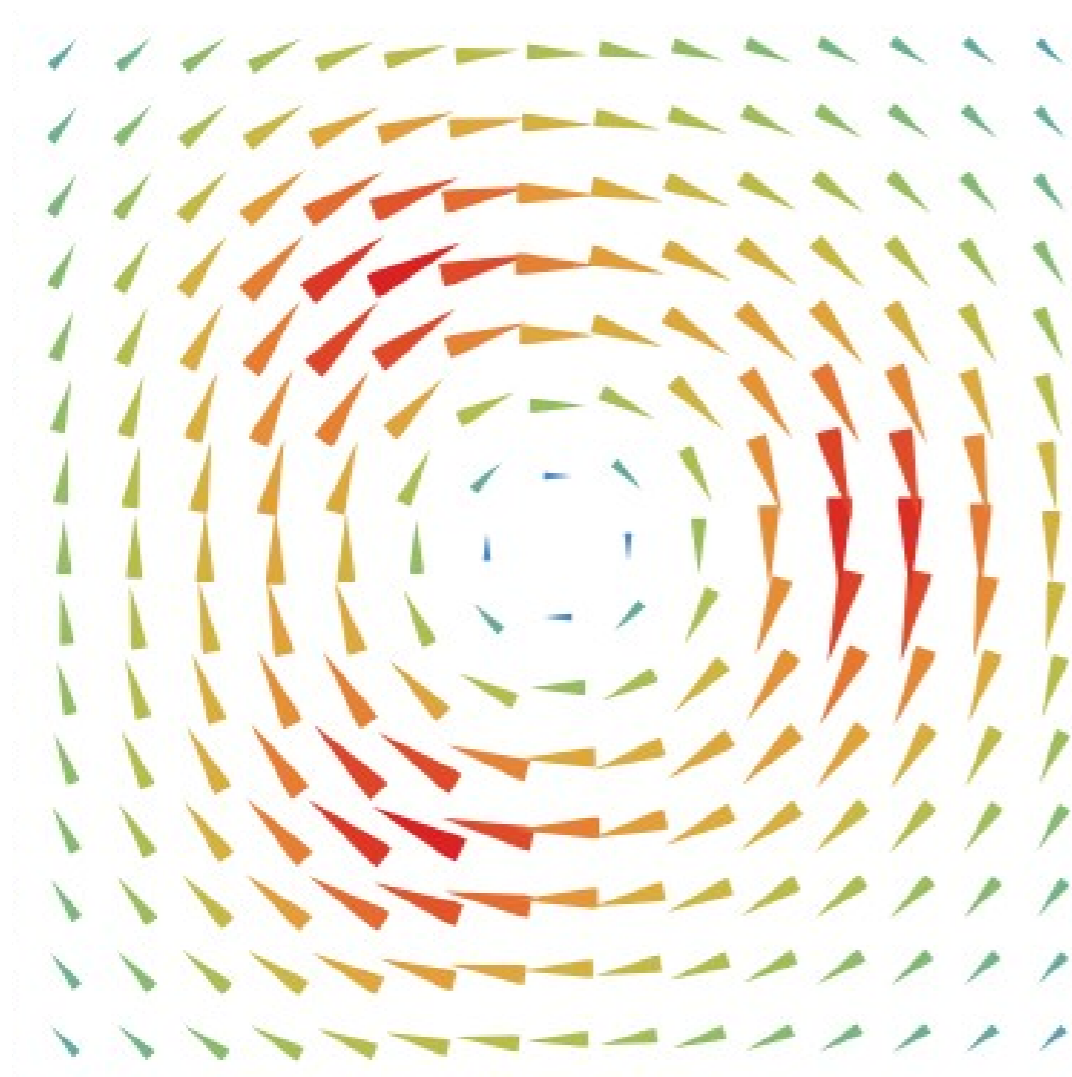


Spin coherence in graphene



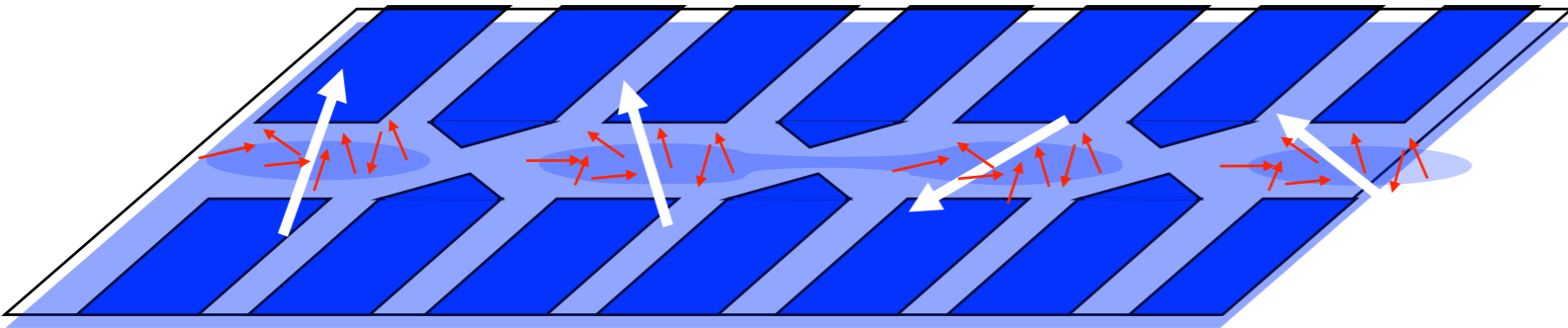
Guido Burkard

University of Konstanz, Germany

Universität
Konstanz

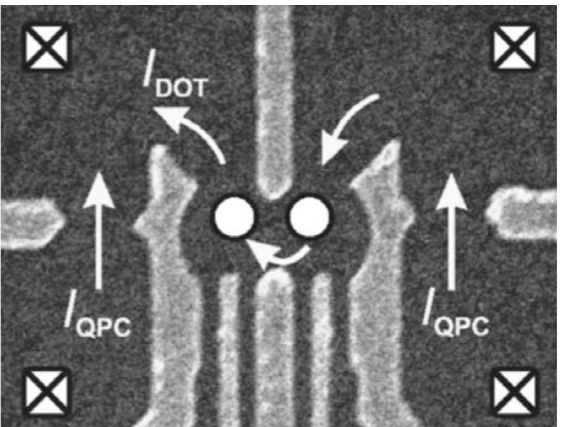
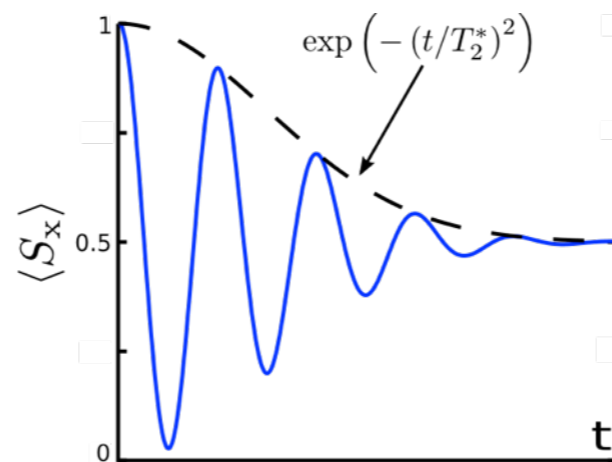


Spin coherence & spin qubits



$$H_{\text{exch}} = \sum_{\langle i,j \rangle} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j$$

nuclear spins
spin decoherence



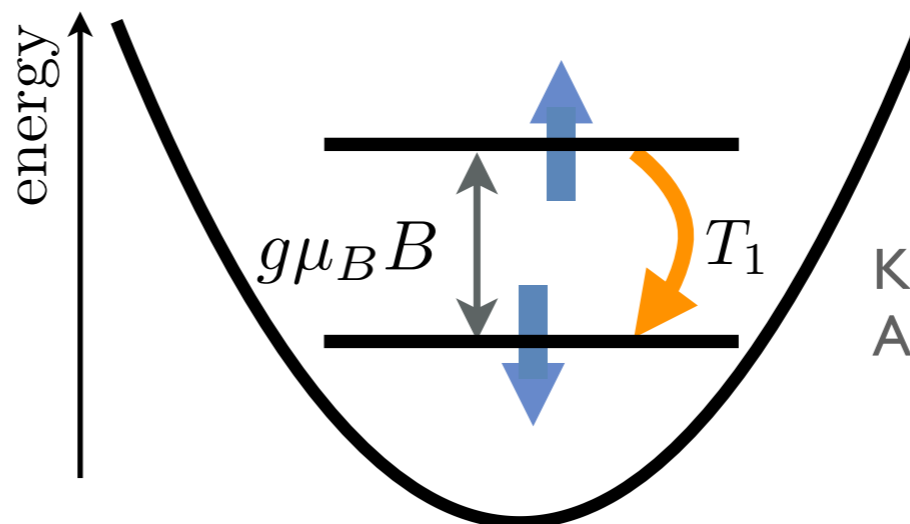
Elzermann *et al.*, PRB 2003
see also Hanson *et al.*, RMP 2007

$$B \gg \delta B_{\text{nuc}}$$

$$T_2^* = \frac{\hbar}{\delta B_{\text{nuc}}}$$

Petta *et al.*, Science 2005

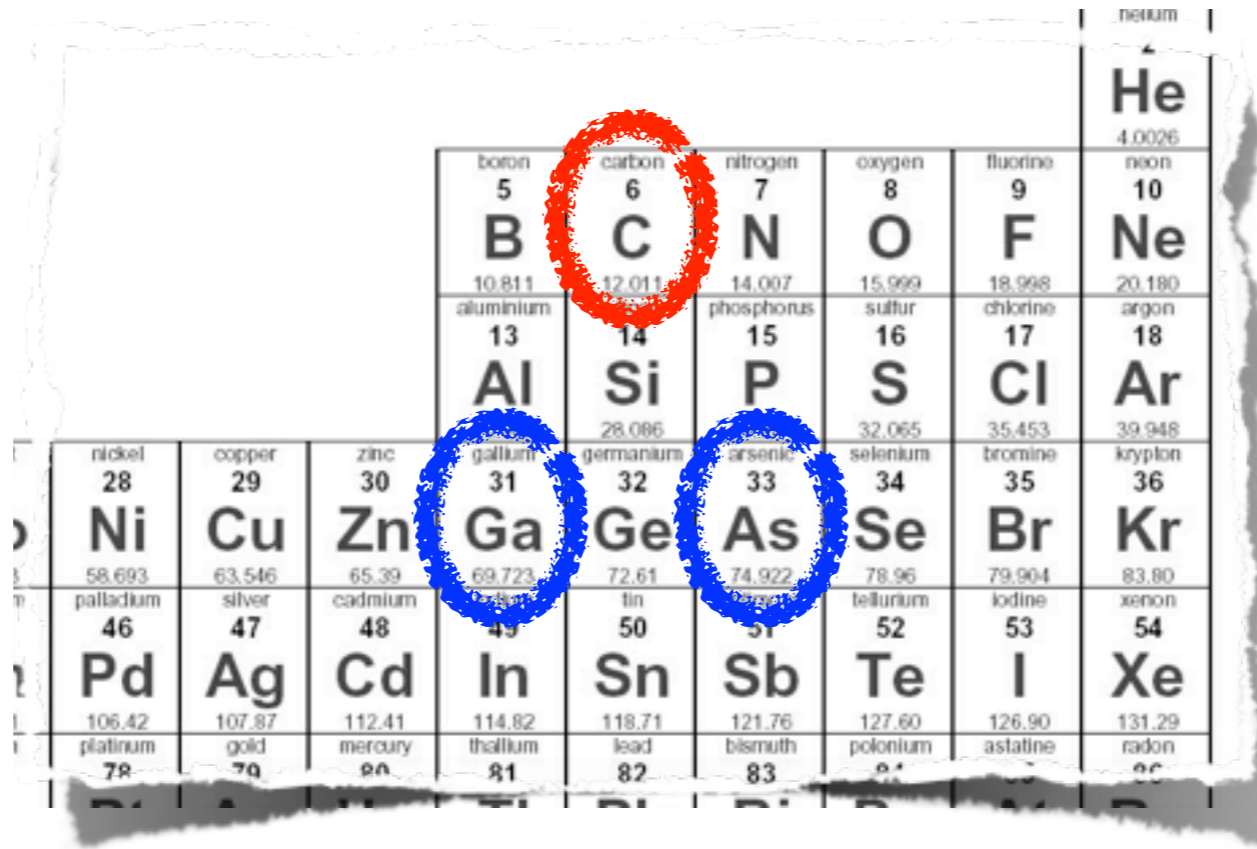
spin orbit interaction
electron phonon coupling
spin relaxation



$$T_1 \propto \frac{1}{B^5}$$

Khaetskii & Nazarov, PRB 2001
Amasha *et al.*, PRL 2008

Carbon as a material for spin qubits



A periodic table of elements with Carbon (C), Gallium (Ga), and Arsenic (As) highlighted. Carbon is circled in red, while Gallium and Arsenic are circled in blue. The table includes element symbols, atomic numbers, and names.

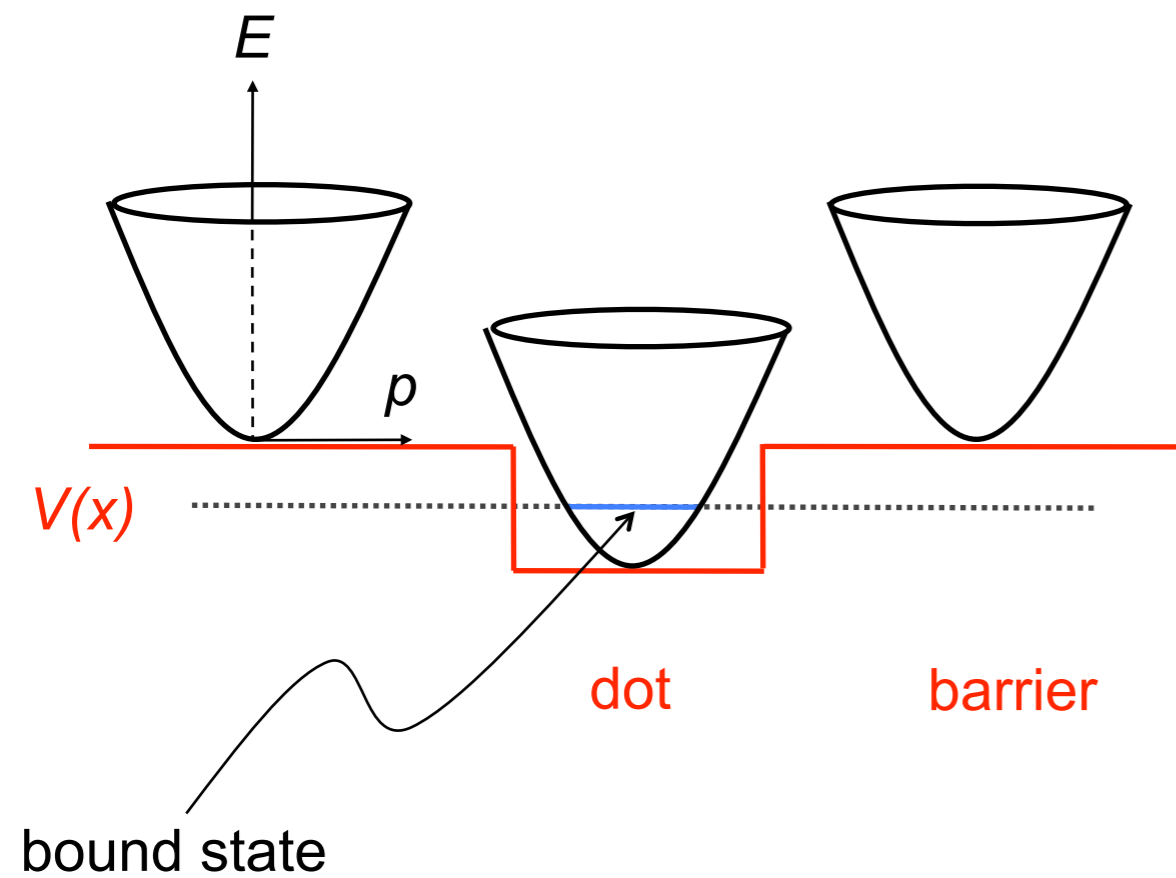
										helium 2 He 4.0026
										neon 10 Ne 20.180
										argon 18 Ar 39.948
										krypton 36 Kr 83.80
										xenon 54 Xe 131.29
										radon 86 Rn 222
										francium 87 Fr [223]
										actinium 89 Ac [227]
										thorium 90 Th 232.0377
										protactinium 91 Pa [231]
										uranium 92 U 238.02891
										neptunium 93 Np [237]
										plutonium 94 Pu [244]
										americium 95 Am [243]
										curium 96 Cm [247]
										berkelium 97 Bk [247]
										californium 98 Cf [251]
										einsteinium 99 Es [252]
										mendelevium 101 Md [258]
										lawrencium 103 Lr [260]
										roentgenium 105 Rg [272]
										dubnium 107 Db [268]
										seaborgium 108 Sg [271]
										bohrium 109 Bh [270]
										hassium 110 Hs [277]
										meitnerium 111 Mt [276]
										darmstadtium 112 Ds [285]
										roentgenium 113 Rg [284]
										copernicium 114 Cn [285]
										nihonium 115 Nh [286]
										flerovium 116 Fl [289]
										tennessine 117 Ts [294]
										oganesson 118 Og [294]
										unbinilium 120 Ubn [298]
										unhexilium 122 Uhl [304]
										unseptilium 124 Uus [310]
										unnonium 126 Unn [315]
										unhennium 128 Uhn [321]
										unquadium 130 Uuq [327]
										unpentium 132 Uup [332]
										unhexium 134 Uuh [338]
										unseptium 136 Uus [344]
										unoganium 138 Uuo [350]
										unnilium 140 Uun [356]
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										untrium 144 Uut [368]
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										unoganium 186 Uuo [494]
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										untrium 192 Uut [512]
										unquadium 194 Uuq [518]
										unpentium 196 Uup [524]
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										unoganium 202 Uuo [542]
										unnilium 204 Uun [548]
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Outline

