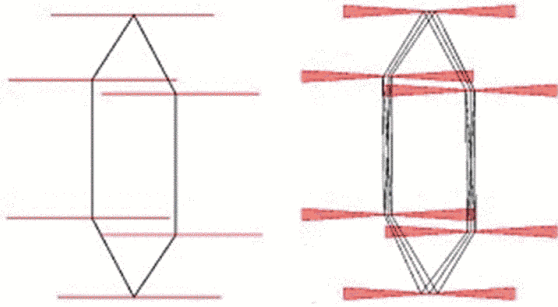


26° tilt

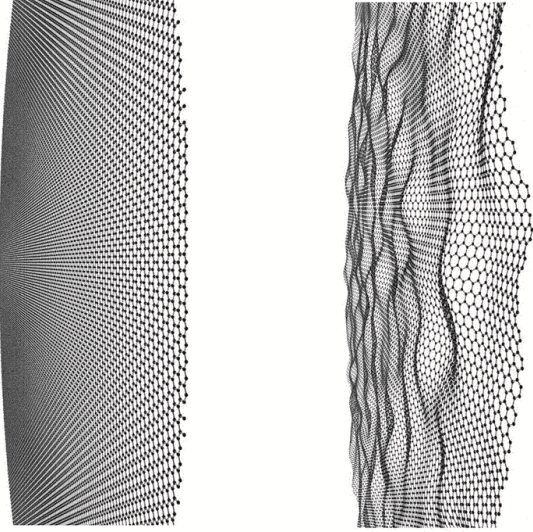


## Structure Of Suspended Graphene

reciprocal space

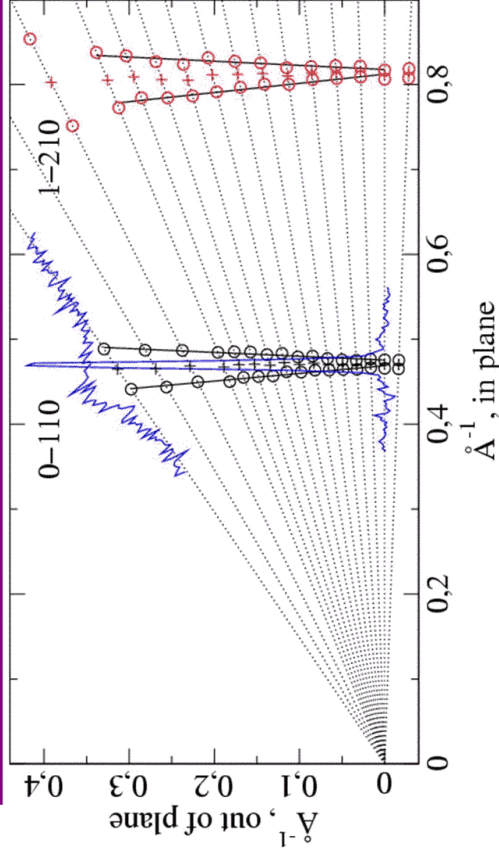


real space



cones in Fourier space rather than rods

## Intrinsic Microscopic Crumpling



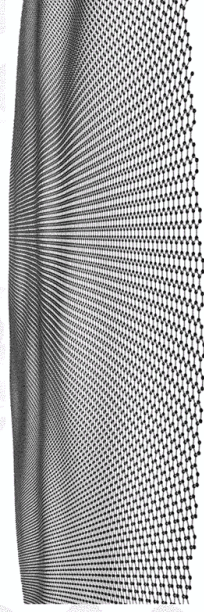
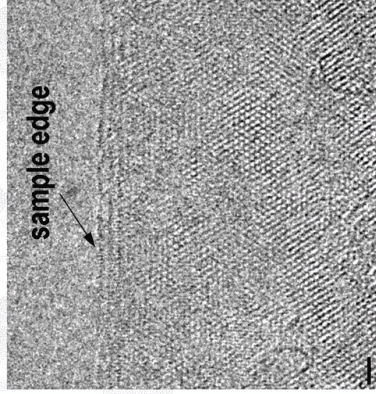
5° angle, and highly reproducible for different samples  
 twice smaller ( $\approx 2^\circ$ ) for bilayer graphene  
 disappears for few-layer samples

isotropic: local strain up to 1%



## Intrinsic Microscopic Crumpling

atomic resolution TEM  
ripple contrast appears for >1 layer



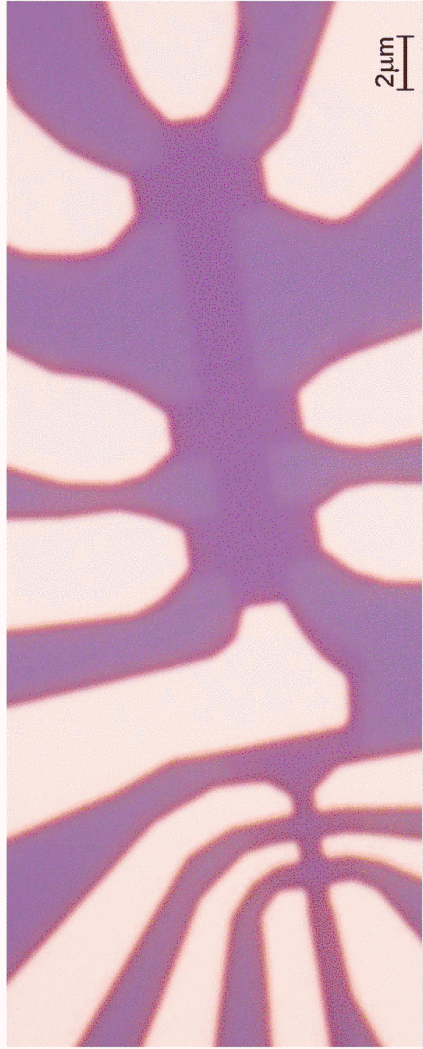
height  $\approx 5\text{\AA}$ ; size  $\approx 5\text{nm}$ ;  
elastic energy  
few  $kT$  per ripple

➤ ***crumpling in third-D may stabilize 2D sheet***

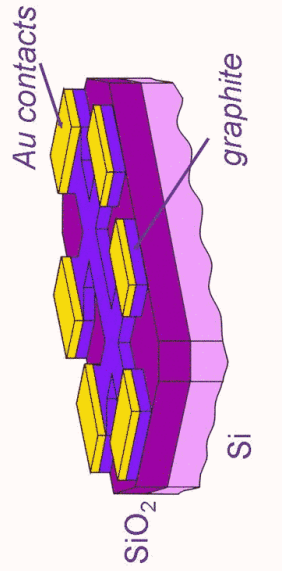
loose in elastic energy but gain in entropy; Nelson (1987)

