GRAPHENE: SYMMETRIES, TRANSITIONS
HALL EFFECT

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- IH, PRL 97, 146401 '06
- IH, COND-MAT/0610349

ISSUES:

- $U \sim 10\,eV$: WHY NOT STRONGLY CORRELATED?
- WHAT IF IT WERE? (GROSS-NEVEU CRIT. POINT)
- WEAK INTERACTION & $B \neq 0$: $\nu = 0$, EVEN, 1

LINEARIZE:

$$H_0 = \int_{\vec{k}=\vec{K}+\vec{p}} d^2\vec{k}\, (U^+(\vec{p}), V^+(\vec{p}))\left(v_F p_x \sigma_x - v_F p_y \sigma_y \right)\begin{pmatrix} U(\vec{p}) \\ V(\vec{p}) \end{pmatrix}$$

$$+ \int_{\vec{k}=-\vec{K}+\vec{p}} d^2\vec{k}\, (U^+(\vec{p}), V^+(\vec{p}))\left(- v_F p_x \sigma_x - v_F p_y \sigma_y \right)\begin{pmatrix} U(\vec{p}) \\ V(\vec{p}) \end{pmatrix}$$

OR, IN ACTION: $S = \int_0^{1/k_B T} d\tau\, d^2\vec{x}\, L_0$

$$L_0 = \sum_{z=\pm 1} \Psi_z^+(\vec{x},\tau)\left(\frac{\partial}{\partial \tau} + M_1 \frac{\partial}{\partial x} + M_2 \frac{\partial}{\partial y} \right) \Psi_z(\vec{x},\tau)$$

WHERE

$$\Psi_z(\vec{x},\tau) = T \sum_{\omega_n} \int_0^{\Lambda} \frac{d\vec{q}}{(2\pi a)^2} e^{i\omega_n \tau + i\vec{q}\cdot\vec{x}} \begin{pmatrix} U_z(\vec{K}+\vec{q}, \omega_n) \\ V_z(\vec{K}+\vec{q}, \omega_n) \\ U_z(-\vec{K}+\vec{q}, \omega_n) \\ V_z(-\vec{K}+\vec{q}, \omega_n) \end{pmatrix}$$

$\Lambda \approx \frac{1}{a}$, $v_F = t a \frac{\sqrt{3}}{2} (\to 1)$

MATRICES:

$$M_1 = -i \begin{pmatrix} \sigma_x & \\ & -\sigma_x \end{pmatrix}, \quad M_2 = i \begin{pmatrix} \sigma_y & \\ & \sigma_y \end{pmatrix} \quad ; \{M_1, M_2\} = 0$$

FIND <u>SOME</u> γ_0 : $\gamma_0^2 = 1$, $\{M_i, \gamma_0\} = 0$

SO THAT:

$$L_0 = \sum_{z=\pm} \Psi_z^+ \gamma_0 \left(\gamma_0 \partial_\tau + \gamma_1 \partial_x + \gamma_2 \partial_y \right) \Psi_z$$

$$\gamma_1 = \gamma_0 M_1$$
$$\gamma_2 = \gamma_0 M_2$$

FREEDOM OF CHOICE (IN γ_0):

$$\gamma_0 = a\, I_2 \otimes \sigma_z + b\, \sigma_x \otimes \sigma_x + c\, \sigma_y \otimes \sigma_x ,$$

OR

$$\gamma_0 = \sigma_z \otimes \sigma_z ,$$

$a^2 + b^2 + c^2 = 1$.