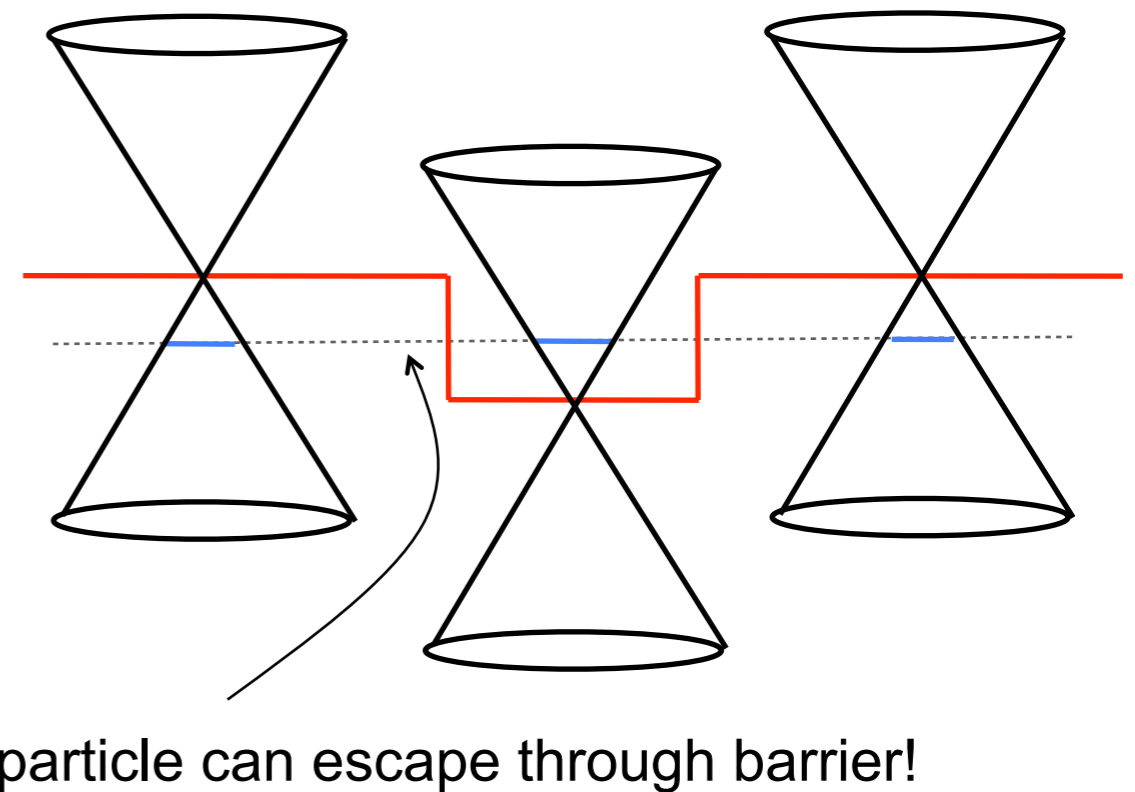
- **quantum dots in graphene**
- **spin-valley hyperfine interaction in graphene**
- **spin relaxation of localized electrons**
- **spin relaxation of mobile electrons (spin transport)**

Quantum dots in graphene

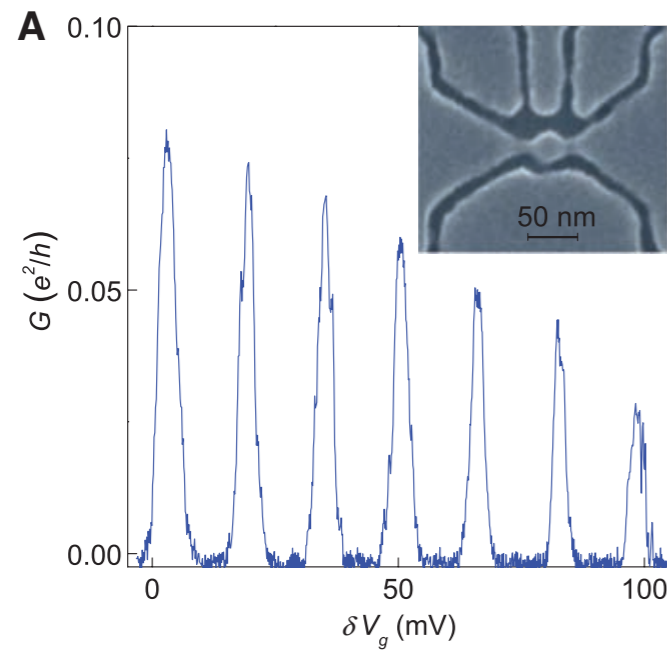
semiconductor



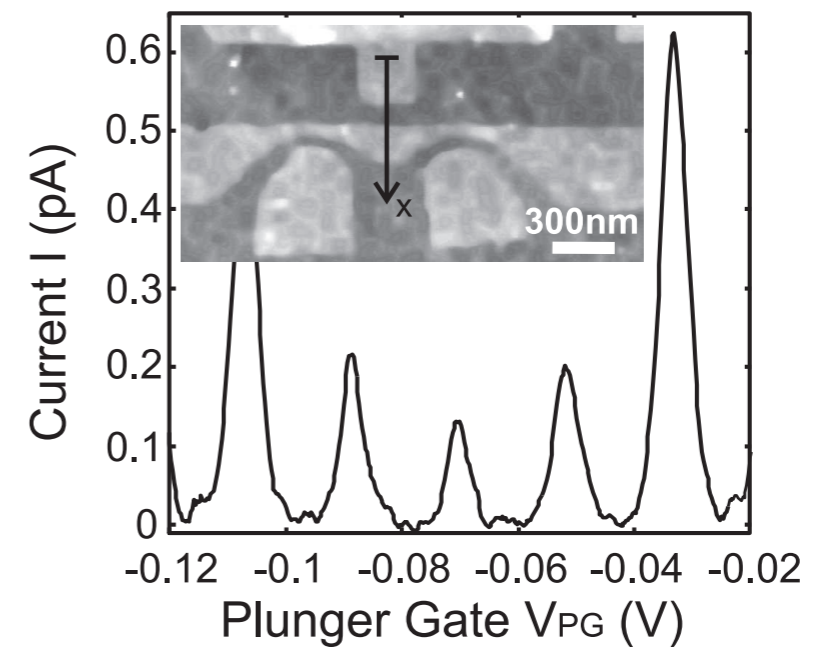
graphene



Quantum dots in nanostructured graphene



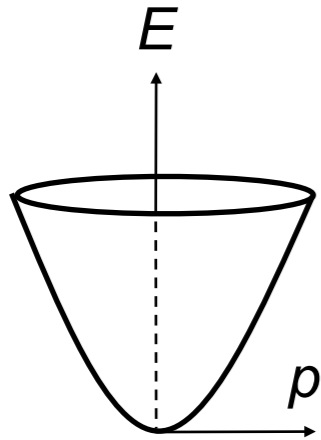
Ponomarenko *et al.*, Science 2008



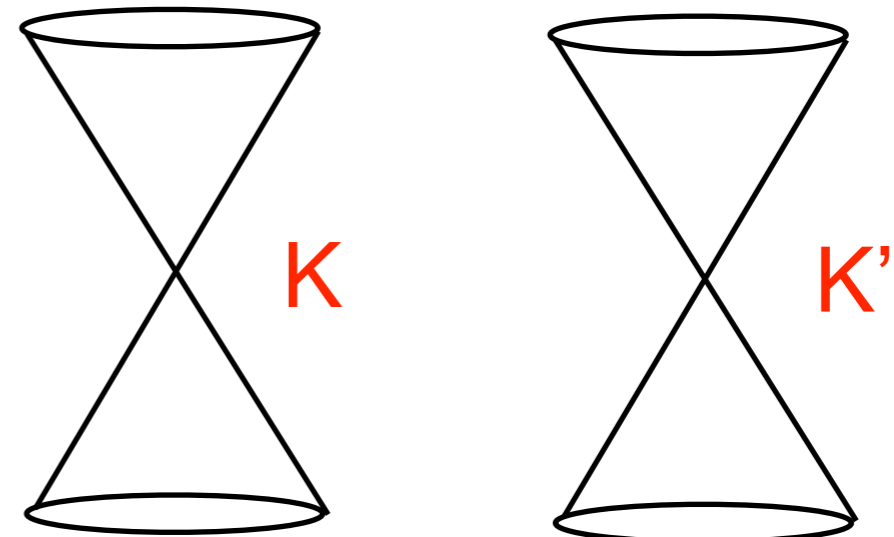
Stampfer *et al.*, APL 2008

Valley degeneracy and exchange

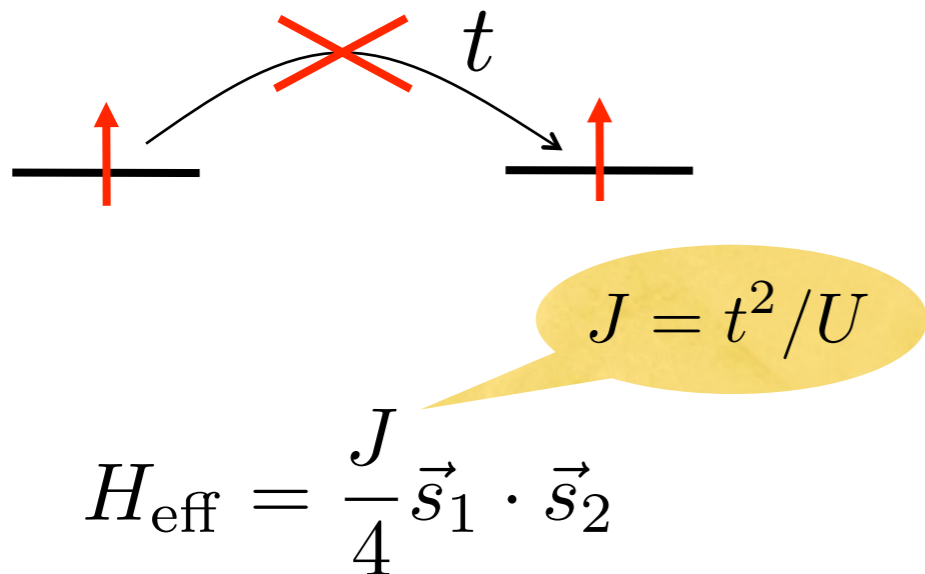
direct-gap semiconductor



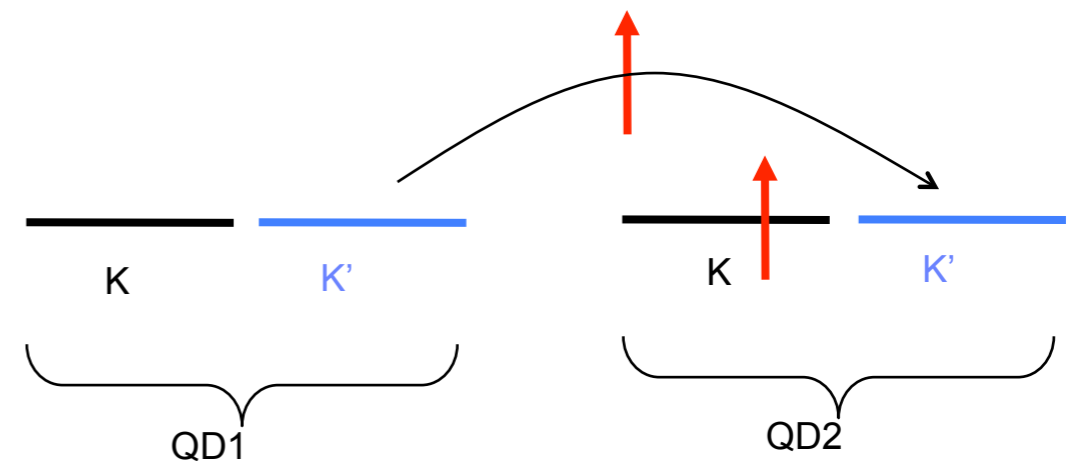
graphene



Pauli principle, exchange coupling



valley degeneracy



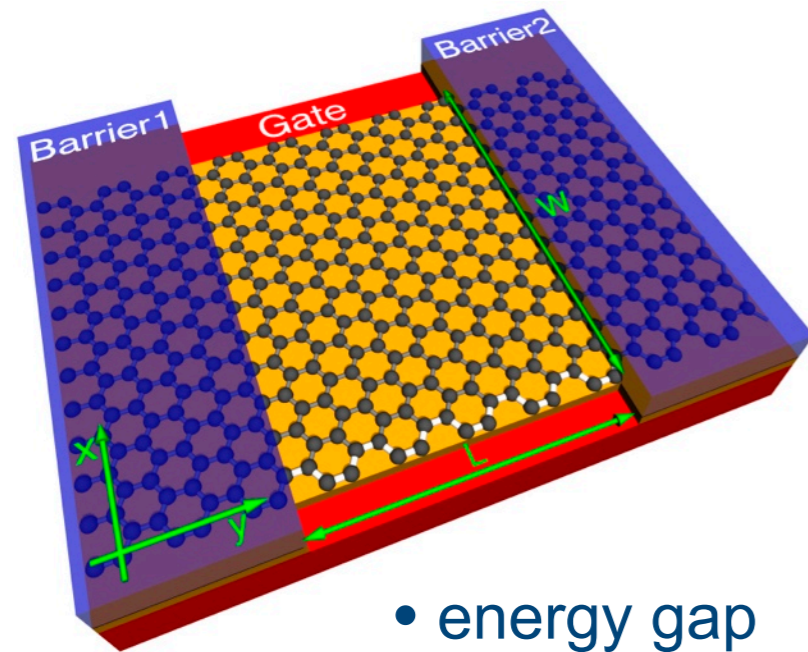
$$H_{\text{eff}} = \frac{J}{8} ((\vec{s}_1 \cdot \vec{s}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) + \vec{s}_1 \cdot \vec{s}_2 + \vec{\tau}_1 \cdot \vec{\tau}_2 - 3)$$

