Zigzag graphene ribbons and coupled chains

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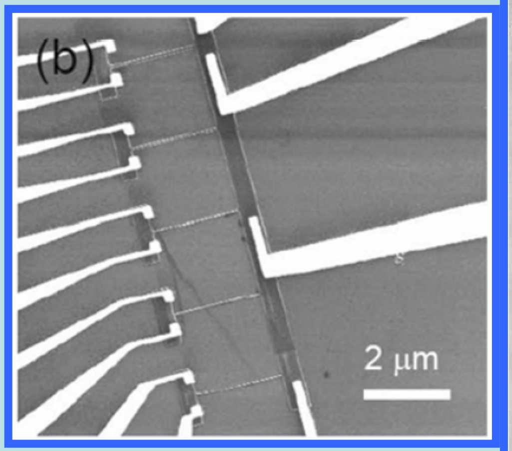
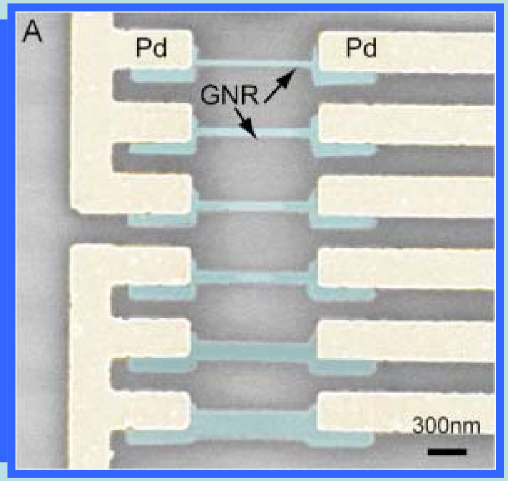
In collaboration with: Medhi Zarea

Ribbons: some methods of fabrication

Lithographic patterning

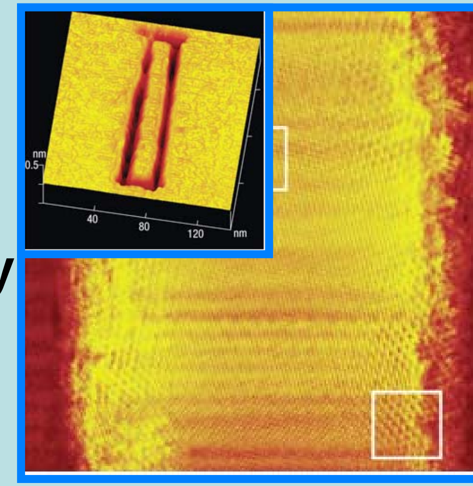


Kim PRL (2007)



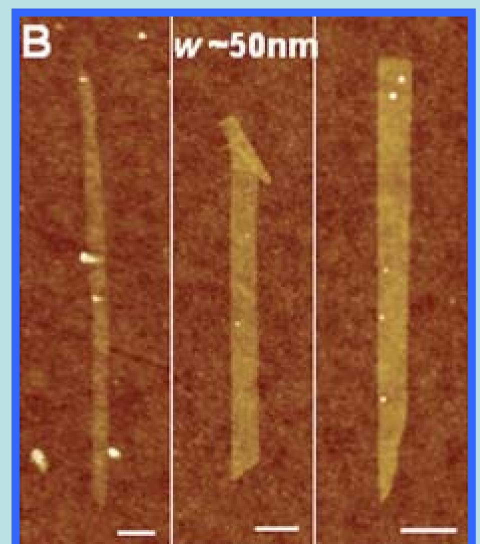
Avouris Physica E (2007)

STM lithography



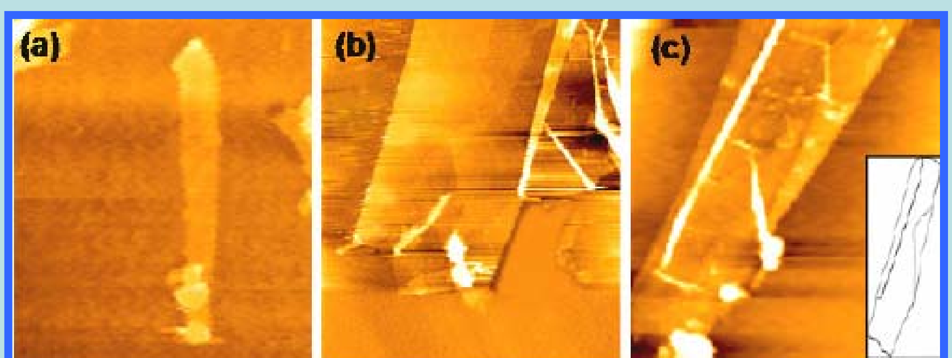
Biro Nat. Nanot. (2008)

Exfoliated graphite in solution



Dai
Science (2008)

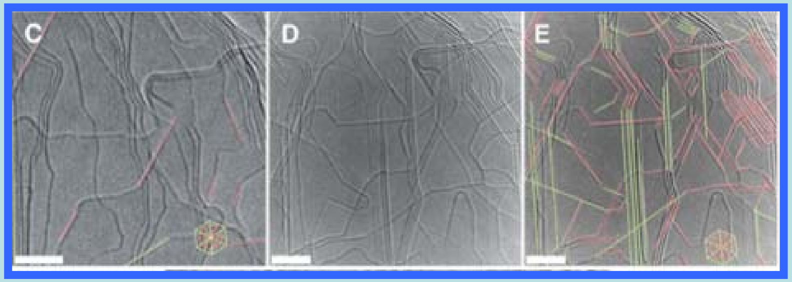
Electrostatic deposition with gas

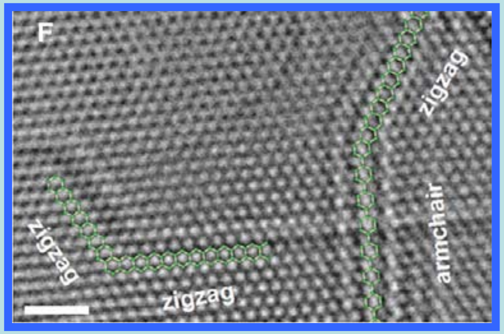


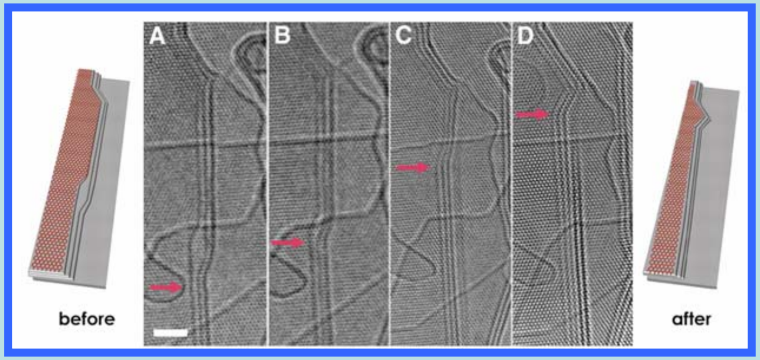
Wu Nanotechnology (2009)