"CHIRAL" SYMMETRY : (ROTATIONS OF (a, b, c))

- FOR EACH SPIN : $SU(2) = \{\gamma_3, \gamma_5, ; \gamma_3\gamma_5\}$

EXAMPLE: $\gamma_0 = \begin{pmatrix} \sigma_z & \\ & \sigma_z \end{pmatrix}$, $\gamma_3 = \begin{pmatrix} & \sigma_y \\ \sigma_y & \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} & -i\sigma_y \\ i\sigma_y & \end{pmatrix}$

- SPIN - 1/2 : $U(4)$, 16 GENER.

$$\underbrace{\{I_2, \vec{\sigma}\}}_{2\times 2} \otimes \underbrace{\{I_4, \gamma_3, \gamma_5, \gamma_{35}\}}_{4\times 4}$$

EXACT : $\vec{\sigma} \otimes I_4$; SPIN ROTAT. $SU(2)$

$I_2 \otimes I_4$; $U(1)$, NUMBER CONS.

THE REST "EMERGING" !

BROKEN BY :

$$L \sim m\, \psi_\sigma^+ \gamma_0 \psi_\sigma$$

(- γ_3, γ_5 BROKEN
- γ_{35} PRESERVED)

- $a = 1$ ⇒ CDW
- $a = 0$ ⇒ "KEKULE" (HOU, CHAMON, MUDRY, COND-MAT/0609740)
- $a = 3$, AF

INTERACTION :

$$H_1 = \frac{U}{2} \sum_{\vec{x}, \sigma, \sigma'} n_\sigma(\vec{x})\, n_{\sigma'}(\vec{x}) \quad ; \quad \vec{X} = \vec{A},\ \vec{B} = \vec{A} + \vec{b}$$

DECOMPOSITION : n - density
m - magnetization

$$H_1 = \frac{U}{8} \sum_{\vec{A}} \Big\{ \big(\overset{\text{density}}{n(\vec{A}) + n(\vec{A}+\vec{b})}\big)^2 + \big(\overset{\text{CDW}}{n(\vec{A}) - n(\vec{A}+\vec{b})}\big)^2 - \big(\overset{\text{ferro}}{m(\vec{A}) + m(\vec{A}+\vec{b})}\big)^2 - \big(\underset{\text{AF}}{m(\vec{A}) - m(\vec{A}+\vec{b})}\big)^2 \Big\}$$

NEAR $\pm\vec{K}$:

$$v_\sigma^+(\vec{x},\tau)\, v_\sigma(\vec{x},\tau) \approx \frac{1}{2} \bar{\Psi}_\sigma(\vec{x},\tau) \left(I_2 \otimes e^{i2\vec{K}\cdot\vec{x}\,\sigma_z} \sigma_x\right) \otimes \left(\sigma_z \underset{r=v}{\overset{r=u}{\pm}} I_2\right) \Psi_\sigma$$

WITH INTERACTION: $L = L_0 + \sum_{x=d,c,f,a} L_x$

$$L_x = g_x\Big(\sum_z W_{x,z}\,\bar{\Psi}_z M_x \Psi_z\Big)^2 + \tilde{g}_x \sum_{\mu=3,5}\Big(\sum_{z=\pm 1} W_{x,z}\,\bar{\Psi}_z M_x \gamma_1\gamma_\mu \Psi_z\Big)^2$$

$$\boxed{\begin{aligned} &W_{d,z} = W_{c,z} = 1 \\ &W_{f,z} = W_{a,z} = z \\ &M_d = M_f = \gamma_0 \\ &M_c = M_a = I \end{aligned}}$$

("RELATIVISTIC MASS")

d - density

f - ferromagnetic

c - charge dens. wave

a - anti ferro mag.

$$g_c = (U - V)\frac{a^2}{8}$$

$$g_d = (U + V)\frac{a^2}{8}$$

$$g_f = -U\,\frac{a^2}{8}$$

$$g_a = -U\,\frac{a^2}{8}$$

V - NEXT NN REPULSION

$$\Big(\tilde{g}_x = -\frac{g_x}{2}\Big)$$

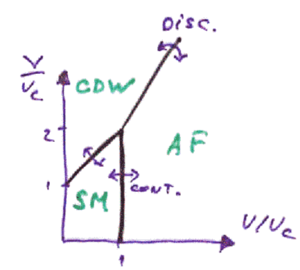
RG FOR LARGE N (HARTREE): $\Lambda \to \frac{\Lambda}{b}$