spin-valley coupling

N. Rohling & GB, unpublished

Graphene Nanoribbon QDs

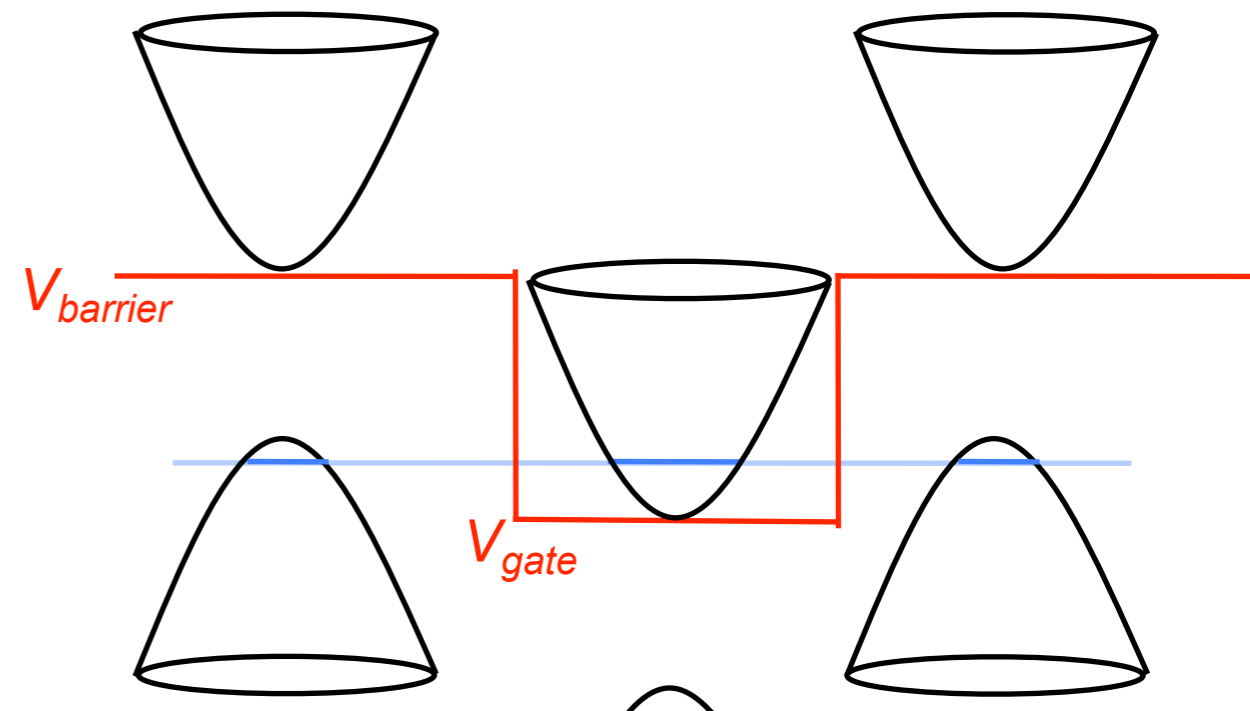
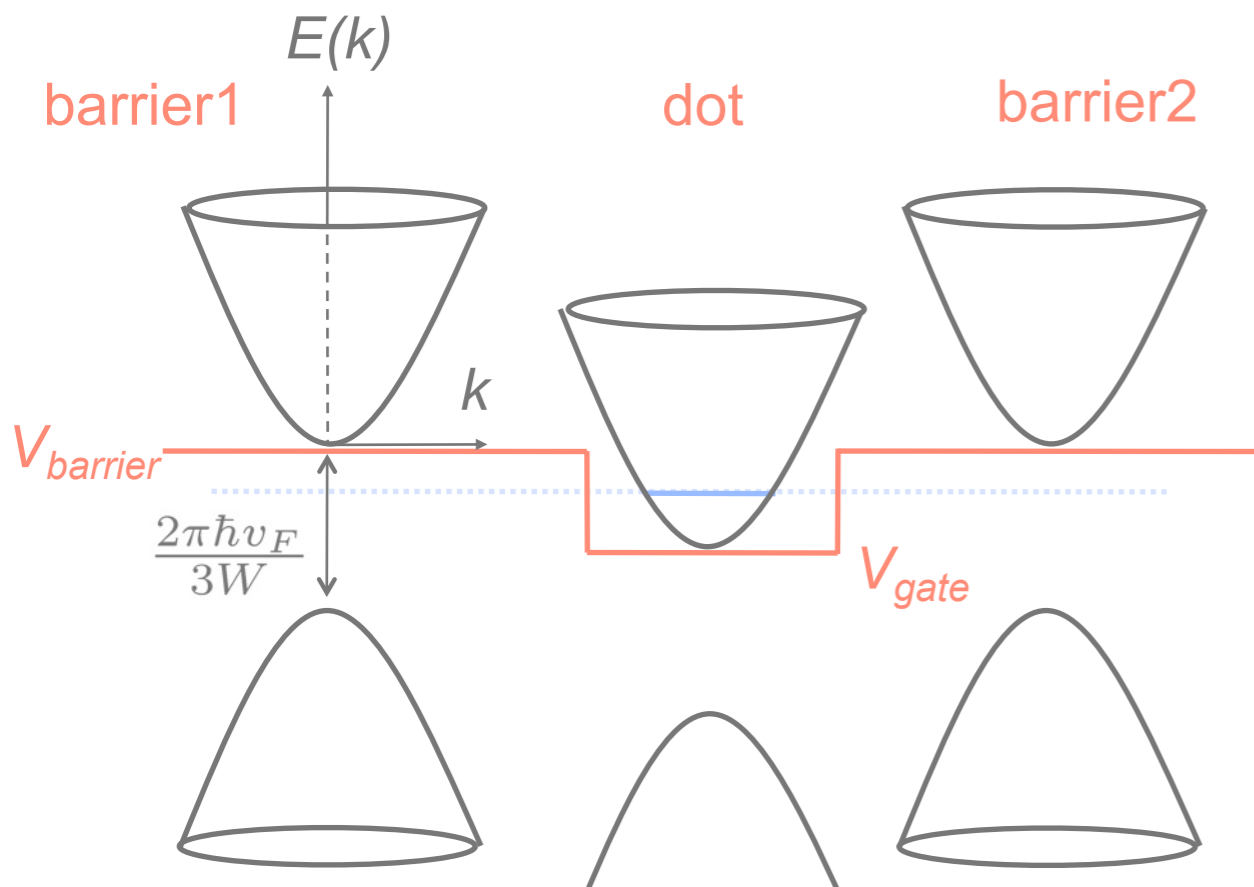
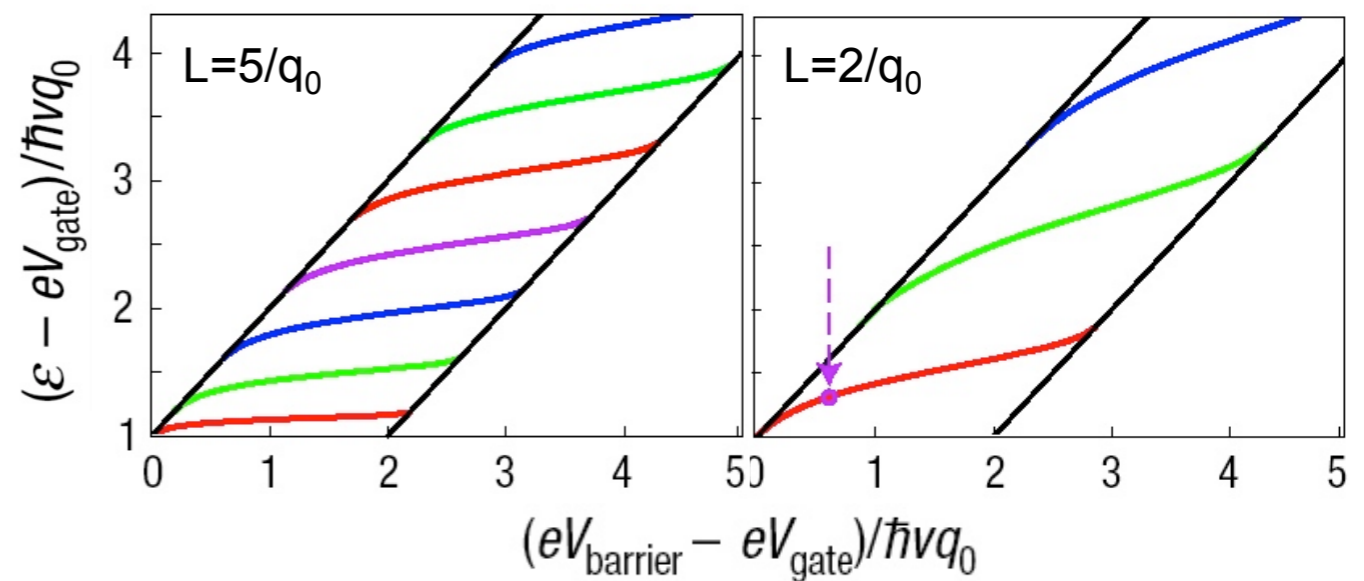
Trauzettel, Bulaev, Loss & GB, Nature Physics (2007)



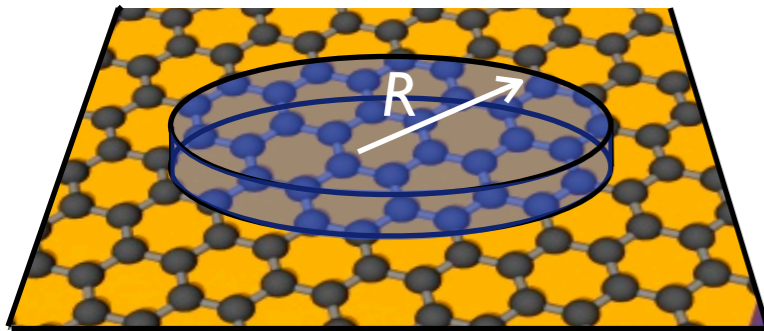
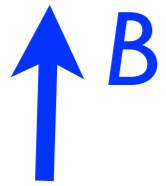
- energy gap
- broken valley degeneracy

Dirac particle in a box: solve transcendental equation for ε

$$\tan(\tilde{k}L) = \frac{\hbar v \tilde{k} \sqrt{(\hbar v q_0)^2 - (\varepsilon - eV_{\text{barrier}})^2}}{(\varepsilon - eV_{\text{barrier}})(\varepsilon - eV_{\text{gate}}) - (\hbar v q_0)^2}$$



Quantum dots in gapped graphene



single layer: substrate-induced gap

Giovannetti *et al.*, Phys. Rev. B **76**, 073103 (2007).

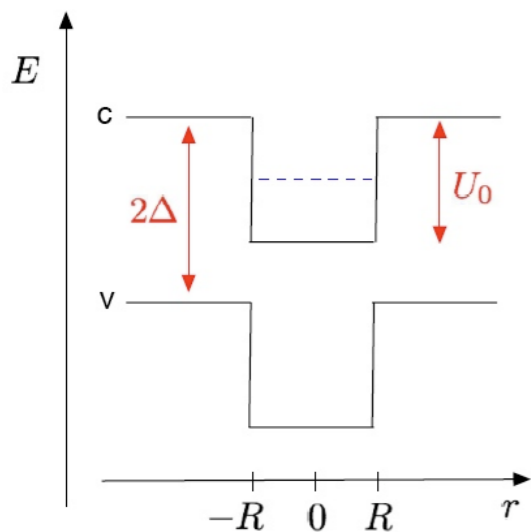
Zhou *et al.*, Nature Mat. **6**, 770 (2007).

bilayer: electrically induced gap

McCann, PRB (2006).

Ohta *et al.*, Science (2006).

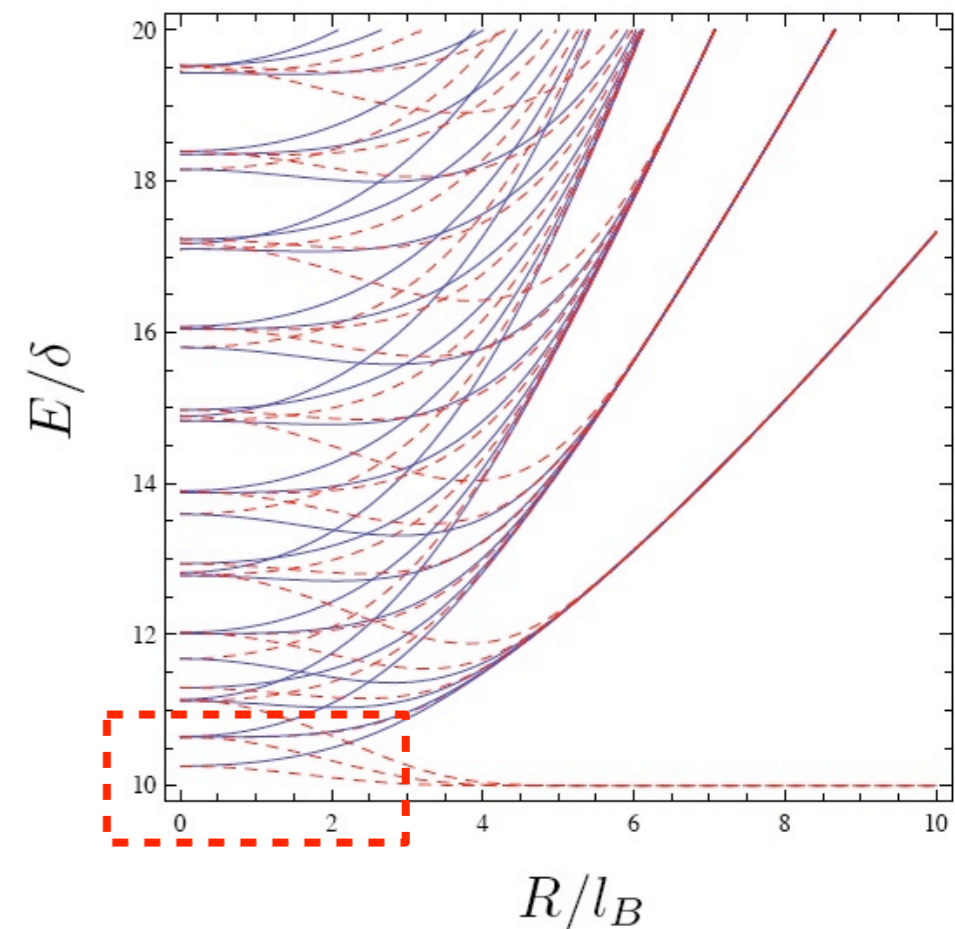
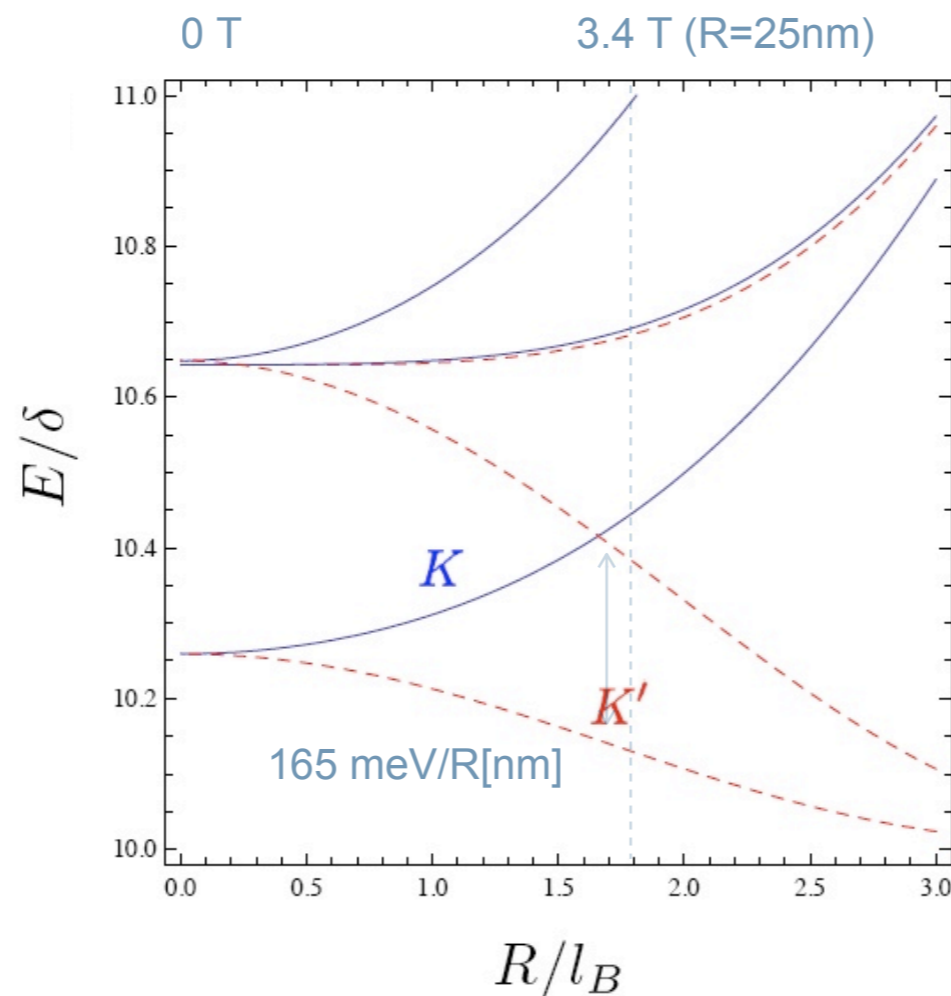
single layer case:



$$U_0 = \Delta$$

$$\Delta = 10 \delta$$

$$l_B = (\hbar/eB)^{1/2}$$



Recher, Nilsson, GB & Trauzettel, PRB **79**, 085407 (2009)