Edges… Last two: cleaner edges of armchair or zigzag

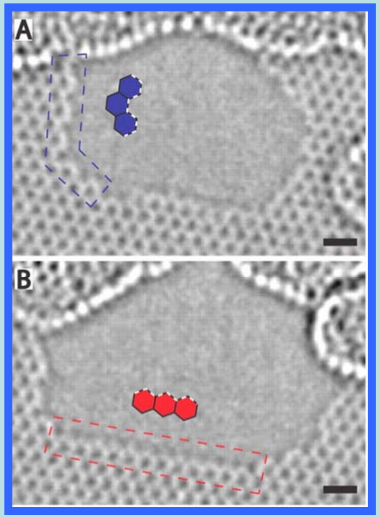
Also: edge reconstruction

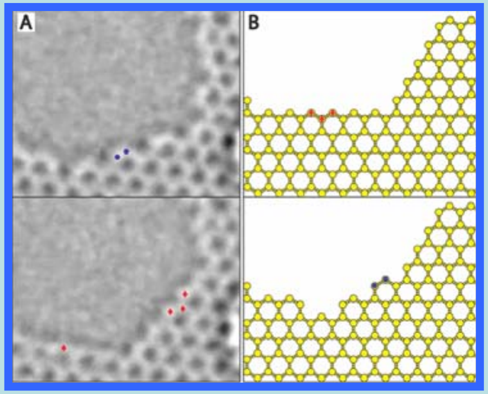






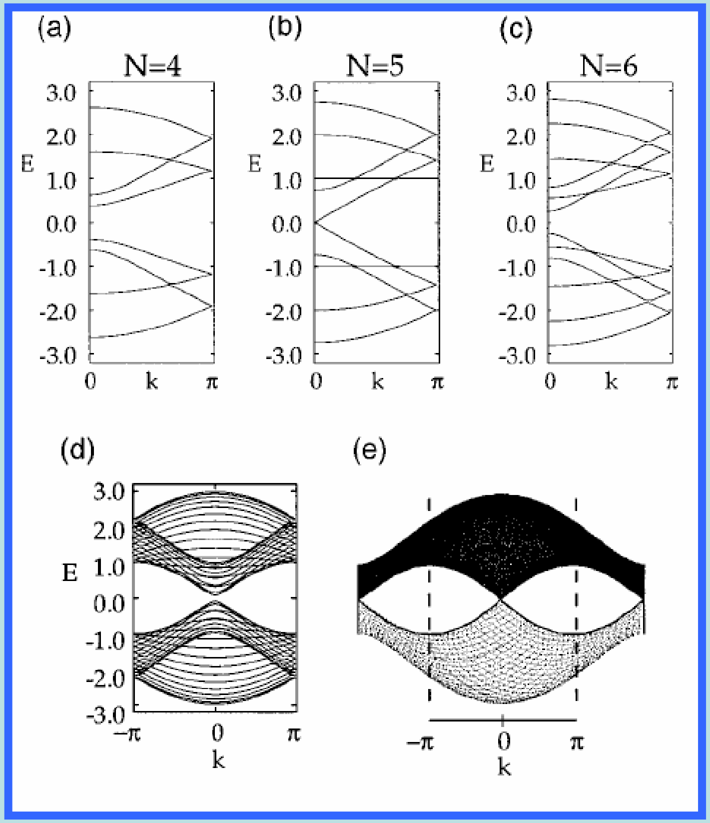
Dresselhaus Science (2009)

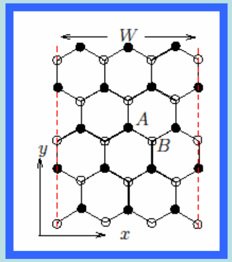




Crommie Science (2009)

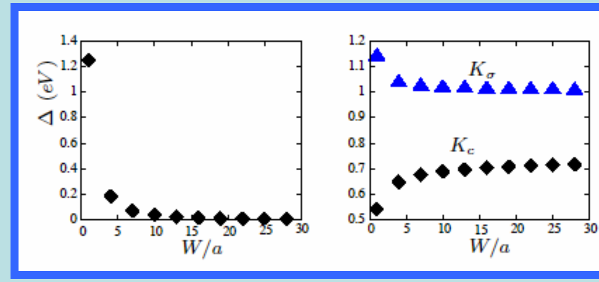
Armchair ribbons





TB + Hard Wall BC= Wavefunctions Effective 1d Coulomb + Bosonization

- Long-range Coulomb

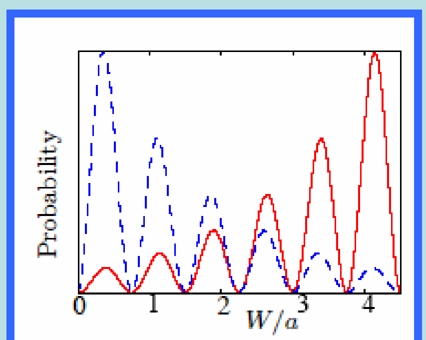


$$\Delta \propto \frac{1}{W}$$

- Intrinsic Spin-orbit

$$\psi_{\pm}^{s} = N e^{\pm s\gamma_0 (x - W/2)} \sin\left(\frac{4\pi x}{3a}\right)\frac{e^{ik_y y}}{\sqrt{2L}}\begin{pmatrix}1\\ \pm i\end{pmatrix}$$

Nakada et al, PRB (1996)