## ELECTRON TRANSPORT IN GRAPHENE

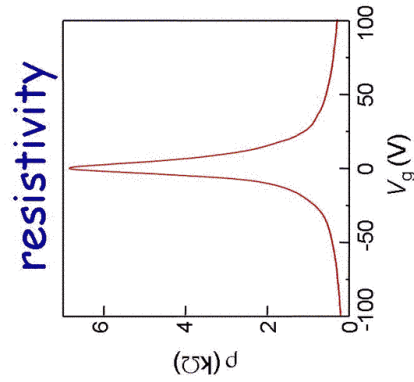
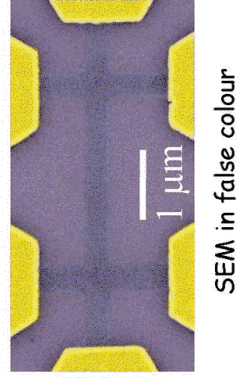
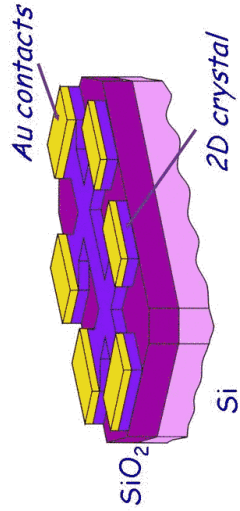
# Graphene Devices



- optical image
- SEM image
- design
- contacts and mesa



# Electric Field Effect in Graphene



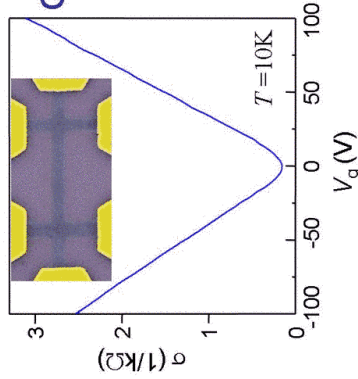
resistivity

graphene: currently  
 up to 10,000 cm<sup>2</sup>/V·s at 300K  
*ballistic transport*  
*on submicron scale*  
 under ambient conditions



# Electric Field Effect in Graphene

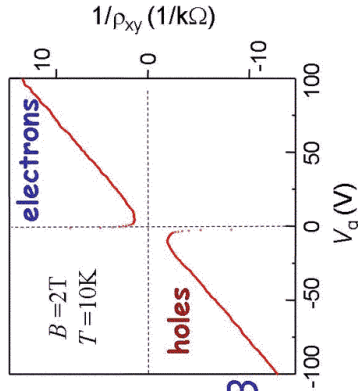
conductivity



$$\sigma = n(V_g)e\mu$$

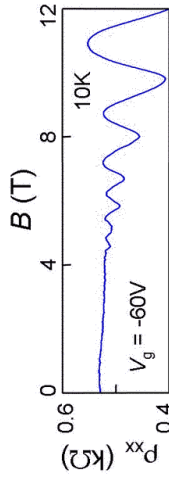
$$1/\rho_{xy} = ne/B$$

Hall effect

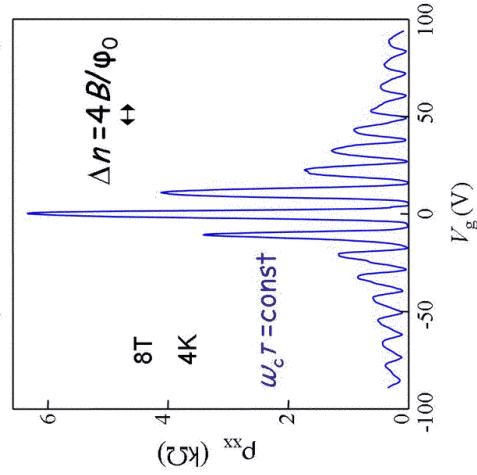


simple behaviour;  
practically constant mobility;  
no trapped carriers

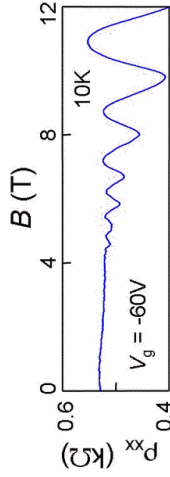
# Quantum Oscillations in Graphene



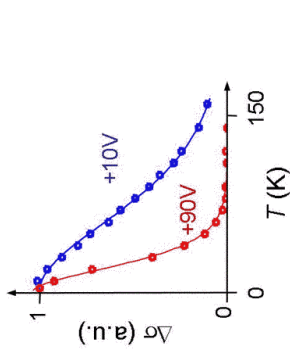
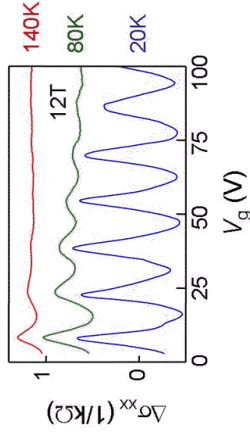
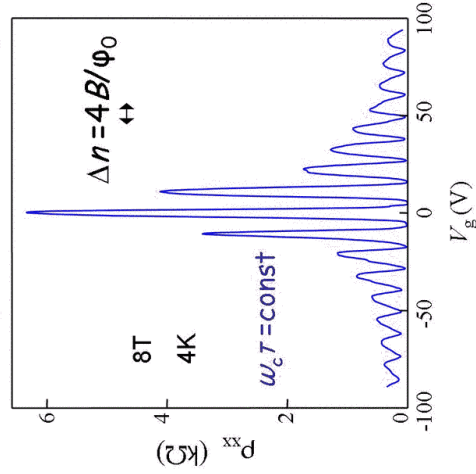
degeneracy  $f=4$   
two spins & two valleys



# Quantum Oscillations in Graphene



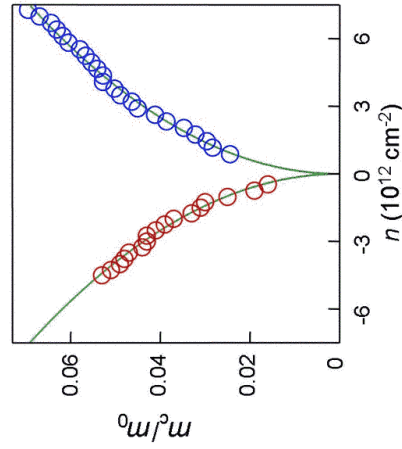
degeneracy  $f=4$   
two spins & two valleys



$$\Delta\sigma_{xx} \propto T \cdot \sinh\left(\frac{2\pi^2 k_B T m_c}{\hbar e B}\right)$$