$$\frac{d g_x}{d\ln b} = -g_x - C_x g_x^2$$

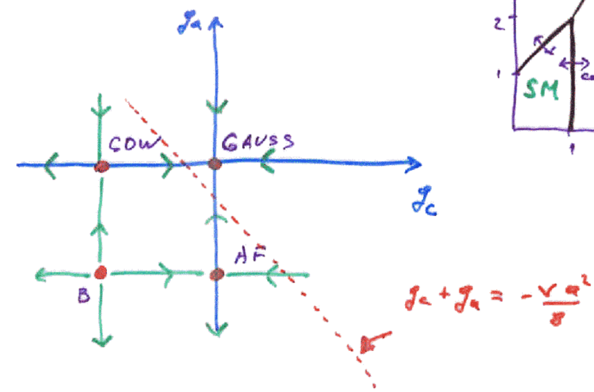
$$\Big(\frac{d\tilde{g}_x}{d\ln b} = -\tilde{g}_x + 2\tilde{g}_x^2\Big)$$

$$\boxed{\begin{aligned} &C_{d,f} = 0 \\ &C_{c,a} = 4 \end{aligned}}$$

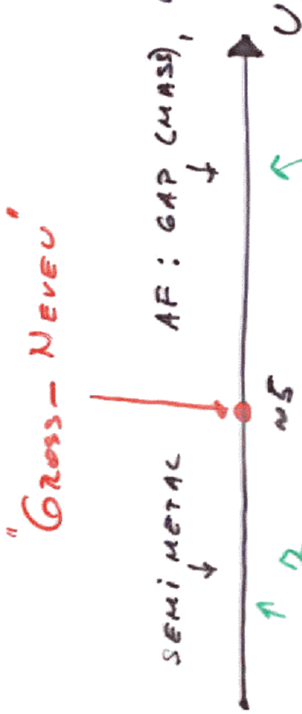
$$\Big(\frac{2}{N} g_x \frac{\Lambda}{\pi} \to g_x\Big)$$



FLOW:



(IH, PRL '06)



$\langle \bar{\psi}\psi \rangle \sim \frac{Z}{i\not{\partial}}$

BROKEN GEN:

$(I_2, \tau_z) \otimes (\gamma_3, \gamma_5)$

$(\tau_x, \tau_y) \otimes (I_4, \gamma_{35})$

EXACT

2 MAGNONS!

$Z \sim (U_c - U)^{\eta_\psi}$

$$\eta_\psi = \frac{2}{3\pi^2 N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

HIGGS-LIKE PHENOMENON!

(AT STRONG COUPLING)

MAGNETIC FIELD: QHE

- $B \approx 10\,T \Rightarrow \frac{l_B}{a} = \sqrt{\frac{B_0}{B}} \approx 100$,

$\left(B_0 = \frac{1}{a^2} \approx 10^5\,T\right)$

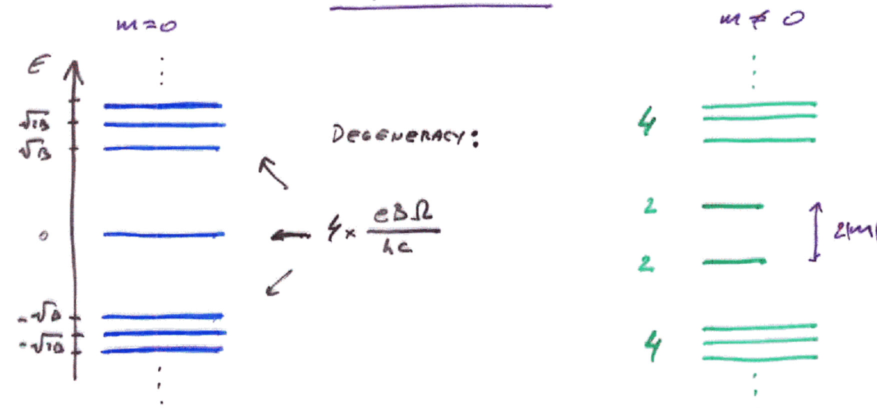
SO LAB. FIELDS ARE WEAK!