Outline

- quantum dots in graphene
- **spin-valley hyperfine interaction in graphene**
- **spin relaxation of localized electrons**
- **spin relaxation of mobile electrons**

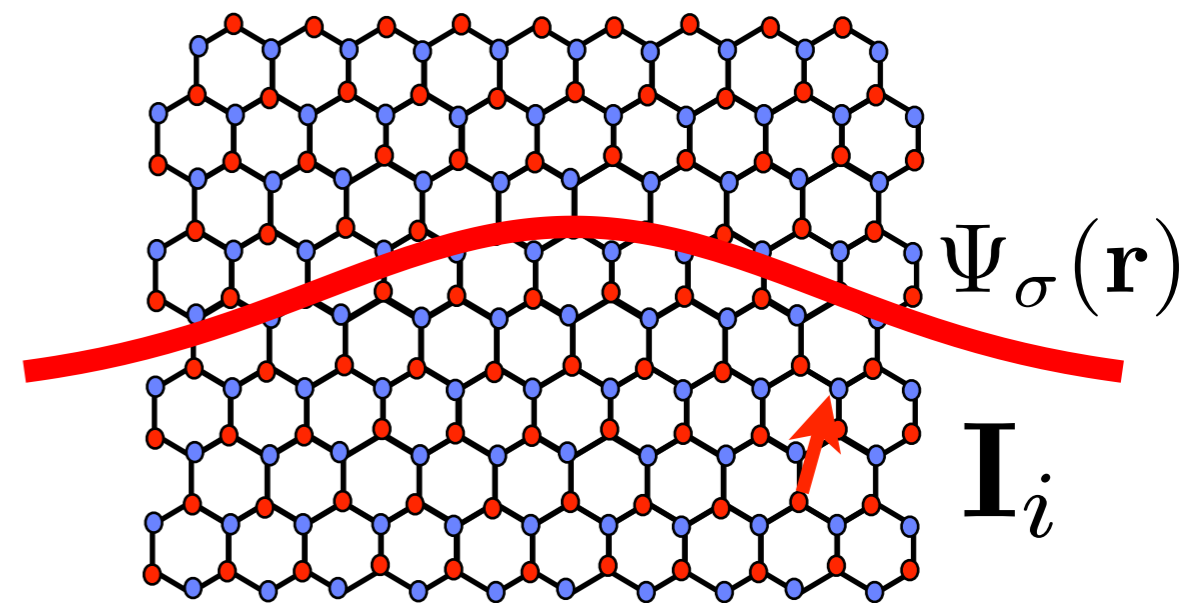
Hyperfine interaction in Graphene

main idea

- ^{13}C atom is an atomically sharp impurity

$$H_{\text{hf},ij} = \delta_{ij} S A \mathbf{I}_i.$$

- can take up momentum on the order of $2K$, thus leading to **inter-valley scattering**
- invisible to electron charge
- visible to spin via hyperfine coupling
- strength of coupling: same as hyperfine



Hyperfine interaction in Graphene

hyperfine Hamiltonian for Dirac wavefunctions:

$$H'_{\text{hf}} = \Omega_{\text{cell}} S A \sum_i \mathbf{I}_i \begin{pmatrix} \boxed{|\Psi_{\sigma}(\mathbf{r}_i)|^2} & \boxed{e^{-i2Kx_i} \Psi_{\sigma}^*(\mathbf{r}_i) \Psi'_{\sigma}(\mathbf{r}_i)} \\ \boxed{e^{i2Kx_i} \Psi'_{\sigma}{}^*(\mathbf{r}_i) \Psi_{\sigma}(\mathbf{r}_i)} & \boxed{|\Psi'_{\sigma}(\mathbf{r}_i)|^2} \end{pmatrix}$$

unit cell
sublattice
 $i = (l\sigma)$

valley mixing

K

K'

hyperfine Hamiltonian for Dirac electrons in graphene

Palyi & GB, PRB **80**, 201404(R) (2009)

$$H_{\text{hf}} = \mathbf{S} \cdot \left(\mathbf{h}^{(0)} \tau_0 + \sum_{i=x,y,z} \mathbf{h}^{(i)} \tau_i \right)$$

valley-conserving terms
(same form as e.g. in GaAs)

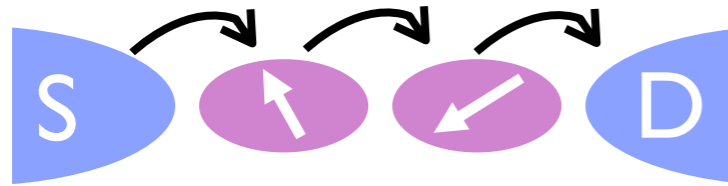
valley-mixing terms (x,y)
valley dephasing term (z)

four 'nuclear fields' instead of one

$$\begin{aligned} \mathbf{h}^{(0)} &= \frac{1}{2} \Omega_{\text{cell}} A \sum_{l\sigma} \mathbf{I}_{l\sigma} \sum_v |\Psi_{\sigma}^{(v)}(\mathbf{r}_{l\sigma})|^2 & \mathbf{h}^{(x)} &= \Omega_{\text{cell}} A \sum_{l\sigma} \mathbf{I}_{l\sigma} \text{Re} [e^{2i\mathbf{K} \cdot \mathbf{r}_{l\sigma}} \Psi_{\sigma}^*(\mathbf{r}_{l\sigma}) \Psi'_{\sigma}(\mathbf{r}_{l\sigma})] \\ \mathbf{h}^{(z)} &= \frac{1}{2} \Omega_{\text{cell}} A \sum_{l\sigma} \mathbf{I}_{l\sigma} \sum_v v |\Psi_{\sigma}^{(v)}(\mathbf{r}_{l\sigma})|^2 & \mathbf{h}^{(y)} &= \Omega_{\text{cell}} A \sum_{l\sigma} \mathbf{I}_{l\sigma} \text{Im} [e^{2i\mathbf{K} \cdot \mathbf{r}_{l\sigma}} \Psi_{\sigma}^*(\mathbf{r}_{l\sigma}) \Psi'_{\sigma}(\mathbf{r}_{l\sigma})] \end{aligned}$$

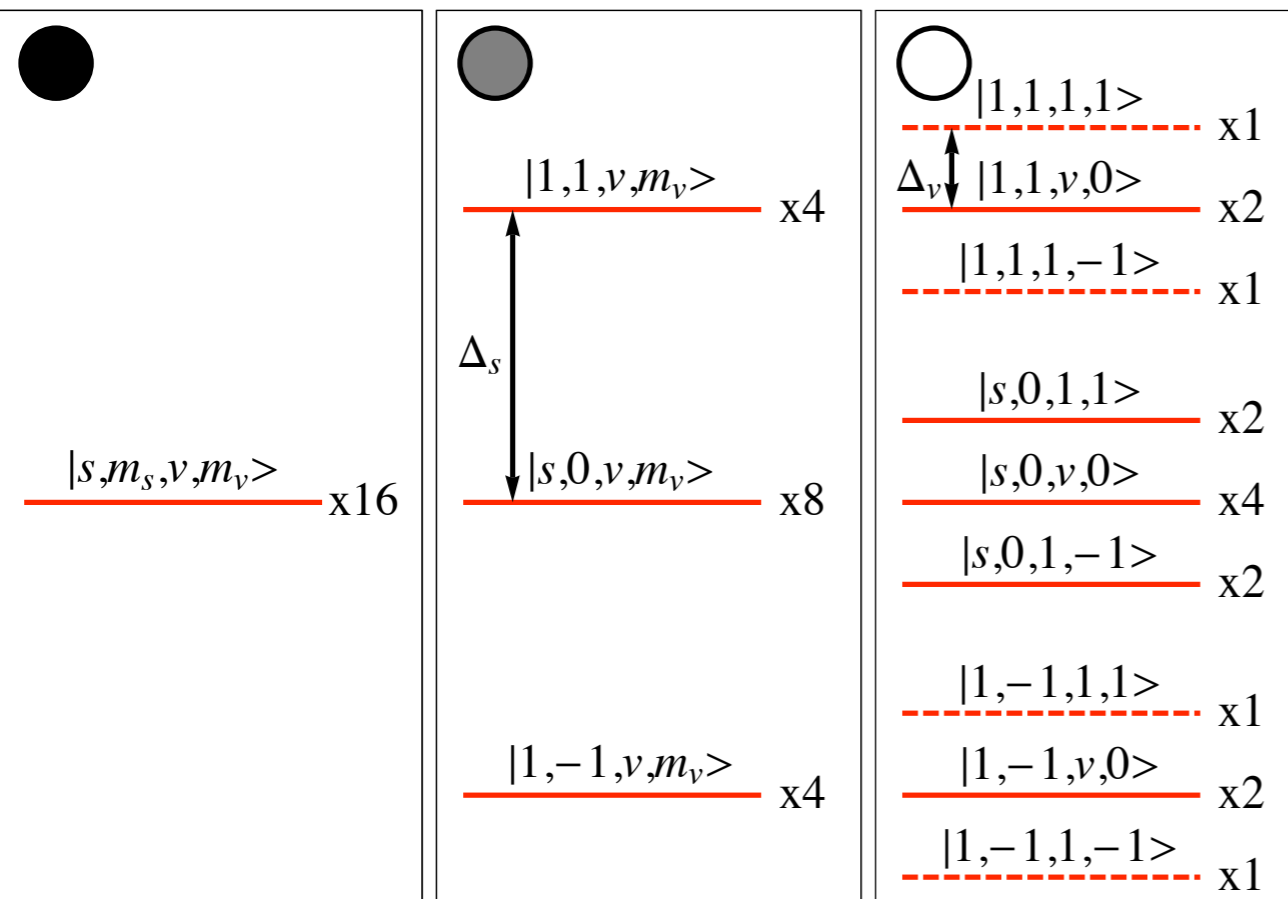
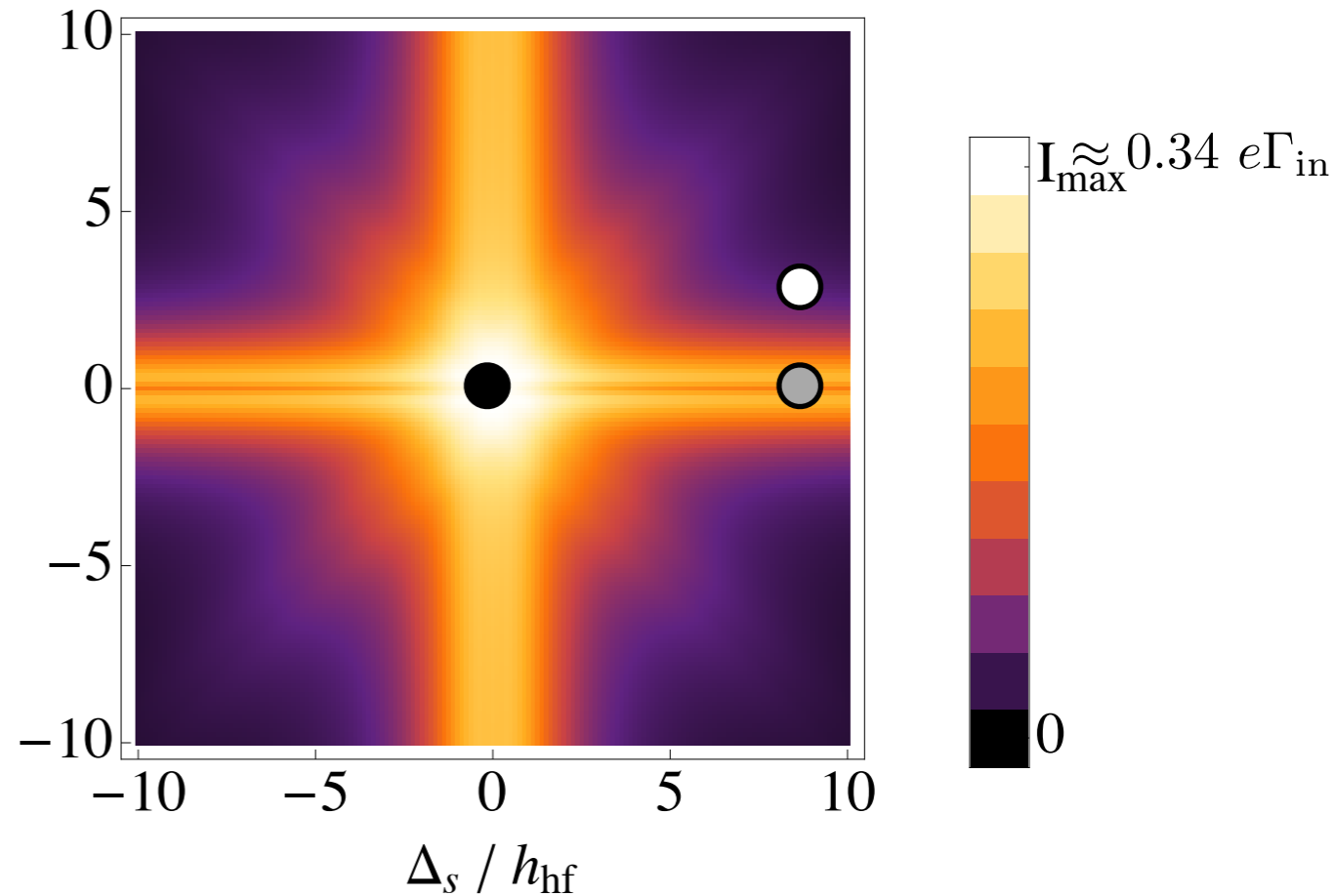
Spin blockade in graphene dots

Pályi & GB, PRB 2009



- effect of **hyperfine** interaction on leakage current?
- no disorder and no spin-orbit interaction
- incoherent interdot tunneling

Δ_v / h_{hf}



Energy level diagram in the (1,1) charge configuration

● $\Delta_s = 0$ $\Delta_v = 0$ hf-induced **spin and valley** mixing
no pure supertriplets
blockade lifted

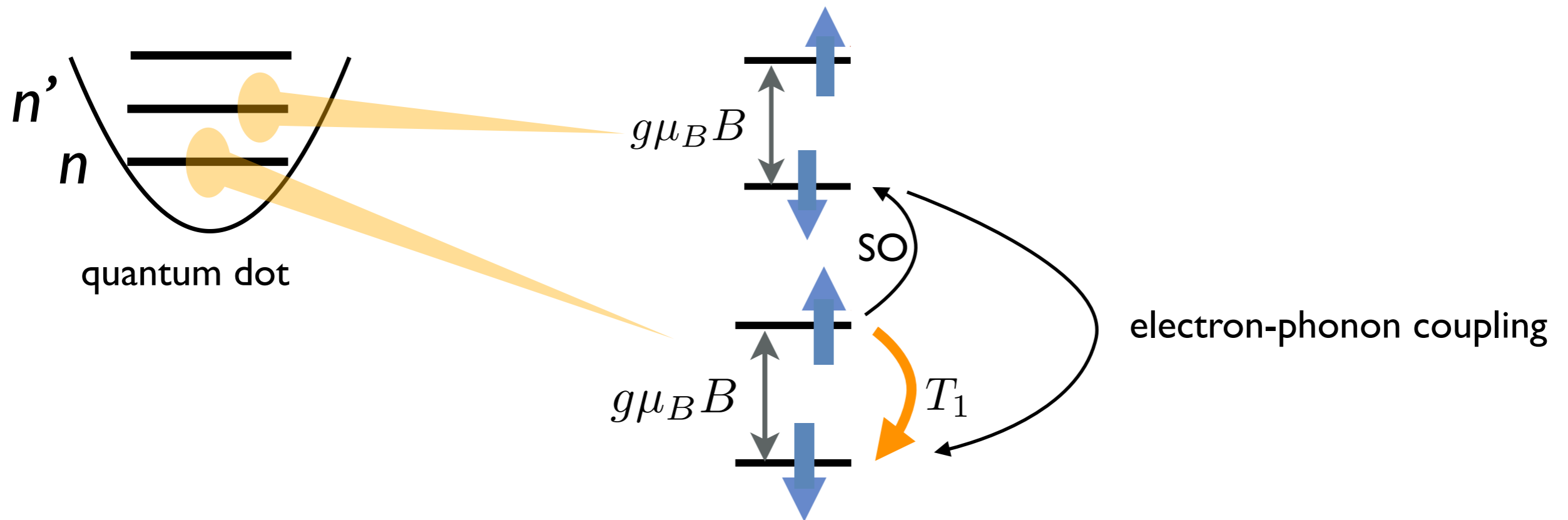
● $\Delta_s \gg h_{\text{hf}}$ $\Delta_v = 0$ hf-induced **valley** mixing
no pure supertriplets
blockade lifted