# Band Structure of Graphene

cyclotron mass strongly depends on concentration

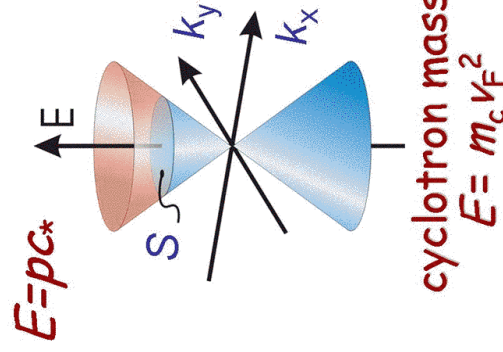


$$B_F = (\hbar/2\pi e) S \text{ and } m_c = (\hbar^2/2\pi) \partial S/\partial E$$

experimental dependences

$$B_F \sim n \text{ and } m_c \sim n^{1/2}$$

necessitates  $S \sim E(k)^2$  or  $E \sim k$



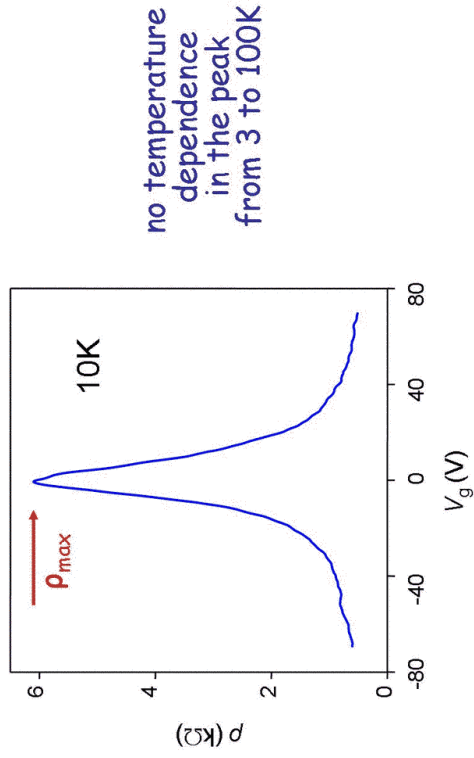
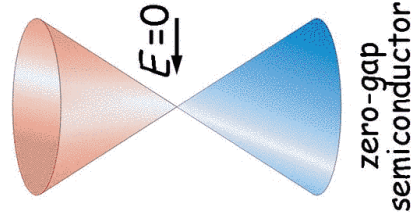
$$v_F = 10^6 \text{ m/s } \pm 5\%$$

previously seen in zero-gap 3D semiconductors (Zawadzki 1974)



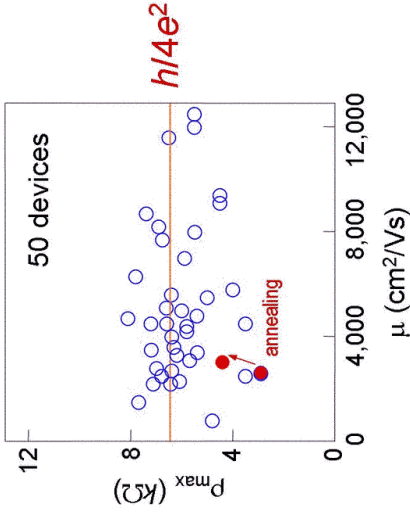
# CONDUCTIVITY "WITHOUT" CHARGE CARRIERS

## Minimum Quantum Conductivity



# Minimum Quantum Conductivity

most theories predict  $\pi$ -times larger value



- Fradkin 1986
- Lee 1993
- Ludwig 1994
- Morita 1997
- Ziegler 1998
- Peres 2005
- Gusynin 2005
- Katsnelson 2006
- Tworzydło 2006
- Cserti 2006
- Ostrovsky 2006

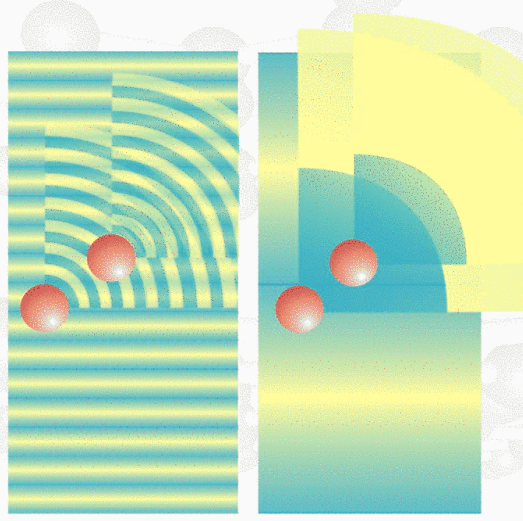
.....

quantized **resistivity** NOT resistance  
( $h/e^2$  per spin and valley)

# Minimum Quantum Conductivity

Mott's argument:  $l \geq \lambda_F$

$$\sigma = ne\mu = \frac{e^2}{h} k_F l \geq \frac{e^2}{h}$$



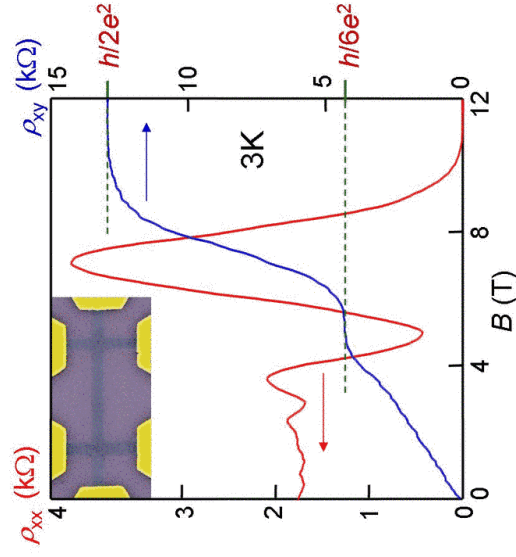
**NO LOCALIZATION**  
as observed in graphene