CONTINUUM (LOW-ENERGY) APPROX:

(NON-INTERACTING, NO ZEEMAN FIRST)

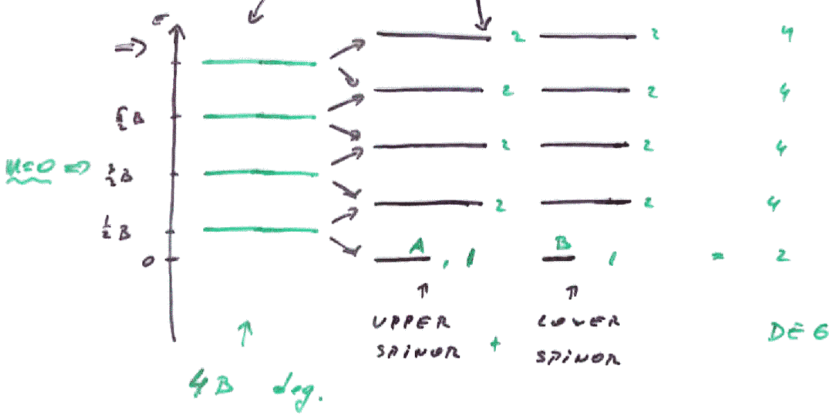
$$H_0 \to M_1(-i\partial_x - A_x) + M_2(-i\partial_y - A_y) + m\,\gamma_0$$

SPECTRUM



WHY? TAKE ONE SPIN STATE:

$$H_0^2 = -(\nabla - i\vec{A})^2 + B(\sigma_z \otimes \sigma_z) + m^2 \rightarrow nB + m^2$$

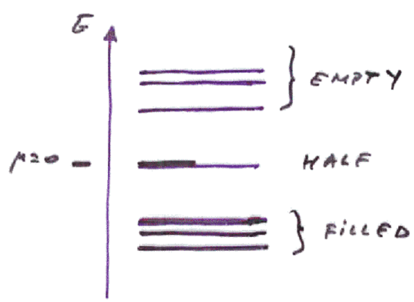


$H_0 \rightarrow \pm\sqrt{nB + m^2}$

a) $n \neq 0$ 2 = deg

b) $n = 0$: $+m \Rightarrow$ deg = 1
$-m \Rightarrow$ deg = 1

SO FOR $m = 0$ (GAPLESS), WITH SPIN:



$$\Rightarrow f = \frac{N/\Omega}{Be/hc} = 2, 6, 10, 14$$

$= 2 + 4n$, IQHE

IQHE AT $f = \pm 1 \Rightarrow$ INTERACTIONS

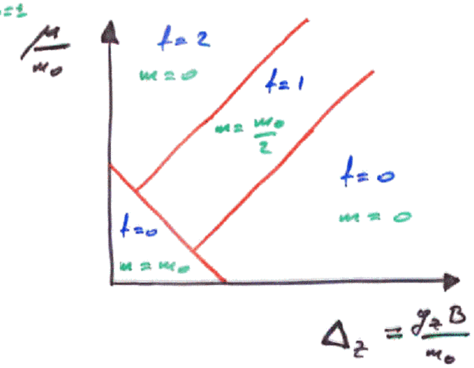
FOR $l_B \gg a$, FIRST RESCALE TO $\Lambda = \frac{1}{l_B}$,

$\Rightarrow$ SINGLE DOMINANT (LEAST IRRELEVANT) INTERACTION!

