Fermi liquid theory of a Fermi ring

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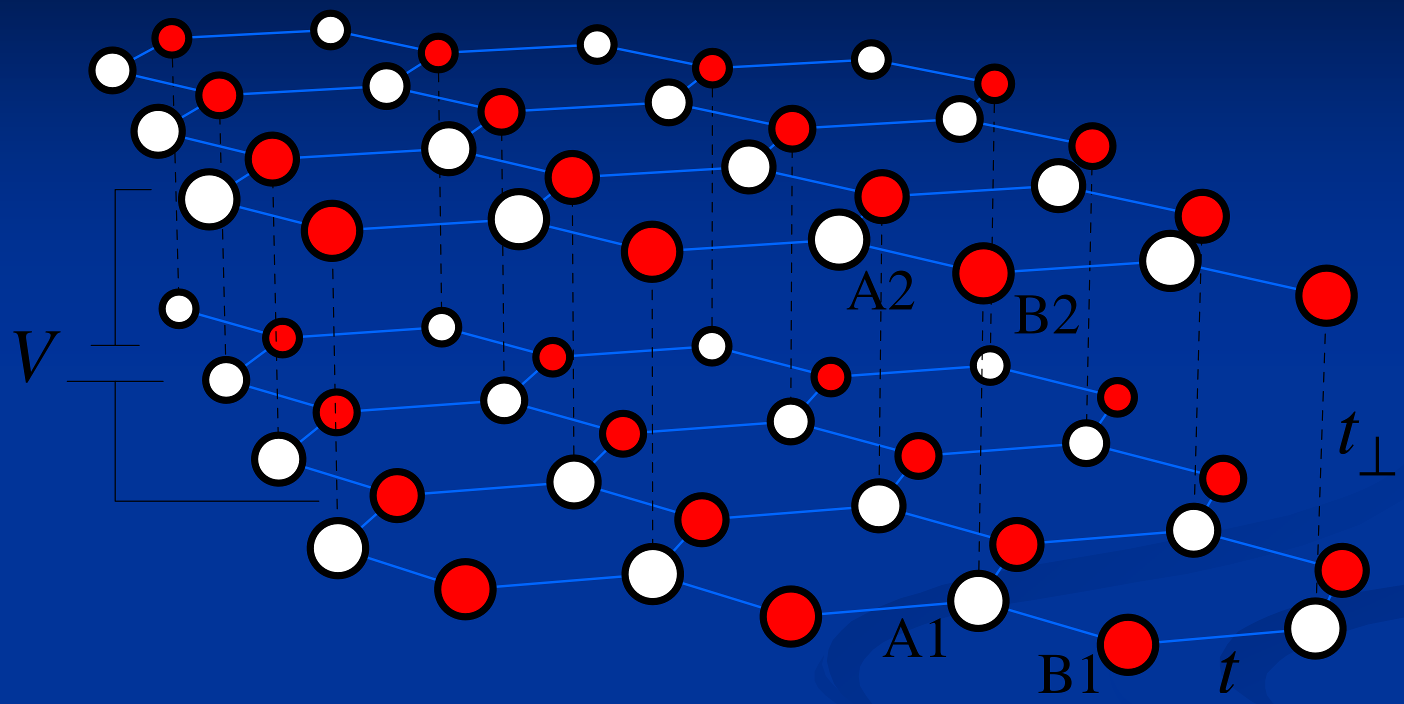
Overview

- Biased bilayer graphene
- One band approximation: Mexican hat dispersion
- Coulomb interaction: Self-energy correction
- Ferromagnetism: Ginzburg-Landau functional
- Polarizability

  A) High energies (Non-Fermi liquid behaviour)

  B) Low energies (Fermi liquid with modified Friedel oscillations)
- Plasmon dispersion