# CHIRAL Quantum Hall Effects

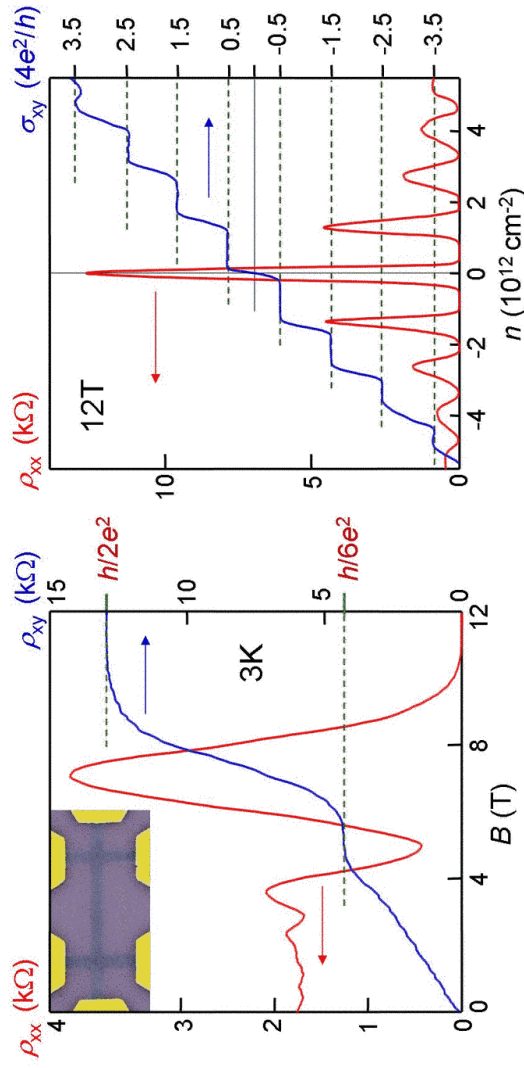
## Quantum Hall Effect in Graphene

quadruple degeneracy:  
plateaus are expected at  $h/4Ne^2$



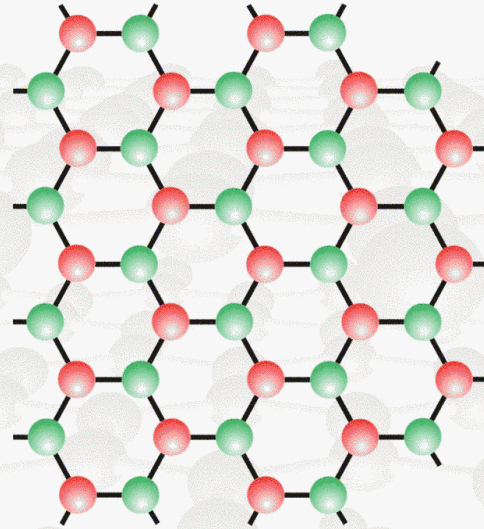
# Quantum Hall Effect in Graphene

quadruple degeneracy:  
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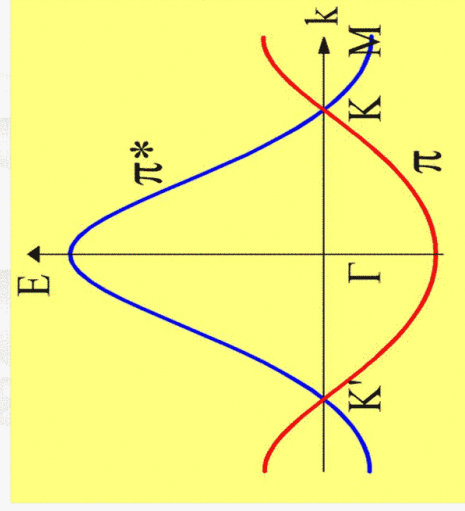
"half-integer" QHE

# Chiral Fermions in Graphene



two sublattices

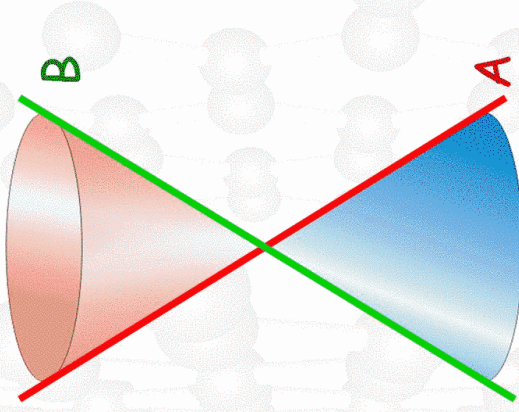
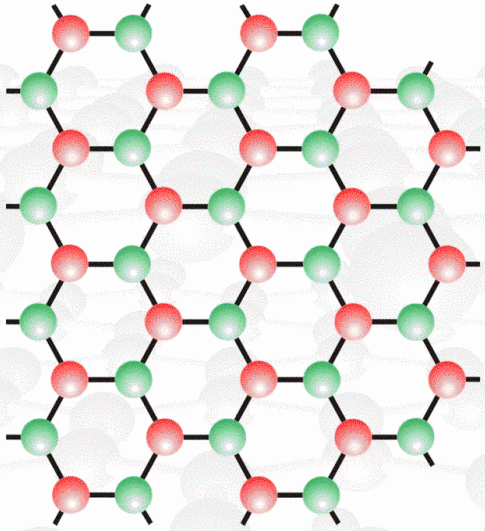
superposition of their wavefunctions



spinors  
(2 projections of pseudospin)