IF $-g_a > -g_c$ ($V = 0$):

$$L = i\sum_\sigma \bar{\Psi}_\sigma \gamma_\mu D_\mu \Psi_\sigma - \sum_\sigma (\mu + \sigma g_z B)\Psi_\sigma^\dagger \Psi_\sigma + g_a \sum_\sigma (\sigma \bar{\Psi}_\sigma \Psi_\sigma)^2$$

$$m = g_a \sum_{\sigma=\pm} \sigma \langle \bar{\Psi}_\sigma \Psi_\sigma \rangle$$



DEGENERACY

$$m_0 = g_a \left(\frac{2B_\perp}{\pi}\right)$$

Y. Zhang et al, PRL '06:

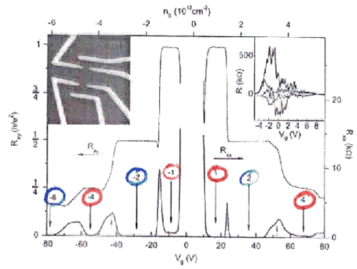


FIG. 1 (color online). R_{xx} and R_{xy} measured in the device shown in the left inset, as a function of V_g at $B = 45$ T and $T = 1.4$ K. $-R_{xy}$ is plotted for $V_g > 0$. The numbers with the vertical arrows indicate the corresponding filling factor ν. Gray arrows indicate developing QH states at $\nu = \pm 3$. n_s is the sheet carrier density derived from the geometrical consideration. Right inset: R_{xx} (dark solid lines) and R_{xy} (light solid lines) for another device measured at $B = 30$ T and $T = 1.4$ K in the region close to the Dirac point. Two sets of R_{xx} and R_{xy} are taken at different time under the same condition. Left inset: an optical microscope image of a graphene device used in this experiment.

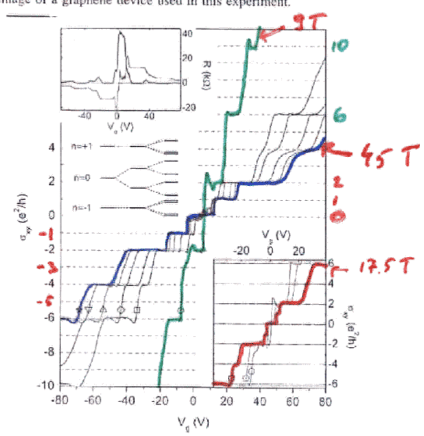


FIG. 2 (color online). σ_{xy}, as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{xy} for the same device measured at $B = 25$ T. Left lower inset: a sche-

Energy scales:

- LL separation $\sim t\sqrt{\frac{B_\perp}{B_0}} \sim 100\,\mathrm{meV}$
- Zeeman: $\sim g_z \left(\frac{B}{10^4 T}\right) eV \sim 1\,\mathrm{meV}$
- interaction $\omega_0 \sim U \frac{B_\perp}{B_0} \sim 1\,\mathrm{meV}$

At a large Zeeman term $(\sim\sqrt{B_\perp^2 + B_\parallel^2})$:

- $\Delta_{t=0} \sim B_\parallel$
- $\Delta_{t=1} \sim B_\perp$

SUMMARY:

1) $B=0$: METAL - INS, HIGGS-LIKE TRANSITION AT LARGE INTERACTION

2) $B\neq 0$: "CATALYSIS" OF THE "RELATIVISTIC" GAP $\Rightarrow$ IQHE AT $f=\pm 1$, f - EVEN

3) ZEEMAN + INTERACTION : FINE STRUCTURE!

FUTURE ISSUES ($U \neq 0$)

- $U/t \gtrsim 5$: - OTHER ORDERS, SPIN LIQUID?
  - FATE OF FERMIONS?
- TOPOLOGICAL DEFECTS AND ZERO MODES
- COULOMB:

$$\frac{de^2}{d\ln b} = -\frac{e^4}{8\pi}$$

LOG-CORRECTIONS?

- DOPING : SUPERCONDUCTIVITY, ... ?
- MASS VS. QH FERROMAGNETISM, $B\neq 0$?
- TUNING U/t IN GRAPHENE?