Biased bilayer graphene



$$V = t_\perp = 0.3eV$$

Controlling the Electronic Structure of Bilayer Graphene

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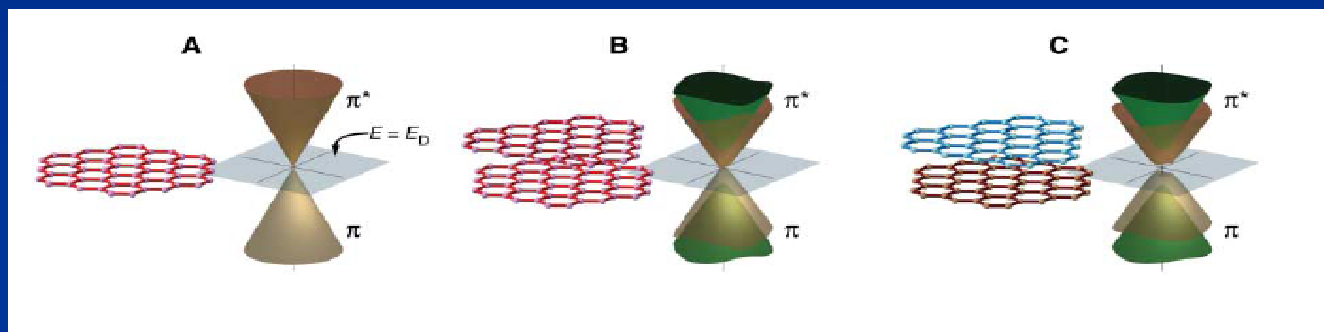
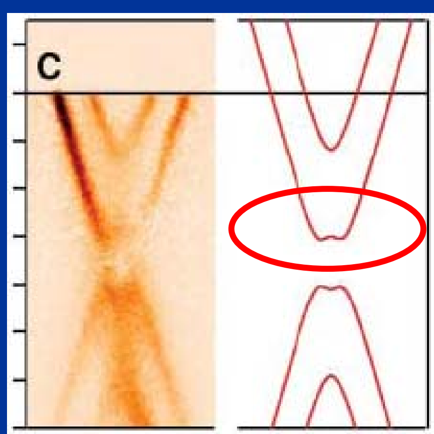
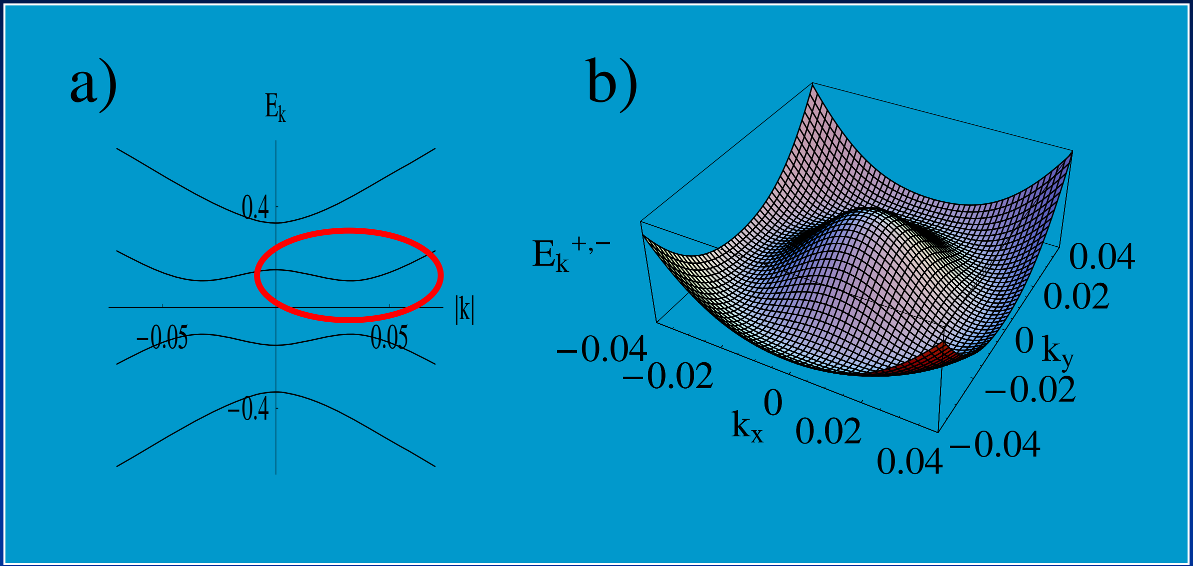
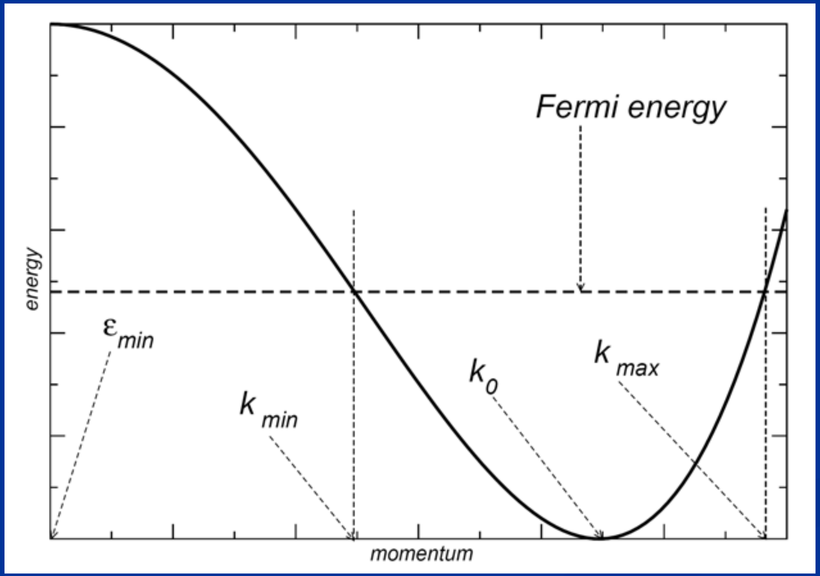