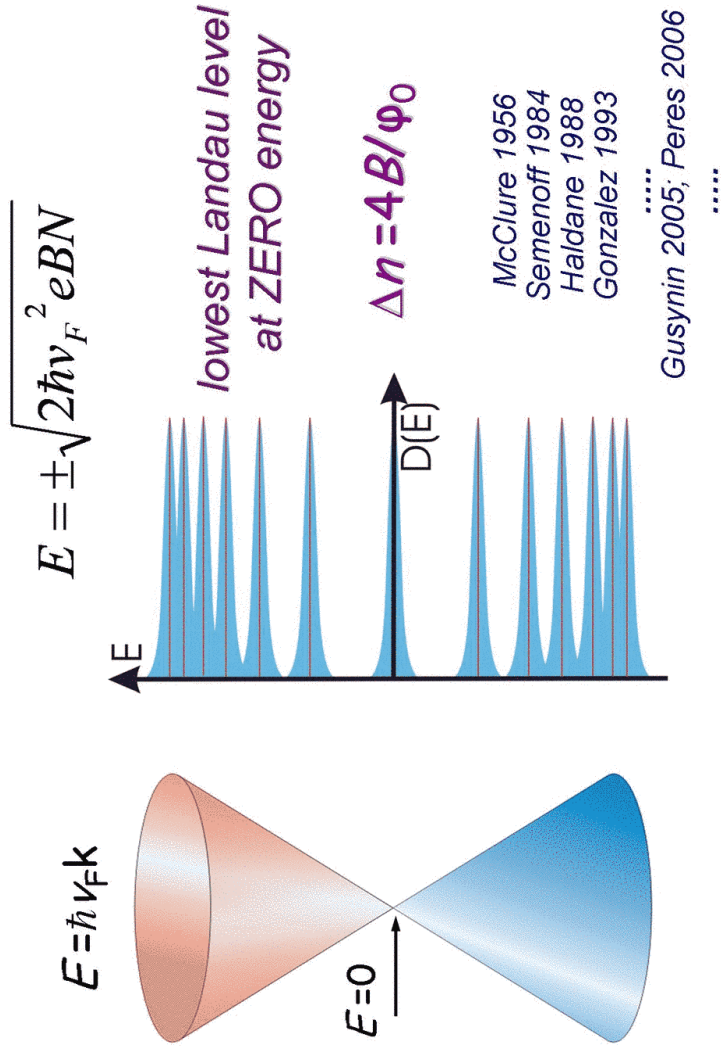
## Chiral Fermions in Graphene

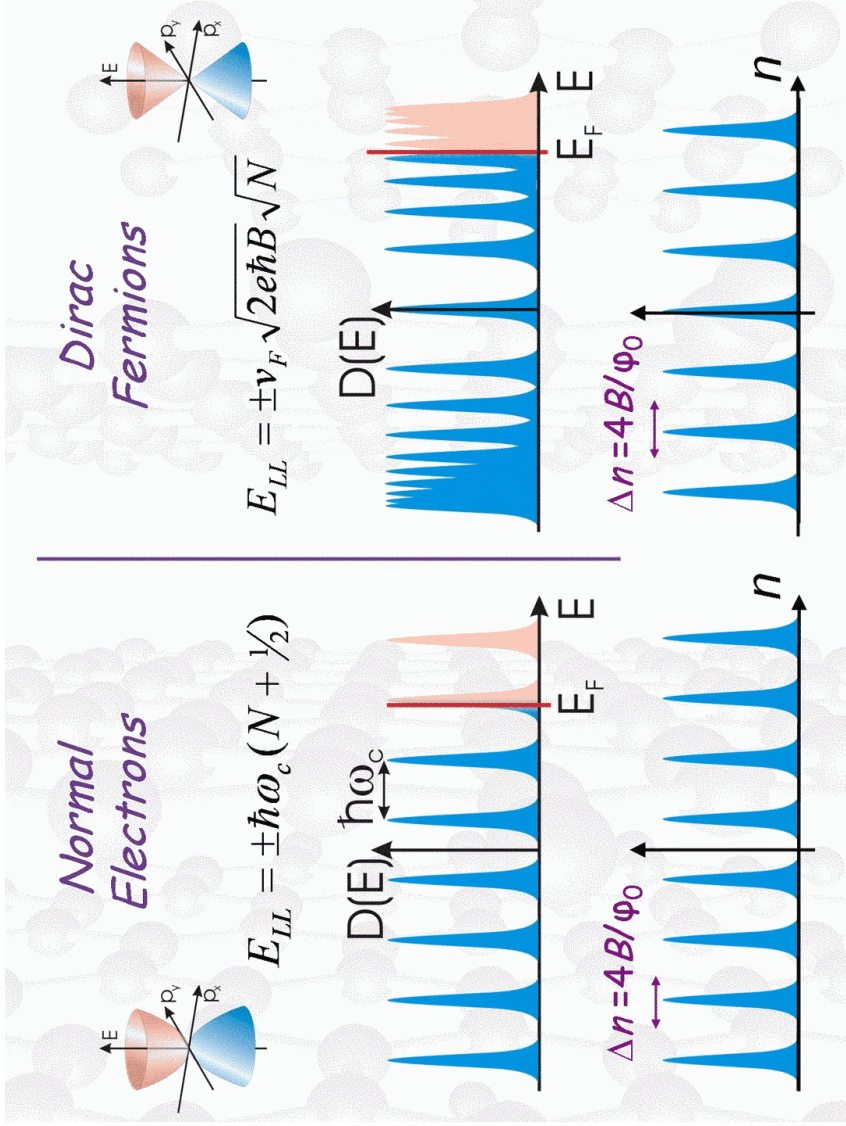


Dirac equation:

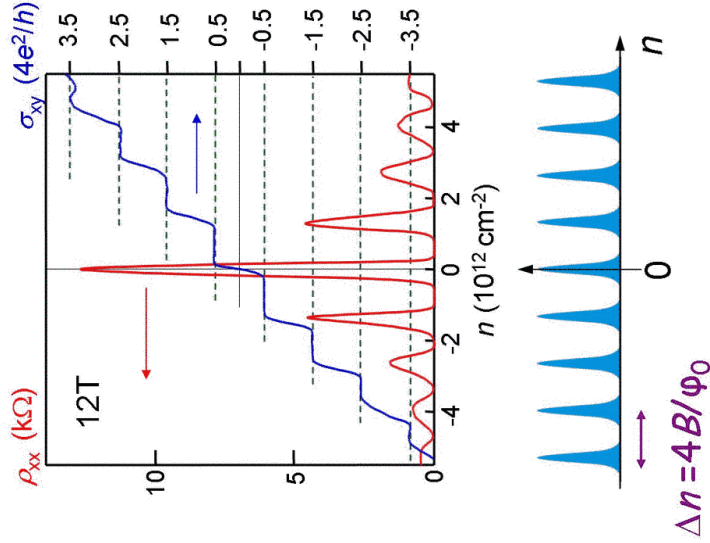
$$\hat{H} = v_F \begin{pmatrix} 0 & \hat{p}_x - i\hat{p}_y \\ \hat{p}_x + i\hat{p}_y & 0 \end{pmatrix} = v_F \vec{\sigma} \cdot \vec{p}$$

## quantization of Dirac fermions





## half-integer quantum Hall effect



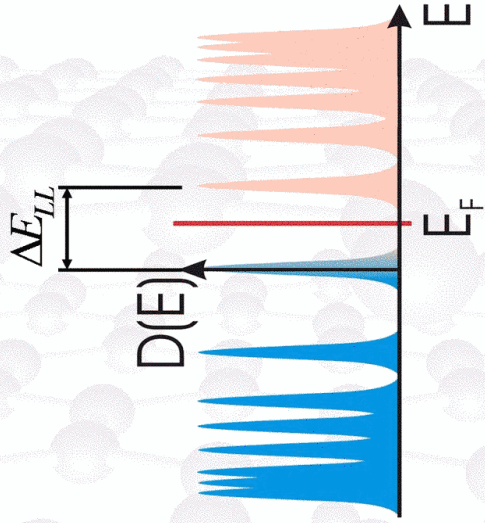
relativistic analogue of the integer QHE

*Nature* **438**, 197 (2005)

also Zhang *et al*, *ibid* 201 (2005)