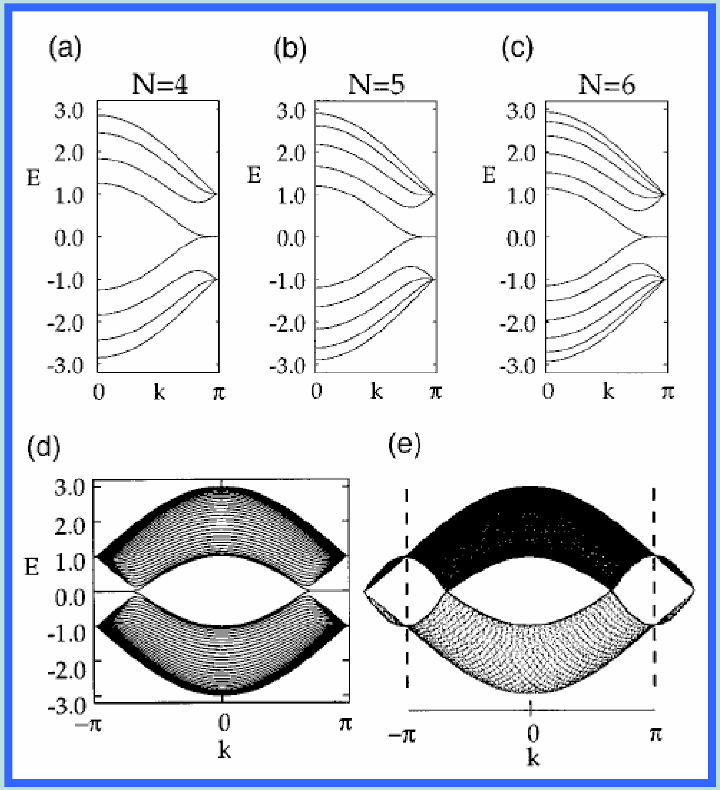
$\gamma_0 \approx t'$
±: left/right movers
s: spin

Spin-filtered edge states.

K. Nakada et al. PRB (1996)
M. Fujita et al. J. Phys. Soc. Jpn. (1996)
T. Hikihara et al. PRB (2003)
L. Brey and H. Fertig PRB (2006)

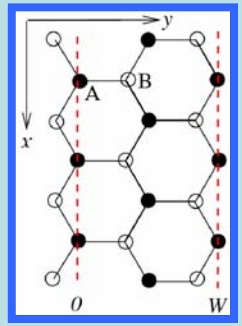
M. Zarea, N.S. PRL (2007)

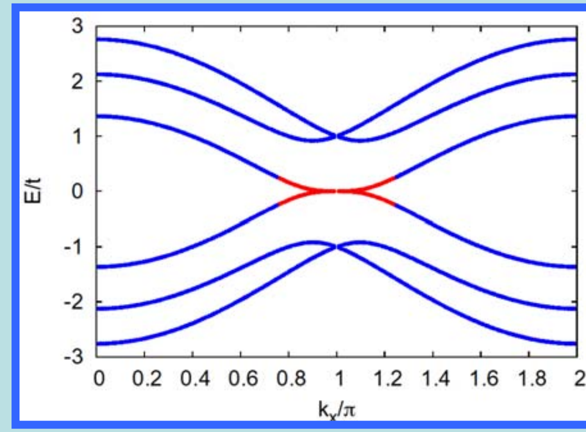
Zigzag Ribbons



Nakada et al, PRB (1996)

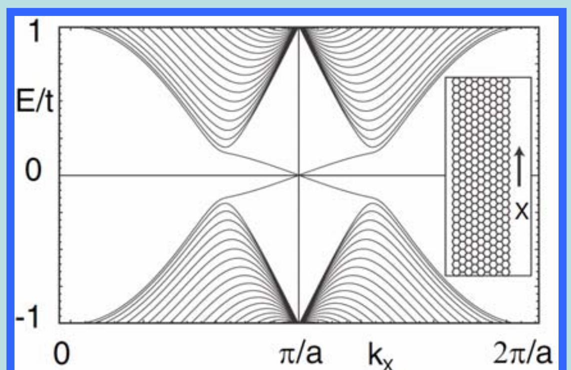
TB + Hard Wall= Band-structure and wave- functions.





$$\psi = e^{ik_x x}\left[\begin{pmatrix}1\\ e^{-k_y W}\end{pmatrix} e^{ik_y y} - \begin{pmatrix}1\\ e^{k_y W}\end{pmatrix} e^{-ik_y y}\right]$$