○ $\Delta_s \gg h_{\text{hf}}$ $\Delta_v \gg h_{\text{hf}}$ 4 pure supertriplets (-----)
transport blocked

Outline

- quantum dots in graphene
- spin-valley hyperfine interaction in graphene
- **spin relaxation of localized electrons**
- **spin relaxation of mobile electrons**

Spin relaxation of localized electrons



- spin-orbit (SO) interaction + electron phonon coupling (EPC), piezo-phonons

- van Vleck cancellation at $B=0$: qubit states form Kramers doublet van Vleck, Phys. Rev. 1940

at $B=0$: $T_1 \rightarrow \infty$

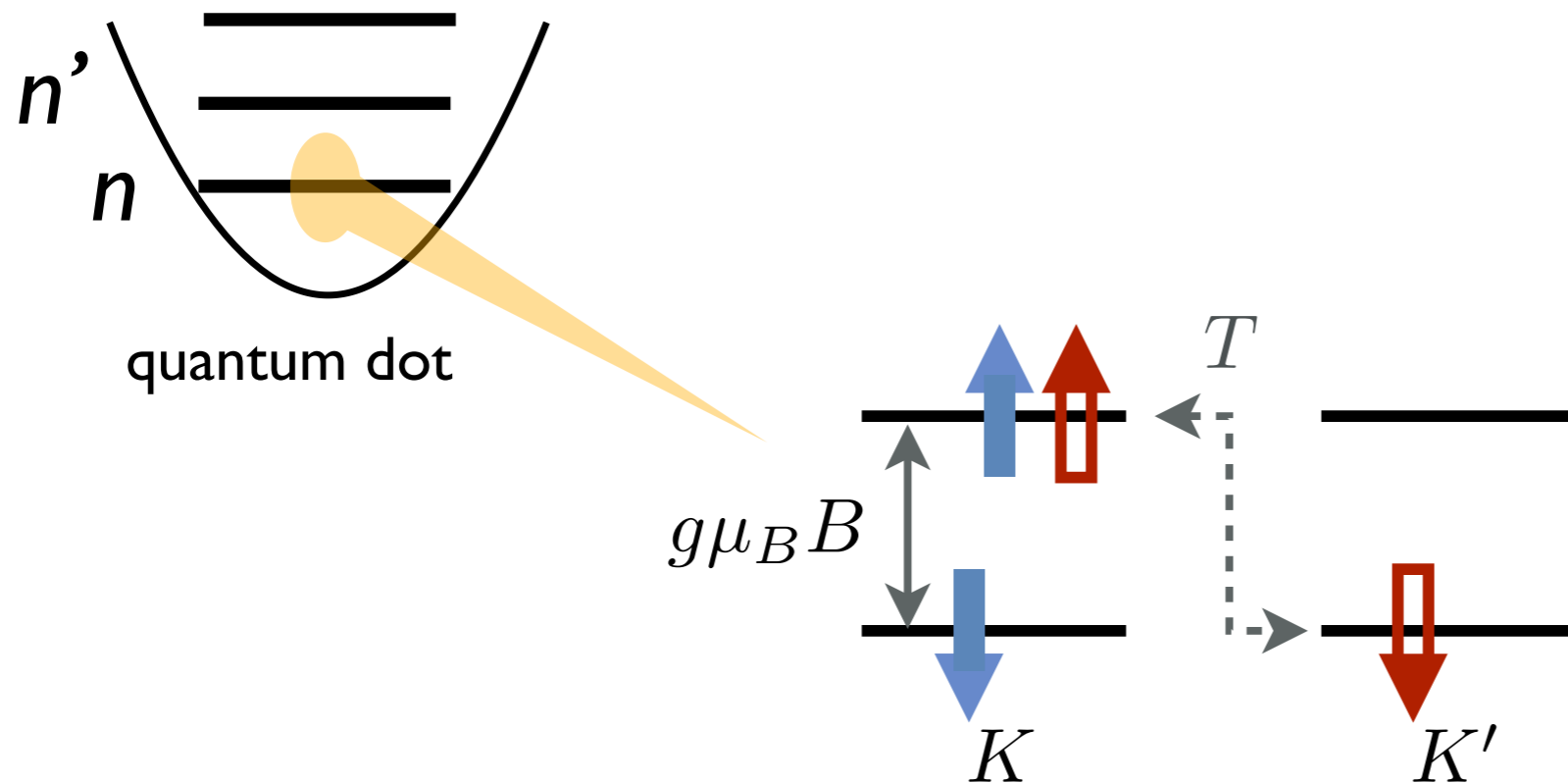
$B \neq 0$: “at least” $T_1 \propto B^{-2}$

typical regime (GaAs): $T_1 \propto B^{-5}$

graphene QDs?

Khaetskii & Nazarov, PRB 2001
Amasha et al., PRL 2008

Spin relaxation in graphene QDs



in some graphene QDs: valley degeneracy (K, K'):

(1) time reversal invariance at $B=0$ intact,
but Kramers pair resides in different valleys

(2) either: **Kramers qubit**
or: pure **spin qubit** in one valley

spin qubit
there is **no van Vleck cancellation**

Spin-orbit interaction in graphene

$$H_{\text{SO}} = H_{\text{i}} + H_{\text{R}} = \Delta_{\text{i}} \tau \sigma_z s_z + \Delta_{\text{R}} (\tau \sigma_x s_y - \sigma_y s_x)$$

Kane & Mele, PRL 2005

Huertas-Hernando, Guinea & Brataas, PRB 2006

Castro Neto & Guinea PRL 2009

Gmitra *et al.* PRB 2009

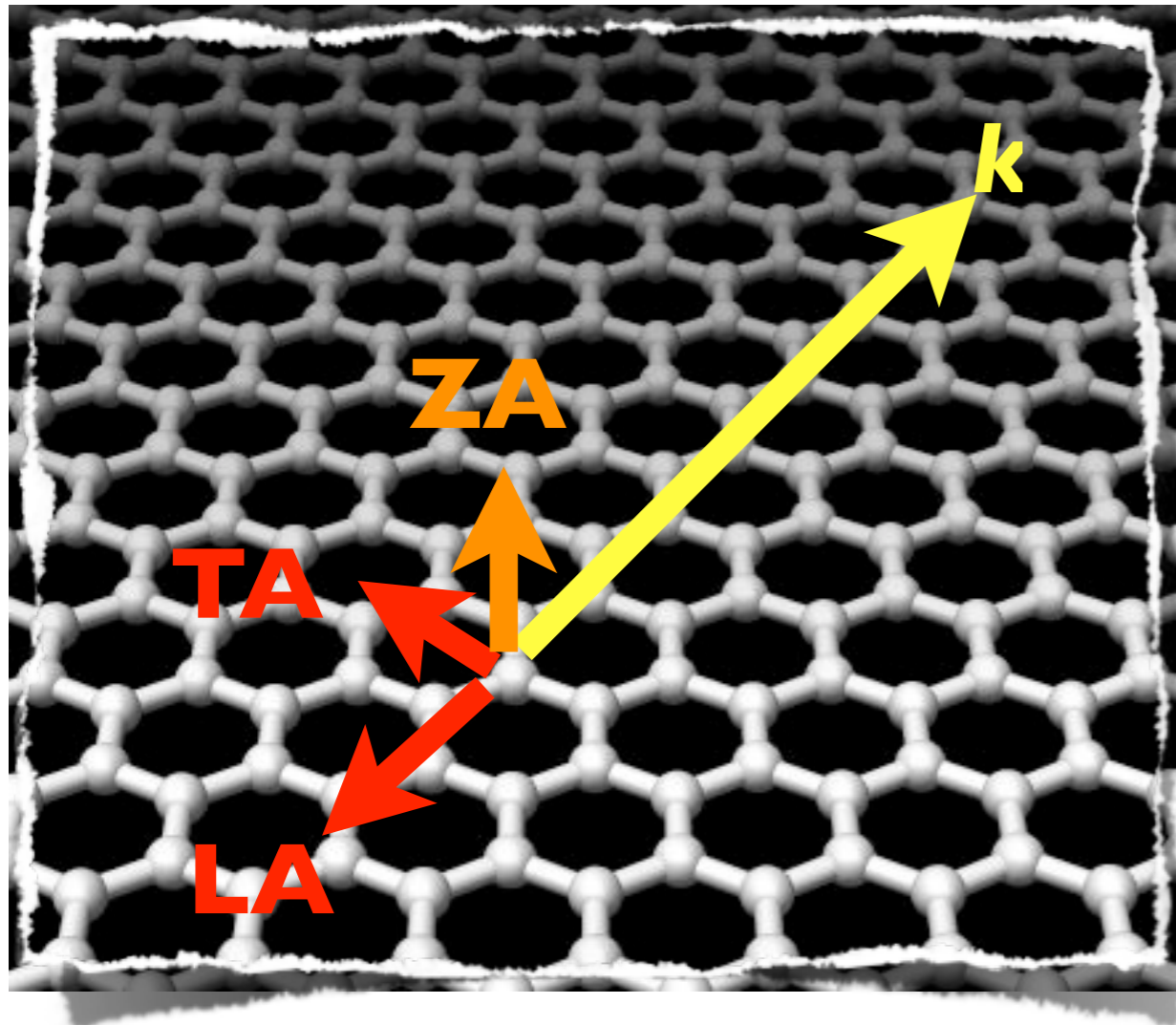
Rashba SOI

- tunable with external E-field
- selection rule $\Delta j = \pm 1$

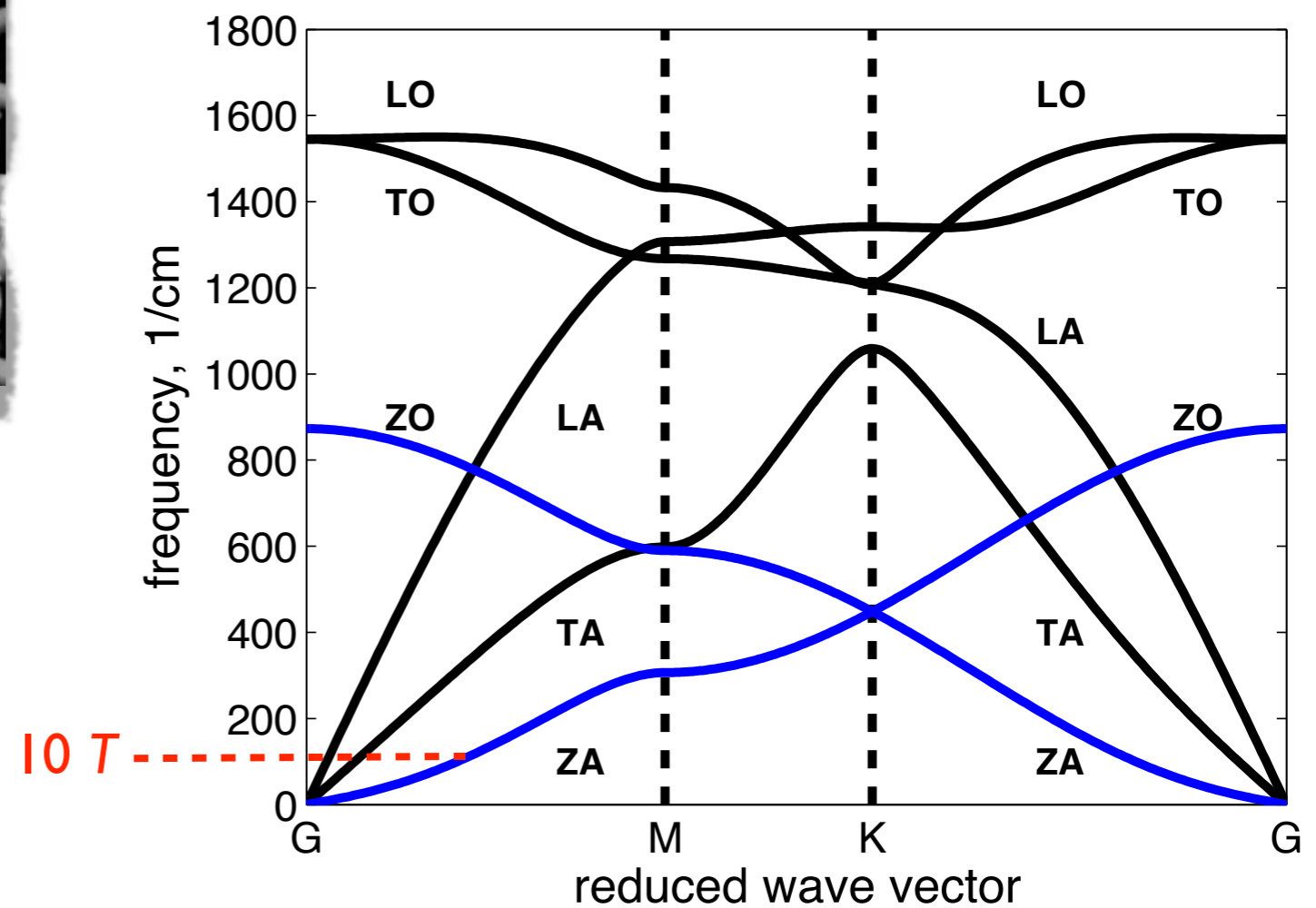
intrinsic SOI

- selection rule $\Delta j = 0$

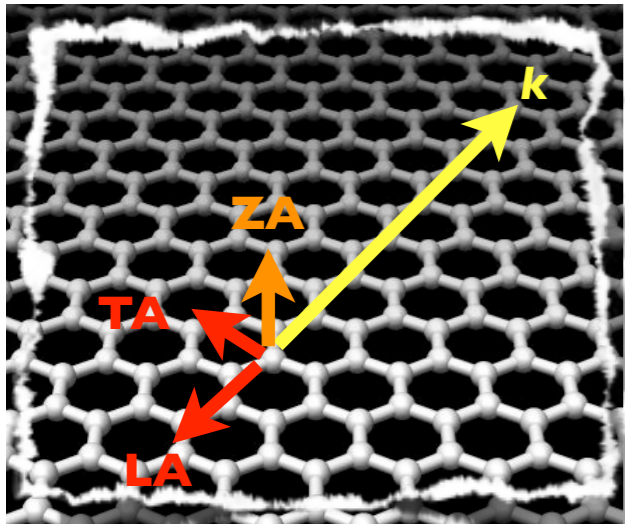
Phonons in Graphene



Falkovsky, JETP 2007 [cond-mat/0702409]



Electron-phonon coupling



(0) no piezo phonons!

(1) deformation potential
(only LA phonons, $g_1 \sim 30$ eV)

$$H_{\text{EPC},1} = \frac{ig_1q}{\sqrt{A\rho\omega_{\mathbf{q},\mu}}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (e^{i\mathbf{q}\mathbf{r}}b^\dagger - e^{-i\mathbf{q}\mathbf{r}}b)$$

(2) bond-length change
(LA, and TA, $g_2 \sim 1.5$ eV)

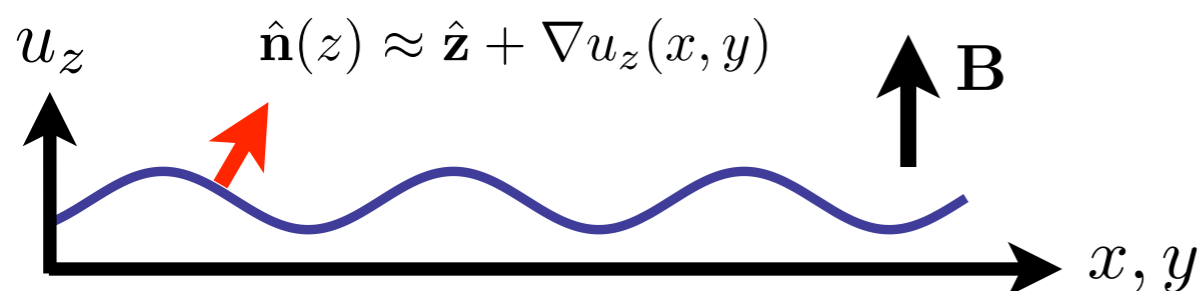
$$H_{\text{EPC},2} = \frac{ig_2q}{\sqrt{A\rho\omega_{\mathbf{q},\mu}}} \begin{pmatrix} 0 & e^{2i\phi_q} \\ e^{-2i\phi_q} & 0 \end{pmatrix} (e^{i\mathbf{q}\mathbf{r}}b^\dagger - e^{-i\mathbf{q}\mathbf{r}}b)$$

Ando, J. Phys. Soc. Jpn. 2005
Suzuura & Ando, PRB 2002
Mariani & von Oppen, PRL 2008.