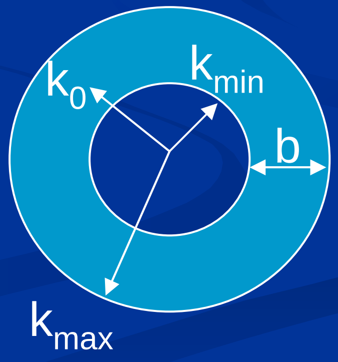
Mexican hat dispersion

Mexican hat dispersion

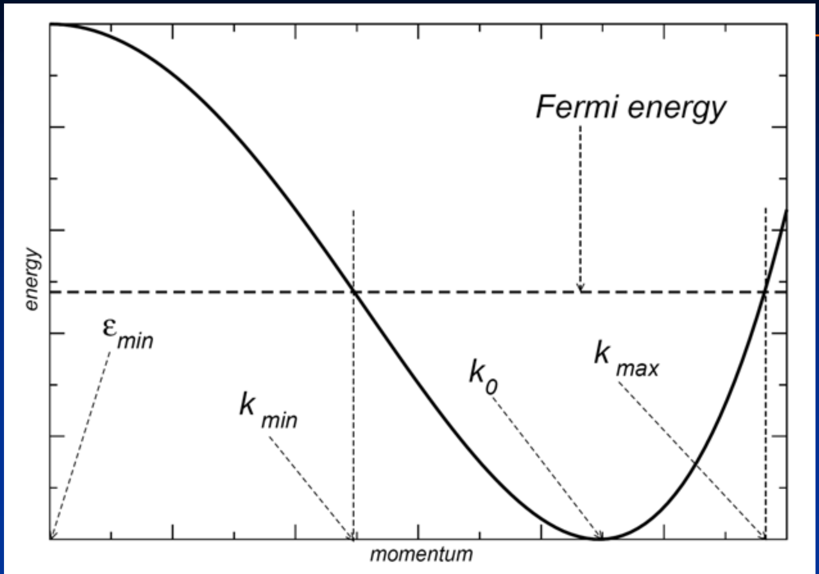




Fermi ring:



Mexican hat dispersion



$$E(\vec{k}) = \Delta - \alpha k^2 + \lambda k^4$$

$$k^2_{\min,\sigma} = \frac{\alpha}{2\lambda} - 2\pi n_\sigma$$

$$k^2_{\max,\sigma} = \frac{\alpha}{2\lambda} + 2\pi n_\sigma$$

$$\begin{aligned} \Delta &= \frac{V}{2}, \\ \alpha &= \frac{V}{t_\perp^2} v_F^2, \\ \lambda &= V\left(\frac{(t_\perp^2 + V^2)^2}{V^2 t_\perp^6} - \frac{1}{t_\perp^4}\right) v_F^4 \end{aligned}$$

Coulomb interaction

$$H_{\text{int}} = \frac{1}{2A} \sum_{\vec{k},\vec{p},\vec{q}} \sum_{\sigma,\sigma'} V(q) c^{+}_{\vec{k}+\vec{q},\sigma} c^{+}_{\vec{p}-\vec{q},\sigma'} c_{\vec{p},\sigma'} c_{\vec{k},\sigma}$$

$$V(q) = \frac{2\pi e^2}{\varepsilon_0 q}$$

$$a = \frac{k_{\min}}{k_{\max}}$$

a) Self-energy corrections to the dispersion

$$I_1(a,y) = \int_0^{2\pi} \int_a^1 \frac{x\,dx\,d\theta}{\sqrt{x^2 + y^2 \pm 2xy\cos\theta}}$$

b) Exchange energy

$$I_2(a) = \int_a^1 y\,dy\,I_1(a,y)$$

Self-energy corrections

$$\Delta_1 = \Delta - \frac{e^2}{\epsilon_0}(k_{\max} - k_{\min})\,,$$

$$\alpha_1 = \alpha + \frac{e^2}{4\epsilon_0}\left(\frac{k_{\max} - k_{\min}}{k_{\max}k_{\min}}\right)$$

$$\lambda_1 = \lambda - \frac{3e^2}{64\epsilon_0}\left(\frac{k_{\max}^3 - k_{\min}^3}{k_{\max}^3 k_{\min}^3}\right)$$

$n < n_c$

$$\Delta_1 = \Delta - \frac{e^2}{\epsilon_0}k_{\max}\,,$$

$$\alpha_1 = \alpha - \frac{e^2}{4\epsilon_0}k_{\max}^{-1}$$

$$\lambda_1 = \lambda + \frac{3e^2}{64\epsilon_0}k_{\max}^{-3}$$

$n > n_c$

$n_c = 1.5 \times 10^{12}$ cm^{-2} --> "exactly" filled Mexican hat

Density of states:

$$\rho(\varepsilon) = \frac{1}{4\pi\sqrt{\lambda}} \frac{1}{\sqrt{\varepsilon - \varepsilon_{\min}}}$$