- Intrinsic spin orbit



- Gap opens: insulating bulk
- Spin-filtered edge states

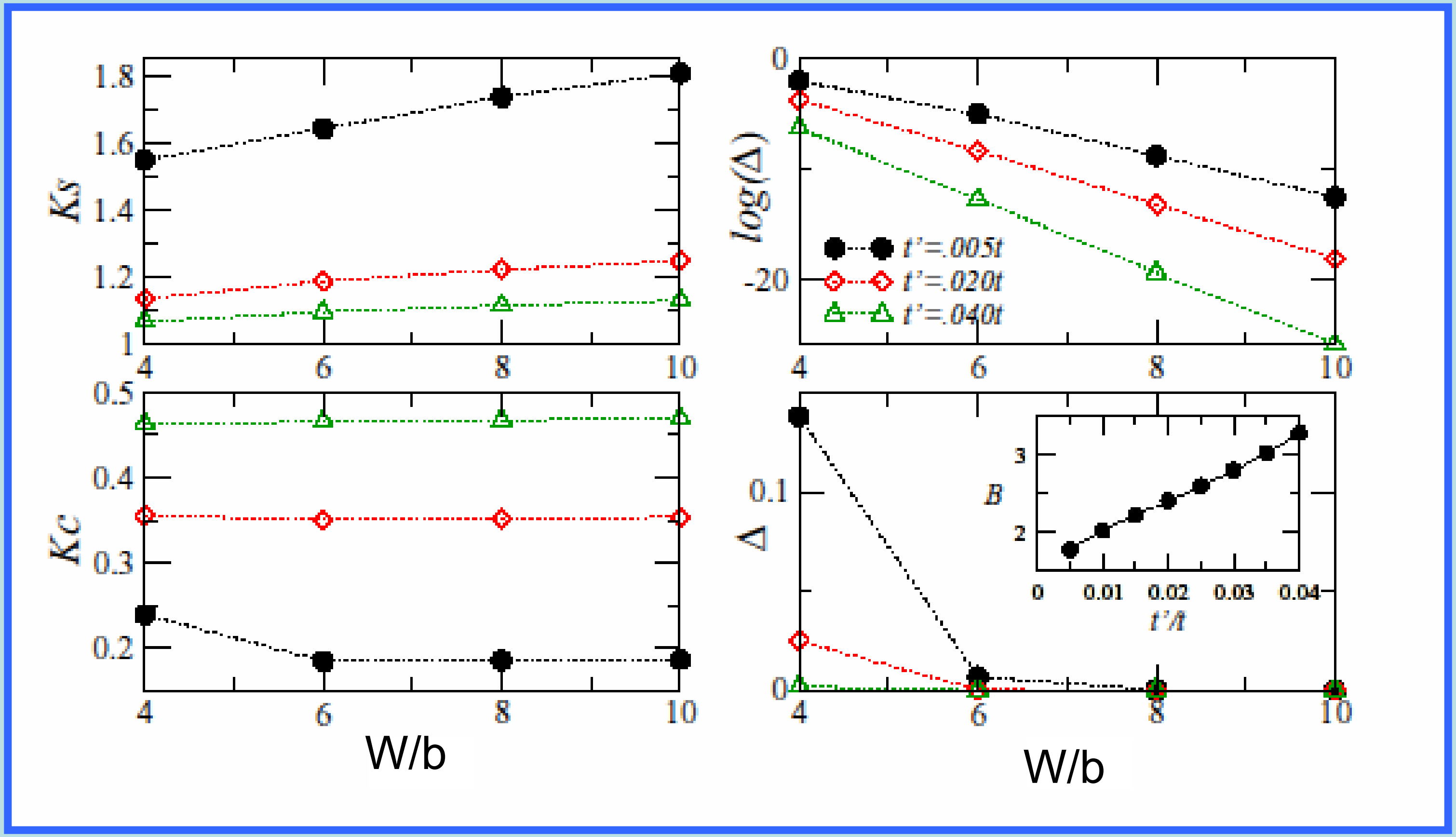
Quantum Spin Hall Phase

K. Nakada et al. PRB (1996)
M. Fujita et al. J. Phys. Soc. Jpn. (1996)
T. Hikihara et al. PRB (2003)
L. Brey and H. Fertig PRB (2006)

C. Kane and E. Mele PRL (2005)
H. Min et al. PRB (2006)
J. Boettger et al. PRB (2006)
D. Huertas-Hernando et al PRB (2006)

What about Coulomb interactions?

With I-SO interaction and bosonization

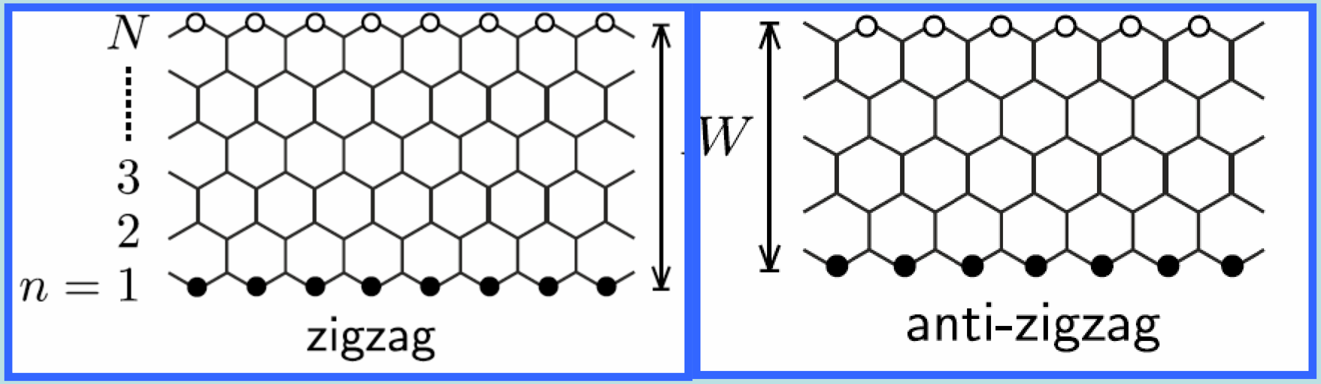


Increasing magnetic order ($K_s > 1$)

$$\Delta \propto e^{-BW}$$

M. Zarea, C. Büsser and N.S., PRL(2008)

Even-Odd Feature:



Hikihara et al PRB (2003)
Kohmoto and Hasegawa PRB (2007)
Akhmerov et al PRB (2008)
Li et al PRL (2008)
Yao et al PRL (2009)

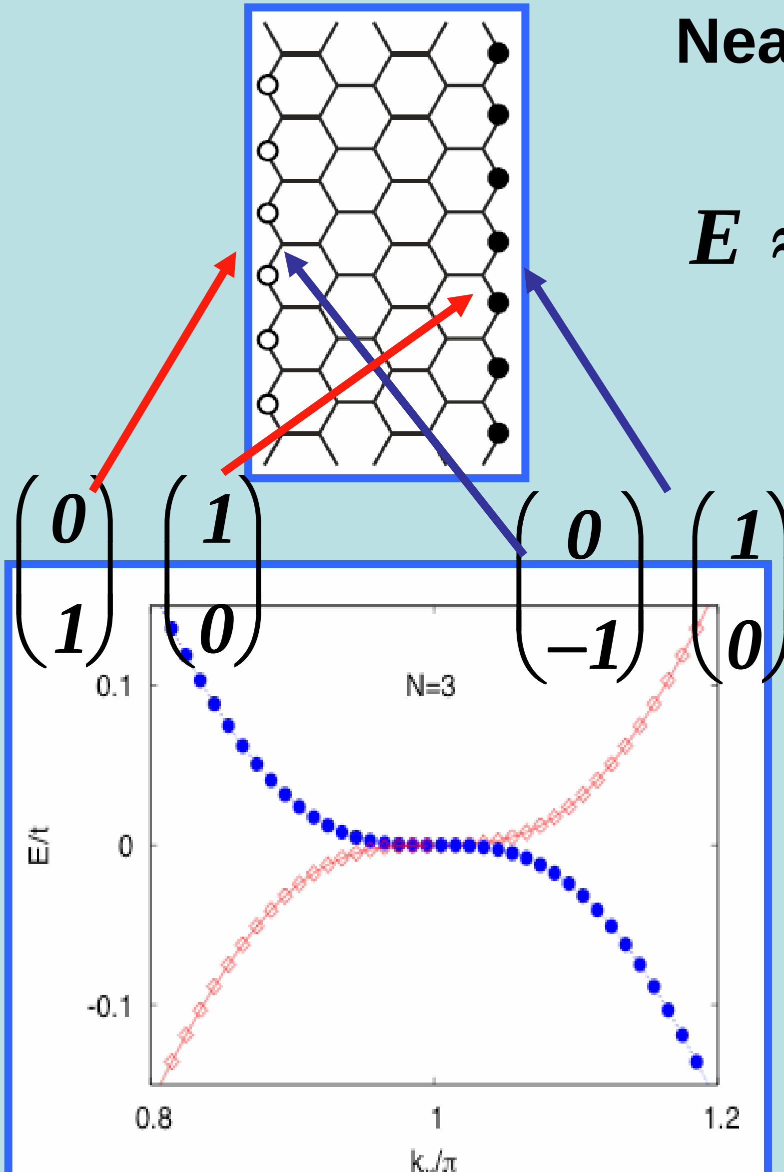
TB + Hard Wall BC = Wavefunctions finite system