(3) direct spin-phonon coupling (ZA phonons) Struck & GB, PRB 2010.

$$H_{\text{SO}} = H_i + H_R = \Delta_i \tau \sigma_z s_z + \Delta_R (\tau \sigma_x s_y - \sigma_y s_x)$$

deflection coupling in CNTs: Rudner & Rashba, PRB 2010.



$$H_i = H_i^{(0)} + \Delta_i (\partial_x u_z s_x + \partial_y u_z s_y) \sigma_z \tau$$

$$H_R = H_R^{(0)} + \Delta_R (-\sigma_y \partial_x u_z + \tau \sigma_x \partial_y u_z) s_z$$

ZA phonons: $u_z = \sqrt{1/A\rho\omega_{\mathbf{q}}} (e^{i\mathbf{q}\cdot\mathbf{r}}b^\dagger + e^{-i\mathbf{q}\cdot\mathbf{r}}b)$

Relaxation rate

Fermi's Golden Rule

$$\Gamma = 2\pi A \int \frac{d^2 q}{(2\pi)^2} \left| (H_{\text{EPC}})_{nn}^{\uparrow\downarrow} \right|^2 \delta(\omega_{\mathbf{q}} - g\mu_B B)$$

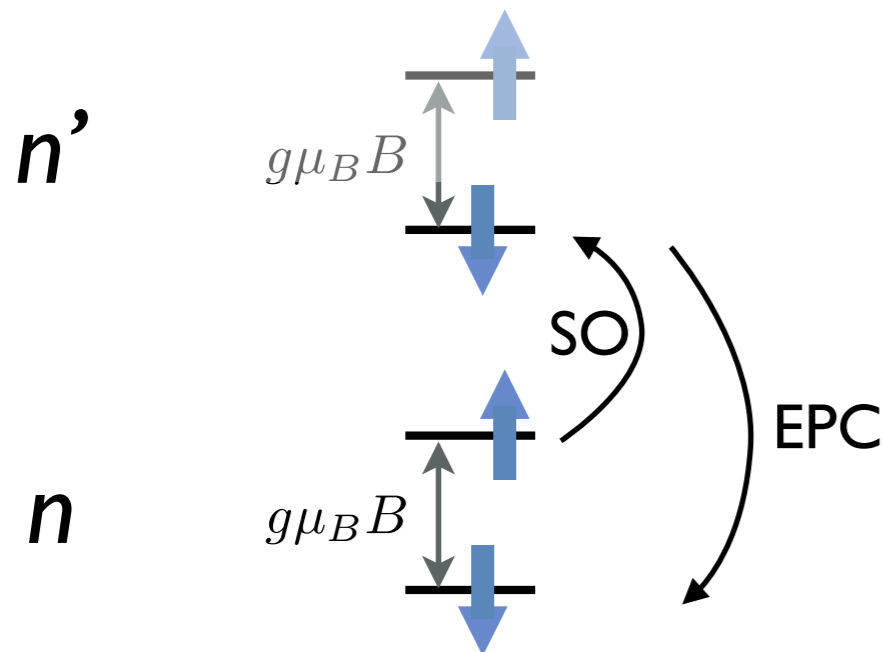
matrix elements

$$(H_{\text{EPC}})_{nn}^{\uparrow\downarrow} = \sum_{n' \neq n} \left[\frac{(H_{\text{SO}})_{nn'}^{\uparrow\downarrow} (H_{\text{EPC}})_{n'n}}{E_n - E_{n'} - \frac{1}{2}g\mu_B B} + \frac{(H_{\text{EPC}})_{nn'} (H_{\text{SO}})_{n'n}^{\uparrow\downarrow}}{E_n - E_{n'} + \frac{1}{2}g\mu_B B} \right]$$

(LA,TA)

$$i\Delta_i \sqrt{1/A\rho\omega_{\mathbf{q}}} (q_x \langle \uparrow | s_x | \downarrow \rangle + q_y \langle \uparrow | s_y | \downarrow \rangle) \langle n | \sigma_z e^{i\mathbf{q} \cdot \mathbf{r}} | n \rangle$$

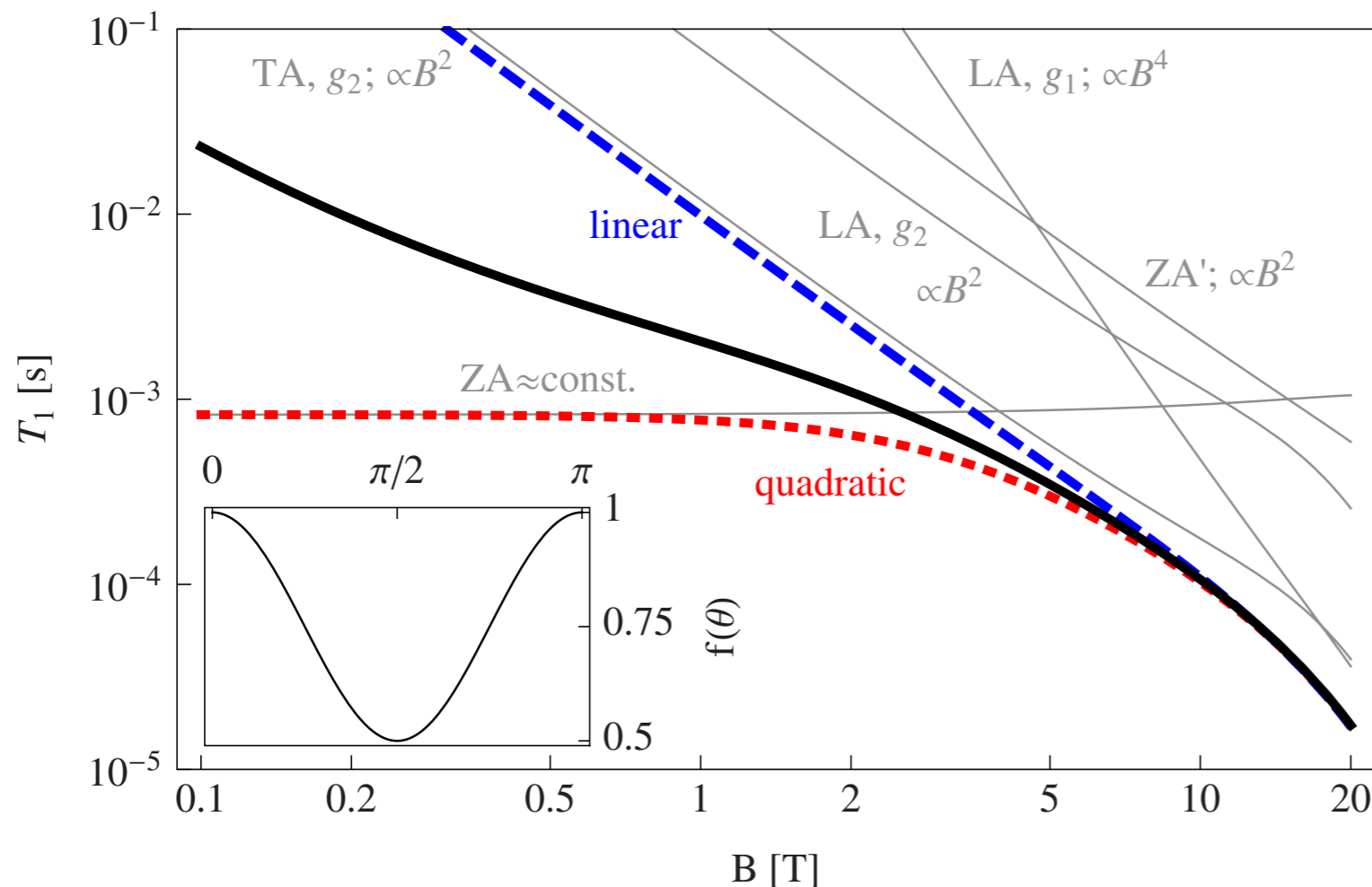
(ZA)



Spin relaxation in graphene QDs

Struck & GB, PRB (2010)

- spin relaxation T_1 in circular graphene QD
- no van Vleck cancellation
- B-independent T_1 at low B for quadratic dispersion of ZA mode, crossover to B^2 or B^4



$$U_0 = \Delta = 260 \text{ meV}$$
$$R = 25 \text{ nm}$$

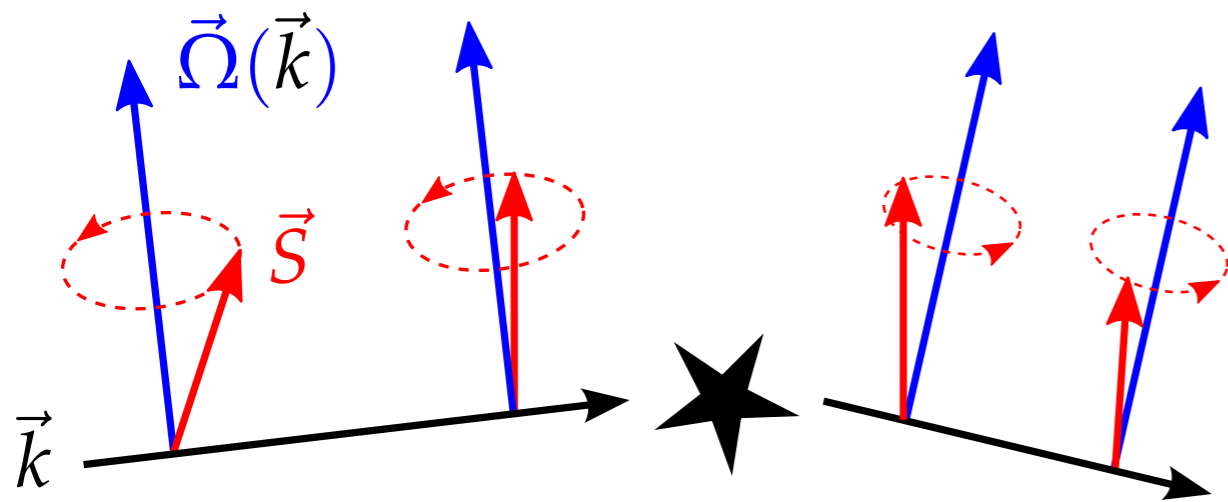
Outline

- quantum dots in graphene
- spin-valley hyperfine interaction in graphene
- spin relaxation of localized electrons
- **spin relaxation of mobile electrons**

Spin transport relaxation mechanisms

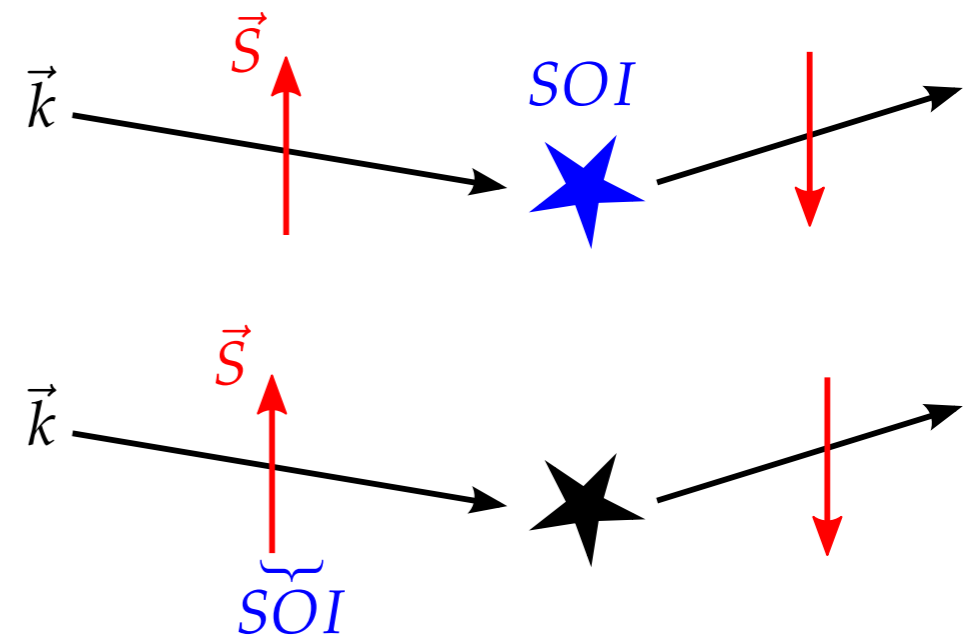
D'yakonov-Perel'

$$\tau_S \propto 1/\tau_p$$



Elliot-Yafet

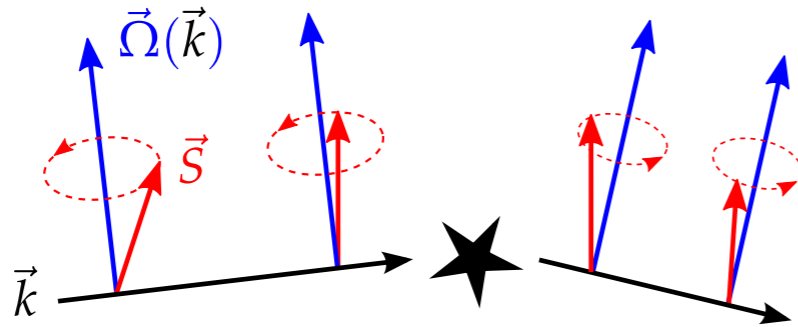
$$\tau_S \propto \tau_p$$



Summary of experimental observations

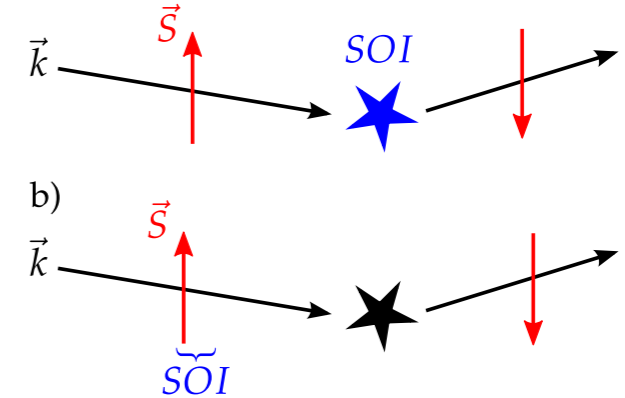
D'yakonov-Perel'

$$\tau_S \propto 1/\tau_p$$



Elliot-Yafet

$$\tau_S \propto \tau_p$$



N. Tombros *et al.*, Nature 2007

single layer

coherence time ~ 150 ps

impurity limited

Elliot-Yafet (EY) ?