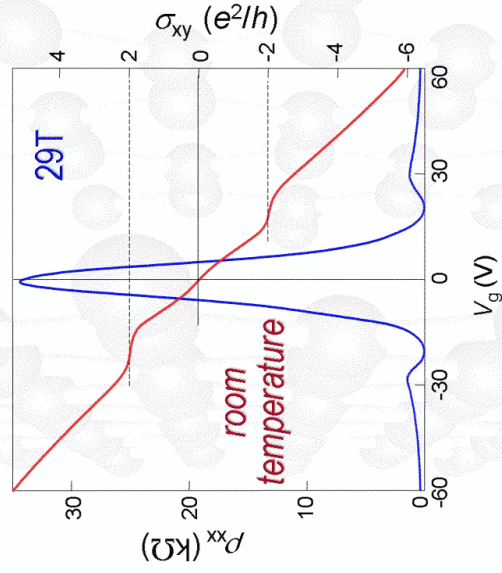


# room-temperature QHE



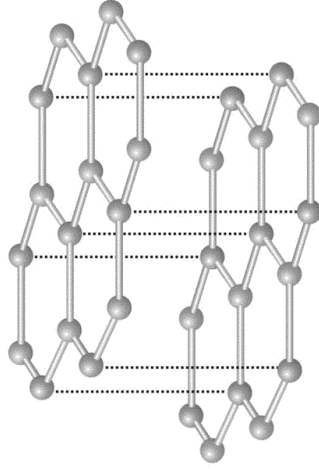
$$\Delta E_{LL} = v_F \sqrt{2e\hbar B}$$

$$\Delta E_{LL} (K) = 420 \sqrt{B(T)}$$

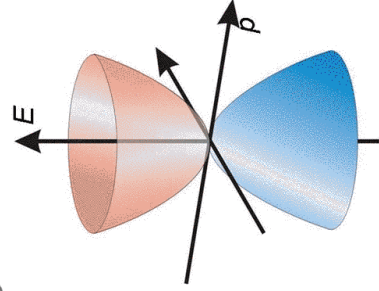


previously,  
only below 30K

# BILAYER GRAPHENE

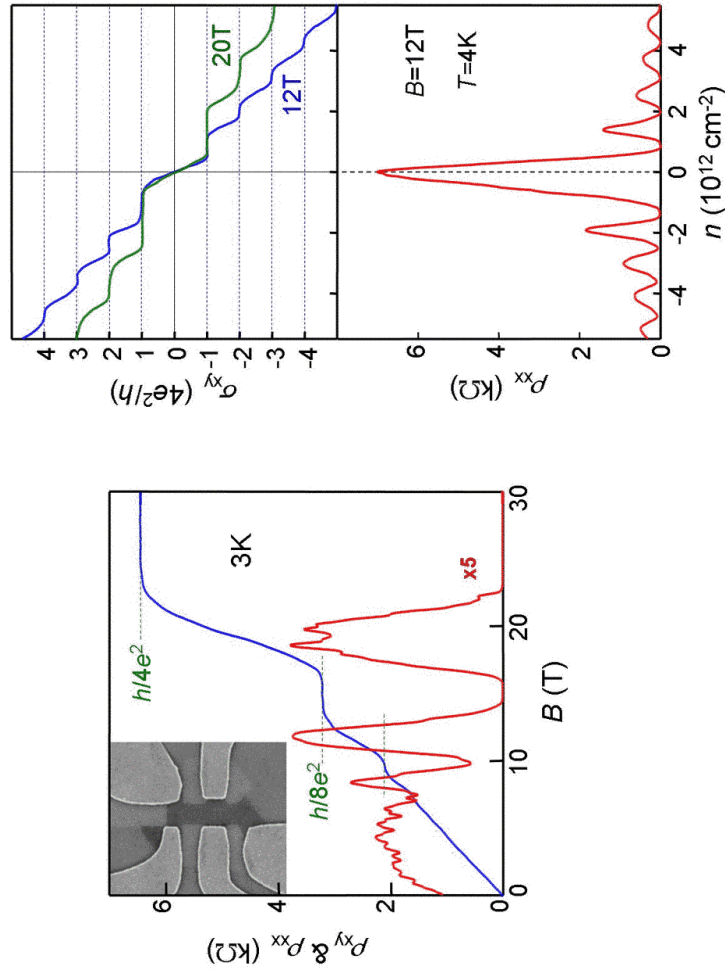


$$E(p) = \pm \frac{1}{2} \gamma_1 \pm \sqrt{\frac{1}{4} \gamma_1^2 + v_F^2 p^2}$$

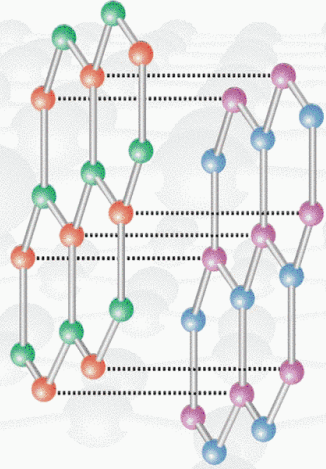


Nature Phys 2, 177 (2006)

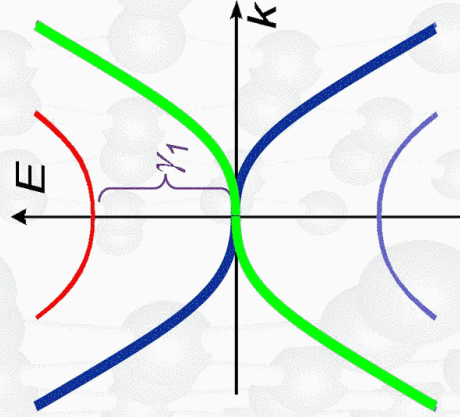
# QHE in bilayer graphene



# massive chiral fermions



$$E(p) = \pm \frac{1}{2} \gamma_1 \pm \sqrt{\frac{1}{4} \gamma_1^2 + v_F^2 p^2}$$



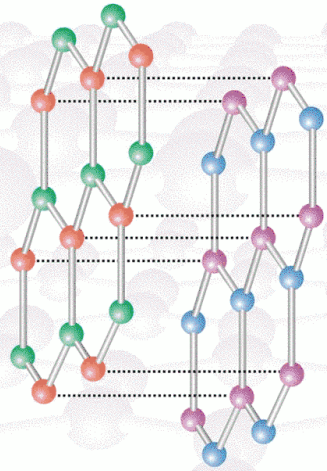
$$\hat{F} = -\frac{1}{2m} \begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix}$$

$$E_N = \pm \hbar \omega_c \sqrt{N(N-1)}$$

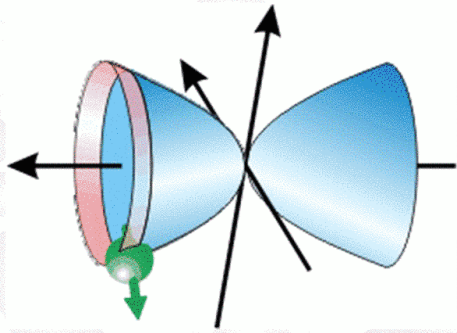
McCann & Falcko 2006



# massive chiral fermions



$$E(\mathbf{p}) = \pm \frac{1}{2} \gamma_1 \pm \sqrt{\frac{1}{4} \gamma_1^2 + v_F^2 \mathbf{p}^2}$$