Stoner criterion:

$$\varepsilon_F = \varepsilon_{\min} + (2\pi n_\sigma \sqrt{\lambda})^2$$

$$\rho(\varepsilon_F)U > 1$$

Ginzburg-Landau functional

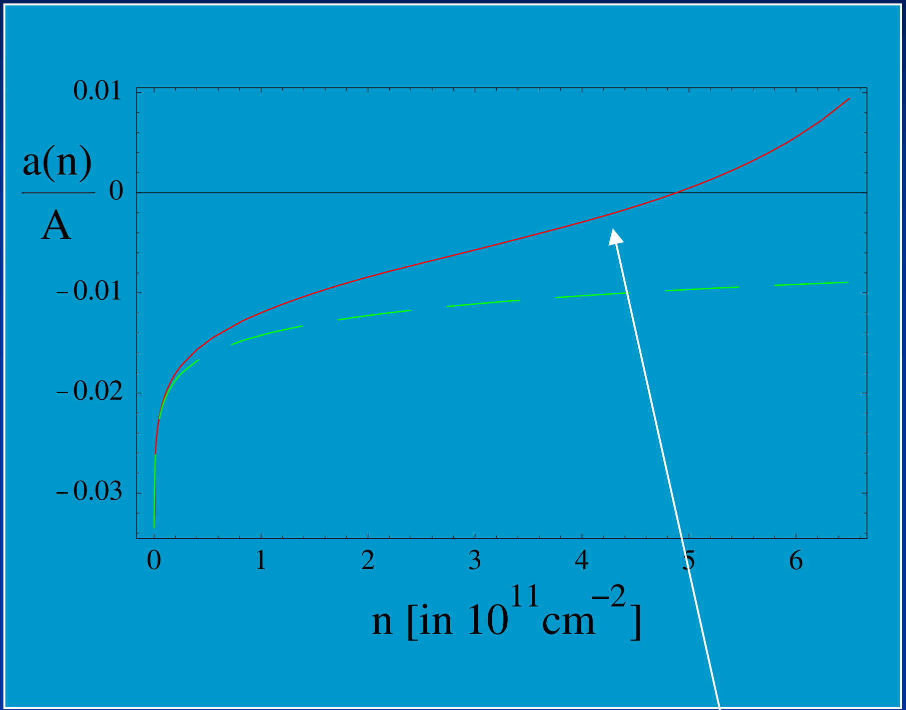
$$F[\alpha, \lambda, n, m] = \sum_{\sigma} [K_{\sigma}(n, m) + E_{\sigma}^{\mathrm{Exc}}(n, m) - K_{\sigma}(n, 0) - E_{\sigma}^{\mathrm{Exc}}(n, 0)]$$