$$\psi_{\pm} = Ce^{ik_x x}\begin{pmatrix} \sin k_y (y + W/2) \\ \pm \sin[k_y (y - W/2) - \beta] \end{pmatrix}$$

$$k_x a > \pi \quad \beta = 0 \quad k_y = iq$$

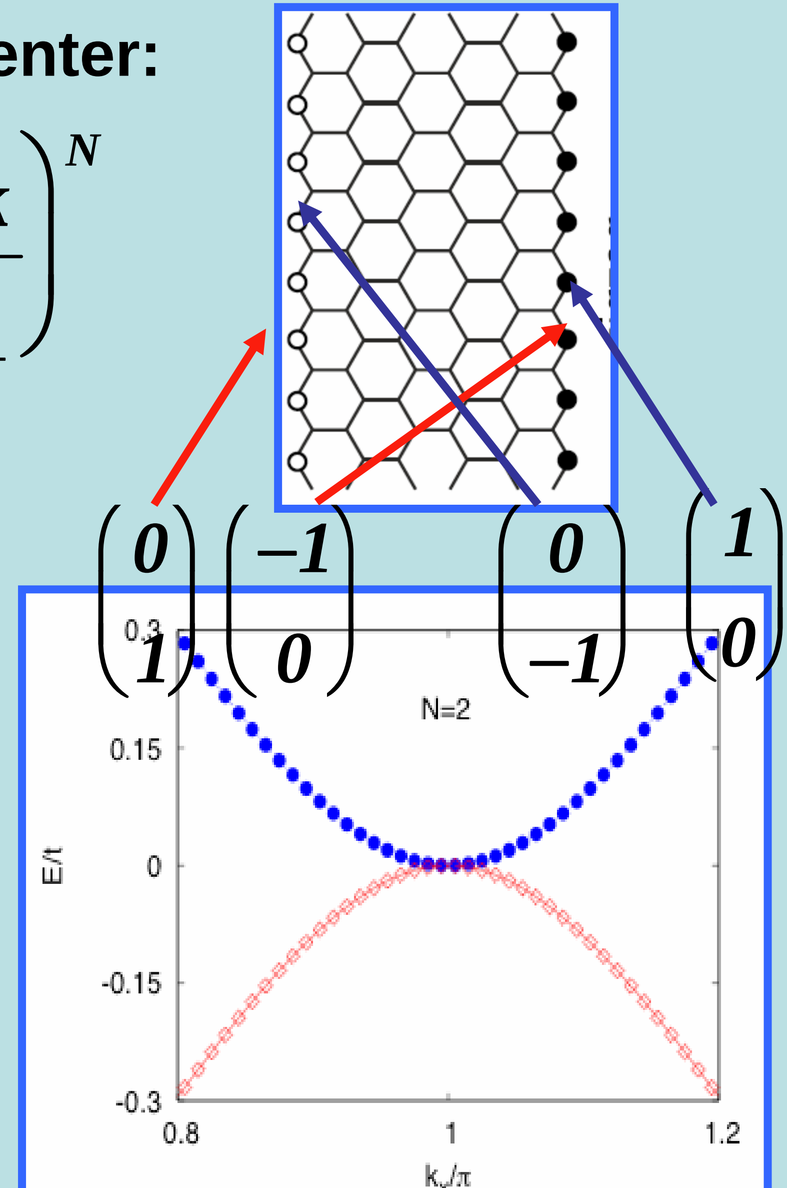
$$k_x a < \pi \quad \beta = \pi N \quad k_y = iq - \pi$$

Could it be **so** important???

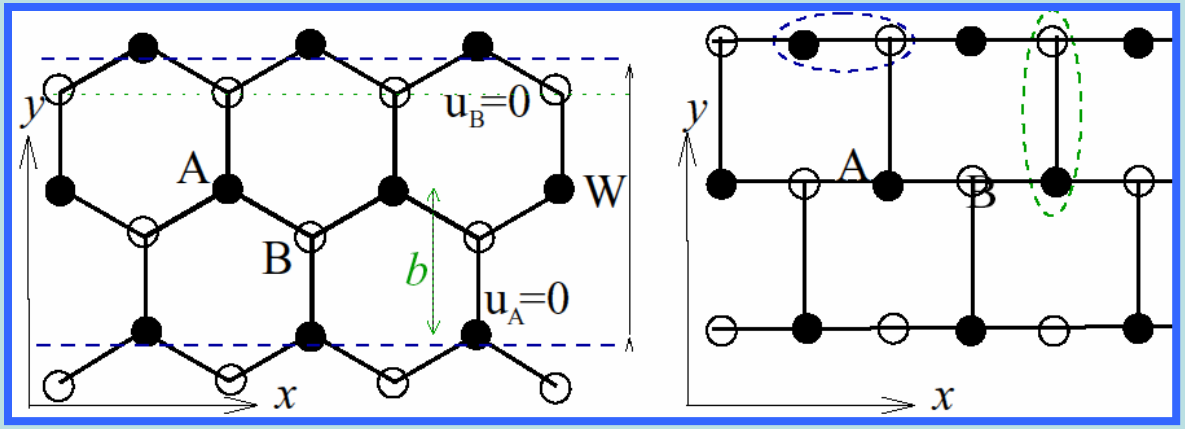


Near band center:

$$E \approx \pm t_{\perp} \left(\frac{tk}{t_{\perp}} \right)^{N}$$



Coupled chains:



$$H_N = \sum_{n=1}^{N} H_n + t_\perp H'$$ ← Coupling between chains

$$H_n = \int dx[tc^+_{A_n}(x)(c_{B_n}(x) + c_{B_n}(x-a)) + h.c.]$$

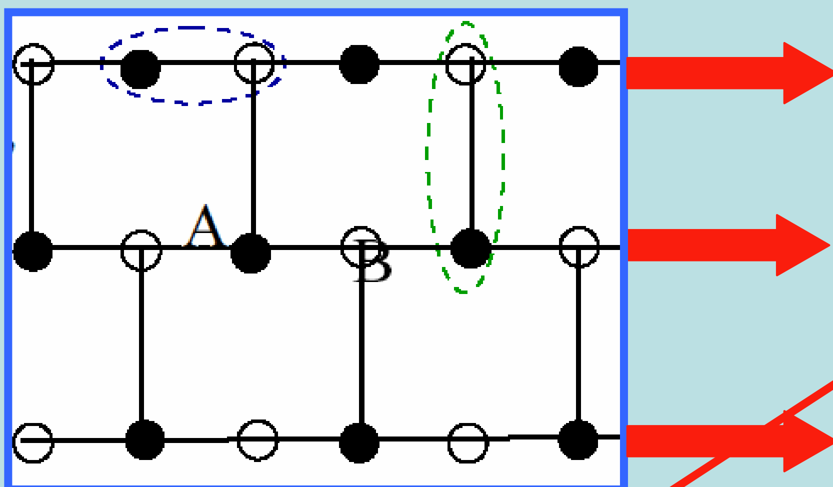
Right and Left moving operators: even and odd sites

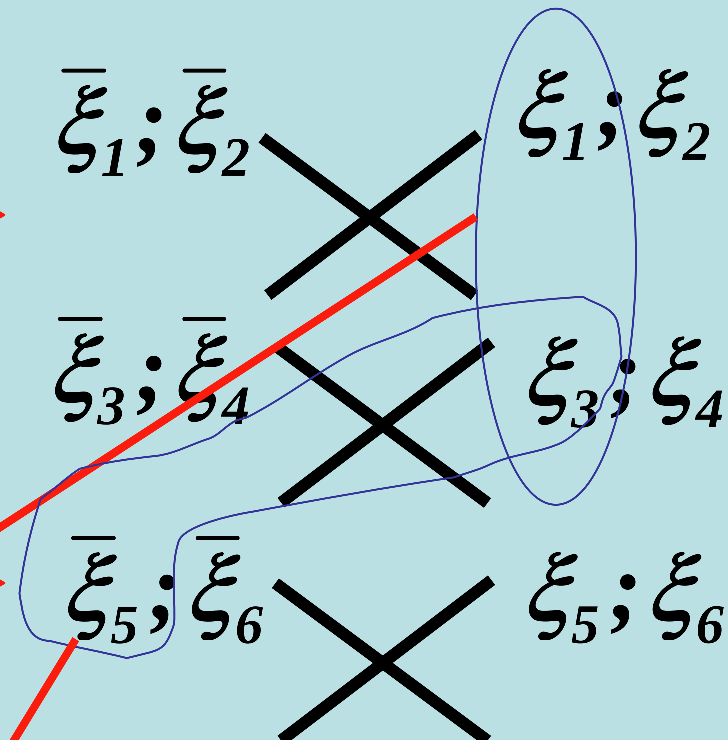
$$H_{n,0} = -iv(R_n^+\partial_x R_n - L_n^+\partial_x L_n)$$

Introduce Majorana representation

$$R_n = \left(\xi_{2n-1} + i\xi_{2n}\right)/\sqrt{2}$$

$$L_n = \left(\bar{\xi}_{2n-1} + i\bar{\xi}_{2n}\right)/\sqrt{2}$$



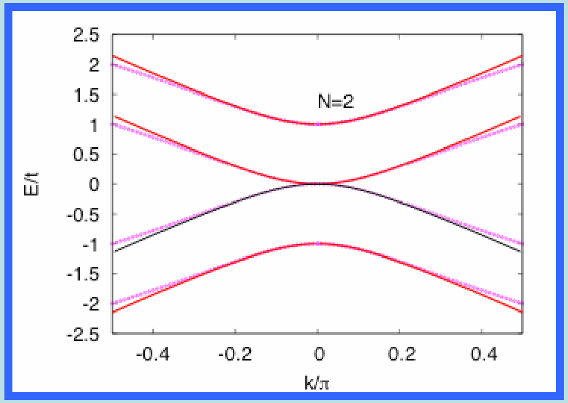


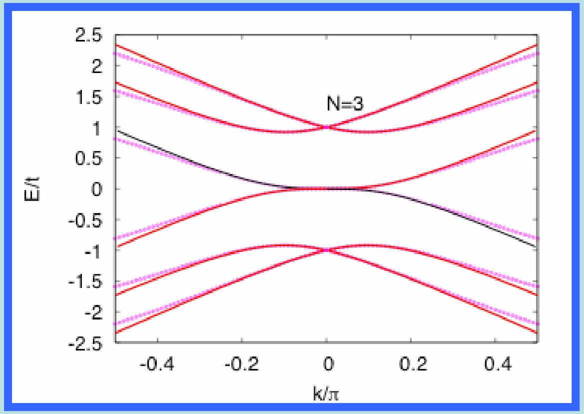
$$H_0 = \sum_{n=1}^{2N} iv\left(\xi_n \partial_x \xi_n - \bar{\xi}_n \partial_x \bar{\xi}_n\right)$$

$$h = m = \frac{t}{2} = \frac{t_\perp}{2}$$

$$H' = \sum_{n=1}^{2N-2} ih\left(\xi_n \xi_{n+2} - \bar{\xi}_n \bar{\xi}_{n+2}\right) + im\left(\xi_n \bar{\xi}_{n+2} - \bar{\xi}_n \xi_{n+2}\right)$$