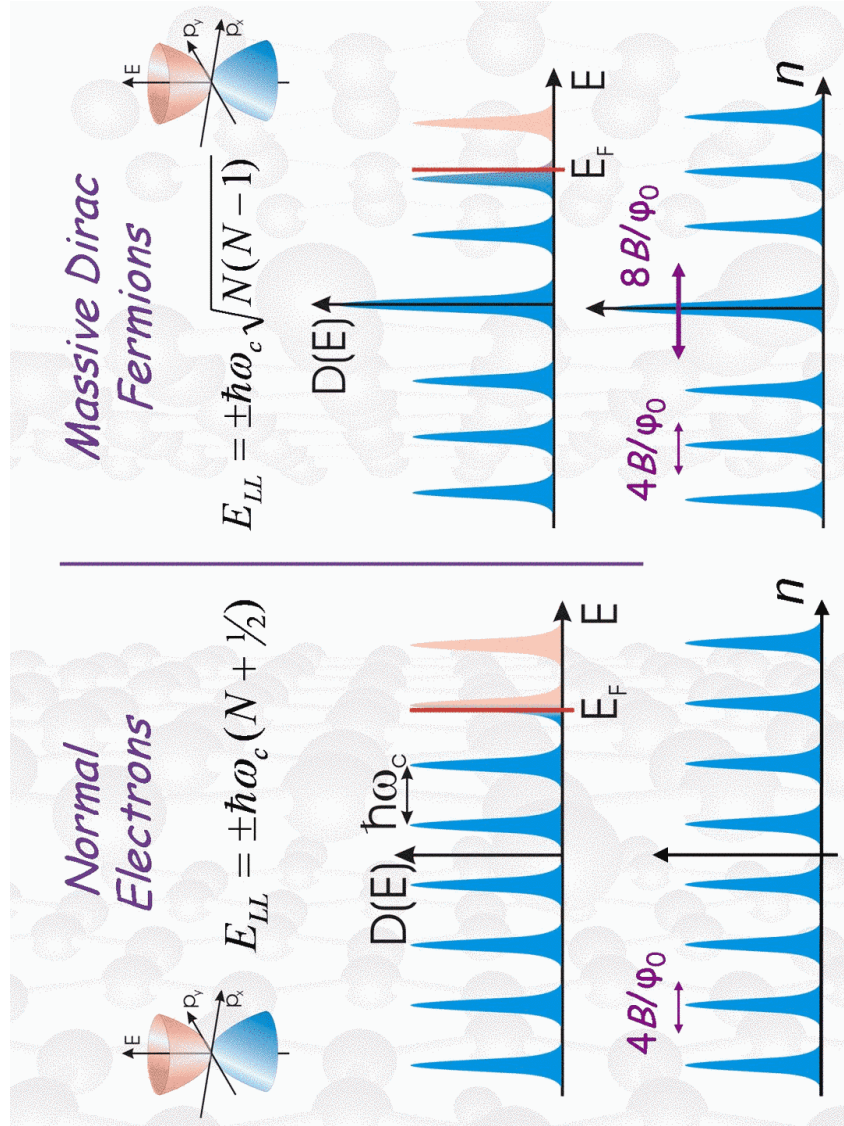


topological phase  
of  $2\pi$

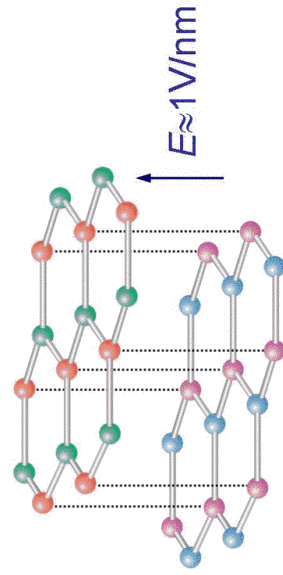
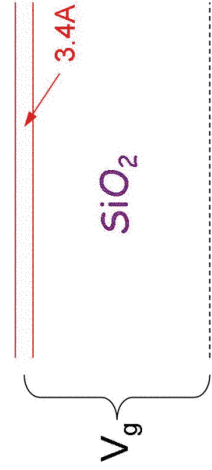
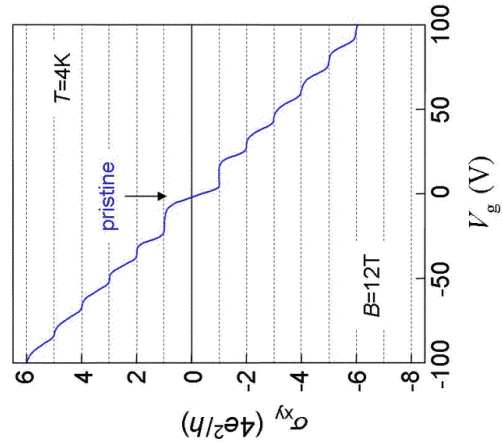
$$\hat{H} = -\frac{1}{2m} \begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix}$$

$$E_N = \pm \hbar \omega_c \sqrt{N(N-1)}$$

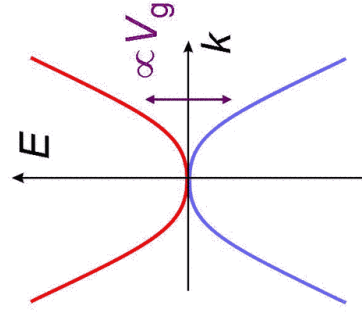
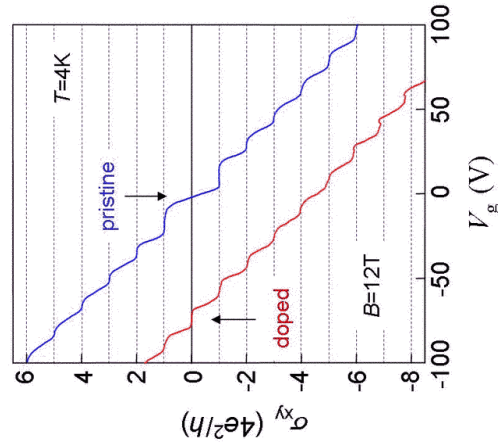
McCann & Falcko, PRL 2006