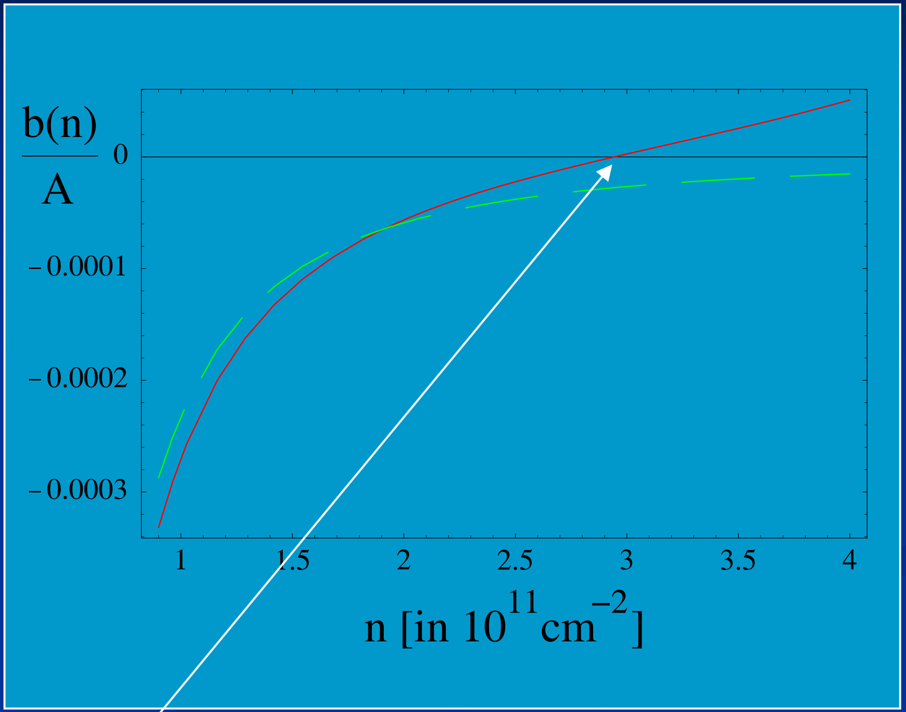
$$n = n_{+} + n_{-}$$

$$m = n_{+} - n_{-}$$

$$F = a(n)m^{2} + b(n)m^{4} + \ldots$$

Ginzburg-Landau functional

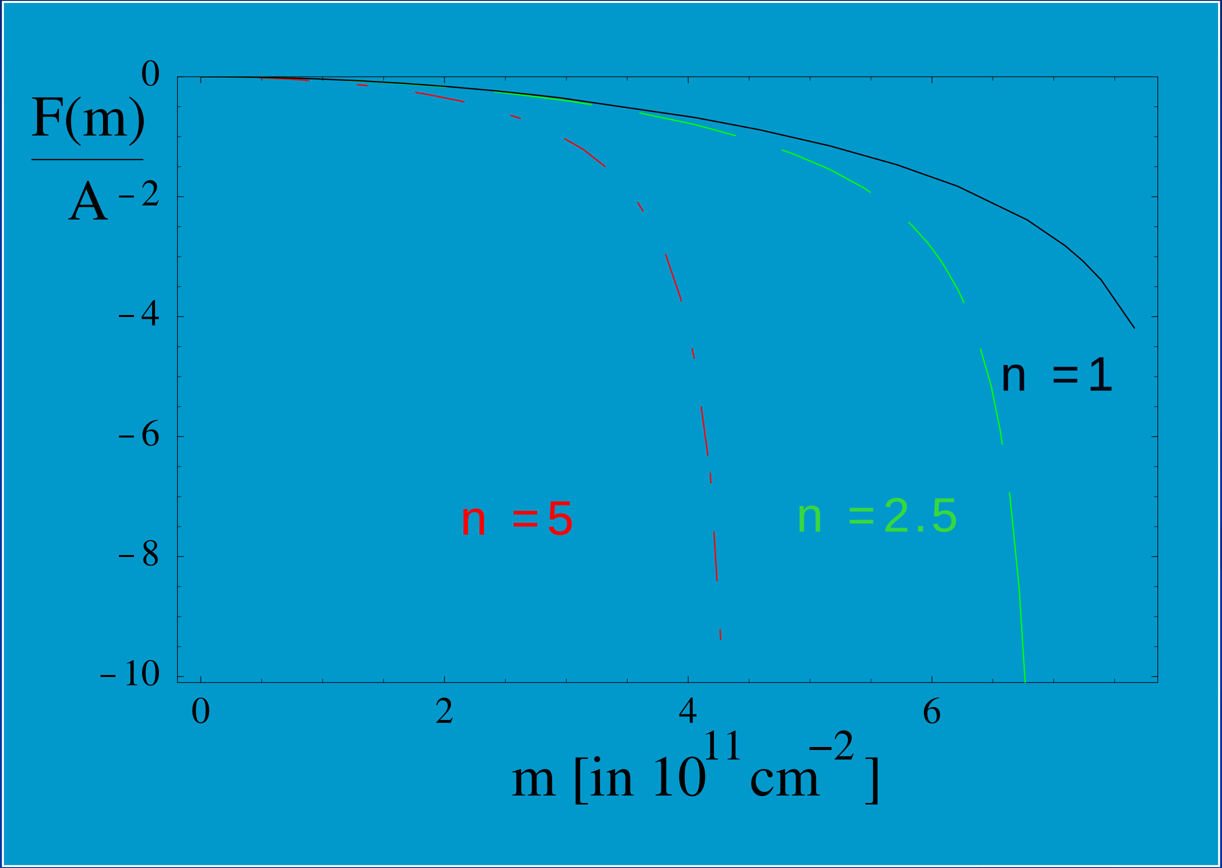




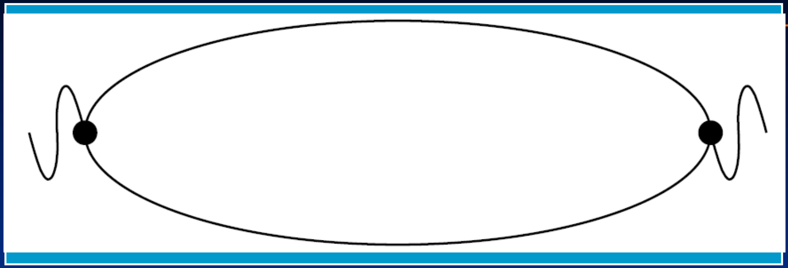
with self-energy correction

$$m = n$$

Ginzburg-Landau functional



Polarizability



$$P^{(1)}(\mathbf{q},\omega) = \frac{2}{(2\pi)^2}\int d^2k \frac{n_F(E(\boldsymbol{k})) - n_F(E(\boldsymbol{k}+\boldsymbol{q}))}{E(\boldsymbol{k}) - E(\boldsymbol{k}+\boldsymbol{q}) - \omega - i\delta},$$

$$\mathrm{Im}P^{(1),ret}(\mathbf{q},\omega) = \frac{1}{2\pi}\int d^2k n_F(E(\boldsymbol{k}))\left[\delta(\omega - (E(\boldsymbol{k}) - E(\boldsymbol{k}+\boldsymbol{q}))) - \delta(-\omega - (E(\boldsymbol{k}) - E(\boldsymbol{k}-\boldsymbol{q})))\right]$$

"Mexican hat dispersion"

$$E(\vec{k}) = (|\vec{k}| - k_0)^2$$

High energy regime ($b \to 0$)