N. Tombros *et al.*, PRL 2008

single layer

anisotropy

T.-J. Yang *et al.*, PRL 2011

bilayer

up to 2 ns

D'yakonov-Perel' (DP)

W. Han and R. K. Kawakami,
PRL 2011

single layer

~ 1 ns

bilayer

up to 6 ns

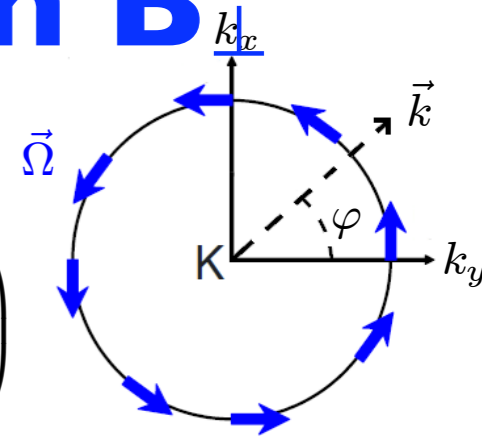
D'yakonov-Perel' (DP)

DP Spin relaxation in SLG with B

spin-orbit: $H_{SO} = \lambda_I \tau \sigma_z s_z + \lambda_R (\tau \sigma_x s_y - \sigma_y s_x)$

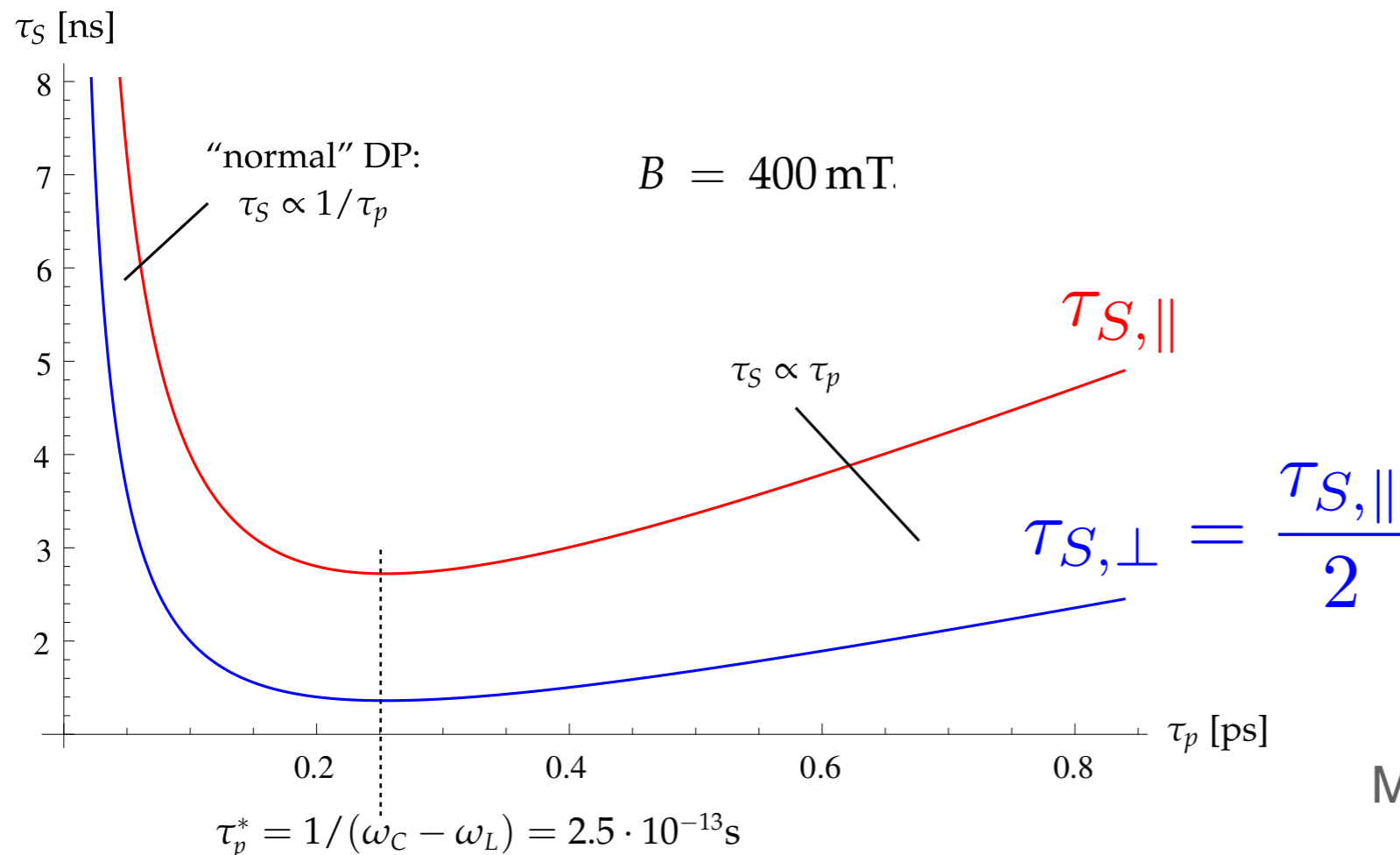
effective electron Hamiltonian: $H_{\text{eff}} = \vec{\Omega} \cdot \vec{s}$

$$\vec{\Omega}(\vec{k}) = m \underbrace{\frac{2\lambda_R}{\hbar}}_{\omega_R} \begin{pmatrix} -\sin(\phi_{\vec{k}}) \\ \cos(\phi_{\vec{k}}) \\ 0 \end{pmatrix}$$



kinetic spin Bloch equation: $\frac{\partial \vec{s}_{\vec{k}}}{\partial t} - \underbrace{\vec{\Omega}(\vec{k})}_{\text{SOI} + \text{Larmor}} \times \vec{s}_{\vec{k}} = - \int_0^{2\pi} \frac{d\phi'}{2\pi} W(\phi - \phi') (\vec{s}_{\vec{k}} - \vec{s}_{\vec{k}'})$

longitudinal spin relaxation (decoherence): $\frac{1}{\tau_{S,\parallel}} = \frac{1}{T_2} = \frac{1}{2} \frac{\omega_R^2 \tau_p}{1 + (\omega_C - \omega_L)^2 \tau_p^2}$



frequencies:

Rashba $\omega_R = \frac{2\lambda_R}{\hbar}$

cyclotron $\omega_C = \frac{ev_F^2}{E_F} B$

Larmor $\omega_L = 2 \frac{\mu_B}{\hbar} B$

M. Diez & GB, manuscript in preparation

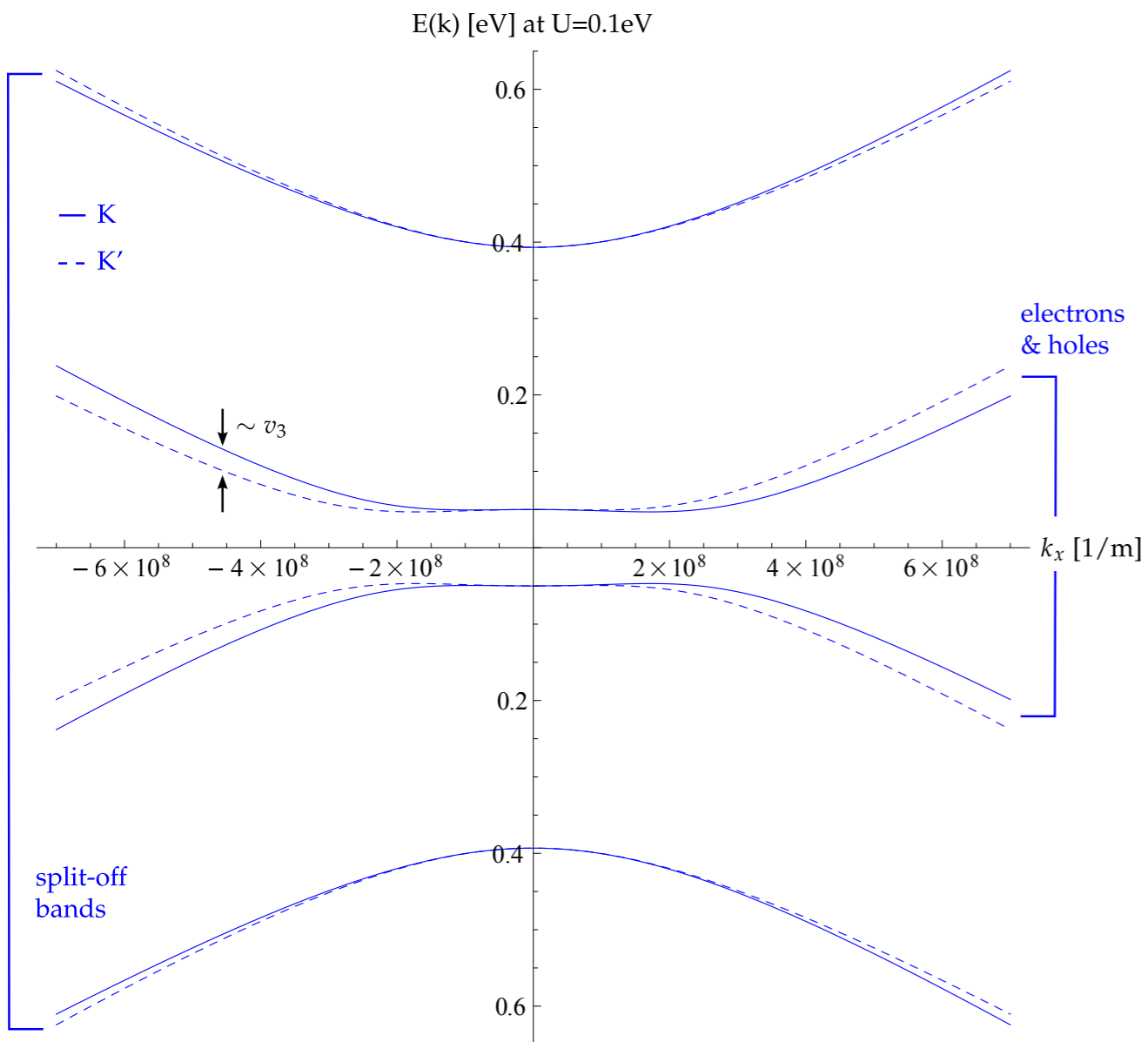
DP spin relaxation in BLG

$$H_0^\tau = \begin{pmatrix} \frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} & 0 & \tau \hbar k v_3 e^{-i\tau\phi} \\ \tau \hbar k v_F e^{-i\tau\phi} & \frac{U}{2} & \gamma_1 & 0 \\ 0 & \gamma_1 & -\frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} \\ \tau \hbar k v_3 e^{i\tau\phi} & 0 & \tau \hbar k v_F e^{-i\tau\phi} & -\frac{U}{2} \end{pmatrix}$$

AI BI A2 B2

interlayer potential (points to $\frac{U}{2}$)
 from skew interlayer hopping (points to $\tau \hbar k v_3 e^{-i\tau\phi}$)
 interlayer hopping (points to γ_1)

McCann & Fal'ko, PRL 2006



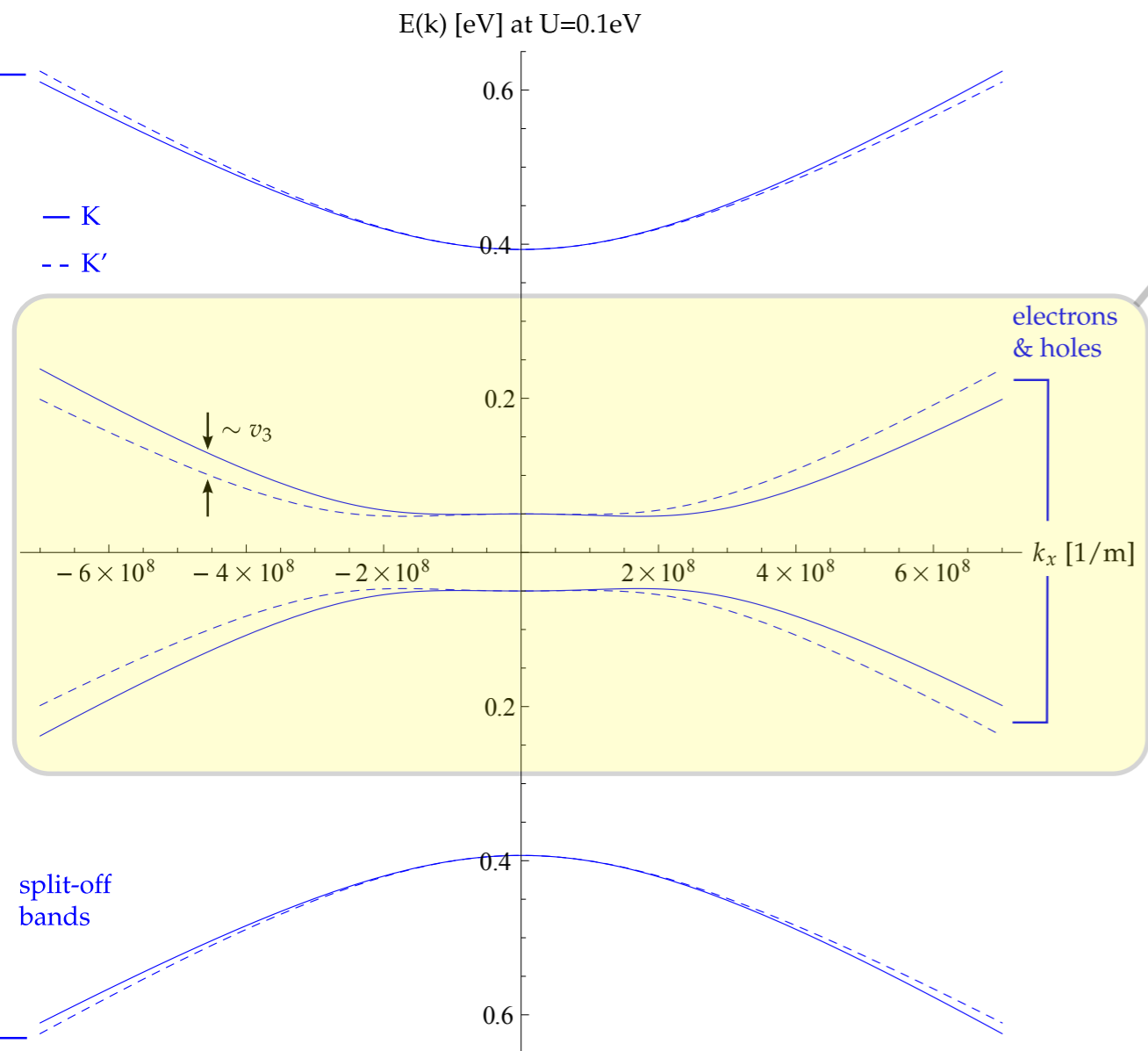
DP spin relaxation in BLG

$$H_0^\tau = \begin{pmatrix} \frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} & 0 & \tau \hbar k v_3 e^{-i\tau\phi} \\ \tau \hbar k v_F e^{-i\tau\phi} & \frac{U}{2} & \gamma_1 & 0 \\ 0 & \gamma_1 & -\frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} \\ \tau \hbar k v_3 e^{i\tau\phi} & 0 & \tau \hbar k v_F e^{-i\tau\phi} & -\frac{U}{2} \end{pmatrix}$$

AI BI A2 B2

interlayer potential (points to $\frac{U}{2}$)
 from skew interlayer hopping (points to $\tau \hbar k v_3 e^{-i\tau\phi}$)
 interlayer hopping (points to γ_1)

McCann & Fal'ko, PRL 2006



$$H_0^{\text{eff}} = \begin{pmatrix} U \left(\frac{1}{2} - \frac{p^2 v_F^2}{\gamma_1^2} \right) & -\frac{p^2 v_F^2 e^{i2\tau\phi}}{\gamma_1} + \tau p v_3 e^{-i\tau\phi} \\ -\frac{p^2 v_F^2 e^{-i2\tau\phi}}{\gamma_1} + \tau p v_3 e^{i\tau\phi} & -U \left(\frac{1}{2} - \frac{p^2 v_F^2}{\gamma_1^2} \right) \end{pmatrix}$$

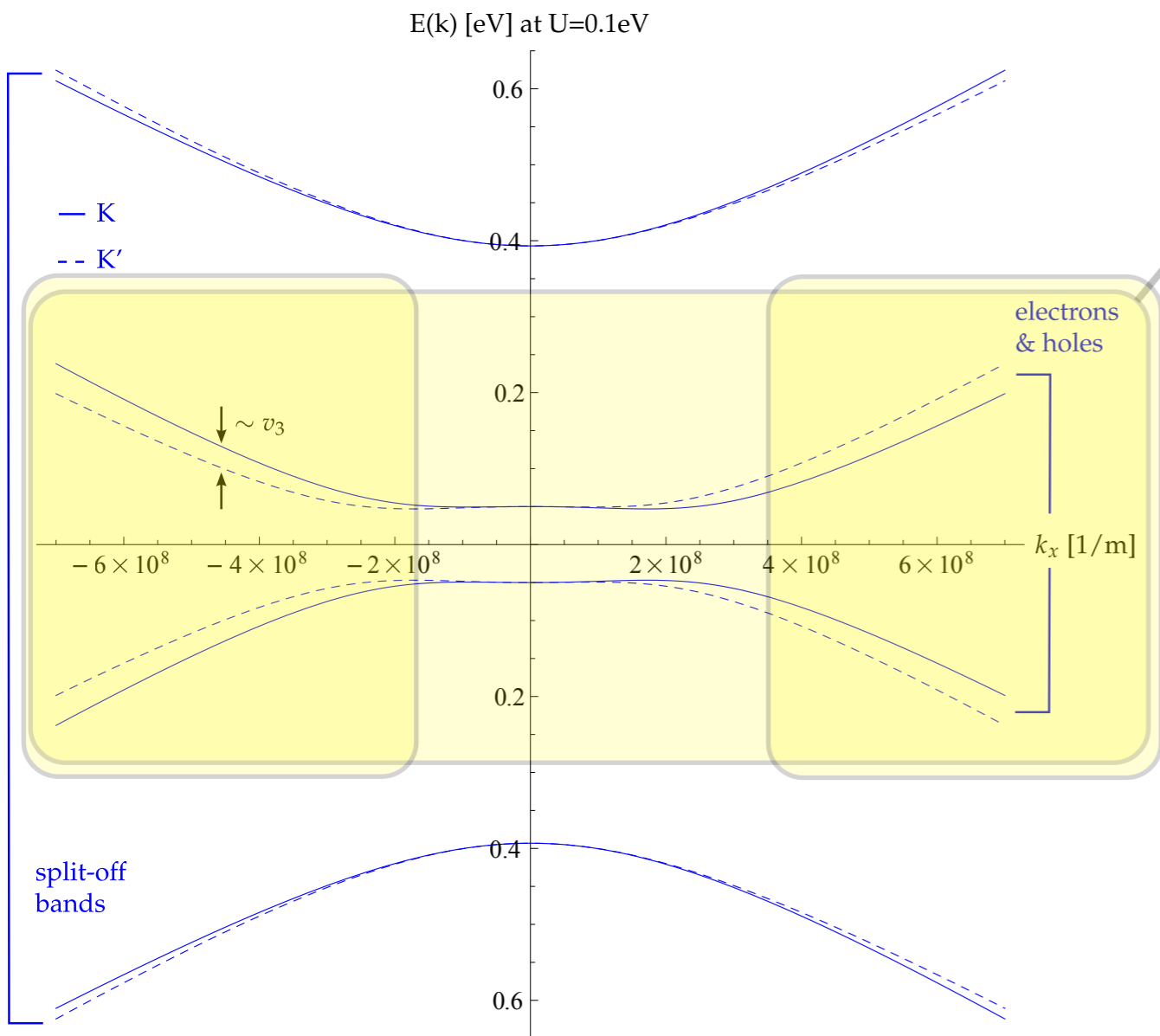
DP spin relaxation in BLG

$$H_0^\tau = \begin{pmatrix} \frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} & 0 & \tau \hbar k v_3 e^{-i\tau\phi} \\ \tau \hbar k v_F e^{-i\tau\phi} & \frac{U}{2} & \gamma_1 & 0 \\ 0 & \gamma_1 & -\frac{U}{2} & \tau \hbar k v_F e^{i\tau\phi} \\ \tau \hbar k v_3 e^{i\tau\phi} & 0 & \tau \hbar k v_F e^{-i\tau\phi} & -\frac{U}{2} \end{pmatrix}$$