## Tunable-Gap Semiconductor



## Tunable-Gap Semiconductor

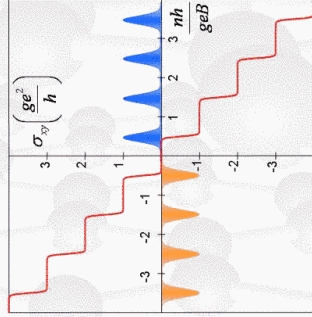


semiconductor with  
electrically controlled gap  
up to 0.3 eV



# three types of IQHE

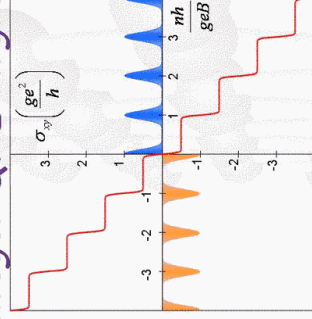
*conventional IQHE*



all LL at non-zero E  
zero Berry phase

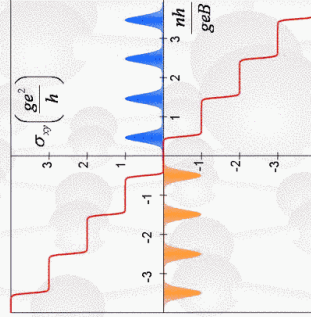
# three types of IQHE

*half-integer QHE in graphene*



Dirac fermions  
one LL at zero E  
Berry phase  $\pi$   
metallic at  $\nu=0$

*conventional IQHE*

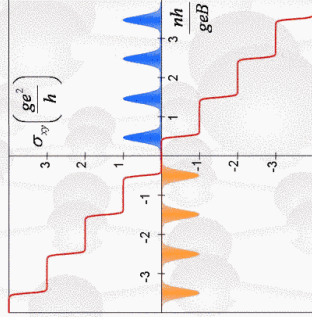


all LL at non-zero E  
zero Berry phase

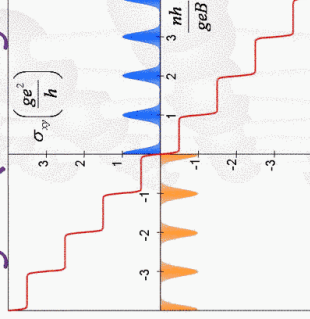
# three types of IQHE

*half-integer QHE in graphene*

*conventional IQHE*

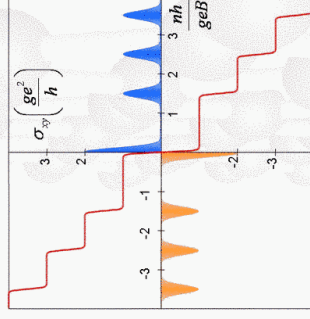


all LL at non-zero E  
zero Berry phase



**Dirac fermions**  
one LL at zero E  
Berry phase  $\pi$   
metallic at  $\nu = 0$

*chiral IQHE in bilayer graphene*



**massive Dirac fermions**  
two LLs at zero E  
Berry phase  $2\pi$   
metallic at  $\nu = 0$

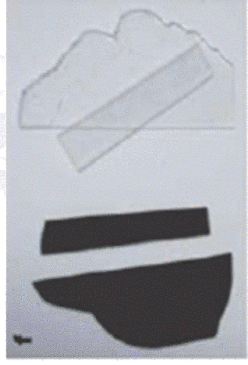
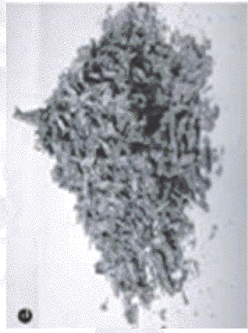
# POTENTIAL APPLICATIONS

*just think of carbon nanotubes  
but ... in large wafers  
with fully reproducible properties*

*unless specifically 1D phenomena*



**composite materials**



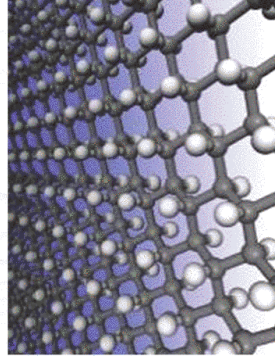
*graphene-based composite*  
Ruoff 2006

**field-emitters**



*very thin graphite flakes*  
(already used; PFE Ltd)

**hydrogen storage**



"GRAPHANE"

**electrical batteries**

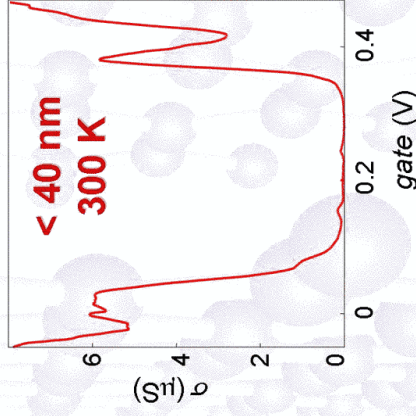
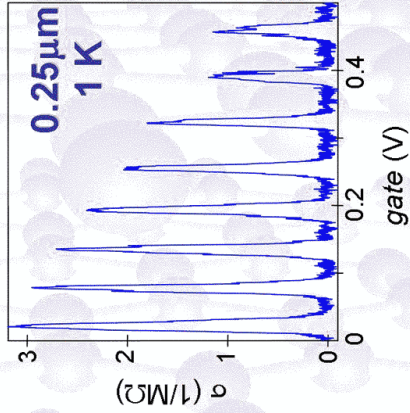
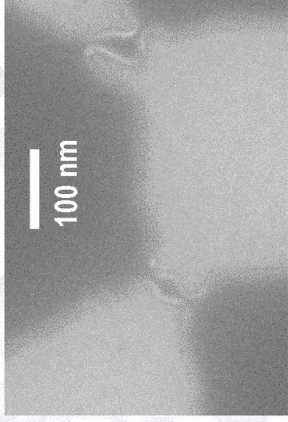
**GRAPHENE-BASED ELECTRONICS**

- ✓ *ballistic field-effect transistor*
  - ⊕ THz-frequency operation  
(despite low on-off ratio ~10 at room T)
- ✓ *chemical sensors*  
(detection of a single gas molecule!)
- ✓ *superconducting FETs*  
(Delft group)
- ⊕ *spintronics with gate control*

## GRAPHENE DREAMS

*entire SET-like circuitry  
carved in graphene*

stable down to a single benzene ring  
molecular electronics  
but with the top-down approach

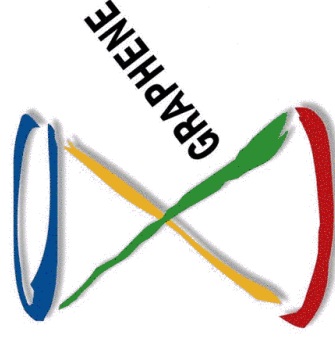


## CONCLUSIONS

**strictly-2D crystals do exist**

“relativistic”  
condensed matter  
physics

(addressing QED phenomena such as  
zitterbewegung, Klein paradox, etc)



**REALISTIC POSSIBILITY  
OF VARIOUS APPLICATIONS**