Hamiltonian: solved recursively



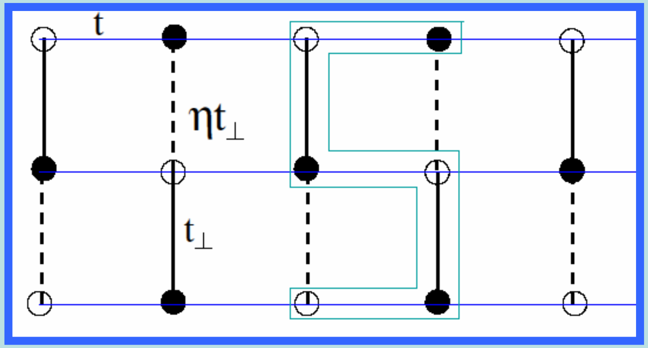


Can the model be generalized?

$$h \neq m \Rightarrow t \neq t_{\perp}$$

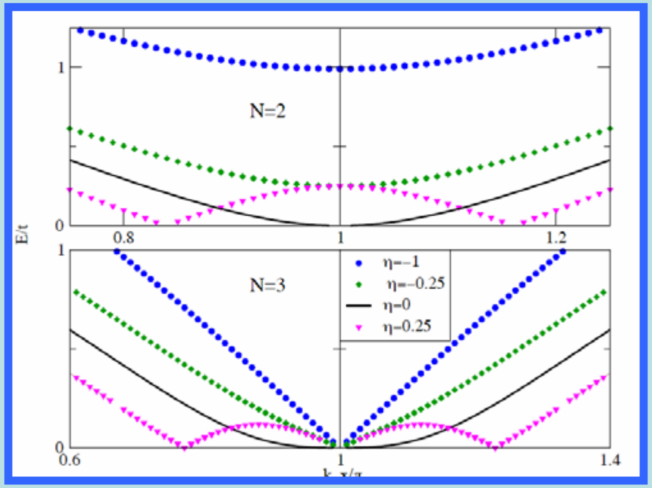
$$h = \frac{t_{\perp}}{2}(1+\eta)$$

$$m = \frac{t_{\perp}}{2}(1-\eta)$$



Square: η = 1
Graphene: η = 0
π-Flux: η = -1

Affleck and Marston PRB (1988)
Ludwig et al PRB (1994)



Band structure:

$$E = \pm\sqrt{\left(t\cos\frac{k_x a}{2} + 2h\cos k_y b\right)^2 + 4m^2\sin^2 k_y b}$$

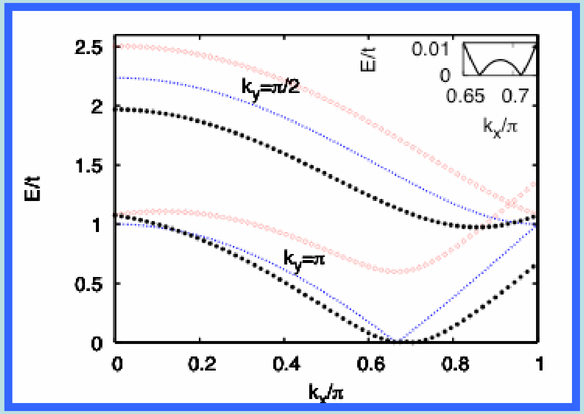
Rashba interactions?

$$H_R = \sum_{<ij>} i c_i^{+} \left(\vec{u}_{ij}.\sigma\right) c_j + h.c.$$

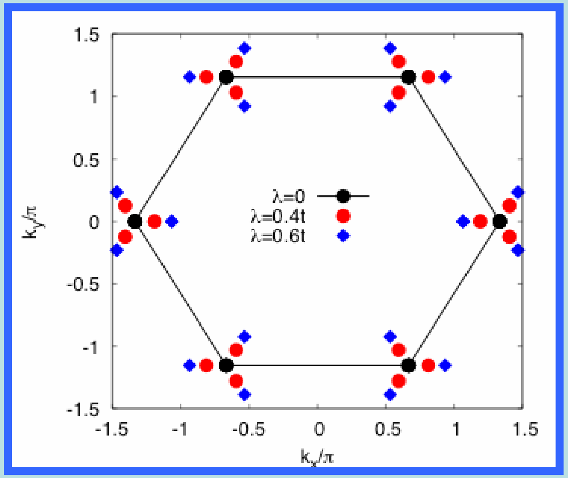
$$\vec{u}_{ij} = -\frac{\lambda_R}{d}\,\hat{z}\times\vec{\delta}_{ij}$$

$$H = \begin{pmatrix} 0 & \varphi_0 & 0 & i\varphi_+ \\ \overline{\varphi}_0 & 0 & -i\overline{\varphi}_- & 0 \\ 0 & i\varphi_- & 0 & \varphi_0 \\ -i\overline{\varphi}_+ & 0 & \overline{\varphi}_0 & 0 \end{pmatrix} \qquad \Psi = \begin{pmatrix} u_{A\uparrow} \\ u_{B\uparrow} \\ u_{A\downarrow} \\ u_{B\downarrow} \end{pmatrix}$$

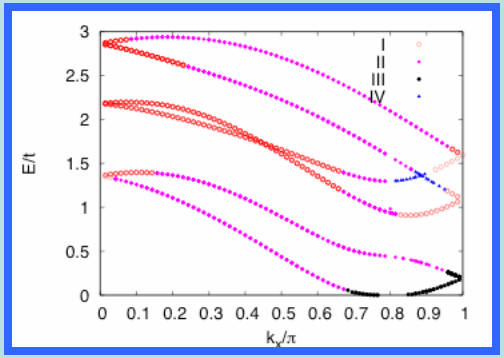
Bulk:



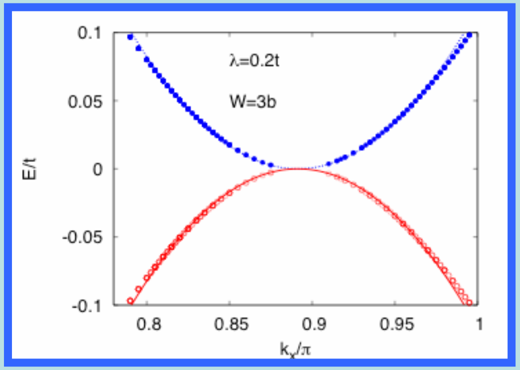
New Dirac points!