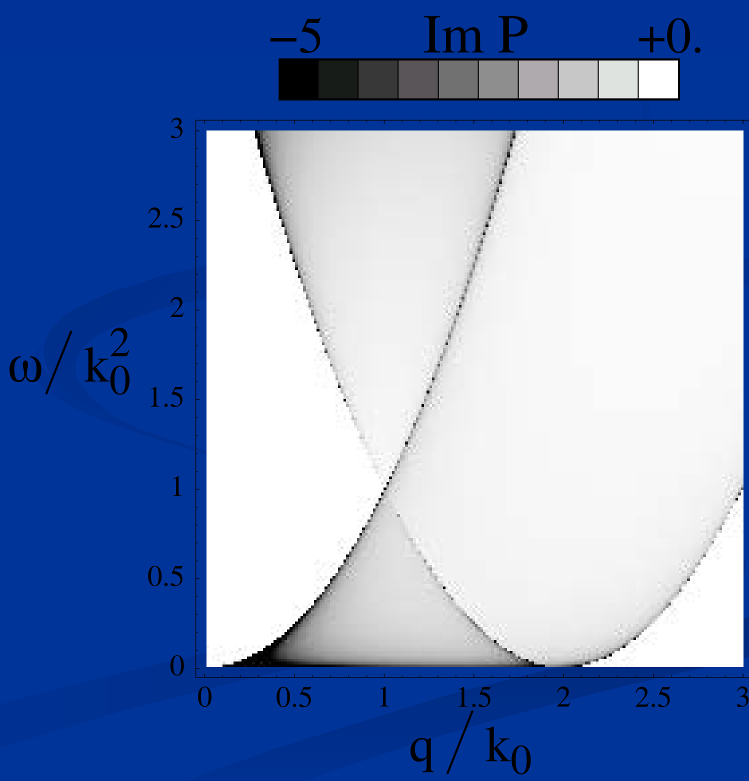
$$\mathrm{Im}P^{(1),\mathrm{ret}}(\mathbf{q},\omega) = -(2b)\frac{2}{2\pi}\sum_{\gamma=\pm}\frac{|1+\gamma\sqrt{\omega}|}{|1-|1+\gamma\sqrt{\omega}||}\mathrm{Re}\frac{1}{\sqrt{(1+q)^2-(1+\gamma\sqrt{\omega})^2}}\mathrm{Re}\frac{1}{\sqrt{-(1-q)^2+(1+\gamma\sqrt{\omega})^2}}\,.$$

Asymptotic behaviour:

$$\mathrm{Im}\,P(q,\omega) \to \frac{1}{\sqrt{\omega}}$$

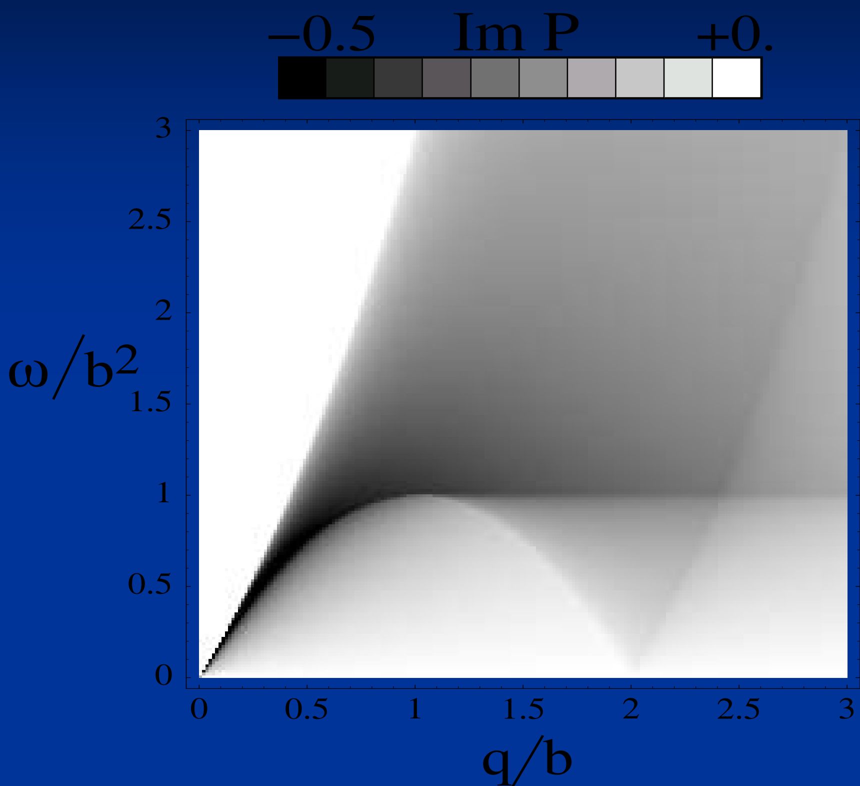
Quasiparticle lifetime:

$$\Gamma(\varepsilon) \propto \sqrt{|\,\varepsilon - \varepsilon_F\,|}$$



Low-energy regime ($\omega \ll b^2$)