AI
BI
A2
B2

interlayer potential
from skew interlayer hopping
interlayer hopping

McCann & Fal'ko, PRL 2006



$$H_0^{\text{eff}} = \begin{pmatrix} U \left(\frac{1}{2} - \frac{p_F^2 v_F^2}{\gamma_1^2} \right) & -\frac{p_F^2 v_F^2 e^{i2\tau\phi}}{\gamma_1} + \tau p v_3 e^{-i\tau\phi} \\ -\frac{p_F^2 v_F^2 e^{-i2\tau\phi}}{\gamma_1} + \tau p v_3 e^{i\tau\phi} & -U \left(\frac{1}{2} - \frac{p_F^2 v_F^2}{\gamma_1^2} \right) \end{pmatrix}$$

$$\frac{p_F^2 v_F^2}{\gamma_1^2} = \frac{U^2 + \sqrt{4E_F^2(U^2 + \gamma_1^2) - U^2\gamma_1^2}}{2(U^2 + \gamma_1^2)} \approx \frac{E_F}{\gamma_1}$$

$\gamma_1 \gg E_F > U/2$

DP spin relaxation in BLG

$$H_{SO} = \lambda_1 \tau \sigma_z s_z + \lambda_2 \tau \mu_z s_z + \lambda_3 \mu_z (\sigma_y s_x - \tau \sigma_x s_y) + \lambda_4 \sigma_z (\mu_y s_x + \tau \mu_x s_y)$$

↑
pseudospin
↑
spin
↑
layer
↑
valley

F. Guinea, N. J. Phys. **12**, 083063 (2010)

λ_1 [meV]	λ_2 [meV]	λ_3 [meV]	λ_4 [meV]
0.014	0.008	0.0055	0.48

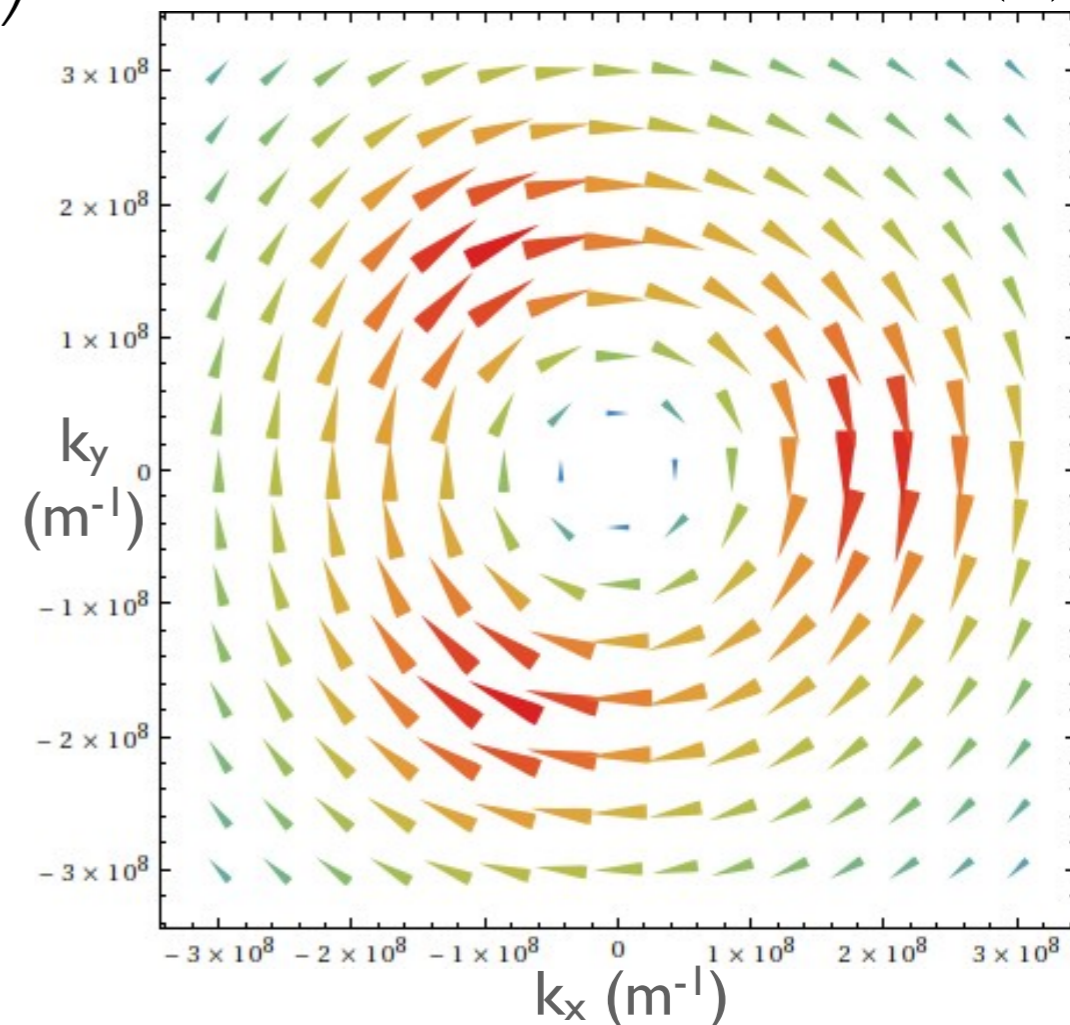
largest contribution

$$H_{\lambda_4}^e = \lambda_4 \frac{U}{E_{\text{eff}}^0} \frac{p v_F}{\gamma_1} \begin{pmatrix} 0 & ie^{-i\phi} \left(1 - \frac{p^2 v_F^2}{\gamma_1^2}\right) - \tau ie^{2i\phi} \frac{p v_3}{\gamma_1} \\ -ie^{i\phi} \left(1 - \frac{p^2 v_F^2}{\gamma_1^2}\right) + \tau ie^{-2i\phi} \frac{p v_3}{\gamma_1} & 0 \end{pmatrix} = \mathbf{\Omega}_{\lambda_4}^e \cdot \mathbf{s} \quad \mathbf{\Omega}(\mathbf{k})$$

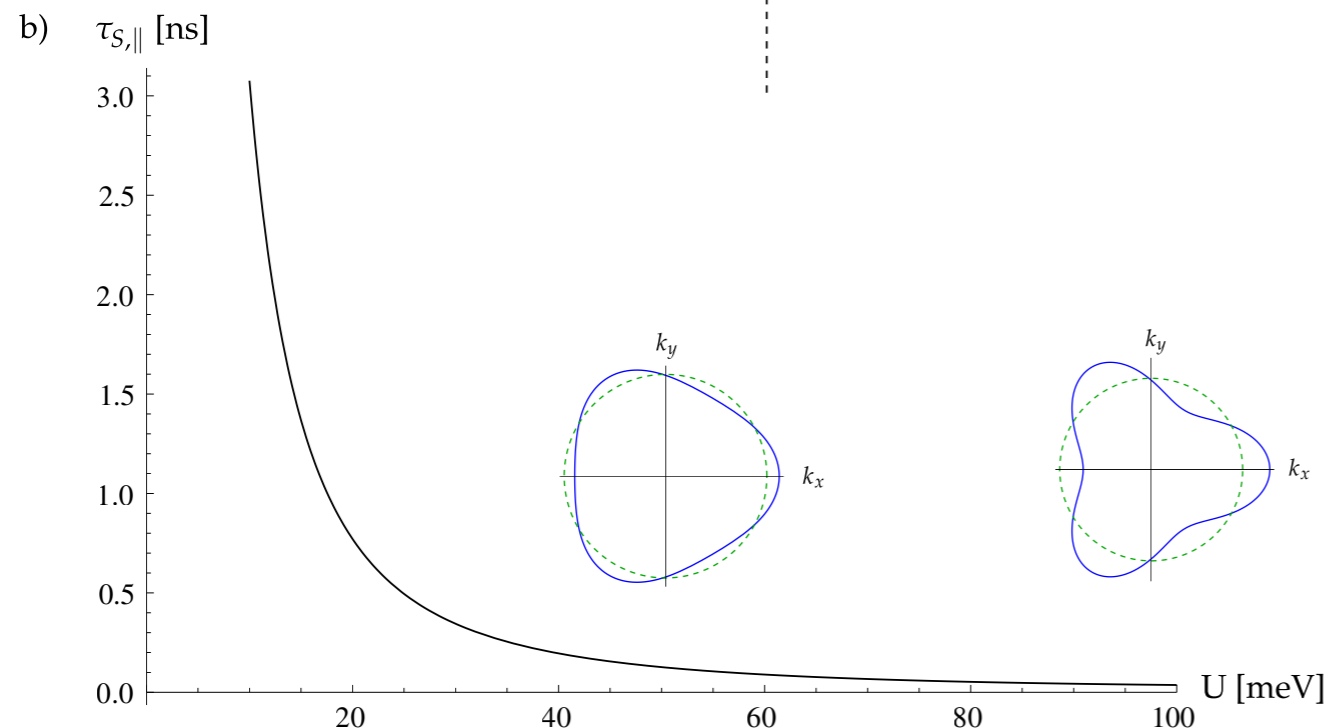
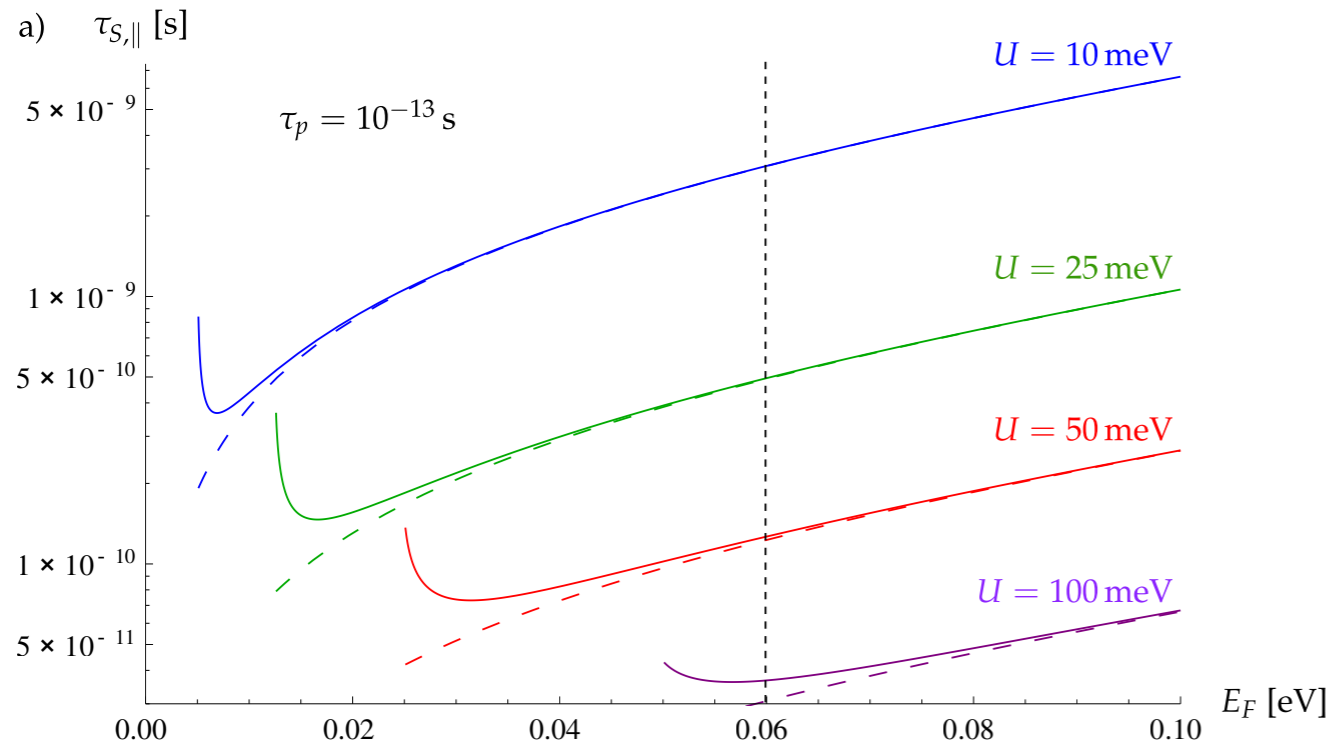
$$\Omega_{\lambda_4, x}^e = \frac{2\lambda_4}{\hbar} \frac{U}{E_{\text{eff}}^0} \frac{p v_F}{\gamma_1} \left[+ \left(1 - \frac{p^2 v_F^2}{\gamma_1^2}\right) \sin \phi + \tau \frac{p v_3}{\gamma_1} \sin 2\phi \right]$$

$$\Omega_{\lambda_4, y}^e = \frac{2\lambda_4}{\hbar} \frac{U}{E_{\text{eff}}^0} \frac{p v_F}{\gamma_1} \left[- \left(1 - \frac{p^2 v_F^2}{\gamma_1^2}\right) \cos \phi + \tau \frac{p v_3}{\gamma_1} \cos 2\phi \right]$$

$$\Omega_{\lambda_4, z}^e = 0.$$



DP spin relaxation in BLG



from kinetic spin Bloch equation:

$$\frac{1}{\tau_{S,\parallel}^0} = \frac{1}{2} \omega_{\lambda_4}^2 \tau_p = \frac{2\lambda_4^2}{\hbar^2} \frac{U^2}{E_F^2} \frac{p_F^2 v_F^2}{\gamma_1^2} \left(1 - \frac{p_F^2 v_F^2}{\gamma_1^2}\right)^2 \tau_p$$

$$\frac{1}{\tau_{S,\perp}^0} = \omega_{\lambda_4}^2 \tau_p = \frac{2}{\tau_{S,\parallel}},$$

in the regime $\gamma_1 \gg E_F > U/2$

$$\frac{p_F^2 v_F^2}{\gamma_1^2} = \frac{U^2 + \sqrt{4E_F^2(U^2 + \gamma_1^2) - U^2\gamma_1^2}}{2(U^2 + \gamma_1^2)} \approx \frac{E_F}{\gamma_1}$$

$$\frac{1}{\tau_{S,\parallel}^0} = \frac{1}{2\tau_{S,\perp}^0} \approx \frac{2\lambda_4^2}{\hbar^2} \frac{(\gamma_1 - E_F)^2 U^2}{E_F \gamma_1^3} \tau_p$$

M. Diez & GB, manuscript in preparation

Summary

- **spin-valley hyperfine interaction in graphene**
couples nuclear spin to both electron spin and valley
- **spin relaxation of localized electrons**
effect of flexural phonons saturates at low B fields
- **spin relaxation of mobile electrons**
SLG: crossover $\tau_s \sim 1/\tau_p$ to $\tau_s \sim \tau_p$
BLG: gate tunability of τ_s