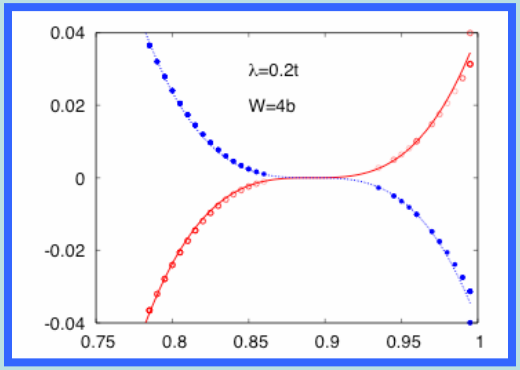


Zigzag ribbons:



Even-odd feature:





M. Zarea and N.S. PRB (2009)

Relation to Quantum Spin Chains:

N= 2 ZGR ⇔ Anisotropic 2 coupled Spin 1/2 chains

$$H = t\sum_{i} \vec{S}_{1i} \bullet \vec{S}_{1j} + t_{\perp} S^{z}_{1i} . S^{z}_{2j}$$

N= 3 ZGR ⇔ Anisotropic biquadratic Spin 1 chain with in plane B

Tsvelik PRB (1990)
Shelton et al PRB (1996)

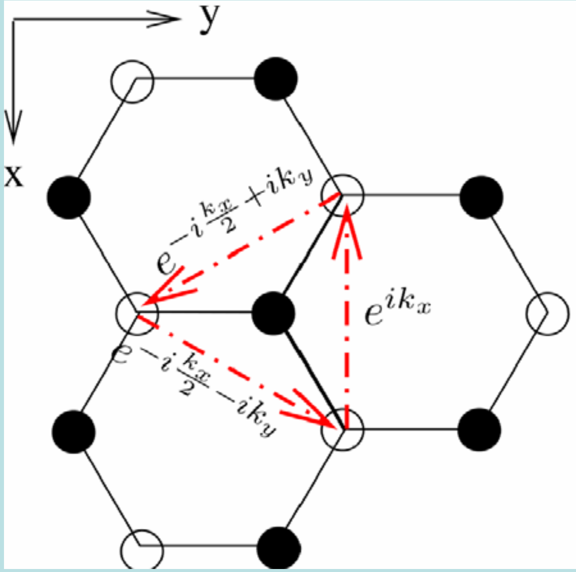
Advantages:

- **Chemical potential $\xi_1 \xi_2$ (gap on chain)**
- **Second neighbor hopping = chemical potential**
- **Intrinsic spin-orbit (gap for even N)** $t'\left(\xi_1 \bar{\xi}_3 + \bar{\xi}_1 \xi_3\right)$
- **Staggered sublattice potential (gap for all N)**
- **Electron-electron interactions**
- **…**

Conclusions

- Zigzag ribbons: ‘peculiar’ edge states
- Mapping to coupled chains: metal or insulator as N changes
- Many lattices share similar properties
- Connections to Quantum Spin Chains.

The symmetries of graphene allow for: the intrinsic spin-orbit interaction.



$$H_\uparrow = \begin{pmatrix} \gamma & \Phi \\ \overline{\Phi} & -\gamma \end{pmatrix}$$

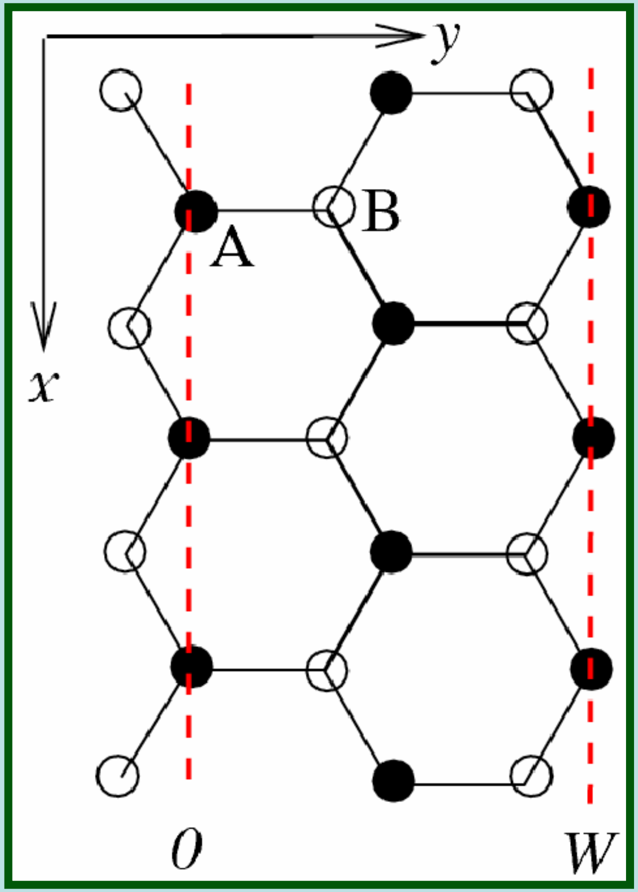
$$H_\downarrow = \begin{pmatrix} -\gamma & \Phi \\ \overline{\Phi} & \gamma \end{pmatrix}$$

$$\gamma = 2t'\left[\sin(k_x a) - 2\sin(k_x a/2)\cos(k_y b)\right]$$

$$E = \sqrt{|\Phi|^2 + \gamma^2}$$

C. Kane and E. Mele PRL 95 226801 (2005)
H. Min et al. PRB 74, 165310 (2006)
J. Boettger et al. PRB 75, 121402(R) (2006)

Zigzag ribbons: tight-binding and hard-wall boundary conditions



Wavefunction:

$$\psi = e^{ik_x x}\left[\begin{pmatrix} 1 \\ e^{-k_y W} \end{pmatrix} e^{ik_y y} - \begin{pmatrix} 1 \\ e^{k_y W} \end{pmatrix} e^{-ik_y y}\right]$$

$$k_y = k_y(W, k_x)$$

$$\tan\left[k_y\left(W - \frac{1}{2}\right)\right] = \frac{1 - 2\cos\left(\frac{k_x}{2}\right)}{1 + 2\cos\left(\frac{k_x}{2}\right)} \tan\left(\frac{k_y}{2}\right)$$

K. Nakada et al. PRB 54, 17954 (1996)
M. Fujita et al. J. Phys. Soc. Jpn. 65, 1920 (1996)
T. Hikihara et al. PRB 68, 035432 (2003)
L. Brey and H. Fertig PRB 73, 235411 (2006)