A) Forward scattering

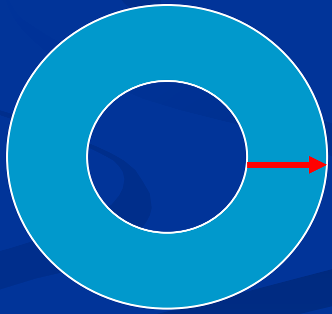


Asymptotic behaviour

$-4\pi q \mathrm{Im} P^{(1),\mathrm{ret}}(\mathbf{q},\omega))$:

$$P = \begin{cases} (\omega/b^2) & ;\ 0 < q < 2b \\ (\omega/b^2)^{1/2}/\sqrt{2} & ;\ q = 2b \\ (\omega/b^2)(1 + 1/\sqrt{1-(2b/q)^2}) & ;\ q > 2b \end{cases}$$

Perfect nesting at q=2b:

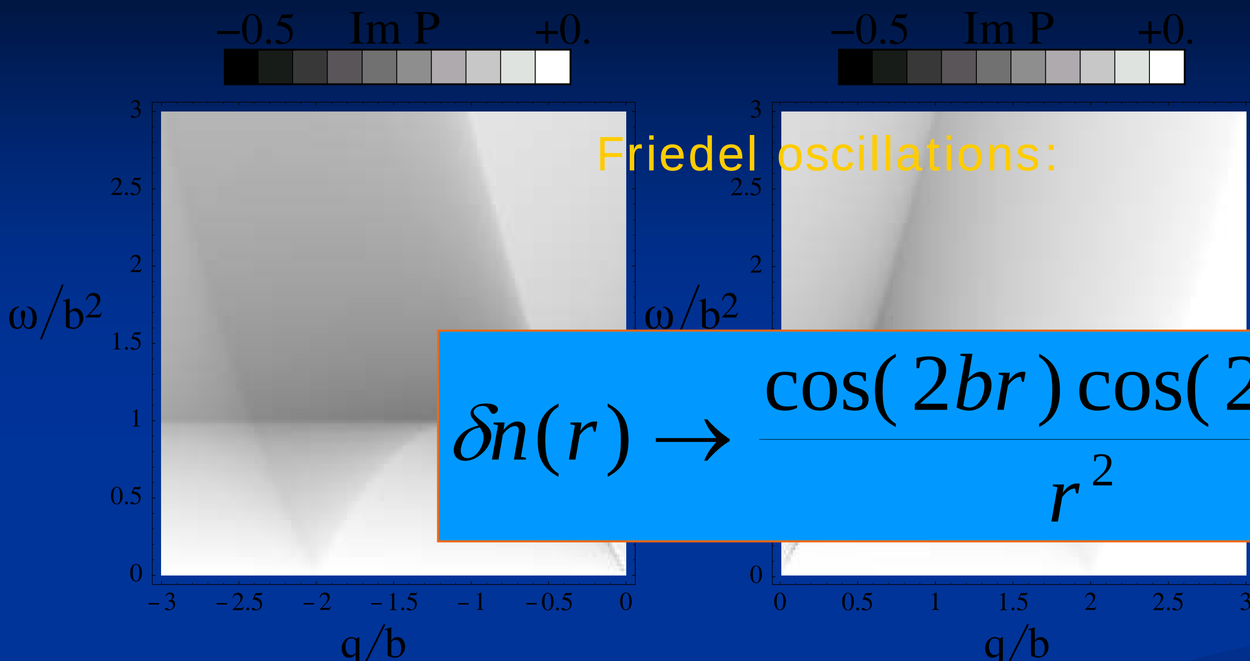


Friedel oscillations:

$$\delta n(r) \rightarrow \frac{\cos(2br)}{r^2}$$

B) Backward scattering

Low-energy regime ($\omega \ll b^2$)



Friedel oscillations:

$$\delta n(r) \rightarrow \frac{\cos(2br)\cos(2k_0 r)}{r^2}$$

$$q \rightarrow 2k_0 + q$$

Asymptotic behaviour:

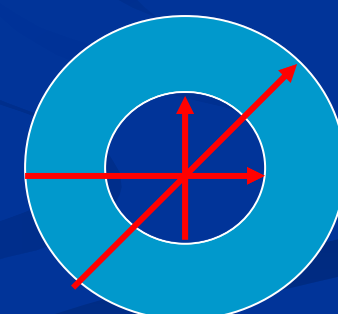
$$P^2 = \begin{cases} (\omega/b^2)/2\sqrt{|q|-2b} & ;\ q < -2b \\ (\omega/b^2)^{1/2}/\sqrt{2b} & ;\ q = -2b \\ (\omega/b^2)/2\sqrt{|q|} & ;\ -2b < q < 0 \\ (\omega/b^2)^{1/2}/\sqrt{2b} & ;\ q = 0 \\ (\omega/b^2)/2\sqrt{2b-q} & ;\ 0 < q < 2b \\ (\omega/b^2)^{1/2}/\sqrt{2b} & ;\ q = 2b \\ 0 & ;\ q > 2b \end{cases}$$

Perfect nesting:

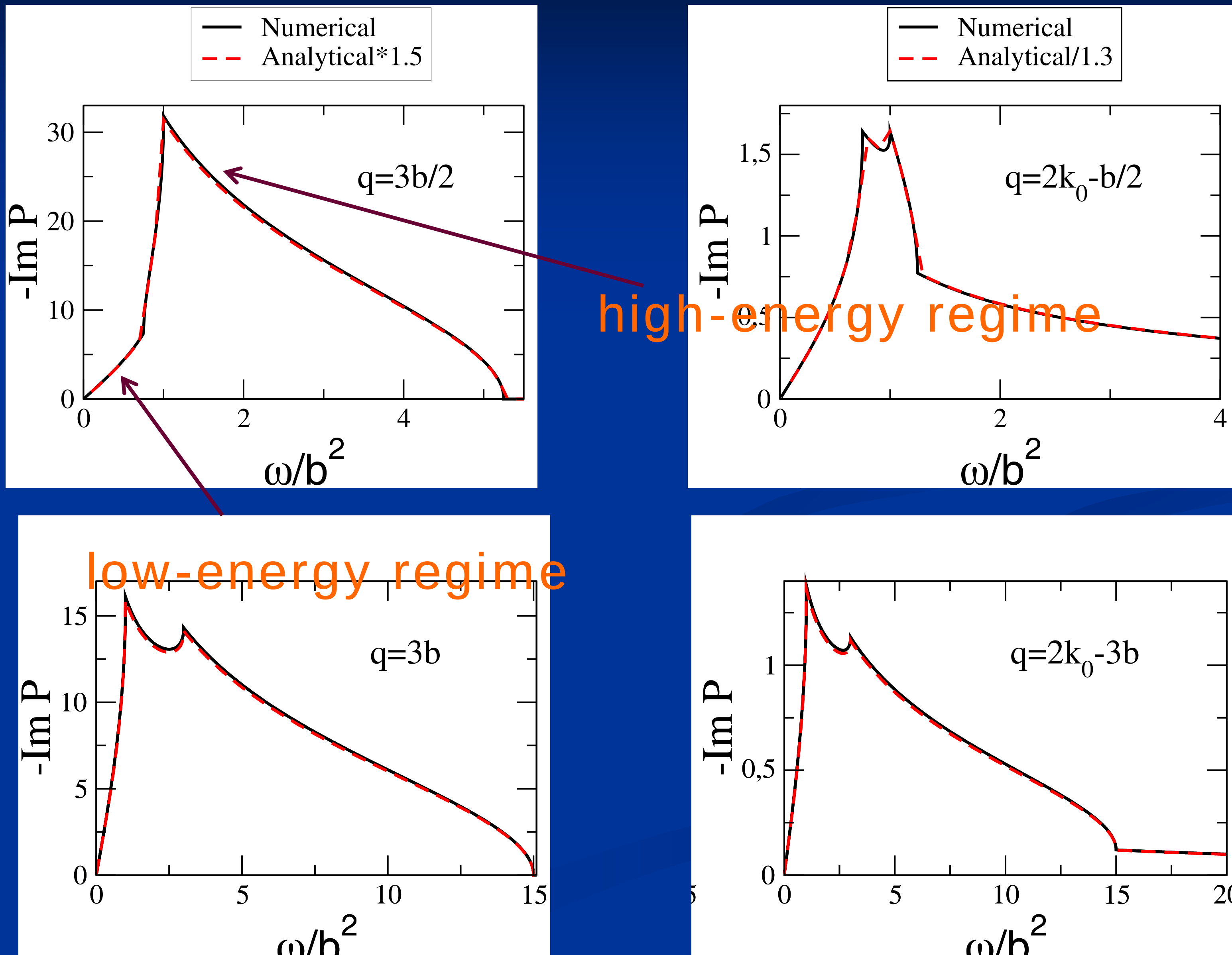
$q = 2k_0$

$q = 2k_0 - 2b$

$q = 2k_0 + 2b$



Example


Numerical
Analytical*1.5
q=3b/2
-Im P
ω/b2
Numerical
Analytical/1.3
q=2k0-b/2
high-energy regime
low-energy regime
q=3b
q=2k0-3b

Random-phase approximation:

$$\varepsilon(\vec{q},\omega) = 1 - V(q)P(\vec{q},\omega)$$

Plasmon dispersion:

$$\varepsilon(\vec{q},\Omega_q) = 0$$

2 DEG:

$$\Omega_q = \left(\frac{2\pi n e^2}{m^*} q\right)^{1/2}$$

Bilayer graphene:

$$\Omega_q = \frac{n e^2 v_F}{t_\perp}\sqrt{dq}$$

Summary

- ✓ **Biased bilayer graphene at low densities leads to a topologically non-trivial Fermi-surface.**
- ✓ **At intermediate electronic densities, the one-band approximation becomes unstable.**
- ✓ **The ground state is unstable toward ferromagnetic ordering.**
- ✓ **Including self-energy effects leads to a saturation in the magnetisation.**
- ✓ **At low energies, the system is a Fermi-liquid, albeit with peculiar Friedel oscillations.**
- ✓ **At high energies or low densities, the system shows non-Fermi liquid behavior.**
- ✓ **The plasmons show the familiar two-dimensional square-root disperion, but with a larger energy scale.**